1 Introduction

CONCEPT CHECK

1.1 | Significant Figures

Quantity	Length or Distance in Meters (m)	Number of Significant Figures
Diameter of a proton	1×10^{-15}	1
Diameter of a red blood cell	8×10^{-6}	1
Diameter of a human hair	5.5×10^{-5}	2
Thickness of a piece of paper	6.4×10^{-5}	2
Diameter of a compact disc	0.12	2
Height of the author	1.80	3
Height of the Empire State Building	443.2	4
Distance from New York City to Chicago	1,268,000	4
Circumference of the Earth	4.00×10^{7}	3
Distance from Earth to the Sun	1.5×10^{11}	2

1.2 | Using Prefixes and Powers of 10

Quantity	Length or Distance in Meters (m)	Same Length or Distance Using Prefix
Diameter of a proton	1×10^{-15}	1 femtometers = 1 fm
Diameter of a red blood cell	8×10^{-6}	8 micrometers = 8 mm
Diameter of a human hair	5.5×10^{-5}	55 micrometers = 55 mm
Thickness of a sheet of paper	6.4×10^{-5}	64 micrometers = 64 mm
Diameter of a compact disc	0.12	1.2 decimeters = 1.2 dm = 12 centimeters = 12 cm
Height of the author	1.80	1.80 meters = 1.80 m
Height of the Empire State Building	443.2	443.2 meters = 443.2 m
Distance from New York City to Chicago	1,268,000	1.268 megameters = 1.268 Mm
Circumference of the Earth	4.00 × 107	40.0 megameters = 40.0 Mm
Distance from Earth to the Sun	1.5×10^{11}	150 gigameters = 150 Gm

1.3 | (a), (c), or (d) could all be used as they have dimensions of length squared.

QUESTIONS

Q1.1 The radius is a dimension of length, so $\frac{4}{3}\pi r^4$ has dimension of L^4. Density has dimensions of M/L^3, so this cannot be the expression for density.

Q1.2 Since time is a basic physical quantity like space and mass, it is very difficult to define one of these quantities without the use of the other. Most definitions of time are in terms of a sequence of events (or cause and effect), generally where something (oscillating springs, hands on a clock, or a pendulum) moves through space. Electronics appear to avoid this at first glance, but clocks based on time to charge a capacitor depend on the voltage, which is related to force (and therefore mass). The current standard of time avoids this by defining a second in terms of an average number of atomic energy level cycles. The direction of spontaneous movement through time (forward) is defined by reactions (such as burning) which run spontaneously in one direction.

[SSM] Q1.3 Using dimensional analysis we can determine which expressions have units of volume, since volume has a dimension equal to L^3.

meters^3: Dimensions of $\mathbf{L}^3$, so this is a volume.
millimeters·miles^2: Dimensions of $\mathbf{L} \times \mathbf{L}^2$ or $\mathbf{L}^3$, so this is a volume.
kiloseconds·feet^2: Dimensions of $\mathbf{T} \times \mathbf{L}^2$, so this is *not* a volume.
kilograms^2·centimeters: Dimensions of $\mathbf{M}^2 \times \mathbf{L}$, so this is *not* a volume.
acres/meter^2: Dimensions of L^2/L^2 or dimensionless, so this is *not* a volume.
hours·millimeter^3/second: Dimensions of $\mathbf{T} \times \mathbf{L}^3/\mathbf{T}$ or $\mathbf{L}^3$, so this is a volume.
millimeters·centimeters·meters^2/feet: Dimensions of $\mathbf{L} \times \mathbf{L} \times \mathbf{L}^2/\mathbf{L}$ or $\mathbf{L}^3$ so this is a volume.

Q1.4 m/s: This is velocity which describes rate of change of position.

m^3/s: Volume per unit time. This combination can describe the rate of flow of water from a faucet, or rate of loading grain onto train cars.

kg/m: Mass per unit length. Called linear density, this concept is a useful way of describing how much mass a rope, chain, or cable has for a given length.

m/s^2: This quantity will soon be very familiar. It is the units of acceleration, the rate at which the velocity changes (m/s)/s.

m^2/s: Area per unit time. This could be the units for how fast carpet is installed, or how fast painters can paint a wall.

Q1.5 All physical objects, such as bars of metal, change and react to the environment. Minute changes in temperature make measurable changes in the bar's length due to thermal expansion. All metals react with air and chemical changes can alter the bar's length. By contrast, the interferometer measurement relies on physical quantities (such as the speed of light and certain radiation frequencies) that we don't believe ever change.

[SSM] Q1.6 Vectors: displacement, velocity. These all have directions as well as magnitudes.
Scalars: mass, density, temperature. These quantities have magnitudes only.

Q1.7 The main advantage of the metric system over the U.S. customary system of units is that all conversions are some power of 10 in the metric system (examples:

10 mm = 1 cm, 1 km = 1000 m). Measurements are easily expressed in decimals and scientific notation, making them more suitable for electronic calculation. Conversion factors vary widely in the U.S. customary system (1 ft = 12 in, 1 mi = 5280 ft), resulting in much more to remember. There are also advantages in better standards with the metric system, which is now accepted internationally by the scientific community.

The main disadvantage of the metric system is that it is still not used commonly in many parts of the U.S., so a significant number of Americans are unfamiliar with basic units and amounts. (Typical highway speeds, weights, and distances are all more commonly known in U.S. customary units.) Many tools and parts have been standardized using the U.S. customary system, so change would require a large investment in retooling.

Q1.8 The use of ratios is appropriate here. First look at the ratio of the galaxy diameters to that of the pie tins. We can use this ratio as a conversion factor to determine the distance between pie tins.

$$\frac{20\text{ cm}}{2 \times 10^5\text{ ly}} = 10^{-4}\text{ cm/ly}$$

$$(10^{-4}\text{ cm/ly})(2.5 \times 10^6) = 250\text{ cm} = \boxed{2.5\text{ m}}$$

Q1.9 At first glance one might be prone to say only two significant figures, since the value appears to have only two significant figures. However the football field is defined to be this length and so it should be at least be accurate down to the yard. This means this should be written with three significant figures. This shows the advantage of scientific notation—this value would best be written as 1.20×10^2 m to show the third significant figure.

PROBLEMS

[Reasoning] P1.1 Recognize the principle. Apply the concepts of scientific notation and significant figures.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. The height of the top floor of the Empire State Building can be found by multiplying the number of floors (102) by the height of each floor. The height of a floor is not given, so we need to estimate it. Based on personal experience, the height of a single floor is about 4 m. Once we have a value of the height of the 102nd floor, we can then divide by the thickness of a sheet of paper (which is given) to find the number of sheets.

Solve. The height of floor 102 of the Empire State Building is approximately $102 \times 4 = 400$ m tall. Dividing this by the thickness of paper, 6×10^{-5} m, we can find how many pieces of paper there would be:

$$\frac{400\text{ m}}{6 \times 10^{-5}\text{ m}} = \boxed{7 \times 10^6\text{ pieces of paper}}$$

What does it mean? It would take about 7 million pieces of paper to make a stack as tall as the Empire State Building. Since the thickness of a piece of paper was only given to one significant figure and we only estimated the height of a single floor, the final answer should also have only one significant figure.

P1.2 Recognize the principle. The number of grains of sand in the shoe can be found by dividing the total volume by the volume of a typical grain of sand.

Sketch the problem. No sketch needed.

Identify the relationships. Mathematically, we can express our relationship as:

$$\text{number of grains of sand} = \frac{\text{total volume}}{\text{grain volume}}$$

Solve. We first convert the total volume of the grain of sand into the same units as the volume of one grain of sand:

$$= 1.5 \text{ cm}^3 \left(\frac{10^3 \text{ mm}^3}{1 \text{ cm}^3}\right) = 1.5 \times 10^3 \text{ mm}^3$$

Then, we insert the values into our relationship above:

$$\text{number of grains of sand} = \frac{1.5 \times 10^3 \text{ mm}^3}{0.1 \text{ mm}^3} = 1.5 \times 10^4 \approx \boxed{20{,}000}$$

What does it mean? This calculation estimates the number of grains of sand as 15,000. Since the volume of one grain of sand has only one significant figure, it is safer to say that there are about 20,000 grains of sand in the shoe—especially since our volume assumes that there is no air or other volume that is not sand.

[Reasoning] P1.3 Recognize the principle. We need to make reasonable estimates and use ratios to solve this problem.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. Distance traveled: The distance between Chicago and New York is about 800 miles or about 1300 km, and the length per step is about 0.7 m. Dividing the distance traveled by the length of each step, one can find an estimate of the number of steps taken on this journey.

$$\text{number of steps} = \frac{\text{distance traveled}}{\text{step distance}}$$

Solve. We need to have the step distance and distance traveled in the same unit, so we first convert:

$$1300 \text{ km} \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 1.3 \times 10^6 \text{ m}$$

Then, inserting this value gives:

$$\text{number of steps} = \frac{1{,}300{,}000 \text{ m}}{0.7 \text{ m}} \approx \boxed{2 \times 10^6}$$

What does it mean? A person who walks from Chicago to New York would take approximately 2 million steps!

[Life Sci] P1.4 Recognize the principle. The total length divided by the diameter of each blood cell gives the number of blood cells.

Sketch the problem. No sketch needed.

Identify the relationships. We can write our relationship mathematically as:

$$\text{number of blood cells} = \frac{\text{total distance}}{\text{diameter of cell}}$$

Solve. Since both distances are already in meters, we need only insert values:

$$\text{number of blood cells} = \frac{1 \text{ m}}{8 \times 10^{-6} \text{ m}} = 125{,}000 \approx \boxed{1 \times 10^5}$$

What does it mean? It would take about 100,000 blood cells lined up to measure 1 meter in length. Since the measurement of a single cell is accurate to only one significant figure, our answer should also only contain one significant figure.

P1.5 Recognize the principle. Apply the concept of significant figures as discussed in Section 1.3.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. We can use the guidelines from Section 1.3 to find the appropriate number of significant figures in each case.

Solve.
For Table 1.2:

Quantity	Number of Significant Figures for Each Time Listed
Time for light to travel 1 meter	3
Time between heartbeats (approximate)	1
Time for light to travel from the Sun to Earth	1
1 day	exact
1 month (30 days)	exact
Human lifespan (approximate)	1
Age of the universe	1

For Table 1.3:

Object	Number of Significant Figures for Each Mass Listed
Electron	2
Proton	2
Red blood cell	1
Mosquito	1
Typical person	1
Automobile	2
Earth	2
Sun	2

P1.6 Recognize the principle. Determine the appropriate number of significant figures when applying addition as discussed in Section 1.3.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. When adding, the number of significant figures in the answer is determined by the value accurate to the least number of decimal places.

Solve. The total mass will be the mass of the bucket and rocks, plus the additional rocks added.

$$\begin{array}{r} 4.55\text{ kg} \\ +\ 0.224\text{ kg} \\ \hline \boxed{4.77\text{ kg}} \end{array} \quad \leftarrow \text{least accurate number (hundredths place)}$$

What does it mean? The last decimal place of the rock (thousandths place) should be added, but the final answer should be rounded and expressed only to the hundredths place, since the other measurement (the bucket) was only accurate to the hundredths place.

P1.7 Recognize the principle. Here we need to determine the appropriate number of significant figures when applying addition as discussed in Section 1.3.

Sketch the problem. See Figure P1.7.

Identify the relationships. When adding, the number of significant figures in the answer is determined by the value accurate to the least decimal places.

Solve. The total distance from A to C will be the distance from A to B, plus the distance from B to C.

$$\begin{array}{r l} 3.45\text{ km} & \\ +\ 5.4\text{ km} & \leftarrow \text{least accurate number (tenths place)} \\ \hline \boxed{8.9\text{ km}} & \end{array}$$

What does it mean? The distances should be added including the hundredths place, but must be rounded to the tenths place for final expression because one of the values is only accurate to the tenths place.

P1.8 Recognize the principle. Keep track of significant figures in addition and subtraction. See Section 1.3.

Sketch the problem. See Figure P1.7.

Identify the relationships. The numbers should be subtracted as usual, and rounded to the smallest decimal place that is significant in all values.

Solve. The total distance from A to D will be the distance from A to B, minus the distance from B to D.

$$\begin{array}{r l} 3.45\text{ km} & \\ -\ 3.15\text{ km} & \text{(both numbers known to same accuracy)} \\ \hline \boxed{0.30\text{ km}} & \end{array}$$

What does it mean? Since both measured distances are accurate to the hundredths place, we can express our answer with that accuracy as well.

P1.9 Recognize the principle. Here we need to determine the appropriate number of significant figures when applying division as discussed in Section 1.3.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. The density is defined as the mass of the object divided by its volume. Numbers should be divided as usual, then rounded to the smallest number of significant figures among all the values used.

Solve. Density is equal to $\frac{m}{v}$, so simply insert the values.

$$\text{density} = \frac{m}{v}$$

$$\text{density} = \frac{23\text{ kg} \leftarrow 2\text{ significant figures}}{0.005\text{ m}^3 \leftarrow \text{only 1 significant figure}}$$

$$\text{density} = 4.6 \times 10^3\text{ kg/m}^3 \approx \boxed{5 \times 10^3\text{ kg/m}^3}$$

What does it mean? Since one of the numbers used in the division only has a single significant figure, our answer should only be expressed to one significant figure.

P1.10 Recognize the principle. This problem requires knowing the rules for significant figures in multiplication.

Sketch the problem. No sketch needed.

Identify the relationships. Numbers should be multiplied with all given digits, then rounded to the smallest number of significant figures among all the values used.

Solve. Momentum is equal to mv, so simply insert the values.

$$\text{momentum} = mv$$
$$\text{momentum} = (6.5\text{ kg})(1.54\text{ m/s})$$
$$\text{momentum} = 10.01\text{ kg}\cdot\text{m/s}$$
$$\text{momentum} = \boxed{1.0 \times 10^{1}\text{ kg}\cdot\text{m/s}}$$

What does it mean? The starting value with the least number of significant figures is the mass with two significant figures. Therefore the answer will only have two significant figures. It is most clear to write this in scientific notation so that the zero is known to be significant.

P1.11 Recognize the principle. Know the correct way to write numbers in scientific notation, and how to count significant figures.

Sketch the problem. No sketch needed.

a) Identify the relationships. Scientific notation requires placing the decimal point after the first digit in the number and multiplying by 10 to the power equal to the number of decimal places the decimal was moved.

a) Solve. We move the decimal place 8 places to the left in order to make it follow the first digit, so we can write: $\boxed{2.99792458 \times 10^{8}\text{ m/s}}$

b) Identify the relationships/solve. To ensure four significant figures, we write the number in scientific notation to four digits and round the 4th digit appropriately.

$$\boxed{c = 2.998 \times 10^{8}\text{ m/s}}$$

What does it mean? Writing numbers in scientific notation makes rounding to the appropriate number of significant figures easier and (when zeros are involved) more precise.

[SSM] P1.12 Recognize the principle. Significant figures regarding the multiplication of numbers must be followed when finding areas and volumes. (Parts a, b, and c.) Significant figure rules for addition must be followed when finding the perimeter. (Part d.)

Sketch the problem. No sketch needed.

a,b,c) Identify the relationships. Numbers should be multiplied as usual, then rounded to the smallest number of significant figures in any value used.

a,b,c) Solve.

$$\text{(a)}\quad l = 2.34\text{ m},\ w = 1.874\text{ m}\quad A = l \times w = 4.38516\text{ m}$$

Since l has three significant figures, round to: $\boxed{A = 4.39\text{ m}^2}$

$$\text{(b)}\quad r = 0.0034\text{ m}\quad A = \pi r^2 = \pi(0.0034)^2 = 3.63168 \times 10^{-5}\text{ m}^2$$

Since r has two significant figures, round to: $\boxed{A = 3.6 \times 10^{-5}\ \text{m}^2}$

(c) $h = 1.94 \times 10^{-2}$ m, $r = 1.878 \times 10^{-4}$ m $V = \pi r^2 h = 2.14953 \times 10^{-9}\ \text{m}^3$

The height has the least significant figures (three), so round to: $\boxed{V = 2.15 \times 10^{-9}\ \text{m}^3}$

d) Identify the relationships. The numbers should be added as normal, and rounded to the smallest decimal place that is significant in all added values.

d) Solve. $l = 207.1$ m, $w = 28.07$ m, $P = 2l + 2w = 470.34$ m
The length is measured to fewer decimal places (the tenths place), so we round to the tenths place: $\boxed{P = 470.3\ \text{m}}$

What does it mean? Following rules for significant figures when combining quantities is important to avoid overstating the accuracy of the result.

[Reasoning] P1.13 Recognize the principle. We need a conversion factor to scale the apple to an appropriate size.

Sketch the problem. No sketch needed.

Identify the relationships. We can use the ratio of the atom to an apple diameter to find a conversion factor, and then use this same conversion factor to scale a real apple. An apple has a diameter of about 7.0 cm.

Solve. The conversion factor is the ratio of the apple to the atom:

$$\frac{7.0\ \text{cm}\ (1\ \text{m}/100\ \text{cm})}{10^{-10}\ \text{m}} = 7.0 \times 10^8$$

Scaling the apple up means multiplying it by this same factor:

$$(0.070\ \text{m})(7.0 \times 10^8) = 4.9 \times 10^7\ \text{m} \approx \boxed{5 \times 10^4\ \text{km}}$$

What does it mean? This apple would have diameter $\boxed{\text{of } 1 \times 10^8 \text{ m, about the size of Uranus!}}$ This kind of scaling helps us to appreciate just how small atoms are compared to common objects in the world around us.

[Life Sci] [Reasoning] P1.14 Recognize the principle. Unit conversion can be used to determine both the number of heart beats in a year, and the longer lifespan with a reduced heart rate.

Sketch the problem. No sketch is needed for this problem.

a) Identify the relationships. We can estimate the average heart rate for a student to be about 70 beats per minute. This can be converted to a number of beats per year.

a) Solve.

$$70\ \frac{\text{beats}}{\text{min}} \times \frac{60\ \text{min}}{\text{hour}} \times \frac{24\ \text{hours}}{\text{day}} \times \frac{365\ \text{days}}{\text{year}} = 3.6792 \times 10^7$$
$$\approx \boxed{3.7 \times 10^7\ \text{beats/year}}$$

b) Identify the relationships. We can find the total number of beats in an average lifetime by multiplying the number of beats in a year by the lifetime. Someone with a lower pulse rate will take longer to reach that total, and hence they will live a longer life.

b) Solve. A person with a 12% lower heart rate has a heart rate of:

$$(1.00 - 0.12)\ 70\ \text{beats/min} = 61.6\ \text{beats/min}$$

Since the total number of beats in a lifetime must be the same for both people, we know that:

$$70 \text{ beats/min} \times 77.9 \text{ years} = 61.6 \text{ beats/min} \times X$$

$$X = 88.5 \text{ years} = \boxed{89 \text{ years}}$$

What does it mean? Assuming that each heart has a limited number beats, reducing one's heart rate by 12% could lead to an additional 11 years of life.

P1.15 Recognize the principle. We need to apply the concepts of scientific notation.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. Scientific notation requires placing the decimal point after the first digit in the number and multiplying by 10 to the power equal to the number of decimal places the decimal was moved. Unless told otherwise, we assume place-holder zeros (to the right of the decimal) are not significant.

Solve.

(a) $\boxed{6.37 \times 10^{6} \text{ m}}$

(b) $\boxed{3.84 \times 10^{5} \text{ km}}$

(c) $\boxed{5.97 \times 10^{24} \text{ kg}}$

What does it mean? Scientific notation makes it much easier to write large numbers in a compact way.

P1.16 Recognize the principle. Distances can be converted from one unit to another using appropriate conversion factors.

Sketch the problem. No sketch needed.

Identify the relationships. Appropriate conversion factors can be found from a variety of sources. Here we have $1 \text{ m} = 6.21 \times 10^{-4} \text{ mi} = 39.37 \text{ in} = 0.001 \text{ km} = 1 \times 10^{6}\ \mu\text{m}$. The distance from Chicago to New York is about 1,268,000 m.

Solve. We can multiply this by the appropriate conversion factor in each case:

(a) $1{,}268{,}000 \text{ m} \times \left(\dfrac{6.21 \times 10^{-4} \text{ miles}}{1 \text{ m}}\right) = \boxed{787 \text{ miles}}$

(b) $1{,}268{,}000 \text{ m} \times \left(\dfrac{39.37 \text{ inches}}{1 \text{ m}}\right) = \boxed{4.992 \times 10^{7} \text{ inches}}$

(c) $1{,}268{,}000 \text{ m} \times \left(\dfrac{0.001 \text{ km}}{1 \text{ m}}\right) = \boxed{1268 \text{ km}}$

(d) $1{,}268{,}000 \text{ m} \times \left(\dfrac{1 \times 10^{6}\ \mu\text{m}}{1 \text{ m}}\right) = \boxed{1.268 \times 10^{12}\ \mu\text{m}}$

What does it mean? Note that the metric conversion factors are all powers of 10, while the U.S. customary unit conversions vary.

[Reasoning] P1.17 Recognize the principle. This problem requires taking measurements, keeping track of significant figures, and converting from inches to meters and then to centimeters.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. We will assume that a common ruler marked in inches is used to take the measurements. The conversion factor for converting inches to meters is 0.0254 m/in and for converting meters to centimeters is 100 cm/m. Volume is found by multiplying the quantities of length, width, and thickness:

$$V = l \times w \times t$$

The measurements taken should be approximately as follows:

length = 11 in, width = 8.5 in, thickness = 1.5 in.

Solve. We insert our values into our formula for volume:

$$V = (11 \text{ in}) \times (8.5 \text{ in}) \times (1.5 \text{ in})$$
$$V = 140.25 \text{ in}^3$$

Then, converting to cubic meters,

$$V = 140.25 \text{ in}^3 \times (0.0254 \text{ m/in})^3$$
$$V = 0.002298 \text{ m}^3$$

Since the measurements were taken to two significant figures, the answer should be rounded to two significant figures,

$$V \approx \boxed{2.3 \times 10^{-3} \text{ m}^3}$$

Converting to cubic centimeters,

$$V = 2.3 \times 10^{-3} \text{ m}^3 \times (100 \text{ cm/1 m})^3$$
$$\boxed{V \approx 3000 \text{ cm}^3}$$

What does it mean? The volume of a typical textbook is about 2500 cm^3.

[Reasoning] P1.18 Recognize the principle. This problem requires estimating measurements, keeping track of significant figures, and conversion from inches to meters.

Sketch the problem. No sketch needed.

Identify the relationships. The conversion factor for converting inches to meters is 0.0254 m/in. The volume of a sphere is $\frac{4}{3}\pi r^3$, and the diameter of an official NBA basketball is 9.39 in.

Solve. First, we convert the diameter of the basketball to meters and find the radius, keeping the extra significant digits:

$$D = (9.39 \text{ in})\,(0.0254 \text{ m/in}) = 0.2385 \text{ m}$$
$$r = D/2 = 0.11925 \text{ m}$$

Inserting this radius into our volume formula yields:

$$\text{volume} = \frac{4}{3}\pi\,(0.11925 \text{ m})^3$$
$$= \boxed{7.10 \times 10^{-3} \text{ m}^3}$$

Or in U.S. customary units:

$$\text{volume} = \frac{4}{3}\pi\,(4.70 \text{ in})^3$$
$$= \boxed{434 \text{ in}^3}$$

What does it mean? A basketball holds about 434 in^3 or 7100 ml of air.

[SSM] P1.19 Recognize the principle. This problem requires keeping track of significant figures and conversion from yards to meters to millimeters to feet and then to inches.

Sketch the problem. No sketch needed.

Identify the relationships. The conversion factor for converting yards to meters is 0.9144 meters/yard, yards to millimeters is 914.4 millimeters/yard, yards to feet is 3 feet/yard, then yards to inches is 36 inches/yard.

Solve. A U.S. football field is 120 yards long. We assume this has three significant figures—that is—it is accurate to within 0.5 yards. Converting,

(a) 120 yards $\times$ (0.9144 m/yard) = 109.7 m; rounded to three significant figures, this is $\boxed{1.10 \times 10^2 \text{ m}}$

(b) 120 yards $\times$ (914.4 mm/yard) = 109,728 mm; rounded to three significant figures, this is $\boxed{1.10 \times 10^5 \text{ mm}}$

(c) 120 yards $\times$ (3 ft/yard) = 360 ft; or rounded to three significant figures, this is $\boxed{3.60 \times 10^2 \text{ ft}}$

(d) 120 yards $\times$ (36 in/yard) = 4320 in; or rounded to three significant figures, this is $\boxed{4.32 \times 10^3 \text{ in}}$

What does it mean? There are many ways to express the length of a football field. That's why including a unit with an answer is so important!

P1.20 Recognize the principle. Apply the concepts of scientific notation, significant figures, and conversion factors.

Sketch the problem. No sketch is needed for this problem.

Identify the relationships. We can convert to years through a series of steps, since 3600 s = 1 hour, 24 hours = 1 day, and 365 days = 1 year.

Solve. We can use these conversion factors,

$$5 \times 10^{17} \text{ s} \times \left(\frac{1 \text{ hour}}{3600 \text{ s}}\right) \times \left(\frac{1 \text{ day}}{24 \text{ hours}}\right) \times \left(\frac{1 \text{ year}}{365 \text{ days}}\right) = \boxed{2 \times 10^{10} \text{ years}}$$

What does it mean? This is about 20 billion years!

P1.21 Recognize the principle. We can use the appropriate conversion factors and round to the correct number of significant figures in each case.

Sketch the problem. No sketch needed.

Identify the relationships. 5.0 m has two significant figures as given. We can use several different conversion factors here, since 1 m = 100 cm = 3.281 ft = 39.37 in = 6.21×10^{-4} mi.

Solve. Applying the correct conversion factor in each case, and rounding to two significant figures, yields:

(a) $5.0 \text{ m} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = \boxed{5.0 \times 10^2 \text{ cm}}$

(b) $5.0 \text{ m} \times \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = \boxed{16 \text{ ft}}$

(c) $5.0 \text{ m} \times \left(\frac{39.37 \text{ inches}}{1 \text{ m}}\right) = \boxed{2.0 \times 10^2 \text{ inches}}$

(d) $5.0 \text{ m} \times \left(\frac{6.21 \times 10^{-4} \text{ miles}}{1 \text{ m}}\right) = \boxed{3.1 \times 10^{-3} \text{ miles}}$

What does it mean? We can express a distance in many different units, which is why including a unit is so important!

P1.22 Recognize the principle. Apply the concepts of significant figures and conversion factors.

Sketch the problem. No sketch needed.

Identify the relationships. 75 feet has two significant figures as written. We can use these conversion factors: 1 meter = 3.281 ft = 1000 mm.

Solve. Applying the appropriate conversion factor and rounding to two significant figures in each case yields:

$$\text{(a)}\quad 75\ \text{ft} \times \left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right) = \boxed{23\ \text{m}}$$

$$\text{(b)}\quad 75\ \text{ft} \times \left(\frac{1000\ \text{mm}}{3.281\ \text{ft}}\right) = \boxed{2.3 \times 10^4\ \text{mm}}$$

What does it mean? 75 feet is about 23 meters, but 23,000 mm!

P1.23 Recognize the principle. After considering significant figures, we just need a correct conversion factor.

Sketch the problem. No sketch needed.

Identify the relationships. 273 g has three significant digits as written. Also, 1 kg = 1000 g.

Solve. Applying this conversion factor and maintaining three significant figures gives:

$$273\ \text{g} \times \left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) = \boxed{0.273\ \text{kg}}$$

What does it mean? 273 g is just over one-quarter kilogram.

P1.24 Recognize the principle. When converting areas, the appropriate conversion factor must be applied twice, once for each dimension.

Sketch the problem. No sketch needed.

Identify the relationships. Use the conversion 1 m = 100 cm = 1000 mm. These conversion factors must be squared since we are converting areas.

Solve.

$$\text{(a)}\quad 1\ \text{m}^2 \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^2 = \boxed{10{,}000\ \text{cm}^2 = 10^4\ \text{cm}^2}$$

$$\text{(b)}\quad 1\ \text{m}^2 \times \left(\frac{1000\ \text{mm}}{1\ \text{m}}\right)^2 = \boxed{1{,}000{,}000\ \text{mm}^2 = 10^6\ \text{mm}^2}$$

What does it mean? Area and volume conversion factors can always be determined from the linear conversion factor. The linear conversion factor is raised to the power of 2 to find an area conversion factor.

P1.25 Recognize the principle. We can apply the proper conversion factors to convert to the desired unit.

Sketch the problem. No sketch needed.

Identify the relationships. There are 1000 cm^3 in 1 l and 100 cm in 1 m. Since we will be converting cubic centimeters to cubic meters, we need to cube these conversion factors.

Solve. The first conversion factor can be applied straightforwardly:

(a) $V = 1.2\ \text{l} \times (1000\ \text{cm}^3/\text{l}) = \boxed{1.2 \times 10^3\ \text{cm}^3}$

But in the second case, we must cube the linear conversion factor:

(b) $V = 1.2 \times 10^3\ \text{cm}^3 \times (1\ \text{m}/100\ \text{cm})^3 = \boxed{1.2 \times 10^{-3}\ \text{m}^3}$

What does it mean? When converting volumes, either an appropriate direct conversion factor can be used (as in part a) or the linear conversion factor can be cubed and then used (as in part b).

P1.26 Recognize the principle. After considering significant figures, we just need a correct conversion factor.

Sketch the problem. No sketch needed.

Identify the relationships. The value given has two significant figures, so the converted value is only accurate to two significant figures. The conversion factor for converting feet to mm is 304.8 mm/ft.

Solve. We can use our conversion factor directly, then round to two significant figures:

$$\text{Height} = 6.5\ \text{ft} \times (304.8\ \text{mm/ft}) = 1981.2\ \text{mm}$$
$$\text{Height} = \boxed{2.0 \times 10^3\ \text{mm}}$$

What does it mean? Michael Jordan is about 2000 mm tall.

P1.27 Recognize the principle. Cubic units require metric conversions for each dimension.

Sketch the problem. No sketch necessary.

Identify the relationships. The linear conversion factor given (1 m = 100 cm = 1000 mm) must be cubed for cubic units.

Solve. Using these conversion factors:

(a) $1\ \text{m}\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = \boxed{1 \times 10^6\ \text{cm}^3}$

(b) $1\ \text{m}\left(\frac{1000\ \text{mm}}{1\ \text{m}}\right)^3 = \boxed{1 \times 10^9\ \text{mm}^3}$

What does it mean? Imagining a cubic meter with 100 cm (or 1000 mm) along each side may be helpful in visualizing the need for cubing the conversion factor.

P1.28 Recognize the principle. After considering significant figures, we just need a correct conversion factor.

Sketch the problem. No sketch needed.

Identify the relationships. The conversion factor relating minutes to seconds is 1 min/60 s.

Solve. Using this conversion factor yields:

$$(950\ \text{s}) \times \frac{1\ \text{min}}{60\ \text{s}} = \boxed{16\ \text{min}}$$

What does it mean? While the speed of light is very high, the distances between bodies in the solar system is very large, so it can take many minutes for light to travel from the Sun to a planet or between planets. Radio signals travel at approximately the speed of light, and this result also explains why the signals from spacecraft on the surface of Mars take many minutes to reach the Earth.

P1.29 Recognize the principle. After considering significant figures, we just need a correct conversion factor.

Sketch the problem. No sketch needed.

Identify the relationships. To make a valid comparison, we need to convert from the U.S. customary system volume units of cubic inches to SI units of liters. Note: 1 l = 1000 cm^3 and 1 in = 2.54 cm. Since we are converting cubic inches to cubic centimeters, we must cube the conversion factor as well.

Solve. First, we find a conversion factor from cubic inches to cubic centimeters:

$$1\ \text{in}^3 \times \left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^3 = 16.387\ \text{cm}^3$$

Then, keeping the above answer to three decimal places to avoid rounding error, we can convert to liters:

$$454\ \text{in}^3\left(\frac{16.387\ \text{cm}^3}{1\ \text{in}^3}\right) = 7440\ \text{cm}^3\left(\frac{1\ \text{l}}{1000\ \text{cm}^3}\right) = \boxed{7.44\ \text{l}}$$

What does it mean? We see that the older car's engine is 0.44 l larger that the new model. The 19% increase in power of the new smaller V8 engine comes from a number of engineering improvements, including fuel injection (instead of a carburetor) and compression of the gas-air mixture through the use of a turbo charger.

P1.30 Recognize the principle. We can use dimensional analysis to make sure both sides of the equation have the same dimensions.

Sketch the problem. No sketch needed.

Identify the relationships/solve. We convert each given relationship to its unit and simplify:

$$mgh = \frac{1}{2}mv^2$$

$$\text{(a)}\quad \mathrm{M}\frac{\mathrm{L}}{\mathrm{T}^2}\mathrm{L} = \mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2$$

$$\mathrm{M}\frac{\mathrm{L}^2}{\mathrm{T}^2} = \mathrm{M}\frac{\mathrm{L}^2}{\mathrm{T}^2}$$

a) What does it mean? The first equation is dimensionally correct.

$$v^2/h = g$$

$$\text{(b)}\quad \frac{\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2}{\mathrm{L}} = \frac{\mathrm{L}}{\mathrm{T}^2}$$

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \frac{\mathrm{L}}{\mathrm{T}^2}$$

b) What does it mean? The second equation is dimensionally correct.

c) Identify the relationships. In a similar way, we can determine the units for the given expression by inserting units.

c) Solve.

$$g/v^2$$

$$\frac{[\mathrm{L}/\mathrm{T}^2]}{[\mathrm{L}/\mathrm{T}]^2} = \boxed{[1/\mathrm{L}]}$$

What does it mean? The units for the given expression are inverse length, or L^{-1}.

P1.31 Recognize the principle. We can use dimensional analysis to find and simplify the units.

Sketch the problem. No sketch needed.

Identify the relationships/solve. We can substitute units for each quantity in the relationships given above, and then simplify:

$$my/t_2$$

$$\text{(a)}\quad \frac{[\text{kg}][\text{m}]}{[\text{s}]} = \text{kg} \cdot \text{m/s} = \boxed{\text{ML/T}}$$

$$hy/t_1^{\,2}$$

$$\text{(b)}\quad \frac{[\text{m}][\text{m}]}{[\text{s}]^2} = \text{m}^2/\text{s}^2 = \boxed{\text{L}^2/\text{T}^2}$$

$$y^3/(ht_1)$$

$$\text{(c)}\quad \frac{[\text{m}^3]}{[\text{ms}]} = \text{m}^2/\text{s} = \boxed{\text{L}^2/\text{T}}$$

What does it mean? All of these unit combinations could represent physical quantities. Part a represents a momentum, part b a velocity squared, and part c an area per unit time.

P1.32 Recognize the principle. We can use algebra to solve for x.

Sketch the problem. No sketch needed.

Identify the relationships. To isolate x, we must group like terms and divide by factors that multiply x.

Solve. First, we add and subtract terms from both sides to group all integers and terms containing x:

$$5x - 17 = 10x - 27$$
$$27 - 17 = 10x - 5x$$

We then simplify and divide both sides by the factor multiplying x.

$$10 = 5x$$
$$x = \boxed{2}$$

What does it mean? The only number that can be inserted for x and make this equation true is 2.

P1.33 Recognize the principle. We can substitute the value of t and solve for v.

Sketch the problem. No sketch needed.

Identify the relationships. The velocity, v, should have units of m/s.

Solve. Solving for v,

$$v = (95\ \text{m})/t$$

Inserting t,

$$v = 95\ \text{m}/0.25\ \text{s}$$
$$v = \boxed{380\ \text{m/s}}$$

What does it mean? We would interpret this to mean that over a time period of $t = 0.25$ s, an object is moving with a velocity of 380 m/s.

[SSM] * P1.34 Recognize the principle. We can use algebra to get the equations into the desired format. If we then graph the lines, the point of intersection (on both lines) is the single solution to both equations.

Sketch the problem. See Figure Ans1.34 for the needed graphs of the two equations to answer part (a).

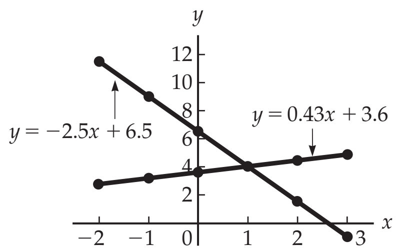

Figure Ans 1.34

Identify the relationships. We can solve each given equation for y by subtracting the x term from both sides and then dividing by the factor multiplying the y.

Solve. (a) Solving for y in each of the equations we find they yield equations of a straight line given by,

$$y = -2.5x + 6.5$$
$$y = 0.43x + 3.6$$

Figure Ans1.34 shows the plot of these two lines. Reading from the graph, we find that the two lines cross at

$$\boxed{(x, y) = (1, 4)}$$

(b) Applying techniques from Section 1.6 we find a common value for the x term,

$$(5x + 2y = 13)\,(3) \Rightarrow 15x + 6y = 39$$
$$(-3x + 7y = 25)\,(5) \Rightarrow -15x + 35y = 125$$

Then add these two equations to eliminate x:

$$\begin{array}{r} 15x + 6y = 39 \\ -15x + 35y = 125 \\ \hline 0x + 41y = 164 \end{array}$$

$$y = \frac{164}{41} = \boxed{4}$$

Now solve one of the equations for x and substitute $y = 4$:

$$15x + 6y = 39$$
$$\Rightarrow x = \frac{39 - 6y}{15} = \frac{39 - 6(4)}{15} = \boxed{1}$$

Thus: $\boxed{(x, y) = (1, 4)}$

What does it mean? In the graph in part (a), the lines cross at point (1, 4). This is the same solution found in part (b), so the methods agree on this single point that solves both equations.

*** P1.35 Recognize the principle.** We can write simultaneous equations using the molecular weight of hydrogen atoms and the molecular weight of carbon atoms in each molecule.

Sketch the problem. No sketch needed.

Identify the relationships. We can call x = g/mol of carbon, and y = g/mol of hydrogen. For benzene:

$$(6\ \text{C})x + (6\ \text{H})y = 78.11\ \text{g/mol}$$

For propane:

$$(3\ \text{C})x + (8\ \text{H})y = 44.096\ \text{g/mol}$$

We have two equations and two unknowns, and can solve for x and y using the algebra discussed in Section 1.6.

Solve. We start with the equations in simpler form:

$$6x + 6y = 78.11$$
$$3x + 8y = 44.096$$

Multiplying the second equation through by -2 yields:

$$6x + 6y = 78.11$$
$$-6x - 16y = -88.192$$

These equations can then be added to eliminate the x term, and isolate y:

$$-10y = -10.082$$
$$y = 1.0082$$

This value for y can then be inserted back into the first equation to find:

$$6x + 6(1.0082) = 78.11$$
$$x = 12.01$$

What does it mean? The molecular weights for each atom are therefore given by:

$$\boxed{\begin{aligned} x &= 12.01\ \text{g/mol (carbon)} \\ y &= 1.008\ \text{g/mol (hydrogen)} \end{aligned}}$$

These values agree with the periodic table.

[SSM] * P1.36 Recognize the principle. The concepts needed here are trigonometry, angular units, and conversion factors.

Sketch the problem. See Figure Ans1.36

Figure Ans 1.36

Identify the relationships. Take measurements directly off the clock face in Figure Ans1.36. Note that at 3 o'clock the hour hand position makes a right angle with the minute hand at the 12 o'clock position which allows us to derive a conversion factor:

$$\frac{90°}{3\ \text{h}} = 30°/\text{h}$$

Using radians instead of degrees the conversion factor is written as:

$$\frac{\pi/2}{3\ \text{h}} = \frac{\pi}{6}\ \text{h}^{-1}$$

Solve. We can then apply this conversion factor to each given situation:

(a) 3 o'clock $\rightarrow 3(30°) = \boxed{90° \text{ or } \pi/2 = 1.57 \text{ rad}}$

(b) 6:00 $\rightarrow 6(30°) = \boxed{180° \text{ or } \pi = 3.14 \text{ rad}}$

(c) 6:30 $\rightarrow 6(30°) + 30°/2 = \boxed{195°}$ or $\pi + (1/2)(\pi/6) = \boxed{13\pi/12 = 3.40 \text{ rad}}$

(Remember that the hour hand would be halfway between the six and seven at 6:30.)

(d) 9:00 $\rightarrow 9(30°) = \boxed{270° \text{ or } 3\pi/2 = 4.71 \text{ rad}}$

(e) 11:10 $\rightarrow (11)(30°) + (10/60)(30°) = \boxed{335°}$ or $335°\left(\frac{2\pi}{360°}\right) =$
$(1.87)\,\pi = \boxed{5.85 \text{ rad}}$

What does it mean? All of these angles have magnitudes that are less than 360°, or 2π radians.

P1.37 Recognize the principle. Given the two sides of a right triangle, we can find the hypotenuse using the Pythagorean theorem. The angles can then be calculated using trigonometry.

Sketch the problem. No sketch needed.

a) Identify the relationships. (a) For a right triangle, we know $r^2 = x^2 + y^2$

a) Solve. Solving for r and inserting the values for x and y,

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(4.5\ \text{m})^2 + (3.7\ \text{m})^2}$$

$$r = \boxed{5.8\ \text{m}}$$

b) Identify the relationships. Since this is a right triangle, the angle opposite the hypotenuse is 90° or $\pi/2$ radians. Labeling the angle opposite the y-side α, and the angle opposite the x-side β, then $\sin\alpha = y/r$ and $\alpha + \beta = 90°$.

b) Solve. Solving these equations,

$$\alpha = \sin^{-1}\left(\frac{y}{r}\right) = \sin^{-1}\left(\frac{3.7\ \text{m}}{5.8\ \text{m}}\right) = \boxed{40°}$$

And $\alpha + \beta = 90° \Rightarrow \beta = \boxed{50°}$
Since π is 180°, multiplying each angle by $\pi/180°$ will give the angle in radians,

$$\alpha = 40°\,(\pi/180°) = \boxed{0.70\ \text{rad}}$$

$$\beta = 50°\,(\pi/180°) = \boxed{0.87\ \text{rad}}$$

c) Identify the relationship. The smaller interior angle is α, and:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r},\ \cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r},\ \text{and}\ \tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

c) Solve. Inserting values yields:

$$\sin \alpha = \frac{y}{r} = \frac{3.7}{5.8} = \boxed{0.64}$$
$$\cos \alpha = \frac{x}{r} = \frac{4.5}{5.8} = \boxed{0.78}$$
$$\tan \alpha = \frac{y}{x} = \frac{3.7}{4.5} = \boxed{0.82}$$

d) Identify the relationships. The larger interior angle is β, and:

$$\sin \beta = \frac{\text{opp}}{\text{hyp}} = \frac{x}{r}, \cos \beta = \frac{\text{adj}}{\text{hyp}} = \frac{y}{r}, \text{ and } \tan \beta = \frac{\text{opp}}{\text{adj}} = \frac{x}{y}; \text{ therefore,}$$
$$\cos \beta = \frac{y}{r} = \frac{3.7}{5.8} = \boxed{0.64}$$
$$\sin \beta = \frac{x}{r} = \frac{4.5}{5.8} = \boxed{0.78}$$
$$\tan \beta = \frac{x}{y} = \frac{4.5}{3.7} = \boxed{1.2}$$

What does it mean? Given two sides of a right triangle, we can determine the third side and all of the angles.

P1.38 Recognize the principle. The sum of the interior angles of a triangle sum to 180°.

Sketch the problem. No sketch needed.

Identify the relationships. Including the right angle as 90° and calling the unknown angle θ, we know that:

$$90° + 25° + \theta = 180°$$

Solve. Solving for θ yields:

$$\boxed{\theta = 65°}$$

What does it mean? Since the angle measure you need is not always given in a physics problem, this type of geometry is often needed to find needed angle measurements.

*** P1.39 Recognize the principle.** The sum of the angles in a triangle must be equal to 180° which is π radians.

Sketch the problem. No sketch needed.

Identify the relationships. By definition, a right triangle has one angle that measures 90°, or $\frac{\pi}{2}$ radians. We are given one other angle, and can therefore find the third angle by requiring the sum to equal π radians:

$$\theta + 0.70 + \frac{\pi}{2} = \pi$$

If we then choose a value for the hypotenuse ($r = 1.0$ m), we can use trigonometric relationships to find the other two sides.

Solve. The remaining angle can therefore be found by summing the known and unknown angles:

$$\theta = \frac{\pi}{2} - 0.70 = 0.87 \text{ radians}$$

Now: $\sin \theta = \frac{\text{opp}}{\text{hyp}} \Rightarrow (\text{hyp}) \sin \theta = \text{opp}$

By choosing $r = 1.0$ (the hypotenuse), the side opposite each angle must just be the sin of that angle. That is:

$$\text{side opposite 0.87 radian angle} = \sin(0.87) = \boxed{0.76 \text{ m}}$$

$$\text{side opposite 0.70 radian angle} = \sin(0.70) = \boxed{0.64 \text{ m}}$$

What does it mean? Many answers are possible here, but a right triangle with a 0.70 radian angle and hypotenuse 1.0 m long has sides of 0.76 m and 0.64 m. The sides of all such triangles should be in the same ratio as the sides given here.

P1.40 Recognize the principle. We can convert angles from degrees to radians.

Sketch the problem. No sketch needed.

Identify the relationships. Since π is equivalent to 180°, multiplying the angle by $\pi/180°$ will give the angle in radians.

Solve. $73° (\pi/180°) = \boxed{1.3 \text{ rad}}$

What does it mean? A 73° angle is equivalent to an angle of 1.3 radians.

P1.41 Recognize the principle. We can convert angles from radians to degrees.

Sketch the problem. No sketch needed.

Identify the relationships. Since π is equivalent to 180°, multiplying the angle by $180°/\pi$ will give the angle in radians.

Solve. Using this conversion yields: $1.25 \text{ rad } (180°/\pi) = \boxed{71.6°}$

What does it mean? 1.25 radians is equivalent to 71.6°. This makes sense, since $\pi/2$ (which is 1.57) radians is 90°.

P1.42 Recognize the principle. The angle around a circle is related to the path distance around its edge by the radius.

Sketch the problem. See Figure P1.42.

Identify the relationships. Mathematically, the relationship between the path distance (s) and the angle in radians within the circle (θ) can be expressed as:

$$s = \theta\, r$$

Solve. Solving this expression for the angle and inserting values gives:

$$\theta = \frac{s}{r} = \frac{210 \text{ m}}{38 \text{ m}} = 5.5 \text{ rad}$$

What does it mean? Radian measurement of angles makes it very convenient to find path distances at a given radius.

P1.43 Recognize the principle. The angle around a circle is related to the path distance around its edge by the radius.

Sketch the problem. See Figure P1.43.

Identify the relationships. We must first convert the angle given in degrees to radians using the relationship:

$$\theta_{\text{rad}} = \theta_{\text{deg}} \times \frac{2\pi}{360°}$$

Mathematically, the relationship between the path distance(s) and the angle in radians within the circle (θ) can be expressed as:

$$s = \theta r$$

Solve. Converting the angle to degrees therefore yields:

$$\theta_{rad} = 450 \times \frac{2\pi}{360°} = 7.85 \text{ rad}$$

Solving this expression for the radius and inserting values gives:

$$r = \frac{s}{\theta_{rad}} = \frac{750 \text{ m}}{7.85 \text{ rad}} = \boxed{96 \text{ m}}$$

What does it mean? Angles must be measured in radius to easily relate them to the path distance.

P1.44 Recognize the principle. The ladder forms the hypotenuse of a right triangle, while the wall forms one side and the floor the other.

Sketch the problem. See Figure P1.44.

Identify the relationships. The angle α between the ladder and the wall is formed with the hypotenuse (ladder, l) and the height along the wall (h), which is adjacent to this angle. We can therefore write:

$$\cos \alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{h}{l}, \alpha = \cos^{-1}\left(\frac{h}{l}\right)$$

Similarly, for the angle β between the ladder and the floor, the opposite side is the height. With the ladder still as the hypotenuse, we can write:

$$\sin \beta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{l}, \beta = \sin^{-1}\left(\frac{h}{l}\right)$$

Solve. Inserting values in each case,

$$\alpha = \cos^{-1}\left(\frac{1.7 \text{ m}}{2.5 \text{ m}}\right) = \boxed{47°}$$

$$\beta = \sin^{-1}\left(\frac{1.7 \text{ m}}{2.5 \text{ m}}\right) = \boxed{43°}$$

What does it mean? Since the two angles are nearly equal, we can see that the ladder is placed such that the distance along the floor and the distance along the wall are nearly equal.

P1.45 Recognize the principle. We can create a triangle with the distance traveled along the road as the hypotenuse. We can then use trigonometry to find the height, which is one of the sides of the triangle.

Sketch the problem. No sketch needed.

Identify the relationships. If the distance along the hill is the hypotenuse, then for the angle of the hill with the horizontal, the height is the opposite side of the triangle. Then we can write:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{l}$$

Solve. Solving for h, $h = l \sin\theta$. Then inserting the given values,

$$h = (3.5 \text{ km}) \sin(8.5°)$$

$$\boxed{h = 0.52 \text{ km}}$$

What does it mean? The bicycle experiences a rise in elevation of about one-half of one kilometer.

P1.46 Recognize the principle. We can use vector addition and vector multiplication by a scalar to sketch these.

Sketch the problem. See Figure P1.46 and Figure Ans1.46.

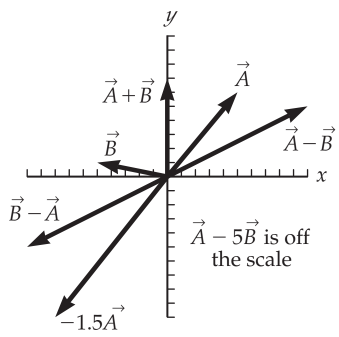

Figure Ans 1.46

Identify the relationships. After drawing the original vectors to scale, we can multiply by the needed factors and extend the length proportionally. For example, if you measured $\vec{A}$ as 42 mm, and $\vec{B} - \vec{A}$ as 60 mm since we were told that $\vec{A}$ symbolizes 2.3 m, so we can set up a ratio:

$$\frac{42}{60} = \frac{2.3}{|\vec{B} - \vec{A}|}$$

We can change signs for the negative vectors by flipping the direction of the vectors. Finally, we add by placing one vector "tip to tail" with the other and then connecting the back tip to the foremost tail to graph the various combinations. The magnitude of each resultant vector can then be found by measuring. (Note that a separate graph with a different scale must be drawn to measure $|\vec{A} - 5\vec{B}|$ as the coordinates are [30, 1].)

Solve.

(a) See Figure Ans1.46.

(b) Drawing, measuring, and setting proportions as described above,

$$\begin{aligned}
|\vec{A} + \vec{B}| &= \boxed{2.1 \text{ m}} \\
|\vec{A} - \vec{B}| &= \boxed{3.3 \text{ m}} \\
|\vec{B} - \vec{A}| &= \boxed{3.3 \text{ m}} \\
|\vec{A} - 5\vec{B}| &= \boxed{8.8 \text{ m}} \\
|-1.5\vec{A}| &= \boxed{3.5 \text{ m}}
\end{aligned}$$

What does it mean? A graphical approach is one way to find the length of vector combinations.

P1.47 Recognize the principle. The length of a vector is found by adding the components. This can be done using the Pythagorean theorem since components are always perpendicular. The angle of the vector can be found from the components using trigonometry.

Sketch the problem. No sketch needed.

Identify the relationships. The length of vector $\vec{A}$ can be found from $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}$, and the angle can be found from $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$.

Solve. We insert the values for the components into these two expressions and solve:

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2} = \sqrt{(54\text{ m})^2 + (23\text{ m})^2} = \boxed{59\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{23\text{ m}}{54\text{ m}}\right) = \boxed{23°}$$

What does it mean? This vector has a length of 59 m and points 23° above the x axis.

P1.48 Recognize the principle. Vector components can be found using trigonometry.

Sketch the problem. See Figure Ans1.48.

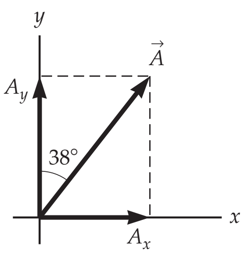

Figure Ans 1.48

Identify the relationships. Since the angle is measured from the vertical, we see that for the angle given, the y-component is the adjacent side and the x-component will be the opposite side of the right triangle. The hypotenuse is the magnitude of the given vector.

Solve. Inserting the magnitude and angle into each expression yields:

$$A_x = A\sin\theta = (2.5\text{ m})\sin(38°) = \boxed{1.5\text{ m}}$$

$$A_y = A\cos\theta = (2.5\text{ m})\cos(38°) = \boxed{2.0\text{ m}}$$

What does it mean? Note that the x-component of a vector is *not* always associated with the cosine of the given angle. You must determine the correct trig operator for each component. A drawing is always useful in determining the correct relationship.

P1.49 Recognize the principle. The length of a vector represents its magnitude.

Sketch the problem. Figure P1.49 serves as our sketch.

Identify the relationships/solve. From the figure, $\boxed{\text{vectors (a), (b), (c), and (f)}}$ have the same length, and therefore the same magnitudes. Similarly, $\boxed{\text{vectors (e) and (h)}}$ have the same magnitudes, and $\boxed{\text{vectors (d) and (g)}}$ have the same magnitudes.

What does it mean? Graphically, any vector with the same measured length has the same magnitude.

P1.50 Recognize the principle. Graphically, the direction of a vector is represented with the vector's arrow.

Sketch the problem. Figure P1.50 serves as our sketch.

Identify the relationships. The key to this problem is to understand that the magnitude (length) of the vector can be ignored and only consider if any two vectors are oriented in the same direction.

Solve. By inspection of Figure P1.50, there are two pairs of vectors that share the same direction:

$\boxed{\text{(a) and (e), and (f) and (h).}}$

What does it mean? The magnitude of a vector can be entirely ignored when focusing on the direction.

P1.51 Recognize the principle. The graphical sum of vectors is identified by orienting the vectors from tip to tail.

Sketch the problem.

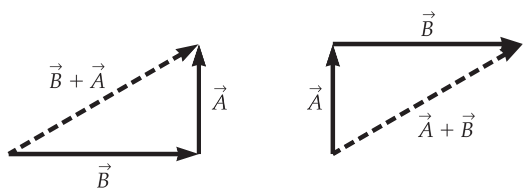

Figure Ans 1.51

Identify the relationships. Examining the sketch, the vector sum should point upward and to the right and be longer than either $\vec{A}$ or $\vec{B}$ by itself.

Solve. The only vector which points upward to the right and is longer than either $\vec{A}$ or $\vec{B}$ is (c).

What does it mean? Note from the sketch that we can add the vectors in either order and get the same result! Vector addition is therefore commutative, just like scalar addition. Note also that the x-component of vector (c) is 4 (the same as $\vec{B}$) while the y-component is 2 (the same as $\vec{A}$).

***P1.52 Recognize the principle.** We view the full vector as the hypotenuse of a triangle and the y-component as one side of the triangle.

Sketch the problem. See Figure Ans1.52.

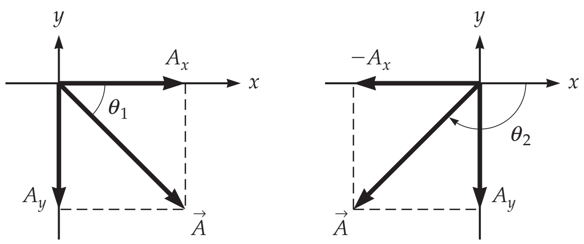

Figure Ans 1.52

Identify the relationships. The sketch shows two possible answers, one with a positive x-component, and one with a negative x-component. We can use trigonometry to determine the relationships of components of vectors. Since we are given the side opposite the angle of interest (y-component) and the hypotenuse (length), we can write:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{A_y}{A}$$

Solve. We can solve this expression for the angle:

$$\theta = \arcsin\left(\frac{A_y}{A}\right)$$

Inserting the given values, we find the angle in quadrant IV:

$$\theta_1 = \arcsin\left(\frac{-2.7\text{ m}}{4.5\text{ m}}\right) = \boxed{-37°}$$

Your calculator is programmed to find the smallest angle that satisfies this ratio, but our sketch has determined there is a second such angle. By inspection of Figure Ans1.52, we see that the angle in quadrant III is just as far from the $-x$ axis as the angle in quadrant IV is from the $+x$ axis. That means:

$$\theta_2 = -(180° - 37°) = \boxed{-143° \text{ or } +217°}$$

What does it mean? Your calculator will not always tell you the whole story. A good sketch and some thought shows that there are two values for the angle, since $\boxed{\text{the } x\text{-component can be either positive or negative.}}$

P1.53 Recognize the principle. Using geometry and a sketch, we can find the vector sum of the westward and southward trips.

Sketch the problem. See Figure Ans 1.53.

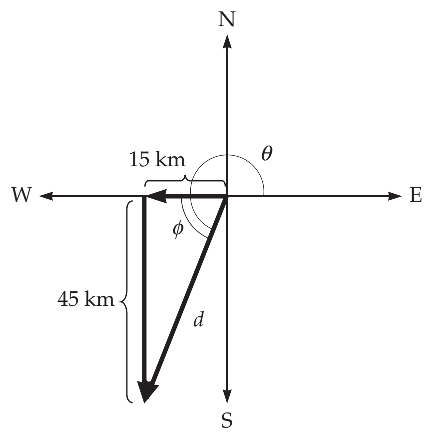

Figure Ans 1.53

Identify the relationships. In the sketch, we can see the distance traveled must be the hypotenuse of a triangle with legs of 15 km and 45 km. The angle can be found using the tangent function since we know the opposite and adjacent sides of the triangle.

Solve. Using the Pythagorean theorem, the hypotenuse is therefore:

$$d = \sqrt{(15\text{ km})^2 + (45\text{ km})^2} = \boxed{47\text{ km}}$$

And the angle is:

$$\theta = \tan^{-1}\frac{45\text{ km}}{15\text{ km}} = 72°$$

We note that this angle is referenced to the negative x (or westward) axis. More conventionally, this would be referenced to the positive x (or eastward) axis and expressed as:

$$\theta_{\text{east}} = 72° + 180° = \boxed{252°}$$

What does it mean? The straight westward and southward trips serve as components of a vector showing the net displacement.

P1.54 Recognize the principle. Using geometry and a sketch, we can find the vector sum of the three trips.

Sketch the problem. See Figure Ans1.54.

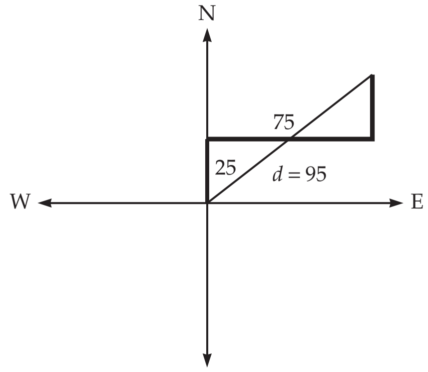

Figure Ans 1.54

Identify the relationships. In the sketch, we can see that the total distance traveled must be the vector sum of the two northward trips and the eastward trip. Since these vectors form a right triangle, of which we know the eastward side and the hypotenuse, we can find the northward side.

Solve. Using the Pythagorean Theorem:

$$95\text{ km} = \sqrt{(75\text{ km})^2 + ((z + 25)\text{ km})^2}$$

This simplifies to:

$$(95\text{ km})^2 - (75\text{ km})^2 = 3400\text{ km}^2 = ((z + 25)\text{ km})^2$$

And: $(z + 25)\text{ km} = \sqrt{3400\text{ km}^2} = 58\text{ km}$

$$z = 58\text{ km} - 25\text{ km} = 33\text{ km}$$

What does it mean? The unknown northward distance must be 33 km to give this total distance from the starting point.

P1.55 Recognize the principle. A scalar multiplying a vector just lengthens the vector proportionally, and a negative sign points the vector in the opposite direction.

Sketch the problem. The sketch in Figure Ans1.55 shows all four vectors.

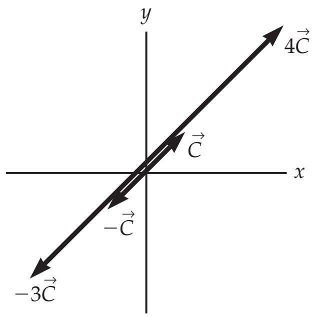

Figure Ans 1.55

What does it mean? Note that $-\vec{C}$ and $-3\vec{C}$ point in the opposite direction of the original vector $\vec{C}$ and $4\vec{C}$.

P1.56 Recognize the principle. These vectors can best be calculated by adding components of the original vectors.

Sketch the problem. No sketch needed.

Identify the relationships. We can calculate the components of each given vectors:

$$A_x = A\cos\theta = (15)\cos(25°) = 13.6$$
$$A_y = A\sin\theta = (15)\sin(25°) = 6.34$$
$$B_x = B\cos\theta = (25)\cos(70°) = 8.55$$
$$B_y = B\sin\theta = (25)\sin(70°) = 23.5$$

We then apply the appropriate operations to these vector components to find the components of $\vec{C}$. Finally we "reassemble" the vector by finding the magnitude $C = \sqrt{C_x^2 + C_y^2}$ and angle $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$ for each $\vec{C}$.

Solve

(a) $\vec{C} = \vec{A} + \vec{B}$

$$C_x = A_x + B_x = 22.2$$
$$C_y = A_y + B_y = 29.8$$
$$C = \sqrt{C_x^2 + C_y^2} = \boxed{37}$$
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{53°}$$

(b) $\vec{C} = \vec{A} - \vec{B}$

$$C_x = A_x - B_x = 5.04$$
$$C_y = A_y - B_y = -17.2$$
$$C = \sqrt{C_x^2 + C_y^2} = \boxed{18}$$
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{-74°}$$

(c) $\vec{C} = \vec{A} + 4\vec{B}$

$$C_x = A_x + 4B_x = 47.8$$
$$C_y = A_y + 4B_y = 100$$

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{110}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{64°}$$

(d) $\vec{C} = -\vec{A} - 7\vec{B}$

$$C_x = -A_x - 7B_x = -73.5$$
$$C_y = -A_y - 7B_y = -171$$

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{190}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{67°}$$

In this case, since both components are negative, the vector $\vec{C}$ will be in quadrant III. This means the angle from the x axis will be $180° + \theta$, or $\boxed{250°}$ to the correct number of significant figures.

What does it mean? Many vector operations can be easily done by breaking vectors into components, then reassembling them after adding, multiplying, or subtracting.

P1.57 Recognize the principle. We can find the vector components of the sum by summing the components of the vectors being added or subtracted.

Sketch the problem. No sketch needed.

Identify the relationships/solve. As calculated in Problem 1.57:

(a) $\vec{C} = \vec{A} + \vec{B}$

$$C_x = A_x + B_x = \boxed{22}$$
$$C_y = A_y + B_y = \boxed{30}$$

(b) $\vec{C} = \vec{A} - \vec{B}$

$$C_x = A_x - B_x = \boxed{5.0}$$
$$C_y = A_y - B_y = \boxed{-17}$$

(c) $\vec{C} = \vec{A} + 4\vec{B}$

$$C_x = A_x + 4B_x = \boxed{48}$$
$$C_y = A_y + 4B_y = \boxed{100}$$

(d) $\vec{C} = -\vec{A} - 7\vec{B}$

$$C_x = A_x - 7B_x = \boxed{-73}$$
$$C_y = A_y - 7B_y = \boxed{-171}$$

What does it mean? We can work with just the x- or y-components of a vector if we only need to know the result along one axis.

P1.58 Recognize the principle. We are given the horizontal and vertical components of motion, and need to find the resultant distance and direction.

Sketch the problem. See Figure P1.58.

Identify the relationships. Using d as the magnitude of the vector and θ as the angle, the magnitude can be found from $d = \sqrt{d_x^2 + d_y^2}$, and the angle can be found from, $\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right)$.

Solve. We can insert the horizontal distance d_x and the vertical the distance d_y to find the distance:

$$d = \sqrt{(77\text{ m})^2 + (95\text{ m})^2} = \boxed{120\text{ m}}$$

And the angle:

$$\theta = \tan^{-1}\left(\frac{95}{77}\right) = \boxed{51°}$$

What does it mean? By considering the horizontal and vertical distances as the components of a vector, we can find the overall distance and direction.

P1.59 Recognize the principle. By considering each part of the trip a vector and adding the two parts, we can find the resultant distance and direction.

Sketch the problem. No sketch needed.

Identify the relationships. The first part of the trip will be labeled as $\vec{A}$ and the second part of the trip will be labeled as $\vec{B}$.

For $\vec{A}$, $\quad A_x = 0$ and $A_y = -1.2$ km

For $\vec{B}$, $\quad B_x = B\cos\theta = (3.1\text{ km})\cos(53°) = 1.9$ km

$\qquad B_y = B\sin\theta = (3.1\text{ km})\sin(53°) = 2.5$ km

Solve. Then we add components along each axis:

$$C_x = A_x + B_x = 1.9\text{ km}$$
$$C_y = A_y + B_y = 1.3\text{ km}$$

And finally, we reassemble the vector to find a distance and magnitude:

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{2.3\text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{34° \text{ north of east}}$$

What does it mean? The car ends up 2.3 km from its starting point, along a line 34° N of E.

[SSM] * P1.60 Recognize the principle. The same vector algebra we've used in two dimensions can be applied in three dimensions.

Sketch the problem See Figure Ans1.60.

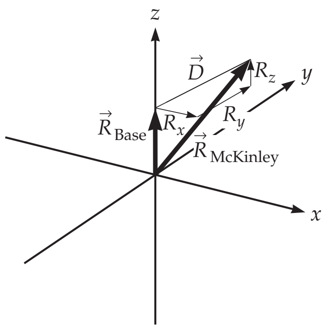

Figure Ans 1.60

Identify the relationships. The three-dimensional displacement vector has a magnitude given by $D = \sqrt{R_x^2 + R_y^2 + R_z^2}$, where the displacement vector $\vec{D}$ is equal to the difference between the position vector to the top of the mountain, $\vec{R}_{\text{McKinley}}$, and that of the position vector to the base camp, $\vec{R}_{\text{Base}}$. This difference can be written as the vector equation:

$$\vec{D} = -\vec{R}_{\text{Base}} + \vec{R}_{\text{McKinley}} = \vec{R}_{\text{McKinley}} - \vec{R}_{\text{Base}}$$

As in two dimensions, the components of the displacement vector from the base camp to the top of Mount McKinley can be found by subtracting each component of the Mount McKinley position vector from that of the base camp position vector. The third (z) coordinate represents the altitude change.

Solve. We subtract each dimension:

$$D_x = R_{\text{McKinley}, x} - R_{\text{Base}, x} = 1600 \text{ m} - 0 \text{ m} = 1600 \text{ m}$$

$$D_y = R_{\text{McKinley}, y} - R_{\text{Base}, y} = 4200 \text{ m} - 0 \text{ m} = 4200 \text{ m}$$

$$D_z = R_{\text{McKinley}, z} - R_{\text{Base}, z} = 6200 \text{ m} - 4300 \text{ m} = 1900 \text{ m}$$

The magnitude of the displacement vector is then,

$$D = \sqrt{(D_x^2 + D_y^2 + D_z^2)} = \sqrt{(1600 \text{ m})^2 + (4200 \text{ m})^2 + (1900 \text{ m})^2} = \boxed{4900 \text{ m}}$$

What does it mean? If you could run a cable from the top of Mount McKinley directly to base camp, it would be 4.9 km long.

[Life Sci] [Reasoning] P1.61 Recognize the principle. We can estimate an average student heart rate and use unit conversions to find the number of beats over 4 years.

Sketch the problem. No sketch needed.

Identify the relationships. To get a conservative estimate one can use the low-end value of the resting heart rate of 60 bpm (beats per minute) which simplifies the calculations (this is one heart beat per second). This means a low-end approximation is just the number of seconds in 4 years.

Solve. We then use unit conversions to find the number of seconds in 1 year:

$$1 \text{ year}\left(\frac{365 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ h}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 31{,}536{,}000 \text{ s} = 3.15 \times 10^7.$$

Over 4 years, this means:

$$4 \text{ year}\left(\frac{3.15 \times 10^7 \text{ s}}{1 \text{ year}}\right)\left(\frac{1 \text{ beat}}{1 \text{ s}}\right) = \boxed{1.3 \times 10^8 \text{ beats}}$$

What does it mean? This very conservative estimate gives 130 million heartbeats during college.

[SSM] * [Reasoning] P1.62 Recognize the principle. After considering significant figures, we can determine a conversion factor for our model.

Sketch the problem. No sketch needed.

Identify the relationships. We can use the ratio of the peppercorn to Earth diameter as a conversion factor:

$$\frac{3.7 \text{ mm}}{1.3 \times 10^7 \text{ m}} = 2.846 \times 10^{-7} \text{ mm/m}$$

Thus 2.846×10^{-7} mm in our model represents 1 m in the solar system. We also need several real distances:

Diameter of the sun $\approx 1.4 \times 10^{9}$ m

Typical Earth–Sun distance is $\approx 1.5 \times 10^{11}$ m

Typical Pluto–Sun distance $\approx 5.9 \times 10^{12}$ m

Solve. We can then apply this conversion factor to determine distances in our model.

(a) $(1.4 \times 10^{9}\text{ m})(2.846 \times 10^{-7}\text{ mm/m}) = 398\text{ mm} = \boxed{40\text{ cm}}$

This is the size of a large beach ball.

(b) $(1.5 \times 10^{11}\text{ m})(2.846 \times 10^{-7}\text{ mm/m}) = 4.3 \times 10^{4}\text{ mm} = \boxed{43\text{ m}}$

The peppercorn earth would orbit 43 m away from the center of the beach ball sun.

(c) $(5.9 \times 10^{12}\text{ m})(2.846 \times 10^{-7}\text{ mm/m}) = 1.7 \times 10^{6}\text{ mm} = \boxed{1.7\text{ km} \approx 1\text{ mi}}$

The grain of sand-sized Pluto would orbit approximately 1 mile from the center of the beachball sun.

What does it mean? It is a challenge to depict a scale model of the solar system where the planet diameters and orbit distances are both to scale simultaneously. Such an accurate model will not fit on a page in a book, or in a classroom!

P1.63 Recognize the principle. We can use several known unit conversions in each case.

Sketch the problem. No sketch needed.

Identify the relationships. We will need several conversion factors (from the inside cover or the internet):

1 mi = 1760 yd = 5280 ft

1 yd = 0.915 m = 3 ft (exactly)

Solve.

(a) We can first find the area of an acre in square yards:

$$A_{\text{acre}} = (220\text{ yd})(22\text{ yd}) = 4840\text{ yd}^2$$

A square mile, in square yards, is:

$$A_{\text{mile}} = (1760\text{ yd})^2 = 3{,}097{,}600\text{ yd}^2$$

Therefore, 1 square mile is:

$$\frac{3{,}097{,}600\text{ yd}^2/\text{mile}^2}{4840\text{ yd}^2/\text{acre}^2} = \boxed{640\text{ acre}^2/\text{mile}^2}$$

(b) Keeping in mind that we need to square the linear conversion factor and keep only as many significant digits as the conversion, we can combine the first calculation from part b and the yards to meters conversion to find:

$$4840\text{ yd}^2\left(\frac{0.915\text{ m}}{1\text{yd}}\right)^2 = \boxed{4050\text{ m}^2/\text{mile}^2}$$

(c) And similar to part b, we have:

$$1\text{ acre} = 4840\text{ yd}^2\left(\frac{3\text{ ft}}{1\text{ yd}}\right)^2 = \boxed{43{,}560\text{ ft}^2}$$

$$1\text{ mile}^2\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2 = \boxed{27{,}878{,}400\text{ ft}^2}$$

Note that all significant figures are kept here, since these conversions are exact.

What does it mean? These types of conversions require careful attention to both the dimensions of the conversions and the number of significant figures.

[Reasoning] P1.64 Recognize the principle. We can use the tip-to-tail method of graphically adding the vectors.

Sketch the problem. First, we sketch the three vectors tip to tail and show the resultant.

(a)

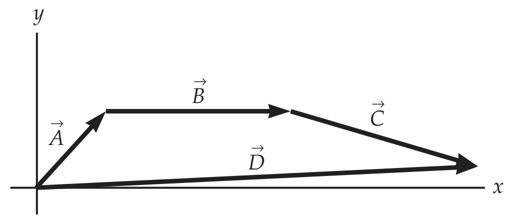

Figure Ans 1.64a

Figure Ans1.64a shows the sum of the three vectors.
Next, we show the sum of $\vec{A}$ and $\vec{B}$ added to $\vec{C}$. The resultant doesn't change.

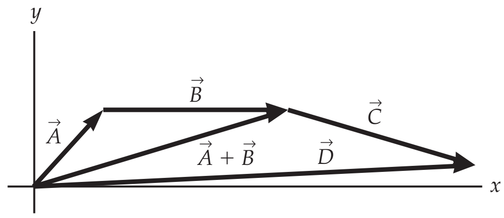

Figure Ans 1.64b

Figure Ans1.64b shows the vector $\vec{A} + \vec{B}$ added to $\vec{C}$.
Finally, we show the sum of $\vec{B}$ and $\vec{C}$ added to $\vec{A}$. The resultant still doesn't change.

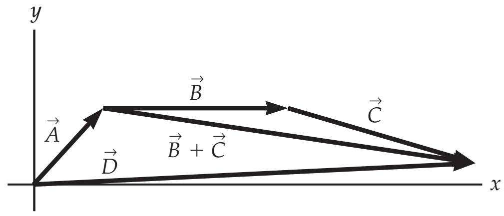

Figure Ans 1.64c

Figure Ans1.64c shows the vector $\vec{A}$ added to the vector $\vec{B} + \vec{C}$.

Solve. From the diagram, the length of $\vec{D}$ is about 4 times as long as $\vec{A}$, or 20 cm. We can also estimate the angle at about $+3°$.

What does it mean? Vector addition is distributive. This means when adding three (or more) vectors, you can add any two, then add the third, and get the same answer.

*** P1.65 Recognize the principle.** Since each tile is 1 square foot, we need to use conversion factors to find the number of square feet on the floor.

Sketch the problem. No sketch needed.

Identify the relationships. We can use conversion factors for linear distances to find conversion factors for areas by squaring them. Here 1 yd = 3 ft.

Solve. We then use this conversion factor to find the number of square feet:

$$10.7\ \text{yd}^2\left(\frac{3\ \text{ft}}{1\ \text{yd}}\right)^2 = 96.3\ \text{ft}^2$$

What does it mean? Rounding here is inappropriate since you would leave a small part of your floor uncovered with 96 tiles. You would need $\boxed{97}$ tiles to do the job with a little left over.

*** P1.66 Recognize the principle.** The velocity of the boat with respect to still water and the velocity of the river with respect to the land can be added as vectors to give the velocity of the boat with respect to land.

Sketch the problem. See Figures Ans1.66a and Ans1.66b.

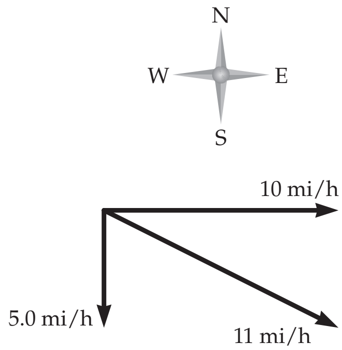

Figure Ans 1.66a

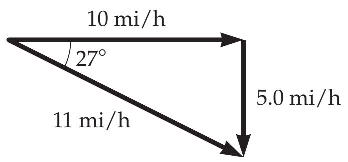

Figure Ans 1.66b

Identify the relationships. Each velocity is a vector. Figure Ans1.66a shows the orientation of each vector with respect to the compass directions. The two vectors should be added to obtain the total velocity by placing them tip to tail as shown in Figure Ans1.66b.
Let v be the total velocity vector and θ be the angle with respect to the horizontal. The magnitude of v can then be found using $v = \sqrt{v_x^2 + v_y^2}$, and the angle of the velocity can be found with, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$.

Solve. Inserting the components, v_x and v_y as given:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10\ \text{mi/h})^2 + (-5.0\ \text{mi/h})^2} = \boxed{11\ \text{mi/h}}$$

$$\theta = \arctan\left(\frac{-5\ \text{mi/h}}{10\,\text{mi/h}}\right) = \boxed{27°}\ \text{south of east.}$$

What does it mean? The boat is carried downstream by the river and therefore moves more quickly with respect to the ground than it can in still water.

*** [Life Sci] [Reasoning] P1.67 Recognize the principle.** Since we are given the volume of an average cell, we need only estimate the average volume of a human body to find an estimate for the number of cells.

Sketch the problem. No sketch needed.

Identify the relationships. For the sake of simplicity, we assume that a human is approximately a box shape approximately 1.5 m tall, 0.5 m wide, and 0.2 m thick.

Solve. The volume will be then be the height times the width times the thickness,

$$V = (1.5 \text{ m}) \times (0.5 \text{ m}) \times (0.2 \text{ m})$$
$$V = 0.15 \text{ m}^3$$

Dividing the volume of a human by the volume of a cell will give an approximate value to the number of cells in a human body,

$$\frac{V_{\text{human}}}{V_{\text{cell}}} = \frac{0.15 \text{ m}^3}{1 \times 10^{-16} \text{ m}^3} = 1.5 \times 10^{15} \text{ cells} \approx \boxed{2 \times 10^{15} \text{ cells}}$$

What does it mean? There are a very large number of cells in the human body!

*** [Life Sci] [Reasoning] P1.68 Recognize the principle.** We need to make estimates in familiar units and then convert them to m/s.

Sketch the problem. No sketch needed.

Identify the relationships. On an average, human hair grows about an inch a month. We will need the following conversion factors: 1 month = 30 days, 1 day = 24 hours, 1 hour = 60 min, 1 min = 60 s, and 1 in = 0.0254 m.

Solve. We can then convert this rate of growth to inches per second:

$$\frac{1 \text{ in}}{1 \text{ month}} \times \frac{1 \text{ month}}{30 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}}$$
$$\approx 4 \times 10^{-7} \text{ in/s}$$

And then we can convert this rate to m/s:

$$4 \times 10^{-7} \text{ in/s} \times \frac{0.0254 \text{ m}}{1 \text{ in}} \approx \boxed{1 \times 10^{-8} \text{ m/s}}$$

What does it mean? The typical hair grows an average of 10 nanometers every second. In Problem 1.67, we are told that a single cell has a volume of 1×10^{-16} m^3. If a cell were cubical, this would imply that each side is 4.6×10^{-6} m long. This means only about 1 out of 500 hairs on your head adds a cell each second!

P1.69 Recognize the principle. We can estimate the height of an adult, match units, and then take the ratio.

Sketch the problem. No sketch needed.

Identify the relationships. An average adult female has a height of about 5'6".

Solve. The ratio requires both quantities to be in the same units, so we first convert the adult height to inches:

5 feet = 60 inches, so 5'6" = 66 inches

Then the ratio is:

$$\frac{66 \text{ inches}}{20 \text{ inches}} = \boxed{3.3}$$

What does it mean? An adult grows to a little more than 3 times longer than a baby.

[SSM] * [Life Sci] [Reasoning] P1.70 Recognize the principle. We can use the molecular mass of water to find the number of molecules in 1 gram of water, then use unit conversions to find the volume of each molecule.

Sketch the problem. No sketch needed.

a) Identify the relationships. The molar mass of water is about 18 g/mol.

a) Solve. First, we find the number of water molecules in 1.0 grams of water by dividing 1.0 grams by the molar mass and multiplying by Avogadro's number.

$$1.0 \text{ g} \times \frac{1 \text{ mol}}{18 \text{ g}} \times \frac{6.023 \times 10^{23} \text{ molecules}}{1 \text{ mol}} = 3.3 \times 10^{22} \text{ molecules in 1 g}$$

Since 1 g has a volume of 1 cm^3, the approximate volume of one water molecule is:

$$\frac{1.0 \text{ cm}^3}{3.3 \times 10^{22} \text{ molecules}} \boxed{\approx 3.0 \times 10^{-23} \text{ cm}^3/\text{molecule}}$$

b) Identify the relationships. For simplicity, we assume an average human is a box approximately 1.5 m tall, 0.5 m wide, and 0.2 m thick.

b) Solve. Using our box-like estimation for the body, the volume is:

$$V = (1.5 \text{ m}) \times (0.5 \text{ m}) \times (0.2 \text{ m})$$
$$V = 0.15 \text{ m}^3$$

Dividing this volume by the volume of a water molecule, an approximate value for the number of water molecules in a human body can be found.

$$\frac{V_{\text{human}}}{V_{\text{water molecule}}} = \frac{0.15 \text{ m}^3}{3 \times 10^{-23} \text{ cm}^3} \times \frac{(100 \text{ cm})^3}{\text{m}^3} \boxed{\approx 5 \times 10^{27} \text{ water molecules}}$$

What does it mean? These values show the exceptionally small size of water molecules, and therefore the extremely large number that are present in any typical quantity of water.

P1.71 Recognize the principle. We can set the length of the rope as the hypotenuse of a right triangle, with the distance along the ground and the height as the sides.

Sketch the problem. See Figure P1.71.

Identify the relationships. Let the height of the building be h, the length of rope equal l, and the distance from the tree to the building be d, then for this right triangle: $l^2 = h^2 + d^2$.

Solve. We can solve this expression for the height:

$$l^2 = h^2 + d^2$$

$$h = \sqrt{l^2 - d^2}$$

Then inserting values,

$$h = \sqrt{(55 \text{ m})^2 - (25 \text{ m})^2}$$

$$\boxed{h = 49 \text{ m}}$$

What does it mean? The building must be 49 m high—at approximately 4 m per story, this is a 12 story building!

*** P1.72 Recognize the principle.** The volume of a spherical object is related to its radius, so we can find a ratio of the volumes in terms of the given radii.

Sketch the problem. No sketch needed.

Identify the relationships. The volume of a sphere is $\frac{4}{3}\pi r^3$.

Solve. If the radius of the Earth is R_E, then the ratio of the volumes will be

$$\frac{V_{\text{Jupiter}}}{V_{\text{Earth}}} = \frac{\cancel{\frac{4}{3}}\cancel{\pi}(11\ \cancel{R_E})^3}{\cancel{\frac{4}{3}}\cancel{\pi}\cancel{R_E^3}} = (11)^3 = \boxed{1300}$$

What does it mean? About 1300 Earths could fit inside Jupiter's volume.

*** P1.73 Recognize the principle.** Appendix A gives planetary masses and radii. We can use these to calculate the density of each planet.

Sketch the problem. No sketch needed.

Identify the relationships. The density is the mass divided by the volume, and the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

The mass and radius of Earth, Mars, Jupiter, and Venus are in Appendix A.

Solve. Using our volume formula, we can then find each density:

$$\text{Earth:}\quad D_{\text{Earth}} = \frac{M_{\text{Earth}}}{V_{\text{Earch}}} = \frac{5.98\times10^{24}\ \text{kg}}{\frac{4}{3}\pi(6.38\times10^6\ \text{m})^3} = \boxed{5500\ \text{kg/m}^3}$$

$$\text{Mars:}\quad D_{\text{Mars}} = \frac{M_{\text{Mars}}}{V_{\text{Mars}}} = \frac{6.42\times10^{23}\ \text{kg}}{\frac{4}{3}\pi(3.4\times10^6\ \text{m}^3)} = \boxed{3900\ \text{kg/m}^3}$$

$$\text{Jupiter:}\quad D_{\text{Jupiter}} = \frac{M_{\text{Jupiter}}}{V_{\text{Jupiter}}} = \frac{1.90\times10^{27}\ \text{kg}}{\frac{4}{3}\pi(7.14\times10^7\ \text{m})^3} = \boxed{1200\ \text{kg/m}^3}$$

$$\text{Saturn:}\quad D_{\text{Saturn}} = \frac{M_{\text{Saturn}}}{V_{\text{Saturn}}} = \frac{5.69\times10^{26}\ \text{kg}}{\frac{4}{3}\pi(6.03\times10^7\ \text{m})^3} = \boxed{620\ \text{kg/m}^3}$$

What does it mean? Jupiter and Saturn are categorized as *gas giant* planets. This means they are mainly composed of gas and have no discernable hard surface. Their resulting density is much lower than the inner planets, which are more rocky and solid. For comparison, the density of water is 1000 kg/m^3. Since Saturn's density is lower than water, it would indeed float if there were a lake large enough!

P1.74 Recognize the principle. The maximum and minimum magnitudes occur when the two vectors lie along the same line—parallel or anti-parallel.

Sketch the problem. No sketch needed.

a) Identify the relationships. The magnitude would be maximized if the two vectors pointed in the same direction, and would then be the scalar sum of the magnitudes. The two vectors would be minimized if they pointed in exactly opposite directions. In this case, the vector sum would then be the scalar difference of the two magnitudes.

a) Solve. The maximum magnitude is therefore:

$$12 \text{ m} + 17 \text{ m} = \boxed{29 \text{ m}}$$

While the minimum magnitude is:

$$17 \text{ m} - 12 \text{ m} = \boxed{5 \text{ m}}$$

b) Identify the relationships. The magnitude would still be maximized if the two vectors pointed in the same direction, and minimized when the vectors are opposite. Since $-\vec{B}$ is just $\vec{B}$ pointing in the opposite direction, these two conditions occur again when the vectors are parallel and anti-parallel.

b) Solve. The maximum magnitude is therefore again $\boxed{29 \text{ m}}$, while the minimum magnitude is again $\boxed{5 \text{ m}}$.

What does it mean? Because negative vectors are anti-parallel, these two combinations have the same minimum and maximum.

P1.75 Recognize the principle. After estimating typical highway speed, we can find the distance traveled in 30 min.

Sketch the problem. No sketch needed.

Identify the relationships. Typical expressway speed is about 60 mi/h, or about 30 m/s.

Solve. At 60 mi/h, in 30 min (1/2 hour) we will have traveled:

$$60 \text{ mi/h } (0.5 \text{ h}) = \boxed{30 \text{ mi}}$$

At 30 m/s, in 30 min (1800 s) we will have traveled:

$$30 \text{ m/s } (1800 \text{ s}) = 54{,}000 \text{ m} = 54 \text{ km} \approx \boxed{50 \text{ km}}$$

What does it mean? Note that these speed estimates are only given to one significant figure, so the distances can only be expected to be accurate to one significant figure as well. Even though these speeds are different—they give the same distance to this accuracy.