$$\frac{E_1 - E_2}{E_1 + E_2} = 0.662 .$$

2.4. The Lorentz factor for E_μ=5 GeV is $\gamma = E_\mu / m_\mu = 47$. In its rest frame the distance of the Earth surface is $l_0 = l/\gamma = 630$ m. For E_μ=5 TeV, the distance of the Earth is $l_0 = l/\gamma = 0.63$ m. The first muon travels in a lifetime $\gamma\beta c\tau \simeq \gamma c\tau = 28$ km, the second would travel 28 000 km if it did not hit the surface first.

2.5. The Lorentz factor for E_π=5 GeV is $\gamma = E_\pi / m_\pi = 36$. In its rest frame it sees the Earth's surface at the distance $l_0 = l/\gamma = 830$ m. In a lifetime it travels $\gamma c\tau = 280$ m. We see that only a few such pions survive. To find them we must go to high altitude.

2.6. The momenta of the electrons are $p = 0.3B\rho$=12 MeV. The gamma energy is E_γ=24 MeV.

2.8. Since the decay is isotropic, the probability of observing a photon is a constant $P(\cos\theta^*, \phi^*) = K$. We determine K by imposing that the probability of observing a photon at any angle is 2, i. e. the number of photons.

We have $2 = \int K \sin\theta^* d\theta^* d\phi = \int_0^{2\pi} d\phi \int_0^{\pi} K d(\cos\theta^*) = K 4\pi$. Hence $K = 1/2\pi$ and $P(\cos\theta^*, \phi^*) = 1/2\pi$.

The distribution is isotropic in azimuth in L too. To have the dependence of θ, that is given by $P(\cos\theta) \equiv \frac{dN}{d\cos\theta} = \frac{dN}{d\cos\theta^*}\frac{d\cos\theta^*}{d\cos\theta}$, we must calculate the 'Jacobian' $J = \frac{d\cos\theta^*}{d\cos\theta}$.

Calling β and γ the Lorentz factors of the transformation and taking into account that $p^* = E^*$, we have

$$p\cos\theta = \gamma\left(p^*\cos\theta^* + \beta E^*\right) = \gamma p^*\left(\cos\theta^* + \beta\right)$$

$$E = p = \gamma\left(E^* + \beta p^*\cos\theta^*\right) = \gamma p^*\left(1 + \beta\cos\theta^*\right).$$

We differentiate the first and third members of these relationships, taking into account that p^* is a constant. We obtain

$$dp \times \cos\theta + p \times d(\cos\theta) = \gamma p^* d(\cos\theta^*) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*}\cos\theta + p\frac{d\cos\theta}{d\cos\theta^*} = \gamma p^* .$$

$$dp = \gamma\beta p^* d(\cos\theta^*) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*} = \gamma\beta p^*$$

and $J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma\frac{p^*}{p}(1 - \beta\cos\theta)$.

The inverse transformation is $E^* = \gamma(E - \beta p\cos\theta)$, i. e. $p^* = \gamma p(1 - \beta\cos\theta)$, giving

$$J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma^2(1 - \beta\cos\theta)^2 .$$

Finally we obtain $P(\cos\theta) \equiv \frac{dN}{d\cos\theta} = \frac{1}{2\pi}\gamma^{-2}(1 - \beta\cos\theta)^{-2}$

2.10. $\mu_e / \mu_\mu = m_\mu / m_e = 207$; $\mu_e / \mu_\tau = m_\tau / m_e = 3477$.

2.11. The energy needed to produce an antiproton is minimum when the Fermi motion is opposite to the beam direction. If E_f is the total energy of the target proton and p_f its momentum, the threshold condition is $(E_p+E_f)^2-(p_p-p_f)^2=(4m_p)^2$. From this we have $E_pE_f + p_pp_f = 7m_p^2$. We simplify by setting $p_p \simeq E_p$ obtaining

$$E_p = \frac{7m_p^2}{E_f + p_f} \simeq \frac{7m_p^2}{m_p + p_f} \simeq 7m_p\left(1 - \frac{p_f}{m_p}\right) = 5.5 \text{ GeV}.$$

This value should be compared to E_p=6.6 GeV on free protons.

2.12. By differentiating (1.79) we obtain $\Delta\theta = 0.3BL\Delta p / p^2$. The slit of opening d at the distance l defines the angle within $\Delta\theta = d / l$. The requested distance is then $l = \dfrac{d \times p}{0.3BL\Delta p / p} = 3.3$ m.

2.13. Considering the beam energy and the event topology, the event is probably an associate production of a K^0 and a Λ. Consequently the V^0 may be one of these two particles. The negative track is in both cases a π, while the positive track may be a π or a proton. We need to measure the mass of the V. With the given data we start by calculating the Cartesian components of the momenta

$p_x^- = 121 \times \sin(-18.2°)\cos 15° = -36.5$ MeV; $p_y^- = 121 \times \sin(-18.2°)\sin 15° = -9.8$ MeV;

$p_z^- = 121 \times \cos(-18.2°) = 115$ MeV.

$p_x^+ = 1900 \times \sin(20.2°)\cos(-15°) = 633.7$ MeV; $p_y^- = 1900 \times \sin(20.2°)\sin(-15°) = -169.8$ MeV;

$p_z^- = 1900 \times \cos(20.2°) = 1783.1$ MeV.

Summing the components, we obtain the momentum of the V, i. e. $p = 1998$ MeV.

The energy of the negative pion is $E^- = \sqrt{(p^-)^2 + m_\pi^2} = 185$ MeV. If the positive track is a π its energy is $E_\pi^+ = \sqrt{(p^+)^2 + m_\pi^2} = 1905$ MeV, while if it is a proton its energy is $E_p^+ = 2119$ MeV. The energy of the V is $E_\pi^V = 2090$ MeV in the first case, $E_p^V = 2304$ MeV in the second case. The mass of the V is consequently $m_\pi^V = \sqrt{E_\pi^{V2} - p^2} = 620$ MeV in the first hypothesis, $m_p^V = 1150$ MeV in the second. Within the ±4% uncertainty, the first hypothesis is incompatible with any known particle, while the second is compatible with the particle being a Λ.

2.14.

1. The CM energy squared is $s = (E_\nu + m_n)^2 - p_\nu^2 = m_n^2 + 2m_nE_\nu$. The threshold condition is

$$s = (m_e + m_p)^2 = m_p^2 + m_e^2 + 2m_em_p.$$

Hence, the threshold condition is $E_\nu = \dfrac{(m_e + m_p)^2 - m_n^2}{2m_n} < 0$, meaning that there is no threshold, the reaction proceeds also at zero neutrino energy.

2. The threshold condition is $s=\left(m_\mu+m_p\right)^2=m_p^2+m_\mu^2+2m_\mu m_p$. The threshold energy is

$$E_\nu=\frac{\left(m_\mu+m_p\right)^2-m_n^2}{2m_n}=\frac{(105.7+938.3)^2-939.6^2}{2\times 939.6}=110\text{ MeV}$$

3. The threshold energy is

$$E_\nu=\frac{\left(m_\tau+m_p\right)^2-m_n^2}{2m_n}=\frac{(1777+938.3)^2-939.6^2}{2\times 939.6}=3.45\text{ GeV}.$$

2.15. We first find an expression valid in both cases. Call $E_{\gamma1}$ and $p_{\gamma1}=E_{\gamma1}$ the energy and momentum of the initial photon and $E_{\gamma2}$ and $p_{\gamma2}=E_{\gamma2}$ those of the final one. Similarly E_{e1}, p_{p1} and E_{e2}, p_{p2} for the electron.

The initial values of energies and momenta are given; hence the total energy and momentum and CM energy squared

$E_T=E_{e1}+E_{\gamma1}$ $\qquad p_T=p_{e1}-E_{\gamma1}$ $\qquad s=E_T^2-p_T^2$.

Energy conservation gives $E_T=E_{e2}+E_{\gamma2}$ $\qquad p_T=p_{e2}-E_{\gamma2}$.

We can eliminate the final energy and momentum of the electron by imposing $E_{e2}^2-p_{e2}^2=m_e^2$.

$E_{e2}=E_T-E_{\gamma2}$ $\qquad p_{e2}=p_T+E_{\gamma2}$. Hence: $\left(E_T-E_{\gamma2}\right)^2-\left(p_T+E_{\gamma2}\right)^2=m_e^2$. Solving for $E_{\gamma2}$ we have $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}$.

1. We have, in MeV: $E_{\gamma1}=0.511$, E_{e1}=0.511, p_{e1}=0.

E_T=1.02, p_T=0.511, s=0.78 and $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{0.78-0.511^2}{2(1.02+0.511)}=0.170\text{ MeV}$.

2. We have $E_{\gamma1}=0.511$, E_{e1}=1.02, $p_{e1}=\sqrt{1.02^2-0.511^2}=0.88$.

E_T=1.53, p_T=0.511, s=2.08 and $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{2.08-0.511^2}{2(1.53+0.511)}=0.446\text{ MeV}$.

2.16. The LASER photon energy is $E_{\gamma i}=\dfrac{h}{\lambda}=\dfrac{1240\text{ eV nm}}{694\text{ nm}}=1.79\text{ eV}$.

The electron initial momentum (we shall need its difference from energy) is

$$p_{ei}=\sqrt{E_{ei}^2-m_e^2}\simeq E_{ei}-\frac{m_e^2}{2E_{ei}}$$

The total energy and momentum are $E_T=E_{ei}+E_{\gamma i}$ $\qquad p_T=p_{ei}-E_{\gamma i}$

Energy conservation gives $E_T=E_{ef}+E_{\gamma f}$ $\qquad p_T=E_{\gamma f}-p_{ef}$

We can eliminate the final energy and momentum of the electron by imposing $E_{ef}^2-p_{ef}^2=m_e^2$.

$E_{ef}=E_T-E_{\gamma f}$ $\qquad p_{ef}=E_{\gamma f}-p_T$. Hence: $\left(E_T-E_{\gamma f}\right)^2-\left(E_{\gamma f}-p_T\right)^2=m_e^2$. Solving for $E_{\gamma f}$ we have $E_{\gamma f}=\dfrac{s-m_e^2}{2\left(E_T-p_T\right)}$.

$$E_T - p_T = \left(E_{ei} + E_{\gamma i}\right) - \left(p_{ei} - E_{\gamma i}\right) \simeq \frac{m_e^2}{2E_{ei}} + 2E_{\gamma i} =$$

$$= \frac{0.5^2 \times 10^{-6}}{2 \times 20} + 2 \times 1.79 \times 10^{-9} = \left(6.25 + 3.58\right)10^{-9} \text{ GeV} = 9.83 \text{ eV}$$

$s = \left(E_{\gamma i} + E_{ei}\right)^2 - \left(E_{\gamma i} - p_{ei}\right)^2 = m_e^2 + 4E_{\gamma i}E_{ei}$. Hence

$s - m_e^2 = 4E_{\gamma i}E_{ei} = 4 \times 1.79 \times 20 \times 10^9 \text{ eV}^2 = 14.3 \times 10^{10} \text{ eV}^2$, and

$$E_{\gamma f} = \frac{s - m_e^2}{2\left(E_T - p_T\right)} = \frac{14.3 \times 10^{10}}{2 \times 9.83} = 7.3 \text{ GeV}$$

2.17. The kinetic energy is $T = \sqrt{p^2 + m^2} - m$

For a proton we have $T = \sqrt{23^2 + 938.3^2} - 938.3 = 280$ keV

For a positron we have $T = \sqrt{23^2 + 0.51^2} - 0.51 = 22.5$ MeV

2.18. $E \simeq p = 0.3BR = 0.3 \times 0.3 \times 0.14 = 12.6$ MeV.

2.19. In problem 2.1 we already calculated the CM momentum $p^* = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8$ MeV. The CM muon energy is $E_\mu^* = \sqrt{p^{*2} + m_\mu^2} = 110$ MeV. For the Lorentz transformation to the L frame we have $\beta \simeq 1$ and $\gamma = \dfrac{E_\pi}{m_\pi} = \dfrac{200}{0.14} == 1400$. The maximum and minimum muon