Chapter 2

1. (a) The tenth datum is unknown, so the population mean can't be computed. The nine known values are not a random sample. Since they are the nine smallest values, the sample mean is below the population mean by an unknown amount.

(b) The population median *can* be computed because the rank of the missing value is known. The median (the expected survival time) is $\mu_{½}$ = (2.6 + 2.9)/2 = 2.75.

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2. (a) $\bar{x} = x_{½} = 5$. This is a sample, so

$$s^2 = \frac{1}{10}\{2(25+16+9+4+1)\} = 11, \text{ and } s = 3.31.$$

(b) $\bar{x} = x_{½} = 5$.

$$s^2 = \frac{2}{24}\{16+2\cdot 9+3\cdot 4+4\cdot 1\} = 4.167, \text{ and } s = 2.04.$$

3. (a) is a uniform distribution, with

$$P(i) = \frac{1}{11}, \text{ for integers } 0 \le i \le 10. \text{ (else } P(i) = 0)$$

(b) is a "triangular" distribution with

$$P(i) = \frac{1}{25}\left[5 - |x-5|\right], \text{ for integers } 0 \le i \le 10. \text{ (else } P(i) = 0)$$

4. (a) In the absence of further information, this appears to be a Poisson process, with a mean rate of 1.7 events per year per square km or $r = 1.7 \times 10^{-4}$ events per yr per 100 m^2 roof.

(b) The probability of zero events per year on the roof is given by the Poisson distribution:

$$P = P_P(0, r) = \frac{1}{1}\exp(-1.7\times 10^{-4}) = .99983$$

(c) The probability of more than one penetration in 40 years will be:

$$P = 1 - P_P(0, 40r) - P(1, 40r) = 1 - \exp(-6.8\times 10^{-3}) - \frac{6.8\times 10^{-3}}{1}\exp(-6.8\times 10^{-3})$$

$$= 1 - 0.99322(1 + .0068) = 2.6\times 10^{-5}$$

5. Because the integral of this function between the limits of zero and infinity diverges, it cannot be interpreted as a valid probability function. (The integral of a normalized continuous probability function equals unity.) This distribution with $\gamma > 0$ becomes arbitrarily large as *a* approaches zero. A student might argue that the mean will be driven this limit, and so in a sense this represents a "typical" member of the population. But the function also permits indefinitely large values *a,* so in another sense a mean of zero does not represent the population at all. Imposing upper and lower limits on the permitted values of the random variable will allow a computation of a valid mean and standard deviation. (If one investigates the mathematical definitions of the mean and standard deviations of continuous distributions even slightly, it is easy to show that imposing only a lower limit on a —and no upper limit — leads to a well-defined mean if $\gamma > 2$, and a well-defined variance if $\gamma > 3$.)

6. Follow the reasoning in the example on page 48. Transform the variable q into the standard normal variable z:

$$z = (q - 0.8)/0.6, \quad z_{low} = -1.50, \quad z_{high} = -0.833$$

Now compute the probability that a single trial will result in the discovery of an earth-like planet. Use the tabulation of the standard normal distribution $P_{SN}(z)=G(z)$ and its integral given in *Appendix C:*

$$\text{Prob}(\textit{earthlike}) = \int_{-1.5}^{-0.833} G(z)\,dz = P(1.5) - P(.833) = .933 - .796 = .137$$

So in 500 trials, on should expect 68.5 earthlike planets discovered.

7. Counting photons is a Poisson process. The fractional uncertainty in counting N events (equation 2.15) is

$$\text{(a) } 0.05 = \frac{1}{\sqrt{N}}, \text{ so } N\text{=400. (b) } N\text{=40,000.}$$

9. The standard deviation of the sample of four measurements is 13.6 km/s, which implies an uncertainty in the mean of $13.6/\sqrt{4} = 6.8$ km/s. If the astronomer measures N additional stars, to reach an uncertainty of 2.0 km/s, then:

$$2.0 = \frac{13.6}{\sqrt{N+4}}$$

and therefore N=43.

8. Assume she spends equal amounts of time measuring the target and the background. The counts are $N_* = N_b = N_{meas}/2$. The uncertainty in the star brightness can be computed from the variance:

$$\sigma_*^2 = \sigma_{meas}^2 + \sigma_b^2 = 3N_*$$

So the relative uncertainty is

$$\frac{\sigma_*}{N_*} = \frac{\sqrt{3}}{\sqrt{N_*}} = \frac{\sqrt{6}}{\sqrt{N_{meas}}}$$

and (a) for 5% uncertainty $N_{meas} = 2400$, (b) $N_{meas} = 240{,}000$.

9. The uncertainty in the mean is related to the scatter in the population and the number of samples, N, by :

$$\sigma_{mean} = \frac{\sigma}{\sqrt{N}}$$

We estimate the scatter in the population from the standard deviation (N-1 weighting) of the four measurements as 13.6 km/s. The above equation then implies a sample size of N = 46 would yield an uncertainty in the mean of 2 km/s.

10. Because we know that the fluctuations in the value of *r* in 10 second exposures are normally distributed with scatter of .05 mV, we can use the Central Limit Theorem to conclude that uncertainty of *r* in 100-s exposures will be reduced by a factor of $1/\sqrt{10}$. We will assume that this detector accumulates voltage in a linear fashion as exposure time increases, so the values for each of the Ns in equation 2.39 on the longer exposures will each increase by a factor of 10. Making these substitutions in equation 2.39:

$$\sigma_*^2 = 1660 + 850 + \left(\frac{.05}{5}\right)^2 \left(\frac{1}{10}\right)\left[\left(2.7556 + .7225\right)\times 10^6\right] = 2510 + 3.5 = 2513$$

$$\frac{N_*}{\sigma_*} = \frac{810}{50.1} = 16.2$$

11. (a) A straight-forward average of the five trials and a treatment of the five results as random variables gives: $\bar{x} = 21.2,\ \ \sigma_{mean} = 37.2/\sqrt{5} = 16.6$

(b) An alternative approach is to use that fact that these are (presumably) counts of photons, so an arrival 106 photons in five seconds suggests (Poisson distribution) an uncertainty in the average of $\sqrt{106}/5 = 2.06$

The divergence of these two methods suggests that something may be amiss, as does the fact that the mean and median values are quite different. The fact that the result of trial 2 differs from the mean by more than almost 2σ in method (a) is a little suspect. However, if we really believe that we are counting photons (as in method b) then the second trial is 29σ larger than the mean, and it is highly unlikely that we are correct in including it as a valid measurement of the same process as the other trials. Often a single deviant event like trial number 2 will alert us to a systematic error.

12. As in the Chapter 1 problem, it is important that the aperture used for each star be identical in size; i.e. use the same total number of pixels for each star image, and centering of the apertures will require the use of fractional pixels. Repeating the measurements made in the solutions for Chapter 1:

34	16	26	33	37	22	25	25	29	19	28	25

22	20	44	34	22	26	14	30	30	20	19	17
31	70	98	66	37	25	35	36	39	39	23	20
34	99	229	107	38	28	46	102	159	93	37	22
33	67	103	67	36	32	69	240	393	248	69	30
22	33	34	29	36	24	65	241	363	244	68	24
28	22	17	16	32	24	46	85	157	84	42	22
18	25	27	26	17	18	30	29	35	24	30	27
32	23	16	29	25	24	30	28	20	35	22	23
28	28	28	24	26	26	17	19	30	35	30	26

Background: the 18 pixels 3 X 6 box in the lower left corner has a total count of 440 and a mean value of 24.44. Uncertainty in the total count is $\sqrt{440} = 20.98$ and uncertainty in the average background is $\sqrt{440}/18 = 1.165$. Note that the standard deviation of the sample of 18 pixels is s =4.49, which implies an uncertainty in their mean value of 1.06. This is consistent with the uncertainty computed under the assumption of Poisson statistics.

Brighter star: This star is very symmetric around a point midway between the pixels with values 393 and 363. Add the values of the 16 shaded pixels: Sum of star and background = 2680, so the total number of counts for the brighter star alone is F_{STD} =2680 – (16 x 24.44) = 2288.9 with an uncertainty given by:

$$\sigma_{std}^2 = 2680 + \left(\frac{16}{18}\right)^2 440 = 3027.7$$

so $\sigma_{std} = 55.0$.

Fainter star: Also very symmetric, but around the center of the pixel with value 229. Take the 21 pixels indicated, but give 13 pixels full weight (dark shading) and give the 8 outer pixels a weight of 3/8. Thus the effective number of pixels in both star apertures are the same. For the fainter star alone the total counts are: F_f = 1056 + ((3/8) x 253) - (16 x 24.44) = 759.8 and the uncertainty is:

$$\sigma_f^2 = 1056 + \left(\frac{3}{8}\right)^2 253 + \left(\frac{16}{18}\right) 440 = 1439.25$$

so $\sigma_f = 37.9$

Magnitude: If the magnitude of the brighter star is 9.000, the magnitude of the follows from the ratio $R = F_f / F_{STD}$ = 759.8/2288.9 = 0.332:

$$m_f = -2.5\log(759.8/2288.9) + 9.0 = 10.197$$

To get the uncertainty in the magnitude of the fainter star, first note that for the uncertainty in the ratio, *R* (for a product or ratio, the relative variances add), we have

$$\frac{\sigma^2_{F_f/F_{STD}}}{\left(F_f/F_{STD}\right)^2} = \frac{\sigma^2_{F_f/F_{STD}}}{\left(0.332\right)^2} = \left(\frac{37.9}{759.8}\right)^2 + \left(\frac{55}{2288.9}\right)^2 = (0.055)^2$$

Then either use the result from Problem 1.13, or note, since

$$m_f = -2.5\log(F_f / F_{STD}) + 9.0 = -2.5\log(R) + 9.0$$

$$\sigma_m^2 = (-2.5)^2\left(\frac{\sigma_R \log e}{R}\right)^2 = (1.086)^2\left(\frac{\sigma_R}{R}\right)^2 = (0.060)^2$$

This assumes that the cataloged standard magnitude of the brighter star is perfectly known. (Not always the case!). So m_f= 10.197 ± 0.060.