Chapter 2 Problems

(2-1) Use the conversion formula $\overline{V} = V\angle\phi$

(a) $\overline{V_1} = 100\angle -30^\circ$ V

(b) $\overline{V_2} = 100\angle 30^\circ$ V

■

(2-2) We have,

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = \sqrt{2}V cos(\omega t).$$

Taking Laplace Transform we get,

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = \sqrt{2}V \times \frac{s}{s^2+\omega^2}$$
$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \sqrt{2}V \times \frac{s}{s^2+\omega^2}$$

$$\therefore I(s) = \frac{\sqrt{2}V \times Cs^2}{(s^2LC + RCs + 1)(s^2+\omega^2)}$$
$$= \frac{\sqrt{2}\frac{V}{L}s^2}{(s^2 + (\frac{R}{L})s + \frac{1}{LC})(s^2+\omega^2)}$$

Solving the above form by partial fractions and then taking the inverse Laplace transform we get

$$i(t) = 0.0368cos(\omega t) + 0.466sin(\omega t) = 0.468cos(\omega t - 85.48)A$$

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(2-3)

$$\begin{aligned}
Z_L &= |Z_L|\angle\phi \\
|Z_L| &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\
\phi &= \tan^{-1}\left[\frac{\omega L - \frac{1}{\omega C}}{R}\right] \\
&= 85.48^\circ
\end{aligned}$$

$$\begin{aligned}
\overline{I} &= \frac{\overline{V}}{|Z_L|}\angle - \phi \\
&= \frac{120\angle 0}{1.5^2 + \left(377 \times 20 \times 10^{-3} - \frac{1}{377\times 100\times 10^{-6}}\right)^2}\angle - 85.48^\circ \\
&= 0.33\angle - 85.48^\circ \\
\therefore i(t) &= 0.33\sqrt{2}cos(\omega t - 85.48)A \\
&= 0.467cos(\omega t - 85.48)\ A
\end{aligned}$$

■

(2-4)

$$\begin{aligned}
\overline{V} &= 90\angle 30^\circ \\
\overline{I} &= 5\angle 15^\circ \\
\text{If }\ \overline{V}_{new} &= 100\angle 0^\circ
\end{aligned}$$

$$\overline{I}_{new} = (\frac{\overline{V}_{new}}{\overline{V}})\overline{I} = 5.56\angle - 15^\circ$$

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(2-5)

$$\overline{V} = 100\angle 0^\circ V$$
$$Z = j0.1 + \frac{(2)(-j5)}{2-j5}$$
$$= 1.82\angle -18.9^\circ A$$
$$\overline{I} = \frac{100\angle 0^\circ}{1.82\angle -18.9^\circ}$$
$$= 54.95\angle 18.9^\circ A$$

$$P = VI\cos\phi = 5198.74W$$
$$Q = VI\sin\phi = 1779.93VAR$$

As the current leads the voltage ,so the net load impedance is capacitive hence it draws negative reactive power
So net $Q = -1779.93VAR$.

$$p.f. = \cos\phi = 0.946(Leading)$$

Let $X_1 = 0.1$ and $X_2 = 5$. Then,

$$Q_{X_1} = I^2 X_1 = 301.95VAR$$
$$Q_{X_2} = I_2^2 X_2$$

where I_2 is the current through the capacitance.

$$\overline{I_2} = \overline{I} \times \frac{2}{2-j5} = 20.41\angle 87.1^\circ A$$
$$Q_{X_2} = 5 \times (20.41)^2 = 2082.84VAR$$

We further find that,

$$Q_{X_1} + (-Q_{X_2}) = -1780.57VAR \approx Q$$

Q_{X_2} takes a negative sign as the capacitor supplies reactive power.

$$\therefore Q = \sum_k I_k^2 X_k$$

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(2-6)

$$|I_s| = \frac{V}{\sqrt{R_s^2 + X_s^2}}$$

For the series circuit,the real power and the reactive power supplied to the load are

$$P_s = |I_s|^2 R_s = \frac{V^2}{R_s^2 + X_s^2} R_s$$
$$Q_s = |I_s|^2 X_s = \frac{V^2}{R_s^2 + X_s^2} X_s$$

For the parallel circuit, since the real and reactive powers of both circuits will be the same, we have

$$R_p = \frac{V^2}{P} = \frac{R_s^2 + X_s^2}{R_s}$$
$$X_p = \frac{V^2}{Q} = \frac{R_s^2 + X_s^2}{X_s}$$

■

(2-7) As calculated in Example 2-2,

$$Z_{in} = 6.775\angle 29.03^\circ$$
$$= 5.924 + j3.289$$

Then, we will have,

$$R_P = \frac{R_S^2 + X_S^2}{R_S} = \frac{5.924^2 + 3.289^2}{5.924} = 7.75\Omega$$
$$X_P = \frac{R_S^2 + X_S^2}{X_S} = \frac{5.924^2 + 3.289^2}{3.289} = 13.96\Omega$$

■

(2-8) In Example 2-2

$$\overline{I_1} = 17.172\angle -29.03^\circ A$$
$$\overline{I_{Rp}} = \frac{jX_P}{R_P + jX_P}\overline{I_1} = \frac{j13.96}{7.75 + j13.96} \times 17.172\angle -29.03 = 15.483\angle 0^\circ A$$
$$\overline{I_{Xp}} = \frac{R_P}{R_P + jX_P}\overline{I_1} = \frac{7.75}{7.75 + j13.96} \times 17.172\angle -29.03 = 8.595\angle 89.99^\circ A$$

$$\therefore P = I_{Rp}^2 R_P = 15.483^2 \times 7.75 = 1857.86W$$
$$Q = I_{Xp}^2 X_P = 8.595^2 \times 13.96 = 1031.28VAR$$

Thus, P and Q are the same as calculated in Ex. 2-3. Hence the R_P and X_P calculations are correct. ■

(2-9) Referring to Figure 1,

$$\overline{V} = V\angle 0^\circ,\ I_{net} = I\angle\phi,$$
$$Z_{net} = \frac{\overline{V}}{I} = |Z|\angle-\phi, cos\phi = 0.9$$
$$sin(\phi) = \sqrt{1-cos^2\phi} = 0.436$$
$$\therefore Q_{net} = |P_L tan\phi| = 900.29VAR$$

$$Q_C = Q_{net} + Q_L = 1931.59VAR$$
$$X_C = \frac{V^2}{Q_C}$$
$$\therefore \frac{1}{(2\pi f)C} = \frac{V^2}{Q_C}$$
$$C = \frac{Q_C}{(2\pi f)V^2} = 356\mu F$$

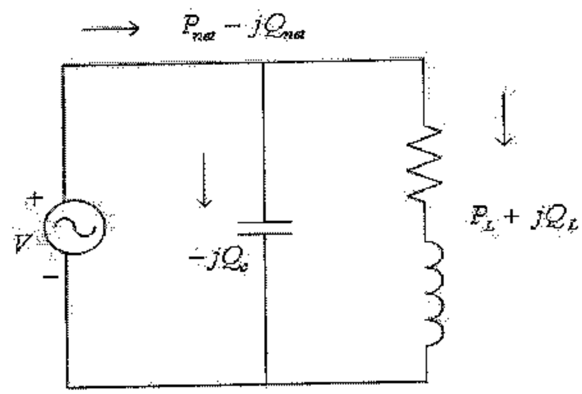

Figure 1:

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(2-10) As shown in Figure 2,

$$Q_{net} = 5000\,tan[cos^{-1}(0.95)] = 1643.42VAR$$
$$Q_{net} = Q_L - Q_C$$
$$Q_C = Q_L - Q_{net} = 2106.58VAR$$
$$X_C = \frac{V^2}{Q_C} = 6.835\Omega$$
$$\therefore C = \frac{1}{(2\pi f)X_C} = 388\mu F$$

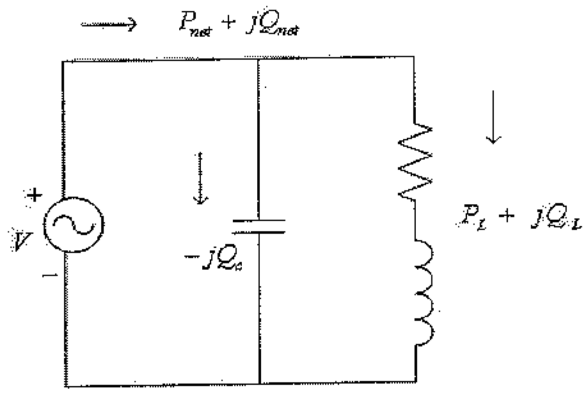

Figure 2:

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(2-11) Note that

$$\overline{V}_a = 100\angle 30^\circ V$$

Thus,

$$\begin{aligned}
\overline{V}_b &= 100\angle -90^\circ V, \\
\overline{V}_c &= 100\angle 150^\circ V \\
v_a(t) &= \sqrt{2} \times 100cos(\omega t + 30) = 141.4cos(\omega t + 30) \\
v_b(t) &= \sqrt{2} \times 100cos(\omega t + 30 - 120) = 141.4cos(\omega t - 90) \\
v_c(t) &= \sqrt{2} \times 100cos(\omega t + 30 + 120) = 141.4cos(\omega t + 150)
\end{aligned}$$

$$\begin{aligned}
\overline{V_{ab}} &= \overline{V}_a - \overline{V}_b = 173.2\angle 60^\circ \\
v_{ab}(t) &= \sqrt{2} \times 173.2cos(\omega t + 60) = 244.9cos(\omega t + 60)
\end{aligned}$$

■

(2-12) Given a Y-connected inductive load,

$$\begin{aligned}
3V_a I_a cos\phi &= 10000 \\
\therefore I_a &= \frac{10000}{3 \times 120 \times 0.9} \\
&= 30.86A
\end{aligned}$$

Assuming $V_a = 120\angle 0°$

$$Z = |Z|\angle cos^{-1}(0.9) = |Z|\angle 25.84\Omega$$

$$\therefore |Z| = \frac{V_a}{I_a} = 3.89\Omega$$

$$\overline{I_a} = \frac{\overline{V_a}}{Z} = \frac{120\angle -25.84°}{3.89} = 30.86\angle -25.84° A$$

$$\overline{I_b} = 3.86\angle -25.84 - 120°3.89 = 30.86\angle -145.84° A$$

$$\overline{I_c} = 3.86\angle -25.84 + 120°3.89 = 30.86\angle 94.16° A$$

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(2-13) Same as above , only magnitude of load impedance changes

$$\therefore |Z| = 3.89 \times 3 = 11.67\Omega$$

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(2-14)

$$\overline{V_{aA}} = (Z_{self} - Z_{mutual})\overline{I_a} = (0.3 + j1) \times 10\angle -30°$$
$$= 10.44\angle 43.3° V$$

$$\overline{V_{aA}} = \overline{V_a} - \overline{V_A}$$
$$= 1000\angle 0° - \overline{V_A}$$
$$\therefore \overline{V_A} = 1000\angle 0° - 10.44\angle 43.3°$$
$$= 992.43\angle -0.413° V$$

■

(2-15) In a balanced circuit,

$$\overline{V_a} + \overline{V_b} + \overline{V_c} = 0$$
$$\overline{V_b} + \overline{V_c} = -\overline{V_a}$$
$$\overline{I_a} = \frac{\overline{V_a}}{-jX_{c2}} + \frac{\overline{V_a} - \overline{V_b}}{-jX_{c1}} + \frac{\overline{V_a} - \overline{V_c}}{-jX_{c1}}$$
$$= \frac{3\overline{V_a}}{-jX_{c1}} + \frac{\overline{V_a}}{-jX_{c2}}$$
$$= \frac{\overline{V_a}}{Z_{eq}} \text{ where } Z_{eq} = \frac{1}{\frac{3}{-jX_{c1}} + \frac{1}{-jX_{c2}}}$$

■

(2-16)

$$P_R = \frac{V_S V_R}{X} sin\delta$$
$$sin\delta = \frac{1000 \times 1.5}{100 \times 95} = 0.158$$
$$\therefore \delta = 9.091^\circ$$
$$\overline{I} = \frac{\overline{V_S} - \overline{V_R}}{jX} = \frac{100\angle\delta - 95\angle 0}{j1.5}$$
$$= \frac{100\angle 9.091 - 95\angle 0}{1.5\angle 90}$$
$$= 10.83\angle - 13.39^\circ$$

From Eq 2-46

$$Q_R = \frac{V_S V_R}{X} cos\delta - \frac{V_R^2}{X} = 237.11VAR$$

As a check

$$P_R + jQ_R = \overline{V_R}\,\overline{I}^* = 1028.85\angle 13.39^\circ$$
$$\therefore Q_R = 238.3VAR$$

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(2-17)

$$P = |V_S||I|cos\phi = 100|I|cos\phi$$
$$\overline{V_R} = \overline{V_S} - jX\overline{I}$$

Solving the above two equations we get an expression for $\frac{|V_S|}{|V_R|}$ and a corresponding plot in Matlab as shown above

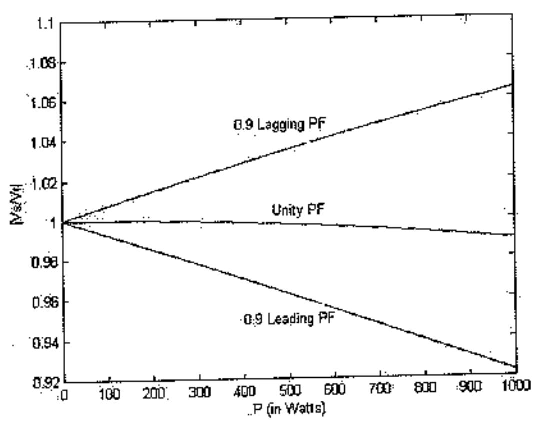

Figure 3:

(2-18) $V_{LL,base} = 208$ V, $V_{ph,base} = \frac{208}{\sqrt{3}} = 120$ V, $P_{ph,base} = 1200$ W

$$I_{base} = \frac{P_{ph,base}}{V_{ph,base}} = 10A$$
$$Z_{base} = \frac{V_{ph,base}}{I_{base}} = 12\Omega$$

With these base values, we calculate the per unit values as,

$$\overline{V_{ph}} = \frac{120\angle 0}{120} = 1.0\angle 0^\circ pu$$
$$\overline{I_L} = \frac{12\angle -36.87^\circ}{10} = 1.2\angle -36.87^\circ pu$$
$$\overline{Z_L} = \frac{10\angle 36.87^\circ}{12} = 0.83\angle 36.87^\circ pu$$
$$P_L = \frac{1152}{1200} = 0.96pu$$
$$Q_L = \frac{864}{1200} = 0.72pu$$

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(2-19) $V_{LL,base} = 240$ V, $V_{ph,base} = 138.57$ V, $P_{ph,base} = 1800$ W

$$I_{base} = \frac{P_{ph,base}}{V_{ph,base}} = 12.99A$$
$$Z_{base} = \frac{V_{ph,base}}{I_{base}} = 10.67\Omega$$

With these base values, we calculate the per unit values as,

$$\overline{V_{ph}} = 0.867\angle 0^\circ pu$$
$$\overline{I_L} = \frac{138.57}{|Z_L|}\angle -36.87^\circ = 13.86\angle -36.87^\circ$$
$$\overline{I} = \frac{13.86\angle 36.87^\circ}{12.99} = 1.067\angle -36.87^\circ pu$$
$$\overline{Z_L} = \frac{10\angle 36.87^\circ}{10.67} = 0.937\angle 36.87^\circ pu$$
$$P_L = \frac{V^2}{|Z_L|}cos\,\phi = 1536.13W = 0.853pu$$
$$Q_L = \frac{V^2}{|Z_L|}sin\,\phi = 1152.1VAR = 0.64pu$$

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(2-20) $V_{LL,base} = 240$ V, $V_{ph,base} = 138.57$ V, $P_{ph,base} = 1200$ W

$$I_{base} = \frac{P_{ph,base}}{V_{ph,base}} = 8.66A$$
$$Z_{base} = \frac{V_{ph,base}}{I_{base}} = 16\Omega$$

With these base values, we calculate the per unit values as,

$$\overline{V_{ph}} = 0.867\angle 0^\circ pu$$
$$\overline{I_L} = \frac{138.57}{|Z_L|}\angle -36.87^\circ = 13.86\angle -36.87^\circ$$
$$\overline{I} = \frac{13.86\angle 36.87^\circ}{8.66} = 1.6\angle -36.87^\circ pu$$
$$\overline{Z_L} = \frac{10\angle 36.87^\circ}{16} = 0.625\angle 36.87^\circ pu$$
$$P_L = \frac{V^2}{|Z_L|}\cos\phi = 1536.13W = 1.28pu$$
$$Q_L = \frac{V^2}{|Z_L|}\sin\phi = 1152.1VAR = 0.96pu$$

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(2-21)*(a)*

$$l_m = 2\pi r_m$$
$$H_{ID} = \frac{Ni}{l_m} = 477.71A/m$$

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(b)

$$l_m = \pi OD$$
$$H_{OD} = \frac{Ni}{l_m} = 433.53A/m$$

■

(c)

$$Error_{ID} = \frac{H_{ID} - H_m}{H_m} \times 100 = 5.107\%$$
$$Error_{OD} = \frac{H_{OD} - H_m}{H_m} \times 100 = 4.614\%$$

■

(2-22)

$$\begin{aligned}\mathcal{R}_m &= \frac{l_m}{\mu_0\mu_r A_m}\\ &= \frac{0.165}{4\pi \times 10^{-7} \times 2000 \times \frac{\pi}{4}\left(\frac{OD-ID}{2}\right)^2}\\ &= 1.337 \times 10^7 A/Wb\end{aligned}$$

■

(2-23) Using the $\mathcal{R}_m$ from Prob 2-22,

$$L = \frac{N^2}{\mathcal{R}_m}$$
$$\therefore N = \sqrt{L\mathcal{R}_m} = 19$$

$$B_m = \mu_0 \mu_r \frac{NI}{l_m}$$
$$\therefore \mu_r = \frac{B_m l_m}{\mu_0 NI} = \frac{1.3 \times 0.165}{4\pi \times 10^{-7} \times 19 \times 3} = 2994.63$$

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(2-24)

$$\mathcal{R} = \mathcal{R}_m + \mathcal{R}_{gap} \approx \mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} \text{ [Using } A_{gap} \text{ from Prob 2-22]}$$
$$= \frac{l_{gap}}{4\pi \times 10^{-7} \times 4.91 \times 10^{-6}} = 16.2 \times 10^{10} l_{gap}$$

$$\lambda = N\varphi = LI$$
$$\therefore NB_{max}A_m = LI$$
$$N = \frac{LI}{B_{max}A_m}$$
$$= \frac{25 \times 10^{-6} \times 3}{1.3 \times 4.91 \times 10^{-6}} = 12$$
$$\mathcal{R} = \frac{N^2}{L}$$
$$\therefore 16.2 \times 10^{10} l_{gap} = \frac{N^2}{25 \times 10^{-6}}$$
$$l_{gap} = \frac{N^2}{25 \times 10^{-6} \times 16.2 \times 10^{10}} = 0.00356\ cm$$

■