Chapter 2 End of Chapter Problem Solutions

2.1

Assume Ideal Gas Behavior

$$\frac{dP}{dy} = -\rho g = -\frac{Pg}{RT}$$

For T = a + by

$\Rightarrow$ T= 530 - 24 y/n

$$\frac{dP}{P} = -\frac{g}{R} - \frac{dy}{530-24(\frac{y}{n})}$$

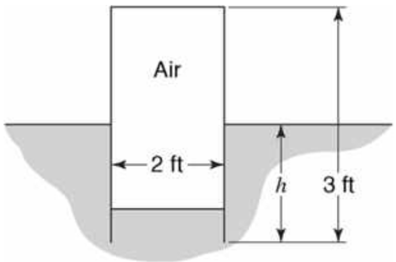
$$\int_{P_o}^{P} \frac{dP}{P} = \frac{gh}{24R} \int_0^1 \frac{-24\, d(\frac{y}{n})}{530-24(\frac{y}{n})}$$

$$\ln\frac{P}{P_o} = \frac{gh}{24R} \ln\frac{506}{530}$$

With P = 10.6 PSIA, P_o = 30.1 in Hg

h = 9192 ft.

2.2



(a) $\sum$F=0 on tank

$$P\frac{\pi d^2}{4} - P_{atm}\frac{\pi d^2}{4} - 250 = 0 \qquad (1)$$

At H_20 Level in Tank: $P=P_{atm}+ \rho_w g(h-y)$ (2)

From (1) & (2): h-y=1.275 ft. (3)

For Isothermal Compression of Air
$P_{atm}V_{tank}=P(V_{air})$

$$P = \frac{3}{3-y} P_{atm} \qquad (4)$$

Combing (1) & (4): y=0.12 ft. and h=1.395 ft.

(b) For Top of Tank Flush with H_20 Level

$\sum$F=0
$P=P_{atm}+ \frac{250+F}{\pi d^2/4}$

At H_20 Level in Tank: $P=P_{atm}+ \rho_w g(3-y)$

Combining Equations:
F= 196(3-y)- 250

For Isothermal Compression of Air:
(As in Part (a))
3-y = 2.8 ft.

F = 196(2.8) - 250 = 293.6 LB_f

2.3

When New Force on Tank = 0

Wt. = Buoyant Force = 250 Lb_f

V_w Displaced = 250/ρ_wg = 4.01 ft^3

Assuming Isothermal Compression
P_{atm}A(3ft.)= P(4.01 ft^3) = (P_{atm}+ρgy)(4.01)
y=45.88 ft.

Top is at Level: $y-\frac{4.01}{\pi d^2/4}$

or at 44.6 ft. Below Surface

2.4

$\frac{dP}{dy} = \rho g = \rho_o e^{\Delta P/\beta}$

$\int_0^{\Delta P} \frac{-\Delta P/\beta}{e^{\Delta P/\beta}} = \int_0^{y} \frac{-\rho_o g \Delta y}{\beta}$

$e^{-\Delta P/\beta} = 1 - \frac{\rho_o g y}{\beta g}$

$\Delta P = -\beta \ln(1 - \frac{\rho_o y}{\beta})$ = 300,000 ln(1-0.0462) = 14190 Psi

Density Ratio:

$\frac{\rho}{\rho_o} = e^{-\Delta P/\beta} = 1.0484$

so $P = 1.0484\rho_o$

2.5

Buoyant Force:

$F_B = \rho V = \frac{PV}{RT}$

For constant volume: F varies inversely with T

2.6

Sea H_20: S.G.=1.025

At Depth y=185m

P_g= 1.025$\rho_w$gy = 1.025(1000)(9.81)(185) = 1.86x10^6 Pa = 1.86 MPa

2.7

r = Measured from Earth's Surface
R = Radius of Earth

$\frac{dP}{dr} = \rho g = \rho g_o \frac{r}{R}$

$P - P_{atm} = \frac{\rho g_o r^2}{2R}$

At Center of Earth: r = R

$P_{Ctr} - P_{atm} = \frac{\rho g_o R}{2}$

Since $P_{Ctr} >> P_{atm}$

$P_{Ctr} \cong \frac{\rho g_o R}{2} = \frac{(5.67)(1000)(9.81)(6330x10^3)}{2} = 176x10^9$ Pa = 176 MPa

2.8

$$\frac{dP}{dy} = -\rho g$$

$$\int_{P_{atm}}^{P} dP = -\rho g \int_{0}^{-h} dy$$

$P - P_{atm} = \rho g(+h) = (1050)(9.81)(11034) = 113.7$ MPa $\cong$ 1122 Atmospheres

2.9

As in Previous Problem

$P\text{-}P_{atm} = \rho gh$

For $P\text{-}P_{atm} = 101.33$ kPa

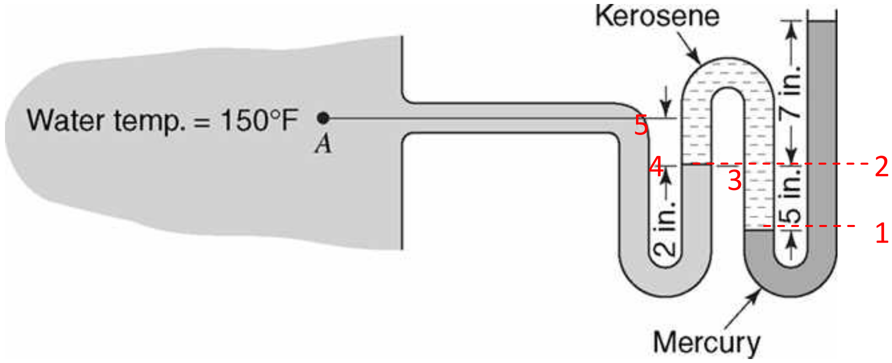
$h = 101.33 / \rho g$

for H_20: $h = \frac{101.33}{(1000)(9.81)} = 10.33\text{m}$

Sea H_20: $h = \frac{101.33}{(1.025)(1000)(9.81)} = 10.08\text{m}$

Hg: $h = \frac{101.33}{(13.6)(1000)(9.81)} = 0.80\text{m}$

2.10



$P_1=P_{atm}+\rho_{Hg}\ g(12'')$ $P_1=P_2$

$P_2=P_3+\rho_K\ g(5'')$ $P_3=P_4$

$P_4=P_A+\rho_w\ g(2'')$ $P_4=P_5$

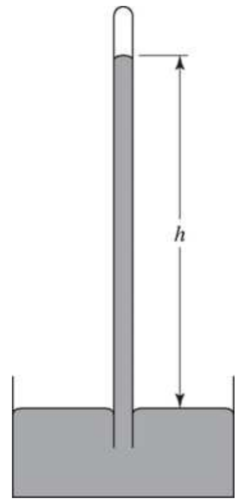
$P_{atm}+\rho_{Hg}\ g(12'')= P_A+\rho_w\ g(2'') + \rho_K\ g(5'')$

$P_A= P_{atm}+\rho_w g[(13.6)(12)-2-0.75(5)]=P_{atm}+5.81$

P_A = 5.81 PSIG

2.11

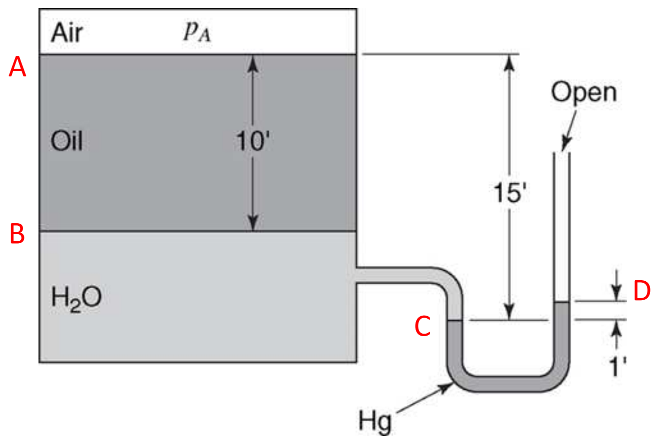
Force Balance on Liquid Column: A=Area of Tube



$-3A + 14.7A - \rho ghA = 0$

$h = \frac{11.7(144)}{62.4(12.2)} = 26.6$ in.

2.12



$P_A = P_B - \rho_o\, g(10\ \text{ft.})$

$P_C = P_B + \rho_w\, g(5\ \text{ft.})$

$P_D = P_C - \rho_{Hg}\, g(1\ \text{ft.})$

$P_A - P_D = \rho_{Hg}\, g(1) - \rho_w\, g(5) - \rho_o\, g(10)$

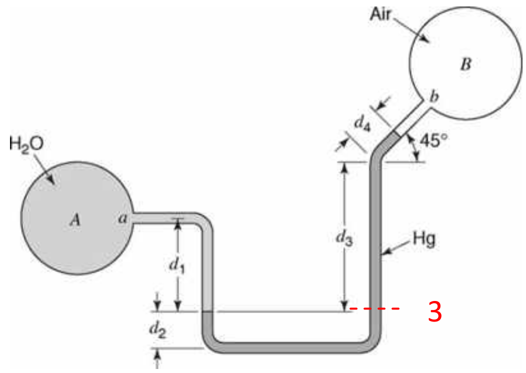
$P_A - P_{Atm} = \rho_w\, g(13.6 \times 1 - 5 - 0.8 \times 10 \times 1) = 37.4\ LB_f/ft^2$

2.13

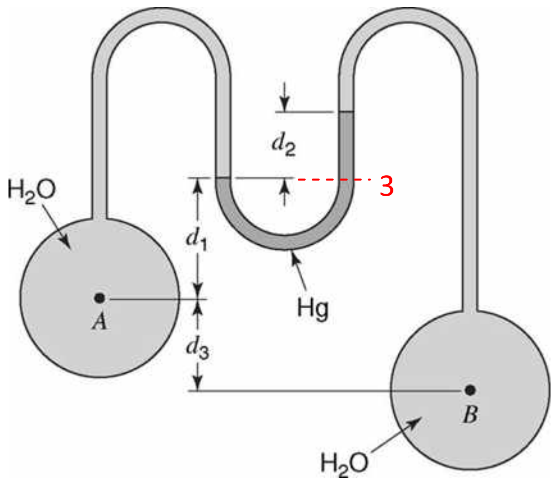
$P_3 = P_A - d_1 g\, \rho_w = P_B + (\rho_{Hg}\, g) x (d_3 + d_4 \sin 45)$

$P_A - P_B \;= \frac{(62.4)(32.2)}{32.2}\left[\frac{(2.4+4\sin 45)}{12}\ 13.6 - 2\right]$

$= 245\ LB_f/ft^2 = 1.70$ Psi



2.14



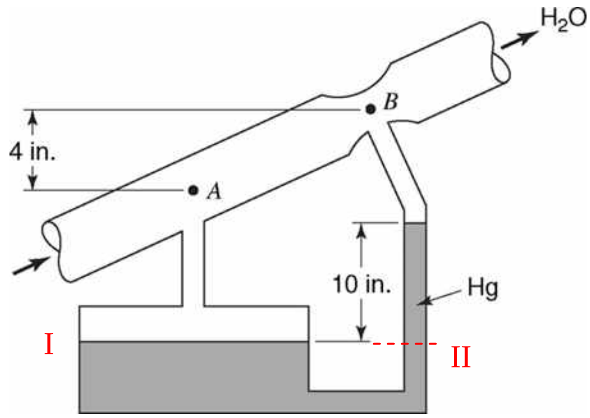
$P_3 = P_A - \rho_w g d_1$

$P_3 = P_B - \rho_w g(d_1+d_2+d_3) + \rho_{Hg}\, g d_2$

Equating:

$P_A - P_B = \rho_{Hg}\, g d_2 - \rho_w g(d_2+d_3) = \rho_w g[(13.6)(1/12)-7.3/12] = 32.8\ LB_f/ft^2 = 0.227$ Psi

2.15



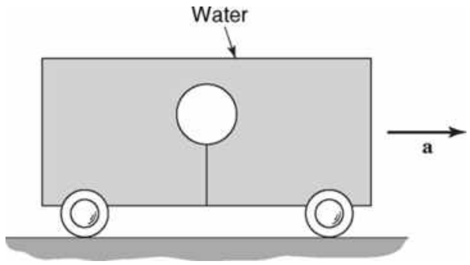
$P_I = P_A + \rho_w g(10")$

$P_{II} = P_B + \rho_w g(4'') + \rho_{Hg} g(10")$

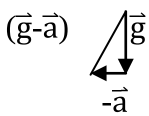
$P_I = P_{II}$

$P_A - P_B = \rho_w g[-6+13.6(10)] = 56.3$ psi

2.16

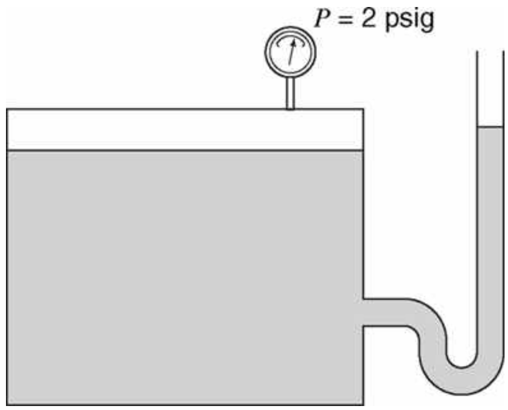


Pressure Gradient is in direction of $\vec{g}-\vec{a}$ & isobars are perpendicular to $(\vec{g}-\vec{a})$



String will assume the $(\vec{g}-\vec{a})$ direction & Balloon will move <u>forward.</u>

2.17



At Rest: $P=\rho g y_o$

Accelerating: $P=\rho|(\vec{g}-\vec{a})|=\rho(g+a)y_a$

Equating: $y_a=\frac{g}{g-a}$ which $<y_o$

Level goes <u>down</u>.

2.18

$F = P_{6.6}A - P_{atm}A = \rho gh(\pi r^2)$

h=2m
r=0.3m

<u>F=5546N</u>

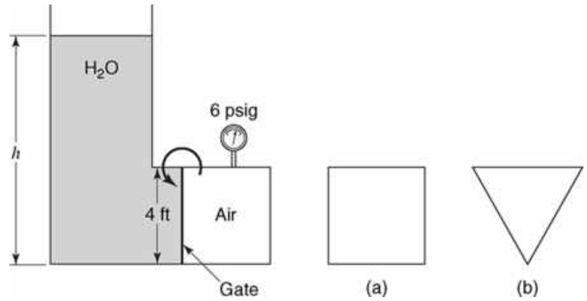
$y_{C.P.} = \bar{y} + I_{bb}/A\bar{y}$

For a circle: $I_{bb} = \pi r^4/4$

$y_{C.P.} = 2m + \frac{\pi(0.3m)^4}{4\pi(0.3m)^2(2m)} = 2.011m$

2.19

Height of H_20 column above differential element= h-4+y



For (a) - Rectangular gate- dA = 4dy

$dF_w=[\rho_w g(h-4+y)+P_{atm}]dA$

$dF_A=[P_{atm}+(6Psig)(144)]dA$

$\sum M_o= \int_A^0 y(dF_w - dF_A)=0$

$\int_0^4 y[\rho g(h-4+y)-864](4dy)=0$

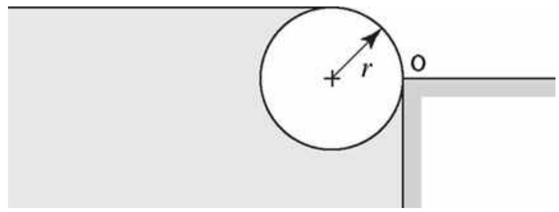
<u>h=15.18 ft.</u>

For (b): dA=(4-y)dy

$\int_0^4 y[\rho g(h-4+y)-864](4-y)dy=0$

<u>h=15.85 ft.</u>

2.20



Per unit depth: $\sum F_y = 0$

$F_y|_{up} = \rho_w g\ \pi r^2/2$ {buoyancy}

$F_y|_{down} = \rho g\ \pi r^2 + \rho_w g(r^2 - \pi r^2/4)$

Equating: $\frac{\rho_w g\ \pi r^2}{2} = \rho g\ \pi r^2 + \rho_w g r^2(1 - \pi/4)$

$\rho = \rho_w\left(\frac{\pi}{2} - 1 + \frac{\pi}{4}\right)/\pi = \rho_w\left(\frac{3}{4} - \frac{1}{\pi}\right) = 0.432\rho_w = 432\ \text{kg/m}^3$

2.21

a) To lift block from bottom

F = {wt. of concrete}+{wt. of H_20}

= $\rho_c gV$+[$\rho_w g$(22.75')+P_{atm}]A

= (150)g(3x3x0.5)+[62.4g(22.75)+14.7(144)]x(3x3)

= 675+31828 = 32503 lb_f

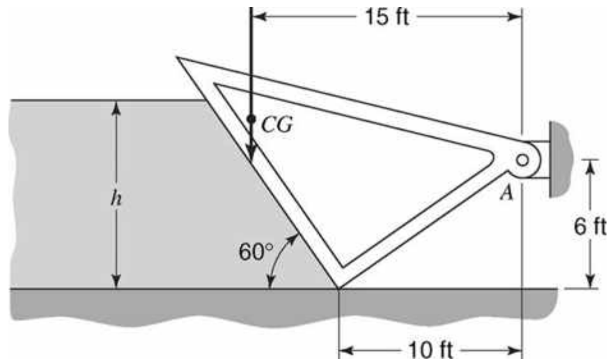
b) To maintain block in free position

F = {wt. of concrete}-{Buoyant force of H_20)}

= 675- $\rho_w gV$ = 675-[62.4g(3x3x0.5)] = 675 - 281 = 394 lb_f

2.22

Distance z measured along gate surface from bottom



$\sum M_A$=500(15)- $\int_0^{h/\sin 60} z\,\rho g(h\text{-}z\sin 60)dz$=0

$\rho g \int_0^{h/\sin 60} (zh - z^2 \sin 60)\, dz$=7500

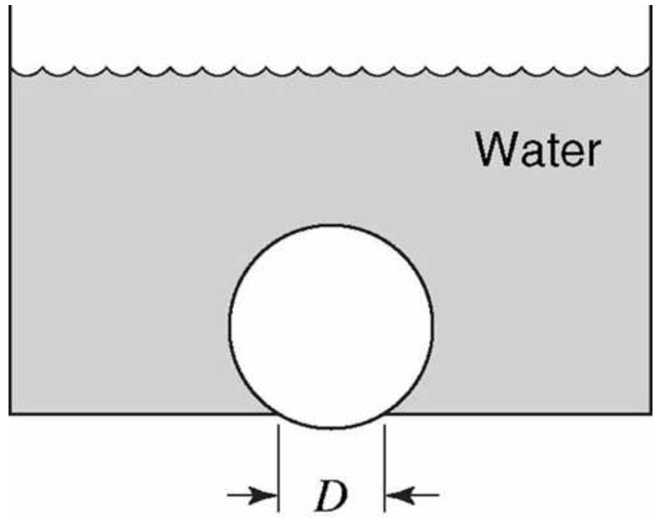
$\rho g\left[h\frac{z^2}{2} - \frac{z^3}{3}\sin 60\right]_0^{h/\sin 60}$=7500

(62.4)g$\left[\frac{h^3}{(\sin 60)^2}\left(\frac{1}{2} - \frac{1}{3}\right)\right]$=7500

$h^3 = \frac{7500(6)(\sin 60)^2}{62.4g} = 541$

h=8.15 ft.

2.23



Using spherical coordinates for a ring
At y=constant

$dA=2\pi r^2 \sin\theta d\theta$

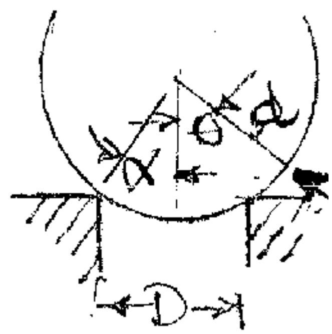
$P= \rho g[h-r\cos\alpha+r\cos\theta]$

$dF_y=dF\cos\theta$

$F_y =\int \rho g(h-r\cos\alpha+r\cos\theta)(2\pi r^2\sin\theta\cos\theta d\theta)$
$=2\pi \rho g r^2\int_x^{\pi}(h - r\cos\alpha + r\cos\theta)(\sin\theta\cos\theta d\theta)$

Let: $c=2\pi \rho g r^2$

$= c[\int_\alpha^{\pi}(h - r\cos\alpha) \sin\theta\cos\theta d\theta+r \int_\alpha^{\pi} \sin\theta \cos^2 \theta d\theta]$



$= c[(h - r\cos\alpha) \sin^2 \theta |_\alpha^\pi+r(-\frac{1}{3}\cos^3 \theta)|_\alpha^\pi]$

$= c[(h- r\cos\alpha)(1-\sin^2 \alpha)- \frac{r}{3}(0-\cos^3\alpha)]$

Now for $F_y=0$

$\sin \alpha= \frac{D}{d} \cos\alpha=[1-\left(\frac{D}{d}\right)^2]^{1/2}$ & $r=d/2$

$0=(h-\frac{d}{2} \cos\alpha)(\cos^2\alpha)+\frac{d}{6}(\cos^3\alpha) = h-\frac{d}{2} \cos\alpha+ \frac{d}{6}\cos\alpha$

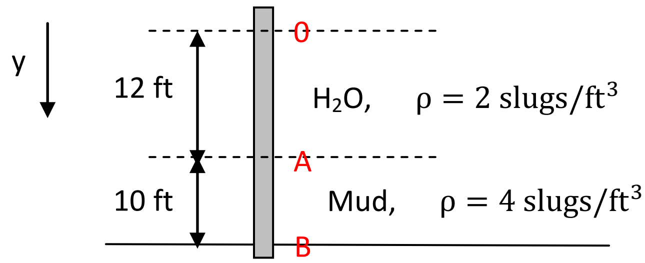
Giving $h=\frac{d}{3} \cos\alpha$

$\frac{h}{d}= \frac{\cos\alpha}{3}= \frac{1}{3}\left[1 - \left(\frac{D}{d}\right)^2\right]^{1/2}$

For d=0.6m

$h = \frac{0.6}{3}= \frac{1}{3}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2} = \frac{1}{5}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2}$

2.24



$P_A - P_{atm} = \rho_w g(12) = 24g$
$P_B - P_{atm} = 24g + 40g = 64g$

Between 0 & A: $P - P_{atm} = \rho_w g y$
Between A & B: $P = \rho_w g(12) + \rho_m g(y-12)$

Per unit depth:
$F = \int_{12}^{0} (P - P_{atm}) dA$

$= \int_0^{12} \rho_w g y dy + \int_{12}^{22} [\rho_w g(12) + \rho_m g(y - 12)] dy$

$= \rho_w g(192) + \rho_m g(50)$

$= 18790$ lb$_f$

Force Location:
$Fxy = \int_0^{22} y(P - P_{atm}) dA$

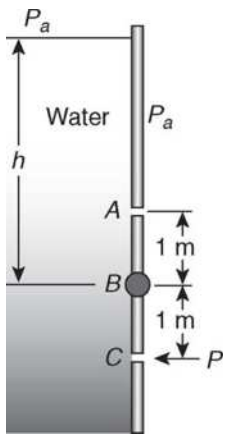
$= \int_0^{12} \rho_w g y^2 dy + \int_{12}^{22} \rho_w g 12 y dy + \int_{12}^{22} \rho_m g(y^2 - 12y) dy$

$= \rho_w g(576+2040) + \rho_m g(2973-2040)$

$= 288{,}400$ ft. LB$_f$

$\bar{\bar{y}} = \frac{288{,}400}{18{,}790} = 15.35$ ft.

2.25



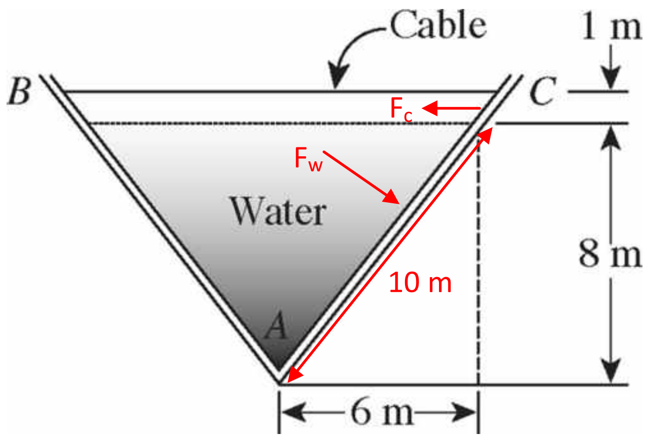
Force on gate= $\rho g\bar{y}A = (1000)(9.81)(12)\frac{\pi}{4}(2)^2 = 369.8$ kN

$y_{C.P.} = \frac{I_{\infty}}{\bar{y}A} = \frac{\frac{\pi}{4}(1)^4}{(12)\frac{\pi}{4}(2)^2} = 0.0208$m (Below axis B)

$\sum M_B = 0$

$P(1) = (369.8 \times 10^3)(0.0208) = 7.70$ kN

2.26



$F_w = \rho_g g\bar{\bar{y}}A = (1000)(9.81)(4)(10)(1) = 392$ kN

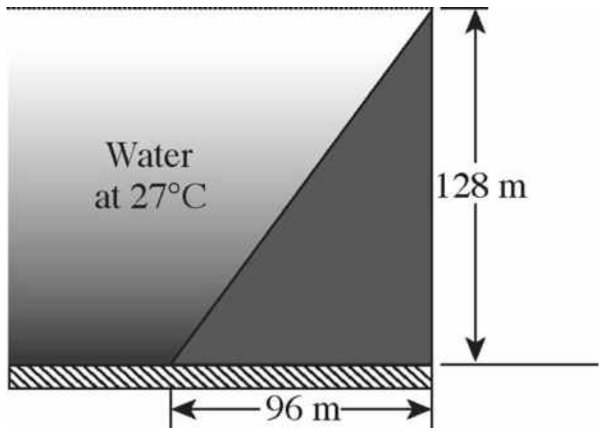
y_{cp} is 2/3 distance form water line to A

~ 6.66m down form H_20 line
~ 3.33m up from A

$\sum M_A = F_c(9) = 392(3.33)$

$F_c = 145.2$ kN

2.27



Width = 100m

H_20@27℃   ρ=997 kg/m^3

F = $\rho_g g\bar{y}A$ = (997)(9.81)(64)x(160)(100) = 10.016 x 10^9 N = 10.02x10^3 MN

For a free H_20 surface

y_{cp}= $\frac{2}{3}$(128m) = 85.3m {below H_20 surface} =106.7 m {measured along dam surface}

2.28 Spherical Float

Upward forces ~ $F + F_{Buoyant}$

Downward forces ~ W_T

$W = \rho g V = \rho g(\frac{4}{3}\pi R^3)$

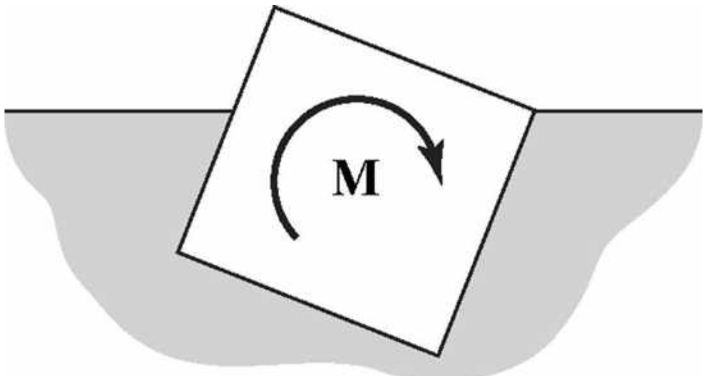
$F_b = \rho_w g V z = \rho_w g(\frac{4}{3}\pi R^3) z$

z= fraction submerged

$F = \rho g(\frac{4}{3}\pi R^3) - \rho_w g z(\frac{4}{3}\pi R^3)$

$z = \frac{\rho g\left(\frac{4}{3}\pi R^3\right) - F}{\rho_w g(\frac{4}{3}\pi R^3)}$

2.29



$\theta = \tan^{-1}\left(\frac{0.1}{0.5}\right)$

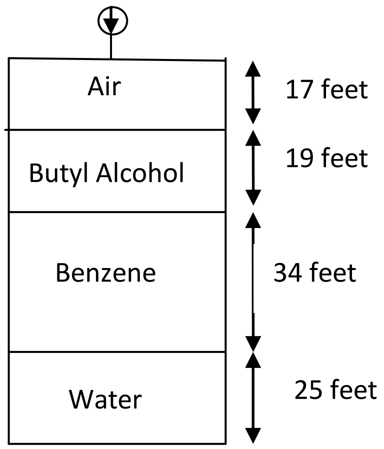
G is center of mass of solid

$\sum M_G = 2[1/2(L/2)(0.1L)(L)\ \rho g(\frac{2}{3}\frac{L}{2}\sin\theta) - (0.9L)(L)(L)\ \rho g(0.05L\sin\theta)] + M$

{Part of original submerged volume is now out of H_20}
(Part that was originally out is now submerged}

$M = \rho g\ L^4 \sin\theta[-\frac{1}{60} + 0.045] = \rho g L^4 \sin\theta(0.02833) = 0.00556\ \rho g L^4$

2.30



(a) the pressure at the butyl alcohol/benzene interface

$\Delta P = \rho g h$

$P_{top} - P_{Air-ButylAlcohol\ Interface} = -\rho_{air} g(17feet)$

$P_{Air-ButylAlcohol\ Interface} - P_{ButylAlcohol-Benzene\ Interface} = -\rho_{ButylAlcohol} g(19feet)$

$P_{ButylAlcohol-Benzene\ Interface} = P_{top} + \rho_{air} g(17feet) + \rho_{ButylAlcohol} g(19feet)$

$$= 2116\frac{lb_f}{ft^3} + \frac{\left(0.0735\frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(17\ ft)}{32.174\frac{lb_m ft}{lb_f s^2}} + \frac{\left(50.0\frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(19\ ft)}{32.174\frac{lb_m ft}{lb_f s^2}}$$

$$= 3068.0\frac{lb_f}{ft^3} = 21.3\ psi$$

(b) the pressure at the benzene/water interface

$P_{ButylAlcohol-Benzene\ Interface} - P_{Benzene-Water\ Interface} = -\rho_{Benzene} g(34feet)$

$P_{Benzene-Water\ Interface} = P_{ButylAlcohol-Benzene\ Interface} + \rho_{Benzene} g(34feet)$

$$= 3068.0\frac{lb_f}{ft^3} + \frac{\left(54.6\frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(34\ ft)}{32.174\frac{lb_m ft}{lb_f s^2}} = 4925.9\frac{lb_f}{ft^3} = 34.2\ psi$$

(c) the pressure at the bottom of the tank

$P_{ButylAlcohol-Benzene\ Interface} - P_{Bottom\ of\ Tank} = -\rho_{Water} g(25feet)$

$P_{Benzene-Water\ Interface} = P_{ButylAlcohol-Benzene\ Interface} + \rho_{Benzene} g(25feet)$

$$= 4925.9\frac{lb_f}{ft^3} + \frac{\left(62.2\frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(25\ ft)}{32.174\frac{lb_m ft}{lb_f s^2}} = 6482.16\frac{lb_f}{ft^3} = 45\ psi$$

2.31

$\Delta P = \rho g h$

For Blood: $P = \rho_B g h_B$

For Mercury: $P = \rho_M g h_M$

So,
$\Delta P = \rho_B g h_B = \rho_M g h_M$

Solve for the height of the blood,

$\rho_B g h_B = \rho_M g h_M$
Eliminate the gravity terms,

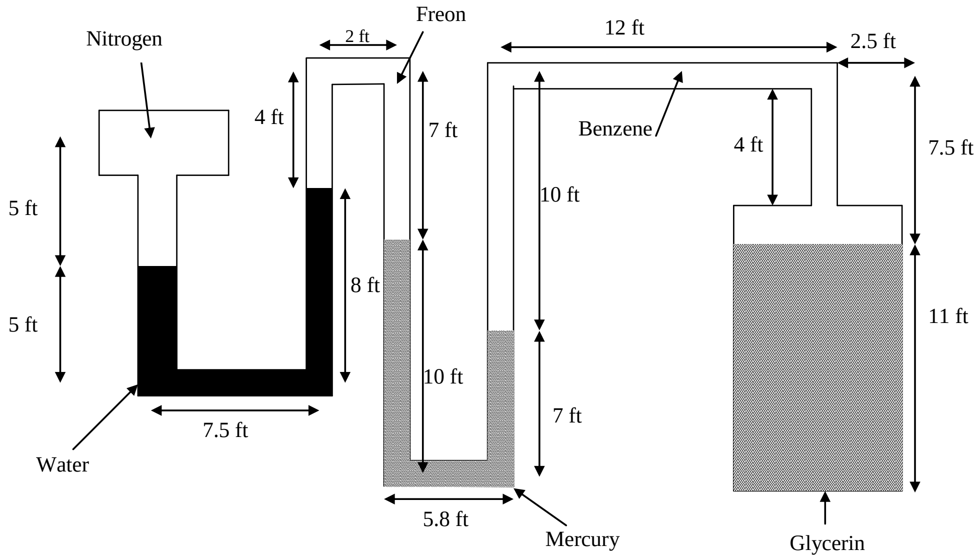
$\rho_B h_B = \rho_M h_M$

$$h_B = \frac{\rho_M}{\rho_B} h_M$$

We take 120 mm Hg as the height here, and 120 mm = 0.12 m, and the density of mercury is 845 lb_m/ft^3 = 13535.6 kg/m^3 :

$$h_B = \frac{(13535.6\ kg/m^3)}{(1060\ kg/m^3)}(0.12\ m) = 1.55\ m$$

2.32



$$P_N - P_1 = -\rho_{Nitrogen} g(5\ \text{ft}) \quad \text{so} \quad P_1 = P_N + \rho_{Nitrogen} g(5\ \text{ft})$$

$$P_2 - P_1 = -\rho_{H2O} g(8 - 5\ \text{ft}) \quad \text{so} \quad P_2 = P_1 - \rho_{H2O} g(8 - 5\ \text{ft})$$

$$P_2 - P_3 = -\rho_{Freon} g(7 - 4\ \text{ft}) \quad \text{so} \quad P_3 = P_2 + \rho_{Freon} g(7 - 4\ \text{ft})$$

$$P_3 - P_4 = -\rho_{Mercury} g(10 - 7\ \text{ft}) \text{ so } P_4 = P_3 + \rho_{Mercury} g(10 - 7\ \text{ft})$$

$$P_5 - P_4 = -\rho_{Benzene} g(10 - 7.5\ \text{ft}) \text{ so } P_5 = P_4 - \rho_{Benzene} g(10 - 7.5\ \text{ft})$$

$$P_5 - P_A = -\rho_A g(11\ \text{ft}) \text{ so } P_A = P_6 + \rho_A g(11\ \text{ft})$$

$$P_A = \rho_A g(11\ \text{ft}) - \rho_{Benzene} g(10 - 7.5\ \text{ft}) + \rho_{Mercury} g(10 - 7\ \text{ft}) + +\rho_{Freon} g(7 - 4\ \text{ft}) - \rho_{H2O} g(8 - 5\ \text{ft}) + \rho_{Nitrogen} g(5\ \text{ft}) + P_N$$

$$P_A = \frac{(78.2\ \text{lb}_m/\text{ft}^3)}{(32.174\ \text{lb}_m\text{ft}/\text{lb}_f\text{s}^2)}(32.2\ \text{ft/s}^2)(11\ \text{ft}) - \frac{(53.6\ \text{lb}_m/\text{ft}^3)}{(32.174\ \text{lb}_m\text{ft}/\text{lb}_f\text{s}^2)}(32.2\ \text{ft/s}^2)(10 - 7.5\ \text{ft}) + \frac{(843\ \text{lb}_m/\text{ft}^3)}{(32.174\ \text{lb}_m\text{ft}/\text{lb}_f\text{s}^2)}(32.2\ \text{ft/s}^2)(10 - 7\ \text{ft}) + \frac{\left(78.7\frac{\text{lb}_m}{\text{ft}^3}\right)}{\left(32.174\frac{\text{lb}_m\text{ft}}{\text{lb}_f\text{s}^2}\right)}\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(7 - 4\ \text{ft}) - \frac{\left(62.1\frac{\text{lb}_m}{\text{ft}^3}\right)}{\left(32.174\frac{\text{lb}_m\text{ft}}{\text{lb}_f\text{s}^2}\right)}\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(8 - 5\ \text{ft}) + \frac{\left(0.0685\frac{\text{lb}_m}{\text{ft}^3}\right)}{\left(32.174\frac{\text{lb}_m\text{ft}}{\text{lb}_f\text{s}^2}\right)}\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(5\ \text{ft}) + 4500\frac{\text{lb}_f}{\text{ft}^2} = 7808.0\ \text{lb}_f/\text{ft}^2$$