

# Chapter 1

1-1

$$1 \text{ Yr} = 365 \frac{\text{days}}{\text{yr}} \times 24 \frac{\text{hrs}}{\text{day}} = 8760 \text{ hrs}$$

Average  
Wasted

$$\text{Power } P_{\text{avg}} = \frac{100 \times 10^9 \text{ kWh/yr}}{8760 \text{ hr/yr}} = 11.41 \times 10^3 \text{ MW}$$

(a)

$\therefore \sim 11 \frac{1}{2}$  1000-MW generating plants running continuously provide this power.

$$\begin{aligned} \text{(b) Annual Savings} &= 0.10 \frac{\$}{\text{kWh}} \times 100 \times 10^9 \frac{\text{kWh}}{\text{yr}} \\ &= 10 \text{ Billion } \$/\text{yr} \end{aligned}$$

1-5

For the ease of solving this problem, let us assume that the system operates for 100 hrs and draws 1 kW while delivering 100% flow rate.

The percentage reduction in the power consumption is the same as the percentage reduction in the energy consumption.

The following Table shows the energy consumption using each of the three methods:

<u>hrs of Operation</u>	<u>Drive</u>	<u>Outlet</u>	<u>Inlet</u>
20 hrs @ 100%	1 x 20 kWh	1 x 20 kWh	1 x 20 kWh
20 hrs @ 80%	0.5 x 20	0.92 x 20	0.81 x 20
30 hrs @ 60%	0.3 x 30	0.87 x 30	0.7 x 30
10 hrs @ 30%	0.1 x 10	0.72 x 10	0.65 x 10
Total Energy Consumption	$E = 40 \text{ kWh}$ Drive	$E = 71.7 \text{ kWh}$ Outlet	$E = 63.7 \text{ kWh}$ Inlet

Using an adjustable speed drive,

$$(a) \text{ \% reduction over outlet damper} = \frac{E_{\text{outlet}} - E_{\text{Drive}}}{E_{\text{outlet}}} = \frac{71.7 - 40}{71.7} \times 100 \approx 44\%$$

$$(b) \text{ \% reduction over inlet vanes} = \frac{E_{\text{inlet}} - E_{\text{Drive}}}{E_{\text{inlet}}} = \frac{63.7 - 40}{63.7} \times 100 \approx 37\%$$

1-5

$$\lambda = k \frac{\omega_m r}{V_{\text{wind}}}$$

$$V_{\text{wind, rated}} = 12 \text{ m/s}$$

$$V_{\text{wind, cut-in}} = 4 \text{ m/s}$$

For maximum value of  $C_p$ ,  $\omega_m = 20 \text{ rpm}$   
at the rated wind.

$\therefore$   $\omega_m$  at the cut-in wind speed is

$$20 \times \frac{4}{12} = 6.66 \text{ rpm}$$

$\therefore$  the blade rotational speed should vary between 6.66 rpm to 20 rpm between the cut-in and the rated wind speeds, in order to keep  $C_p$  at its maximum value.

## Chapter 2

2-1

$$J \frac{d\omega_m}{dt} = T$$

$$T = 5 \text{ Nm}$$

$$\begin{aligned} \frac{d\omega_m}{dt} &= \frac{1800 - 0}{3} \frac{\text{rpm}}{\text{s}} = \frac{1800}{3} \times \frac{1}{60} \times 2\pi \text{ rad/s}^2 \\ &= 62.83 \frac{\text{rad}}{\text{s}^2} \end{aligned}$$

$$J = \frac{T}{\frac{d\omega_m}{dt}} = \frac{5 \text{ Nm}}{62.83 \frac{\text{rad}}{\text{s}^2}} \approx 0.08 \text{ kg}\cdot\text{m}^2$$

2-2

Modifying Eq. 2-16 for a hollow cylinder

$$\left( \rho \int_{r_2}^{r_1} r^3 dr \int_0^{2\pi} d\theta \int_0^l dl \right) \frac{d\omega_m}{dt} = T$$

or

$$\underbrace{\left( \rho \frac{r^4}{4} \Big|_{r_2}^{r_1} 2\pi l \right)}_{J_{\text{cyl, hollow}}} \frac{d\omega_m}{dt} = T$$

$$\begin{aligned} \therefore J_{\text{cyl, hollow}} &= \frac{\pi}{2} \rho l (r_1^4 - r_2^4) \\ &= \frac{\pi}{2} \times 7.85 \times 10^3 \times 0.18 \times (0.06^4 - 0.04^4) \\ &= 0.0174 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

2-3

$$50 \frac{\text{km}}{\text{hr}} = \frac{50 \times 10^3}{3600} \frac{\text{m}}{\text{s}} = 13.89 \frac{\text{m}}{\text{s}}$$

$$\text{and, } 10 \frac{\text{km}}{\text{hr}} = 2.78 \frac{\text{m}}{\text{s}}$$

$$\text{Eq 2-9 } W_m = \frac{1}{2} M u^2$$

$$\begin{aligned} \Delta W_{m, \text{recovered}} &= \frac{1}{2} M u^2 \Big|_{u=13.89 \frac{\text{m}}{\text{s}}} - \frac{1}{2} M u^2 \Big|_{u=2.78 \frac{\text{m}}{\text{s}}} \\ &= \frac{1}{2} \times 1500 \left( 13.89^2 - 2.78^2 \right) \\ &= 138.9 \text{ kJ} \end{aligned}$$

2-4

$$u = r \omega_m$$

$$\therefore \omega_m = \frac{u}{r} = \frac{1 \text{ m/s}}{0.09 \text{ m}} = 11.11 \text{ rad/s}$$

$$\frac{d\omega_m}{dt} = \frac{11.11 - 0}{4} = 2.777 \frac{\text{rad}}{\text{s}^2} \text{ (linear acceleration due to constant torque)}$$

Eq. 2-38

$$\begin{aligned} T_{em} &= (J_m + r^2 M) \frac{d\omega_m}{dt} \\ &= (0.01 + 0.09^2 \times 1) \times 2.777 \\ &= 0.05 \text{ Nm} \end{aligned}$$

2-5 The load-speed profile is given -  
that is,  $\frac{du}{dt}$  is given.

$$u = r \omega_m \Rightarrow \omega_m = \frac{u}{r}$$

$$\text{and } \frac{d\omega_m}{dt} = \frac{du}{dt} \frac{1}{r}$$

Substituting for  $\frac{d\omega_m}{dt}$  in Eq. 2-38 ( $f_L=0$ )

$$\begin{aligned} T_{em} &= \left( \frac{J_m}{r} + rM \right) \frac{du}{dt} \\ &= r \left[ M + \frac{J_m}{r^2} \right] \frac{du}{dt} \end{aligned}$$

To minimize  $T_{em}$  for a given  $\frac{du}{dt}$ , we will take a partial derivative of  $T_{em}$  (with respect to  $r$ ) and set it to zero -

$$\frac{\partial T_{em}}{\partial r} = \left\{ \left( M + \frac{J_m}{r^2} \right) + r \left[ -2 \frac{J_m}{r^3} \right] \right\} \frac{du}{dt} = 0$$

$$\therefore M + \frac{J_m}{r^2} - 2 \frac{J_m}{r^2} = 0$$

$$\text{or, } M = \frac{J_m}{r^2} \Rightarrow r = \sqrt{\frac{J_m}{M}}$$

We can take a second derivative with respect to  $r$  to confirm that what we have is the minimum (not the maximum).

$$= \sqrt{\frac{40 \times 10^{-3} \times 10^{-4}}{0.02}}$$

$$= 0.0141 \text{ m}$$

$$= 1.41 \text{ cm}$$

2-6

The length of the arc between two teeth is the same in two gears. Therefore, for the motor-side gear

$$\text{arc length} \times n_M = 2\pi r_M$$

Similarly,

$$\text{arc length} \times n_L = 2\pi r_L$$

$$\therefore \frac{n_M}{n_L} = \frac{r_M}{r_L} = \frac{\omega_L}{\omega_M} \quad (\text{using Eq. 2-41})$$
$$= \frac{1}{3}$$

$$\therefore \omega_M = 3\omega_L$$

during  $0 \leq t \leq 1$  s

$$\frac{d\omega_L}{dt} = 100 \text{ rad/s}^2 \quad \therefore \frac{d\omega_M}{dt} = 300 \text{ rad/s}^2$$

From Eq. 2-43

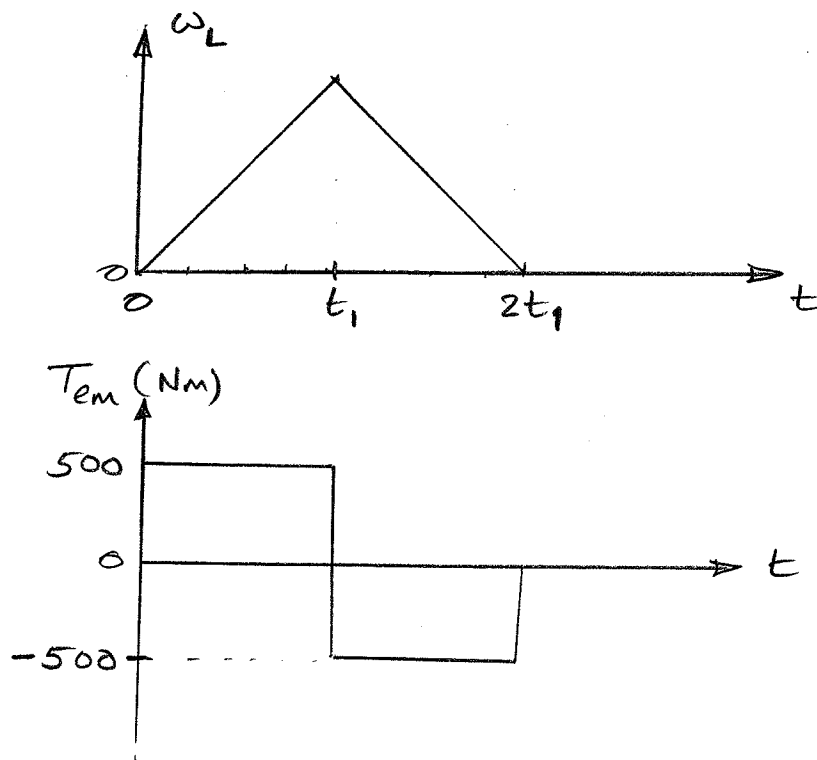
$$T_{em} = \left[ J_M + \left( \frac{\omega_L}{\omega_M} \right)^2 J_L \right] \frac{d\omega_M}{dt} = \left[ 1.2 + \left( \frac{1}{3} \right)^2 10.0 \right] \times 300$$
$$= 693.33 \text{ Nm}$$

$$1 \leq t \leq 2 \quad T_{em} = 0 \quad (\text{since } \frac{d\omega_L}{dt} = 0)$$

$$2 \leq t \leq 3 \quad T_{em} = -693.33 \text{ Nm}$$

$$3 \leq t \leq 4 \quad T_{em} = 0$$

2-7



During  $0 \leq t < t_1$ , from Eq. 2-43

$$T_{em} = 500 = \left[ 1.2 + \left(\frac{1}{3}\right)^2 10.0 \right] \frac{d\omega_m}{dt}$$

$$\therefore \frac{d\omega_m}{dt} = \frac{500}{\left[ 1.2 + \left(\frac{1}{3}\right)^2 \times 10 \right]} = 216.35 \text{ rad/s}^2$$

$$\Rightarrow \frac{d\omega_L}{dt} = \frac{1}{3} \frac{d\omega_m}{dt} = 72.11 \frac{\text{rad}}{\text{s}^2}$$

$\therefore$  during  $0 \leq t \leq t_1$ ,

$$\omega_L = 72.11 t \text{ rad/s}$$

Due to the symmetry of the speed profile during acceleration and deceleration, the load over the interval  $t_1$  rotates by an angle  $= \frac{30^\circ}{2} = 15^\circ$ .

$$\therefore \theta_L(t_1) = \int_0^{t_1} \omega_L(\tau) \cdot d\tau = \int_0^{t_1} 72.11 \tau \cdot d\tau = \frac{72.11}{2} t_1^2$$

$$= \frac{15 \times \pi}{180} \text{ rad}$$

(assuming that  $\theta_L(0) = 0$ )



$$\therefore t_1 = \sqrt{\frac{2 \times 15 \times \pi}{72.11 \times 130}} = 85.2 \text{ ms}$$

Therefore, the time required to rotate the load by an angle of  $30^\circ$  is

$$2t_1 = 0.17 \text{ s.}$$

2-8

$$\text{Vehicle maximum speed} = 150 \frac{\text{km}}{\text{hr}} = \frac{150 \times 10^3}{3600}$$

$$\therefore v_L = 41.667 \frac{\text{m}}{\text{s}}$$

$\therefore$  wheel's maximum rotational speed

$$\omega_L = \frac{v_L}{r_L} = \frac{41.667}{(0.6/2)} = 138.89 \frac{\text{rad}}{\text{s}}$$

$$\text{wheel radius} \quad = \frac{138.89}{2\pi} \times 60$$

$$= 1326.29 \text{ rpm}$$

This should occur at the maximum motor speed of 5000 rpm. Therefore, using the relationship derived in problem 2-6,

$$(a) \quad \frac{\eta_m}{\eta_L} = \frac{\omega_L}{\omega_m} = \frac{1326.29}{5000} = 0.265$$

$$(b) \quad \text{From Table 2-1, } f_L = 930.9 \text{ N}$$

$$\therefore f_{L, \text{ per wheel}} = \frac{f_L}{4} = \frac{930.9}{4} \text{ N}$$

$$T_{L, \text{ per wheel}} = r_L f_{L, \text{ wheel}} = 0.3 \times \frac{930.9}{4} = 69.82 \text{ Nm}$$

$$\therefore T_{M, \text{ per motor}} = \frac{\omega_L}{\omega_m} T_{L, \text{ per wheel}} = 0.265 \times 69.82 = 18.5 \text{ Nm}$$

2-9

From Eq. 2-45a in Fig. 2-15,

$$\left(\frac{r_1}{r_2}\right)_{\text{opt.}} = \sqrt{\frac{J_M}{J_L}} = \sqrt{\frac{40}{60}} = 0.816$$

2-10

In the Lead-screw system, a rotation of  $\theta_m (= 2\pi \text{ rad})$  corresponds to a linear movement  $x_L (= 5 \text{ m})$ .

$$\text{Therefore, } \frac{x_L}{\theta_m} = \frac{5}{2\pi} = n \quad (1)$$

At a speed  $u_L$ , the stored kinetic energy by the mass  $(M_T + M_w)$  is  $\frac{1}{2}(M_T + M_w) u_L^2$ . In terms of an equivalent rotating mass of inertia  $J_L'$ , the kinetic energy would be  $\frac{1}{2} J_L' \omega_m^2$ . Equating these two energy expressions,

$$\frac{1}{2} J_L' \omega_m^2 = \frac{1}{2} (M_T + M_w) u_L^2$$

$$\text{or } J_L' = (M_T + M_w) \left(\frac{u_L}{\omega_m}\right)^2 \quad (2)$$

Differentiating both sides of Eq.(1) with respect to time,

$$\frac{u_L}{\omega_m} = n \quad (3)$$