Problem 2.2

The deepest known spot in the oceans is the Challenger Deep in the Mariana Trench of the Pacific Ocean and is approximately $11,000 \text{ m}$ below the surface. Assume that the salt-water density is constant at $1025 \,\text{kg/m}^3$ and determine the pressure at this depth.

Solution 2.2

GIVEN: Density of the fluid is $1025 \dfrac{\text{kg}}{\text{m}^3}$, and the depth of the Challenger Deep is 11000 m.

FIND: Pressure at the depth of 11000 m.

SOLUTION:

$$p = \rho g h = \left(\frac{1025\text{kg}}{\text{m}^3}\right)\left(\frac{9.807\text{m}}{\text{s}^2}\right)(11000\text{m})\left(\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}}\right)\left(\frac{1\text{ Pa}}{1\,\frac{\text{N}}{\text{m}^2}}\right)$$

$$= 1.11\times 10^8 \text{ Pa}$$

$$\boxed{p = 111 \text{ MPa gage}}$$

Problem 2.3

A closed tank is partially filled with glycerin. If the air pressure in the tank is $6\ \text{lb}/\text{in.}^2$ and the depth of glycerin is $10\,\text{ft}$, what is the pressure in lb/ft^2 at the bottom of the tank?

Solution 2.3

$$p = \gamma h + p_0 = \left(78.6\frac{\text{lb}}{\text{ft}^3}\right)(10\text{ft}) + \left(6\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{144\,\text{in.}^2}{1\ \text{ft}^2}\right)$$

$$\boxed{p = 1{,}650\frac{\text{lb}}{\text{ft}^2}}$$

Problem 2.4

A 3-m-diameter vertical cylindrical tank is filled with water to a depth of $11\,\text{m}$. The rest of the tank is filled with air at atmospheric pressure. What is the absolute pressure at the bottom of the tank?

Solution 2.4

Known:water filled tank, $\text{dia.} = 3\,\text{m}$, $\text{depth} = 11\,\text{m}$

Determine:absolute pressure at tank bottom

Strategy: insert information into hydrostatic pressure distribution

Solution:

$$\begin{aligned} p_{\text{bottom}} &= p_{\text{atmos}} + \gamma_{\text{water}} h_{\text{bottom}} \\ &= 101\,\text{kPa} + \left(9.80\frac{\text{kN}}{\text{m}^3}\right)(11\,\text{m}) = 208.80\,\text{kPa} \end{aligned}$$

$$\boxed{p_{\text{bottom}} = 209\,\text{kPa}}$$

Problem 2.5

Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). Such pressures are commonly measured with a mercury manometer. A typical value for this ratio for a human would be 120/70, where the pressures are in mm Hg. **(a)** What would these pressures be in pascals? **(b)** If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

Solution 2.5

$$p = \gamma h$$

(a) For $120\,\text{mm Hg}$: $p = \left(133\times10^{3}\,\dfrac{\text{N}}{\text{m}^3}\right)(0.120\,\text{m}) \rightarrow \boxed{p = 16.0\text{ kPa}}$

For $70\,\text{mm Hg}$: $p = \left(133\times10^{3}\,\dfrac{\text{N}}{\text{m}^3}\right)(0.070\,\text{m}) \rightarrow \boxed{p = 9.31\text{ kPa}}$

(b) For $120\,\text{mm Hg}$: $p = \left(16.0\times10^{3}\,\dfrac{\text{N}}{\text{m}^2}\right)\left(1.450\times10^{-4}\,\dfrac{\text{lb/in.}^2}{\text{N/m}^2}\right) = 2.32\,\text{psi}$

Typical tire pressure is $30-35\,\text{psi}$, therefore.

$\boxed{\text{120 mm is insufficent inflation for normal driving}}$

Problem 2.6

An unknown immiscible liquid seeps into the bottom of an open oil tank. Some measurements indicate that the depth of the unknown liquid is $1.5\ \mathrm{m}$ and the depth of the oil $\left(\text{specific weight} = 8.5\ \mathrm{kN/m^3}\right)$ floating on top is $5.0\ \mathrm{m}$. A pressure gage connected to the bottom of the tank reads $65\ \mathrm{kPa}$. What is the specific gravity of the unknown liquid?

Solution 2.6

$$p_{\text{bottom}} = (\gamma_{\text{oil}})(5\mathrm{m}) + (\gamma_{\text{u}})(1.5\mathrm{m}) \quad \text{where } \gamma_{\text{u}} \text{ □ unknown liquid } \gamma$$

$$\gamma_{\text{u}} = \frac{p_{\text{botton}} - (\gamma_{\text{oil}})(5\ \mathrm{m})}{1.5\ \mathrm{m}} = \frac{65\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^2} - \left(8.5\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^3}\right)(5\ \mathrm{m})}{1.5\ \mathrm{m}} = 15\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^3}$$

$$SG = \frac{\gamma_{\text{u}}}{\gamma_{\mathrm{H_2O}} @ 4^\circ C} = \frac{15\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^3}}{9.81\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^3}} \rightarrow \boxed{SG = 1.53}$$

Problem 2.7

A 30-ft- high downspout of a house is clogged at the bottom. Find the pressure at the bottom if the downspout is filled with 60 °F rainwater.

Solution 2.7

$P = \rho g h$

Inserting the density f water and the specified column height:

$$P = \frac{\left(62.4\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(30\text{ft})}{\left(32.2\frac{\text{ft}\cdot\text{lbm}}{\text{lb}\cdot\text{s}^2}\right)\left(144\frac{\text{in}^2}{\text{ft}^2}\right)} \rightarrow \boxed{P = 13.0\text{ psig}}$$

Problem 2.8

How high a column of SAE 30 oil would be required to give the same pressure as 700 mm Hg?

Solution 2.8

$$p = \gamma h$$

For $p_{\mathrm{Hg}} = p_{\mathrm{oil}}$

$$\gamma_{\mathrm{Hg}} h_{\mathrm{Hg}} = \gamma_{\mathrm{oil}} h_{\mathrm{oil}}$$

or

$$h_{\mathrm{oil}} = \frac{\gamma_{\mathrm{Hg}}}{\gamma_{\mathrm{oil}}} h_{\mathrm{Hg}} = \frac{\left(133 \frac{\mathrm{kN}}{\mathrm{m}^3}\right)}{\left(8.95 \frac{\mathrm{kN}}{\mathrm{m}^3}\right)} (0.700\mathrm{m}) \quad \rightarrow \quad \boxed{h_{\mathrm{oil}} = 10.4 \text{ m}}$$

Problem 2.9
Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of $10.1\ \mathrm{kN/m^3}$? Express your answer in pascals and psi.

Solution 2.9

$$p = \gamma h + p_0$$

At the surface $p_0 = 0$ so that

$$p = \left(10.1\times 10^3\ \frac{\mathrm{N}}{\mathrm{m}^3}\right)\left(5\times 10^3\,\mathrm{m}\right) = 50.5\times 10^6\ \frac{\mathrm{N}}{\mathrm{m}^2} \quad \rightarrow \quad \boxed{p = 50.5\,\mathrm{MPa}}$$

$$p = \left(50.5\times 10^6\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\frac{1.450\times 10^{-4}\ \dfrac{\mathrm{lb}}{\mathrm{in.}^2}}{1\ \dfrac{\mathrm{N}}{\mathrm{m}^2}}\right) \quad \rightarrow \quad \boxed{p = 7320\ \mathrm{psi}}$$

Problem 2.10

The deepest known spot in the oceans is the Challenger Deep in the Mariana Trench of the Pacific Ocean and is approximately $11,000\text{ m}$ below the surface. For a surface density of 1030 kg/m^3, a constant water temperature, and an isothermal bulk modulus of elasticity of $2.3\times10^9\text{ N/m}^2$, find the pressure at this depth.

Solution 2.10

GIVEN: Ocean depth of 11,000 m, surface density of 1030 kg/m^3, constant water temperature, and isothermal bulk modulus of elasticity $E_{V,T} = 2.3 \text{ x } 10^9\text{ N/m}^2$.

FIND: Pressure at this depth

SOLUTION:

$$E_v \equiv -\frac{dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho}$$

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_{p_0}^{p}\frac{dp}{E_v} \rightarrow \ln\left(\frac{\rho}{\rho_0}\right) = \frac{p-p_0}{E_v} \rightarrow \rho = \rho_0 e^{\left(\frac{p-p_0}{E_v}\right)}$$

where: z is positive upward, z=z_0 at the surface, and $p(z=0)=p_0$, $\rho(z=0)=\rho_0$

Substitution into the hydrostatic pressure equation yields:

$$\frac{dp}{dz} = -\gamma = -\rho g = -\rho_0 g e^{\left(\frac{p-p_0}{E_v}\right)}$$

$$\int_{p_0}^{p} e^{-\left(\frac{p-p_0}{E_v}\right)}dp = -E_v\left[e^{-\left(\frac{p-p_0}{E_v}\right)}\right]_{p_0}^{p} = -E_v\left(e^{-\left(\frac{p-p_0}{E_v}\right)}-1\right) = -\rho_0 g\int_0^z dz = -\rho_0 gz$$

$$e^{-\left(\frac{p-p_0}{E_v}\right)} = 1+\frac{\rho_0 gz}{E_v}$$

$$-\left(\frac{p-p_0}{E_v}\right) = \ln\left(1+\frac{\rho_0 gz}{E_v}\right)$$

$$p = p_0 - E_v\ln\left(1+\frac{\rho_0 gz}{E_v}\right) = 0-\left(2.3\times10^9\frac{\text{N}}{\text{m}^2}\right)\ln\left(1+\frac{\left(1030\frac{\text{kg}}{\text{m}^3}\right)\left(9.807\frac{\text{m}}{\text{s}^2}\right)(-11000\text{ m})}{\left(2.3\times10^9\frac{\text{N}}{\text{m}^2}\right)\left(\frac{1\text{ kg}\cdot\text{m}}{1\text{ N}\cdot\text{s}^2}\right)}\right)$$

$$p = 1.14\times10^8\frac{\text{N}}{\text{m}^2} \rightarrow \boxed{p = 114\text{ MPa gage}}$$

Problem 2.11

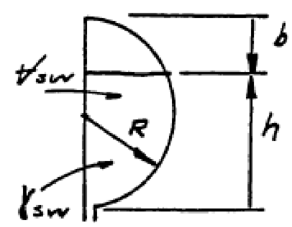
A submarine submerges by admitting seawater S= 1.03 into its ballast tanks. The amount of water admitted is controlled by air pressure, because seawater will cease to flow into the tank when the internal pressure (at the hull penetration) is equal to the hydrostatic pressure at the depth of the submarine. Consider a ballast tank, which can be modeled as a vertical half-cylinder ($R = 8$ ft, $L = 20$ ft) for which the air pressure control valve has failed shut. The failure occurred at the beginning of a dive from 60 ft to 1000 ft. The tank was initially filled with seawater to a depth of 2 ft and the air was at a temperature of 40 °F. As the weight of water in the tank is important in maintaining the boat's attitude, determine the weight of water in the tank as a function of depth during the dive. You may assume that tank internal pressure is always in equilibrium with the ocean's hydrostatic pressure and that the inlet pipe to the tank is at the bottom of the tank and penetrates the hull at the "depth" of the submarine.

Solution 2.11

GIVEN: Ballast tank, $\mathrm{L} = 20\,\mathrm{ft}$, $\mathrm{R} = 8\,\mathrm{ft}$, initial condition of $\mathrm{d} = 60\,\mathrm{ft}$, $\mathrm{h} = 2\,\mathrm{ft}$., $\mathrm{T_{air}} = 40°\mathrm{F}$,
Final condition d= 1,000 ft.

FIND: Weight of water in ballast tank as a function of depth d during dive.

The water volume in the ballast tank is determined using given information about the ballast tank,



$$V_{\mathrm{sw\,i}} = \frac{L}{2}\left[R^2 \sin^{-1}\left(\frac{\sqrt{2Rh - h^2}}{R}\right) - (R - h)\sqrt{2Rh - h^2}\right]$$

$$= \frac{20\mathrm{ft}}{2}\left[(8\,\mathrm{ft})^2 \sin^{-1}\left(\frac{\sqrt{2\cdot 8\cdot 2 - 2^2}\,\mathrm{ft}}{8\,\mathrm{ft}}\right) - \left[(8-2)\,\mathrm{ft}\right]\cdot\sqrt{2\cdot 8\cdot 2 - 2^2}\,\mathrm{ft}\right]$$

$$= 145\ \mathrm{ft}^3$$

The initial air volume is

$$V_{\mathrm{air\,i}} = \frac{\pi R^2 L}{2} \cdot \cancel{V}_{\mathrm{wi}} = \frac{\pi}{2}(8\ \mathrm{ft})^2 (20\ \mathrm{ft}) \cdot 145\ \mathrm{ft}^3 = 1866\ \mathrm{ft}^3$$

During the dive the ballast tank air pressure is assumed to be in equilibrium with the ocean hydrostatic pressure. Then

$$P_{\mathrm{air}} = P_{\mathrm{atm}} + \gamma_{\mathrm{sw}}(d - h)$$

Using the ideal gas law,

$$\frac{M_{\mathrm{air}} R T_{\mathrm{air}}}{V_{\mathrm{air}}} = \frac{M_{\mathrm{air}} R T_{\mathrm{air}}}{V_{\mathrm{tank}} - V_{\mathrm{sw}}} = P_{\mathrm{atm}} + \gamma_{\mathrm{sw}}(h - d)$$

Solving for h,

$$h = d + \frac{P_{\text{atm}}}{\gamma_{\text{sw}}} - \frac{M_{\text{air}} R T_{\text{air}}}{\gamma_{\text{sw}} \left(V_{\text{tank}} - V_{\text{sw}} \right)} \tag{2}$$

V_{sw} is a function of h, given by equation (1) for $h < R$. For $h > R$, define $b = 2R - h$ and

$$V_{\text{sw}} = \frac{\pi R^2 L}{2} - \frac{L}{2} \left[R^2 \sin^{-1} \left(\frac{\sqrt{2Rb - b^2}}{R} \right) - (R - b) \sqrt{2Rb - b^2} \right] \tag{3}$$

The air mass M_{air} is

$$M_{\text{air}} = \rho_{\text{air i}} V_{\text{air i}} = \left(0.219 \frac{\text{lbm}}{\text{ft}^3} \right) \left(1866\,\text{ft}^3 \right)$$
$$= 408.7\ \text{lbm}$$

The initial weight of the water is

$$W_{\text{sw i}} = \gamma_{\text{sw}} V_{\text{sw i}} = \left(62.4 \times 1.03 \frac{\text{lb}}{\text{ft}^3} \right) \left(145\,\text{ft}^3 \right) = 9320\,\text{lb}\,.$$

Pseudocode for procedural language:

$h_{\min} = 0$

$h_{\max} = 2R$

$h_{\text{new}} = \dfrac{(h_{\min} + h_{\max})}{2}$

$p_{\text{tank}} = p_{\text{air}} + \gamma_{\text{sw}} h$

$p_{\text{ocean}} = p_{\text{atm}} + \gamma_{\text{sw}} d$

$p_{\text{tank}} < p_{\text{ocean}} \Rightarrow h_{\min} = h_{\text{new}}$

$p_{\text{tank}} > p_{\text{ocean}} \Rightarrow h_{\max} = h_{\text{new}}$

TRUE

If $\left(\dfrac{(h_{\max} - h_{\min})}{h_{\min}} < \text{errtol} \right)$

FALSE

$h = \dfrac{(h_{\min} + h_{\max})}{2}$

Problem 2.12

Determine the pressure at the bottom of an open 5-m- deep tank in which a chemical process is taking place that causes the density of the liquid in the tank to vary as

$$\rho = \rho_{\text{surf}} \sqrt{1 + \sin^2\left(\frac{h}{h_{\text{bot}}} \frac{\pi}{2}\right)},$$

where h is the distance from the free surface and $\rho_{\text{surf}} = 1700 \text{ kg/m}^3$.

Solution 2.12

GIVEN:

$$H_{\text{bot}} = 5\,\text{m},\; \rho_{\text{surf}} = 1700\ \frac{\text{kg}}{\text{m}^3};\; \rho = \rho_{\text{surf}} \sqrt{1 + \sin^2\left(\frac{h}{h_{\text{bot}}} \frac{\pi}{2}\right)},$$

FIND: Pressure at bottom of tank.

SOLUTION:

The pressure gradient is

$$\frac{dP}{dh} = \rho = \gamma$$

Separating variables, substituting for ρ, and integrating give

$$\int_{P_{\text{surf}}}^{P} dP = \rho_{\text{surf}}\, g \int_0^h \sqrt{1 + \sin^2\left(\frac{h}{h_{\text{bot}}} \frac{\pi}{2}\right)}\,dh$$

This integral must be solved numerically.

The Romberg method gives for $h = h_{\text{bot}}$

$$P_{\text{bot}} - P_{\text{surf}} = \rho_{\text{surf}}\, g\,(6.080\,\text{m})$$

Setting $P_{\text{surf}} = 0$

$$P_{\text{bot}} = \left(1700 \frac{\text{N}}{\text{m}^3}\right)\left(9.807 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right)(6.080\,\text{m}) \rightarrow \boxed{P_{\text{bot}} = 101 \times 10^3\ \frac{\text{N}}{\text{m}^2}}$$

Problem 2.13

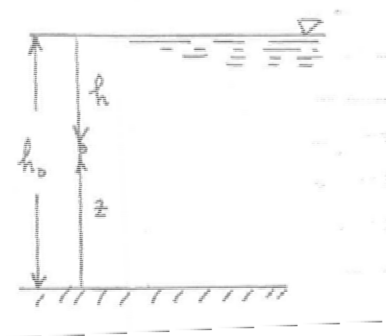
In a certain liquid at rest, measurements of the specific weight at various depths show the following variation:

h (ft)	γ (lb/ft^3)
0	70
10	76
20	84
30	91
40	97
50	102
60	107
70	110
80	112
90	114
100	115

The depth $h = 0$ corresponds to a free surface at atmospheric pressure. Determine, through numerical integration of $\frac{dp}{dz} = -\gamma$, the corresponding variation in pressure and show the results on a plot of pressure (in psf) versus depth (in feet).

Solution 2.13

$$\frac{dp}{dz} = -\gamma$$



Let $z = h_0 - h$ (see figure) so

that $dz = -dh$ and therefore

$$dp = -\gamma dz = \gamma dh$$

Thus,

$$\int_0^{p_i} dp = \int_0^{h_i} \gamma dh$$

or

$$p_i = \int_0^{h_i} \gamma dh \qquad (1)$$

where p_i is the pressure at depth h_i.

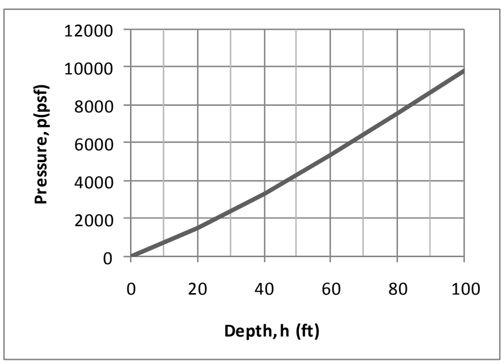
Equation (1) can be integrated numerically using the trapezoidal rule, i.e.,

$$I = \frac{1}{2}\sum_{i=1}^{n-1}\left(y_i + y_{i+1}\right)\left(x_{i+1} - x_i\right)$$

where y □ γ, x □ h, and n = number of data points.

The tabulated results are given below, along with the corresponding plot of pressure vs. depth.

h (ft)	ɣ (lb/ft^3)	Pressure, psf
0	70	0
10	76	730
20	84	1530
30	91	2405
40	97	3345
50	102	4340
60	107	5385
70	110	6470
80	112	7580
90	114	8710
100	115	9855



Problem 2.15

Under normal conditions the temperature of the atmosphere decreases with increasing elevation. In some situations, however, a temperature inversion may exist so that the air temperature increases with elevation. A series of temperature probes on a mountain give the elevation–temperature data shown in the table below. If the barometric pressure at the base of the mountain is 12.1 psia, determine by means of numerical integration the pressure at the top of the mountain.

Elevation (ft)	Temperature (°F)
5000	50.1 (base)
5500	55.2
6000	60.3
6400	62.6
7100	67.0
7400	68.4
8200	70.0
8600	69.5
9200	68.0
9900	67.1 (top)

Solution 2.15

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

In the table below the temperature in $°R$ is given and the integrand $\dfrac{1}{T(°R)}$ tabulated.

Elevation, ft	T, ˚F	T, ˚R	1/T(˚R)
5000	50.1	509.8	0.001962
5500	55.2	514.9	0.001942
6000	60.3	520.0	0.001923
6400	62.6	522.3	0.001915
7100	67.0	526.7	0.001899
7400	68.4	528.1	0.001894
8200	70.0	529.7	0.001888
8600	69.5	529.2	0.00189
9200	68.0	527.7	0.001895
9900	67.1	526.8	0.001898

The approximate value of the integral in $\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$ is 9.34 obtained using the trapezoidal rule, i.e.,

$$I = \frac{1}{2} \sum_{i=1}^{n-1} (y_i + y_{i+1})(x_{i+1} - x_i) \text{ where } y \equiv \frac{1}{T},\ x \equiv \text{elevation},$$

and $n =$ number of data points. Thus,

$$\int_{5000\,\text{ft}}^{9900\,\text{ft}} \left(\frac{1}{T}\right) dz = 9.34 \frac{\text{ft}}{°\text{R}}$$

so that (with $g = 32.2 \text{ft/s}^2$ and $R = 1716\,\text{ft} \cdot \text{lb/slug} \cdot °\text{R}$)

$$\ln \frac{p_2}{p_1} = -\frac{\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)\left(9.34\,\frac{\text{ft}}{°\text{R}}\right)}{1716\,\text{ft}\cdot\text{lb/slug}\cdot°\text{R}} = -0.1753 \qquad (1)$$

It follows from Eq.(1) with $p_1 = 12.1\,\text{psia}$ that

$$p_2 = (12.1\,\text{psia})e^{-0.1753} = \underline{\underline{10.2\,\text{psia}}}$$

(Note: Since the temperature variation is not very large, it would be expected that the assumption of a constant temperature would give good results. If the temperature is assumed to be constant at the base temperature ($50.1°\text{F}$), $p_2 = 10.1\,\text{psia}$, which is only slightly different from the result given above.)

Problem 2.16

Often young children drink milk ($\rho = 1030\ \text{kg/m}^3$) through a straw. Determine the maximum length of a vertical straw that a child can use to empty a milk container, assuming that the child can develop $75\ \text{mm Hg}$ of suction, and use this answer to determine if you think this is a reasonable estimate of the suction that a child can develop.

Solution 2.16

Known: $\rho_{\text{milk}} = 1030\ \dfrac{\text{kg}}{\text{m}^3}$, suction $= 75$ mm Hg

Determine: maximum length of vertical straw, is this reasonable?

Strategy: compute height of equivalent milk column

Solution:

$h_{\text{max}} =$ height of milk column lifted by suction

$\Delta p_{\text{max}} = \gamma_{\text{Hg}} h_{\text{Hg}} = 75$ mm Hg

$\Delta p_{\text{max}} = \gamma_{\text{milk}} h_{\text{milk}}$

$$h_{\text{max,milk}} = \frac{\Delta p_{\text{max}}}{\gamma_{\text{milk}}} = \frac{\gamma_{\text{Hg}} h_{\text{Hg}}}{\gamma_{\text{milk}}} = \left(\frac{\rho_{\text{Hg}}}{\rho_{\text{milk}}}\right) h_{\text{Hg}} = \left(\frac{13600\,\dfrac{\text{kg}}{\text{m}^3}}{1030\,\dfrac{\text{kg}}{\text{m}^3}}\right)(75\ \text{mm})$$

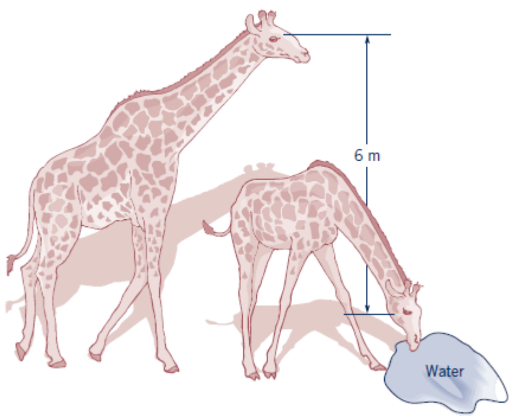
$h_{\text{max,milk}} = 990.3$ mm milk

$\boxed{\text{max. length straw} \approx 1\ \text{m}}$

Although this may seem large, adults can routinely lift water much higher through a straw. Therefore, a 1 m draw seems large, but within reason for a child.

Problem 2.17

(a) Determine the change in hydrostatic pressure in a giraffe's head as it lowers its head from eating leaves 6 m above the ground to getting a drink of water at ground level as shown in the figure below. Assume the specific gravity of blood is SG = 1. (b) Compare the pressure change calculated in part (a) to the normal 120 mm of mercury pressure in a human's heart.



Solution 2.17

(a) For hydrostatic pressure change,

$$\Delta p = \gamma h = \left(9.80\frac{\text{kN}}{\text{m}^3}\right)(6\text{m}) = 58.8\frac{\text{kN}}{\text{m}^2} \quad \rightarrow \quad \boxed{\Delta p = 58.8\text{ kPa}}$$

(b) To compare with pressure in human heart convert pressure in part (a) to mm Hg:

$$58.8\frac{\text{kN}}{\text{m}^2} = \gamma_{\text{Hg}} h_{\text{Hg}} = \left(133\frac{\text{kN}}{\text{m}^3}\right)h_{\text{Hg}}$$

$$h_{\text{Hg}} = (0.442\text{m})(10^3\frac{\text{mm}}{\text{m}}) = 442\,\text{mm}\,\text{Hg}$$

giraffe $h_{\text{Hg}} = 442$ mmHg
human $h_{\text{Hg}} = 120$ mmHg
girrafe more than 3.5 times greater

Problem 2.18

What would be the barometric pressure reading, in mm Hg, at an elevation of 4 km in the U.S. standard atmosphere? Refer to Table C.2 Properties of the U.S. Standard Atmosphere (SI Units).

Solution 2.18

At an elevation of 4 km,

$p = 6.166 \times 10^4 \dfrac{\text{N}}{\text{m}^2}$ (from the table given in the Problem). Since

$$p = \gamma h$$

$$h = \frac{p}{\gamma} = \frac{6.166 \times 10^4 \dfrac{\text{N}}{\text{m}^2}}{133 \times 10^3 \dfrac{\text{N}}{\text{m}^3}} = 0.464 \text{ m} \quad \rightarrow \quad \boxed{h = 464 \text{ mm}}$$

Problem 2.19

Denver, Colorado, is called the "mile-high city" because its state capitol stands on land 1 mi above sea level. Assuming that the Standard Atmosphere exists, what is the pressure and temperature of the air in Denver? The temperature follows the lapse rate ($T = T_0 - Bz$).

Solution 2.19

GIVEN: Denver altitude = $1\,\text{mile} = 5280\text{ ft}$ and standard atmosphere. $T = T_0 - Bz$

FIND: Temperature and pressure in Denver.

SOLUTION:

The Lapse rate gives:

$$T = T_0 - Bz = 518.67\ °\text{R} - \left(0.003566\frac{°\text{R}}{\text{ft}}\right)(5280\text{ ft})$$

$$\boxed{T = 500\ °\text{R} = 40\ °\text{F}}$$

Using Equation and Table:

$$P = P_0\left(1 - \frac{\beta z}{T_0}\right)^{\frac{g}{\beta B}}$$

$$= (14.696\,\text{psia})\left(1 - \frac{\left(0.003566\frac{°\text{R}}{\text{ft}}\right)(5280\,\text{ft})}{518.67°\text{R}}\right)^{\frac{\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(32.2\frac{\text{lb}\cdot\text{s}^2}{\text{ft}\cdot\text{lbm}}\right)}{\left(53.35\frac{\text{ft}\cdot\text{lb}}{\text{lbm}\cdot°\text{R}}\right)\left(\frac{0.003566°\text{R}}{\text{ft}}\right)}}$$

$$\boxed{p = 12.10\text{ psia}}$$

Note: In reasonably good agreement with table in appendix of text.

Problem 2.20

Assume that a person skiing high in the mountains at an altitude of 15,000 ft takes in the same volume of air with each breath as she does while walking at sea level. Determine the ratio of the mass of oxygen inhaled for each breath at this high altitude compared to that at sea level.

Solution 2.20

Let $(\ )_0$ denote sea level and $(\ )_{15}$ denote $15{,}000\text{ ft}$ altitude.

Thus, since $m = mass = \rho V$, where $V = volume$,

$$m_0 = \rho_0 V_0$$

$$m_{15} = \rho_{15} V_{15}, \text{ where } V_0 = V_{15}$$

$$\frac{m_{15}}{m_0} = \frac{\rho_{15} V_{15}}{\rho_0 V_0} = \frac{\rho_{15}}{\rho_0}$$

If it is assumed that the air composition (e.g., % of air that is oxygen) is the same at sea level as it is at $15{,}000\text{ ft}$, use the density values from table of Properties of the U.S. Standard Atmosphere (BG/EE Units)

$$\rho_0 = 2.377 \times 10^{-3}\,\frac{\text{slugs}}{\text{ft}^3} \text{ and } \rho_{15} = 1.496 \times 10^{-3}\,\frac{\text{slugs}}{\text{ft}^3} \text{ so that}$$

$$\frac{m_{15}}{m_0} = \frac{1.496 \times 10^{-3}\,\dfrac{\text{slugs}}{\text{ft}^3}}{2.377 \times 10^{-3}\,\dfrac{\text{slugs}}{\text{ft}^3}} = 0.629 \quad \rightarrow \quad \boxed{m_{15} = 62.9\% \text{ of } m_0}$$

Problem 2.21

Pikes Peak near Denver, Colorado, has an elevation of $14{,}110\ \text{ft}$. (a) Determine the pressure at this elevation, based on the equation below. (b) If the air is assumed to have a constant specific weight of $0.07647\ \text{lb/ft}^3$, what would the pressure be at this altitude? (c) If the air is assumed to have a constant temperature of $59\,°\text{F}$, what would the pressure be at this elevation? For all three cases assume standard atmospheric conditions at sea level as provided in the table of Properties of U.S. Standard Atmosphere at Sea Level).

Solution 2.21

(a)

$$p = p_a\left(1-\frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}$$

$$\frac{g}{R\beta} = \frac{32.174\dfrac{\text{ft}}{\text{s}^2}}{\left(1716\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°\text{R}}\right)\left(0.00357\dfrac{°\text{R}}{\text{ft}}\right)} = 5.252$$

$$p = \left(2116.2\frac{\text{lb}}{\text{ft}^2}\right)\left[1-\frac{\left(0.00357\dfrac{°\text{R}}{\text{ft}}\right)(14110\text{ft})}{518.67°\text{R}}\right]^{5.252} \quad\rightarrow\quad \boxed{p = 1240\frac{\text{lb}}{\text{ft}^2}\ \text{(abs)}}$$

(b) $$p = p_a - \gamma h = 2116.2\frac{\text{lb}}{\text{ft}^2} - \left(0.07647\frac{\text{lb}}{\text{ft}^3}\right)(14110\text{ft}) \quad\rightarrow\quad \boxed{p = 1040\frac{\text{lb}}{\text{ft}^2}\ \text{(abs)}}$$

(c) $$p = p_a e^{-\frac{gh}{RT_a}}$$

$$p = \left(2116.2\frac{\text{lb}}{\text{ft}^2}\right)e^{-\left[\frac{\left(32.174\frac{\text{ft}}{\text{s}^2}\right)(14110\text{ft})}{\left(1716\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°\text{R}}\right)(518.67°\text{R})}\right]} \quad\rightarrow\quad \boxed{p = 1270\frac{\text{lb}}{\text{ft}^2}\ \text{(abs)}}$$

Problem 2.22

Equation $p = p_a\left(1-\frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}$ provides the relationship between pressure and elevation in the atmosphere for those regions in which the temperature varies linearly with elevation. Derive this equation and verify the value of the pressure given in the table of Properties of the U.S. Standard Atmosphere (SI Units) for an elevation of $5\,\text{km}$.

Solution 2.22

$$\int_{p_1}^{p_2}\frac{dp}{p} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{T}$$

Let p_1 □ p_a for $z_1 = 0$, p_2 □ p for $z_2 = z$, and $T = T_a - \beta z$.

$$\int_{p_a}^{p}\frac{dp}{p} = -\frac{g}{R}\int_{0}^{z}\frac{dz}{T_a - \beta z}$$

$$\ln\frac{p}{p_a} = -\frac{g}{R}\left[-\frac{1}{\beta}\ln\left(T_a - \beta z\right)\right]_0^z = \frac{g}{R\beta}\left[\ln\left(T_a - \beta z\right) - \ln T_a\right] = \frac{g}{R\beta}\ln\left(1 - \frac{\beta z}{T_a}\right)$$

$$\boxed{p = p_a\left(1-\frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}}$$

For $z = 5\,\text{km}$:

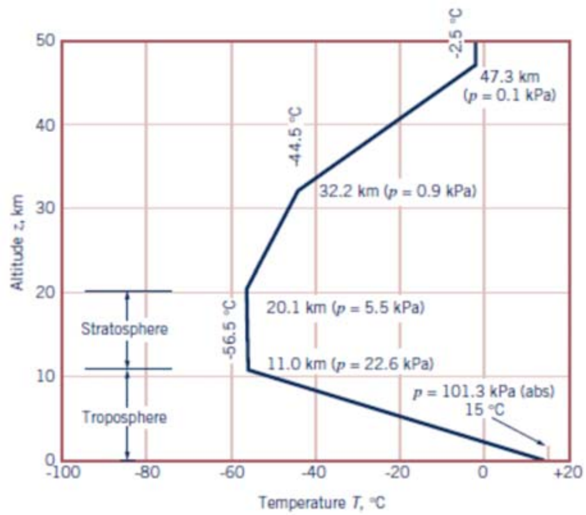
$p_a = 101.33\,\text{kPa}$, $T_a = 288.15\,\text{K}$, $g = 9.807\,\frac{\text{m}}{\text{s}^2}$, $\beta = 0.00650\,\frac{\text{K}}{\text{m}}$, $R = 287\,\frac{\text{J}}{\text{kg}\cdot\text{K}}$,

$$p = \left(101.33\,\text{kPa}\right)\left[1 - \frac{\left(0.0065\,\frac{\text{K}}{\text{m}}\right)\left(5\times10^3\,\text{m}\right)}{288.15\,\text{K}}\right]^{\frac{9.807\,\frac{\text{m}}{\text{s}^2}}{\left(287\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(0.0065\,\frac{\text{K}}{\text{m}}\right)}}$$

$$\rightarrow \boxed{\begin{array}{l} p_{\text{computed}} = 5.40\times10^4\,\frac{\text{N}}{\text{m}^2} \\ p_{tabulated} = 5.405\times10^4\,\frac{\text{N}}{\text{m}^2} \end{array}}$$

Problem 2.23

As shown in the figure below for the U.S. standard atmosphere, the troposphere extends to an altitude of $11\,\text{km}$ where the pressure is $22.6\,\text{kPa}\,(\text{abs})$. In the next layer, called the stratosphere, the temperature remains constant at $-56.5\,°\text{C}$. Determine the pressure and density in this layer at an altitude of $15\,\text{km}$. Assume $g = 9.77\,\text{m/s}^2$ in your calculations. Compare your results with those given in Table C.2 Properties of the U.S. Standard Atmosphere (SI Units).



Solution 2.23

For isothermal conditions,

$$p_2 = p_1 e^{\frac{-g(z_2 - z_1)}{RT_0}}$$

Let $z_1 = 11\,\text{km}$, $p_1 = 22.6\,\text{kPa}$, $R = 287\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$, $g = 9.77\dfrac{\text{m}}{\text{s}^2}$,

$T_0 = -56.5\,°\text{C} + 273.15 = 216.65\,\text{K}$.

$$p_2 = (22.6\,\text{kPa})\,e^{-\left[\frac{\left(9.77\frac{\text{m}}{\text{s}^2}\right)\left(15\times10^3\,\text{m}-11\times10^3\,\text{m}\right)}{\left(287\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(216.65\,\text{K})}\right]} = 12.1\,\text{kPa}$$

Using the ideal gas model:

$$\rho_2 = \frac{p}{RT} = \frac{12.1\times10^3\,\dfrac{\text{N}}{\text{m}^2}}{\left(287\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(216.65\,\text{K})} = 0.195\,\frac{\text{kg}}{\text{m}^3}$$

In comparison to published values:

$$\boxed{\begin{array}{ll} p_{\text{computed}} = 12.1\,\text{kPa}, & \rho = 0.195\dfrac{\text{kg}}{\text{m}^3} \\ p_{\text{tabulatted}} = 12.11\,\text{kPa}, & \rho = 0.1948\dfrac{\text{kg}}{\text{m}^3} \end{array}}$$

Problem 2.24
The record low sea-level barometric pressure ever recorded is $25.8\,\text{in.}$ of mercury. At what altitude in the standard atmosphere is the pressure equal to this value?

Solution 2.24
For record low pressure,

$$p = \gamma_{Hg} h_{Hg} = \left(847\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{25.8\,\text{in.}}{12\dfrac{\text{in.}}{\text{ft}}}\right)\left(\frac{\text{ft}^2}{144\,\text{in.}^2}\right) = 12.6\frac{\text{lb}}{\text{in.}^2}$$

From Table C.1 Properties of the U.S. Standard Atmosphere (BG/EE Units)

$$@\ 0\,\text{ft}\ \ \text{altitude}\ \ p = 14.696\frac{\text{lb}}{\text{in.}^2}$$

$$@\ 5000\,\text{ft altitude}\ \ p = 12.228\frac{\text{lb}}{\text{in.}^2}$$

Assume linear variation change in pressure per foot.

$$\text{Thus, pressure change per foot} = \frac{14.696\dfrac{\text{lb}}{\text{in.}^2} - 12.228\dfrac{\text{lb}}{\text{in.}^2}}{5000\,\text{ft}} = 4.936\times 10^{-4}\,\frac{\dfrac{\text{lb}}{\text{in.}^2}}{\text{ft}}$$

$$14.696\frac{\text{lb}}{\text{in.}^2} - a(\text{ft})\left[4.936\times 10^{-4}\,\frac{\dfrac{\text{lb}}{\text{in.}^2}}{\text{ft}}\right] = 12.6\frac{\text{lb}}{\text{in.}^2}$$

$$a = \frac{14.696 - 12.6}{4.936\times 10^{-4}}\,\text{ft} \quad\rightarrow\quad \boxed{a = 4250\ \text{ft}}$$

Problem 2.25

On a given day, a barometer at the base of the Washington Monument reads 29.97 in. of mercury. What would the barometer reading be when you carry it up to the observation deck 500 ft above the base of the monument?

Solution 2.25

Let $(\ )_{\text{b}}$ and $(\ )_{\text{od}}$ correspond to the base and observation deck, respectively.

Thus, with H = height of the monument,

$$p_{\text{b}} - p_{\text{od}} = \gamma_{\text{air}} H = 7.65\times 10^{-2}\,\frac{\text{lb}}{\text{ft}^3}(500\,\text{ft}) = 38.5\frac{\text{lb}}{\text{ft}^2}$$

$p = \gamma_{\text{Hg}} h$, where $\gamma_{\text{Hg}} = 847\frac{\text{lb}}{\text{ft}^3}$ and h = barometer reading.

$$\gamma_{\text{Hg}}\left(\frac{29.97}{12}\text{ft}\right) - \gamma_{\text{Hg}} h_{\text{od}} = 38.5\frac{\text{lb}}{\text{ft}^2}$$

$$h_{\text{od}} = \left(\frac{29.97}{12}\text{ft}\right) - \frac{38.5\frac{\text{lb}}{\text{ft}^2}}{847\frac{\text{lb}}{\text{ft}^3}} \times 12\frac{\text{in.}}{\text{ft}} = (29.97 - 0.545)\text{in.} \quad \rightarrow \quad \boxed{h_{\text{od}} = 29.43\,\text{in.}}$$

Problem 2.26

Aneroid barometers can be used to measure changes in altitude. If a barometer reads 30.1 in. Hg at one elevation, what has been the change in altitude in meters when the barometer reading is 28.3 in. Hg? Assume a standard atmosphere and that the equation below is applicable over the range of altitudes of interest.

Solution 2.26

$$p = p_a\left(1-\frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}$$

$$z_1 : p_1 = p_a\left(1-\frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \rightarrow \left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} = 1-\frac{\beta z_1}{T_a} \rightarrow z_1 = \frac{T_a}{\beta}-\frac{T_a}{\beta}\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}}$$

$$z_2 = \frac{T_a}{\beta}-\frac{T_a}{\beta}\left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}}$$

$$z_2 - z_1 = \frac{T_a}{\beta}\left[\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} - \frac{T_a}{\beta}\left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}}\right]$$

For $T_a = 288\text{K}$, $\beta = 0.00650\dfrac{\text{K}}{\text{m}}$, $P_a = 101\text{kPa}$, $g = 9.81\dfrac{\text{m}}{\text{s}^2}$, $R = 287\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$

$$\frac{R\beta}{g} = \frac{\left(287\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(0.00650\dfrac{\text{K}}{\text{m}}\right)}{9.81\dfrac{\text{m}}{\text{s}^2}} = 0.190$$

$$p_1 = \gamma_{\text{Hg}}h_1 = \left(133\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(30.1\,\text{in.})\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right) = 102\,\text{kPa}$$

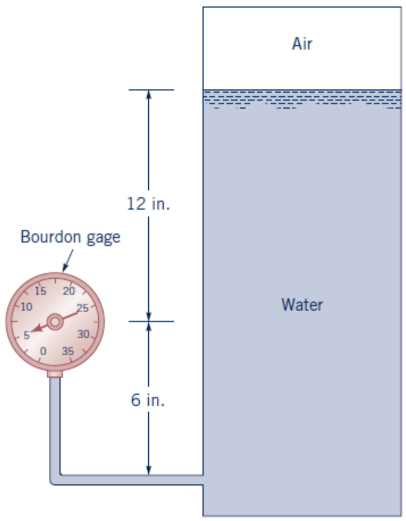
$$p_2 = \gamma_{\text{Hg}}h_2 = \left(133\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(28.3\,\text{in.})\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right) = 95.6\,\text{kPa}$$

Substitution yields:

$$z_2 - z_1 = \frac{288\text{K}}{0.00650\dfrac{\text{K}}{\text{m}}}\left[\left(\frac{102\,\text{kPa}}{101\text{kPa}}\right)^{0.190} - \left(\frac{95.6\,\text{kPa}}{101\text{kPa}}\right)^{0.190}\right] \rightarrow \boxed{z_2 - z_1 = 543\text{ m}}$$

Problem 2.27

Bourdon gages (see the figure below) are commonly used to measure pressure. When such a gage is attached to the closed water tank of figure below the gage reads 5 psi. What is the absolute air pressure in the tank? Assume standard atmospheric pressure of 14.7 psi.



Solution 2.27

$$p = \gamma h + p_0$$

$$p_{gage} - \left(\frac{12}{12}\text{ft}\right)\gamma_{H_2O} = p_{air}$$

$$p_{air} = \left(5\frac{\text{lb}}{\text{in.}^2} + 14.7\frac{\text{lb}}{\text{in.}^2}\right) - \frac{(1\,\text{ft})\left(62.4\frac{\text{lb}}{\text{ft}^2}\right)}{144\frac{\text{in.}^2}{\text{ft}^2}} \rightarrow \boxed{p = 19.3\text{ psia}}$$

Problem 2.28

On the suction side of a pump, a Bourdon pressure gage reads 40 kPa vacuum. What is the corresponding absolute pressure if the local atmospheric pressure is 100 kPa (abs)?

Solution 2.28

$$p(abs) = p(gage) + p(atm) = -40\text{ kPa} + 100\text{ kPa} \quad \rightarrow \quad \boxed{p(abs) = 60\,\text{kPa}}$$

Problem 2.29

A Bourdon pressure gage attached to the outside of a tank containing air reads 77.0 psi when the local atmospheric pressure is 760 mm Hg. What will be the gage reading if the atmospheric pressure increases to 773 mm Hg?

Solution 2.29

$$p(abs) = p(gage) + p(atm)$$

Assuming the absolute pressure of the air in the tank remains constant,

$$\left[p(gage) + p(atm)\right]_i = \left[p(gage) + p(atm)\right]_f$$

Where i □ initial state and f □ final state. Thus,

$$p_f(gage) = p_i(gage) + p_i(atm) - p_f(atm)$$

Since,

$$p_i(atm) = \gamma_{\mathrm{Hg}} h_i = \left(847 \frac{\mathrm{lb}}{\mathrm{ft}^3}\right)(0.760\,\mathrm{m})\left(3.281 \frac{\mathrm{ft}}{\mathrm{m}}\right)\left(\frac{1\,\mathrm{ft}^2}{144\,\mathrm{in.}^2}\right) = 14.7\,\mathrm{psia}$$

$$p_f(atm) = \left(\frac{773\,\mathrm{mm}}{760\,\mathrm{mm}}\right)(14.7\,\mathrm{psia}) = 14.9\,\mathrm{psia}$$

$$p(gage) = 77.0\,\mathrm{psi} + 14.7\,\mathrm{psi} - 14.9\,\mathrm{psia} \quad \rightarrow \quad \boxed{p(gage) = 76.8\,\mathrm{psi}}$$

Problem 2.31

A U-tube manometer is used to check the pressure of natural gas entering a furnace. One side of the manometer is connected to the gas inlet line, and the water level in the other side open to atmospheric pressure rises 3 in. What is the gage pressure of the natural gas in the inlet line in in. H_2O and in lb/in^2 gage?

Solution 2.31

$$P_{\text{atm}} + \rho_{H_2O} g \Delta h = P_{\text{gas}}$$

$$P_{\text{gas}} = 0 + \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.174 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{6}{12}\text{ft}\right)\left(\frac{\text{lb}\cdot\text{s}^2}{32.174\,\text{ft}\cdot\text{lbm}}\right)$$

$$\boxed{P_{\text{gas}} = 31.2 \frac{\text{lb}}{\text{ft}^2}\text{gage} = 6\text{ in. H}_2\text{O gage}}$$

Problem 2.32

A barometric pressure of 29.4 in. Hg corresponds to what value of atmospheric pressure in psia, and in pascals?

Solution 2.32

(psi) $p = \gamma h = \left(847\dfrac{\text{lb}}{\text{ft}^3}\right)\left(\dfrac{29.4}{12}\text{ft}\right)\left(\dfrac{1\,\text{ft}^2}{144\text{in.}^2}\right) \quad \rightarrow \quad \boxed{p = 14.4\,\text{psia}}$

(Pa) $p = \gamma h = \left(133\times 10^3\dfrac{\text{N}}{\text{m}^3}\right)(29.4\text{in.})\left(2.540\times 10^{-2}\dfrac{\text{m}}{\text{in.}}\right) \quad \rightarrow \quad \boxed{p = 99.3\,\text{kPa}\ (abs)}$

Problem 2.33

For an atmospheric pressure of 101 kPa (abs) determine the heights of the fluid columns in barometers containing one of the following liquids: (**a**) mercury, (**b**) water, and (**c**) ethyl alcohol. Calculate the heights including the effect of vapor pressure and compare the results with those obtained neglecting vapor pressure. Do these results support the widespread use of mercury for barometers? Why?

Solution 2.33

(Including vapor pressure)

$$p(\text{atm}) = \gamma h + p_v\text{, where } p_v \text{ } \square \text{ vapor pressure} \rightarrow \quad h = \frac{p(\text{atm}) - p_v}{\gamma}$$

$$\text{(a)}\quad h_{\text{mercury}} = \frac{101\times10^3\ \frac{\text{N}}{\text{m}^2} - 1.6\times10^{-1}\ \frac{\text{N}}{\text{m}^2}}{133\times10^3\ \frac{\text{N}}{\text{m}^3}} = 0.759\text{m}$$

$$\text{(b)}\quad h_{\text{water}} = \frac{101\times10^3 - 1.77\times10^3}{9.80\times10^3}\text{m} = 10.1\text{ m}$$

$$\text{(c)}\quad h_{\text{alchohol}} = \frac{101\times10^3 - 5.9\times10^3}{7.74\times10^3} = 12.3\text{ m}$$

$$\text{(Without vapor pressure):}\quad p(atm) = \gamma h \quad \rightarrow \quad h = \frac{p(atm)}{\gamma}$$

$$\text{(a)}\quad h_{\text{mercury}} = \frac{101\times10^3\ \frac{\text{N}}{\text{m}^2}}{133\times10^3\ \frac{\text{N}}{\text{m}^3}} = 0.759\text{m}$$

$$\text{(b)}\quad h_{\text{water}} = \frac{101\times10^3}{9.80\times10^3}\text{m} = 10.3\text{ m}$$

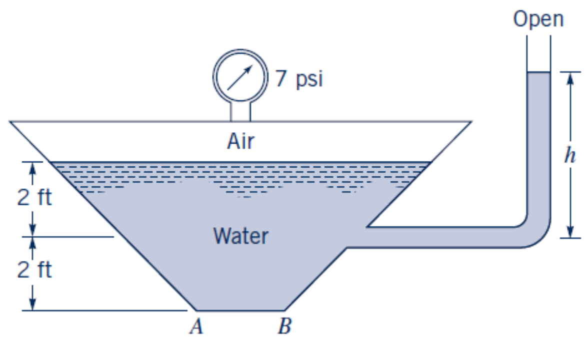
$$\text{(c)}\quad h_{\text{alchohol}} = \frac{101\times10^3}{7.74\times10^3}\text{m} = 13.0\text{ m}$$

$$\boxed{\left.\begin{aligned} h_{\text{mercury}} &= 0.759\text{ m} \\ h_{\text{water}} &= 10.1\text{ m} \\ h_{\text{alchohol}} &= 12.3\text{ m}\end{aligned}\right|_{\text{w/vapor pressure}} \quad .vs. \quad \left.\begin{aligned} h_{\text{mercury}} &= 0.759\text{ m} \\ h_{\text{water}} &= 10.3\text{ m} \\ h_{\text{alchohol}} &= 13.0\text{ m}\end{aligned}\right|_{\text{wo/vapor pressure}}}$$

Mercury is a better choice because it requires less height to represent a comparable pressure difference and the effect of vapor pressure is mush smaller.

Problem 2.34

The closed tank of the figure below is filled with water and is 5 ft long. The pressure gage on the tank reads 7 psi. Determine: (**a**) the height, *h*, in the open water column, (**b**) the gage pressure acting on the bottom tank surface *AB*, and (**c**) the absolute pressure of the air in the top of the tank if the local atmospheric pressure is 14.7 psia.



Solution 2.34

$$p = \gamma h + p_0$$

(a) $p_1 = \gamma_{H_2O}(2\text{ ft}) + p_{air}$

Also $p_1 = \gamma_{H_2O} h$ so that

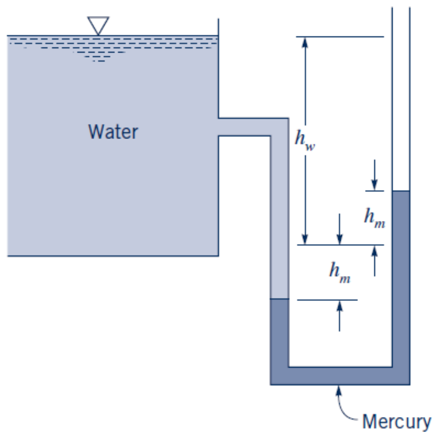
$$\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)h = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft}) + \left(7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{144\text{in.}^2}{\text{ft}^2}\right) \rightarrow \boxed{h = 18.2\text{ ft}}$$

(b) $$p_{AB} = \left[\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ft}) + \left(7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{144\text{in.}^2}{\text{ft}^2}\right)\right]\left(\frac{1\text{ ft}^2}{144\text{in.}^2}\right) \rightarrow \boxed{p = 8.73\text{ psi}}$$

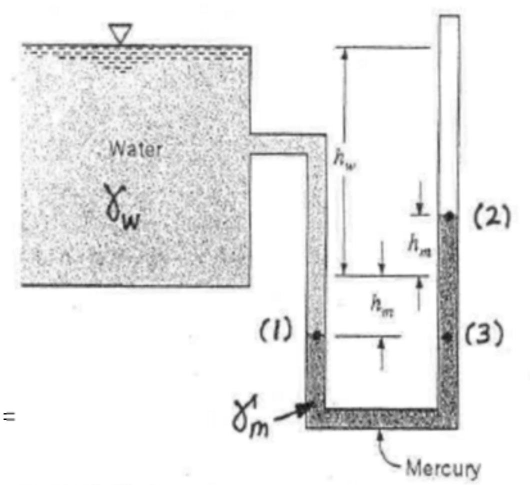
(c) $p_{air} = 7\text{psi} + 14.7\text{ psia} = \rightarrow \boxed{p_{air} = 21.7\text{psia}}$

Problem 2.35

A mercury manometer is connected to a large reservoir of water as shown in the figure below. Determine the ratio, $\frac{h_w}{h_m}$, of the distances h_w and h_m indicated in the figure.



Solution 2.35



$$p_1 = \gamma_w h_w + \gamma_w h_m$$

$$p_1 = p_3 = \gamma_m (2h_m)$$

$$\gamma_w h_w + \gamma_w h_m = 2\gamma_m h_m$$

$$(\gamma_w) h_w = (2\gamma_m - \gamma_w) h_m$$

so that

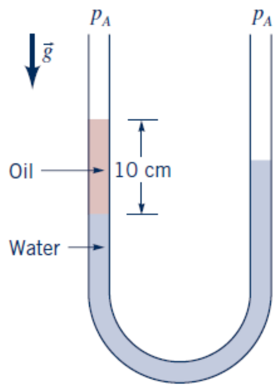
$$\frac{h_w}{h_m} = \frac{(2\gamma_m - \gamma_w)}{\gamma_w} = 2SG_m - 1, \text{ where } SG_m =$$

Thus,

$$\frac{h_w}{h_m} = 2(13.56) - 1 = \underline{\underline{26.1}}$$

Problem 2.36

The U-tube manometer shown in the figure below has two fluids, water and oil ($S = 0.80$). Find the height difference between the free water surface and the free oil surface with no applied pressure difference.



Solution 2.36

GIVEN: $S_{\text{oil}} = 0.8$ (see the figure in the problem)

FIND: Free surface height difference.

SOLUTION:

$$P_A + \rho_0 g h_0 - \rho_w g h_w = P_A$$

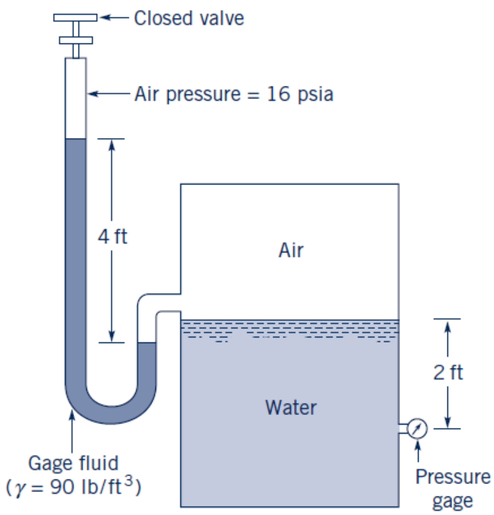
$$h_w = h_0 \left(\frac{\rho_0}{\rho_w} \right) = h_0 S_0$$
$$= (10 \text{ cm})(0.8) = 8 \text{ cm}$$

$$\Delta h = h_0 - h_w$$
$$= 10 \text{ cm} - 8 \text{ cm}$$

$$\boxed{\Delta h = 2 \text{ cm}}$$

Problem 2.37

A U-tube manometer is connected to a closed tank containing air and water as shown in the figure below. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure and neglect the weight of the air columns in the manometer.



Solution 2.37

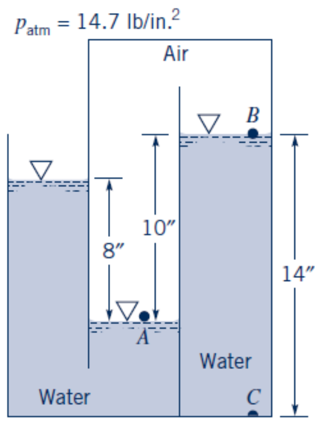
$$p_1 + \gamma_{gf}(4\,\text{ft}) + \gamma_{H_2O}(2\,\text{ft}) = p_{gage}$$

$$p_{gage} = \left(16\frac{\text{lb}}{\text{in.}^2} - 14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(90\frac{\text{lb}}{\text{ft}^3}\right)(4\,\text{ft}) + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(2\,\text{ft})$$

$$= 627\frac{\text{lb}}{\text{ft}^2} \times \frac{1\,\text{ft}^2}{144\,\text{in.}^2} \quad \rightarrow \quad \boxed{p_{gage} = 4.67\text{ psi}}$$

Problem 2.38

The container shown in the figure below holds 60 °F water and 60 °F air as shown. Find the absolute pressures at locations *A*, *B*, and *C*.



Solution 2.38

GIVEN: Figure, water and air at 60 °F

FIND: Absolute pressures at points A, B, C.

SOLUTION:

$$\gamma_w = \left(62.3\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\text{ft}^3}{1728\,\text{in}^3}\right) = 0.0361\frac{\text{lb}}{\text{in}^3}$$

Modelling the air as an ideal gas:

$$\rho_a = \frac{P}{RT} = \frac{\left(14.7\dfrac{\text{lb}}{\text{in}^2}\right)\left(\dfrac{\text{ft}}{12\,\text{in}}\right)}{\left(53.35\dfrac{\text{ft}\cdot\text{lb}}{\text{lbm}\cdot{}^\circ\text{R}}\right)(520^\circ\text{R})} = 0.0000442\frac{\text{lbm}}{\text{in}^3} \quad\rightarrow\quad \gamma_a = 0.0000442\frac{\text{lb}}{\text{in}^3}$$

The hydrostatic equation gives:

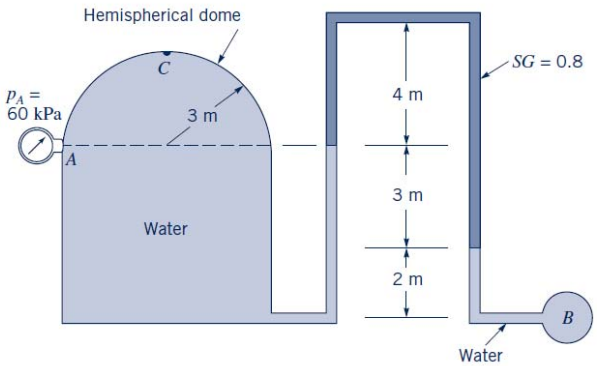
$$P_A = P_{atm} + \gamma_w h_A = 14.7\frac{\text{lb}}{\text{in}^2} + \left(0.0361\frac{\text{lb}}{\text{in}^3}\right)(8\,\text{in}) \quad\rightarrow\quad \boxed{P_A = 15.0\text{ psia}}$$

$$P_B = P_A - \gamma_a h_B = 15.0\frac{\text{lb}}{\text{in}^2} - \left(0.0000442\frac{\text{lb}}{\text{in}^3}\right)(10\,\text{in}) \quad\rightarrow\quad \boxed{P_B = 15.0\text{ psia}}$$

$$P_C = P_B + \gamma_w h_C = 15.0\frac{\text{lb}}{\text{in}^2} + \left(0.0361\frac{\text{lb}}{\text{in}^3}\right)(14\,\text{in}) \quad\rightarrow\quad \boxed{P_C = 15.5\text{ psia}}$$

Problem 2.39

A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in the figure below. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at *A* is 60 kPa, determine (**a**) the pressure in pipe *B*, and (**b**) the pressure head, in millimeters of mercury, at the top of the dome (point *C*).



Solution 2.39

(a) $p_A + (SG)(\gamma_{H_2O})(3\text{m}) + \gamma_{H_2O}(2\,\text{m}) = p_B$

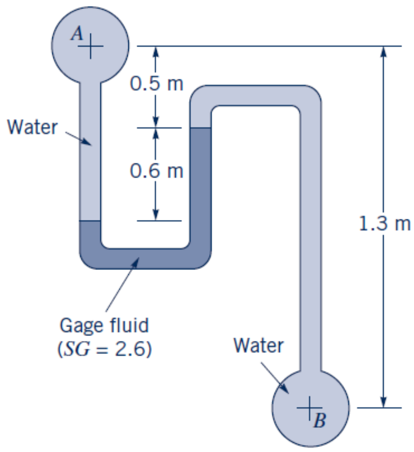
$$p_B = 60\,\text{kPa} + (0.8)\left(9.81\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(3\,\text{m}) + \left(9.81\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(2\,\text{m}) \quad \rightarrow \quad \boxed{p_B = 103\text{ kPa}}$$

(b) $p_c = p_A - \gamma_{H_2O}(3\,\text{m}) = 60\,\text{kPa} - \left(9.81\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(3\,\text{m}) = 30.6\times10^3\,\frac{\text{N}}{\text{m}^2}$

$$h = \frac{p_c}{\gamma_{Hg}} = \frac{30.6\times10^3\,\frac{\text{N}}{\text{m}^2}}{133\times10^3\,\frac{\text{N}}{\text{m}^3}} = 0.230\,\text{m} = 0.230\,\text{m}\left(\frac{10^3\text{mm}}{\text{m}}\right) \quad \rightarrow \quad \boxed{h = 230\text{ mm}}$$

Problem 2.40

Two pipes are connected by a manometer as shown in the figure below. Determine the pressure difference $p_A - p_B$, between the pipes.



Solution 2.40

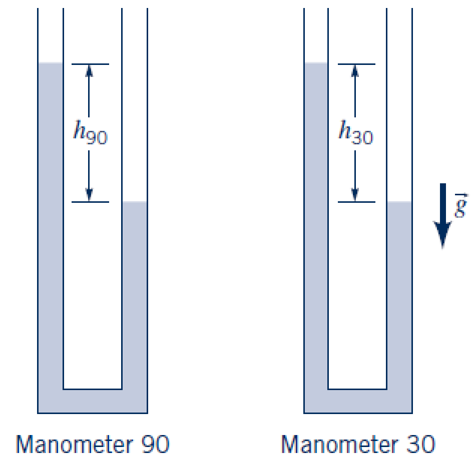
$$p_A + \gamma_{H_2O}(0.5\text{m} + 0.6\text{m}) - \gamma_{gf}(0.6\text{m}) + \gamma_{H_2O}(1.3\text{m} - 0.5\text{m}) = p_B$$

$$p_A - p_B = \gamma_{gf}(0.6\text{m}) - \gamma_{H_2O}(0.5\text{m} + 0.6\text{m} + 1.3\text{m} - 0.5\text{m})$$

$$= (2.6)\left(9.81\frac{\text{kN}}{\text{m}^3}\right)(0.6\text{m}) - \left(9.81\frac{\text{kN}}{\text{m}^3}\right)(1.9\text{m}) \quad \rightarrow \quad \boxed{p_A - p_B = -3.32\text{ kPa}}$$

Problem 2.41

Find the percentage difference in the readings of the two identical U-tube manometers shown in the figure below. Manometer 90 uses 90°C water and manometer 30 uses 30°C water. Both have the same applied pressure difference. Does this percentage change with the magnitude of the applied pressure difference? Can the difference between the two readings be ignored?



Solution 2.41

GIVEN: Figure, Two identical U-tube manometers. Manometer 90 uses 90°C water while manometer 30 uses 30°C water. Same pressure difference applied across each manometer.

FIND: Percent difference in readings. Does this percent difference change with the applied pressure difference? Can the difference in the two manometer readings be ignored?

SOLUTION:

Apply the manometer rule:

$$P_B = P_A + \rho_w gh \quad \rightarrow \quad h_{90} = \frac{P_B - P_A}{\rho_{90w} g}$$

$$h_{30} = \frac{P_B - P_A}{\rho_{30w} g}$$

Using the 30°C water as a reference

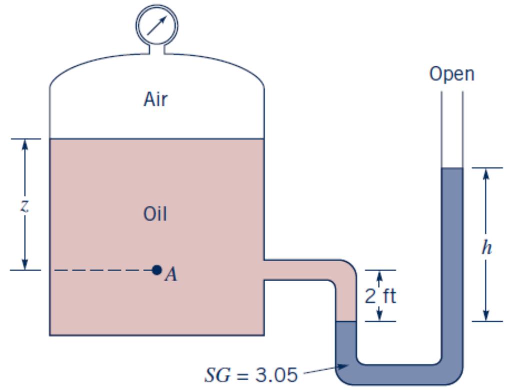
$$\left(\frac{h_{90} - h_{30}}{h_{30}}\right) \times 100 = \left[\frac{\dfrac{1}{\rho_{90w}} - \dfrac{1}{\rho_{30w}}}{\dfrac{1}{\rho_{30w}}}\right] \times 100 = \left(\frac{\rho_{30w}}{\rho_{90w}} - 1\right) \times 100$$

$$\left(\frac{h_{90} - h_{30}}{h_{30}}\right) \times 100 = \left(\frac{996}{965} - 1\right) \times 100 \rightarrow \boxed{\left(\frac{h_{90} - h_{30}}{h_{30}}\right) = 3.2\%}$$

Note that this percent difference does not change with the applied pressure difference and the difference in the two manometer readings cannot be ignored in most cases.

Problem 2.42

A U-tube manometer is connected to a closed tank as shown in the figure below. The air pressure in the tank is 0.50 psi and the liquid in the tank is oil ($\gamma = 54.0\dfrac{\text{lb}}{\text{ft}^3}$). The pressure at point A is 2.00 psi. Determine: (**a**) the depth of oil, z, and (**b**) the differential reading, h, on the manometer.



Solution 2.42

(a) $p_A = \gamma_{oil} z + p_{air}$

$$z = \frac{p_A - p_{air}}{\gamma_{oil}} = \frac{\left(2\dfrac{\text{lb}}{\text{in.}^2} - 0.5\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{144\,\text{in.}^2}{\text{ft}^2}\right)}{54.0\dfrac{\text{lb}}{\text{ft}^3}} \rightarrow \boxed{z = 4.00\text{ ft}}$$

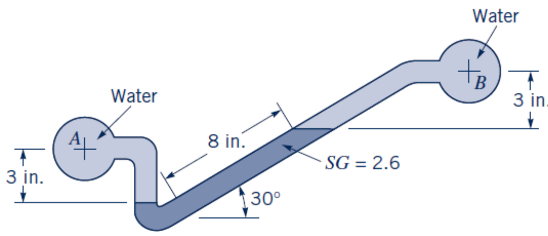
$$(a)\quad p_A + \gamma_{oil}(2\text{ ft}) - (SG)(\gamma_{H_2O})h = 0$$

(b)

$$h = \frac{p_A + \gamma_{oil}(2\text{ ft})}{(SG)(\gamma_{H_2O})} = \frac{\left(2\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{144\,\text{in.}^2}{\text{ft}^2}\right) + \left(54.0\dfrac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft})}{(3.05)\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)} \rightarrow \boxed{h = 2.08\text{ ft}}$$

Problem 2.43

For the inclined-tube manometer of the figure below, the pressure in pipe *A* is 0.6 psi. The fluid in both pipes *A* and *B* is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe *B* corresponding to the differential reading shown?



Solution 2.43

$$p_A + \gamma_{H_2O}\left(\frac{3}{12}\text{ft}\right) - \gamma_{gf}\left(\frac{8}{12}\text{ft}\right)\sin 30° - \gamma_{H_2O}\left(\frac{3}{12}\text{ft}\right) = p_B$$

$$p_B = p_A - \gamma_{gf}\left(\frac{8}{12}\text{ft}\right)\sin 30°$$

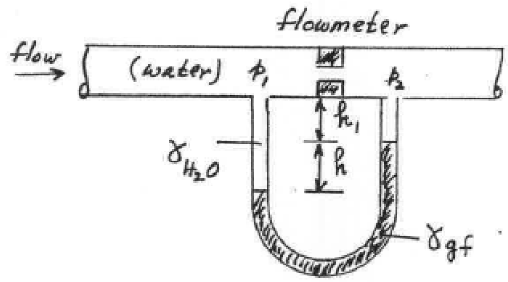
$$= \left(0.6\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) - (2.6)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ft}\right)(0.5)$$

$$= 32.3\frac{\text{lb}}{\text{ft}^2} \times \frac{1\ \text{ft}^2}{144\ \text{ft}^2} \quad \rightarrow \quad \boxed{p_B = 0.224\text{ psi}}$$

Problem 2.44

A flowrate measuring device is installed in a horizontal pipe through which water is flowing. A U-tube manometer is connected to the pipe through pressure taps located 3 in. on either side of the device. The gage fluid in the manometer has a specific weight of $122\,\frac{\text{lb}}{\text{ft}^3}$. Determine the differential reading of the manometer corresponding to a pressure drop between the taps of $0.5\,\frac{\text{lb}}{\text{in.}^2}$.

Solution 2.44



Let p_1 and p_2 be pressures at pressure taps. Apply manometer equation between p_1 and p_2.

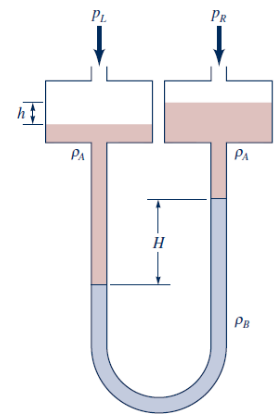
$$p_1 + \gamma_{\text{H}_2\text{O}}\left(h_1 + h\right) - \gamma_{\text{gf}} h - \gamma_{\text{H}_2\text{O}} h_1 = p_2$$

$$h = \frac{p_1 - p_2}{\gamma_{\text{gf}} - \gamma_{\text{H}_2\text{O}}} = \frac{\left(0.5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{122\,\frac{\text{lb}}{\text{ft}^3} - 62.4\,\frac{\text{lb}}{\text{ft}^3}} \rightarrow \boxed{h = 1.45\text{ ft}}$$

Problem 2.45

The sensitivity Sen of the micromanometer shown in the figure below is defined as

$$Sen = \frac{H}{p_L - p_R}.$$



Find the sensitivity of the micromanometer in terms of the densities ρ_A and ρ_B. How can the sensitivity be increased?

Solution 2.45

GIVEN: Figure and sensitivity defined as: $Sen = \dfrac{H}{p_L - p_R}$.

DETERMINE: Sensitivity as a function of fluid densities. How can the sensitivity increase?

SOLUTION:

Apply manometer rule,

$$P_R + \gamma_A h + (\gamma_B - \gamma_A) H = P_L$$

$$P_L - P_R = \gamma_A h + (\gamma_B - \gamma_A) H$$

$$Sen = \frac{H}{P_L - P_R} = \frac{H}{\gamma_A h + (\gamma_B - \gamma_A) H} \rightarrow \boxed{Sen = \frac{1}{\gamma_A \left(\frac{h}{H}\right) + (\gamma_B - \gamma_A)}}$$

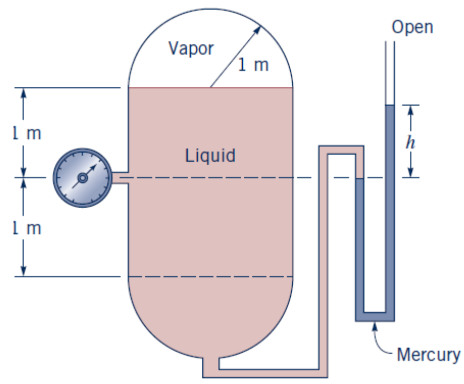
The sensitivity can be increased by decreasing the denominator.

→ decrease density difference or

→ decrease $\dfrac{h}{H}$ by increasing the ratio of reservoir area to tube area.

Problem 2.46

The cylindrical tank with hemispherical ends shown in the figure below contains a volatile liquid and its vapor. The liquid density is $800\ \dfrac{\text{kg}}{\text{m}^3}$, and its vapor density is negligible. The pressure in the vapor is $120\ \text{kPa}\ (\text{abs})$ and the atmospheric pressure is $101\ \text{kPa}\ (\text{abs})$. Determine: (**a**) the gage pressure reading on the pressure gage, and (**b**) the height, h, of the mercury, in the manometer.



Solution 2.46

(a) Let γ_ℓ = specific weight of liquid $=\left(800\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)=7850\dfrac{\text{N}}{\text{m}^3}$

$$p_{\text{vapor}}(\text{gage})=120\,\text{kPa(abs)}-101\,\text{kPa}(\text{abs})=19\,\text{kPa}$$

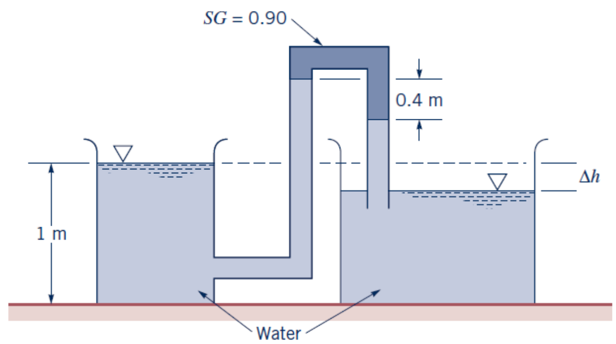
$$p_{\text{gage}}=p_{\text{vapor}}+\gamma_\ell(1\text{m})=19\times10^3\frac{\text{N}}{\text{m}^2}+\left(7850\frac{\text{N}}{\text{m}^3}\right)(1\text{m}) \rightarrow \boxed{p_{\text{gage}}=26.9\ \text{kPa}}$$

(b) $p_{\text{vapor}}(\text{gage})+\gamma_\ell(1\text{m})-\gamma_{\text{Hg}}(h)=0$

$$19\times10^3\frac{\text{N}}{\text{m}^2}+\left(7850\frac{\text{N}}{\text{m}^3}\right)(1\text{m})-\left(133\times10^3\frac{\text{N}}{\text{m}^3}\right)(h)=0 \rightarrow \boxed{h=0.202\ \text{m}}$$

Problem 2.47

Determine the elevation difference, Δh, between the water levels in the two open tanks shown in the figure below.



Solution 2.47

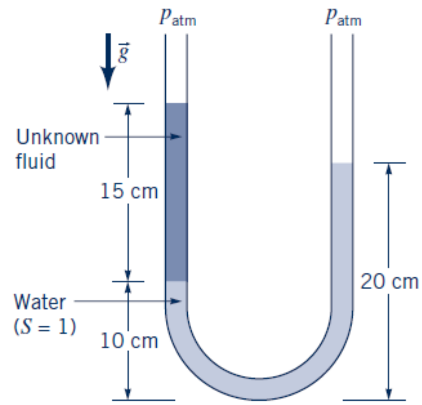
Let subscript 1 indicate the surface of the left tank, and subscrip 2 the surface of the right tank.

$$\cancel{p_1} - \gamma_{\mathrm{H_2O}} h + (SG)\gamma_{\mathrm{H_2O}}(0.4\,\mathrm{m}) + \gamma_{\mathrm{H_2O}}(h - 0.4\,\mathrm{m}) + \gamma_{\mathrm{H_2O}}(\Delta h) = \cancel{p_2}$$

$$\Delta h = 0.4\,\mathrm{m} - (0.9)(0.4\,\mathrm{m}) \quad \rightarrow \quad \boxed{\Delta h == 0.040\ \mathrm{m}}$$

Problem 2.48

What is the specific gravity of the liquid in the left leg of the U-tube manometer shown in the figure below?



Solution 2.48

GIVEN: Figure

FIND: Specific gravity S of unknown fluid

SOLUTION:

$$\text{Let } \begin{cases} h_{10} = 10\text{cm} \\ h_{15} = 15\text{cm} \\ h_{20} = 20\text{cm} \end{cases}$$

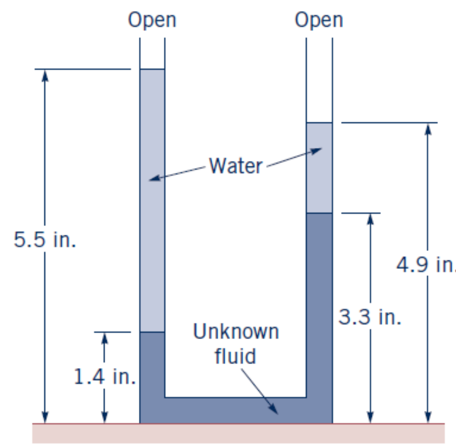
Apply manometer rule,

$$\cancel{P_{atm}} + \rho_w g\left(h_{20} - h_{10}\right) - \rho_u g h_{15} = \cancel{P_{atm}}$$

$$S = \frac{\rho_u}{\rho_w} = \frac{h_{20} - h_{10}}{h_{15}} = \frac{20\text{cm} - 10\text{cm}}{15\text{cm}} \quad \rightarrow \quad \boxed{S = 0.667}$$

Problem 2.49

For the configuration shown in the figure below what must be the value of the specific weight of the unknown fluid? Express your answer in $\frac{\text{lb}}{\text{ft}^3}$.



Solution 2.49

Let γ be specific weight of unknown fluid.

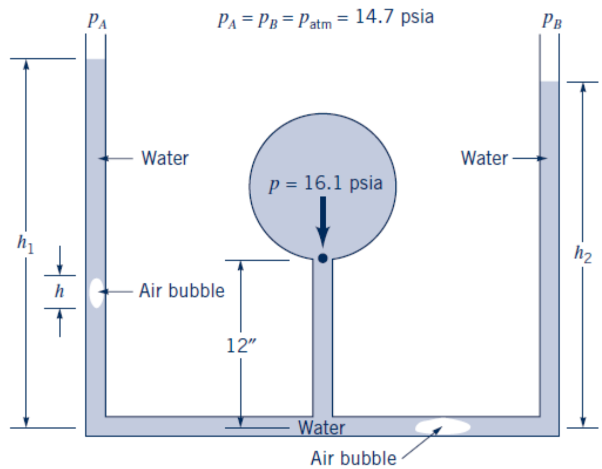
Applying the manometer rule:

$$\cancel{p_{\text{atm}}} + \gamma_{H_2O}\left[\frac{(5.5-1.4)}{\cancel{12}}\text{ft}\right] - \gamma\left[\frac{(3.3-1.4)}{\cancel{12}}\text{ft}\right] - \gamma_{H_2O}\left[\frac{(4.9-3.3)}{\cancel{12}}\text{ft}\right] = \cancel{p_{\text{atm}}}$$

$$\gamma = \frac{\gamma_{H_2O}\left[(5.5-1.4)-(4.9-3.3)\right]\text{in.}}{(3.3-1.4)\text{in.}} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{4.1-1.6}{1.9}\right) \rightarrow \boxed{\gamma = 82.1\frac{\text{lb}}{\text{ft}^3}}$$

Problem 2.50

The manometer shown in the figure below has an air bubble either in **(a)** the right horizontal line or **(b)** the left vertical leg. Find $h_1 - h_2$ for both cases if $p_A = p_B$.



Solution 2.50

GIVEN: Figure, manometer with small pockets of air.

FIND: $h_1 = h_2$ if (a) air pocket in horizontal line and (b) air pocket in vertical line.

SOLUTION:

(a) Air pocket in horizontal line. Apply the manometer rule between the left liquid surface (A) and the right liquid surface (B),

$$P_A = P_B + \gamma h_2 - \gamma h_1$$

$$h_2 - h_1 = \frac{1}{\gamma}(P_A - P_B)$$

$P_A = P_B = 14.7\,\text{psia}$ $\boxed{h_1 = h_2}$

(b) Air pocket in (left) vertical line. Apply the manometer rule

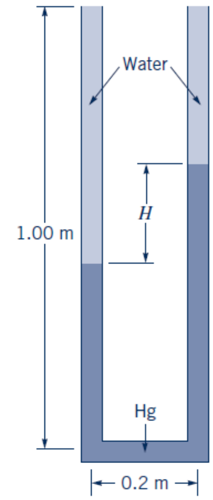
$$P_A = P_B + \gamma h_2 - \gamma(h_1 - h)$$

$P_A = P_B = 14.7\,\text{psia}$ $\boxed{h_1 - h_2 = h}$

NOTE: For above analyses, the hydrostatic pressure of the air pocket has been neglected.

Problem 2.51

The U-tube manometer shown in the figure below has legs that are $1.00\,\text{m}$ long. When no pressure difference is applied across the manometer, each leg has $0.40\ \text{m}$ of mercury. What is the maximum pressure difference that can be indicated by the manometer?



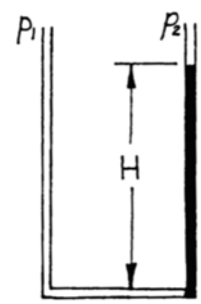
Solution 2.51

GIVEN: Manometer in the figure in the problem. With no pressure difference applied across manometer, each mercury leg is $0.40\,\text{m}$ high.

FIND: Maximum pressure difference that can be indicated by the manometer.

SOLUTION:

The maximum pressure difference that can be indicated is illustrated by the sketch on the right. Applying the manometer rule,



$$P_1 = P_2 + \gamma_{\text{Hg}} H - \gamma_w H$$

$$P_1 - P_2 = (\gamma_{\text{Hg}} - \gamma_w)H = \left(\rho_{\text{Hg}} - \rho_w\right) gH$$

Using table in the Appendix, and assuming $20\,^\circ\text{C}$ fluid

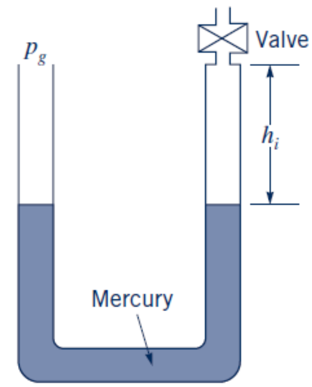
$$P_1 - P_2 = (13550 - 998)\frac{\text{kg}}{\text{m}^3}\left(9.807\frac{\text{m}}{\text{s}^2}\right)(1.0\text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 123100\frac{\text{N}}{\text{m}^2}$$

$$\boxed{P_1 - P_2 = 123.1\,\text{kPa}}$$

Problem 2.52

Both ends of the U-tube mercury manometer of the figure below are initially open to the atmosphere and under standard atmospheric pressure. When the valve at the top of the right leg is open, the level of mercury below the valve is h_1. After the valve is closed, air pressure is applied to the left leg. Determine the relationship between the differential reading on the manometer and the applied gage pressure, p_g. Show on a plot how the differential reading varies with p_g for $h_1 = 25, 50, 75,$ and 100 mm over the range $0 \leq p_g \leq 300\text{ kPa}$. Assume that the temperature of the trapped air remains constant.

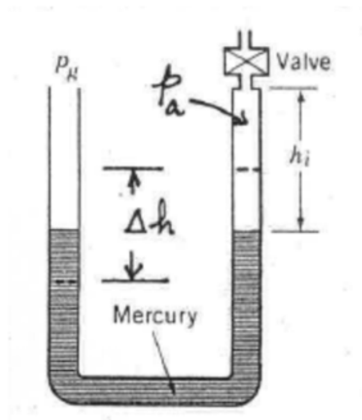


Solution 2.52

With the valve closed and a pressure, p_g, applied,

$$p_g - \gamma_{Hg}\Delta h = p_a$$

$$\Delta h = \frac{p_g - p_a}{\gamma_{Hg}} \qquad (1)$$



Where p_g and p_a are gage pressures.

For isothermal compression of trapped air

$$\frac{p}{\rho} = \text{Constant} \quad \rightarrow \quad p_i \mathcal{V}_i = p_f \mathcal{V}_f$$

where $\mathcal{V}$ is air volume, p is absolute pressure, i & f refer to initial and final states., respectively.

$$p_{atm} \mathcal{V}_i = \left(p_a - p_{atm}\right)\mathcal{V}_f \qquad (2)$$

For air trapped in right leg, $\mathcal{V}_i = h_i$(Area of tube) so that Eq.(2) can be written as

$$p_a = p_{atm}\left[\frac{h_i}{h_i - \frac{\Delta h}{2}} - 1\right] \qquad (3)$$

Substitute Eq.(3) into Eq.(1) to obtain: $\Delta h = \frac{1}{\gamma_{Hg}}\left[p_g + p_{atm}\left(1 - \frac{h_i}{h_i - \frac{\Delta h}{2}}\right)\right]$ (4)

Eq.(4) can be expressed in the form: $(\Delta h)^2 - \left(2h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right)\Delta h + \frac{2 p_g h_i}{\gamma_{Hg}} = 0$

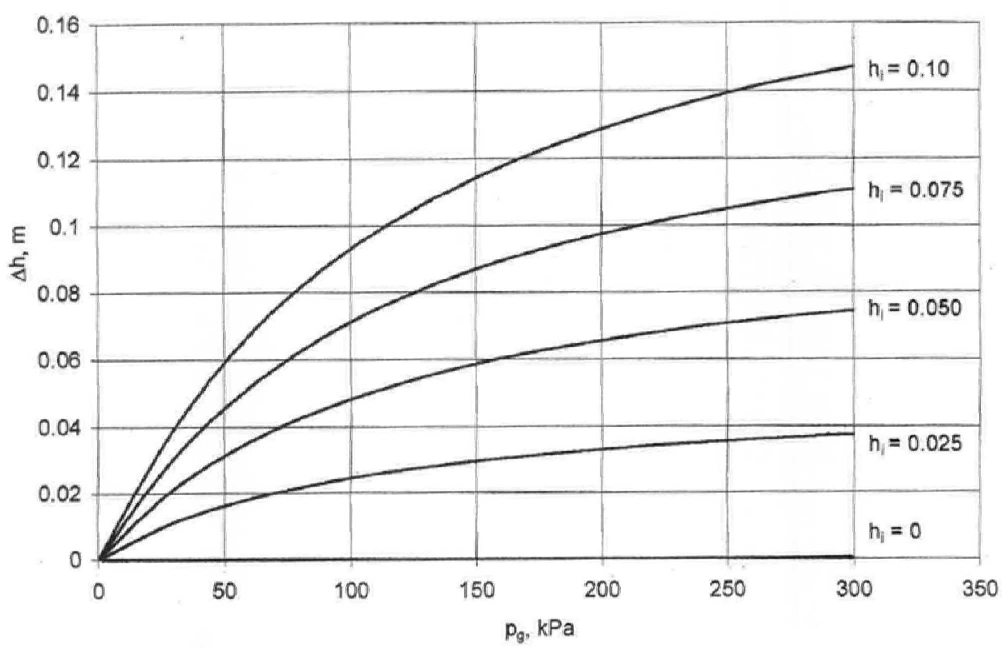
The roots of this quadratic equation are

$$\Delta h = \left(h_\mathrm{i} + \frac{p_\mathrm{g} + p_\mathrm{atm}}{2\gamma_\mathrm{Hg}} \right) \pm \sqrt{\left(h_\mathrm{i} + \frac{p_\mathrm{g} + p_\mathrm{atm}}{2\gamma_\mathrm{Hg}} \right)^2 - \frac{2 p_\mathrm{g} h_\mathrm{i}}{\gamma_\mathrm{Hg}}} \qquad (5)$$

To evaluate Δh, the negative sign is used since $\Delta h = 0$ for $p_\mathrm{g} = 0$.

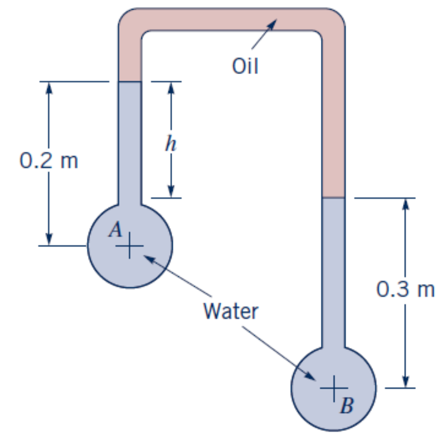
Tabulated values of Δh for various values of p_g are given in the following table for different values of h_i (with $p_\mathrm{atm} = 101\,\mathrm{kPa}$ and $\gamma_\mathrm{Hg} = 133\,\mathrm{kN/m^3}$). A plot of the data follows.

hi (m)	patm (kPa)	γhg (kN/m3)	p_g (kPa)	$\Delta h(h_i = 0)$ (m)	$\Delta h(h_i=0.025)$ (m)	$\Delta h(h_i=0.05)$ (m)	$\Delta h(h_i=0.075)$ (m)	$\Delta h(h_i=0.1)$ (m)
0.025	101	133	0	0	0	0	0	0
0.05	101	133	30	0	0.0110	0.0212	0.0306	0.0394
0.075	101	133	60	0	0.0182	0.0354	0.0517	0.0672
0.1	101	133	90	0	0.0231	0.0454	0.0668	0.0874
	101	133	120	0	0.0268	0.0528	0.0781	0.1026
	101	133	150	0	0.0296	0.0585	0.0867	0.1143
	101	133	180	0	0.0318	0.0630	0.0936	0.1236
	101	133	210	0	0.0335	0.0666	0.0991	0.1312
	101	133	240	0	0.0350	0.0696	0.1037	0.1374
	101	133	270	0	0.0362	0.0721	0.1075	0.1426
	101	133	300	0	0.0372	0.0742	0.1108	0.1470



Problem 2.53

The inverted U-tube manometer of the figure below contains oil ($SG = 0.9$) and water as shown. The pressure differential between pipes A and B, $p_A - p_B$, is -5 kPa . Determine the differential reading h .



Solution 2.53

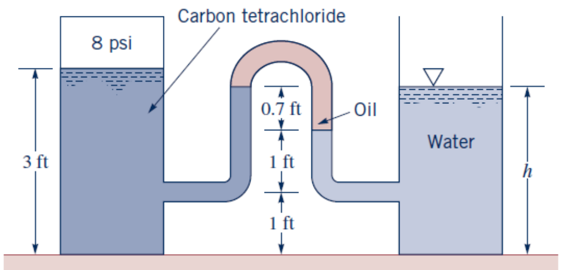
$$p_A - \gamma_{H_2O}(0.2\,m) + \gamma_{oil}(h) + \gamma_{H_2O}(0.3\,m) = p_B$$

$$h = \frac{(p_B - p_A) + \gamma_{H_2O}(0.2\,m) - \gamma_{H_2O}(0.3\,m)}{\gamma_{oil}}$$

$$= \frac{5\times10^3\,\frac{N}{m^2} - \left(9.80\times10^3\,\frac{N}{m^3}\right)(0.1\,m)}{8.95\times10^3\,\frac{N}{m^3}} \rightarrow \boxed{h = 0.449\text{ m}}$$

Problem 2.54

An inverted U-tube manometer containing oil ($SG = 0.8$) is located between two reservoirs as shown in the figure below. The reservoir on the left, which contains carbon tetrachloride, is closed and pressurized to 8 psi. The reservoir on the right contains water and is open to the atmosphere. With the given data, determine the depth of water, h, in the right reservoir.



Solution 2.54

Let p_A be the air pressure in left reserviour. Manometer equation can be written as

$$p_A + \gamma_{CCl_4}(3\text{ft} - 1\text{ft} - 1\text{ft} - 0.7\text{ft}) + \gamma_{oil}(0.7\text{ft}) - \gamma_{H_2O}(h - 1\text{ft} - 1\text{ft}) = 0$$

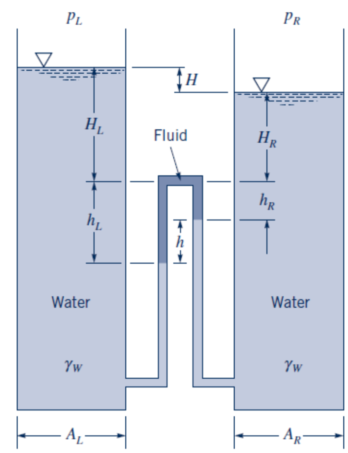
$$h = \frac{p_A + \gamma_{CCl_4}(0.3\text{ft}) + \gamma_{oil}(0.7\text{ft})}{\gamma_{H_2O}} + 2\text{ft}$$

$$= \frac{\left(8\dfrac{\text{lb}}{\text{in.}^2}\right)\left(144\dfrac{\text{in.}^2}{\text{ft}^2}\right) + \left(99.5\dfrac{\text{lb}}{\text{ft}^3}\right)(0.3\text{ft}) + \left(57.0\dfrac{\text{lb}}{\text{ft}^3}\right)(0.7\text{ft})}{62.4\dfrac{\text{lb}}{\text{ft}^3}} + 2\text{ft}$$

$$\boxed{h = 21\text{ ft}}$$

Problem 2.55

The sensitivity Sen of the manometer shown in the figure below can be defined as: $Sen = \dfrac{h}{p_L - p_R}$.



Three manometer fluids with the listed specific gravities S are available:

Kerosene, $S = 0.82$;

SAE 10 oil, $S = 0.87$; and

Normal octane, $S = 0.71$.

Which fluid gives the highest sensitivity? The areas A_R and A_L are much larger than the cross-sectional area of the manometer tube, so $H << h$.

Solution 2.55

GIVEN: The figure in the problem, three manometer fluids, kerosene ($S =$ 0.82), SAE 10 oil ($S = 0.87$), and normal octane ($S = 0.71$). $H << h$.

FIND: Manometer fluid that gives highest sensitivity.

SOLUTION:

Apply manometer rule,

$$P_R + \gamma_w(H_R + h_R) - \gamma_f(h_R) + \gamma_f(h_L) - \gamma_w(H_L + h_L) = P_L$$
$$P_R + \gamma_w(H_R + h_R - H_L - h_L) + \gamma_f(h_L - h_R) = P_L$$
$$P_R + \gamma_w(H_R - H_L + h_R - h_L) + \gamma_f(h) = P_L$$
$$P_R - \gamma_w(H_L - H_R + h_L - h_R) + \gamma_f(h) = P_L$$
$$P_R - \gamma_w(H + h) + \gamma_f(h) = P_L$$
$$P_R - P_L = \gamma_w(H + h) + \gamma_f(h)$$

$$\text{H} << \text{h} \rightarrow P_R - P_L \approx (\gamma_w - \gamma_f)h$$

$$Sen = \frac{h}{P_R - P_L} \approx \frac{h}{(\gamma_w - \gamma_f)h} = \frac{1}{\rho_w g(S_w - S_f)}$$

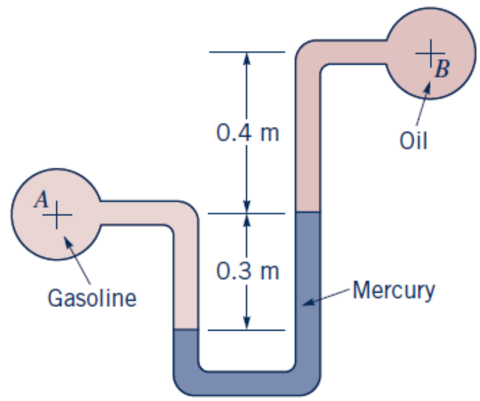
Sensitivity maximized for S_f closest to $S_w = 1 \rightarrow$ SAE 10 oil.

$$Sen = \frac{1}{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)(1-0.87)} = \frac{1}{(9810)(1-0.87)}\frac{\text{m}\cancel{^3}}{\cancel{\text{kg}}}\frac{\cancel{\text{s}^2}}{\cancel{\text{m}}}\times\frac{\cancel{\text{N}}}{\text{Pa}\cdot\cancel{\text{m}^2}}\times\frac{\cancel{\text{kg}}\cdot\cancel{\text{m}}}{\cancel{\text{N}}\cdot\cancel{\text{s}^2}}$$

$$Sen = 0.000784\frac{\text{m}}{\text{Pa}} \rightarrow \boxed{Sen = 0.784\frac{\text{mm}}{\text{Pa}}}$$

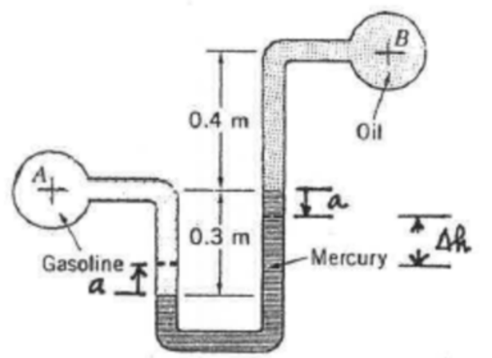
Problem 2.56

In the figure below pipe A contains gasoline ($SG = 0.7$), pipe B contains oil ($SG = 0.9$), and the manometer fluid is mercury. Determine the new differential reading if the pressure in pipe A is decreased 25 kPa, and the pressure in pipe B remains constant. The initial differential reading is 0.30 m as shown.



Solution 2.56

For the initial configuration:



$$p_A + \gamma_{gas}(0.3\,\text{m}) - \gamma_{Hg}(0.3\,\text{m}) - \gamma_{oil}(0.4\,\text{m}) = p_B \quad (1)$$

With a decrease in p_A to p'_A gage fluid levels change as shown on figure.

Thus, for final configuration:

$$p'_A + \gamma_{gas}(0.3 - a) - \gamma_{Hg}(\Delta h) - \gamma_{oil}(0.4 + a) = p_B \quad (2)$$

Where all lengths are in m. Subtract Eq.(2) from Eq.(1) to obtain,

$$p_A - p'_A + \gamma_{gas}(a) - \gamma_{Hg}(0.3 - \Delta h) + \gamma_{oil}(a) = 0 \quad (3)$$

Since $2a + \Delta h = 0.3$ (see figure) then, $a = \dfrac{0.3 - \Delta h}{2}$

and from Eq.(3),

$$p_A - p'_A + \gamma_{gas}\left(\frac{0.3 - \Delta h}{2}\right) - \gamma_{Hg}(0.3 - \Delta h) + \gamma_{oil}\left(\frac{0.3 - \Delta h}{2}\right) = 0$$

$$\Delta h = \frac{p_A - p'_A + \gamma_{gas}(0.15) - \gamma_{Hg}(0.3) + \gamma_{oil}(0.15)}{-\gamma_{Hg} + \dfrac{\gamma_{gas}}{2} + \dfrac{\gamma_{oil}}{2}}$$

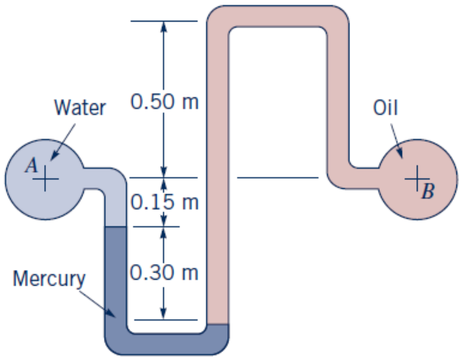
for $p_A - p'_A = 25\,\text{kPa}$

$$\Delta h = \frac{25\dfrac{\text{kN}}{\text{m}^2} + (0.7)\left(9.81\dfrac{\text{kN}}{\text{m}^3}\right)(0.15\,\text{m}) - \left(133\dfrac{\text{kN}}{\text{m}^3}\right)(0.3\,\text{m}) + (0.9)\left(9.81\dfrac{\text{kN}}{\text{m}^3}\right)(0.15\,\text{m})}{-133\dfrac{\text{kN}}{\text{m}^3} + \dfrac{(0.7)}{2}\left(9.81\dfrac{\text{kN}}{\text{m}^3}\right) + \dfrac{(0.9)}{2}\left(9.81\dfrac{\text{kN}}{\text{m}^3}\right)}$$

$$\boxed{\Delta h = 0.100\text{ m}}$$

Problem 2.57

The mercury manometer of the figure below indicates a differential reading of 0.30 m when the pressure in pipe *A* is 30-mm Hg vacuum. Determine the pressure in pipe *B*.



Solution 2.57

$$p_B + \gamma_{oil}(0.15\,\text{m}+0.30\,\text{m}) - \gamma_{Hg}(0.3\,\text{m}) - \gamma_{H_2O}(0.15\,\text{m}) = p_A$$

where $p_A = -\gamma_{Hg}(0.030\,\text{m})$

Thus,

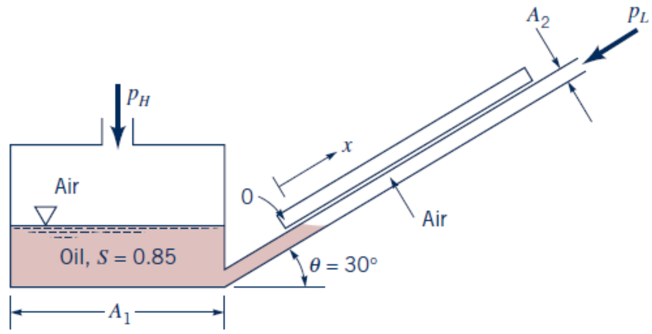
$$p_B = -\gamma_{Hg}(0.030\,\text{m}) - \gamma_{oil}(0.45\,\text{m}) + \gamma_{Hg}(0.3\,\text{m}) + \gamma_{H_2O}(0.15\,\text{m})$$

$$= -\left(133\frac{\text{kN}}{\text{m}^3}\right)(0.030\,\text{m}) - \left(8.95\frac{\text{kN}}{\text{m}^3}\right)(0.45\,\text{m}) + \left(133\frac{\text{kN}}{\text{m}^3}\right)(0.3\,\text{m}) + \left(9.80\frac{\text{kN}}{\text{m}^3}\right)(0.15\,\text{m})$$

$$\boxed{p_B = 33.4\,\text{kPa}}$$

Problem 2.58

Consider the cistern manometer shown in the figure below. The scale is set up on the basis that the cistern area A_1 is infinite. However, A_1 is actually 50 times the internal cross-sectional area A_2 of the inclined tube. Find the percentage error (based on the scale reading) involved in using this scale.



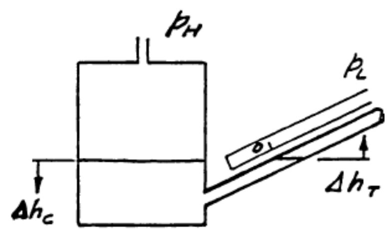
Solution 2.58

GIVEN: The figure in the problem with $A_1 = 50\,A_2$.

FIND: Percent error in using a scale based on A_1 as infinite.

SOLUTION:

Apply manometer rule, using the elevation changes shown in the sketch.



$$P_H - P_L = \gamma\left(\Delta h_c + \Delta h_T\right)$$

$$\frac{P_H - P_L}{\gamma} = \Delta h_c + \Delta h_T \qquad (1)$$

Δh_c and Δh_T are vertical drop & rise of fluid level from the scale's zero.

Conservation of mass requires, $\rightarrow$ $\quad \Delta h_c A_1 = x A_2$ or $\quad \Delta h_c = \dfrac{x A_2}{A_1} \qquad (2)$

Geometry $\rightarrow$ $\quad \Delta h_T = x \sin 30° \qquad (3)$

Eqs. (2) and (3) into Eq. (1) $\quad \dfrac{P_H - P_L}{\gamma} = \dfrac{x A_2}{A_1} + x\sin 30° = x\left(\dfrac{A_2}{A_1} + \dfrac{1}{2}\right)$

$$x = \frac{P_H - P_L}{\gamma\left(\dfrac{A_2}{A_1} + \dfrac{1}{2}\right)}$$

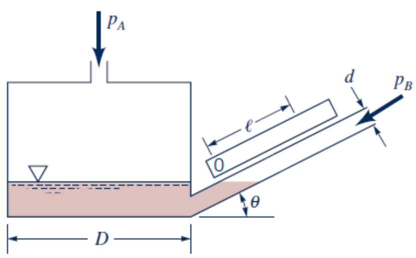
For an infinite cistern area, $A_1 = \infty \rightarrow$ $\quad x_\infty = \dfrac{P_H - P_L}{\gamma\left(\dfrac{A_2}{A_1} + \dfrac{1}{2}\right)} = \dfrac{2(P_H - P_L)}{\gamma}$

Percent error $=$ $\%E = \left(\dfrac{x_\infty - x}{x}\right)100 = \left(\dfrac{x_\infty}{x} - 1\right)100 = \left[2\left(\dfrac{A_2}{A_1} + \dfrac{1}{2}\right) - 1\right]100$

For $\dfrac{A_2}{A_1} = \dfrac{1}{50} \rightarrow \%E = \left[2\left(\dfrac{1}{50} + \dfrac{1}{2}\right) - 1\right]100 \qquad \rightarrow \qquad \boxed{\%E = 4\%}$

Problem 2.59

The cistern shown in the figure below has a diameter D that is 4 times the diameter d of the inclined tube. Find the drop in the fluid level in the cistern and the pressure difference ($p_A - p_B$) if the liquid in the inclined tube rises $l = 20$ in. The angle θ is $20°$. The fluid's specific gravity is 0.85.

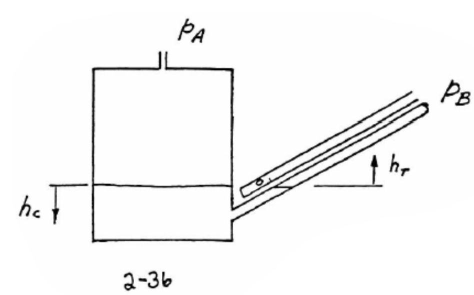


Solution 2.59

GIVEN: The figure in the problem, $D = 4d$, $l = 20\text{in.}$, $\theta = 20^{\circ}$, $\text{S} = 0.85$.

FIND: $P_A - P_B$

SOLUTION:



Conservation of mass requires that cistern level drops as the tube level rises, as show in the sketch.

h_T = vertical rise in tube, h_c = drop in cistern fluid level.

Apply manometer rule,

$$P_B = P_A - \gamma(h_T + h_c)$$

Conservation of mass requires,

$$h_c A_c = l A_T$$

$$h_c = l\left(\frac{A_T}{A_C}\right) = l\left(\frac{d}{D}\right)^2$$

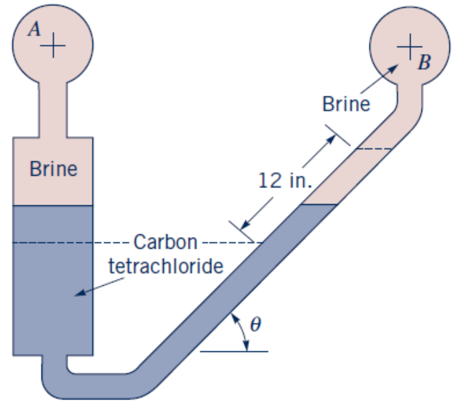
$h_T = l \sin\theta$,

$$P_A - P_B = \gamma\left(h_T + h_c\right) = \gamma l\left[\sin\theta + \left(\frac{d}{D}\right)^2\right]$$

$$P_A - P_B = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(0.85)(20\text{in.})\left[\sin 20^{\circ} + \left(\frac{1}{4}\right)^2\right]\left(\frac{\text{ft}^3}{1728\,\text{in}^3}\right) \rightarrow \boxed{P_A - P_B = 0.248 \text{ psia}}$$

Problem 2.60

The inclined differential manometer of the figure below contains carbon tetrachloride. Initially the pressure differential between pipes A and B, which contain a brine ($SG = 1.1$), is zero as illustrated in the figure. It is desired that the manometer give a differential reading of 12 in. (measured along the inclined tube) fora pressure differential of 0.1 psi. Determine the required angle of inclination, θ.

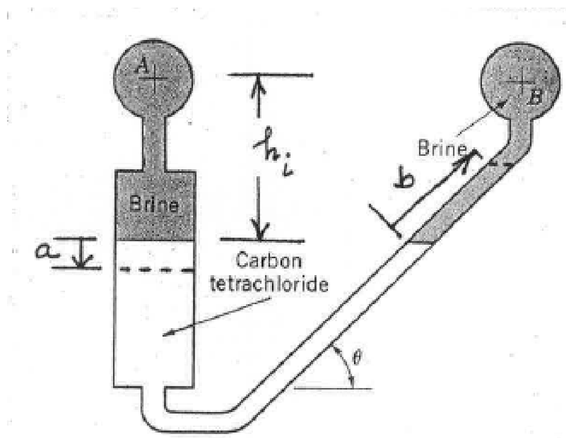


Solution 2.60

When $p_A - p_B$ is increased to $p'_A - p'_B$ the left column falls a distance, a, and the right column rises a distance b along the inclined tube as shown in the figure. For this final configuration:

$$p'_A + \gamma_{br}(h_i + a) - \gamma_{CCl_4}(a + b\sin\theta) - \gamma_{br}(h_i - b\sin\theta) = p'_B$$

$$p'_A - p'_B + (\gamma_{br} - \gamma_{CCl_4})(a + b\sin\theta) = 0 \qquad (1)$$



The differential reading, Δh, along the tube is

$$\Delta h = \frac{a}{\sin\theta} + b$$

Thus, from Eq.(1)

$$p'_A - p'_B + (\gamma_{br} - \gamma_{CCl_4})(\Delta h \sin\theta) = 0$$

$$\sin\theta = \frac{-(p'_A - p'_B)}{(\gamma_{br} - \gamma_{CCl_4})(\Delta h)}$$

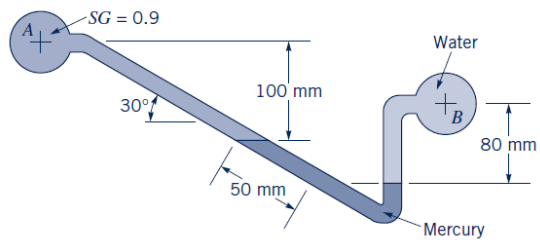
For $p'_A - p'_B = 0.1\,\text{psi}$

$$\sin\theta = \frac{-\left(0.1\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left[(1.1)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right) - 99.5\frac{\text{lb}}{\text{ft}^3}\right]\left(\frac{12}{12}\text{ft}\right)} = 0.466$$

For $\Delta h = 12\,\text{in.}$, $\boxed{\theta = 27.8°}$

Problem 2.61

Determine the new differential reading along the inclined leg of the mercury manometer of the figure below, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

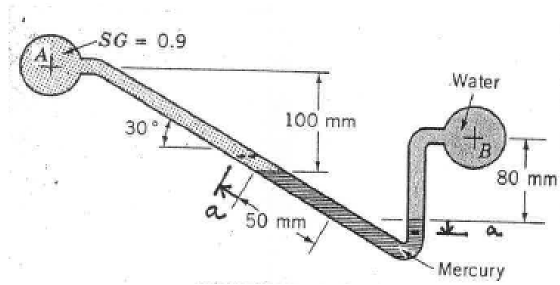


Solution 2.61

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05\sin 30°) - \gamma_{H_2O}(0.08) = p_B \qquad (1)$$

where all length are in m. When p_A decreases, left column moves up a distance, a, and right column moves down a distance, a, as shown in the figure.



For the final configuration:

$$p'_A + \gamma_A(0.1 - a\sin 30°) + \gamma_{Hg}(a\sin 30° + 0.05\sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \qquad (2)$$

Where p'_A is the new pressure in pipe A. Subtract Eq.(2) from Eq.(1) to obtain

$$p'_A - p'_B + \gamma_A(a\sin 30°) - \gamma_{Hg}a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus, $a = \dfrac{-(p_A - p'_A)}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$

For $p_A - p'_A = 10\,\text{kPa}$

$$a = \frac{-10\dfrac{\text{kN}}{\text{m}^2}}{(0.9)\left(9.81\dfrac{\text{kN}}{\text{m}^3}\right)(0.5) - \left(133\dfrac{\text{kN}}{\text{m}^3}\right)(0.5+1) + 9.81\dfrac{\text{kN}}{\text{m}^3}} = 0.0540\,\text{m}$$

New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a = \frac{0.0540\,\text{m}}{0.5} + 0.05\,\text{m} + 0.0540\,\text{m} = \underline{\underline{0.0212\,\text{m}}}$$

Problem 2.62

A student needs to measure the air pressure inside a compressed air tank but does not have ready access to a pressure gage. Using materials already in the lab, she builds a U-tube manometer using two clear 3-ft-long plastic tubes, flexible hoses, and a tape measure. The only readily available liquids are water from a tap and a bottle of corn syrup. She selects the corn syrup because it has a larger density ($SG = 1.4$). What is the maximum air pressure, in psia, that can be measured?

Solution 2.62

Known: two 3-ft-long clear tubes, unknown length flexible hose, tape measure, corn syrup ($SG = 1.4$)

Determine: Maximum compressed air pressure

Strategy: reflect on possible physical considerations, apply hydrostatic pressure equation

Solution:

Form u-tube manometer by connecting bottom of tubes with hose; top of one tube is connected to the tank.

Assume that tubes are of equal diameter.

Assume that hose is not transparent
→ opaque hose cannot contribute to usable manometer "height"

Set tubes at equal elevation.

Fill tubes to bottom of tube with corn syrup

→ maximum difference that can be observed is 3 ft

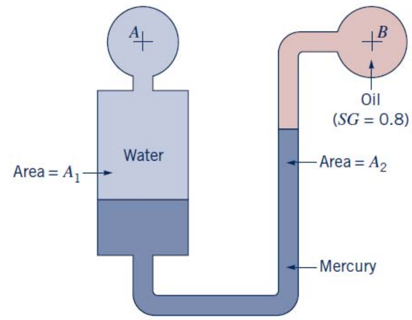
$$\Delta p = \gamma \Delta h = S\gamma_{\mathrm{H_2O}}\Delta h$$

$$= (1.4)\left(62.4\frac{\mathrm{lb}}{\mathrm{ft}^3}\right)(3\ \mathrm{ft})\left(\frac{1\ \mathrm{ft}^2}{144\ \mathrm{in}^2}\right) = 1.82\ \mathrm{psig} = 16.52\ \mathrm{psia}$$

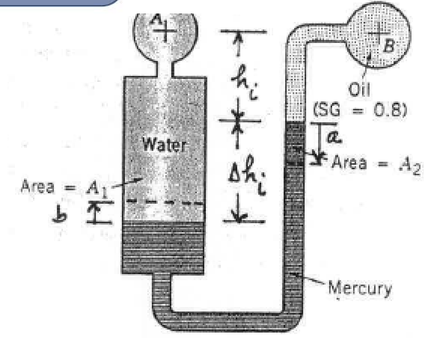
$$\boxed{\Delta p_{\max} = 16.5\ \mathrm{psi}}$$

Problem 2.63

Determine the ratio of areas, $\frac{A_1}{A_2}$, of the two manometer legs of the figure below if a change in pressure in pipe *B* of 0.5 psi gives a corresponding change of 1 in. in the level of the mercury in the right leg. The pressure in pipe *A* does not change.



Solution 2.63



For the initial configuration (see the figure):

$$p_A + \gamma_{H_2O}\left(h_i + \Delta h_i\right) - \gamma_{Hg}\left(\Delta h_i\right) - \gamma_{oil}\left(h_i\right) = p_B \qquad (1)$$

When p_B increases the right column falls a distance, a, and the left column rises a distance, b.
Since the volume of the liquid must remain constant,

$$A_1 b = A_2 a \text{ or } \frac{A_1}{A_2} = \frac{a}{b}.$$

For the final configuration, with pressure in B equal to p'_B:

$$p_A + \gamma_{H_2O}\left(h_i + \Delta h_i - b\right) - \gamma_{Hg}\left(\Delta h_i - a - b\right) - \gamma_{oil}\left(h_i + a\right) = p'_B \qquad (2)$$

Subtract Eq.(1) from Eq.(2) to obtain

$$-\gamma_{H_2O}(b) + \gamma_{Hg}(a+b) - \gamma_{oil}(a) = p'_B - p_B$$

$$b = \frac{\left(p'_B - p_B\right) - \gamma_{Hg}(a) + \gamma_{oil}(a)}{\gamma_{Hg} - \gamma_{H_2O}}$$

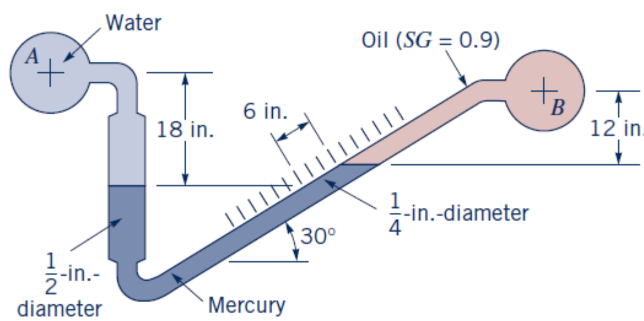
$p'_B - p_B = 0.5\,\text{psi}$ and $a = 1\,\text{in.}$

$$b = \frac{\left(0.5\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) - \left(847\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ft}\right) + (0.8)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ft}\right)}{847\frac{\text{lb}}{\text{ft}^3} - 62.4\frac{\text{lb}}{\text{ft}^3}} = 0.00711\,\text{ft}$$

$$\frac{A_1}{A_2} = \frac{a}{b} = \frac{\frac{1}{12}\text{ft}}{0{,}00711\,\text{ft}} \rightarrow \boxed{\frac{A_1}{A_2} = 11.7}$$

Problem 2.64

Determine the change in the elevation of the mercury in the left leg of the manometer of the figure below as a result of an increase in pressure of 5 psi in pipe *A* while the pressure in pipe *B* remains constant.



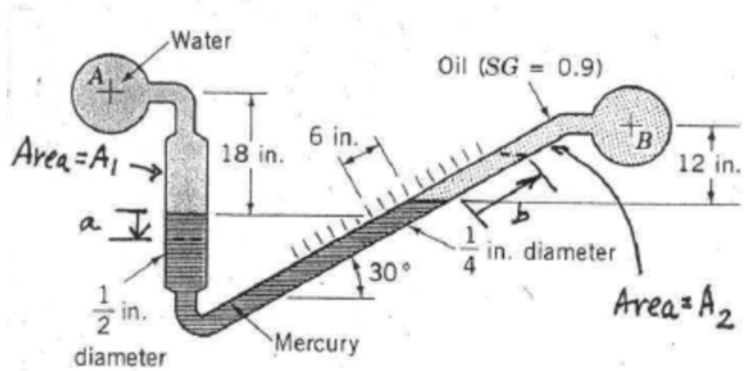
Solution 2.64

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30°\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

Where all lengths are in ft.

When p_A increases to p'_A the left column falls by the distance, a, and the right column moves up the dance, b, as shown in the figure.



For the final configuration:

$$p'_A + \gamma_{H_2O}\left(\frac{18}{12} + a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30° + b\sin 30°\right) - \gamma_{oil}\left(\frac{12}{12} - b\sin 30°\right) = p_B \quad (2)$$

Subtract Eq.(1) from Eq.(2) to obtain

$$p'_A - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30°) + \gamma_{oil}(b\sin 30°) = 0 \qquad (3)$$

The volume of liquid must be constant: $A_1 a = A_2 b$,

$$\left(\frac{1}{2}\text{in.}\right)^2 a = \left(\frac{1}{4}\text{in.}\right)^2 b \rightarrow b = 4a$$

Thus, Eq.(3) can be written as

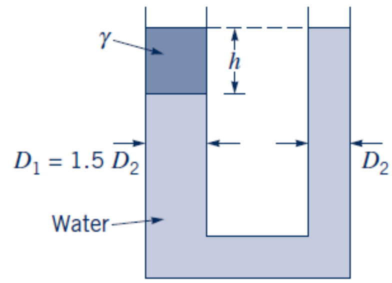
$$p'_A - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30°) + \gamma_{oil}(4a\sin 30°) = 0$$

$$a = \frac{-(p'_A - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{62.4\,\frac{\text{lb}}{\text{ft}^3} - \left(847\,\frac{\text{lb}}{\text{ft}^3}\right)(3) + (0.9)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(2)}$$

$$\boxed{a = 0.304 \text{ ft down}}$$

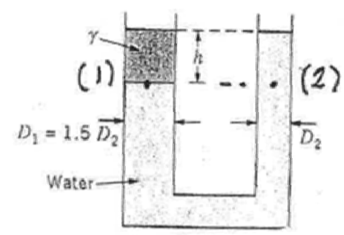
Problem 2.65

The U-shaped tube shown in the figure below initially contains water only. A second liquid with specific weight, γ, less than water is placed on top of the water with no mixing occurring. Can the height, h, of the second liquid be adjusted so that the left and right levels are at the same height? Provide proof of your answer.



Solution 2.65

The pressure at point (1) must be equal to the pressure at point (2) since the pressures at equal elevations in a continuous mass of fluid must be the same.



$$p_1 = \gamma h$$

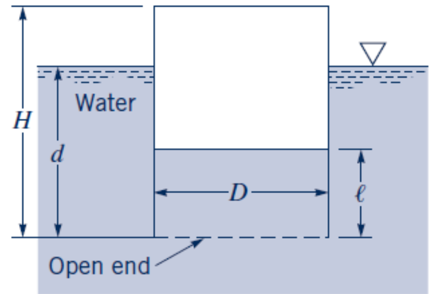
$$p_2 = \gamma_{H_2O} h$$

There two pressures can only be equal if $\gamma = \gamma_{H_2O}$.

Since $\gamma \neq \gamma_{H_2O}$, the configuration shown in the figure is not possible. $\boxed{\text{No}}$

Problem 2.66

An inverted hollow cylinder is pushed into the water as is shown in the figure below. Determine the distance, ℓ, that the water rises in the cylinder as a function of the depth, d, of the lower edge of the cylinder. Plot the results for $0 \leq d \leq H$, when H is equal to 1 m. Assume the temperature of the air within the cylinder remains constant.



Solution 2.66

For constant temperature compression within the cylinder, $p_i \mathcal{V}_i = p_f \mathcal{V}_f$ (1)

where $\mathcal{V}$ is the air volume, and i and f refer to the initial and final states, respectively

$$p_i = p_{\text{atm}} \qquad p_f = \gamma(d-\ell) + p_{\text{atm}}$$

$$\mathcal{V}_i = \frac{\pi}{4} D^2 H \qquad \mathcal{V}_f = \frac{\pi}{4} D^2 (H-\ell)$$

From Eq.(1): $p_{\text{atm}}\left(\frac{\pi}{4}D^2 H\right) = \left(\gamma(d-\ell) + p_{\text{atm}}\right)\frac{\pi}{4}D^2(H-\ell)$ (2)

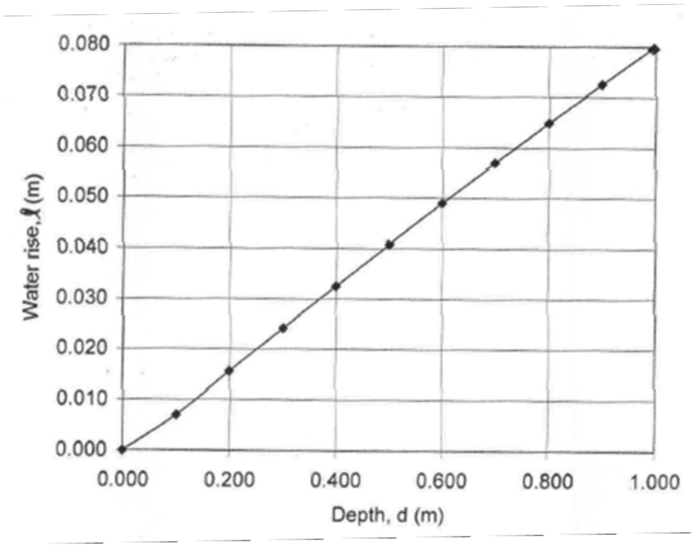
$p_{\text{atm}} = 101\, kPa,\ \gamma = 9.80\frac{\text{kN}}{\text{m}^3},\ H = 1\text{ m},\ \rightarrow\ \ell^2 - (d+11.31)\ell + d(1\text{m}) = 0$

$$\ell = \frac{(d+11.31) \pm \sqrt{d^2 + 18.61d + 128}}{2}$$

For $d = 0$, $\ell = 0$, → use negative sign: $\ell = \dfrac{(d+11.31) - \sqrt{d^2 + 18.61d + 128}}{2}$

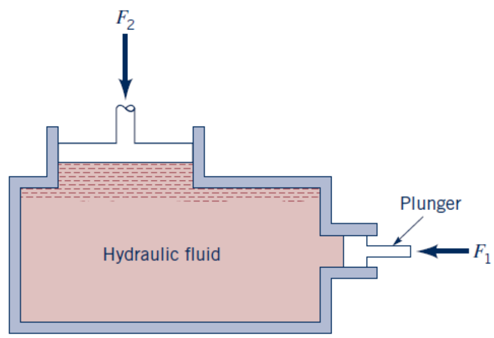
Tabulated data with the corresponding plot are shown below.

Depth, d (m)	Water rise, l (m)
0.000	0.000
0.100	0.007
0.200	0.016
0.300	0.024
0.400	0.033
0.500	0.041
0.600	0.049
0.700	0.057
0.800	0.065
0.900	0.073
0.1000	0.080



Problem 2.68

The basic elements of a hydraulic press are shown in the figure below. The plunger has an area of 1 in.^2, and a force, F_1, can be applied to the plunger through a lever mechanism having a mechanical advantage of 8 to 1. If the large piston has an area of 150 in.^2, what load, F_2, can be raised by a force of 30 lb applied to the lever? Neglect the hydrostatic pressure variation.



Solution 2.68

A force of 30 lb applied to the level results in a plunger force, F_1, of $F_1 = (8)(30) = 240 \text{ lb}$.

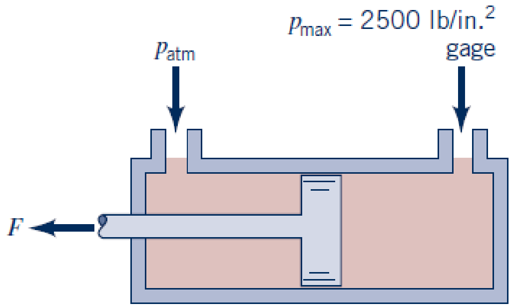
Since $F_1 = pA_1$ and $F_2 = pA_2$ where p is the pressure and A_1 and A_2 are the areas of the plunger and piston, respectively. Since p is constant throughout the chamber,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

so that $F_2 = \dfrac{A_2}{A_1} F_1 = \left(\dfrac{150 \text{ in.}^2}{1 \text{ in.}^2}\right)(240 \text{ lb}) \rightarrow \boxed{F_2 = 36{,}000 \text{ lb}}$

Problem 2.69

The hydraulic cylinder shown in the figure below, with a 4-in.- diameter piston, is advertised as being capable of providing a force of $F = 20$ tons. If the piston has a design pressure (the maximum pressure at which the cylinder should safely operate) of 2500 lb/in^2, gage, can the cylinder safely provide the advertised force?



Solution 2.69

Assuming a "ton" is a "short ton", the advertised force is

$$F_{adv} = 20\,\text{tons}\left(\frac{2000\,\text{lb}}{\text{ton}}\right) = 40000\,\text{lb}$$

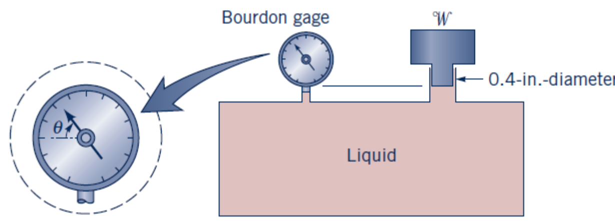
The maximum force that can be safely developed by the piston is

$$F_{safe} = P_{gage}A = P_{gage}\left(\frac{\pi}{4}d^2\right) = \left(2500\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{\pi}{4}(4\text{ in})^2\right) = 31400\,\text{lb}$$

No. The cylinder cannot safely provide 20 tons of force.

Problem 2.70

A Bourdon gage is often used to measure pressure. One way to calibrate this type of gage is to use the arrangement shown in the figure below (a). The container is filled with a liquid and a weight, W, placed on one side with the gage on the other side. The weight acting on the liquid through a 0.4-in.-diameter opening creates a pressure that is transmitted to the gage. This arrangement, with a series of weights, can be used to determine what a change in the dial movement, θ, in the figure below (b), corresponds to in terms of a change in pressure. For a particular gage, some data are given below. Based on a plot of these data, determine the relationship between θ and the pressure, p, where p is measured in psi.



(b) (a)

$\mathcal{W}$ (lb)	0	1.04	2.00	3.23	4.05	5.24	6.31
θ (deg.)	0	20	40	60	80	100	120

Solution 2.70

$$p = \frac{W}{\text{Area}} = \frac{W\,(\text{lb})}{\frac{\pi}{4}(0.4\,\text{in.})^2} = 7.96\,W\,(\text{lb}) \qquad (1)$$

(where p is in psi)

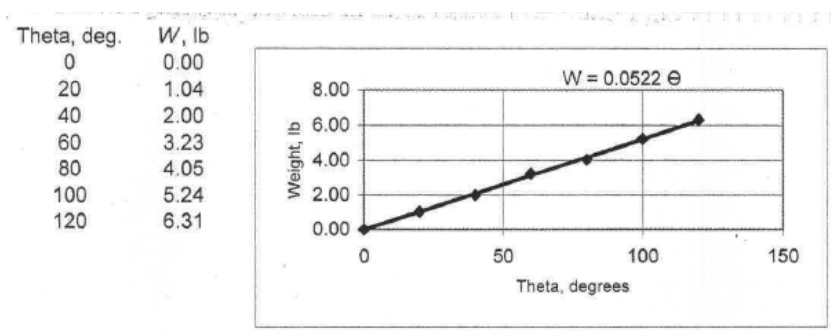
From graph

$W = 0.0522\,\theta$

So that from Eq.(1)

$$\frac{p(\text{psi})}{7.96} = 0.0522\,\theta$$

$$\boxed{p(\text{psi}) = 0.416\,\theta}$$



Theta, deg.	W, lb
0	0.00
20	1.04
40	2.00
60	3.23
80	4.05
100	5.24
120	6.31

Problem 2.71

A bottle jack allows an average person to lift one corner of a 4000-lb automobile completely off the ground by exerting less than 20 lb of force. Explain how a 20-lb force can be converted into hundreds or thousands of pounds of force, and why this does not violate our general perception that you can't get something for nothing (a somewhat loose paraphrase of the first law of thermodynamics). Hint: Consider the work done by each force.

Solution 2.71

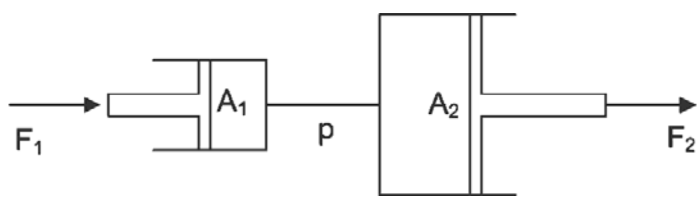
Known: 20 lb applied force lifts corner of 4,000 lb automobile

Determine: Explain

Strategy: Force = (pressure)(area); consider work done by pistons of different size

Solution:

Consider two piston/cylinders of different diameter connected by a rigid tube filled with a liquid.



$$\frac{F_1}{A_1} = p = \frac{F_2}{A_2} \quad \rightarrow F_2 = F_1\left(\frac{A_2}{A_1}\right) = F_1\left(\frac{d_2}{d_1}\right)^2$$

$$\frac{d_2}{d_1} = 4 \rightarrow F_2 = 16F_1$$

$$\frac{d_2}{d_1} = 10 \rightarrow F_2 = 100F_1$$

Therefore producing the required force multiplication is not difficult.

Assuming all solid boundaries are rigid, the volume pushed out of the small cylinder must equal that entering the large cylinder.

$$\Delta \mathcal{V}_1 = \Delta x_1 A_1 = \Delta x_2 A_2 \quad \rightarrow \Delta x_2 = \Delta x_1\left(\frac{A_1}{A_2}\right)$$

Therefore, the larger piston moves a smaller distance than the smaller piston.

Comparing the work done on the smaller piston to the work done by the larger piston:

$$W_2 = F_2 \Delta x_2 = \left[F_1\left(\frac{A_2}{A_1}\right)\right]\left[\Delta x_1\left(\frac{A_1}{A_2}\right)\right] = F_1 \Delta x_1 = W_1$$

$\boxed{\text{Therefore, the work done on the small piston is equal to work done by large piston.}}$

Problem 2.72

Suction is often used in manufacturing processes to lift objects to be moved to a new location. A 4-ft by 8-ft sheet of $\frac{1}{2}$-in. plywood weighs approximately 36 lb. If the machine's end effector has a diameter of 5 in., determine the suction pressure required to lift the sheet, expressed in inches of H_2O suction.

Solution 2.72

Known: $W = 36\text{ lb}$; $D_{\text{CUP}} = 5\text{ in.}$

Determine: Suction required to lift sheet in inches H20

Strategy: Force = (pressure)(area);

Solution:

$$W = F = pA = p\pi R^2$$

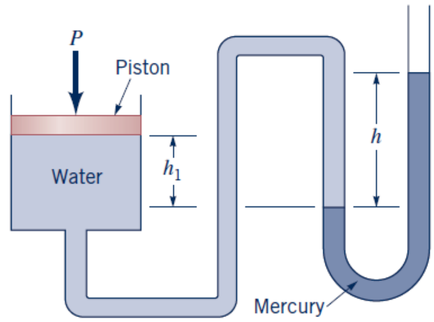
$$p = \frac{W}{\pi R^2} = \frac{36\text{ lb}}{\pi(2.5\text{ in})^2} = 1.833\frac{\text{lb}}{\text{in}^2} = \gamma_{H_2O}h$$

$$h = \frac{1.833\frac{\text{lb}}{\text{in}^2}}{62.4\frac{\text{lb}}{\text{ft}^3}\times\frac{1\text{ ft}^3}{1728\text{ in}^3}} = 50.77\text{ in }H_2O$$

$$\boxed{h = 51\text{ in }H_2O}$$

Problem 2.73

A piston having a cross-sectional area of $0.07\ \text{m}^2$ is located in a cylinder containing water as shown in the figure below. An open U-tube manometer is connected to the cylinder as shown. For $h_1 = 60$ mm and $h = 100$ mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.



Solution 2.73

For equilibrium, $P = p_p A_P$ where p_p is the pressure acting on piston and A_P is the area of the piston. Also,

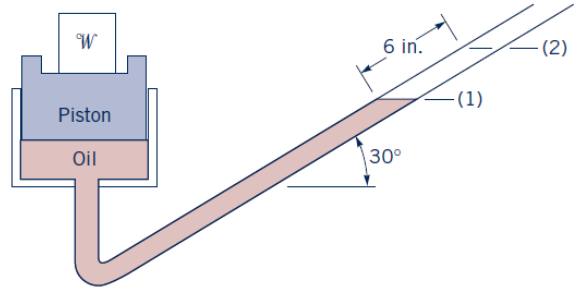
$$p_p + \gamma_{\text{H}_2\text{O}} h_1 - \gamma_{\text{Hg}} h = 0$$

$$p_p = \gamma_{\text{Hg}} h - \gamma_{\text{H}_2\text{O}} h_1 = \left(133 \frac{\text{kN}}{\text{m}^3}\right)(0.100\,\text{m}) - \left(9.80 \frac{\text{kN}}{\text{m}^3}\right)(0.060\,\text{m}) = 12.7 \frac{\text{kN}}{\text{m}^2}$$

$$p = \left(12.7 \times 10^3 \frac{\text{N}}{\text{m}^2}\right)(0.07\,\text{m}^2) \quad \rightarrow \quad \boxed{p = 889\ \text{N}}$$

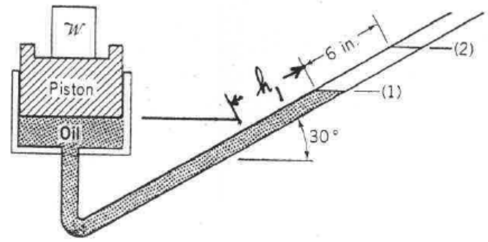
Problem 2.74

A 6-in.-diameter piston is located within a cylinder that is connected to a $\frac{1}{2}$-in.-diameter inclined-tube manometer as shown in the figure below. The fluid in the cylinder and the manometer is oil (specific weight = $59\,\text{lb/ft}^3$). When a weight, W, is placed on the top of the cylinder, the fluid level in the manometer tube rises from point (1) to (2). How heavy is the weight? Assume that the change in position of the piston is negligible.



Solution 2.74

With piston alone let pressure on face of piston $= p_p$. Manometer equation becomes



$$p_p - \gamma_{\text{oil}} h_1 \sin 30° = 0 \qquad (1)$$

With weight added pressure p_p increases to p'_p where

$$p'_p = p_p + \frac{W}{A_P} \qquad (A_P \text{ area of piston})$$

$$p'_p - \gamma_{\text{oil}}\left(h_1 + \frac{6}{12}\text{ft}\right)\sin 30° = 0 \qquad (2)$$

Subtract Eq.(1) from Eq.(2) to obtain

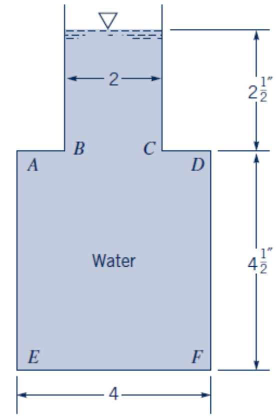
$$p'_p - p_p - \gamma_{\text{oil}}\left(\frac{6}{12}\text{ft}\right)\sin 30° = 0$$

$$\frac{W}{A_P} = \gamma_{\text{oil}}\left(\frac{6}{12}\text{ft}\right)\sin 30°$$

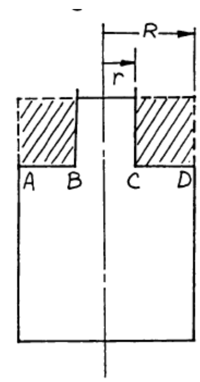
$$\frac{W}{\frac{\pi}{4}\left(\frac{6}{12}\text{ft}\right)^2} = \left(59\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6}{12}\text{ft}\right)(0.5) \rightarrow \boxed{W = 2.90\text{ lb}}$$

Problem 2.75

The container shown in the figure below has square cross sections. Find the vertical force on the horizontal surface, *ABCD*.



Solution 2.75

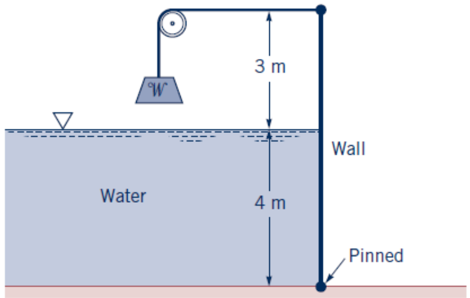


The vertical force on surface ABCD is equal to the weight of the imaginary fluid above ABCD as show on the picture on the right, so

$$
\begin{aligned}
F &= \gamma_W V \\
&= \gamma_W \pi \left(R^2 - r^2\right) h \\
&= \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right) \pi \left(2^2 - 1^2\right) \text{ft}^2 \left(2.5\,\text{ft}\right) \quad \rightarrow \quad \boxed{F = 1470\,\text{lb}}
\end{aligned}
$$

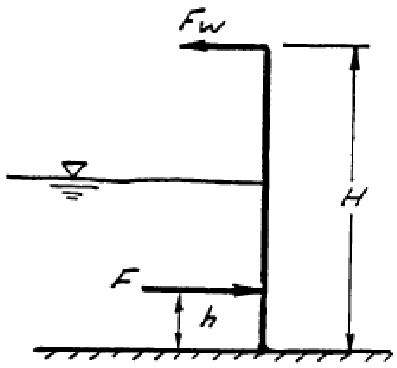
Problem 2.76

Find the weight W needed to hold the wall shown in the figure below upright. The wall is 10 m wide.



Solution 2.76

The hydrostatic force F on the wall is found from



$$
\begin{aligned}
F &= \rho g h_c A \\
&= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2\text{m})\left(4\times 10\text{m}^2\right) \\
&= 78500\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)\left(\frac{\text{kN}}{1000\,\text{N}}\right) \\
&= 785\,\text{kN}
\end{aligned}
$$

The force F is located one-third of the water depth from the bottom of the water.

$$h = \frac{1}{3}(4\text{m}) = 1.33\,\text{m}$$

Summing moments about the pinned joint,

$$F_W = \frac{h}{H}F = \frac{(1.33\,\text{m})}{(7\,\text{m})}(785\,\text{kN}) = 149\,\text{kN}$$

Assuming no friction between the rope and the pulley,

$$W = F_W \quad \rightarrow \quad \boxed{W = 149\text{ kN}}$$

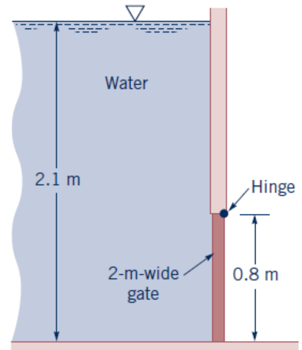
DISCUSSION

Note that the atmospheric pressure acts on both sides of the wall.

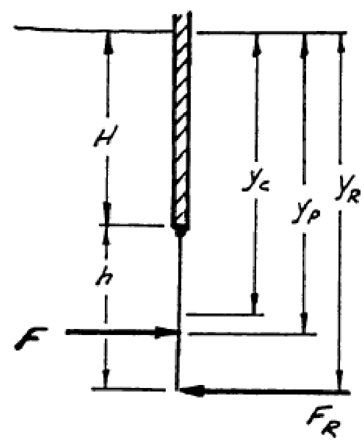
Therefore, the forces due to atmospheric pressure are equal and opposite, and cancel.

Problem 2.77

Determine the magnitude and direction of the force that must be applied to the bottom of the gate shown in the figure below to keep the gate closed.



Solution 2.77



The hydrostatic force on the gate is

$$
\begin{aligned}
F &= \gamma y_c A \\
&= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(1.3\,\text{m}+0.4\,\text{m})(2\,\text{m}\times 0.8\,\text{m}) \\
&= 26700\,\text{N}
\end{aligned}
$$

The location of the force F is

$$y_p = y_c + \frac{I_{xc}}{12 y_c A}$$

Using Appendix,

$$
\begin{aligned}
y_p &= y_c + \frac{bh^3}{12 y_c A} = y_c + \frac{h^2}{12 y_c} \\
&= (1.3+0.4)\,\text{m} + \frac{(0.8\,\text{m})^2}{12(1.3+0.4)\,\text{m}} = 1.73\,\text{m}
\end{aligned}
$$

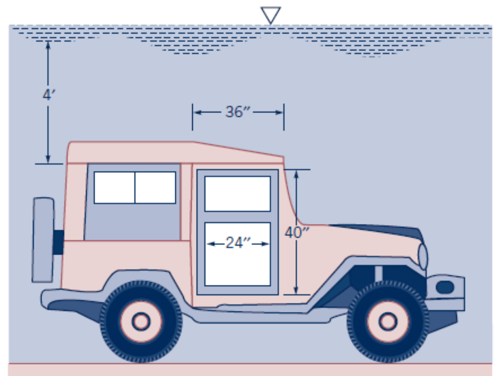
Summing moments about the hinge,

$$\sum M_{\text{hinge}} = F_R h - F\left(y_p - H\right) = 0$$

$$F_R = \frac{F\left(y_p - H\right)}{h} = \frac{(26700\text{N})(1.73-1.3)\text{m}}{0.8\,\text{m}} \quad \rightarrow \quad \boxed{F_R = 14{,}400\ \text{N}}$$

Problem 2.78

An automobile has just dropped into a river. The car door is approximately a rectangle, measures 36 in. wide and 40 in. high, and hinges on a vertical side. The water level inside the car is up to the midheight of the door, and the air inside the car is at atmospheric pressure. Calculate the force required to open the door if the force is applied 24 in. from the hinge line. See the figure below. (The driver did not have the presence of mind to open the window to escape.)



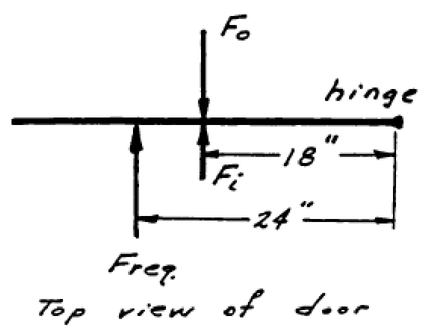
Solution 2.78

Note that the force due to atmospheric pressure acts in equal and opposite directions on two sides of the door. The hydrostatic force on the inside of the door is

$$F_i = \gamma h_c A = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\text{ft}^3}{1728\,\text{in}^3}\right)(10\,\text{in})\left(\frac{36\,\text{in.}\times 40\,\text{in.}}{2}\right) = 260\,\text{lb}$$

The hydrostatic force on the outside of the door is

$$\begin{aligned} F_o &= \gamma h_c A \\ &= \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\text{ft}^3}{1728\,\text{in}^3}\right)\left(4\,\text{ft}\times\frac{12\,\text{in}}{\text{ft}}+20\,\text{in}\right)(36\times 40)\,\text{in}^2 \\ &= 3540\,\text{lb} \end{aligned}$$



Summing moments about the hinge line

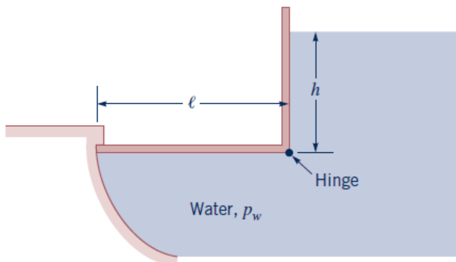
$$\sum M_H = (24\,\text{in.})F_{\text{req}} + (18\,\text{in.})F_i - (18\,\text{in.})F_o$$

$$F_{\text{req}} = (F_o - F_i)\left(\frac{18}{24}\right) = (3540-260)\,\text{lb}\left(\frac{18}{24}\right) \quad \rightarrow \quad \boxed{F_{\text{req}} = 2{,}460\,\text{lb}}$$

Problem 2.79

Consider the gate shown in the figure below. The gate is massless and has a width b (perpendicular to the paper). The hydrostatic pressure on the vertical side creates a counterclockwise moment about the hinge, and the hydrostatic pressure on the horizontal side (or bottom) creates a clockwise moment about the hinge. Show that the net clockwise moment is

$$\sum u = \rho_w ghb\left(\frac{\ell^2}{2}-\frac{h^2}{6}\right).$$



Solution 2.79

The vertical force on the horizontal side is

$$F_v = \rho_w ghA = \rho_w gh(l \times b)$$

Constant force →resultant acts at midpoint

The horizontal force on the vertical side is

$$F_H = \rho_w gh_c A = \rho_w g\left(\frac{h}{2}\right)(h \times b)$$

The resultant acts at

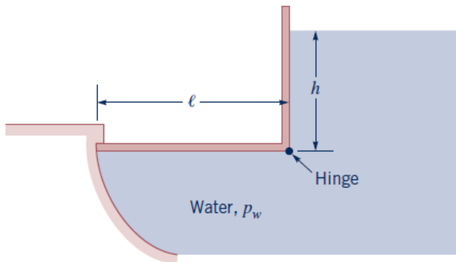
$$y_p = y_c + \frac{I_{xc}}{y_c A} = \frac{h}{2} + \frac{\frac{1}{12}bh^3}{\frac{h}{2}(bh)} = \frac{h}{2} + \frac{h}{6} = \frac{2h}{3}$$

Summing moments about the hinge

$$\overset{\rightarrow}{+}\sum M = F_v x_p - F_H\left(h - y_p\right) = \rho_w ghlb\left(\frac{l}{2}\right) - \frac{\rho_w gh^2 b}{2}\left(\frac{h}{3}\right)$$

$$\rightarrow \quad \boxed{\overset{\rightarrow}{+}\sum M = \rho_w ghb\left(\frac{l^2}{2}-\frac{h^2}{6}\right)}$$

Problem 2.80
Consider the gate shown in the figure below. The gate is massless and has a width b (perpendicular to the paper). The hydrostatic pressure on the vertical side creates a counterclockwise moment about the hinge, and the hydrostatic pressure on the horizontal side (or bottom) creates a clockwise moment about the hinge. Will the gate ever open?



Solution 2.80

Sum moments about hinge

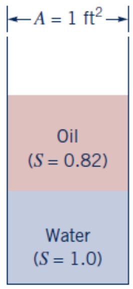
$$\overset{\rightharpoonup}{+}\sum M = F_V L_V - F_H L_H$$
$$= \rho_w g h b l\left(\frac{l}{2}\right) - \rho_w g\left(\frac{h}{2}\right) b h\left(\frac{h}{3}\right)$$
$$= \rho_w g h b\left(\frac{l^2}{2} - \frac{h^2}{6}\right)$$
$$\frac{h^2}{6} > \frac{l^2}{2}, \quad \text{or} \quad h > \sqrt{3}\, l$$

If sum of moments is negative, gate will open

$$\overset{\rightharpoonup}{+}\sum M < 0 \quad \rightarrow \quad \frac{h^2}{6} > \frac{l^2}{2} \quad \rightarrow \quad \boxed{\text{if } h > \sqrt{3}\, l \text{, gate will open}}$$

Problem 2.81

A tank contains 6 in. of oil ($S = 0.82$) above 6 in. of water ($S = 1.00$). Find the force on the bottom of the tank. See the figure below.



Solution 2.81

Assume atmospheric pressure acts on outside of tank.

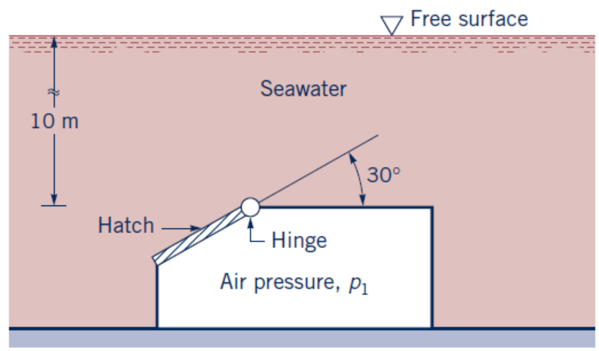
Pressure is constant at constant elevation in a stagnant fluid.

Apply hydrostatic pressure equation

$$
\begin{aligned}
F_{\text{NET}} &= P_{\text{BOTTOM}} A \\
&= \left(\gamma_{oil} h_{oil} + \gamma_w h_w\right) A \\
&= \gamma_w \left(S_{oil} h_{oil} + S_w h_w\right) A \\
&= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left[0.82\left(\frac{1}{2}\text{ ft}\right) + 1\left(\frac{1}{2}\text{ ft}\right)\right]\left(1\text{ ft}^2\right) \quad \rightarrow \quad \boxed{F_{\text{NET}} = 56.7\text{ lb}}
\end{aligned}
$$

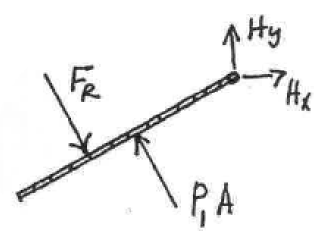
Problem 2.82

A structure is attached to the ocean floor as shown in the figure below. A 2-m-diameter hatch is located in an inclined wall and hinged on one edge. Determine the minimum air pressure, p_1, within the container that will open the hatch. Neglect the weight of the hatch and friction in the hinge.



Solution 2.82

$$F_R = \gamma h_c A \text{ where } h_c = 10\,\text{m} + \frac{1}{2}(2\,\text{m})\sin 30° = 10.5\,\text{m}$$



Thus,

$$F_R = \left(10.1\times10^3\,\frac{\text{N}}{\text{m}^3}\right)(10.5\,\text{m})\left(\frac{\pi}{4}\right)(2\,\text{m})^2 = 3.33\times10^5\,\text{N}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \text{ where } y_c = \frac{10\,\text{m}}{\sin 30°} + 1\,\text{m} = 21\,\text{m}$$

$$y_R = \frac{\left(\frac{\pi}{4}\right)(1\,\text{m})^4}{(21\,\text{m})(\pi)(1\,\text{m})^2} + 21\,\text{m} = 21.012\,\text{m}$$

For equilibrium,

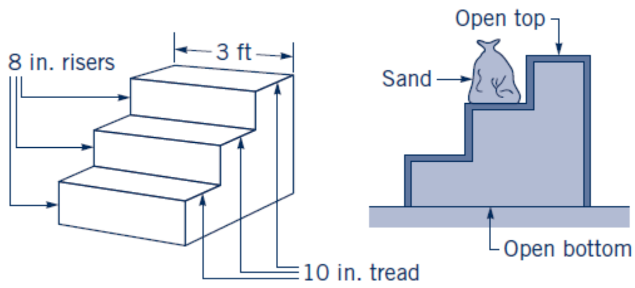
$$\sum M_H = 0$$

$$F_R(21.012\,\text{m} - 20\,\text{m}) = p_1(\pi)(1\,\text{m})^2(1\,\text{m})$$

$$p_1 = \frac{(3.33\times10^5\,\text{N})(1.012\,\text{m})}{\pi(1\,\text{m})^2(1\,\text{m})} \rightarrow \boxed{p_1 = 107\,\text{kPa}}$$

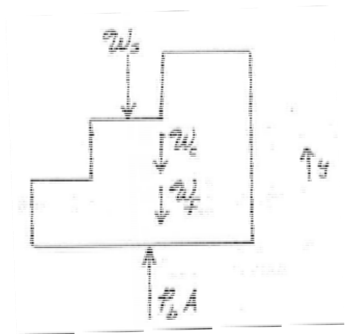
Problem 2.83

Concrete is poured into the forms as shown in the figure below to produce a set of steps. Determine the weight of the sandbag needed to keep the bottomless forms from lifting off the ground. The weight of the forms is 85 lb, and the specific weight of the concrete is $150\ \text{lb}/\text{ft}^3$.



Solution 2.83

From the free-body-diagram



$$(\downarrow^{+})\sum F_y = 0$$

$$W_s + W_c + W_f - p_b A = 0 \qquad (1)$$

$W_s =$ weight of sandbag

$W_c =$ weight of concrete

$W_f =$ weight of forms

$p_b =$ pressure along bottom surface due to concrete

$A =$ area of bottom surface

From the data given:

$$W_c = \left(150\frac{\text{lb}}{\text{ft}^3}\right)(\text{Vol. concrete})$$

$$= \left(150\frac{\text{lb}}{\text{ft}^3}\right)(3\,\text{ft})\frac{\left[(10\,\text{in.})(24\,\text{in.}) + (10\,\text{in.})(16\,\text{in.}) + (10\,\text{in.})(8\,\text{in.})\right]}{144\dfrac{\text{in.}^2}{\text{ft}^2}} = 1500\,\text{lb}$$

$$W_f = 85\,\text{lb}$$

$$p_A = \left(150\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{24}{12}\,\text{ft}\right) = 300\frac{\text{lb}}{\text{ft}^2}$$

$$A = \left(\frac{30}{12}\,\text{ft}\right)(3\,\text{ft}) = 7.5\,\text{ft}^2$$

Thus, from Eq.(1)

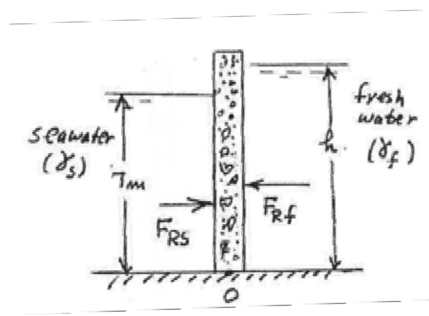
$$W_s = \left(300\frac{\text{lb}}{\text{ft}^2}\right)(7.5\,\text{ft}^2) - 1500\,\text{lb} - 85\,\text{lb} \quad\rightarrow\quad \boxed{W_s = 665\ \text{lb}}$$

Problem 2.84

A long, vertical wall separates seawater from fresh water. If the seawater stands at a depth of 7 m, what depth of freshwater is required to give a zero resultant force on the wall? When the resultant force is zero, will the moment due to the fluid forces be zero? Explain.

Solution 2.84

For a zero resultant force



$$F_{Rs} = F_{Rf}$$

$$\gamma_s h_{cs} A_s = \gamma_f h_{cf} A_f$$

Thus, for a unit length of wall

$$\left(10.1\frac{\text{kN}}{\text{m}^3}\right)\left(\frac{7\,\text{m}}{2}\right)(7\,\text{m}\times 1\,\text{m}) = \left(9.80\frac{\text{kN}}{\text{m}^3}\right)\left(\frac{h}{2}\,\text{m}\right)(h\times 1\,\text{m})$$

$$\boxed{h = 7.11\,\text{m}}$$

In order for moment to be zero, F_{Rs} and F_{Rf} must collinear.

For F_{Rs}: $$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1\,\text{m})(7\,\text{m})^3}{\left(\frac{7}{2}\,\text{m}\right)(7\,\text{m}\times 1\,\text{m})} + \frac{7}{2}\,\text{m} = 4.67\,\text{m}$$

Similarly for F_{Rf}: $$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1\,\text{m})(7.11\,\text{m})^3}{\left(\frac{7.11}{2}\,\text{m}\right)(7.11\,\text{m}\times 1\,\text{m})} + \frac{7.11}{2}\,\text{m} = 4.74\,\text{m}$$

Thus, the distance to F_{Rs} from the bottom (point 0) is $7\,\text{m} - 4.67\,\text{m} = 2.33\,\text{m}$.

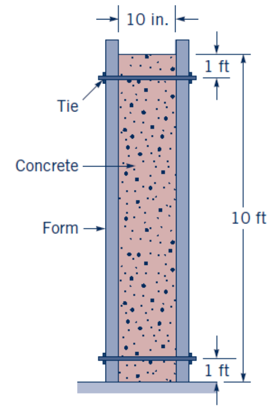
For F_{Rf} this distance is $7.11\,\text{m} - 4.74\,\text{m} = 2.37\,\text{m}$.

The forces are not collinear.

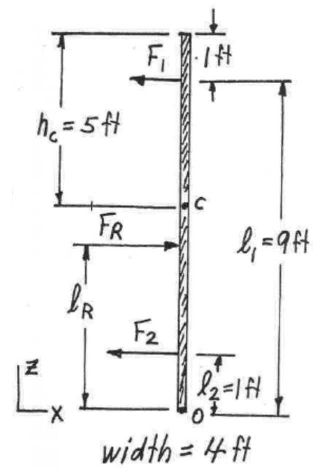
→ No; for zero resultant force, the sum of the moments will not be zero.

Problem 2.85

Forms used to make a concrete basement wall are shown in the figure below. Each 4-ft -long form is held together by four ties—two at the top and two at the bottom as indicated. Determine the tension in the upper and lower ties. Assume concrete acts as a fluid with a weight of $150\ \text{lb}/\text{ft}^3$.



Solution 2.85



(1) $\sum F_x = 0$, or $F_1 + F_2 = F_R$

and

(2) $\sum M_0 = 0$, or $\ell_1 F_1 + \ell_2 F_2 = \ell_R F_R$, where $F_R = p_c A = \gamma h_c A$

Thus,

$$F_R = 150 \frac{\text{lb}}{\text{ft}^3}(5\,\text{ft})(10\,\text{ft})(4\,\text{ft}) = 30000\,\text{lb}$$

Also,

$$\ell_R = 10\,\text{ft} - y_R = 10\,\text{ft} - y_c - (y_R - y_c) = 10\,\text{ft} - h_c - \frac{I_{xc}}{y_c A} = 10\,\text{ft} - 5\,\text{ft} - \frac{\frac{1}{12}(4\,\text{ft})(10\,\text{ft})^3}{5\,\text{ft}(10\,\text{ft})(4\,\text{ft})}$$

$$= 5\ \text{ft} - 1.67\ \text{ft} = 3.33\ \text{ft}$$

Thus, from Eq.(2):

$$(9\,\text{ft})F_1 + (1\,\text{ft})F_2 = (3.33\,\text{ft})(30000\,\text{lb}) = 99{,}900\,\text{ft}\cdot\text{lb}$$

(3) $\quad 9F_1 + F_2 = 99900$

From Eq.(1), $F_1 + F_2 = 30{,}000\,\text{lb}$, or $F_2 = 30000 - F_1$

$$9F_1 + 30000 - F_1 = 99{,}900$$

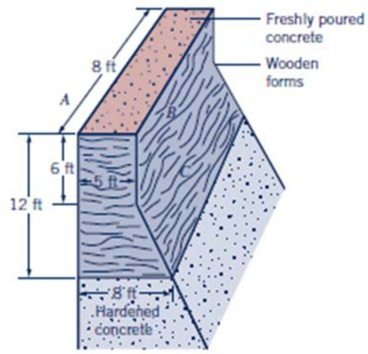
$$\boxed{F_1 = 8{,}740\ \text{lb}}$$

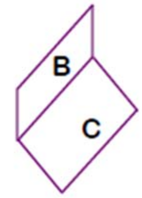
$$F_2 = 30000\,\text{lb} - 8740\,\text{lb}$$

$$\boxed{F_1 = 21{,}260\ \text{lb}}$$

Problem 2.86

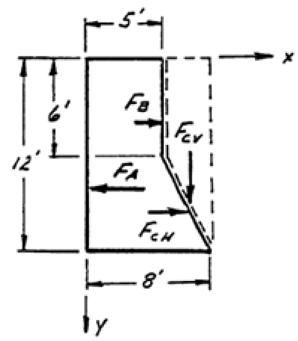
While building a high, tapered concrete wall, builders used the wooden forms shown in the figure below. If concrete has a specific gravity of about 2.5, find the total force on each of the three side sections (A, B, and C) of the wooden forms (neglect any restraining force of the two ends of the forms).





Solution 2.86

The horizontal force F_A



$$F_A = \gamma y_c A$$
$$= 2.5\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(6\,\text{ft})(12\times 8)\,\text{ft}^2 \quad \rightarrow \quad \boxed{F_A = 89900\,\text{lb}}$$

The force F_B is horizontal

$$F_B = \gamma y_c A$$
$$= 2.5\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(3\text{ft})(6\times 8)\,\text{ft}^2 \quad \rightarrow \quad \boxed{F_B = 22500\,\text{lb}}$$

The horizontal force F_{CH}

$$F_{CH} = \gamma y_c A = 2.5\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(9\,\text{ft})(6\times 8)\,\text{ft}^2 = 67400\,\text{lb}$$

The vertical force F_{CV} is the weight of concrete "above" the slanted side (the dashed volume)

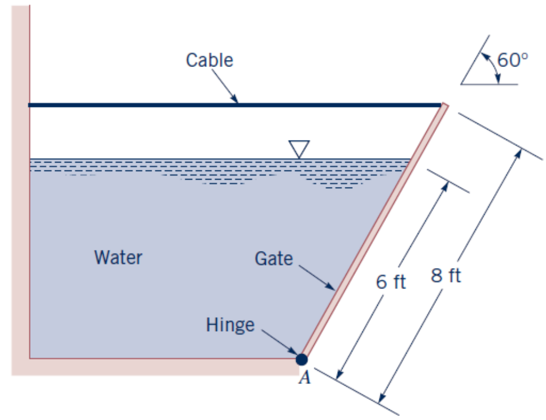
$$F_{CV} = \gamma V = 2.5\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left[(3\times 6\times 8)\,\text{ft}^3 + \frac{1}{2}(3\times 6\times 8)\,\text{ft}^3\right] = 33700\,\text{lb}$$

The total force F_C is

$$F_C = \sqrt{F_{CH}^{\ 2} + F_{CV}^{\ 2}} = \sqrt{(67400)^2 + (33700)^2}\ \text{lb} \quad \rightarrow \quad \boxed{F_C = 75400\,\text{lb}}$$

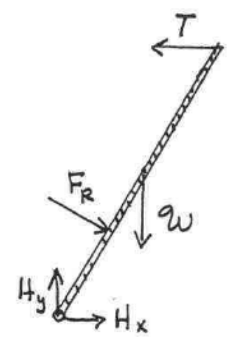
Problem 2.87

A homogeneous, 4-ft -wide, 8-ft -long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in the figure below. Water acts against the gate, which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.



Solution 2.87

$$F_R = \gamma h_c A \text{ where } h_c = \left(\frac{6\text{ ft}}{2}\right)\sin 60°$$



Thus,

$$F_R = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6\text{ ft}}{2}\right)(\sin 60°)(6\text{ ft}\times 4\text{ ft}) = 3890\text{ lb}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \text{ where } y_c = 3\text{ ft}$$

so that

$$y_R = \frac{\frac{1}{12}(4\text{ ft})(6\text{ ft})^3}{(3\text{ ft})(6\text{ ft}\times 4\text{ ft})} + 3\text{ ft} = 4.0\text{ ft}$$

For equilibrium,

$$\sum M_H = 0$$

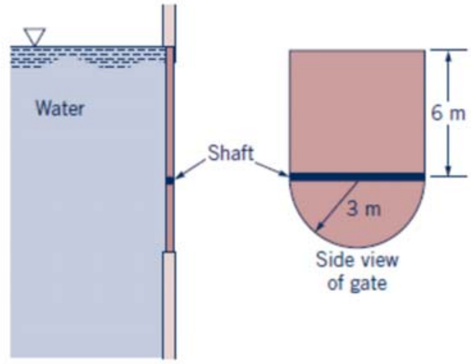
and

$$T(8\text{ ft})(\sin 60°) = W(4\text{ ft})(\cos 60°) + F_R(2\text{ ft})$$

$$T = \frac{(800\text{ lb})(4\text{ ft})(\cos 60°) + (3890\text{ lb})(2\text{ ft})}{(8\text{ ft})(\sin 60°)} \rightarrow \boxed{T = 1350\text{ lb}}$$

Problem 2.88

A gate having the shape shown in the figure below is located in the vertical side of an open tank containing water. The gate is mounted on a horizontal shaft. (**a**) When the water level is at the top of the gate, determine the magnitude of the fluid force on the rectangular portion of the gate above the shaft and the magnitude of the fluid force on the semicircular portion of the gate below the shaft. (**b**) For this same fluid depth determine the moment of the force acting on the semicircular portion of the gate with respect to an axis that coincides with the shaft.



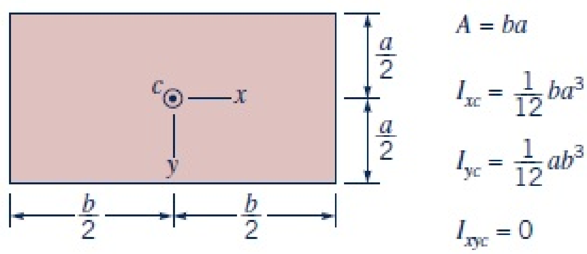
Solution 2.88

(a) For rectangular portion,

$(F_R)_r = \gamma h_c A$ where $h_c = 3\,\text{m}$

$$(F_R)_r = \left(9800\frac{\text{N}}{\text{m}^3}\right)(3\,\text{m})(6\,\text{m}\times 6\,\text{m})$$

$$\boxed{(F_R)_r = 1060\text{ kN}}$$



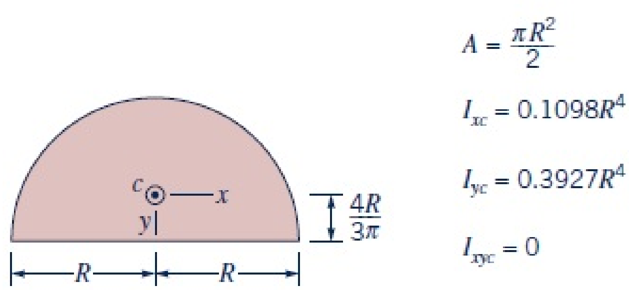
For semi-circular portion,

$(F_R)_{sc} = \gamma h_c A$ where

$$h_c = 6\,\text{m} + \frac{4R}{3\pi} = 6\,\text{m} + \frac{4(3\,\text{m})}{3\pi} = 7.27\,\text{m}$$

$$(F_R)_{sc} = \left(9800\frac{\text{N}}{\text{m}^3}\right)(7.27\,\text{m})\left(\frac{\pi}{2}(3\,\text{m})^2\right)$$

$$\boxed{(F_R)_{sc} = 1010\text{ kN}}$$



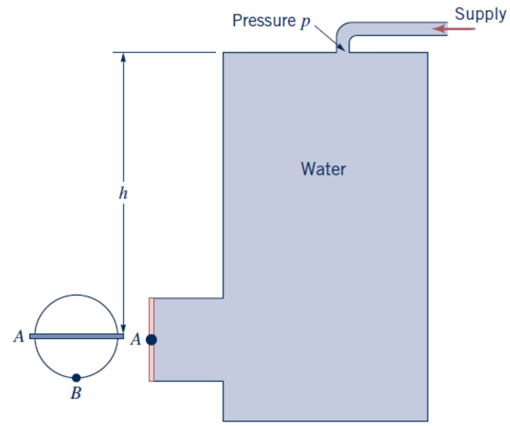
(b) For semi-circular portion $y_R = \dfrac{I_{xc}}{y_c A} + y_c = \dfrac{0.1098\,R^4}{(7.27\,\text{m})\left(\frac{\pi}{2}\right)R^2} + 7.27\,\text{m} = 7.36\text{ m}$

Thus, moment with respect to shaft, M :

$$M = (F_R)_{sc}\times(7.36\,\text{m} - 6.00\,\text{m}) = \left(1010\times 10^3\,\text{N}\right)(1.36\,\text{m}) \rightarrow \boxed{M = 1.37\times 10^6\,\text{N}\cdot\text{m}}$$

Problem 2.89

A pump supplies water under pressure to a large tank as shown in the figure below. The circular-plate valve fitted in the short discharge pipe on the tank pivots about its diameter *A–A* and is held shut against the water pressure by a latch at *B*. Show that the force on the latch is independent of the supply pressure, p , and the height of the tank, h .

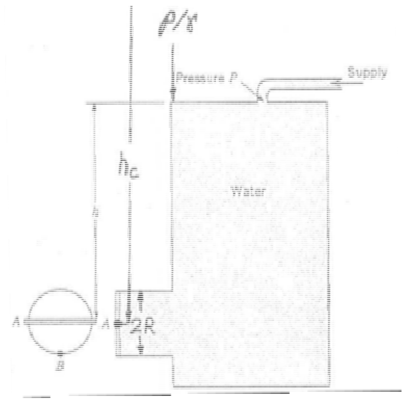


Solution 2.89

The pressure on the gate is the same as it would be for an open tank with a depth of

$$h_c = \frac{p+\gamma h}{\gamma}$$

as shown in the figure.



$$\sum M_A = 0 \text{, or}$$

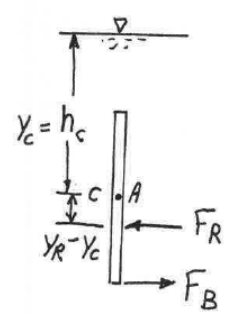
(1) $(y_R - y_c)F_R = RF_B$

where

$$F_R = p_c A = \gamma h_c\left(\pi R^2\right) = (p+\gamma h)\left(\pi R^2\right)$$

and

(2) $$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{\dfrac{\pi R^4}{4}}{\left(\dfrac{p+\gamma h}{\gamma}\right)\pi R^2} = \frac{R^2}{4\left(\dfrac{p}{\gamma}+h\right)}$$



Thus, from Eqs.(1) and (2)

$$F_B = \frac{(y_R - y_c)}{R}F_R = \frac{R}{4\left(\dfrac{p}{\gamma}+h\right)}(p+\gamma h)\left(\pi R^2\right)$$

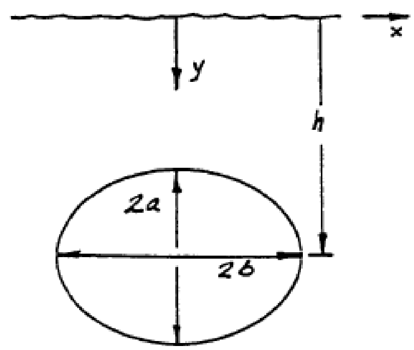
$\boxed{F_B = \gamma \frac{\pi}{4} R^3}$, which is independent of both p and h .

Problem 2.91

Find the center of pressure of an elliptical area of minor axis $2a$ and major axis $2b$ where axis $2a$ is vertical and axis $2b$ is horizontal. The center of the ellipse is a vertical distance h below the surface of the water ($h > a$). The fluid density is constant. Will the center of pressure of the ellipse change if the fluid is replaced by another constant-density fluid? Will the center of pressure of the ellipse change if the vertical axis is tilted back an angle α from the vertical about its horizontal axis? Explain.

Solution 2.91

For a hydrostatic pressure distribution, using geometric information from the Appendix,



$$y_p - y_c = \frac{I_{xc}}{y_c A} = \frac{\left(\dfrac{\pi b a^3}{4}\right)}{h(\pi a b)} = \frac{a^2}{4h}$$

$$y_p = y_c + \frac{a^2}{4h} = h + \frac{a^2}{4h}$$

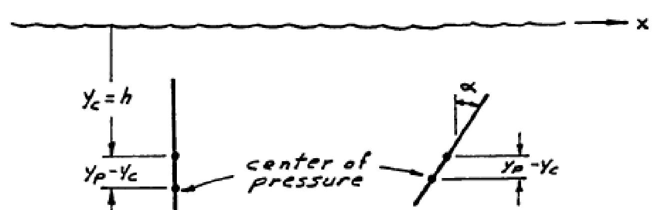
Recognizing symmetry about minor axis,

$$x_p = 0$$

Above expressions for x_p and y_p contain only geometric properties (and not fluid properties)

Location of center of pressure not dependent on density.

Consider the side view of the ellipse.



The equation

$$y_p - y_c = \frac{I_{xc}}{y_c A}$$

Requires that the y-coordinate lie in the plane of the surface.

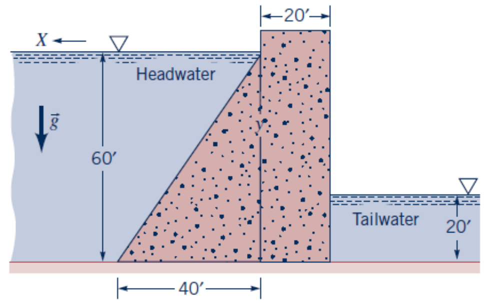
I_{xc} is smaller for the smaller horizontally projected area of the tilted ellipse.

Therefore, for the tilted ellipse,

($y_p - y_c$) is smaller for the tilted ellipse, the center of pressure is higher.

Problem 2.92

The dam shown in the figure below is 200 ft long and is made of concrete with a specific gravity of 2.2. Find the magnitude and y coordinate of the line of action of the net horizontal force.



Solution 2.92

The headwater horizontal force and its line of action are

$$F_{HH} = \gamma y_c A = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(30\,\text{ft})(60\times 200)\,\text{ft}^2 = 2.25\times 10^7\,\text{lb}$$

$$y_{AP} = \frac{2}{3}h_H = \frac{2}{3}(60\,\text{ft}) = 40\,\text{ft}.$$

The tailwater horizontal force and its line of action are

$$F_{TH} = \gamma y_c A = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(10\,\text{ft})(20\,\text{ft}\times 200\,\text{ft}) = 2.50\times 10^6\,\text{lb}$$

$$y_{TP} = \frac{2}{3}h_T = \frac{2}{3}(20\,\text{ft}) = 13.3\,\text{ft}.$$

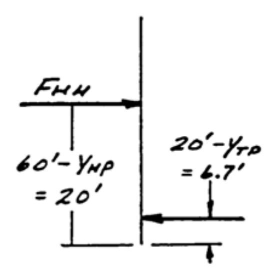
$$F_{\text{net}} = F_{HH} - F_{TH} = 2.25\times 10^7\,\text{lb} - 2.50\times 10^6\,\text{lb}$$

$$\boxed{F_{\text{net}} = 2.00\times 10^7\,\text{lb acting to right.}}$$

The line if action is located by taking moments about the base of the dam.

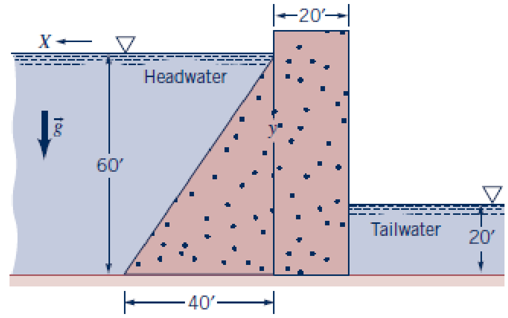
$$y_{\text{net}} = \frac{\left(2.25\times 10^7\,\text{lb}\right)(20\,\text{ft}) - \left(2.50\times 10^6\,\text{lb}\right)(6.7\,\text{ft})}{\left(2.00\times 10^7\,\text{lb}\right)}$$

$$\boxed{y_{\text{net}} = 21.7\,\text{ft above base}}$$



Problem 2.93

The dam shown in the figure below is 200 ft long and is made of concrete with a specific gravity of 2.2. Find the magnitude and x coordinate of the line of action of the vertical force on the dam resulting from the water.



Solution 2.93

The only vertical force due to the water is on the headwater side of the dam. This vertical force equals the weight of the water above the surface and the force acts through the centroid of the water volume. Therefore

$$F_{HV} = \gamma \cancel{V} = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(40\,\text{ft})(60\,\text{ft})(200\,\text{ft})$$

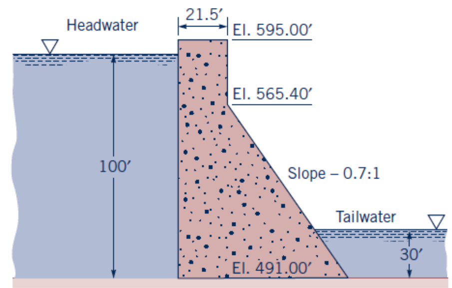
$$\boxed{F_{HV} = 1.50 \times 10^7 \text{ lb}}$$

The x-location of this force is the centroid of the $40\,\text{ft} \times 60\,\text{ft}$ triangle which gives

$$x_p = \frac{2}{3}(40\,\text{ft}) \text{ or } \boxed{x_p = 26.7 \text{ ft}}.$$

Problem 2.94

The figure below is a representation of the Keswick gravity dam in California. Find the magnitudes and locations of the hydrostatic forces acting on the headwater vertical wall of the dam and on the tailwater inclined wall of the dam. Note that the slope given is the ratio of the run to the rise. Consider a unit length of the dam ($b = 1\,\text{ft}$).

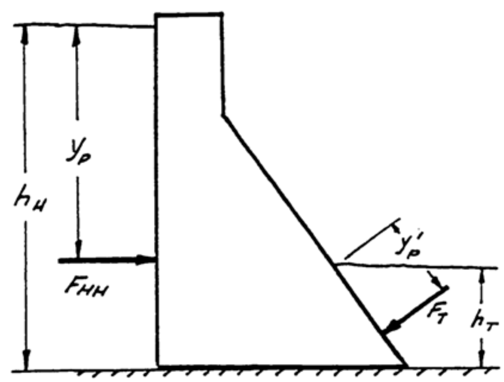


Solution 2.94

Consider a unit length of the dam. The headwater force is

$$F_{HH} = \gamma y_c A = \gamma\left(\frac{h_H}{2}\right)(bh_H) = \frac{\gamma b h_H{}^2}{2} = \frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(100\,\text{ft})^2}{2}$$

$$\boxed{F_{HH} = 312{,}000\ \text{lb}}$$



The location y_p is

$$y_p = \frac{2}{3}h_H = \frac{2}{3}(100\,\text{ft})$$

$$\boxed{y_p = 66.7\ \text{ft}}$$

The tailwater force is

$$F_T = \gamma y_c A = \gamma\left(\frac{h_T}{2}\right)A$$

where

$$A = (30\,\text{ft})(1\,\text{ft})\frac{\sqrt{1.0^2 + 0.7^2}}{1.0} = 36.6\,\text{ft}^2$$

so

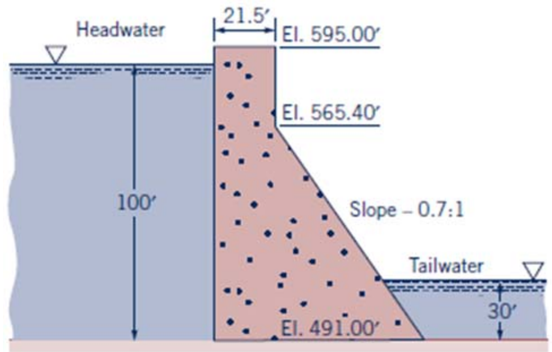
$$F_T = \frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(30\,\text{ft})\left(36.6\,\text{ft}^2\right)}{2} \quad \text{or} \quad \boxed{F_T = 34{,}300\ \text{lb}}.$$

The location y_p' is

$$y_p' = \frac{2}{3}h_T\frac{\sqrt{1.0^2+0.7^2}}{1.0} = \frac{2}{3}(30\,\text{ft})\frac{\sqrt{1.0^2+0.7^2}}{1.0} \quad \text{or} \quad \boxed{y_p' = 24.4\ \text{ft}}$$

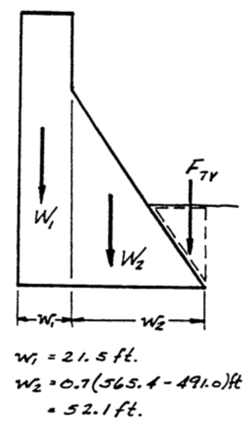
Problem 2.95

The Keswick dam shown in the figure below is made of concrete and has a specific weight of 150 lb/ft^3. The hydrostatic forces and the weight of the dam produce a total vertical force of the dam on the foundation. Find the magnitude and location of this total vertical force. Consider a unit length of the dam ($b = 1$ ft).



Solution 2.95

The hydrostatic vertical force is due to the tailwater. Its magnitude for a dam unit length is



$$F_{TV} = \gamma \cancel{V} = \gamma\left(\frac{1}{2}\right)(b)h_T(0.7h_T) = 0.35\gamma b h_T^2$$

$$= 0.35\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(30\,\text{ft})^2$$

$$F_{TV} = 19700\,\text{lb}$$

The dam weight consists of $W_1 + W_2$. Now

$$W_1 = \gamma_C \cancel{V} = \left(150\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(21.5\,\text{ft})(595.0 - 491.0)\,\text{ft} = 335400\,\text{lb}$$

$$W_2 = \gamma_c \cancel{V}_2 = \left(150\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})\left(\frac{1}{2}\right)(52.1\,\text{ft})\left(\frac{52.1}{0.7}\,\text{ft}\right) = 290800\,\text{lb}.$$

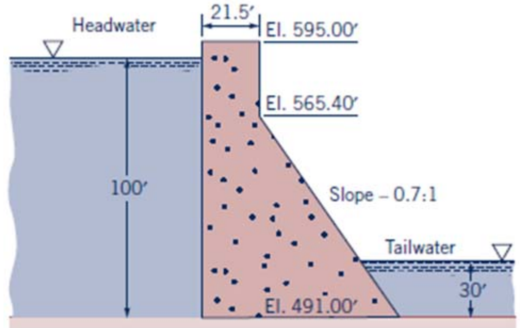
The total force F is

$$F = F_{TV} + W_1 + W_2 = (19700 + 335400 + 290800)\,\text{lb}$$

$$\boxed{F = 645{,}900\ \text{lb}}\,.$$

Problem 2.96

The Keswick dam shown in the figure below is made of concrete and has a specific weight of $150\ \text{lb/ft}^3$. The coefficient of friction μ between the base of the dam and the foundation is 0.65. Is the dam likely to slide downstream? Consider a unit length of the dam ($b = 1$ ft).



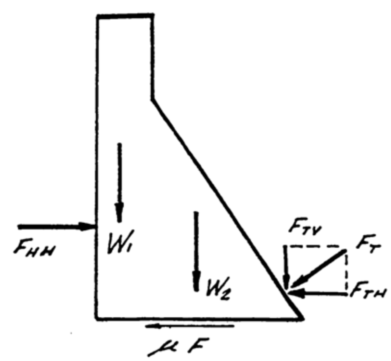
Solution 2.96

The total vertical force acting downward is

$$F = W_1 + W_2 + F_{TV}\,.$$

Using the result of Problem 2.51,

$$F = 645900\ \text{lb}\,.$$



The horizontal force resisting movement of the dam is

$$\mu F = 0.65\left(645900\ \text{lb}\right) = 419800\ \text{lb}$$

The net force causing the dam to move downstream is $\left(F_{HH} - F_{TH}\right)$. Using the result of Problem 2.50,

$$F_{TH} = F_T\left(\frac{1.0}{\sqrt{1.0^2 + 0.7^2}}\right) = \left(34300\ \text{lb}\right)\frac{1.0}{\sqrt{1.0^2 + 0.7^2}} = 28100\ \text{lb}$$

Then

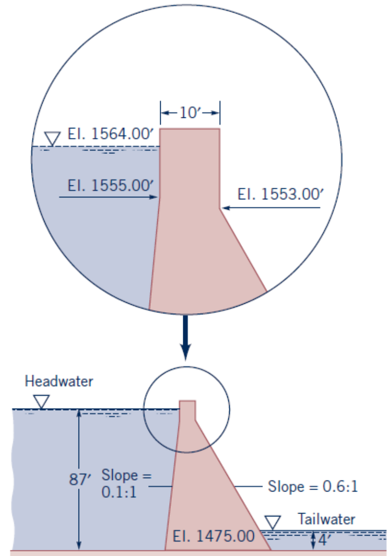
$$F_{HH} - F_{TH} = \left(312000 - 28100\right)\text{lb} = 283900\ \text{lb}\,.$$

Since

$F_{HH} - F_{TH} < \mu F$, $\boxed{\text{the dam will not slide downstream.}}$

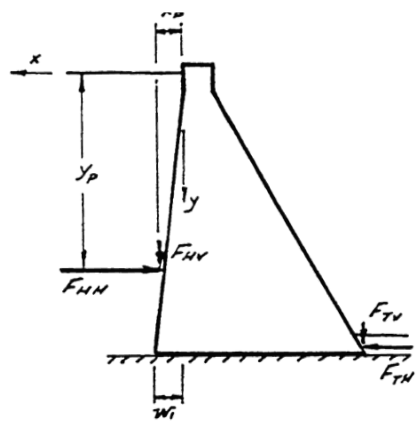
Problem 2.97

The figure below is a representation of the Altus gravity dam in Oklahoma. Find the magnitudes and locations of the horizontal and vertical hydrostatic force components acting on the headwater wall of the dam and on the tailwater wall of the dam. Note that the slope given is the ratio of the run to the rise. Consider a unit length of the dam $(b = 1\,\text{ft})$.



Solution 2.97

First consider the headwater hydrostatic force components.



$$F_{HH} = \gamma y_c A = \gamma\left(\frac{h_H}{2}\right)(b h_H)$$

$$= \frac{\gamma b h_H^2}{2}$$

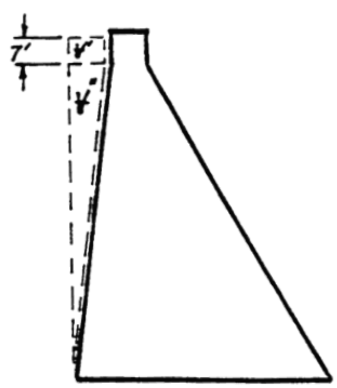
$$= \frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(87\,\text{ft})^2}{2}$$

$$\boxed{F_{HH} = 236{,}200\text{ lb}}\,.$$

$$w_1 = 0.1(1555.0 - 1475.0)\,\text{ft}$$
$$= 8\,\text{ft}.$$

$$y_p = \frac{2}{3}h_H = \frac{2}{3}(87\,\text{ft})$$

$$\boxed{y_p = 58\text{ ft}}\,.$$



$$F_{HV} = \gamma \mathcal{V}' + \gamma \mathcal{V}''$$

$$\gamma \mathcal{V}' = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(7\,\text{ft})(87\,\text{ft})$$
$$= 3490\,\text{lb}$$

$$\gamma \mathcal{V}'' = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\frac{(1\,\text{ft})(8\,\text{ft})(87-7)\,\text{ft}}{2}$$
$$= 19970\,\text{lb}$$

$$F_{HV} = 3490\,\text{lb} + 19970\,\text{lb}$$

$$\boxed{F_{HV} = 23460\ \text{lb}}\ .$$

The location x_p of F_{HV} is found from $x_p = \dfrac{\gamma \cancel{V}' x_p' + \gamma \cancel{V}'' x_p''}{\gamma \cancel{V}' + \gamma \cancel{V}''}$

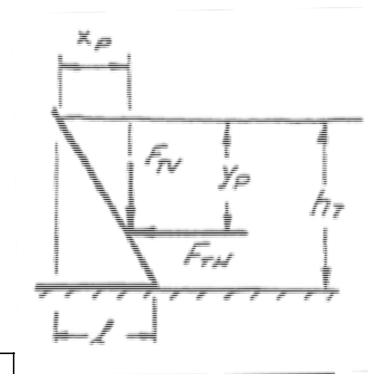
where x_p' and x_p'' are the horizontal locations of $\gamma \cancel{V}'$ and $\gamma \cancel{V}''$ respectively.

The numerical values give

$$x_p = \frac{(3490\,\text{lb})\left(\frac{1}{2}\times 8\,\text{ft}\right) + (19970\,\text{lb})\left(\frac{2}{3}\times 8\,\text{ft}\right)}{(3490 + 19970)\,\text{lb}}$$

$$\boxed{x_p = 5.14\ \text{ft}}\ .$$

The tailwater hydrostatic force components



$$F_{TH} = \gamma y_c A = \gamma\left(\frac{h_T}{2}\right)(b h_T)$$

$$= \frac{\gamma b h_T^2}{2}$$

$$= \frac{\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(4\,\text{ft})^2}{2} \quad \rightarrow \quad \boxed{F_{TH} = 499\ \text{lb}}$$

$$\ell = 0.6(4\,\text{ft}) = 2.4\,\text{ft}.$$

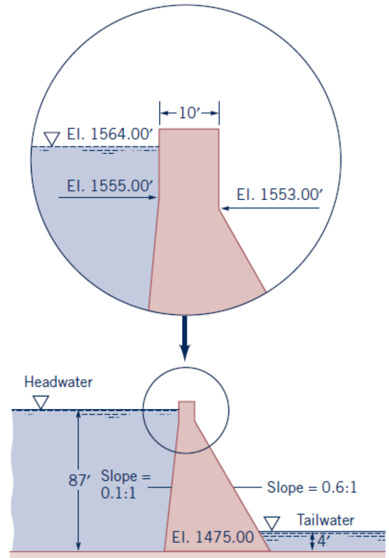
$$y_p = \frac{2}{3}h_T = \frac{2}{3}(4\,\text{ft}) \quad \text{or} \quad \boxed{y_p = 2.67\ \text{ft}}\ .$$

$$F_{TV} = \gamma \cancel{V} = \gamma b\left(\frac{\ell h_T}{2}\right) = \frac{\gamma b \ell h_T}{2} = \frac{\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft})(2.4\,\text{ft})(4\,\text{ft})}{2} \quad \rightarrow \quad \boxed{F_{TV} = 300\ \text{lb}}$$

$$x_p = \frac{2}{3}\ell = \frac{2}{3}(2.4\,\text{ft}) \quad \text{or} \quad \boxed{x_p = 1.6\ \text{ft}}\ .$$

Problem 2.98

The Altus dam in the figure below is made of concrete with a density of 150 lbm/ft^3. The coefficient of friction μ between the base of the dam and the foundation is 0.65. Is the dam likely to slide downstream? Consider a unit length of the dam $(\mathrm{b} = 1\ \mathrm{ft})$.



Solution 2.98

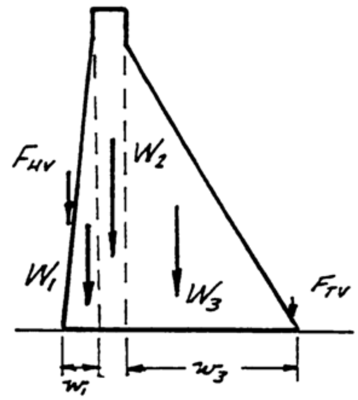
The total vertical force acting downward is

$$F = F_{HV} + F_{TV} + W_1 + W_2 + W_3.$$

Using the results of Problem 2.97

$$F_H = 23{,}460\ \mathrm{lb}$$

$$F_{TV} = 300\,\mathrm{lb}.$$



The dam weights are

$$w_1 = 0.1(1555.0 - 1475.0)\,\mathrm{ft}{=}8\,\mathrm{ft}$$
$$\mathrm{w}_3 = 0.6(1553.0 - 1475.0)\,\mathrm{ft} = 46.8\,\mathrm{ft}$$

$$W_1 = \gamma_c \cancel{V}_1 = \left(150\frac{\mathrm{lb}}{\mathrm{ft}^3}\right)\left(\frac{1}{2}\right)(1\,\mathrm{ft})(8\,\mathrm{ft})(1555.0 - 1475.0)\,\mathrm{ft}{=}48{,}000\,\mathrm{lb}$$

$$W_2 = \gamma_c \cancel{V}_2 = \left(150\frac{\mathrm{lb}}{\mathrm{ft}^3}\right)(1\,\mathrm{ft})(10\,\mathrm{ft})(1564.0 - 1475.0)\,\mathrm{ft} = 133{,}500\,\mathrm{lb}$$

$$W_3 = \gamma_c \cancel{V}_3 = \left(150\frac{\mathrm{lb}}{\mathrm{ft}^3}\right)\left(\frac{1}{2}\right)(1\,\mathrm{ft})(46.8\,\mathrm{ft})(1553.0 - 1475.0)\,\mathrm{ft} = 273{,}800\,\mathrm{lb}.$$

$$F = (23460 + 300 + 48000 + 133500 + 273800)\,\mathrm{lb} = 479{,}000\,\mathrm{lb}$$

Horizontal force resisting sliding movement of the dam is: $\mu F = 0.65(479000\,\mathrm{lb}) = 311{,}400\,\mathrm{lb}$.

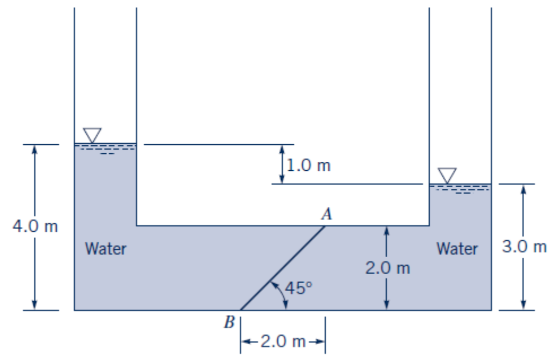
Net force acting to slide dam downstream is $(F_{HH} - F_{TH})$. Using the results of Problem 2.53

$$(F_{HH} - F_{TH}) = (236200 - 499)\,\mathrm{lb}{=}235700\,\mathrm{lb}.$$

$$(F_{HH} - F_{TH}) < \mu F,\quad \boxed{\text{the dam will not slide downstream.}}$$

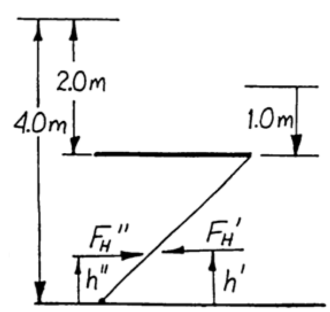
Problem 2.99

Find the magnitude and location of the net horizontal force on the gate shown in the figure below. The gate width is 5.0 m.



Solution 2.99

The hydrostatic horizontal force $F_H^{'}$ is



$$
\begin{aligned}
F_H^{'} &= \gamma_w h_c A = \rho_w g h_c A \\
&= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2.0\,\text{m})(2.0\times 5.0)\,\text{m}^2 \\
&= 196000\,\text{N}=196\,\text{kN}
\end{aligned}
$$

The location h' of $F_H^{'}$ is

$$
\begin{aligned}
h' &= 3.0\,\text{m} - y_p \\
&= 3.0\,\text{m} - \left(y_c + \frac{I_{xc}}{y_c A}\right) \\
&= 3.0\,\text{m} - \left(2.0\,\text{m} + \frac{\frac{1}{12}(5.0\,\text{m})(2.0\,\text{m})^3}{(2.0\,\text{m})(2.0\times 5.0)\,\text{m}^2}\right) \\
&= 0.833\,\text{m}
\end{aligned}
$$

The hydrostatic horizontal force $F_H^{''}$ is

$$
\begin{aligned}
F_H^{''} &= \gamma_w h_c A = \rho_w g h_c A \\
&= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(3.0\,\text{m})(2.0\times 5.0)\,\text{m}^2 \\
&= 294000\,\text{N} = 294\,\text{kN}
\end{aligned}
$$

The location of h of F_H is

$$h'' = 4.0\,\text{m} - y_p = 4.0\,\text{m} - \left(y_c + \frac{I_{xc}}{y_c A} \right)$$

$$= 4.0\,\text{m} - \left(3.0\,\text{m} + \frac{\frac{1}{12}(5.0\,\text{m})(2.0\,\text{m})^3}{(3.0\,\text{m})(2.0 \times 5.0)\,\text{m}^2} \right)$$

$$= 0.889\,\text{m}.$$

The magnitude of the net horizontal force F_H

$$F_H = F_H'' - F_H' = 294\,\text{kN} - 196\,\text{kN}$$

$$\boxed{F_H = 98\ \text{kN}}$$

The location of the net horizontal force F above the base is denoted by h and is found by noting the moment of the resultant is equal to the moment of the components or

$$Fh = F_H'' h'' - F_H' h',$$

$$h = \frac{F_H'' h'' - F_H' h'}{F}$$

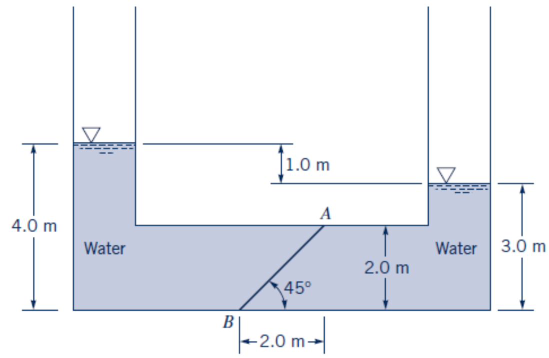
$$= \frac{(294\,\text{kN})(0.889\,\text{m}) - (196\,\text{kN})(0.833\,\text{m})}{98\,\text{kN}}$$

$$\boxed{h = 1.00\ \text{m}}\ .$$

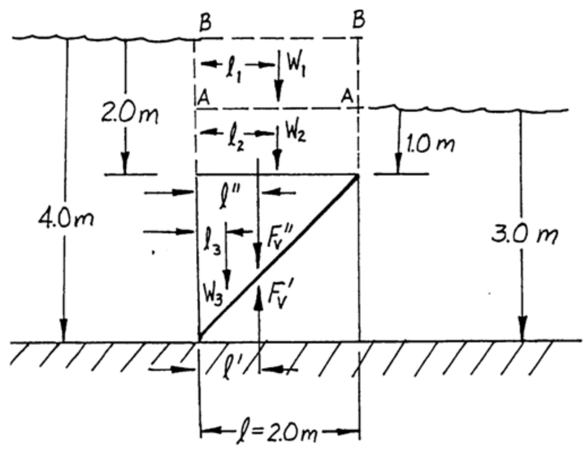
<u>DISCUSSION</u> Note that the resultant of the two opposing forces is <u>not</u> located between the two forces; this is only true for two forces that are acting in the same direction. Again note that the atmospheric pressure force was not considered as it acts uniformly on both sides of the gate and cancels out. Also note that the horizontal hydrostatic forces do not depend on the 45° angle.

Problem 2.100

Find the magnitude and location of the net vertical force on the gate shown in the figure below. The gate width is 5.0 m.



Solution 2.100



The hydrostatic vertical force $F_V^{'}$ is the weight of the water above the gate to the level A-A.

Then

$$F_V^{'} = W_3 + W_2 = \rho_w g\left(\mathcal{V}_3 + \mathcal{V}_2\right)$$

$$= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left[\frac{1}{2}(2.0\times2.0\times5.0)\,\text{m}^3 + (2.0\times1.0\times5.0)\,\text{m}^3\right]\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$F_V^{'} = 98100\,\text{N} + 98100\,\text{N} = 196200\,\text{N}$$

The hydrostatic vertical force $F_V^{''}$ is the weight of the water above the gate to the level B-B.

$$F_V^{''} = W_3 + W_2 + W_1 = \rho_w g\left(\mathcal{V}_3 + \mathcal{V}_2 + \mathcal{V}_1\right)$$

$$F_V^{''} = 98100\,\text{N} + 98100\,\text{N} + \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2.0\times1.0\times5.0)\,\text{m}^3\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 98100\,\text{N} + 98100\,\text{N} + 98100\,\text{N}$$

$$= 294300\,\text{N}$$

The magnitude of the vertical force F_V

$$F_V = F_V^{''} - F_V^{'} = 294300 - 196200\,\text{N}$$

$$\boxed{F_V = 98100\text{ N acting downward.}}$$

The location of F is found by first finding the locations of $F_V^{'}$ and $F_V^{"}$. First

$$\ell^{'} = \frac{W_3\ell_3 + W_2\ell_2}{W_3 + W_2}.$$

Recognizing that $\ell_3 = \frac{\ell}{3}$ and $\ell_2 = \frac{\ell}{2}$ gives

$$\ell^{'} = \frac{98100\,\mathrm{N}\left(\frac{2.0}{3\,\mathrm{m}}\right) + 98100\,\mathrm{N}\left(\frac{2.0}{2\,\mathrm{m}}\right)}{98100\,\mathrm{N} + 98100\,\mathrm{N}}$$

$$\ell^{'} = 0.833\,\mathrm{m}$$

Also

$$\ell^{"} = \frac{W_3\ell_3 + W_2\ell_2 + W_1\ell_1}{W_3 + W_2 + W_1}$$

Recognizing that $\ell_1 = \frac{\ell}{2}$ gives

$$\ell^{"} = \frac{98100\,\mathrm{N}\left(\frac{2.0}{3\,\mathrm{m}}\right) + 98100\,\mathrm{N}\left(\frac{2.0}{2\,\mathrm{m}}\right) + 98100\,\mathrm{N}\left(\frac{2.0}{2\,\mathrm{m}}\right)}{98100\,\mathrm{N} + 98100\,\mathrm{N} + 98100\,\mathrm{N}}$$

$$\ell^{"} = 0.889\,\mathrm{m}.$$

The location of the resultant force from the left side of the gate is denoted by ℓ_V and is found from

$$\ell_V = \frac{F_V^{"}\ell^{"} - F_V^{'}\ell^{'}}{F_V}$$

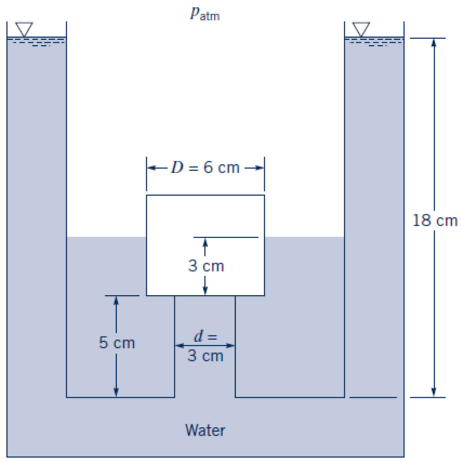
$$= \frac{294300\,\mathrm{N}(0.889\,\mathrm{m}) - 196200\,\mathrm{N}(0.833\,\mathrm{m})}{294300\,\mathrm{N} - 196200\,\mathrm{N}}$$

$\boxed{\ell_V = 1.00\text{ m}}$ from left side of gate.

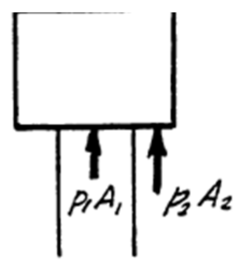
<u>DISCUSSION</u> Noting that the resultant vertical force, $F_V = 98100\,\mathrm{N}$ is the weight of a volume of water measuring $\ell = 2.0\,\mathrm{m}$, width $5.0\,\mathrm{m}$, and height $= 1.0\,\mathrm{m}$, is there a quick way to find F_V?

Problem 2.101

Find the total vertical force on the cylinder shown in the figure below.



Solution 2.101



The net force F on the cylinder is due to the water and is

$$F = F_1 + F_2 = p_1 A_1 + p_2 A_2 .$$

Since the atmospheric pressure does not contribute to the net force, p_1 and p_2 will be considered gage pressures.

$$p_1 = \rho_w g h = \frac{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)(18-5)\,\text{cm}}{\left(100\dfrac{\text{cm}}{\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)} = 1275\frac{\text{N}}{\text{m}^2}$$

$$p_2 = \rho_w g h = \frac{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)(3)\,\text{cm}}{\left(100\dfrac{\text{cm}}{\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)} = 294\frac{\text{N}}{\text{m}^2}$$

Then

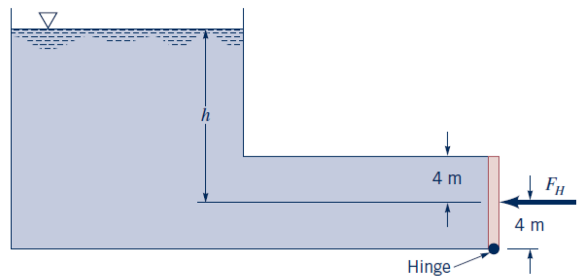
$$F = \left(1275\frac{\text{N}}{\text{m}^2}\right)\frac{\pi}{4}(3\,\text{cm})^2\left(\frac{\text{m}}{100\,\text{cm}}\right)^2 + \left(294\frac{\text{N}}{\text{m}^2}\right)\frac{\pi}{4}\left(6^2-3^2\right)\text{cm}^2\left(\frac{\text{m}}{100\,\text{cm}}\right)^2$$

or

$$\boxed{F = 1.52\,\text{N}}.$$

Problem 2.102

A 3-m -wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in the figure below. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3500 kN. (**a**) Determine the maximum water depth, *h*, above the center of the gate that can exist without the gate opening. (**b**) Is the answer the same if the gate is hinged at the top? Explain your answer.



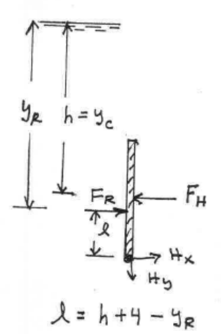
Solution 2.102

For gate hinged at <u>bottom</u>

$$\sum M_H = 0$$

so that

$$(4\text{m})F_H = \ell F_H \qquad \text{(see figure)} \quad (1)$$



and

$$F_R = \gamma h_c A = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right)(h)(3\text{m} \times 8\text{m}) = (9.80 \times 24h)\text{kN}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3\text{m})(8\text{m})^3}{h(3\text{m} \times 8\text{m})} + h = \frac{5.33}{h} + h$$

Thus, $\ell(\text{m}) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$

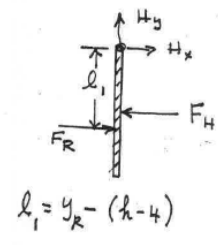
and from Eq.(1)

$$(4\text{m})(3500\text{kN}) = \left(4 - \frac{5.33}{h}\right)(9.80 \times 24)(h)\text{kN}$$

so that

$$\underline{\underline{h = 16.2\text{m}}}$$

For gate hinged at <u>top</u>



$$\sum M_H = 0$$

so that

$$(4\,\text{m})\text{F}_\text{H} = \ell_1 \text{F}_\text{H} \qquad \text{(see figure)} \quad (1)$$

where

$$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + 4\right) - (h-4) = \frac{5.33}{h} + 4$$

Thus, from Eq.(1)

$$(4\,\text{m})(3500\,\text{kN}) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\,\text{kN}$$

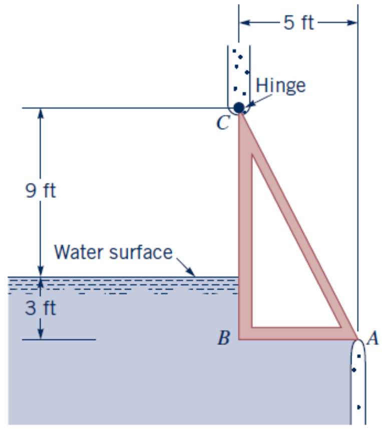
and

$$\underline{\underline{h = 13.5\,\text{m}}}$$

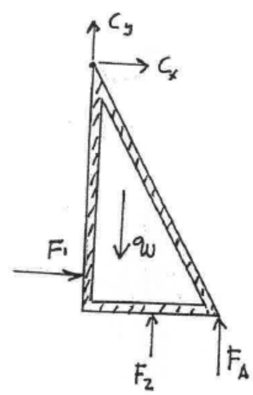
Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

Problem 2.103

A gate having the cross section shown in the figure below is 4 ft wide and is hinged at C. The gate weighs 18,000 lb, and its mass center is 1.67 ft to the right of the plane BC. Determine the vertical reaction at A on the gate when the water level is 3 ft above the base. All contact surfaces are smooth.



Solution 2.103



$F_1 = \gamma h_c A$ where $h_c = 1.5\,\text{ft}$

Thus, $F_1 = \left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)(1.5\,\text{ft})(3\,\text{ft}\times 4\,\text{ft}) = 1120\,\text{lb}$

The force F_1 acts at a distance of 1 ft from the base of the gate.

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}(3\,\text{ft})$

Thus,

$$F_2 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(3\,\text{ft})(5\,\text{ft}\times 4\,\text{ft}) = 3740\,\text{lb}$$

and acts at the center of the bottom gate surface.

For equilibrium,

$$\sum M_c = 0$$

and

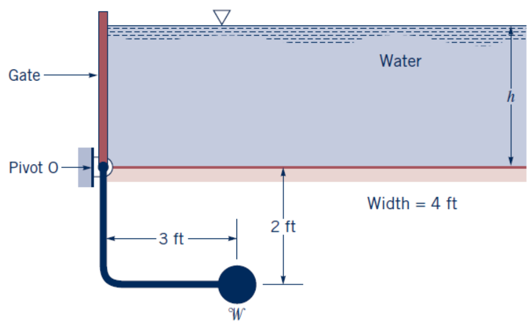
$$F_1(11\,\text{ft}) + F_2(2.5\,\text{ft}) + F_A(5\,\text{ft}) = W(1.67\,\text{ft})$$

so that

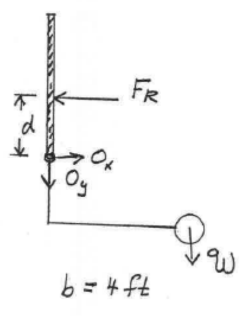
$$F_A = \frac{(18{,}000\,\text{lb})(1.67\,\text{ft}) - (1120\,\text{lb})(11\,\text{ft}) - (3740\,\text{lb})(2.5\,\text{ft})}{5\,\text{ft}} = \underline{\underline{1680\,\text{lb}}}$$

Problem 2.104

The massless, 4-ft -wide gate shown in the figure below pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, h.



Solution 2.104



$F_R = \gamma h_c A$ where $h_c = \dfrac{h}{2}$

Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2}(h \times b) = \gamma_{H_2O} \frac{h^2}{2}(4\,\text{ft})$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,\text{ft})(h^3)}{\frac{h}{2}(4\,\text{ft} \times h)} + \frac{h}{2} = \frac{2}{3}h$$

For equilibrium, $\sum M_0 = 0$

$$F_R d = W(3\,\text{ft}) \text{ where } d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000\,\text{lb})(3\,\text{ft})}{(\gamma_{H_2O})\left(\frac{h^2}{2}\right)(4\,\text{ft})}$$

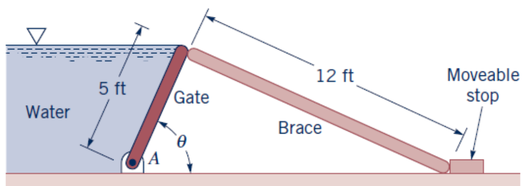
Thus,

$$h^3 = \frac{(3)(2000\,\text{lb})(3\,\text{ft})}{\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(4\,\text{ft})}$$

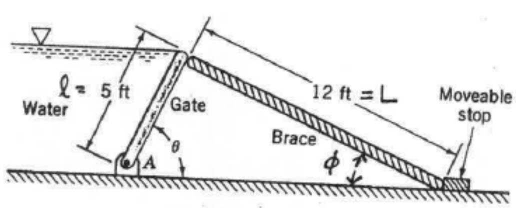
$$\underline{\underline{h = 5.24\,\text{ft}}}$$

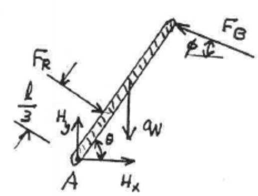
Problem 2.105

A $200-\text{lb}$ homogeneous gate 10 ft wide and 5 ft long is hinged at point A and held in place by a 12-ft -long brace as shown in the figure below. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. (**a**) Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, θ, for $0 \le \theta \le 90°$. (**b**) Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the result as $\theta \to 0$.



Solution 2.105





(a) For the free-body-diagram of the gate (see figure),

$$\sum F_A = 0$$

so that

$$F_R\left(\frac{\ell}{3}\right) + W\left(\frac{\ell}{2}\cos\theta\right) = \left(F_B\cos\phi\right)\left(\ell\sin\theta\right) + \left(F_B\sin\phi\right)\left(\ell\cos\theta\right) \qquad (1)$$

Also,

$\ell\sin\theta = L\sin\phi$ (assuming hinge and end of brace at same elevation)

or

$$\sin\phi = \frac{\ell}{L}\sin\theta$$

and

$$F_R = \gamma h_c A = \gamma\left(\frac{\ell\sin\theta}{2}\right)\left(\ell w\right)$$

where w is the gate width. Thus, Eq.(1) can be written as

$$\gamma\left(\frac{\ell^3}{6}\right)(\sin\theta)\,w+\frac{W\ell}{2}\cos\theta=F_B\ell\left(\cos\phi\sin\theta+\sin\phi\cos\theta\right)$$

so that

$$F_B=\frac{\left(\frac{\gamma\ell^2 w}{6}\right)\sin\theta+\frac{W}{2}\cos\theta}{\cos\phi\sin\theta+\sin\phi\cos\theta}=\frac{\left(\frac{\gamma\ell^2 w}{6}\right)\tan\theta+\frac{W}{2}}{\cos\phi\tan\theta+\sin\phi}\quad(2)$$

For $\gamma=62.4\,\frac{\text{lb}}{\text{ft}^3}$, $\ell=5\,\text{ft}$, $w=10\,\text{ft}$, and $W=200\,\text{lb}$,

$$F_B=\frac{\frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(5\,\text{ft})^2(10\,\text{ft})}{6}\tan\theta+\frac{200\,\text{lb}}{2}}{\cos\phi\tan\theta+\sin\phi}=\frac{2600\tan\theta+100}{\cos\phi\tan\theta+\sin\phi}\qquad(3)$$

Since $\sin\phi=\frac{\ell}{L}\sin\theta$ and $\ell=5\,\text{ft}$, $L=12\,\text{ft}$

$$\sin\phi=\frac{5}{12}\sin\theta$$

and for a given θ, ϕ can be determined. Thus, Eq.(3)

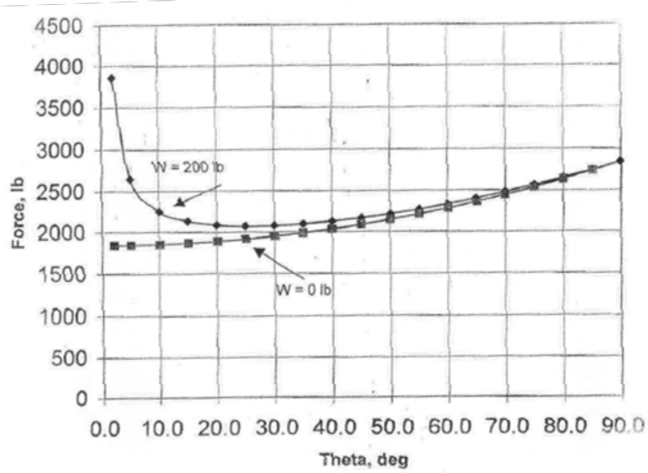
can be used to determine F_B for a given θ.

(b) For $W=0$, Eq.(3) reduces to

$$F_B=\frac{2600\tan\theta}{\cos\phi\tan\theta+\sin\phi}\qquad(4)$$

and Eq.(4) can be used to determine F_B for a given θ. Tabulated data of F_B vs. θ for both $W=200\,\text{lb}$ and $W=0\,\text{lb}$ are given below.

θ, deg	F(B), lb (W=200 lb)	F(B), lb (W=0 lb)
90.0	2843	2843
85.0	2745	2736
80.0	2651	2633
75.0	2563	2536
70.0	2480	2445
65.0	2403	2360
60.0	2332	2282
55.0	2269	2210
50.0	2213	2144
45.0	2165	2085
40.0	2125	2032
35.0	2094	1985
30.0	2075	1945
25.0	2069	1911
20.0	2083	1884
15.0	2130	1863
10.0	2250	1847
5.0	2646	1838
2.0	3858	1836



As $\theta \to 0$ the value of F_B can be determined from Eq.(4),

$$F_B = \frac{2600 \tan\theta}{\cos\phi \tan\theta + \sin\phi}$$

Since

$$\sin\phi = \frac{5}{12}\sin\theta$$

it follows that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{5}{12}\right)^2 \sin^2\theta}$$

and therefore

$$F_B = \frac{2600\tan\theta}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}\tan\theta+\frac{5}{12}\sin\theta} = \frac{2600}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}+\frac{5}{12}\cos\theta}$$

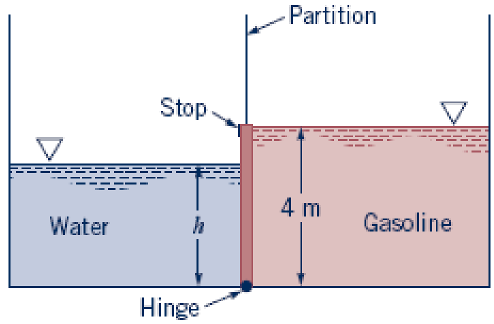
Thus, as $\theta \to 0$

$$F_B \to \frac{2600}{1+\frac{5}{12}} = 1840\,\text{lb}$$

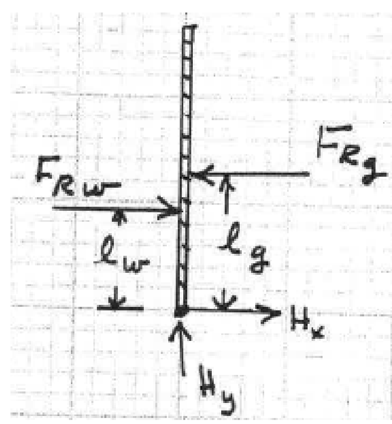
Physically this result means that for $\theta \equiv 0$, the value of F_B is indeterminate, but for any "very small" value of θ, F_B will approach $1840\,\text{lb}$.

Problem 2.106

An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ \text{kg}/\text{m}^3$ at a depth of $4\ \text{m}$, as shown in the figure below. A rectangular gate that is $4\ \text{m}$ high and $2\ \text{m}$ wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?



Solution 2.106



$F_{Rg} = \gamma_g h_{cg} A_g$; where g refers to gasoline.

$$F_{Rg} = \left(700\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2\,\text{m})(4\,\text{m}\times 2\,\text{m})$$
$$= 110\times 10^3\ \text{N} = 110\,\text{kN}$$

$F_{Rw} = \gamma_w h_{cw} A_w$; where w refers to water.

$$F_{Rw} = \left(9.80\times 10^3\,\frac{\text{N}}{\text{m}^3}\right)\left(\frac{h}{2}\right)(2\,\text{m}\times h);$$ where h is depth of water.

$$F_{Rw} = \left(9.80\times 10^3\right)h^2$$

For equilibrium, $\sum M_H = 0 \rightarrow F_{Rw}\ell_w = F_{Rg}\ell_g$

$$\ell_w = \frac{h}{3}\ ;\ \ \ell_g = \frac{4}{3}\text{m}$$

$$\left(9.80\times 10^3\right)\left(h^2\right)\left(\frac{h}{3}\right) = \left(110\times 10^3\ \text{N}\right)\left(\frac{4}{3}\text{m}\right)$$

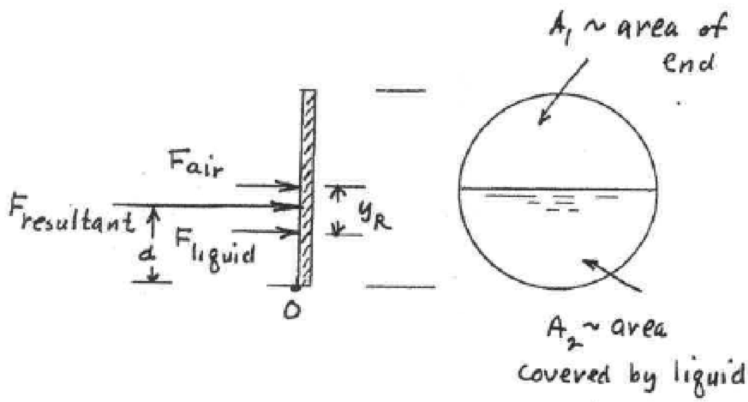
$$h = \underline{\underline{3.55\,\text{m}}}$$

which is the limiting value for h.

Problem 2.107

A horizontal 2-m-diameter conduit is half filled with a liquid ($SG = 1.6$) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 200 kPa. Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit.

Solution 2.107



$F_{air} = pA,$ where p is air pressure

Thus,

$$F_{air} = \left(200 \times 10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}\right)(2\text{ m})^2 = 200\pi \times 10^3 \text{ N}$$

$F_{liquid} = \gamma h_c A_2$ where $h_c = \dfrac{4R}{3\pi}$ (see the figure below)

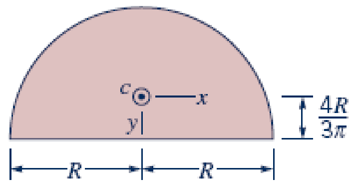
Thus,

$$F_{liquid} = (1.6)\left(9.81 \times 10^3 \frac{\text{N}}{\text{m}^3}\right)\left[\frac{4(1\text{m})}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\text{ m})^2 = 10.5 \times 10^3 \text{ N}$$

For F_{liquid},

$y_R = \dfrac{I_{xc}}{y_c A_2} + y_c$ where $I_{xc} = 0.1098R^4$ (see the figure below)

and $y_c = h_c = \dfrac{4R}{3\pi}$



$$A = \frac{\pi R^2}{2}$$

$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

Thus,

$$y_R = \frac{0.1098(1\,\text{m})^4}{\left[\frac{4(1\,\text{m})}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\,\text{m})^2} + \frac{4(1\,\text{m})}{3\pi} = 0.5891\,\text{m}$$

Since $F_{\text{resultant}} = F_{air} + F_{liquid} = (200\pi + 10.5)\times 10^3\ \text{N} = \underline{\underline{639\,\text{kN}}}$,

we can sum moments about O to locate resultant to obtain

$$F_{\text{resultant}}(d) = F_{air}(1\,\text{m}) + F_{liquid}(1\,\text{m} - 0.5891\,\text{m})$$

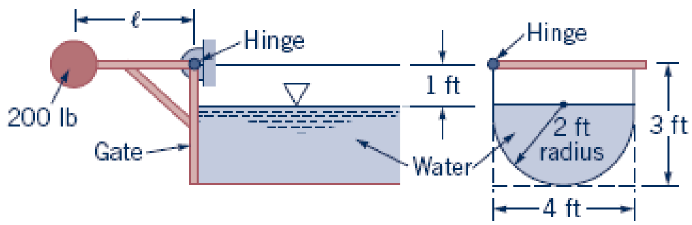
So that

$$d = \frac{\left(200\pi\times 10^3\ \text{N}\right)(1\,\text{m}) + \left(10.5\times 10^3\ \text{N}\right)(0.4109\,\text{m})}{639\times 10^3\ \text{N}}$$

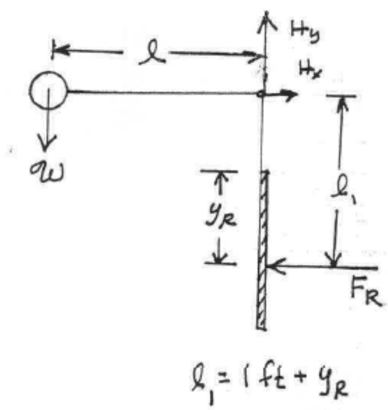
$$= \underline{\underline{0.990\,\text{m}}}\ \underline{\text{above bottom of conduit.}}$$

Problem 2.108

A 4-ft by 3-ft massless rectangular gate is used to close the end of the water tank shown in the figure below. A 200-lb weight attached to the arm of the gate at a distance ℓ from the frictionless hinge is just sufficient to keep the gate closed when the water depth is 2 ft, that is, when the water fills the semicircular lower portion of the tank. If the water were deeper, the gate would open. Determine the distance ℓ.



Solution 2.108



$F_R = \gamma h_c A$ where $h_c = \dfrac{4R}{3\pi}$ (see the figure below)

Thus,

$$F_R = \gamma_{H_2O}\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$$

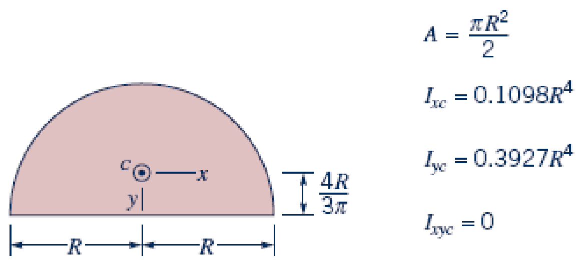
$$= \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{4(2\text{ ft})}{3\pi}\right)\left(\frac{\pi(2\text{ ft})^2}{2}\right) = 333\text{ lb}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{0.1098\,R^4}{\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)} + \frac{4R}{3\pi}$$ (see the figure below)

$$= \frac{(0.1098)(2\,\text{ft})^4}{\left(\frac{4(2\,\text{ft})}{3\pi}\right)\frac{\pi(2\,\text{ft})^2}{2}} + \frac{4(2\,\text{ft})}{3\pi} = 1.178\,\text{ft}$$



For equilibrium,

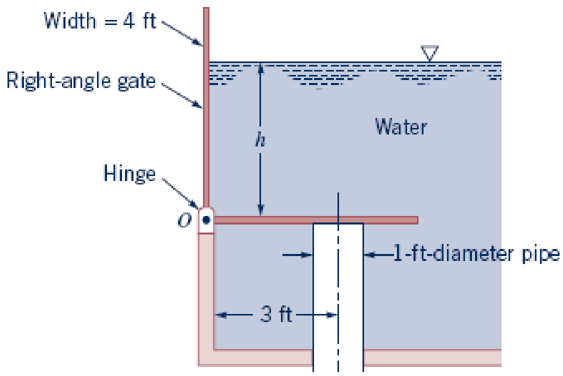
$$\sum M_H = 0$$

So that $W\,\ell = F_R\left(1\,\text{ft} + y_R\right)$

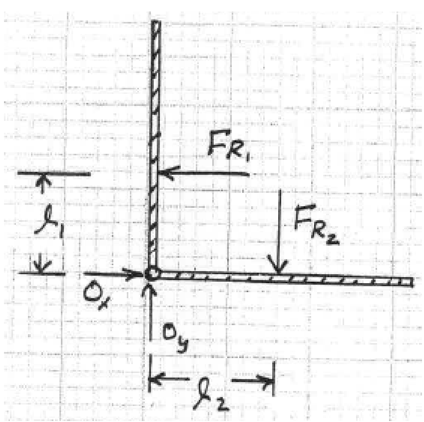
And $\ell = \dfrac{(333\,\text{lb})(1\,\text{ft} + 1.178\,\text{ft})}{200\,\text{lb}} = \underline{\underline{3.63\,\text{ft}}}$

Problem 2.109

A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in the figure below. The horizontal portion of the gate covers a 1-ft-diameter drain pipe that contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.



Solution 2.109



For equilibrium,

$$\sum M_0 = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$

$$F_{R_1} = \gamma h_{c_1} A_1$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{h}{2}\right)(4\,\text{ft} \times h) = 125\,h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h\left(\frac{\pi}{4}\right)(1\,\text{ft})^2 = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(h)\left(\frac{\pi}{4}\right)(1\,\text{ft})^2$$
$$= 49.0\,h$$

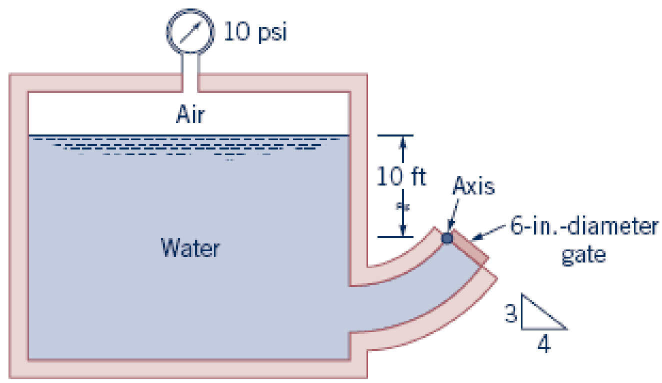
Thus, from Eq. (1) with $\ell_1 = \dfrac{h}{3}$ and $\ell_2 = 3\,\text{ft}$

$$\left(125\,h^2\right)\left(\frac{h}{3}\right) = (49.0\,h)(3\,\text{ft})$$

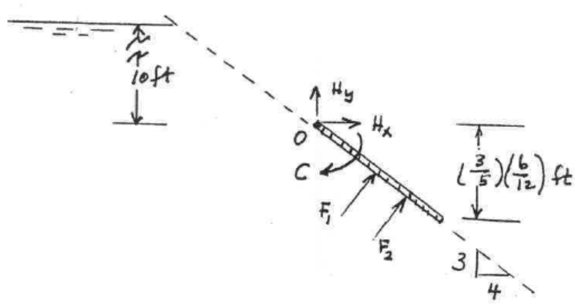
$$\underline{\underline{h = 1.88\,\text{ft}}}$$

Problem 2.110

The closed vessel of the figure below contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.



Solution 2.110



Let F_1 □ force due to air pressure, and F_2 □ force due to hydrostatic pressure distribution of water.

Thus,

$$F_1 = p_{air}A = \left(10\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\text{ft}\right)^2 = 283\,\text{lb}$$

and

$$F_2 = \gamma h_c A \qquad \text{where} \quad h_c = 10\,\text{ft} + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right)\text{ft}\right] = 10.15\,\text{ft}$$

So that

$$F_2 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(10.15\,\text{ft})\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\text{ft}\right)^2 = 124\,\text{lb}$$

Also,

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad y_c = \frac{10\,\text{ft}}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12}\,\text{ft}\right) = 16.92\,\text{ft}$$

So that

$$y_{R_2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\,\text{ft}\right)^4}{(16.92\,\text{ft})\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\,\text{ft}\right)^2} + 16.92\,\text{ft} = 16.92\,\text{ft}$$

For equilibrium,

$$\sum M_0 = 0$$

And

$$C = F_1\left(\frac{3}{12}\,\text{ft}\right) + F_2\left(y_{R_2} - \frac{10\,\text{ft}}{\frac{3}{5}}\right)$$

$$C = (283\,\text{lb})\left(\frac{3}{12}\,\text{ft}\right) + (124\,\text{lb})\left(16.92\,\text{ft} - \frac{10\,\text{ft}}{\frac{3}{5}}\right) = \underline{\underline{102\,\text{ft}\cdot\text{lb}}}$$

Problem 2.111

(a) Determine the horizontal hydrostatic force on the 2309-m-long Three Gorges Dam when the average depth of the water against it is 175 m. **(b)** If all of the 6.4 billion people on Earth were to push horizontally against the Three Gorges Dam, could they generate enough force to hold it in place? Support your answer with appropriate calculations.

Solution 2.111

(a)

$$F_R = \gamma h_c A = \left(9.80\times10^3 \frac{\text{N}}{\text{m}^3}\right)\left(\frac{175\,\text{m}}{2}\right)(175\,\text{m}\times 2309\,\text{m})$$

$$= \underline{\underline{3.46\times10^{11}\ \text{N}}}$$

(b)

$$\text{Required average force per person} = \frac{3.46\times10^{11}\ \text{N}}{6.4\times10^9}$$

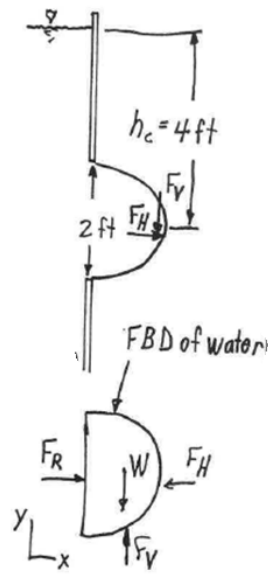
$$= \underline{\underline{54.1\frac{\text{N}}{\text{person}}}}\left(12.2\frac{\text{lb}}{\text{person}}\right)$$

Yes. It is likely that enough force could be generated since required average force per person is relatively small.

Problem 2.113

A 2-ft-diameter hemispherical plexiglass “bubble” is to be used as a special window on the side of an above-ground swimming pool. The window is to be bolted onto the vertical wall of the pool and faces outward, covering a 2-ft-diameter opening in the wall. The center of the opening is 4 ft below the surface. Determine the horizontal and vertical components of the force of the water on the hemisphere.

Solution 2.113



$\sum F_x = 0$, or $F_H = F_R = p_c A$

Thus,

$$F_H = \gamma h_c A = 62.4\frac{\text{lb}}{\text{ft}^3}(4\,\text{ft})\frac{\pi}{4}(2\,\text{ft})^2 = \underline{\underline{784\,\text{lb}}}\ (\text{to right})$$

and

$$\sum F_y = 0,\ \text{or}\ F_V = W = \gamma V = \gamma\frac{4}{3}\frac{\pi R^3}{2},$$

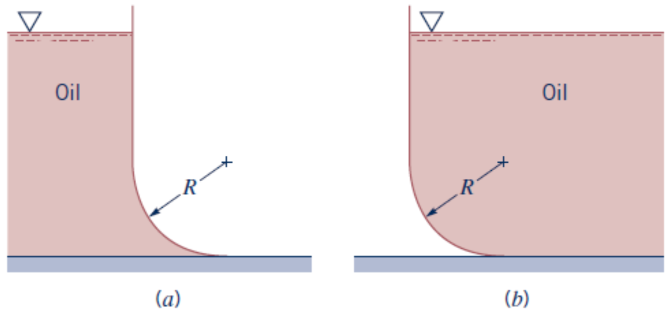
where $R = 1\,\text{ft}$

Thus,

$$F_V = 62.4\frac{\text{lb}}{\text{ft}^3}\left(\frac{4\pi(1\,\text{ft})^3}{6}\right) = \underline{\underline{131\,\text{lb}}}\ (\text{down on bubble})$$

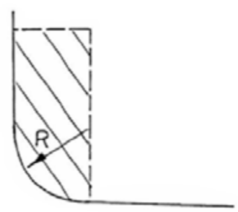
Problem 2.114

Consider the curved surface shown in the figure below (a) and (b). The two curved surfaces are identical. How are the vertical forces on the two surfaces alike? How are they different?



Solution 2.114

In both cases the magnitude of the vertical force is the weight of shaded section shown on the right. In addition, the location of the vertical force is the same (the centroid of the shaded section.) Therefore:



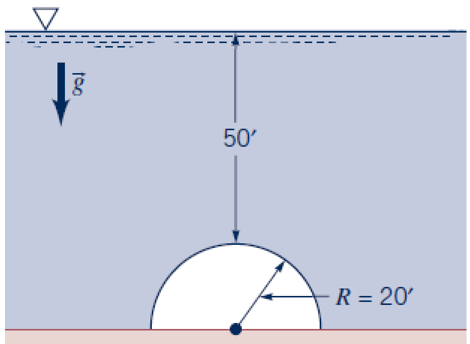
Alike: magnitude and location of vertical forces same.

However, the two vertical forces are different in that the force in (a) is acting upward and the force in (b) is acting downward. Therefore:

Different: direction of vertical forces opposite.

Problem 2.115

The figure below shows a cross section of a submerged tunnel used by automobiles to travel under a river. Find the magnitude and location of the resultant hydrostatic force on the circular roof of the tunnel. The tunnel is 4 mi long.

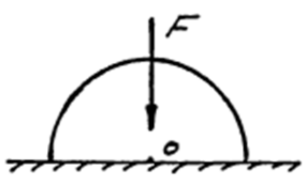


Solution 2.115

Due to symmetry, there is no net horizontal force on the roof. The vertical force is equal to the weight of fluid above the tunnel. This vertical force acts through the centroid of the fluid volume. Then for a tunnel length ℓ,

$$F = \gamma \cancel{V} = \gamma \ell \left(2Rh - \frac{\pi}{2} R^2 \right)$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3} \right) (4 \text{ mi}) \left(5280 \frac{\text{ft}}{\text{mi}} \right) \left[2(20 \text{ ft})(70 \text{ ft}) - \frac{\pi}{2} (20 \text{ ft})^2 \right]$$

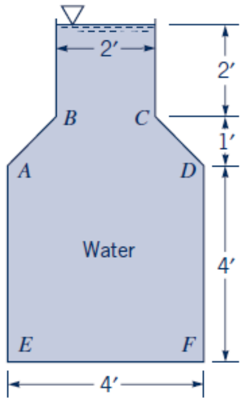


$\boxed{F = 2.86 \times 10^9 \text{ lb}}$.

This force $\boxed{\text{acts downward through the point "O".}}$

Problem 2.116

The container shown in the figure below has circular cross sections. Find the vertical force on the inclined surface. Also find the net vertical force on the bottom, *EF*. Is the vertical force equal to the weight of the water in the container?



Solution 2.116

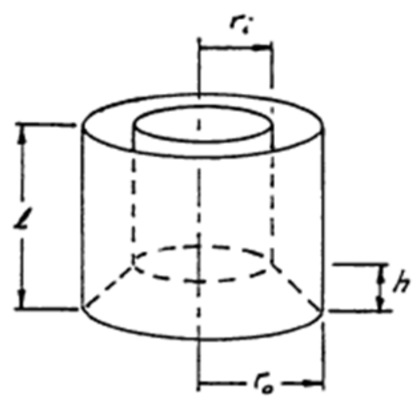
The vertical force on the inclined surface is equal to the weight of the water "above" it. This "water volume" is

$$\forall = \forall_{cyl} + \forall_{hole} - \forall_{\text{frustrum}}$$

$$\forall = \pi r_o^2 \ell - \pi r_i^2 (\ell - h) - \frac{1}{3}\pi h \left(r_o^2 + r_i^2 + r_o r_i\right).$$

$$\forall = \pi (2\,\text{ft})^2 (3\,\text{ft}) - \pi (1\,\text{ft})^2 (3-1)\,\text{ft}$$

$$-\frac{1}{3}\pi(1\,\text{ft})\left[(2\,\text{ft})^2 + (1\,\text{ft})^2 + (2\,\text{ft})(1\,\text{ft})\right] = 24.1\,\text{ft}^3$$



The vertical force F_{Vi} is

$r_i = 1", r_o = 2", h = 1", \ell = 3".$

$$F_{Vi} = \gamma \forall = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(24.1\,\text{ft}^3\right) = \boxed{F_{Vi} = 1500\,\text{lb}}.$$

The pressure is uniform over the bottom *EF* so

$$F_{Vb} = pA = \gamma h A = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(7\,\text{ft})\pi(2\,\text{ft})^2$$

or

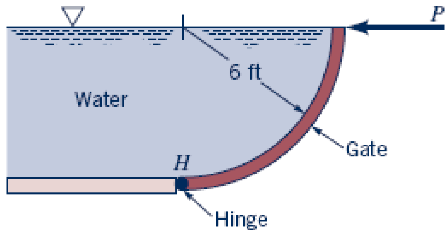
$$\boxed{F_{Vb} = 5490\,\text{lb}}$$

*CRC Standard Math Tables

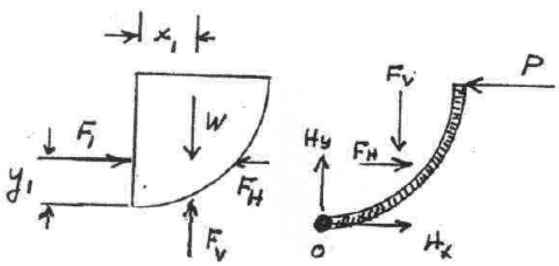
This force F_{Vb} is not equal to the weight of water in the container.

Problem 2.117

The 18-ft-long lightweight gate of the figure below is a quarter circle and is hinged at H. Determine the horizontal force, P, required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.



Solution 2.117



For equilibrium (from free-body-diagram of fluid mass),

$$\sum F_x = 0$$

So that

Similarly,

$$F_H = F_1 = \gamma h_{c_1} A_1$$

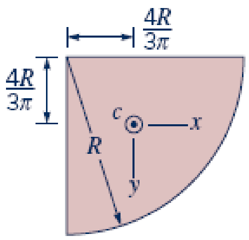
$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6\,\text{ft}}{2}\right)(6\,\text{ft} \times 18\,\text{ft}) = 20200\,\text{lb}$$

$$\sum F_y = 0$$

So that

$$F_V = W = \gamma_{H_2O} \times (\text{volume of fluid}) = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left[\frac{\pi}{4}(6\,\text{ft})^2 \times 18\,\text{ft}\right] = 31800\,\text{lb}$$

Also, $x_1 = \dfrac{4(6\,\text{ft})}{3\pi} = \dfrac{8}{\pi}\,\text{ft}$ (see the figure below) and $\quad y_1 = \dfrac{6\,\text{ft}}{3} = 2\,\text{ft}$



$$A = \frac{\pi R^2}{4}$$

$$I_{xc} = I_{yc} = 0.05488R^4$$

$$I_{xyc} = -0.01647R^4$$

For equilibrium (from free-body-diagram of gate)

$$\sum M_0 = 0$$

So that

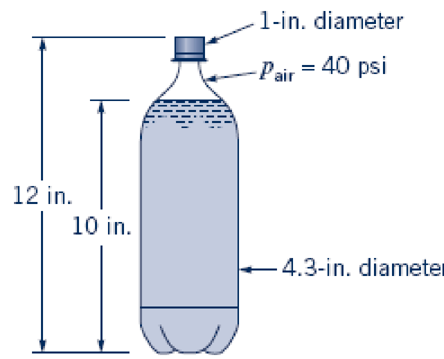
$$P(6\,\text{ft}) = F_H\,(y_1) + F_V\,(x_1)$$

or

$$P = \frac{(20200\,\text{lb})(2\,\text{ft}) + (31800\,\text{lb})\left(\dfrac{8}{\pi}\,\text{ft}\right)}{6\,\text{ft}} = \underline{\underline{20200\,\text{lb}}}$$

Problem 2.118

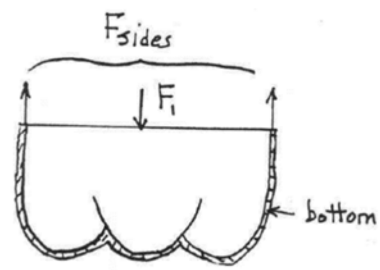
The air pressure in the top of the 2-liter pop bottle and the figure below is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. **(a)** If the bottle cap has a diameter of 1 in. what is the magnitude of the axial force required to hold the cap in place? **(b)** Determine the force needed to secure the bottom 2 in. of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. **(c)** By how much does the weight of the pop increase the pressure 2 in. above the bottom? Assume the pop has the same specific weight as that of water.



Solution 2.118

(a) $$F_{cap} = p_{air} \times \text{Area}_{cap} = \left(40\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{\pi}{4}\right)(1\,\text{in.})^2 = \underline{\underline{31.4\,\text{lb}}}$$

(b)



$$\sum F_{\text{vertical}} = 0$$

$$F_{\text{sides}} = F_1 = (\text{pressure @ 2 in. above bottom }) \times (\text{Area})$$
$$= \left(40\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{\pi}{4}\right)(4.3\,\text{in.})^2$$
$$= \underline{\underline{581\,\text{lb}}}$$

(c)

$$p = p_{air} + \gamma h$$

$$= 40\frac{\text{lb}}{\text{in.}^2} + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\,\text{ft}\right)\left(\frac{1}{144\frac{\text{in.}^2}{\text{ft}^2}}\right)$$

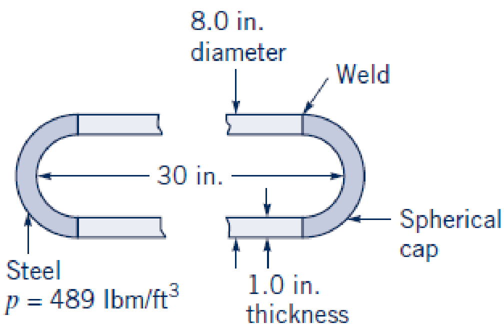
$$= 40\frac{\text{lb}}{\text{in.}^2} + 0.289\frac{\text{lb}}{\text{in.}^2}$$

Thus, the increase in pressure due to weight $= \underline{\underline{0.289\,\text{psi}}}$

(which is less that 1% of air pressure).

Problem 2.119

In drilling for oil in the Gulf of Mexico, some divers have to work at a depth of 1300 ft. (**a**) Assume that seawater has a constant density of 64 lb/ft^3 and compute the pressure at this depth. The divers breathe a mixture of helium and oxygen stored in cylinders, as shown in the figure below, at a pressure of 3000 psia. (**b**) Calculate the force, which trends to blow the end cap off, that the weld must resist while the diver is using the cylinder at 1300 ft. (**c**) After emptying a tank, a diver releases it. Will the tank rise or fall, and what is its initial acceleration?



Solution 2.119

(a) The hydrostatic pressure is

$$p = \gamma_{\text{sw}} h = \left(64\frac{\text{lb}}{\text{ft}^3}\right)(1300\,\text{ft})\left(\frac{\text{ft}^2}{144\,\text{in.}^2}\right) \quad \text{or} \quad \boxed{p = 578\,\text{psig}}$$

(b) The net horizontal force on the end caps is

$$F_N = F_{\text{in}} - F_{\text{out}} = p_{\text{in}} A_{\text{in}} - p_{\text{out}} A_{\text{out}}$$

and

$$\tau = \text{wall stress} = \frac{F_N}{A_{\text{wall}}} = \frac{F_N}{A_{\text{out}} - A_{\text{in}}}$$

$$= \frac{p_{\text{in}} A_{\text{in}} - p_{\text{out}} A_{\text{out}}}{A_{\text{out}} - A_{\text{in}}} = \frac{p_{\text{in}} D_{\text{in}}^2 - p_{\text{out}} D_{\text{out}}^2}{D_{\text{out}}^2 - D_{\text{in}}^2}$$

$$= \frac{\left(3000\frac{\text{lb}}{\text{in.}^2}\right)(6\,\text{in.})^2 - (14.7 - 578)\frac{\text{lb}}{\text{in.}^2}(8\,\text{in.})^2}{(8\,\text{in.})^2 - (6\,\text{in.})^2}$$

and

$$\boxed{\tau = 2500\,\text{psi.}}$$

(c) The net vertical force on an empty tank and Newton's second law give

$$+\uparrow \quad F_{\text{vert}} = F_{\text{Buoy}} - W = m\,a$$

or

$$a = \frac{F_{\text{Buoy}} - W}{m} = \frac{F_{\text{Buoy}}}{m} - g$$

where m is the mass of the tank. Now

$$F_{\text{Buoy}} = \gamma_{\text{sw}} \cancel{V} = \gamma_{\text{sw}} \left[\left(\frac{\pi}{4}\right) \ell\, D_{\text{out}}{}^2 + \left(\frac{\pi}{6}\right) D_{\text{out}}{}^3 \right]$$

where $\ell = 30\,\text{in.} - 6\,\text{in.} = 24\,\text{in.}$ Also

$$m = \rho_{\text{steel}} \left[\left(\frac{\pi}{4}\right) \ell \left(D_{\text{out}}{}^2 - D_{\text{in}}{}^2\right) + \left(\frac{\pi}{6}\right)\left(D_{\text{out}}{}^3 - D_{\text{in}}{}^3\right) \right].$$

Substituting into the equation for a gives

$$a = \frac{\gamma_{\text{sw}} \left[\frac{1}{4} \ell\, D_{\text{out}}{}^2 + \frac{1}{6} D_{\text{out}}{}^3 \right]}{\rho_{\text{steel}} \left[\frac{1}{4} \ell \left(D_{\text{out}}{}^2 - D_{\text{in}}{}^2\right) + \frac{1}{6}\left(D_{\text{out}}{}^3 - D_{\text{in}}{}^3\right) \right]} - g\,.$$

The numerical values give

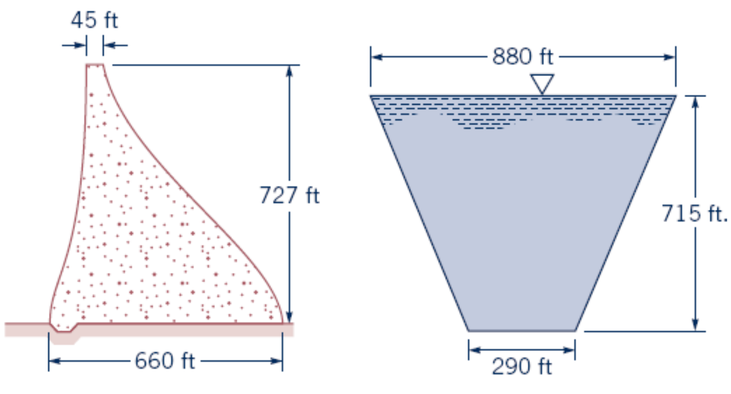
$$a = \frac{\left(64 \frac{\text{lb}}{\text{ft}^3}\right) \left[\frac{1}{4}(24)8^2 + \frac{1}{6}\left(8^3\right) \right] \text{in.}^3 \left(32.2 \frac{\text{ft} \cdot \text{lbm}}{\text{lb} \cdot \text{sec}^2}\right)}{\left(489 \frac{\text{lbm}}{\text{ft}^3}\right) \left[\frac{1}{4}(24)\left(8^2 - 6^2\right) + \frac{1}{6}\left(8^3 - 6^3\right) \right] \text{in.}^3} - 32.2 \frac{\text{ft}}{\text{sec}^2}$$

or

$$\boxed{\begin{array}{l} a = -23.1 \dfrac{\text{ft}}{\text{sec}^2} \\ \text{tank will fall} \\ \text{since } a < 0. \end{array}}$$

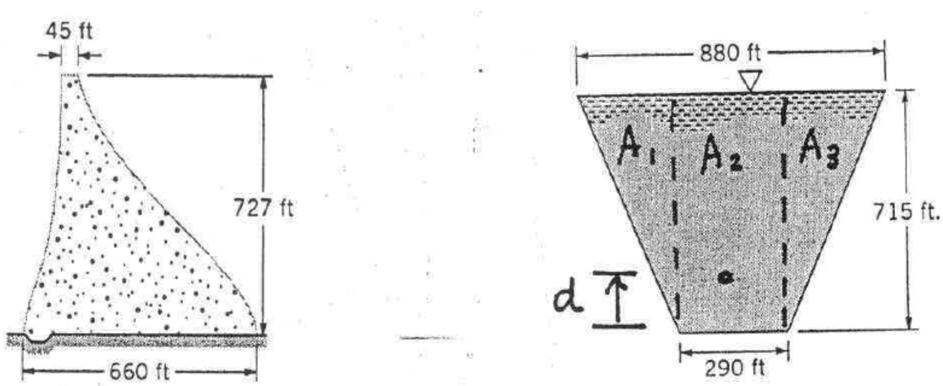
Problem 2.120

Hoover Dam is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in the figure below (a). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in the figure below (b) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.



(a) (b)

Solution 2.120



(a) (b)

Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{3}\right)(715\,\text{ft})\left(\frac{1}{2}\right)(295\,\text{ft})(715\,\text{ft})$$

$$= 1.57 \times 10^9 \text{ lb}$$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9$ lb

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(715\,\text{ft})(290\,\text{ft})(715\,\text{ft})$$
$$= 4.63 \times 10^9\ \text{lb}$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ \text{lb} + 4.63 \times 10^9\ \text{lb} + 1.57 \times 10^9\ \text{lb}$$
$$= 7.77 \times 10^9\ \text{lb}$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\,\text{ft}) + F_{R_2}\left(\frac{1}{2}\right)(715\,\text{ft}) + F_{R_3}\left(\frac{2}{3}\right)(715\,\text{ft})$$

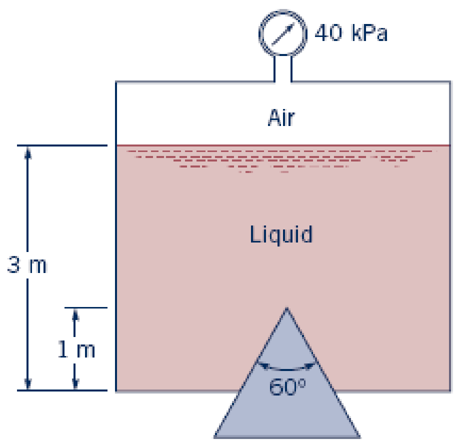
and

$$d = \frac{\left(1.57\times10^9\ \text{lb}\right)\left(\frac{2}{3}\right)(715\,\text{ft}) + \left(4.63\times10^9\ \text{lb}\right)\left(\frac{1}{2}\right)(715\,\text{ft}) + \left(1.57\times10^9\ \text{lb}\right)\left(\frac{2}{3}\right)(715\,\text{ft})}{7.77\times10^9\ \text{lb}}$$
$$= 406\,\text{ft}$$

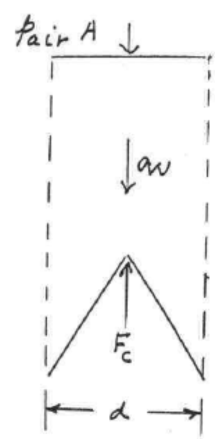
Thus, the resultant horizontal force on the dam is $7.77\times10^9\ \text{lb}$ acting $406\,\text{ft}$ up from the base of the dam along the axis of symmetry of the area.

Problem 2.121

A plug in the bottom of a pressurized tank is conical in shape, as shown in the figure below. The air pressure is $40\ \text{kPa}$, and the liquid in the tank has a specific weight of $27\ \text{kN/m}^3$. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the $40-\text{kPa}$ pressure and the liquid.



Solution 2.121



$$\tan 30^\circ = \frac{\frac{d}{2}}{1}$$

$$d = 2\tan 30^\circ = 1.155\,\text{m}$$

$$\text{volume of cone} = \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1)$$

For equilibrium,

$$\sum F_{vertical} = 0$$

So that

$$F_c = p_{air}A + a_w$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air}A = (40\,\text{kPa})\left(\frac{\pi}{4}\right)(d^2)$$
$$= (40\,\text{kPa})\left(\frac{\pi}{4}\right)(1.155\,\text{m})^2 = 41.9\,\text{kN}$$

And

$$W = \gamma\left[\frac{\pi}{4}d^2(3\,\text{m}) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2(1\,\text{m})\right] = \gamma\pi d^2\left[\frac{3\,\text{m}}{4} - \frac{1\,\text{m}}{12}\right]$$
$$= \left(27\frac{\text{kN}}{\text{m}^3}\right)(\pi)(1.155\,\text{m})^2\left(\frac{2}{3}\text{m}\right) = 75.4\,\text{kN}$$

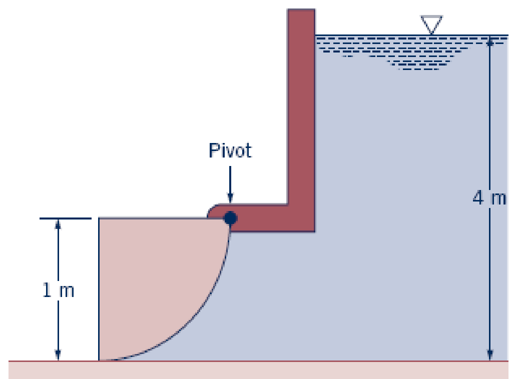
Thus,

$$F_c = 41.9\,\text{kN} + 75.4\,\text{kN} = 117\,\text{kN}$$

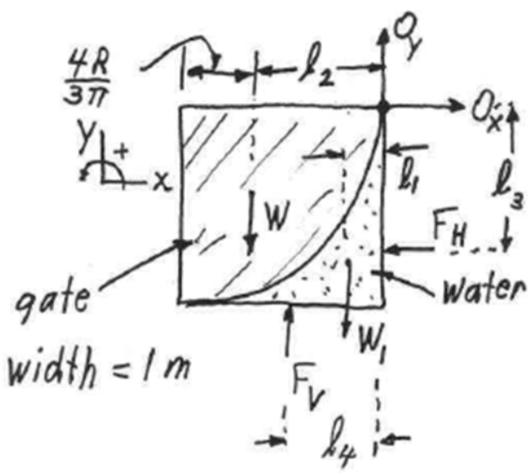
And the force on the cone has a magnitude of $\underline{\underline{117\,\text{kN}}}$ and is <u>directed vertically downward along the cone axis</u>.

Problem 2.122

The homogeneous gate shown in the figure below consists of one quarter of a circular cylinder and is used to maintain a water depth of 4 m. That is, when the water depth exceeds 4 m, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.



Solution 2.122



Consider the free body diagram of the gate and a portion of the water as shown.

$\sum M_o = 0$, or

(1) $\ell_2 W + \ell_1 W_1 - F_H \ell_3 - F_V \ell_4 = 0$, where

(2) $F_H = \gamma h_c A = 9.8\times10^3 \dfrac{\text{N}}{\text{m}^3}(3.5\,\text{m})(1\,\text{m})(1\,\text{m}) = 34.3\,\text{kN}$

since for the vertical side, $h_c = 4\,\text{m} - 0.5\,\text{m} = 3.5\,\text{m}$

Also,

(3) $F_V = \gamma h_c A = 9.8\times10^3 \dfrac{\text{N}}{\text{m}^3}(4\,\text{m})(1\,\text{m})(1\,\text{m}) = 39.2\,\text{kN}$

Also,

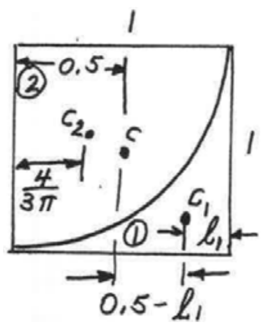
(4) $$W_1 = \gamma(1\text{m})^3 - \gamma\left(\frac{\pi}{4}(1\text{m})^2\right)(1\text{m}) = 9.8\times10^3\,\frac{\text{N}}{\text{m}^3}\left[1-\frac{\pi}{4}\right]\text{m}^3 = 2.10\,\text{kN}$$

(5) Now, $\ell_4 = 0.5\,\text{m}$ and

(6) $$\ell_3 = 0.5\,\text{m}+\left(y_R - y_c\right) = 0.5\,\text{m} + \frac{I_{xc}}{y_c A} = 0.5\,\text{m} + \frac{\frac{1}{12}(1\text{m})(1\text{m})^3}{3.5\,\text{m}(1\text{m})(1\text{m})} = 0.524\,\text{m}$$

(7) and $\ell_2 = 1\,\text{m} - \dfrac{4R}{3\pi} = 1 - \dfrac{4(1\text{m})}{3\pi} = 0.576\,\text{m}$

To determine ℓ_1, consider a unit square that consist of a quarter circle and the remainder as show in the figure. The centroids of areas (1) and (2) are as indicated.



Thus,

$$\left(0.5 - \frac{4}{3\pi}\right)A_2 = \left(0.5 - \ell_1\right)A_1$$

So that with $A_2 = \frac{\pi}{4}(1)^2 = \frac{\pi}{4}$ and $A_1 = 1 - \frac{\pi}{4}$ this gives

$$\left(0.5 - \frac{4}{3\pi}\right)\frac{\pi}{4} = \left(0.5 - \ell_1\right)\left(1 - \frac{\pi}{4}\right)$$

or

(8) $\ell_1 = 0.223\,\text{m}$

Hence, by combining Eqs.(1) through (8):

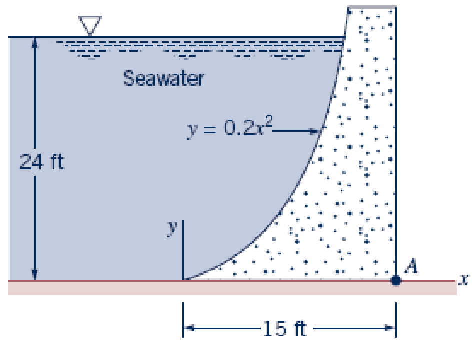
$$(0.576\,\text{m})W + (0.223\,\text{m})(2.10\,\text{kN}) - (34.3\,\text{kN})(0.524\,\text{m}) - (39.2\,\text{kN})(0.5\,\text{m}) = 0$$

or

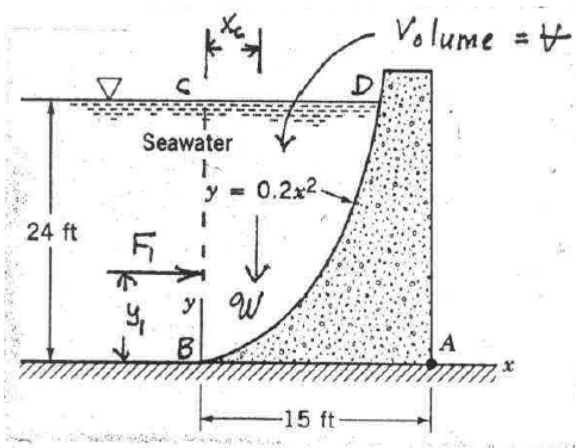
$$W = \underline{\underline{64.4\,\text{kN}}}$$

Problem 2.123

The concrete (specific weight $=150\text{ lb}/\text{ft}^3$) seawall of the figure below has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).



Solution 2.123



The components of the fluid force acting on the wall are F_1 and W as shown on the figure where

$$F_1 = \gamma h_c A = \left(64.0\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{24\text{ ft}}{2}\right)(24\text{ ft}\times 1\text{ ft})$$
$$= 18400\text{ lb}$$

and

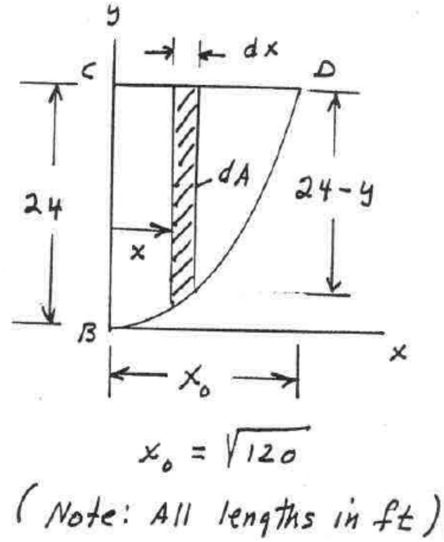
$$y_1 = \frac{24\text{ ft}}{3} = 8\text{ ft}$$

Also,

$$W = \gamma \forall$$

To determine $\forall$ find area BCD.

Thus,



$$A = \int_0^{x_0} (24 - y)\, dx = \int_0^{x_0} \left(24 - 0.2x^2\right) dx$$

$$= \left[24x - \frac{0.2x^3}{3} \right]_0^{x_0}$$

And with $x_0 = \sqrt{120}$, $A = 175\,\text{ft}^2$ so that

$$\cancel{V} = A \times 1\,\text{ft} = 175\,\text{ft}^3$$

Thus,

$$W = \left(64.0\frac{\text{lb}}{\text{ft}^3}\right)\left(175\,\text{ft}^3\right) = 11200\,\text{lb}$$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\, dA = \int_0^{x_0} (24 - y)\, x\, dx = \int_0^{x_0} \left(24x - 0.2x^3\right) dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

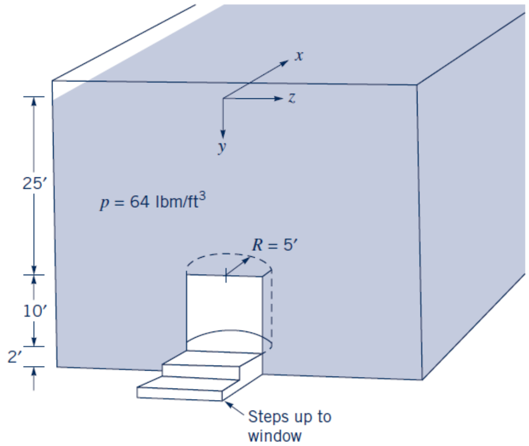
$$x_c = \frac{12\left(\sqrt{120}\right)^2 - \dfrac{0.2\left(\sqrt{120}\right)^4}{4}}{175} = 4.11\,\text{ft}$$

Thus,

$$\begin{aligned} M_A &= F_1 y_1 - W\left(15 - x_c\right) \\ &= (18400\,\text{lb})(8\,\text{ft}) - (11200\,\text{lb})(15\,\text{ft} - 4.11\,\text{ft}) \\ &= \underline{\underline{25200\,\text{ft}\cdot\text{lb}}} \end{aligned}$$

Problem 2.124

A step-in viewing window having the shape of a half-cylinder is built into the side of a large aquarium. See the figure below. Find the magnitude, direction, and location of the net horizontal forces on the viewing window.



Solution 2.124

Due to symmetry, the net force parallel to the wall is zero or

$\boxed{F_z = 0}$

The net horizontal force perpendicular to the wall is

$$F_x = \gamma h_c A = \left(64\frac{\text{lb}}{\text{ft}^3}\right)(25+5)\,\text{ft}\,(10\,\text{ft}\times 10\,\text{ft})$$

$\boxed{F_x = 1.92\times 10^5\ \text{lb}}$

The vertical location of F_x is

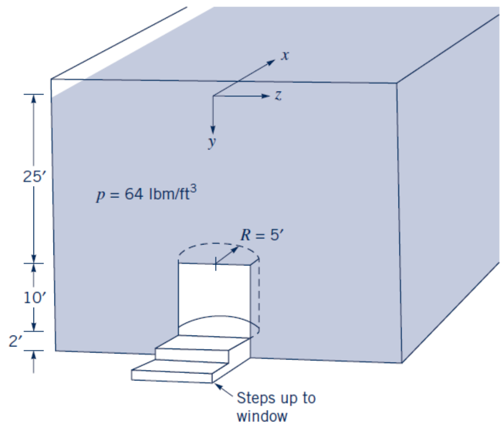
$$y_p = y_c + \frac{I_{xc}}{y_c A} = y_c + \frac{\frac{1}{12}bh^3}{y_c bh} = y_c + \frac{h^2}{12y_c} = 30\,\text{ft} + \frac{(10\,\text{ft})^2}{12(30\,\text{ft})} \quad \text{or} \quad \boxed{y_p = 30.3\,\text{ft}}$$

The net horizontal force also acts through the coordinate

$\boxed{z = 0 \text{ and acts in an outward direction.}}$

Problem 2.125

Find the magnitude, direction, and location of the net vertical force acting on the viewing window in Problem 2.124.



Solution 2.125

The net vertical force must equal the weight of fluid inside the viewing window. Then

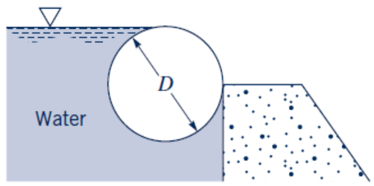
$$F_y = \gamma \cancel{V} = \gamma h\left(\frac{\pi}{2}R^2\right) = \left(64\frac{\text{lb}}{\text{ft}^3}\right)(10\,\text{ft})\left(\frac{\pi}{2}\right)(5\,\text{ft})^2 \text{ or } \boxed{\begin{array}{l}F_y = 25100\,\text{lb},\\ \text{acting upword.}\end{array}}$$

This net vertical force acts through the centroid of the window volume. Using Appendix B gives

$$\bar{x} = \frac{4R}{3\pi} = \frac{4(5\,\text{ft})}{3\pi} \qquad \text{or} \qquad \boxed{\bar{x} = 2.12\,\text{ft}}$$

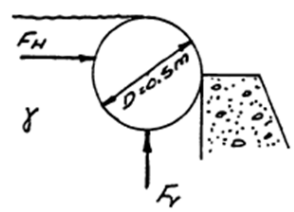
Problem 2.126

A 10-m-long log is stuck against a dam, as shown in the figure below. Find the magnitudes and locations of both the horizontal force and the vertical force of the water on the log in terms of the diameter D. The center of the log is at the same elevation as the top of the dam.



Solution 2.126

Consider the water forces on the log as shown on the right.



The horizontal forces F_H is on the top portion only and is

$$F_H = \gamma\left(\frac{D}{4}\right)\left(\frac{D}{2}\right)\ell$$

where ℓ is the log length. Assuming 10°C water, Table A.5 gives

$$F_H = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.25\,\text{m})(0.5\,\text{m})(10\,\text{m}) = \boxed{12300\text{ N} = F_H}$$

The location of F_H is

$$y_p = \frac{2}{3}\left(\frac{D}{2}\right) = \frac{2}{3}\left(\frac{0.5}{2}\text{m}\right) = \boxed{0.167\,\text{m} = y_p}$$

The vertical force F_V is the weight of water "above» the bottom of the log minus the weight of water above the top half of the log. This is

$$F_V = \gamma\ell\left[\frac{\pi D^2}{8} + \left(\frac{D}{2}\right)D - \left(\frac{D^2}{4} - \frac{\pi D^2}{16}\right)\right] = \frac{\gamma\ell D^3}{4}\left(\frac{3\pi}{4}+1\right)$$

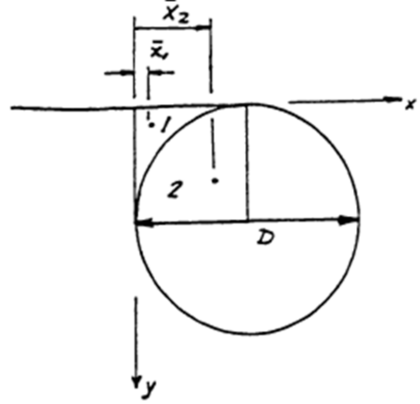
$$= \frac{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(10\,\text{m})(1.0\,\text{m})^2}{4}\left(\frac{3\pi}{4}+1\right)$$

$$\boxed{F_V = 82300\,\text{N}}$$

The location $\bar{x}$ of F_V is found by first locating the centroid of area A_1 by

$$\bar{x}_1 = \frac{A_{1+2}\bar{x}_{1+2} - A_2\bar{x}_2}{A_1}.$$

Using Table B

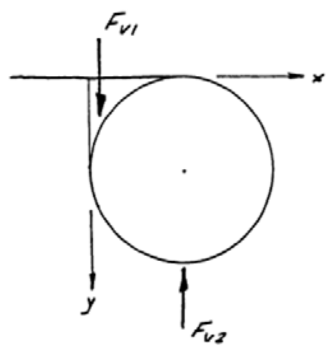


$$\bar{x}_1 = \frac{\left(\frac{D}{2}\right)^2\left(\frac{D}{4}\right) - \left(\frac{\pi D^2}{16}\right)\left(\frac{D}{2} - \frac{2D}{3\pi}\right)}{\left(\frac{D}{2}\right)^2 - \frac{\pi D^2}{16}}$$

$$= \left[\frac{\frac{1}{16} - \frac{\pi}{16}\left(\frac{1}{2} - \frac{2}{3\pi}\right)}{\frac{1}{4} - \frac{\pi}{16}}\right] D$$

$$= 0.112\,D$$

and is the location of F_{V1}. The location of F_{V2} is $\frac{D}{2}$. The location of F_V is



$$\bar{x} = \frac{F_{V2}\left(\frac{D}{2}\right) - F_{V1}(0.112\,D)}{F_V}$$

$$= \frac{\left[\frac{\pi D^2}{B} + \frac{D^2}{2}\right]\left(\frac{D}{2}\right) - \left[\frac{D^2}{4} - \frac{\pi D^2}{16}\right](0.112\,D)}{\frac{\pi D^2}{B} + \frac{D^2}{2} - \left[\frac{D^2}{4} - \frac{\pi D^2}{16}\right]}$$

$$= \frac{\frac{1}{4}\left(\frac{\pi}{4} + 1\right) - 0.112\left(\frac{1}{4}\right)\left(1 - \frac{\pi}{4}\right)}{\frac{1}{2}\left(\frac{\pi}{4} + 1\right) - \frac{1}{4}\left(1 - \frac{\pi}{4}\right)} D$$

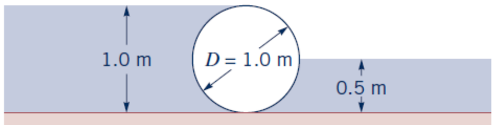
$$\boxed{\bar{x} = 0.525\,D}$$

F_{V2} = weight of water "above" bottom portion of log.

F_{V1} = weight of water above top left portion of log.

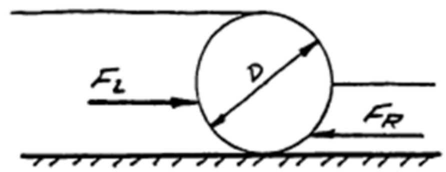
Problem 2.127

Find the net horizontal force on the 4.0-m-long log shown in the figure below.



Solution 2.127

The force F_L on the left side of the log is the horizontal force on the horizontally projected area of the log. This horizontally projected area measures $D = 1.0\,\text{m}$ by $4.0\,\text{m}$ and gives



$$F_L = \rho g h_c A$$

$$= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.5\,\text{m})(1.0\,\text{m}\times 4.0\,\text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 19600\text{ N} = 19.6\text{ kN}$$

The force F_R on the right side of the log is the horizontal force on the horizontally projected area of the lower half of the log. This horizontally projected area measures $\frac{D}{2} = 0.5\text{m}$ by 4.0 m and gives

$$F_R = \rho g h_c A = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.25\,\text{m})(0.5\,\text{m}\times 4.0\,\text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 4910\,\text{N} = 4.91\text{ kN}$$

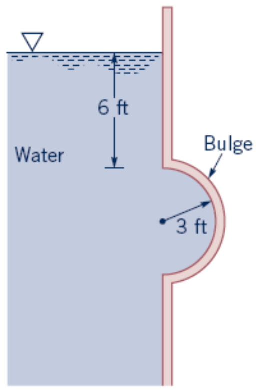
The net horizontal force is

$$F = F_L - F_R = 19.6\,\text{kN} - 4.91\,\text{kN}$$

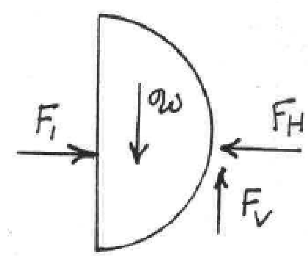
$$\boxed{F = 14.7\,\text{kN},\ \text{acting to right.}}$$

Problem 2.128

An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in the figure below. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.



Solution 2.128



F_H □ horizontal force of wall on fluid

F_V □ vertical force of wall on fluid

$$W = \gamma_{H_2O} V_{vol}$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\pi (3\,\text{ft})^2}{2}\right)(1\,\text{ft})$$

$$= 882\,\text{lb}$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)(6\,\text{ft} + 3\,\text{ft})(6\,\text{ft} \times 1\,\text{ft}) = 3370\,\text{lb}$$

For equilibrium,

$$F_V = W = 882\,\text{lb} \uparrow$$

and $$F_H = F_1 = 3370\,\text{lb} \leftarrow$$

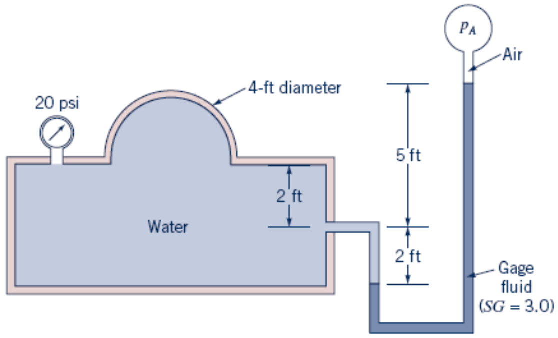
The force the water exerts on the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{\underline{(F_H)_{wall} = 3370\,\text{lb} \rightarrow}}$$

$$\underline{\underline{(F_V)_{wall} = 882\,\text{lb} \downarrow}}$$

Problem 2.129

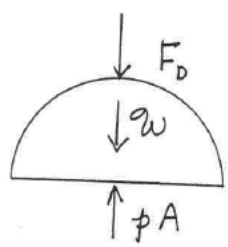
A closed tank is filled with water and has a 4-ft-diameter hemispherical dome as shown in the figure below. A U-tube manometer is connected to the tank. Determine the vertical force of the water on the dome if the differential manometer reading is 7 ft and the air pressure at the upper end of the manometer is 12.6 psi.



Solution 2.129

For equilibrium,

$\sum F_{vertical} = 0$



So that

$F_D = pA - W$ Eq. (1)

Where F_D force exerted by dome on the fluid and p is water pressure at the dome base.

From the manometer, $p_A + \gamma_{gf}(7\,\text{ft}) - \gamma_{H_2O}(4\,\text{ft}) = p$

$$p = \left(12.6\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) + (3.0)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(7\,\text{ft}) - \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(4\,\text{ft})$$

$$= 2880\frac{\text{lb}}{\text{ft}^2}$$

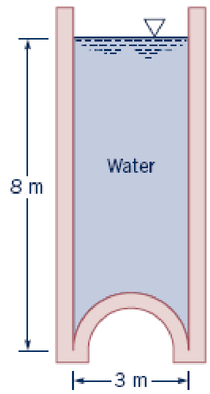
Thus, from Eq.(1) with volume of sphere $= \frac{\pi}{6}(\text{diameter})^3$

$$F_D = \left(2880\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{\pi}{4}\right)(4\,\text{ft})^2 - \frac{1}{2}\left[\frac{\pi}{6}(4\,\text{ft})^3\right]\left(62.4\frac{\text{lb}}{\text{ft}^3}\right) = 35100\,\text{lb}$$

The vertical force that the water exerts on the dome is $\underline{\underline{35100\,\text{lb}\uparrow}}$.

Problem 2.130

A 3-m-diameter open cylindrical tank contains water and has a hemispherical bottom as shown in the figure below. Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.



Solution 2.130

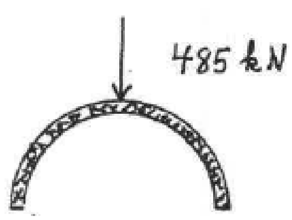
$$\text{Force} = \text{weight of water supported by hemispherical bottom}$$

$$= \gamma_{H_2O}\left[(\text{volume of cylinder}) - (\text{volume of hemisphere})\right]$$

$$= 9.80\frac{\text{kN}}{\text{m}^3}\left[\frac{\pi}{4}(3\,\text{m})^2(8\,\text{m}) - \frac{\pi}{12}(3\,\text{m})^3\right]$$

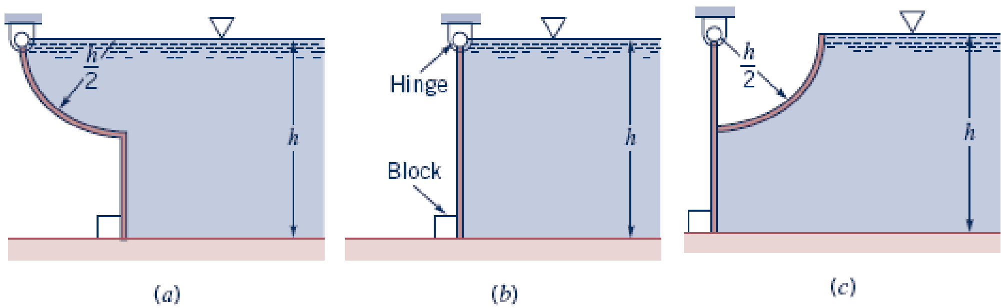
$$= \underline{\underline{485\,\text{kN}}}$$

The force is directed vertically downward, and due to symmetry it acts on the hemisphere along the vertical axis of the cylinder.



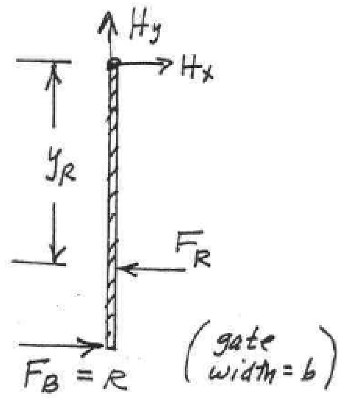
Problem 2.131

Three gates of negligible weight are used to hold back water in a channel of width b as shown in the figure below. The force of the gate against the block for gate (b) is R. Determine (in terms of R) the force against the blocks for the other two gates.



Solution 2.131

For case (b)



$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and

$$y_R = \frac{2}{3}h$$

Thus,

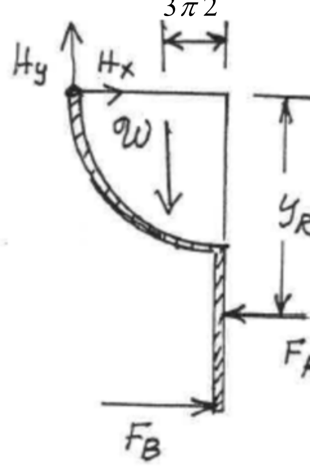
$$\sum M_H = 0$$

So that

$$hR = \left(\frac{2}{3}h\right)F_R = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right) \quad \rightarrow \quad R = \frac{\gamma h^2 b}{3}$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$

For case (a) on free-body-diagram shown $\frac{4h}{3\pi 2}$ (see the figure below)

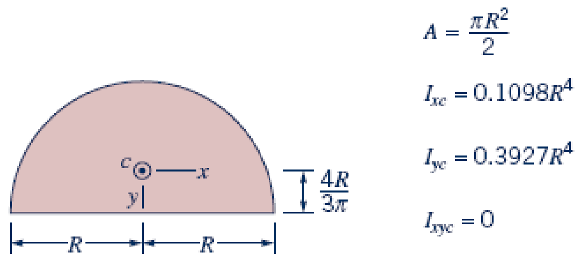


$$F_R = \frac{\gamma h^2 b}{2} \qquad \text{(from above) and}$$

$$y_R = \frac{2}{3}h$$

and

$$W = \gamma \times \cancel{V}_o = \gamma \left[\frac{\pi \left(\frac{h}{2} \right)^2}{4} (b) \right] = \frac{\pi \gamma h^2 b}{16}$$



Thus, $\sum M_H = 0$

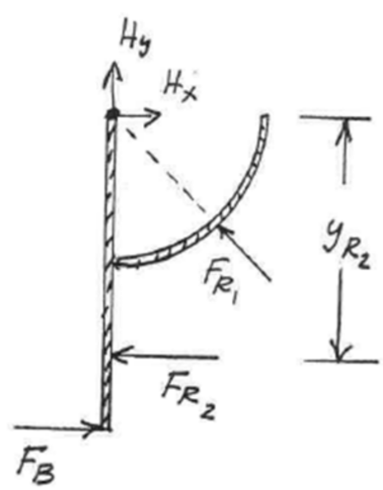
So that $\quad W\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B h$

$$\frac{\pi \gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B h$$

$$F_B = \gamma h^2 b(0.390) = 3R(0.390) \quad \rightarrow \quad \boxed{F_B = 1.17R}$$

For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right) = \frac{3}{8}\gamma h^2 b$$



and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus,

$$\sum M_H = 0$$

So that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B h$$

or

$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

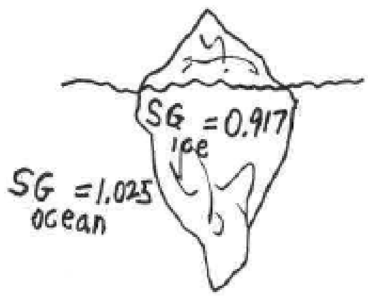
From Eq.(1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

Problem 2.133

An iceberg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?

Solution 2.133



For equilibrium,

W = weight of iceberg = F_B = buoyant force

or

$\cancel{V}_{ice}\gamma_{ice} = \cancel{V}_{\text{sub}}\gamma_{ocean}$, where $\cancel{V}_{sub}$ = volume of ice submerged.

Thus,

$$\frac{\cancel{V}_{sub}}{\cancel{V}_{ice}} = \frac{\gamma_{ice}}{\gamma_{ocean}} = \frac{SG_{ice}}{SG_{ocean}} = \frac{0.917}{1.025} = 0.895 = \underline{\underline{89.5\%}}$$

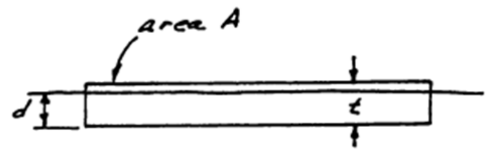
Problem 2.134

A floating 40-in.-thick piece of ice sinks 1 in. with a 500-lb polar bear in the center of the ice. What is the area of the ice in the plane of the water level? For seawater, $S = 1.03$.

Solution 2.134

Without the polar bear on the ice, the submerged depth d of the ice is found by equating the weight of the ice and the buoyant force. Denoting the pure water specific weight by γ and the ice area by A gives

$$F_B = W_{\text{ice}}$$



or

$$W_{\text{ice}} = \gamma S A d\ .$$

The ice sinks an additional depth d' with the bear in the center of the ice. Equating the new buoyant force to the weight of the ice plus bear gives

$$F_B = W_{\text{ice}} + W_{\text{bear}}\ ,$$

$$\gamma S A\left(d + d'\right) = \gamma S A d + W_{\text{bear}}\ ,$$

or

$$A = \frac{W_{\text{bear}}}{\gamma S d'} = \frac{500\,\text{lb}}{\left(62.4\,\dfrac{\text{lb}}{\text{ft}^3}\right)(1.03)\left(\dfrac{1}{12}\,\text{ft}\right)} \quad \text{or} \quad \boxed{A = 93.4\,\text{ft}^2}$$

Problem 2.135

A spherical balloon filled with helium at $40\,°\mathrm{F}$ and $20\ \mathrm{psia}$ has a $25-\mathrm{ft}$ diameter. What load can it support in atmospheric air at $40\,°\mathrm{F}$ and $14.696\ \mathrm{psia}$? Neglect the balloon's weight.

Solution 2.135

For static equilibrium, the buoyant force must equal the load. Neglecting the weight of the balloon and assuming air and helium to be ideal gases, the load is

$$L = F_B = \left(\gamma_{air} - \gamma_{He}\right)\cancel{V} = \left(\rho_{air} - \rho_{He}\right) g \cancel{V}$$
$$= \left[\left(\frac{p}{R}\right)_{air} - \left(\frac{p}{R}\right)_{He}\right]\left(\frac{g}{T}\right)\left(\frac{4}{3}\pi R^3\right)$$

Using Table A.4, the numerical values give

$$L = \left[\frac{\left(14.696\times 144\right)\dfrac{\mathrm{lb}}{\mathrm{ft}^2}}{\left(53.35\dfrac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{lbm}\cdot{}^{\mathrm{o}}\mathrm{R}}\right)} - \frac{\left(20\times 144\right)\dfrac{\mathrm{lb}}{\mathrm{ft}^2}}{\left(386\dfrac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{lbm}\cdot{}^{\mathrm{o}}\mathrm{R}}\right)}\right]\frac{\left(32.2\dfrac{\mathrm{ft}}{\mathrm{sec}^2}\right)\left(\dfrac{4\pi}{3}\right)\left(12.5\,\mathrm{ft}\right)^3}{\left(500\,{}^{\mathrm{o}}\mathrm{R}\right)\left(\dfrac{32.2\,\mathrm{ft}\cdot\mathrm{lbm}}{\mathrm{lb}\cdot\mathrm{sec}^2}\right)}$$

or

$$\boxed{L = 527\,\mathrm{lb}}$$

Problem 2.136

A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded, its draft (depth of submergence) is 5 ft and with the load of grain the draft is 7 ft. Determine: (**a**) the unloaded weight of the barge, and (**b**) the weight of the grain.

Solution 2.136

(a) For equilibrium,

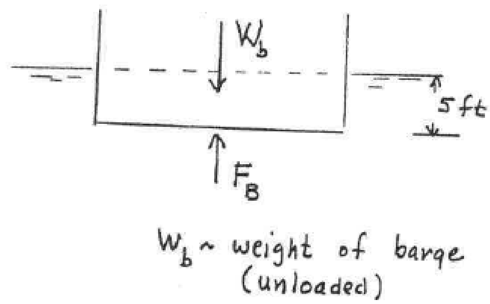
$$\sum F_{vertical} = 0$$

So that

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= (62.4 \frac{\text{lb}}{\text{ft}^3})(5\,\text{ft} \times 28\,\text{ft} \times 90\,\text{ft})$$

$$= \underline{\underline{786000\,\text{lb}}}$$

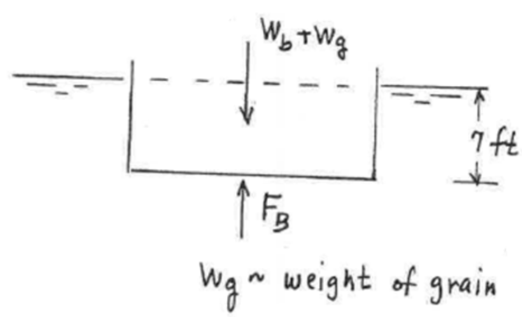


(b) $\sum F_{vertical} = 0$

$$W_B + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = (62.4 \frac{\text{lb}}{\text{ft}^3})(7\,\text{ft} \times 28\,\text{ft} \times 90\,\text{ft}) - 786,000\,\text{lb}$$

$$= \underline{\underline{315000\,\text{lb}}}$$



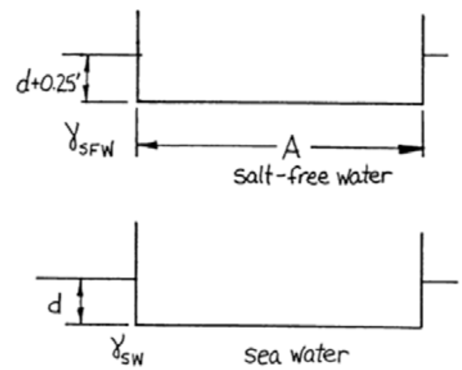
Problem 2.137

A barge is 40 ft wide by 120 ft long. The weight of the barge and its cargo is denoted by W. When in salt-free riverwater, it floats 0.25 ft deeper than when in seawater $\left(\gamma = 64 \text{ lb/ft}^3\right)$. Find the weight W.

Solution 2.137

In both cases, the weight W must equal the weight of the displaced water or

$$W = \gamma_{SFW} A\left(d + 0.25\,\text{ft}\right)$$
$$= \gamma_{SW} Ad$$



Soling for d gives

$$\gamma_{SW} Ad = \gamma_{SFW} A\left(d + 0.25\,\text{ft}\right)$$

or

$$d = \frac{\left(0.25\,\text{ft}\right)\gamma_{SFW}}{\gamma_{SW} - \gamma_{SFW}} = \frac{\left(0.25\,\text{ft}\right)\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)}{\left(64.0 - 62.4\right)\dfrac{\text{lb}}{\text{ft}^3}} = 9.75\,\text{ft}\,.$$

Then

$$W = \gamma_{SW} Ad = \left(64.0\frac{\text{lb}}{\text{ft}^3}\right)\left(40 \times 120\right)\text{ft}^2\left(9.75\,\text{ft}\right)$$

$$W = 3.00 \times 10^6\,\text{lb}\left(\frac{\text{short ton}}{2000\,\text{lb}}\right),$$

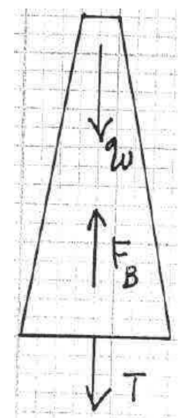
or

$$\boxed{W = 1500\text{ short tons.}}$$

Problem 2.138

When the Tucurui Dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

Solution 2.138



W □ weight, F_B □ buoyant force, T □ tension in ropes

For equilibrium,

$$\sum F_{vertical} = 0$$

$$T = F_B - W$$

For a truncated cone, $\text{Volume} = \dfrac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$

where: r_1 = base radius

r_2 = top radius

h = height

Thus,

$$\cancel{V}_{tree} = \frac{(\pi)(100\,\text{ft})}{3}\left[(4\,\text{ft})^2 + (4\,\text{ft} \times 1\,\text{ft}) + (1\,\text{ft})^2\right] = 2200\,\text{ft}^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \cancel{V}_{tree} = (62.4\frac{\text{lb}}{\text{ft}^3})\left(2200\,\text{ft}^3\right) = 137000\,\text{lb}$$

For weight,

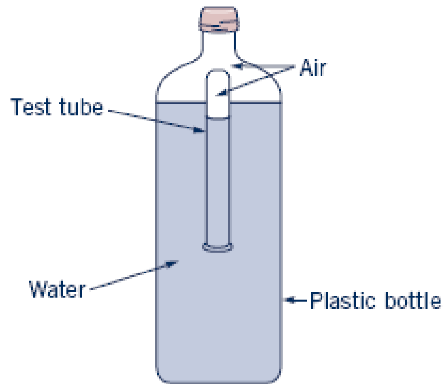
$$W = \gamma_{tree} \times \cancel{V}_{tree} = (0.6)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(2200\,\text{ft}^3\right) = 82400\,\text{lb}$$

Therfore,

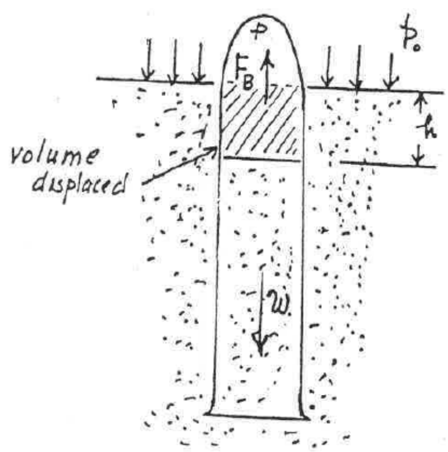
$$T = 137000\,\text{lb} - 82400\,\text{lb} = \underline{\underline{54600\,\text{lb}}}$$

Problem 2.140

An inverted test tube partially filled with air floats in a plastic water-filled soft drink bottle as shown in the figure below. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.



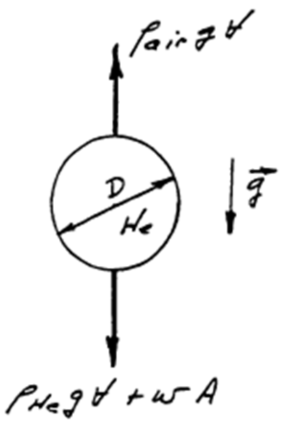
Solution 2.140



Where the test tube is floating the weight of the tube, W , is balanced by the buoyant force, F_B , as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p , trapped in the tube where $p = p_o + \gamma_{H_2O}h.$ When the bottle is squeezed, the air pressure in the bottle, p_o , is increased slightly and this in turn increases p , the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B . Since W is constant, a decrease in F_B will cause the test tube to sink.

Problem 2.141
A child’s balloon is a sphere 1 ft. in diameter. The balloon is filled with helium ($\rho = 0.014\ \text{lbm/ft}^3$). The balloon material weighs $0.008\ \text{lbf/ft}^2$ of surface area. If the child releases the balloon, how high will it rise in the Standard Atmosphere. (Neglect expansion of the balloon as it rises.)

Solution 2.141



w = wt/surface area
A = surface area

A force balance in the vertical direction for the balloon gives

$$+\uparrow\sum F_z = 0 = \rho_{air} g \mathcal{V} - \rho_{He} g \mathcal{V} - wA$$

for the balloon at rest at its highest elevation. Then

$$\rho_{air} = \frac{\rho_{He} g \mathcal{V} + wA}{g \mathcal{V}}$$

$$= \rho_{He} + \frac{wA}{g \mathcal{V}} = \rho_{He} + \frac{w\left(\pi D^2\right)}{g\left(\dfrac{\pi D^3}{6}\right)}$$

$$= \rho_{He} + \frac{6w}{gD} = 0.014\frac{\text{lbm}}{\text{ft}^3} + \frac{6\left(0.008\dfrac{\text{lb}}{\text{ft}^2}\right)\left(32.2\dfrac{\text{ft}\cdot\text{lbm}}{\text{lb}\cdot\text{sec}^2}\right)}{\left(32.2\dfrac{\text{ft}}{\text{sec}^2}\right)(1.0\,\text{ft})}$$

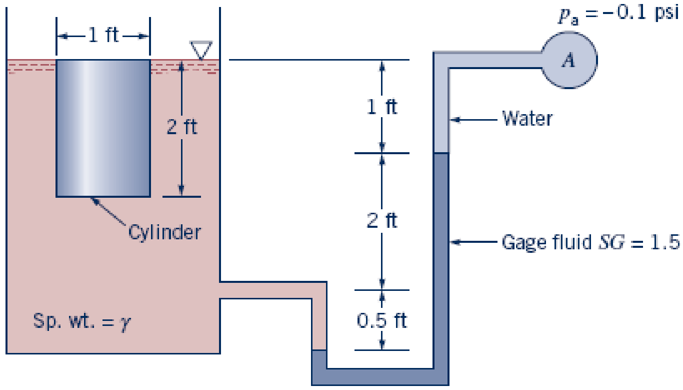
$$= 0.062\frac{\text{lbm}}{\text{ft}^3}.$$

Interpolating Table A.2 for the Standard Atmosphere,

$$z = \text{elevation} = 5000\,\text{ft} + 5000\,\text{ft}\left(\frac{0.06590 - 0.062}{0.06590 - 0.05648}\right) \rightarrow \boxed{z = 7{,}070\,\text{ft}}$$

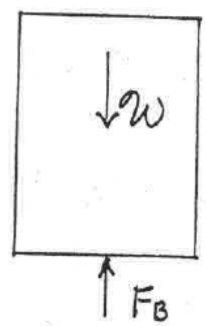
Problem 2.142

A 1-ft-diameter, 2-ft-long cylinder floats in an open tank containing a liquid having a specific weight γ. A U-tube manometer is connected to the tank as shown in the figure below. When the pressure in pipe A is 0.1 psi below atmospheric pressure, the various fluid levels are as shown. Determine the weight of the cylinder. Note that the top of the cylinder is flush with the fluid surface.



Solution 2.142

From a free-body-diagram of the cylinder



$$\sum F_{vertical} = 0$$

So that

$$W = F_B = \gamma\left(\frac{\pi}{4}\right)(1\,\text{ft})^2(2\,\text{ft}) \qquad (1)$$

$$= \frac{\pi\gamma}{2}$$

A manometer equation gives,

$$\gamma(3.5\,\text{ft}) - (SG)(\gamma_{H_2O})(2.5\,\text{ft}) - \gamma_{H_2O}(1\,\text{ft}) = p_A$$

$$\gamma(3.5\,\text{ft}) - (1.5)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(2.5\,\text{ft}) - \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft}) = \left(-0.1\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)$$

$$\gamma = 80.6\frac{\text{lb}}{\text{ft}^3}$$

Therfore, $W = \left(\frac{\pi}{2}\text{ft}^3\right)\left(80.6\frac{\text{lb}}{\text{ft}^3}\right) = \underline{\underline{127\,\text{lb}}}$

Problem 2.143

A not-too-honest citizen is thinking of making bogus gold bars by first making a hollow iridium $(S = 22.5)$ ingot and plating it with a thin layer of gold $(S = 19.3)$ of negligible weight and volume. The bogus bar is to have a mass of $100\,\text{lbm}$. What must be the volumes of the bogus bar and of the air space inside the iridium so that an inspector would conclude it was real gold after weighing it in air and water to determine its density? Could lead $(S = 11.35)$ or platinum $(S = 21.45)$ be used instead of iridium? Would either be a good idea?

Solution 2.143

$S_x = 22.5\,(\text{iridium});\qquad S_G = 19.3\,(\text{gold});\qquad \cancel{V}_{BB} = \cancel{V}_x + \cancel{V}_{AS};\qquad m_{BB} = m_x = 100\,\text{lbm}$

Neglect the weight of air in the air space and the buoyant force of air on the bar.
The volume of a pure gold bar would be

$$\cancel{V}_{GB} = \frac{W_{GB}}{\gamma_G}\ .$$

The bogus bar must have the same volume and weight as the pure gold bar so it will weigh like a solid gold bar in water. The volume condition gives

$$\cancel{V}_{GB} = \cancel{V}_{BB} = \cancel{V}_{AS} + V_x\,.$$

Since $W_{GB} = W_x$,

$$\cancel{V}_{AS} + \cancel{V}x = \cancel{V}_{GB} = \frac{W_{GB}}{\gamma_G} = \frac{W_x}{\gamma_G},\quad \cancel{V}_{AS} + \cancel{V}_x = \frac{\gamma_x \cancel{V}_x}{\gamma_G},$$

$$\cancel{V}_{AS} = \cancel{V}_x\left(\frac{\gamma_x}{\gamma_G} - 1\right) = \cancel{V}_x\left(\frac{S_x}{S_G} - 1\right).$$

The numerical value of the iridium volume is

$$\cancel{V}_x = \frac{W_x}{\gamma_x} = \frac{W_{GB}}{\gamma_x} = \frac{100\,\text{lb}}{\left(22.5\times 62.4\,\dfrac{\text{lb}}{\text{ft}^3}\right)} = 0.0712\,\text{ft}^3\,.$$

The air space volume is $\cancel{V}_{AS} = 0.0712\,\text{ft}^3\left(\dfrac{22.5}{19.3} - 1\right)$ or $\boxed{\cancel{V}_{AS} = 0.0118\,\text{ft}^3.}$

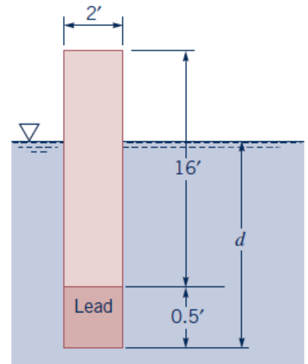
The bogus bar volume is $\cancel{V}_{BB} = \cancel{V}_{AS} + \cancel{V}_x = (0.0118 + 0.0712)\,\text{ft}^3$ or $\boxed{\cancel{V}_{BB} = 0.0830\,\text{ft}^3.}$

Lead will not work because it is less dense than gold.

Platinum could work because it is more dense than gold.
However, platinum is more expensive per unit weight than gold, so it would be a foolish choice.

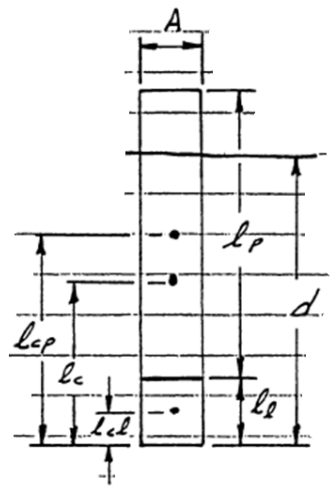
Problem 2.144

A solid cylindrical pine $(S = 0.50)$ spar buoy has a cylindrical lead $(S = 11.3)$ weight attached, as shown in the figure below. Determine the equilibrium position of the spar buoy in seawater (i.e., find d). Is this spar buoy stable or unstable? For seawater, $S = 1.03$.



Solution 2.144

The equilibrium position is found by equating the buoyant force and the body weight.



$$F_B = W$$

$$\gamma_{sw} dA = \gamma_\ell \ell_\ell A + \gamma_p \ell_p A$$

$$d = \frac{\gamma_\ell \ell_\ell + \gamma_p \ell_p}{\gamma_{sw}} = \frac{S_\ell \ell_\ell + S_p \ell_p}{S_{sw}}$$

$$= \frac{11.3(0.5\,\text{ft}) + 0.50(16\,\text{ft})}{1.03} \quad \rightarrow \quad \boxed{d = 13.3\,\text{ft}}$$

Since $d < 13.8\,\text{ft}$ (the total length of the spar buoy), the spar buoy floats. We now have to check the stability of the buoy.

$$I = \frac{\pi}{4}(\text{radius})^4 = \frac{\pi}{4}(1\,\text{ft})^4 = 0.7854\,\text{ft}^4,$$

ℓ_c = distance from bottom of buoy to center of gravity of buoy

$$\ell_c = \frac{\ell_{c\ell} W_\ell + \ell_{cp} W_{cp}}{W_\ell + W_p} = \frac{\left(\frac{\ell_\ell}{2}\right)(\gamma_\ell A \ell_\ell) + \left(\ell_\ell + \frac{\ell_p}{2}\right)(\gamma_p A \ell_p)}{\gamma_\ell A \ell_\ell + \gamma_p A \ell_p}$$

$$= \frac{S_\ell \ell_\ell \left(\frac{\ell_\ell}{2}\right) + S_p \ell_p \left(\ell_\ell + \frac{\ell_p}{2}\right)}{S_\ell \ell_\ell + S_p \ell_p} = \frac{11.3(0.5)(0.25) + 0.5(16)(0.5 + 8)}{11.3(0.5) + 0.5(16)}\text{ft}$$

$$\ell_c = 5.09\text{ ft}$$

$$\frac{d}{2} = \frac{13.3\,\text{ft}}{2} = 6.65\,\text{ft}, \quad n = \ell_c - \frac{d}{2} = 5.09\,\text{ft} - 6.65\,\text{ft} = -1.56\,\text{ft},$$

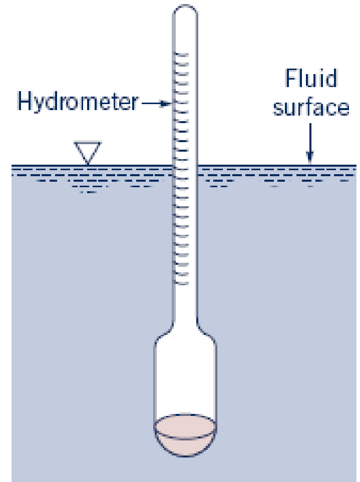
$$\forall_s = Ad = \pi(1\,\text{ft})^2(13.3\,\text{ft}) = 41.8\,\text{ft}^3,$$

$$m = \frac{I}{\forall_s} - n = \frac{0.7854\,\text{ft}^4}{41.8\,\text{ft}^3} - (-1.56\,\text{ft}) = 1.58\,\text{ft}$$

$$m > 0 \quad \rightarrow \quad \boxed{\text{buoy is stable}}$$

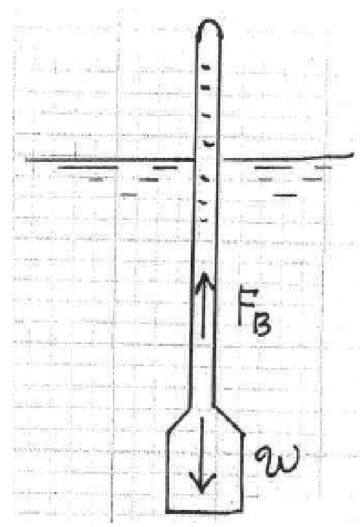
Problem 2.145

When a hydrometer (see the figure below) having a stem diameter of $0.30\,\text{in.}$ is placed in water, the stem protrudes $3.15\,\text{in.}$ above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs $0.042\,\text{lb}$.



Solution 2.145

When the hydrometer is floating its weight, W, is balanced by the buoyant force, F_B.,



$$\sum F_{vertical} = F_{\text{B}} - W = 0$$

$$\left(\gamma_{H_2O}\right)\forall_1 = W = (SG)\left(\gamma_{H_2O}\right)\forall_2$$

$$\forall_2 = \frac{\forall_1}{SG}$$

For water,

$$\forall_1 = \frac{W}{\gamma_{H_2O}} = \frac{0.042\,\text{lb}}{62.4\,\frac{\text{lb}}{\text{ft}^3}} = 6.73\times10^{-4}\,\text{ft}^3$$

For other liquid,

$$\forall_2 = \frac{6.73\times10^{-4}\,\text{ft}^3}{1.10} = 6.12\times10^{-4}\,\text{ft}^3$$

Therefore,

$$\forall_1 - \forall_2 = (6.73-6.12)\times10^{-4}\,\text{ft}^3 = 0.61\times10^{-4}\,\text{ft}^3$$

The change in submergence depth occurs with only the stem protruding from the surface.

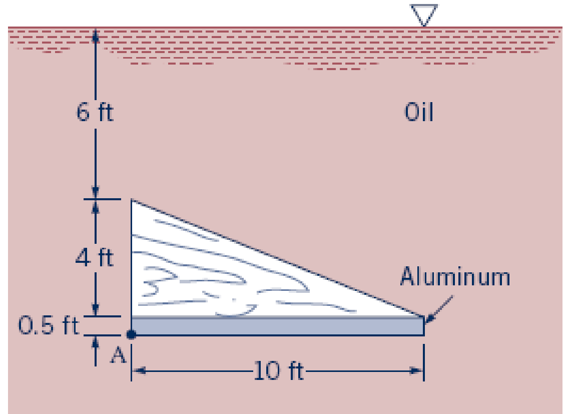
$$\left(\frac{\pi}{4}\right)(0.30\,\text{in.})^2\,\Delta\ell = \left(0.61\times10^{-4}\,\text{ft}^3\right)\left(1728\,\frac{\text{in.}^3}{\text{ft}^3}\right)$$

$$\Delta\ell = 1.49\,\text{in.}$$

With the new liquid the stem would protrude $3.15\,\text{in.} + 1.49\,\text{in.} = \boxed{4.64\,\text{in. above surface}}$

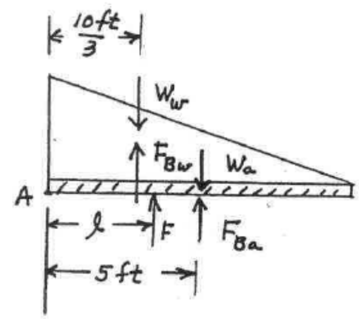
Problem 2.146

A 2-ft-thick block constructed of wood $(SG = 0.6)$ is submerged in oil $(SG = 0.8)$ and has a 2-ft-thick aluminum (specific weight $= 168\,\text{lb}/\text{ft}^3$) plate attached to the bottom as indicated in the figure below. Determine completely the force required to hold the block in the position shown. Locate the force with respect to point *A*.



Solution 2.146

Equilibrium: $\sum F_{vertical} = F - W_w + F_{Bw} - W_a + F_{Ba} = 0$



w ~ wood
a ~ aluminum
F ~ force to hold block

$$W_w = (SG_w)(\gamma_{H_2O})\cancel{V_w}$$

$$= (0.6)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(10\,\text{ft}\times 4\,\text{ft}\times 2\,\text{ft}) = 1500\,\text{lb}$$

$$W_a = \left(168\frac{\text{lb}}{\text{ft}^3}\right)(0.5\,\text{ft}\times 10\,\text{ft}\times 2\,\text{ft}) = 1680\,\text{lb}$$

$$F_{Bw} = (SG_{oil})(\gamma_{H_2O})\cancel{V_w}$$

$$= (0.8)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(10\,\text{ft}\times 4\,\text{ft}\times 2\,\text{ft}) = 2000\,\text{lb}$$

$$F_{Ba} = (SG_{oil})(\gamma_{H_2O})\cancel{V_a} = (0.8)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(0.5\,\text{ft}\times 10\,\text{ft}\times 2\,\text{ft}) = 499\,\text{lb}$$

Therefore, $F = 1500\text{ lb} - 2000\text{ lb} + 1680\text{ lb} - 499\text{ lb} \quad \rightarrow \quad \boxed{F = 681\text{ lb upward}}$

Equilibrium:

$$\sum M_a = 0 \quad \rightarrow \quad \ell F = \left(\frac{10}{3}\text{ft}\right)(W_w - F_{Bw}) + (5\,\text{ft})(W_a - F_{Ba})$$

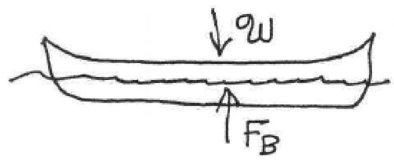
$$\ell(681\,\text{lb}) = \left(\frac{10}{3}\text{ft}\right)(1500\,\text{lb} - 2000\,\text{lb}) + (5\,\text{ft})(1680\,\text{lb} - 499\,\text{lb}) = 6.22\text{ ft}$$

$$\boxed{F \text{ acts 6.22 ft to the right of point A}}$$

Problem 2.147

How much extra water does a $147-\text{lb}$ concrete canoe displace compared to an ultralightweight $38-\text{lb}$ Kevlar canoe of the same size carrying the same load?

Solution 2.147



For equilibrium,

$$\sum F_{vertical} = 0$$

and

$W = F_B = \gamma_{H_2O} \cancel{V}$ and $\cancel{V}$ is displaced volume.

For concrete canoe,

$$147\,\text{lb} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\cancel{V}_c$$

$$\cancel{V}_c = 2.36\,\text{ft}^3$$

For Kevlar canoe,

$$38\,\text{lb} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\cancel{V}_k$$

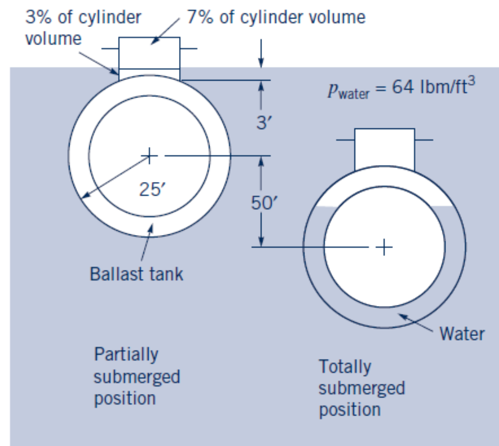
$$\cancel{V}_k = 0.609\,\text{ft}^3$$

$$\text{Extra water displacement} = 2.36\,\text{ft}^3 - 0.609\,\text{ft}^3$$

$$= \underline{\underline{1.75\,\text{ft}^3}}$$

Problem 2 .148

A submarine is modeled as a cylinder with a length of 300 ft , a diameter of 50 ft , and a conning tower as shown in the figure below. The submarine can dive a distance of 50 ft from the floating position in about 30 sec . Diving is accomplished by taking water into the ballast tank so the submarine will sink. When the submarine reaches the desired depth, some of the water in the ballast tank is discharged leaving the submarine in “neutral buoyancy” (i.e., it will neither rise nor sink). For the conditions illustrated, find (**a**) the weight of the submarine and (**b**) the volume (or mass) of the water that must be in the ballast tank when the submarine is in neutral buoyancy. For seawater, $S = 1.03$.



Solution 2.148

(a) Denoting the cylinder radius by R, the submarine weight is equal to the buoyant force so

$$W = F_B = \gamma \cancel{V}_{\text{submerged}}$$
$$= \gamma\left(\pi R^2 \ell\right)(1.03)$$

when the submarine is in the partially submerged position. The numerical values give

$$W = \left(64\frac{\text{lb}}{\text{ft}^3}\right)\pi\left(25\,\text{ft}\right)^2\left(300\,\text{ft}\right)(1.03) \quad \text{or} \quad \boxed{W = 3.88\times 10^7\ \text{lb}}$$

(b) For neutral buoyancy at the lower depth, the submarine weight W plus the ballast weight W_B must equal the buoyant force so

$$W + W_B = F_B = \gamma\left(\pi R^2 \ell\right)(1.10)$$

or

$$W_B = \gamma\left(\pi R^2 \ell\right)(1.10) - W\,.$$

The ballast volume $\cancel{V}_B = \dfrac{W_B}{\gamma}$ so

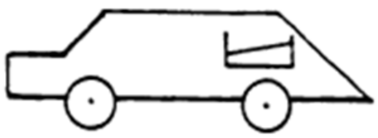
$$\cancel{V}_B = \left(\pi R^2 \ell\right)(1.10) - \frac{W}{\gamma} = \pi\left(25\,\text{ft}\right)^2\left(300\,\text{ft}\right)(1.10) - \frac{3.88\times 10^7\ \text{lb}}{\left(64\dfrac{\text{lb}}{\text{ft}^3}\right)}$$

$$\boxed{\cancel{V}_B = 41700\,\text{ft}^3}$$

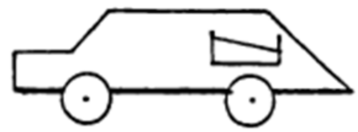
Problem 2.150

When an automobile brakes, the fuel gage indicates a fuller tank than when the automobile is traveling at a constant speed on a level road. Is the sensor for the fuel gage located near the front or rear of the fuel tank? Assume a constant deceleration.

Solution 2.150



accelerating
automobile



decelerating
(braking)
automobile

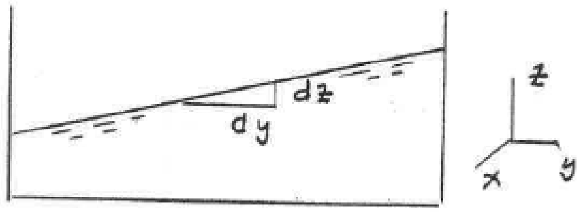
so

sensor located
in front of
fuel tank.

Problem 2.151

An open container of oil rests on the flatbed of a truck that is traveling along a horizontal road at 55 mi/hr. As the truck slows uniformly to a complete stop in 5 s, what will be the slope of the oil surface during the period of constant deceleration?

Solution 2.151



$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

$$a_y = \frac{\text{final velocity - initial velocity}}{\text{time interval}}$$

$$= \frac{0 - (55\,\text{mph})\left(0.4470\,\frac{\frac{\text{m}}{\text{s}}}{\text{mph}}\right)}{5\,\text{s}} = -4.92\,\frac{\text{m}}{\text{s}^2}$$

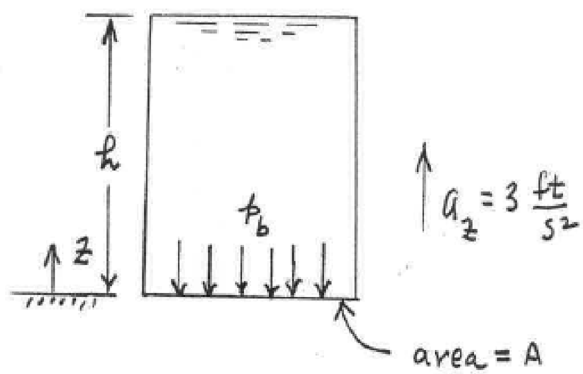
Thus,

$$\frac{dz}{dy} = -\frac{\left(-4.92\,\frac{\text{m}}{\text{s}^2}\right)}{9.81\,\frac{\text{m}}{\text{s}^2} + 0} = \underline{\underline{0.502}}$$

Problem 2.152

A 5-gal, cylindrical open container with a bottom area of 120 in.^2 is filled with glycerin and rests on the floor of an elevator. **(a)** Determine the fluid pressure at the bottom of the container when the elevator has an upward acceleration of 3 ft/s^2. **(b)** What resultant force does the container exert on the floor of the elevator during this acceleration? The weight of the container is negligible. (Note: $1 \text{ gal} = 231 \text{ in.}^3$)

Solution 2.152



$hA = \text{volume}$

$$h\left(120 \text{ in.}^2\right) = (5 \text{ gal})\left(\frac{231 \text{ in.}^3}{\text{gal}}\right)$$

$h = 9.63 \text{ in.}$

(a) $$\frac{\partial p}{\partial z} = -\rho\left(g + a_z\right)$$

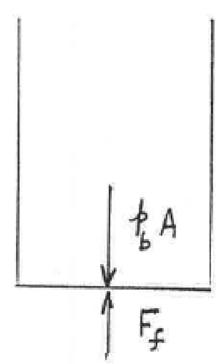
Thus,

$$\int_0^{p_b} dp = -\rho\left(g + a_z\right)\int_h^0 dz$$

and

$$\begin{aligned} p_b &= \rho\left(g + a_z\right)h \\ &= \left(2.44 \frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{s}^2} + 3 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{9.63}{12} \text{ ft}\right) \\ &= \underline{\underline{68.9 \frac{\text{lb}}{\text{ft}^2}}} \end{aligned}$$

(b) From free-body-diagram of container,



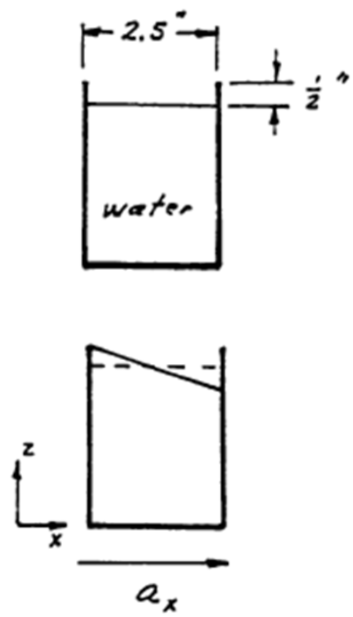
$$\begin{aligned} F_f &= p_b A \\ &= \left(68.9 \frac{\text{lb}}{\text{ft}^2}\right)\left(120 \text{ in.}^2\right)\left(\frac{1 \text{ ft}^2}{144 \text{ in.}^2}\right) \\ &= 57.4 \text{ lb} \end{aligned}$$

Thus, force of container on floor is $\underline{\underline{57.4 \text{ lb downward}}}$.

Problem 2.153

A plastic glass has a square cross section measuring $2\frac{1}{2}$ in. on a side and is filled to within $\frac{1}{2}$ in. of the top with water. The glass is placed in a level spot in a car with two opposite sides parallel to the direction of travel. How fast can the driver of the car accelerate along a level road without spilling any of the water?

Solution 2.153



Slope of water surface

$$= -\frac{a_{car}}{g}$$

or

$$a_{car} = -g(\text{slope})$$

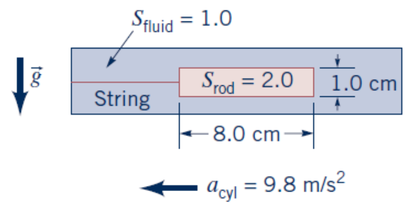
$$= -\left(32.2\,\frac{\text{ft}}{\text{sec}^2}\right)\left(-\frac{1.0\,\text{in.}}{2.5\,\text{in.}}\right)$$

or

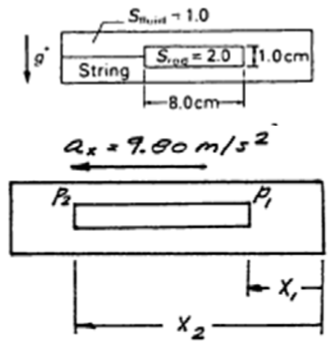
$$\boxed{a_{car} = 12.9\,\frac{\text{ft}}{\text{sec}^2}}$$

Problem 2.154

The cylinder in the figure below accelerates to the left at the rate of $9.80\,\mathrm{m/s^2}$. Find the tension in the string connecting at rod of circular cross section to the cylinder. The volume between the rod and the cylinder is completely filled with water at $10\,°\mathrm{C}$.



Solution 2.154



First find the pressure difference in the water over a length $\ell = 8.0\,\mathrm{cm}$. Since gravity is perpendicular to the rod, Eq.(2.41) gives

$$dp = -\rho a_x d_x$$

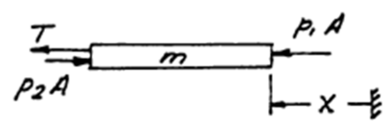
For the x-direction. Integrating gives

$$p_2 - p_1 = -\rho a_x\left(x_2 - x_1\right).$$

For $10\,°\mathrm{C}$ water, Table A.5 gives

$$p_2 - p_1 = -\left(1000\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(9.80\frac{\mathrm{m}}{\mathrm{s^2}}\right)(8.0\,\mathrm{cm})\left(\frac{\mathrm{m}}{100\,\mathrm{cm}}\right) = -784\frac{\mathrm{N}}{\mathrm{m^2}}$$

We next apply Newton's second law to the rod



$$\overset{\leftarrow}{+}\sum F_x = ma_x,$$

$$T + \left(p_1 - p_2\right)A = \mathrm{ma_x},$$

$$T = \left(p_2 - p_1\right)A + \mathrm{ma_x}.$$

Using the specified information,

$$m = \rho_w S_{\text{rod}}\ell A = \rho_w S_{\text{rod}}\left(\frac{\pi D^2}{4}\right)\ell$$

$$= \left(1000\frac{\mathrm{kg}}{\mathrm{m^3}}\right)(2.0)\left(\frac{\pi}{4}\right)(1.0\,\mathrm{cm})^2(8.0\,\mathrm{cm})\left(\frac{1\,\mathrm{m}}{100\ \mathrm{cm}}\right)^3 = 0.0126\,\mathrm{kg}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(1.0\,\mathrm{cm})^2\left(\frac{m}{100\,\mathrm{cm}}\right)^2 = 7.854\times 10^{-5}\,\mathrm{m^2}.$$

Therefore,

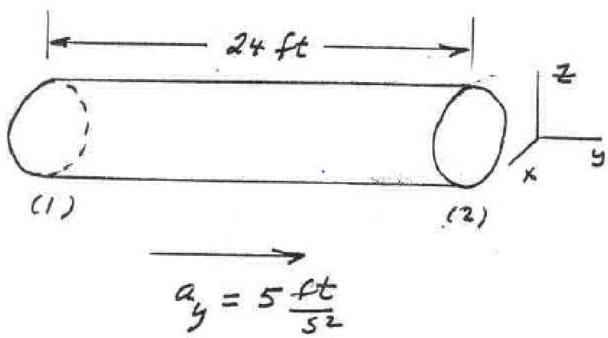
$$T = \left(-784\frac{\mathrm{N}}{\mathrm{m^2}}\right)\left(7.854\times 10^{-5}\,\mathrm{m^2}\right) + (0.0126\,\mathrm{kg})\left(9.80\frac{\mathrm{m}}{\mathrm{s^2}}\right)$$

$$\boxed{T = 0.062\,\mathrm{N}}$$

Problem 2.155

A closed cylindrical tank that is $8\,\text{ft}$ in diameter and $24\,\text{ft}$ long is completely filled with gasoline. The tank, with its long axis horizontal, is pulled by a truck along a horizontal surface. Determine the pressure difference between the ends (along the long axis of the tank) when the truck undergoes an acceleration of $5\,\text{ft}/\text{s}^2$.

Solution 2.155



$$\frac{\partial p}{\partial y} = -\rho a_y$$

Thus,

$$\int_{p_1}^{p_2} dp = -\rho a_y \int_0^{24} dy$$

Where $p = p_1$ at $y = 0$ and $p = p_2$ at $y = 24\,\text{ft}$,

and

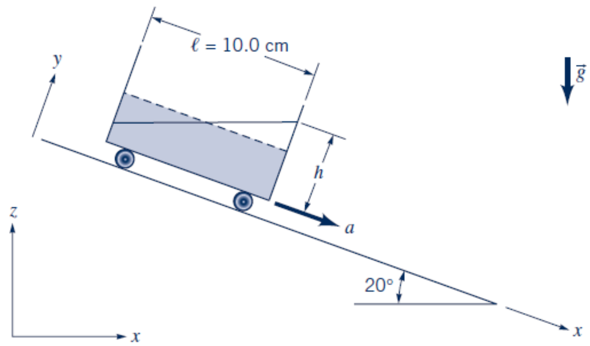
$$\begin{aligned} p_2 - p_1 &= -\rho a_y (24\,\text{ft}) \\ &= -\left(1.32 \frac{\text{slugs}}{\text{ft}^3}\right)\left(5 \frac{\text{ft}}{\text{s}^2}\right)(24\,\text{ft}) \\ &= -158 \frac{\text{lb}}{\text{ft}^2} \end{aligned}$$

or

$$p_1 - p_2 = \underline{\underline{158 \frac{\text{lb}}{\text{ft}^2}}}$$

Problem 2.156

The cart shown in the figure below measures 10.0 cm long and 6.0 cm high and has rectangular cross sections. It is half-filled with water and accelerates down a 20° incline plane at $a = 1.0\ \text{m/s}^2$. Find the height h.



Solution 2.156

Unfortunately, there are 2 x-directions in the problem statement.
Noting that the gravisty vector is in the negative z-direction, change the label on the axis normal to the z-direction to be "n". Resolving the acceleration along the plane into n,z components:

$$a_z = -a\sin\theta\,,\ \ a_n = a\cos\theta\,, \theta = 20^{\circ}$$

For rigid-body motion of the fluid in the n,z coordiantes::

$$dp = -\rho a_n dn - \rho\left(g + a_z\right)dz$$

$$dp = -\rho a\cos\theta\, dn - \rho\left(g - a\sin\theta\right)dz = 0 \ \leftarrow \text{along free surface } p = p_{atm}$$

Using trignometirc relationships this equation can be converted into x,y coordinates.

$$dn = dx\cos\theta + dy\sin\theta$$

$$dz = dy\cos\theta - dx\sin\theta$$

$$-\rho a\cos\theta\, dn - \rho\left(g - a\sin\theta\right)dz = 0$$

$$-\rho a\cos\theta\left[dx\cos\theta + dy\sin\theta\right] - \rho\left(g - a\sin\theta\right)\left[dy\cos\theta - dx\sin\theta\right] = 0$$

$$-\left(\rho a\cos^2\theta\right)dx - \left(\rho a\cos\theta\sin\theta\right)dy - \left[\rho\left(g - a\sin\theta\right)\cos\theta\right]dy - \left[\rho\left(g - a\sin\theta\right)\left(-\sin\theta\right)\right]dx = 0$$

$$\left[-\rho a\cos^2\theta + \rho\left(g - a\sin\theta\right)\sin\theta\right]dx + \left[-\rho a\cos\theta\sin\theta - \rho\left(g - a\sin\theta\right)\cos\theta\right]dy = 0$$

$$\left[-\rho a\left(\cos^2\theta + \sin^2\theta\right) + \rho g\sin\theta\right]dx + \left[-\rho g\cos\theta\right]dy = 0$$

$$\left[-\rho a + \rho g\sin\theta\right]dx - \left[\rho g\cos\theta\right]dy = 0$$

$$\left[-a + g\sin\theta\right]dx - \left[g\cos\theta\right]dy = 0$$

Integration yields:

$$\left(-a + g\sin\theta\right)x - \left(g\cos\theta\right)y = -C$$

$$y = \left(-\frac{a}{g\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)x + C$$

The constant of integration can be determined by noting that the container is ½-full:

$$\forall_{\text{water}} = \int_0^{\ell} y\,dx$$
$$= \int_0^{\ell}\left[\left(-\frac{a}{g\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)x + C\right]dx$$
$$= \left(-\frac{a}{g\cos\theta}+\tan\theta\right)\frac{\ell^2}{2} + C\ell$$

$$C = \frac{\forall_{\text{water}}}{\ell} + \left(\frac{a}{g\cos\theta} - \tan\theta\right)\frac{\ell}{2}$$

$$C = \frac{(10.0\,\text{cm})(6.0\,\text{cm})/2}{(10.0\,\text{cm})} + \left(\frac{1\frac{\text{m}}{\text{s}^2}}{\left(9.81\frac{\text{m}}{\text{s}^2}\right)\cos 20^{\text{o}}} - \tan 20^{\text{o}}\right)\frac{(10.0\,\text{cm})}{2}$$
$$= 1.723\text{ cm}$$

Solving for the the requested length:

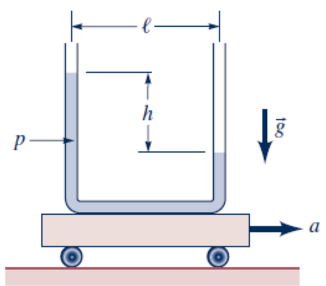
$$y = \left(-\frac{a}{g\cos\theta}+\tan\theta\right)x + C$$

$$h = \left(\frac{-1}{(9.81)\cos 20^{\text{o}}} + \tan 20^{\text{o}}\right)(10\text{ cm}) + 1.723\text{ cm} = 4.277\text{ cm}$$

$$\boxed{h = 4.28\text{ cm}}$$

Problem 2.157

The U-tube manometer in the figure below is used to measure the acceleration of the cart on which it sits. Develop an expression for the acceleration of the cart in terms of the liquid height h, the liquid density ρ, the local acceleration of gravity g, and the length ℓ.

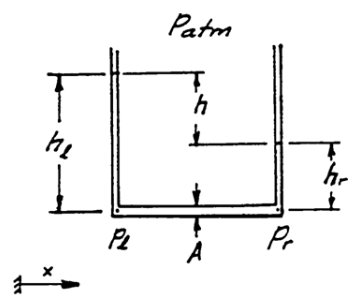


Solution 2.157

Writing Newton's second law in the horizontal direction (x-direction) for the bottom leg of the manometer gives

$$\sum F_x = \mathrm{ma}_\mathrm{x},$$

$$p_\ell A - p_r A = \rho \ell A a,$$



or

$$a = \frac{p_\ell - p_r}{\rho \ell}$$

Applying the manometer rule to the two legs of the manometer gives

$$p_\ell = p_{\mathrm{atm}} + \rho g h_\ell$$

and

$$p_r = p_{\mathrm{atm}} + \rho g h_\ell$$

Subtracting gives

$$p_\ell - p_r = \rho g \left(h_\ell - h_r\right) = \rho g h$$

so

$$a = \frac{\rho g h}{\rho \ell} \qquad \text{or} \qquad \boxed{a = g\left(\frac{h}{\ell}\right)}$$

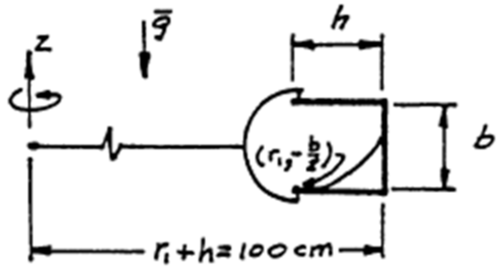
Problem 2.158

A tank has a height of 5.0 cm and a square cross section measuring 5.0 cm on a side. The tank is one third full of water and is rotated in a horizontal plane with the bottom of the tank 100 cm from the center of rotation and two opposite sides parallel to the ground. What is the maximum rotational speed that the tank of water can be rotated with no water coming out of the tank?

Solution 2.158

$$dp = -\rho g dz + \rho\omega^2 r\, dr$$

Since $dp = 0$ along the free surface, the free surface is identified by the equation



$$0 = -\rho g dz + \rho\omega^2 r\, dr$$

or

$$0 = -g dz + \omega^2 r\, dr$$

Integrating gives

$$0 = -g\int_{-\frac{b}{2}}^{z} dz + \omega^2 \int_{r_1}^{r} r\, dr\,,$$

$$0 = -g\left(z + \frac{b}{2}\right) + \frac{\omega^2}{2}\left(r^2 - r_1^2\right),$$

or

$$z = -\frac{b}{2} + \frac{\omega^2}{2g}\left(r^2 - r_1^2\right).$$

Recognizing that the volume of water in the rotating tank must equal $\dfrac{b^2 h}{6}$ gives

$$\frac{b^2 h}{6} = \int_{r_1}^{r_1+h} zb\, dr = b\int_{r_1}^{r_1+h}\left[-\frac{b}{2} + \frac{\omega^2}{2g}\left(r^2 - r_1^2\right)\right] dr\,,$$

$$\frac{b^2 h}{6} = b\left[-\frac{br}{2} + \frac{\omega^2}{2g}\left(\frac{r^3}{3} - r_1^2 r\right)\right]_{r_1}^{r_1+h},$$

$$\frac{b^2 h}{6} = b\left[-\frac{bh}{2} + \frac{\omega^2}{2g}\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right)\right],$$

$$\frac{2bh}{3} = \frac{\omega^2}{2g}\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right),$$

or

$$\omega = \sqrt{\frac{4bhg}{3\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right)}}.$$

The numerical values give

$$\omega = \sqrt{\frac{4(5\,\text{cm})(5\,\text{cm})\left(981\frac{\text{cm}}{\text{s}^2}\right)}{3\left(\frac{(100\,\text{cm})^3}{3} - \frac{(95\,\text{cm})^3}{3} - (95\,\text{cm})^2(5\,\text{cm})\right)}}$$

$$= \left(3.68\frac{\text{rad}}{\text{s}}\right)\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right) \quad \text{or} \quad \boxed{\omega = 35.1\,\text{rpm}}$$

<u>DISCUSSION</u> Note the that when $r = r_1 + h$,

$$z = -\frac{b}{2} + \frac{\omega^2}{2g}\left((r_1+h)^2 - r_1^2\right) = -\frac{b}{2} + \frac{\omega^2}{2g}\left(2r_1 h + h^2\right).$$

The numerical values give

$$z = \frac{5}{2}\text{cm} + \frac{\left(3.68\frac{\text{rad}}{\text{s}}\right)^2}{2\left(981\frac{\text{cm}}{\text{s}^2}\right)}\left[2(95\,\text{cm})(5\,\text{cm}) + (5\,\text{cm})^2\right]$$

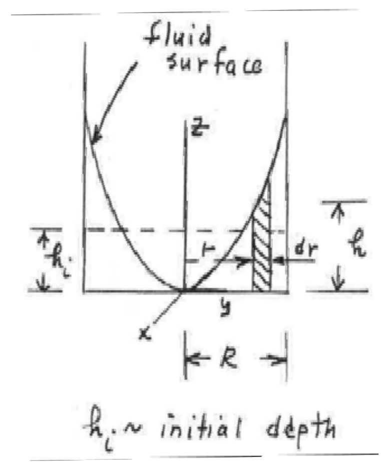
$$= 4.23\,\text{cm}$$

or the assumption indicated in the above figures that the water does not reach the uppermost side of the tank is correct.

Problem 2.159

An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.

Solution 2.159



Equation for surfaces of constant pressure:

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h = 0$ at $r = 0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\cancel{V}_f = \int_0^R 2\pi r h \, dr = \frac{2\pi\omega^2}{2g}\int_0^R r^3 \, dr = \frac{\pi\omega^2 R^4}{4g}$$

Since the initial volume, $\cancel{V}_i = \pi R^2 h_i$, must equal the final volume,

$$\cancel{V}_f = \cancel{V}_i$$

So that

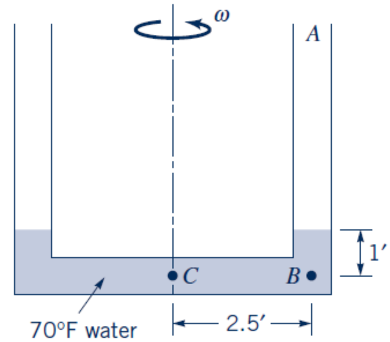
$$\frac{\pi\omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4gh_i}{R^2}} = \sqrt{\frac{4\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.7\text{ m})}{(0.5\text{ m})^2}} = \underline{\underline{10.5\frac{\text{rad}}{\text{s}}}}$$

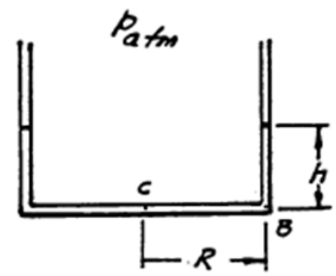
Problem 2.160

The U-tube in the figure below rotates at 2.0 rev/sec. Find the absolute pressures at points C and B if the atmospheric pressure is 14.696 psia . Recall that 70 °F water evaporates at an absolute pressure of 0.363 psia . Determine the absolute pressures at points C and B if the U-tube rotates at 2.0 rev/sec .



Solution 2.160

Applying the manometer rule to one of the legs and using the data in Table A.6,



$$p_B = p_{\text{atm}} + \rho g h$$

$$p_B = 14.696\,\text{psia} + \frac{\left(62.3\dfrac{\text{lb}}{\text{ft}^3}\right)(1\,\text{in.})}{\left(1728\dfrac{\text{in.}^3}{\text{ft}^3}\right)},$$

$$\boxed{p_B = 14.732\text{ psia}}$$

Section 2.6.2 gives: $\dfrac{\partial p}{\partial r} = \rho r \omega^2$.

Integrating from $r = 0$ to $r = R$ gives: $\int_{p_C}^{p_B} dp = \rho\omega^2 \int_o^R r\,dr$ or $p_B - p_C = \dfrac{\rho\omega^2 R^2}{2}$.

Therefore,

$$p_c = p_B - \frac{\rho\omega^2 R^2}{2} = 14.732\,\text{psia} - \frac{\left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)\left(2.0\dfrac{\text{rev}}{\text{sec}}\right)^2 (2.5\,\text{ft})^2}{2\left(\dfrac{144\,\text{in.}^2}{\text{ft}^2}\right)\left(\dfrac{\text{rev}}{2\pi\,\text{rad}}\right)^2\left(\dfrac{32.2\,\text{ft}\cdot\text{lbm}}{\text{lb}\cdot\text{sec}^2}\right)}$$

$$\boxed{p_c = 8.10\text{ psia}}$$

Check for phase change: $p_c > 0.363\,\text{psia}$ → no evaporation → above answer is correct.

<u>DISCUSSION</u> Note that if p_c were calculated to be less than $0.363\,\text{psia}$, some of the water would vaporize and p_c would be $0.363\,\text{psia}$.

Problem 2.161

A child riding in a car holds a string attached to a floating, helium-filled balloon. As the car decelerates to a stop, the balloon tilts backwards. As the car makes a right-hand turn, the balloon tilts to the right. On the other hand, the child tends to be forced forward as the car decelerates and to the left as the car makes a right-hand turn. Explain these observed effects on the balloon and child.

Solution 2.161

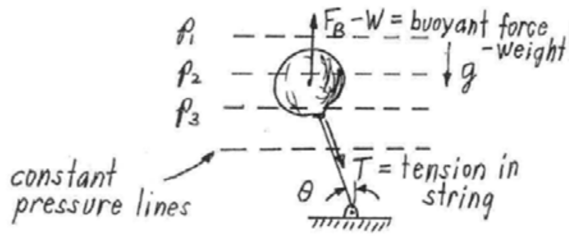


Fig.(1) No acceleration, $\theta = 0$ for equilibrium.

A floating balloon attached to a string will align itself so that the string it normal to lines of constant pressure. Thus, if the car is not accelerating, the lines of $p = \text{constant}$ are horizontal (gravity acts vertically down), and the balloon floats "straight up" (i.e. $\theta = 0$). If forced to the side ($\theta \neq 0$), the balloon will return to the vertical ($\theta = 0$ equilibrium position in which the two forces T and F_B-W line up.

Consider what happens when the car decelerates with an amount $a_y < 0$.

As show by the equation,

$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z},$$

the lines of constant pressure are not horizontal,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} = -\frac{a_y}{g} > 0 \qquad \text{since } a_z = 0 \text{ and}$$

$a_y < 0$.

Again, the balloon's equilibrium position is with the string normal to $p = \text{const.}$ lines. That is, the balloon tilts back as the car stops.

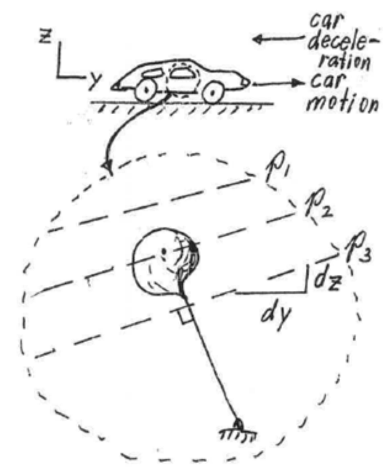


Fig.(2) Balloon aligned so that string is normal to p=constant lines

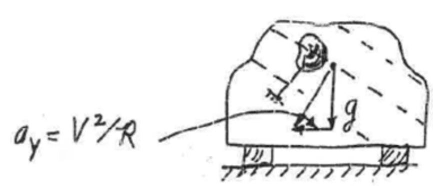


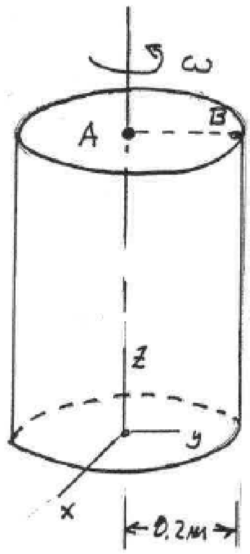
Fig.(3) Left turn; balloon tilts to right

When the car turns, $a_y = \dfrac{V^2}{R}$ (the centrifugal acceleration), the lines of $p = \text{const.}$ are as shown, and the balloon tilts to the outside of the curve.

Problem 2.162

A closed, 0.4-m-diameter cylindrical tank is completely filled with oil $(SG = 0.9)$ and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.

Solution 2.162



Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant}$$

Since $z_A = z_B$,

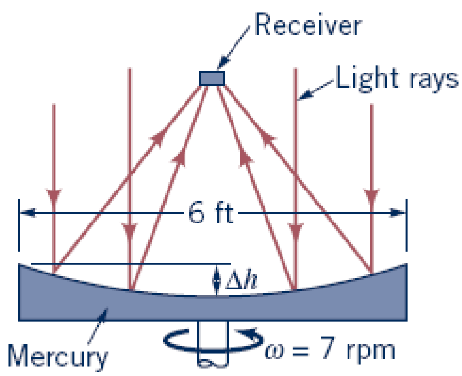
$$p_B - p_A = \frac{\rho\omega^2}{2}\left(r_B^2 - r_A^2\right)$$

$$= \frac{(0.9)\left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(40\,\frac{\text{rad}}{\text{s}}\right)^2}{2}\left[(0.2\,\text{m})^2 - 0\right]$$

$$= \underline{\underline{28.8\,\text{kPa}}}$$

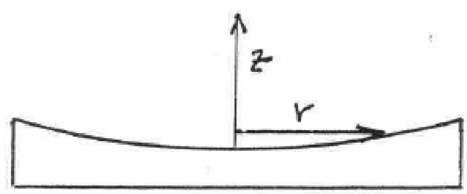
Problem 2.163

The largest liquid mirror telescope uses a 6-ft-diameter tank of mercury rotating at 7 rpm to produce its parabolic-shaped mirror as shown in the figure below. Determine the difference in elevation of the mercury, Δh ,between the edge and the center of the mirror.



Solution 2.163

For free surface of rotating liquid,



$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

Let $z = 0$ at $r = 0$ and therefore constant $= 0$.

Thus, $\Delta h = \Delta z$ for $r = 3\,\text{ft}$ and with

$$\omega = (7\,\text{rpm})\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)$$

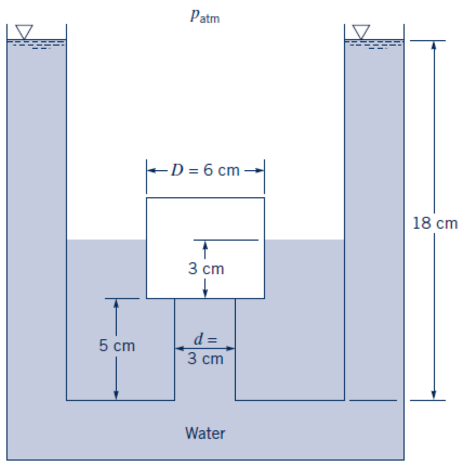
$$= 0.733\frac{\text{rad}}{\text{s}}$$

It follows that

$$\Delta h = \frac{\left(0.733\frac{\text{rad}}{\text{s}}\right)^2 (3\,\text{ft})^2}{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)} = \underline{\underline{0.0751\,\text{ft}}}$$

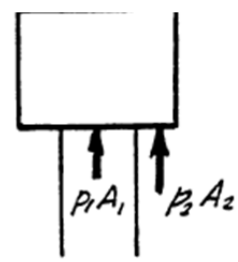
Problem 2.101

Find the total vertical force on the cylinder shown in the figure below.



Solution 2.101

The net force F on the cylinder is due to the water and is



$$F = F_1 + F_2 = p_1 A_1 + p_2 A_2 \,.$$

Since the atmospheric pressure does not contribute to the net force, p_1 and p_2 will be considered gage pressures.

$$p_1 = \rho_w g h = \frac{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)(18-5)\text{cm}}{\left(100\dfrac{\text{cm}}{\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)} = 1275\frac{\text{N}}{\text{m}^2}$$

$$p_2 = \rho_w g h = \frac{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)(3)\text{cm}}{\left(100\dfrac{\text{cm}}{\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)} = 294\frac{\text{N}}{\text{m}^2}$$

Then

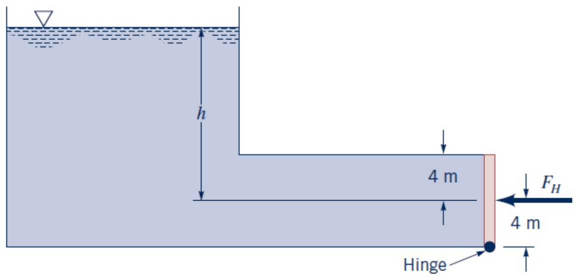
$$F = \left(1275\frac{\text{N}}{\text{m}^2}\right)\frac{\pi}{4}(3\,\text{cm})^2\left(\frac{\text{m}}{100\,\text{cm}}\right)^2 + \left(294\frac{\text{N}}{\text{m}^2}\right)\frac{\pi}{4}\left(6^2-3^2\right)\text{cm}^2\left(\frac{\text{m}}{100\,\text{cm}}\right)^2$$

or

$$\boxed{F = 1.52\,\text{N}}\,.$$

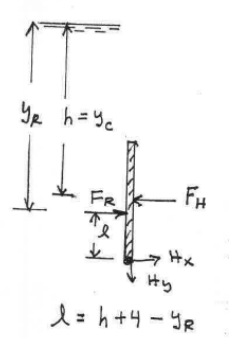
Problem 2.102

A 3-m -wide, 8-m -high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in the figure below. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is $3500\ \text{kN}$. (**a**) Determine the maximum water depth, *h*, above the center of the gate that can exist without the gate opening. (**b**) Is the answer the same if the gate is hinged at the top? Explain your answer.



Solution 2.102

For gate hinged at <u>bottom</u>



$$\sum M_H = 0$$

so that

$$(4\,\text{m})\text{F}_\text{H} = \ell \text{F}_\text{H} \qquad \text{(see figure)}\ \ (1)$$

and

$$F_R = \gamma h_c A = \left(9.80\frac{\text{kN}}{\text{m}^3}\right)(h)(3\,\text{m}\times 8\,\text{m}) = (9.80\times 24h)\,\text{kN}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3\,\text{m})(8\,\text{m})^3}{h(3\,\text{m}\times 8\,\text{m})} + h = \frac{5.33}{h} + h$$

Thus, $\ell(\text{m}) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$

and from Eq.(1)

$$(4\,\text{m})(3500\,\text{kN}) = \left(4 - \frac{5.33}{h}\right)(9.80\times 24)(h)\,\text{kN}$$

so that

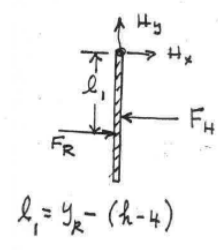
$$\underline{\underline{h = 16.2\,\text{m}}}$$

For gate hinged at top

$\sum M_H = 0$

so that

$(4\,\mathrm{m})\mathrm{F_H} = \ell_1 \mathrm{F_H}$ (see figure) (1)



where

$$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + 4\right) - (h-4) = \frac{5.33}{h} + 4$$

Thus, from Eq.(1)

$$(4\,\mathrm{m})(3500\,\mathrm{kN}) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\,\mathrm{kN}$$

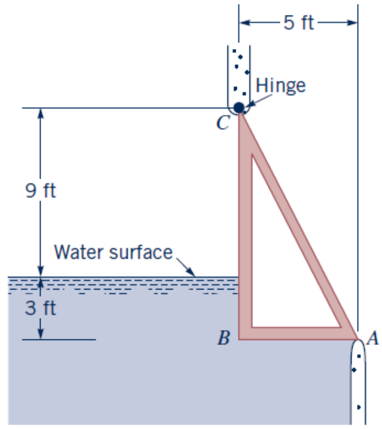
and

$$\underline{\underline{h = 13.5\,\mathrm{m}}}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

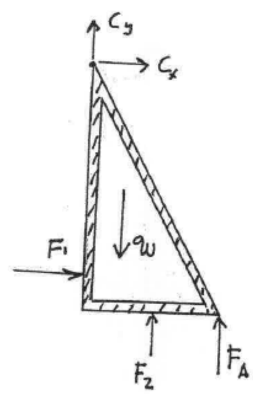
Problem 2.103

A gate having the cross section shown in the figure below is 4 ft wide and is hinged at C. The gate weighs 18,000 lb, and its mass center is 1.67 ft to the right of the plane BC. Determine the vertical reaction at A on the gate when the water level is 3 ft above the base. All contact surfaces are smooth.



Solution 2.103

$F_1 = \gamma h_c A$ where $h_c = 1.5\,\text{ft}$



Thus, $F_1 = \left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)(1.5\,\text{ft})(3\,\text{ft}\times 4\,\text{ft}) = 1120\,\text{lb}$

The force F_1 acts at a distance of 1 ft from the base of the gate.

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}(3\,\text{ft})$

Thus,

$$F_2 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(3\,\text{ft})(5\,\text{ft}\times 4\,\text{ft}) = 3740\,\text{lb}$$

and acts at the center of the bottom gate surface.

For equilibrium,

$$\sum M_c = 0$$

and

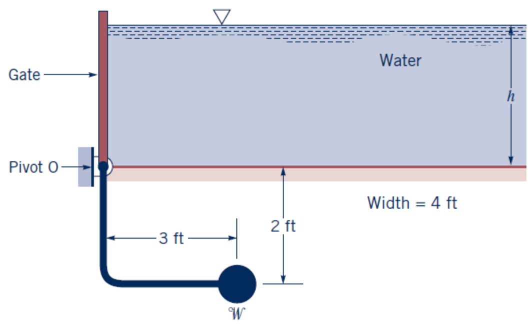
$$F_1(11\,\text{ft}) + F_2(2.5\,\text{ft}) + F_A(5\,\text{ft}) = W(1.67\,\text{ft})$$

so that

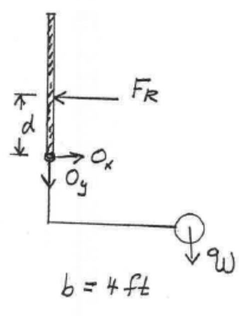
$$F_A = \frac{(18{,}000\,\text{lb})(1.67\,\text{ft}) - (1120\,\text{lb})(11\,\text{ft}) - (3740\,\text{lb})(2.5\,\text{ft})}{5\,\text{ft}} = \underline{\underline{1680\,\text{lb}}}$$

Problem 2.104

The massless, 4-ft -wide gate shown in the figure below pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W . Determine the water depth, h .



Solution 2.104



$$F_R = \gamma h_c A \text{ where } h_c = \frac{h}{2}$$

Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2}(h \times b) = \gamma_{H_2O} \frac{h^2}{2}(4\,\text{ft})$$

To locate F_R ,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,\text{ft})(h^3)}{\frac{h}{2}(4\,\text{ft} \times h)} + \frac{h}{2} = \frac{2}{3}h$$

For equilibrium, $\sum M_0 = 0$

$$F_R d = W(3\,\text{ft}) \text{ where } d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000\,\text{lb})(3\,\text{ft})}{(\gamma_{H_2O})\left(\frac{h^2}{2}\right)(4\,\text{ft})}$$

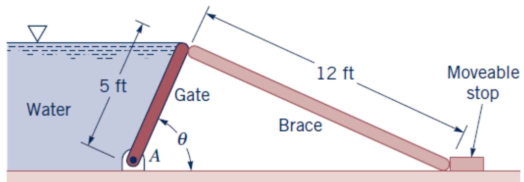
Thus,

$$h^3 = \frac{(3)(2000\,\text{lb})(3\,\text{ft})}{\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(4\,\text{ft})}$$

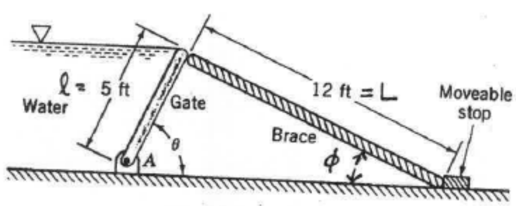
$$h = \underline{\underline{5.24\,\text{ft}}}$$

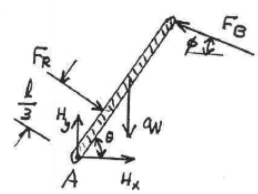
Problem 2.105

A $200-\text{lb}$ homogeneous gate $10\ \text{ft}$ wide and $5\ \text{ft}$ long is hinged at point A and held in place by a 12-ft -long brace as shown in the figure below. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. (**a**) Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, θ, for $0 \le \theta \le 90°$. (**b**) Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the result as $\theta \to 0$.



Solution 2.105





(a) For the free-body-diagram of the gate (see figure),

$$\sum F_A = 0$$

so that

$$F_R\left(\frac{\ell}{3}\right)+W\left(\frac{\ell}{2}\cos\theta\right)=\left(F_B\cos\phi\right)\left(\ell\sin\theta\right)+\left(F_B\sin\phi\right)\left(\ell\cos\theta\right) \qquad (1)$$

Also,

$\ell\sin\theta = L\sin\phi$ (assuming hinge and end of brace at same elevation)

or

$$\sin\phi = \frac{\ell}{L}\sin\theta$$

and

$$F_R = \gamma h_c A = \gamma\left(\frac{\ell\sin\theta}{2}\right)(\ell w)$$

where w is the gate width. Thus, Eq.(1) can be written as

$$\gamma\left(\frac{\ell^3}{6}\right)(\sin\theta)\,w+\frac{W\ell}{2}\cos\theta=F_B\ell\left(\cos\phi\sin\theta+\sin\phi\cos\theta\right)$$

so that

$$F_B=\frac{\left(\frac{\gamma\ell^2 w}{6}\right)\sin\theta+\frac{W}{2}\cos\theta}{\cos\phi\sin\theta+\sin\phi\cos\theta}=\frac{\left(\frac{\gamma\ell^2 w}{6}\right)\tan\theta+\frac{W}{2}}{\cos\phi\tan\theta+\sin\phi}\quad(2)$$

For $\gamma=62.4\,\frac{\text{lb}}{\text{ft}^3}$, $\ell=5\,\text{ft}$, $w=10\,\text{ft}$, and $W=200\,\text{lb}$,

$$F_B=\frac{\frac{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(5\,\text{ft})^2(10\,\text{ft})}{6}\tan\theta+\frac{200\,\text{lb}}{2}}{\cos\phi\tan\theta+\sin\phi}=\frac{2600\tan\theta+100}{\cos\phi\tan\theta+\sin\phi}\qquad(3)$$

Since $\sin\phi=\frac{\ell}{L}\sin\theta$ and $\ell=5\,\text{ft}$, $L=12\,\text{ft}$

$$\sin\phi=\frac{5}{12}\sin\theta$$

and for a given θ, ϕ can be determined. Thus, Eq.(3)

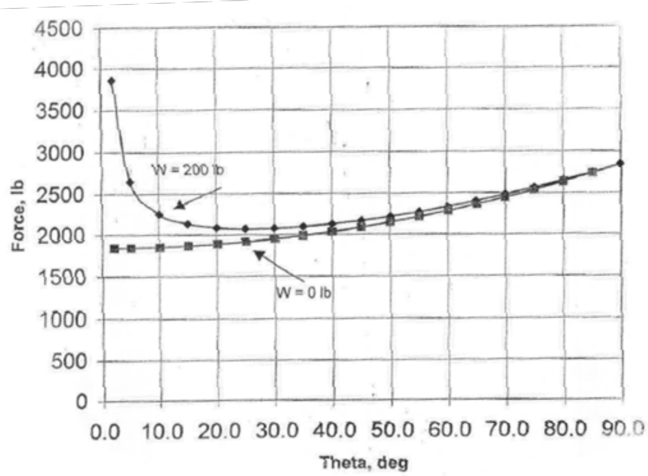
can be used to determine F_B for a given θ.

(b) For $W=0$, Eq.(3) reduces to

$$F_B=\frac{2600\tan\theta}{\cos\phi\tan\theta+\sin\phi}\qquad(4)$$

and Eq.(4) can be used to determine F_B for a given θ. Tabulated data of F_B vs. θ for both $W=200\,\text{lb}$ and $W=0\,\text{lb}$ are given below.

θ, deg	F(B), lb (W=200 lb)	F(B), lb (W=0 lb)
90.0	2843	2843
85.0	2745	2736
80.0	2651	2633
75.0	2563	2536
70.0	2480	2445
65.0	2403	2360
60.0	2332	2282
55.0	2269	2210
50.0	2213	2144
45.0	2165	2085
40.0	2125	2032
35.0	2094	1985
30.0	2075	1945
25.0	2069	1911
20.0	2083	1884
15.0	2130	1863
10.0	2250	1847
5.0	2646	1838
2.0	3858	1836



As $\theta \to 0$ the value of F_B can be determined from Eq.(4),

$$F_B = \frac{2600 \tan\theta}{\cos\phi \tan\theta + \sin\phi}$$

Since

$$\sin\phi = \frac{5}{12}\sin\theta$$

it follows that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{5}{12}\right)^2 \sin^2\theta}$$

and therefore

$$F_B = \frac{2600\tan\theta}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}\tan\theta + \frac{5}{12}\sin\theta} = \frac{2600}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta} + \frac{5}{12}\cos\theta}$$

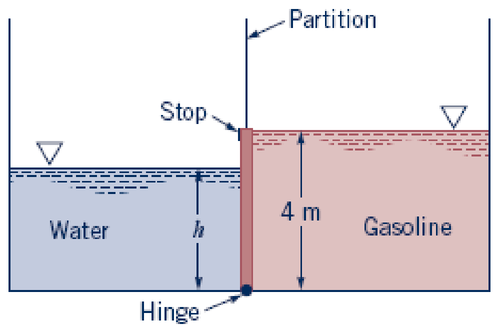
Thus, as $\theta \to 0$

$$F_B \to \frac{2600}{1+\frac{5}{12}} = 1840\,\text{lb}$$

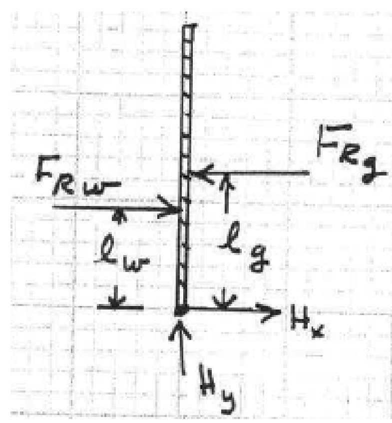
Physically this result means that for $\theta \equiv 0$, the value of F_B is indeterminate, but for any "very small" value of θ, F_B will approach $1840\,\text{lb}$.

Problem 2.106

An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ \text{kg/m}^3$ at a depth of $4\ \text{m}$, as shown in the figure below. A rectangular gate that is $4\ \text{m}$ high and $2\ \text{m}$ wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?



Solution 2.106



$F_{Rg} = \gamma_g h_{cg} A_g$; where g refers to gasoline.

$$F_{Rg} = \left(700\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(2\,\text{m})(4\,\text{m}\times 2\,\text{m})$$

$$= 110\times 10^3\ \text{N}=110\,\text{kN}$$

$F_{Rw} = \gamma_w h_{cw} A_w$; where w refers to water.

$$F_{Rw} = \left(9.80\times 10^3\,\frac{\text{N}}{\text{m}^3}\right)\left(\frac{h}{2}\right)(2\,\text{m}\times h)\text{;}$$ where h is depth of water.

$$F_{Rw} = \left(9.80\times 10^3\right)h^2$$

For equilibrium, $\sum M_H = 0 \rightarrow F_{Rw}\ell_w = F_{Rg}\ell_g$

$$\ell_w = \frac{h}{3}\ ;\ \ \ell_g = \frac{4}{3}\,\text{m}$$

$$\left(9.80\times 10^3\right)\left(h^2\right)\left(\frac{h}{3}\right) = \left(110\times 10^3\ \text{N}\right)\left(\frac{4}{3}\,\text{m}\right)$$

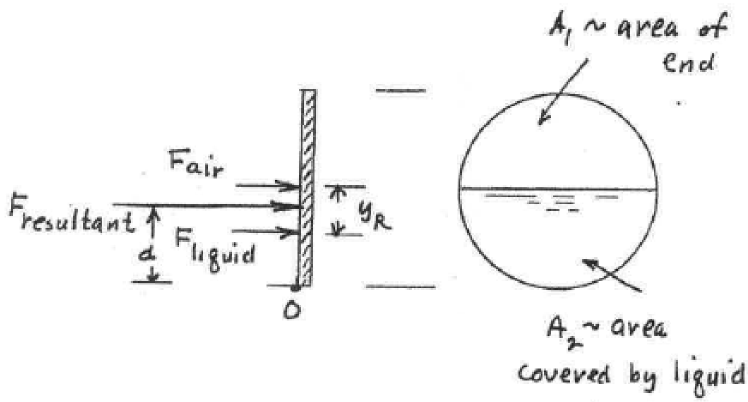
$$h = \underline{\underline{3.55\,\text{m}}}$$

which is the limiting value for h.

Problem 2.107

A horizontal 2-m-diameter conduit is half filled with a liquid ($SG = 1.6$) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 200 kPa . Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit.

Solution 2.107



$F_{air} = pA,$ where p is air pressure

Thus,

$$F_{air} = \left(200 \times 10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}\right)(2\,\text{m})^2 = 200\pi \times 10^3 \text{ N}$$

$F_{liquid} = \gamma h_c A_2$ where $h_c = \dfrac{4R}{3\pi}$ (see the figure below)

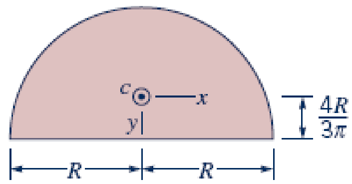
Thus,

$$F_{liquid} = (1.6)\left(9.81 \times 10^3 \frac{\text{N}}{\text{m}^3}\right)\left[\frac{4(1\text{m})}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\,\text{m})^2 = 10.5 \times 10^3 \text{ N}$$

For F_{liquid},

$y_R = \dfrac{I_{xc}}{y_c A_2} + y_c$ where $I_{xc} = 0.1098R^4$ (see the figure below)

and $y_c = h_c = \dfrac{4R}{3\pi}$



$$A = \frac{\pi R^2}{2}$$

$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

Thus,

$$y_R = \frac{0.1098(1\,\text{m})^4}{\left[\frac{4(1\,\text{m})}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2\,\text{m})^2} + \frac{4(1\,\text{m})}{3\pi} = 0.5891\,\text{m}$$

Since $F_{\text{resultant}} = F_{air} + F_{liquid} = (200\pi + 10.5)\times 10^3\ \text{N} = \underline{\underline{639\,\text{kN}}},$

we can sum moments about O to locate resultant to obtain

$$F_{\text{resultant}}(d) = F_{air}(1\,\text{m}) + F_{liquid}(1\,\text{m} - 0.5891\,\text{m})$$

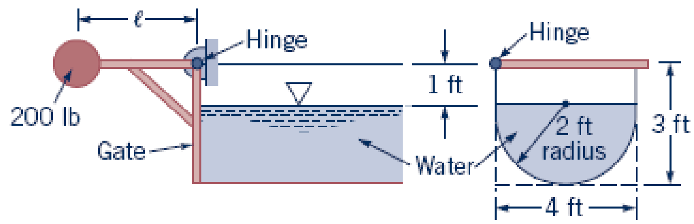
So that

$$d = \frac{(200\pi\times 10^3\ \text{N})(1\,\text{m}) + (10.5\times 10^3\ \text{N})(0.4109\,\text{m})}{639\times 10^3\ \text{N}}$$

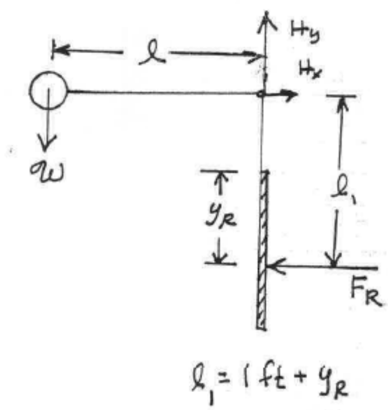
$= \underline{\underline{0.990\,\text{m}}}$ above bottom of conduit.

Problem 2.108

A 4-ft by 3-ft massless rectangular gate is used to close the end of the water tank shown in the figure below. A 200-lb weight attached to the arm of the gate at a distance ℓ from the frictionless hinge is just sufficient to keep the gate closed when the water depth is 2 ft, that is, when the water fills the semicircular lower portion of the tank. If the water were deeper, the gate would open. Determine the distance ℓ.



Solution 2.108



$$F_R = \gamma h_c A \qquad \text{where} \quad h_c = \frac{4R}{3\pi} \qquad \text{(see the figure below)}$$

Thus,

$$F_R = \gamma_{H_2O}\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$$

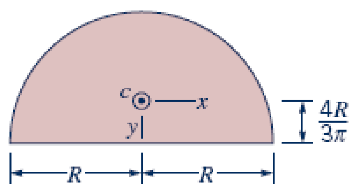
$$= \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{4(2\text{ ft})}{3\pi}\right)\left(\frac{\pi(2\text{ ft})^2}{2}\right) = 333\text{ lb}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{0.1098\,R^4}{\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)} + \frac{4R}{3\pi} \qquad \text{(see the figure below)}$$

$$= \frac{(0.1098)(2\text{ ft})^4}{\left(\frac{4(2\text{ ft})}{3\pi}\right)\frac{\pi(2\text{ ft})^2}{2}} + \frac{4(2\text{ ft})}{3\pi} = 1.178\text{ ft}$$



$$A = \frac{\pi R^2}{2}$$

$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

For equilibrium,

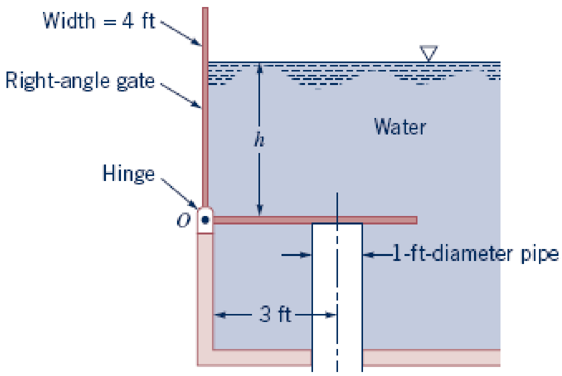
$$\sum M_H = 0$$

So that $\qquad W\,\ell = F_R\left(1\text{ ft}+y_R\right)$

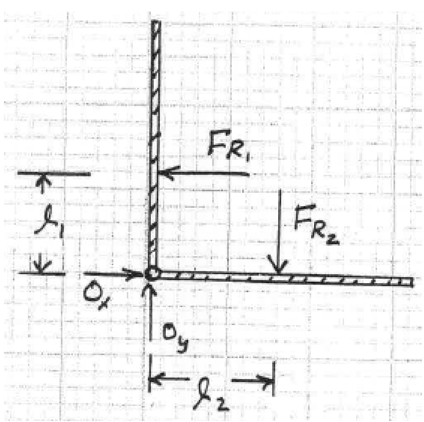
And $\qquad \ell = \frac{(333\text{ lb})(1\text{ ft}+1.178\text{ ft})}{200\text{ lb}} = \underline{\underline{3.63\text{ ft}}}$

Problem 2.109

A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point *O*, as shown in the figure below. The horizontal portion of the gate covers a 1-ft-diameter drain pipe that contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.



Solution 2.109



For equilibrium,

$$\sum M_0 = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$

$$F_{R_1} = \gamma h_{c_1} A_1$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{h}{2}\right)(4\,\text{ft} \times h) = 125\,h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h\left(\frac{\pi}{4}\right)(1\,\text{ft})^2 = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(h)\left(\frac{\pi}{4}\right)(1\,\text{ft})^2$$
$$= 49.0\,h$$

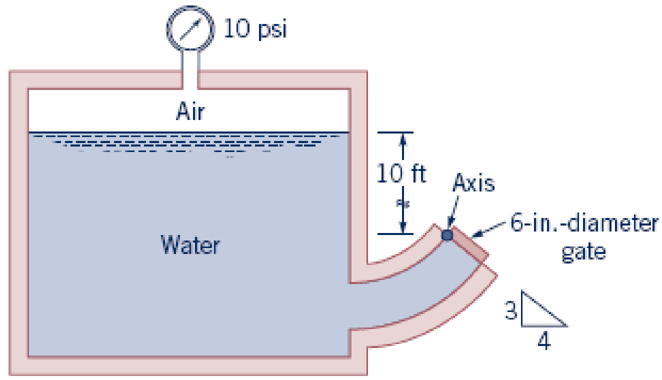
Thus, from Eq. (1) with $\ell_1 = \dfrac{h}{3}$ and $\ell_2 = 3\,\text{ft}$

$$\left(125\,h^2\right)\left(\frac{h}{3}\right) = (49.0\,h)(3\,\text{ft})$$

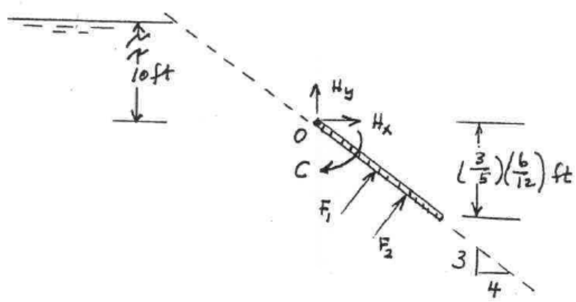
$$\underline{\underline{h = 1.88\,\text{ft}}}$$

Problem 2.110

The closed vessel of the figure below contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.



Solution 2.110



Let F_1 = force due to air pressure, and F_2 = force due to hydrostatic pressure distribution of water.

Thus,

$$F_1 = p_{air}A = \left(10\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\text{ft}\right)^2 = 283\,\text{lb}$$

and

$$F_2 = \gamma h_c A \qquad \text{where} \quad h_c = 10\,\text{ft} + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right)\text{ft}\right] = 10.15\,\text{ft}$$

So that

$$F_2 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(10.15\,\text{ft})\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\text{ft}\right)^2 = 124\,\text{lb}$$

Also,

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad y_c = \frac{10\,\text{ft}}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12}\,\text{ft}\right) = 16.92\,\text{ft}$$

So that

$$y_{R_2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\,\text{ft}\right)^4}{(16.92\,\text{ft})\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\,\text{ft}\right)^2} + 16.92\,\text{ft} = 16.92\,\text{ft}$$

For equilibrium,

$$\sum M_0 = 0$$

And

$$C = F_1\left(\frac{3}{12}\,\text{ft}\right) + F_2\left(y_{R_2} - \frac{10\,\text{ft}}{\frac{3}{5}}\right)$$

$$C = (283\,\text{lb})\left(\frac{3}{12}\,\text{ft}\right) + (124\,\text{lb})\left(16.92\,\text{ft} - \frac{10\,\text{ft}}{\frac{3}{5}}\right) = \underline{\underline{102\,\text{ft}\cdot\text{lb}}}$$

Problem 2.111

(a) Determine the horizontal hydrostatic force on the 2309-m-long Three Gorges Dam when the average depth of the water against it is 175 m. **(b)** If all of the 6.4 billion people on Earth were to push horizontally against the Three Gorges Dam, could they generate enough force to hold it in place? Support your answer with appropriate calculations.

Solution 2.111

(a)

$$F_R = \gamma h_c A = \left(9.80\times10^3 \frac{\text{N}}{\text{m}^3}\right)\left(\frac{175\,\text{m}}{2}\right)(175\,\text{m}\times 2309\,\text{m})$$

$$= \underline{\underline{3.46\times10^{11}\ \text{N}}}$$

(b)

$$\text{Required average force per person} = \frac{3.46\times10^{11}\ \text{N}}{6.4\times10^{9}}$$

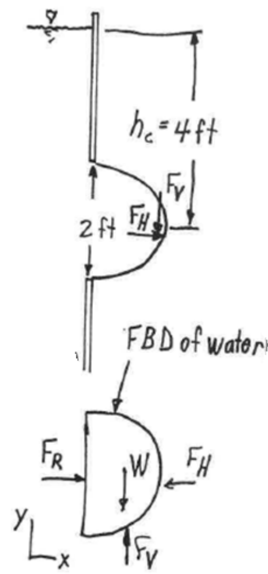
$$= \underline{\underline{54.1\frac{\text{N}}{\text{person}}}}\left(12.2\frac{\text{lb}}{\text{person}}\right)$$

Yes. It is likely that enough force could be generated since required average force per person is relatively small.

Problem 2.113

A 2-ft-diameter hemispherical plexiglass "bubble" is to be used as a special window on the side of an above-ground swimming pool. The window is to be bolted onto the vertical wall of the pool and faces outward, covering a 2-ft-diameter opening in the wall. The center of the opening is 4 ft below the surface. Determine the horizontal and vertical components of the force of the water on the hemisphere.

Solution 2.113



$\sum F_x = 0$, or $F_H = F_R = p_c A$

Thus,

$$F_H = \gamma h_c A = 62.4\frac{\text{lb}}{\text{ft}^3}(4\,\text{ft})\frac{\pi}{4}(2\,\text{ft})^2 = \underline{\underline{784\,\text{lb}}}\ (\text{to right})$$

and

$$\sum F_y = 0,\ \text{or}\ F_V = W = \gamma V = \gamma \frac{4}{3}\frac{\pi R^3}{2},$$

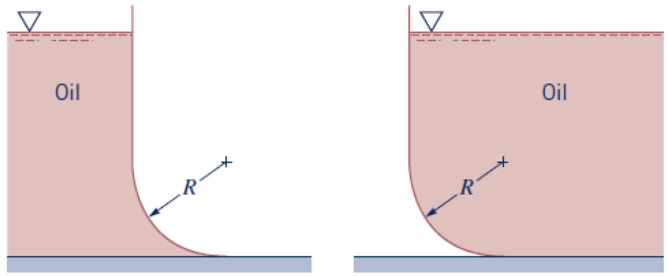
where $R = 1\,\text{ft}$

Thus,

$$F_V = 62.4\frac{\text{lb}}{\text{ft}^3}\left(\frac{4\pi(1\,\text{ft})^3}{6}\right) = \underline{\underline{131\,\text{lb}}}\ (\text{down on bubble})$$

Problem 2.114

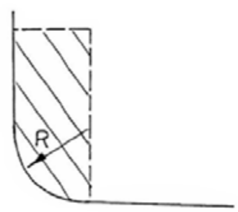
Consider the curved surface shown in the figure below (a) and (b). The two curved surfaces are identical. How are the vertical forces on the two surfaces alike? How are they different?



(a) (b)

Solution 2.114

In both cases the magnitude of the vertical force is the weight of shaded section shown on the right. In addition, the location of the vertical force is the same (the centroid of the shaded section.) Therefore:



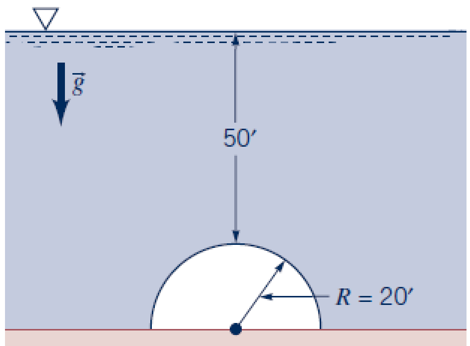
Alike: magnitude and location of vertical forces same.

However, the two vertical forces are different in that the force in (a) is acting upward and the force in (b) is acting downward. Therefore:

Different: direction of vertical forces opposite.

Problem 2.115

The figure below shows a cross section of a submerged tunnel used by automobiles to travel under a river. Find the magnitude and location of the resultant hydrostatic force on the circular roof of the tunnel. The tunnel is 4 mi long.

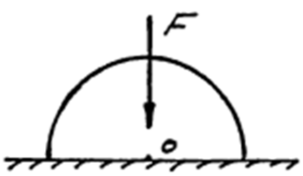


Solution 2.115

Due to symmetry, there is no net horizontal force on the roof. The vertical force is equal to the weight of fluid above the tunnel. This vertical force acts through the centroid of the fluid volume. Then for a tunnel length ℓ,

$$F = \gamma \forall = \gamma \ell \left(2Rh - \frac{\pi}{2} R^2 \right)$$

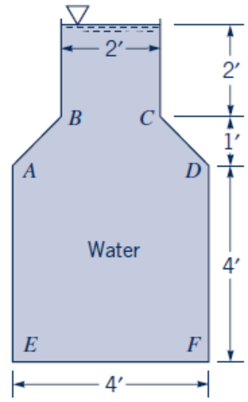
$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3} \right) (4\,\text{mi}) \left(5280 \frac{\text{ft}}{\text{mi}} \right) \left[2(20\,\text{ft})(70\,\text{ft}) - \frac{\pi}{2} (20\,\text{ft})^2 \right]$$



$\boxed{F = 2.86 \times 10^9 \text{ lb}}$.

This force $\boxed{\text{acts downward through the point "O".}}$

Problem 2.116
The container shown in the figure below has circular cross sections. Find the vertical force on the inclined surface. Also find the net vertical force on the bottom, *EF*. Is the vertical force equal to the weight of the water in the container?



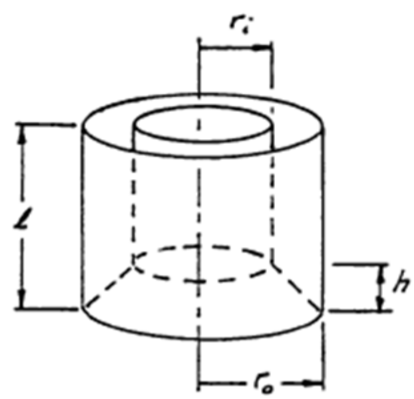
Solution 2.116

The vertical force on the inclined surface is equal to the weight of the water "above" it. This "water volume" is

$$\cancel{V} = \cancel{V}_{cyl} + \cancel{V}_{hole} - \cancel{V}_{\text{frustrum}}$$

$$\cancel{V} = \pi r_o^2 \ell - \pi r_i^2 (\ell - h) - \frac{1}{3}\pi h \left(r_o^2 + r_i^2 + r_o r_i\right).$$

$$\cancel{V} = \pi (2\,\text{ft})^2 (3\,\text{ft}) - \pi (1\,\text{ft})^2 (3-1)\,\text{ft}$$
$$-\frac{1}{3}\pi (1\,\text{ft})\left[(2\,\text{ft})^2 + (1\,\text{ft})^2 + (2\,\text{ft})(1\,\text{ft})\right] = 24.1\,\text{ft}^3$$



The vertical force F_{Vi} is

$r_i = 1"$, $r_o = 2"$, $h = 1"$, $\ell = 3"$.

$$F_{Vi} = \gamma \cancel{V} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(24.1\,\text{ft}^3\right) = \boxed{F_{Vi} = 1500\,\text{lb}}.$$

The pressure is uniform over the bottom *EF* so

$$F_{Vb} = pA = \gamma h A = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(7\,\text{ft})\,\pi\,(2\,\text{ft})^2$$

or

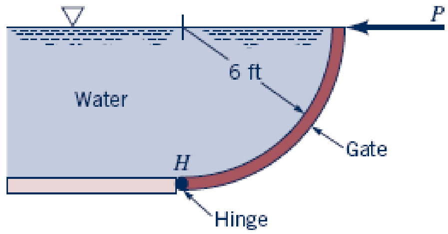
$\boxed{F_{Vb} = 5490\,\text{lb}}$

*CRC Standard Math Tables

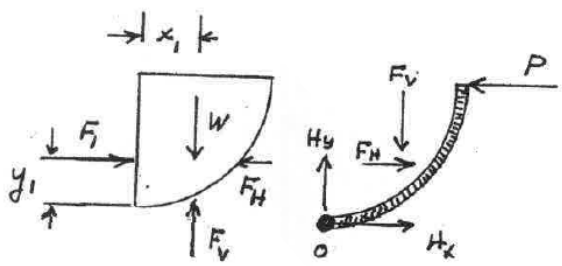
This force F_{Vb} is not equal to the weight of water in the container.

Problem 2.117

The 18-ft-long lightweight gate of the figure below is a quarter circle and is hinged at H. Determine the horizontal force, P, required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.



Solution 2.117



For equilibrium (from free-body-diagram of fluid mass),

$$\sum F_x = 0$$

So that

Similarly,

$$F_H = F_1 = \gamma h_{c_1} A_1$$

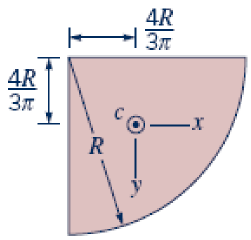
$$= \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6\,\text{ft}}{2}\right)(6\,\text{ft}\times 18\,\text{ft}) = 20200\,\text{lb}$$

$$\sum F_y = 0$$

So that

$$F_V = W = \gamma_{H_2O}\times(\text{volume of fluid}) = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left[\frac{\pi}{4}(6\,\text{ft})^2\times 18\,\text{ft}\right] = 31800\,\text{lb}$$

Also, $x_1 = \dfrac{4(6\,\text{ft})}{3\pi} = \dfrac{8}{\pi}\,\text{ft}$ (see the figure below) and $\quad y_1 = \dfrac{6\,\text{ft}}{3} = 2\,\text{ft}$



$$A = \frac{\pi R^2}{4}$$

$$I_{xc} = I_{yc} = 0.05488 R^4$$

$$I_{xyc} = -0.01647 R^4$$

For equilibrium (from free-body-diagram of gate)

$$\sum M_0 = 0$$

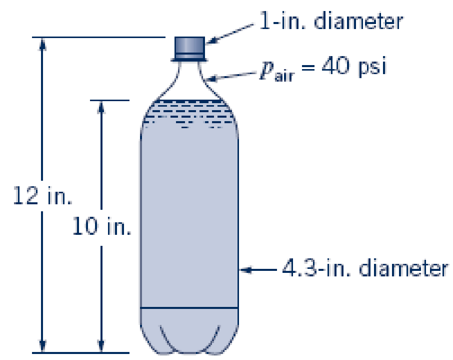
So that

$$P(6\,\text{ft}) = F_H\,(y_1) + F_V\,(x_1)$$

or

$$P = \frac{(20200\,\text{lb})(2\,\text{ft}) + (31800\,\text{lb})\left(\dfrac{8}{\pi}\,\text{ft}\right)}{6\,\text{ft}} = \underline{\underline{20200\,\text{lb}}}$$

Problem 2.118

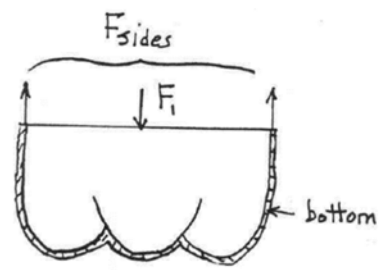


The air pressure in the top of the 2-liter pop bottle and the figure below is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. **(a)** If the bottle cap has a diameter of 1 in. what is the magnitude of the axial force required to hold the cap in place? **(b)** Determine the force needed to secure the bottom 2 in. of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. **(c)** By how much does the weight of the pop increase the pressure 2 in. above the bottom? Assume the pop has the same specific weight as that of water.

Solution 2.118

(a) $$F_{cap} = p_{air} \times \text{Area}_{cap} = \left(40\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{\pi}{4}\right)(1\,\text{in.})^2 = \underline{\underline{31.4\,\text{lb}}}$$

(b)



$$\sum F_{\text{vertical}} = 0$$

$$F_{\text{sides}} = F_1 = (\text{pressure @ 2 in. above bottom}) \times (\text{Area})$$
$$= \left(40\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{\pi}{4}\right)(4.3\,\text{in.})^2$$
$$= \underline{\underline{581\,\text{lb}}}$$

(c)

$$p = p_{air} + \gamma h$$

$$= 40\frac{\text{lb}}{\text{in.}^2} + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ft}\right)\left(\frac{1}{144\frac{\text{in.}^2}{\text{ft}^2}}\right)$$

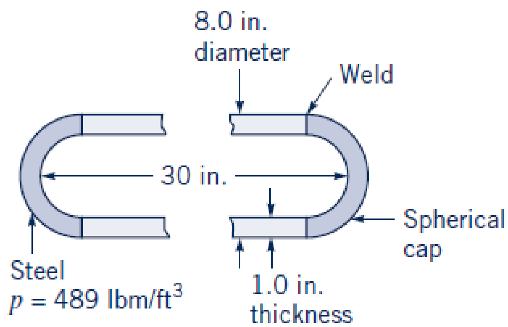
$$= 40\frac{\text{lb}}{\text{in.}^2} + 0.289\frac{\text{lb}}{\text{in.}^2}$$

Thus, the increase in pressure due to weight $= \underline{\underline{0.289\,\text{psi}}}$

(which is less that 1% of air pressure).

Problem 2.119

In drilling for oil in the Gulf of Mexico, some divers have to work at a depth of 1300 ft. (**a**) Assume that seawater has a constant density of 64 lb/ft^3 and compute the pressure at this depth. The divers breathe a mixture of helium and oxygen stored in cylinders, as shown in the figure below, at a pressure of 3000 psia. (**b**) Calculate the force, which trends to blow the end cap off, that the weld must resist while the diver is using the cylinder at 1300 ft. (**c**) After emptying a tank, a diver releases it. Will the tank rise or fall, and what is its initial acceleration?



Solution 2.119

(a) The hydrostatic pressure is

$$p = \gamma_{sw} h = \left(64 \frac{\text{lb}}{\text{ft}^3}\right)(1300\,\text{ft})\left(\frac{\text{ft}^2}{144\,\text{in.}^2}\right) \quad \text{or} \quad \boxed{p = 578\,\text{psig}}$$

(b) The net horizontal force on the end caps is

$$F_N = F_{in} - F_{out} = p_{in} A_{in} - p_{out} A_{out}$$

and

$$\tau = \text{wall stress} = \frac{F_N}{A_{wall}} = \frac{F_N}{A_{out} - A_{in}}$$

$$= \frac{p_{in} A_{in} - p_{out} A_{out}}{A_{out} - A_{in}} = \frac{p_{in} D_{in}^2 - p_{out} D_{out}^2}{D_{out}^2 - D_{in}^2}$$

$$= \frac{\left(3000 \frac{\text{lb}}{\text{in.}^2}\right)(6\,\text{in.})^2 - (14.7 - 578)\frac{\text{lb}}{\text{in.}^2}(8\,\text{in.})^2}{(8\,\text{in.})^2 - (6\,\text{in.})^2}$$

and

$$\boxed{\tau = 2500\,\text{psi.}}$$

(c) The net vertical force on an empty tank and Newton's second law give

$$+\uparrow \quad F_{\text{vert}} = F_{\text{Buoy}} - W = m\,a$$

or

$$a = \frac{F_{\text{Buoy}} - W}{m} = \frac{F_{\text{Buoy}}}{m} - g$$

where m is the mass of the tank. Now

$$F_{\text{Buoy}} = \gamma_{\text{sw}} \cancel{V} = \gamma_{\text{sw}} \left[\left(\frac{\pi}{4}\right) \ell\, D_{\text{out}}{}^2 + \left(\frac{\pi}{6}\right) D_{\text{out}}{}^3 \right]$$

where $\ell = 30\,\text{in.} - 6\,\text{in.} = 24\,\text{in.}$ Also

$$m = \rho_{\text{steel}} \left[\left(\frac{\pi}{4}\right) \ell \left(D_{\text{out}}{}^2 - D_{\text{in}}{}^2\right) + \left(\frac{\pi}{6}\right)\left(D_{\text{out}}{}^3 - D_{\text{in}}{}^3\right) \right].$$

Substituting into the equation for a gives

$$a = \frac{\gamma_{\text{sw}} \left[\frac{1}{4} \ell\, D_{\text{out}}{}^2 + \frac{1}{6} D_{\text{out}}{}^3 \right]}{\rho_{\text{steel}} \left[\frac{1}{4} \ell \left(D_{\text{out}}{}^2 - D_{\text{in}}{}^2\right) + \frac{1}{6}\left(D_{\text{out}}{}^3 - D_{\text{in}}{}^3\right) \right]} \quad - g\,.$$

The numerical values give

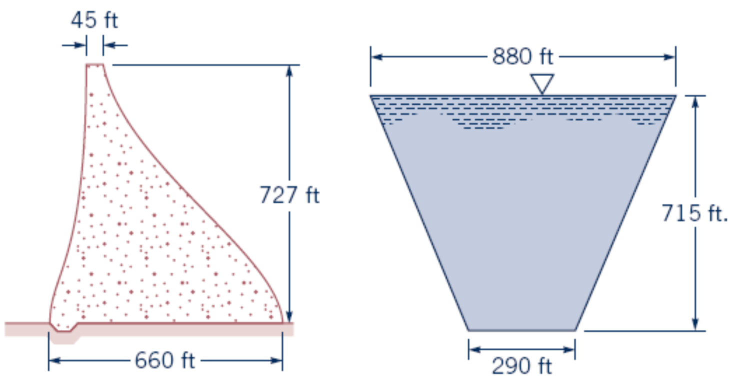
$$a = \frac{\left(64 \frac{\text{lb}}{\text{ft}^3}\right) \left[\frac{1}{4}(24)8^2 + \frac{1}{6}\left(8^3\right) \right] \text{in.}^3 \left(32.2 \frac{\text{ft} \cdot \text{lbm}}{\text{lb} \cdot \text{sec}^2}\right)}{\left(489 \frac{\text{lbm}}{\text{ft}^3}\right) \left[\frac{1}{4}(24)\left(8^2 - 6^2\right) + \frac{1}{6}\left(8^3 - 6^3\right) \right] \text{in.}^3} - 32.2 \frac{\text{ft}}{\text{sec}^2}$$

or

$$\boxed{\begin{array}{l} a = -23.1 \dfrac{\text{ft}}{\text{sec}^2} \\ \text{tank will fall} \\ \text{since } a < 0. \end{array}}$$

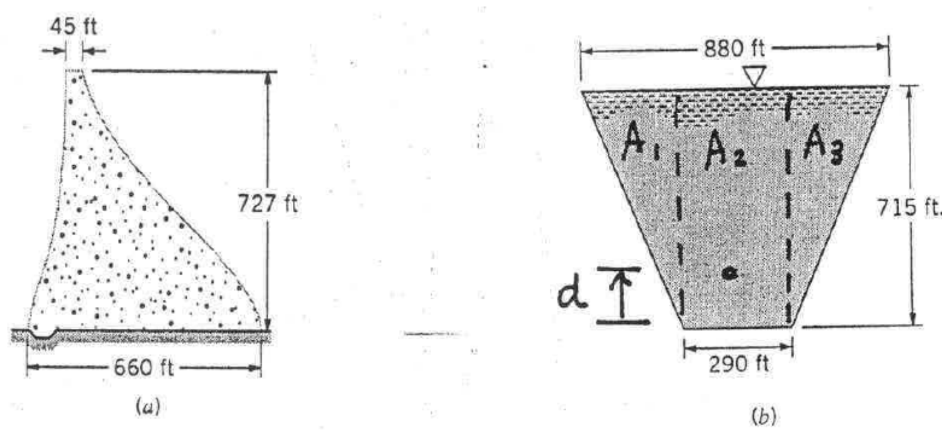
Problem 2.120

Hoover Dam is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in the figure below (a). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in the figure below (b) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.



(a) (b)

Solution 2.120



Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{3}\right)(715\,\text{ft})\left(\frac{1}{2}\right)(295\,\text{ft})(715\,\text{ft})$$

$$= 1.57\times 10^9 \text{ lb}$$

For area 3: $F_{R_3} = F_{R_1} = 1.57\times 10^9$ lb

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(715\,\text{ft})(290\,\text{ft})(715\,\text{ft})$$
$$= 4.63\times10^9\ \text{lb}$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57\times10^9\ \text{lb} + 4.63\times10^9\ \text{lb} + 1.57\times10^9\ \text{lb}$$
$$= 7.77\times10^9\ \text{lb}$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\,\text{ft}) + F_{R_2}\left(\frac{1}{2}\right)(715\,\text{ft}) + F_{R_3}\left(\frac{2}{3}\right)(715\,\text{ft})$$

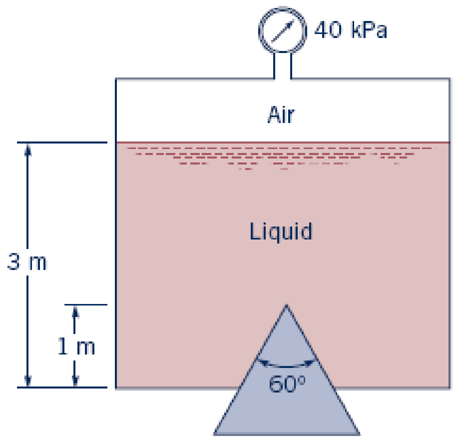
and

$$d = \frac{\left(1.57\times10^9\ \text{lb}\right)\left(\frac{2}{3}\right)(715\,\text{ft}) + \left(4.63\times10^9\ \text{lb}\right)\left(\frac{1}{2}\right)(715\,\text{ft}) + \left(1.57\times10^9\ \text{lb}\right)\left(\frac{2}{3}\right)(715\,\text{ft})}{7.77\times10^9\ \text{lb}}$$
$$= 406\,\text{ft}$$

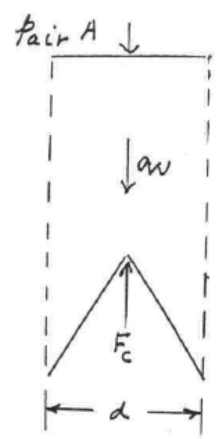
Thus, the resultant horizontal force on the dam is $7.77\times10^9\ \text{lb}$ acting $406\,\text{ft}$ up from the base of the dam along the axis of symmetry of the area.

Problem 2.121

A plug in the bottom of a pressurized tank is conical in shape, as shown in the figure below. The air pressure is $40\ \text{kPa}$, and the liquid in the tank has a specific weight of $27\ \text{kN/m}^3$. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the $40-\text{kPa}$ pressure and the liquid.



Solution 2.121



$$\tan 30^\circ = \frac{\frac{d}{2}}{1}$$

$$d = 2\tan 30^\circ = 1.155\,\text{m}$$

$$\text{volume of cone} = \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1)$$

For equilibrium,

$$\sum F_{vertical} = 0$$

So that

$$F_c = p_{air}A + a_w$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air}A = (40\,\text{kPa})\left(\frac{\pi}{4}\right)\left(d^2\right)$$
$$= (40\,\text{kPa})\left(\frac{\pi}{4}\right)(1.155\,\text{m})^2 = 41.9\,\text{kN}$$

And

$$W = \gamma\left[\frac{\pi}{4}d^2(3\,\text{m}) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2(1\,\text{m})\right] = \gamma\pi d^2\left[\frac{3\,\text{m}}{4} - \frac{1\,\text{m}}{12}\right]$$
$$= \left(27\frac{\text{kN}}{\text{m}^3}\right)(\pi)(1.155\,\text{m})^2\left(\frac{2}{3}\text{m}\right) = 75.4\,\text{kN}$$

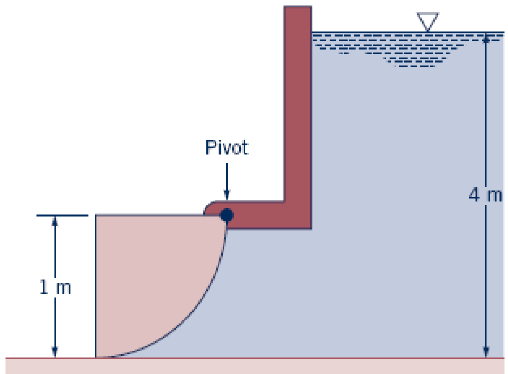
Thus,

$$F_c = 41.9\,\text{kN} + 75.4\,\text{kN} = 117\,\text{kN}$$

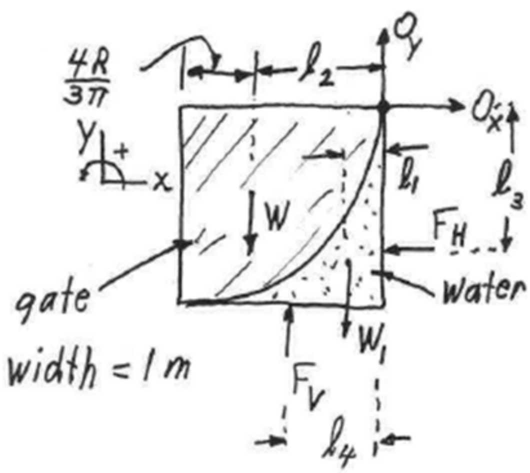
And the force on the cone has a magnitude of $\underline{\underline{117\,\text{kN}}}$ and is <u>directed vertically downward along the cone axis</u>.

Problem 2.122

The homogeneous gate shown in the figure below consists of one quarter of a circular cylinder and is used to maintain a water depth of $4\ \text{m}$. That is, when the water depth exceeds $4\ \text{m}$, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.



Solution 2.122



Consider the free body diagram of the gate and a portion of the water as shown.

$\sum M_o = 0$, or

(1) $\ell_2 W + \ell_1 W_1 - F_H \ell_3 - F_V \ell_4 = 0$, where

(2) $F_H = \gamma h_c A = 9.8 \times 10^3 \dfrac{\text{N}}{\text{m}^3}(3.5\,\text{m})(1\,\text{m})(1\,\text{m}) = 34.3\,\text{kN}$

since for the vertical side, $h_c = 4\,\text{m} - 0.5\,\text{m} = 3.5\,\text{m}$

Also,

(3) $F_V = \gamma h_c A = 9.8 \times 10^3 \dfrac{\text{N}}{\text{m}^3}(4\,\text{m})(1\,\text{m})(1\,\text{m}) = 39.2\,\text{kN}$

Also,

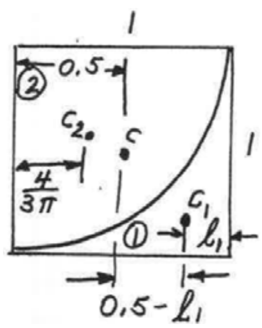
(4) $W_1 = \gamma(1\,\text{m})^3 - \gamma\left(\frac{\pi}{4}(1\,\text{m})^2\right)(1\,\text{m}) = 9.8\times10^3\,\frac{\text{N}}{\text{m}^3}\left[1-\frac{\pi}{4}\right]\text{m}^3 = 2.10\,\text{kN}$

(5) Now, $\ell_4 = 0.5\,\text{m}$ and

(6) $\ell_3 = 0.5\,\text{m} + (y_R - y_c) = 0.5\,\text{m} + \frac{I_{xc}}{y_c A} = 0.5\,\text{m} + \frac{\frac{1}{12}(1\,\text{m})(1\,\text{m})^3}{3.5\,\text{m}(1\,\text{m})(1\,\text{m})} = 0.524\,\text{m}$

(7) and $\ell_2 = 1\,\text{m} - \frac{4R}{3\pi} = 1 - \frac{4(1\,\text{m})}{3\pi} = 0.576\,\text{m}$

To determine ℓ_1, consider a unit square that consist of a quarter circle and the remainder as show in the figure. The centroids of areas (1) and (2) are as indicated.



Thus,

$$\left(0.5 - \frac{4}{3\pi}\right)A_2 = (0.5 - \ell_1)A_1$$

So that with $A_2 = \frac{\pi}{4}(1)^2 = \frac{\pi}{4}$ and $A_1 = 1 - \frac{\pi}{4}$ this gives

$$\left(0.5 - \frac{4}{3\pi}\right)\frac{\pi}{4} = (0.5 - \ell_1)\left(1 - \frac{\pi}{4}\right)$$

or

(8) $\ell_1 = 0.223\,\text{m}$

Hence, by combining Eqs.(1) through (8):

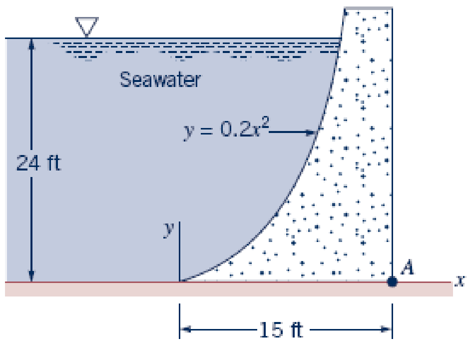
$$(0.576\,\text{m})W + (0.223\,\text{m})(2.10\,\text{kN}) - (34.3\,\text{kN})(0.524\,\text{m}) - (39.2\,\text{kN})(0.5\,\text{m}) = 0$$

or

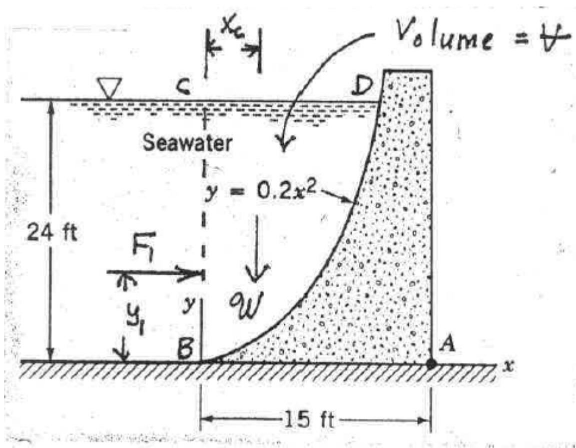
$W = \underline{\underline{64.4\,\text{kN}}}$

Problem 2.123

The concrete (specific weight $=150 \text{ lb/ft}^3$) seawall of the figure below has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).



Solution 2.123



The components of the fluid force acting on the wall are F_1 and W as shown on the figure where

$$F_1 = \gamma h_c A = \left(64.0 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{24 \text{ ft}}{2}\right)(24 \text{ ft} \times 1 \text{ ft})$$
$$= 18400 \text{ lb}$$

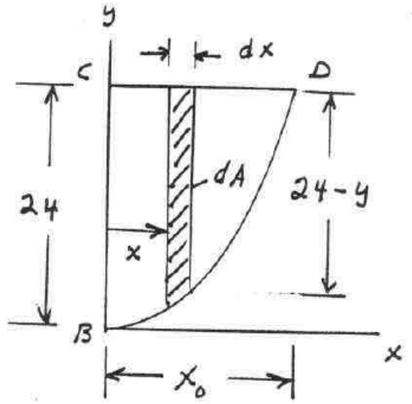
and

$$y_1 = \frac{24 \text{ ft}}{3} = 8 \text{ ft}$$

Also,

$$W = \gamma \forall$$

To determine $\forall$ find area BCD.



$x_0 = \sqrt{120}$

(Note: All lengths in ft)

Thus,

$$A = \int_0^{x_0} (24 - y)\, dx = \int_0^{x_0} \left(24 - 0.2x^2\right) dx$$

$$= \left[24x - \frac{0.2x^3}{3} \right]_0^{x_0}$$

And with $x_0 = \sqrt{120}$, $A = 175\,\text{ft}^2$ so that

$$\cancel{V} = A \times 1\,\text{ft} = 175\,\text{ft}^3$$

Thus,

$$W = \left(64.0\,\frac{\text{lb}}{\text{ft}^3}\right)\left(175\,\text{ft}^3\right) = 11200\,\text{lb}$$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\, dA = \int_0^{x_0} (24 - y)\, x\, dx = \int_0^{x_0} \left(24x - 0.2x^3\right) dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

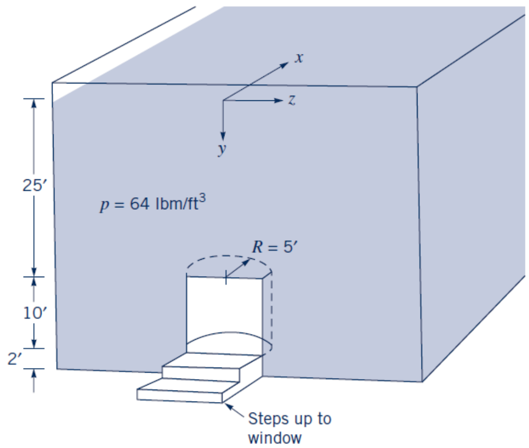
$$x_c = \frac{12\left(\sqrt{120}\right)^2 - \dfrac{0.2\left(\sqrt{120}\right)^4}{4}}{175} = 4.11\,\text{ft}$$

Thus,

$$
\begin{aligned}
M_A &= F_1 y_1 - W\left(15 - x_c\right) \\
&= \left(18400\,\text{lb}\right)\left(8\,\text{ft}\right) - \left(11200\,\text{lb}\right)\left(15\,\text{ft} - 4.11\,\text{ft}\right) \\
&= \underline{\underline{25200\,\text{ft}\cdot\text{lb}}}
\end{aligned}
$$

Problem 2.124

A step-in viewing window having the shape of a half-cylinder is built into the side of a large aquarium. See the figure below. Find the magnitude, direction, and location of the net horizontal forces on the viewing window.



Solution 2.124

Due to symmetry, the net force parallel to the wall is zero or

$$\boxed{F_z = 0}$$

The net horizontal force perpendicular to the wall is

$$F_x = \gamma h_c A = \left(64\,\frac{\text{lb}}{\text{ft}^3}\right)(25+5)\,\text{ft}\,(10\,\text{ft}\times 10\,\text{ft})$$

$$\boxed{F_x = 1.92\times 10^5\ \text{lb}}$$

The vertical location of F_x is

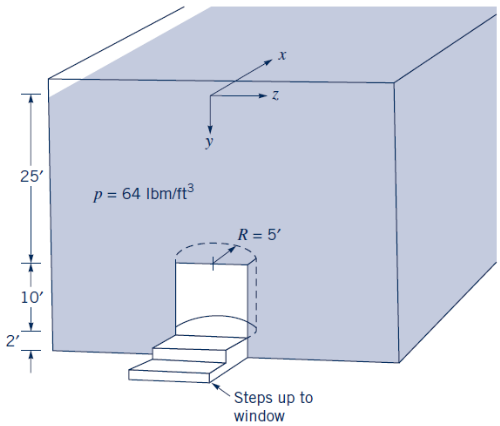
$$y_p = y_c + \frac{I_{xc}}{y_c A} = y_c + \frac{\frac{1}{12}bh^3}{y_c\,bh} = y_c + \frac{h^2}{12 y_c} = 30\,\text{ft} + \frac{(10\,\text{ft})^2}{12(30\,\text{ft})} \quad \text{or} \quad \boxed{y_p = 30.3\,\text{ft}}$$

The net horizontal force also acts through the coordinate

$\boxed{z = 0 \text{ and acts in an outward direction.}}$

Problem 2.125

A step-in viewing window having the shape of a half-cylinder is built into the side of a large aquarium. See the figure below. Find the magnitude, direction, and location of the net horizontal forces on the viewing window. Find the magnitude, direction, and location of the net vertical force acting on the viewing window.



Solution 2.125

The net vertical force must equal the weight of fluid inside the viewing window. Then

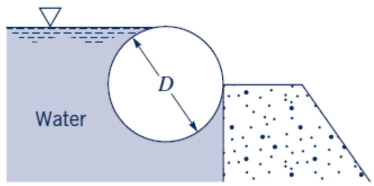
$$F_y = \gamma \cancel{V} = \gamma h\left(\frac{\pi}{2}R^2\right) = \left(64\frac{\text{lb}}{\text{ft}^3}\right)(10\,\text{ft})\left(\frac{\pi}{2}\right)(5\,\text{ft})^2 \quad \text{or} \quad \boxed{\begin{array}{l} F_y = 25100\,\text{lb,} \\ \text{acting upword.} \end{array}}$$

This net vertical force acts through the centroid of the window volume. Using Appendix B gives

$$\bar{x} = \frac{4R}{3\pi} = \frac{4(5\,\text{ft})}{3\pi} \qquad \text{or} \qquad \boxed{\bar{x} = 2.12\,\text{ft}}$$

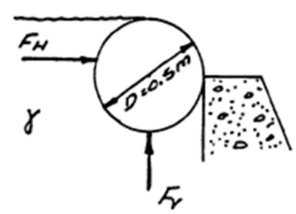
Problem 2.126

A 10-m-long log is stuck against a dam, as shown in the figure below. Find the magnitudes and locations of both the horizontal force and the vertical force of the water on the log in terms of the diameter D. The center of the log is at the same elevation as the top of the dam.



Solution 2.126

Consider the water forces on the log as shown on the right.



The horizontal forces F_H is on the top portion only and is

$$F_H = \gamma\left(\frac{D}{4}\right)\left(\frac{D}{2}\right)\ell$$

where ℓ is the log length. Assuming $10^\circ\mathrm{C}$ water, Table A.5 gives

$$F_H = \left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.25\,\mathrm{m})(0.5\,\mathrm{m})(10\,\mathrm{m}) = \boxed{12300\ \mathrm{N} = F_H}$$

The location of F_H is

$$y_p = \frac{2}{3}\left(\frac{D}{2}\right) = \frac{2}{3}\left(\frac{0.5}{2}\mathrm{m}\right) = \boxed{0.167\,\mathrm{m} = y_p}$$

The vertical force F_V is the weight of water "above» the bottom of the log minus the weight of water above the top half of the log. This is

$$F_V = \gamma\ell\left[\frac{\pi D^2}{8} + \left(\frac{D}{2}\right)D - \left(\frac{D^2}{4} - \frac{\pi D^2}{16}\right)\right] = \frac{\gamma\ell D^3}{4}\left(\frac{3\pi}{4}+1\right)$$

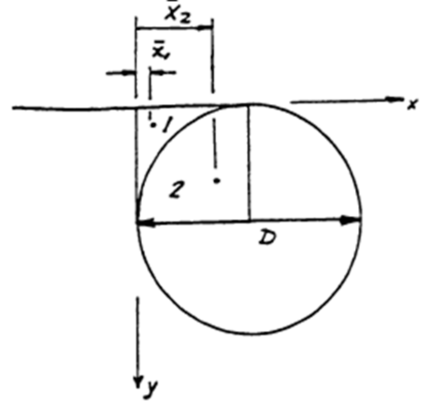
$$= \frac{\left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)(10\,\mathrm{m})(1.0\,\mathrm{m})^2}{4}\left(\frac{3\pi}{4}+1\right)$$

$$\boxed{F_V = 82300\,\mathrm{N}}$$

The location $\bar{x}$ of F_V is found by first locating the centroid of area A_1 by

$$\bar{x}_1 = \frac{A_{1+2}\bar{x}_{1+2} - A_2\bar{x}_2}{A_1}.$$

Using Table B

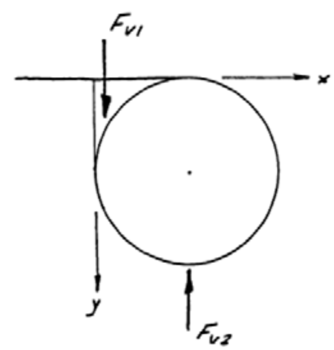


$$\bar{x}_1 = \frac{\left(\frac{D}{2}\right)^2\left(\frac{D}{4}\right) - \left(\frac{\pi D^2}{16}\right)\left(\frac{D}{2} - \frac{2D}{3\pi}\right)}{\left(\frac{D}{2}\right)^2 - \frac{\pi D^2}{16}}$$

$$= \left[\frac{\frac{1}{16} - \frac{\pi}{16}\left(\frac{1}{2} - \frac{2}{3\pi}\right)}{\frac{1}{4} - \frac{\pi}{16}}\right] D$$

$$= 0.112\,D$$

and is the location of F_{V1}. The location of F_{V2} is $\frac{D}{2}$. The location of F_V is



$$\bar{x} = \frac{F_{V2}\left(\frac{D}{2}\right) - F_{V1}(0.112\,D)}{F_V}$$

$$= \frac{\left[\frac{\pi D^2}{B} + \frac{D^2}{2}\right]\left(\frac{D}{2}\right) - \left[\frac{D^2}{4} - \frac{\pi D^2}{16}\right](0.112\,D)}{\frac{\pi D^2}{B} + \frac{D^2}{2} - \left[\frac{D^2}{4} - \frac{\pi D^2}{16}\right]}$$

$$= \frac{\frac{1}{4}\left(\frac{\pi}{4} + 1\right) - 0.112\left(\frac{1}{4}\right)\left(1 - \frac{\pi}{4}\right)}{\frac{1}{2}\left(\frac{\pi}{4} + 1\right) - \frac{1}{4}\left(1 - \frac{\pi}{4}\right)} D$$

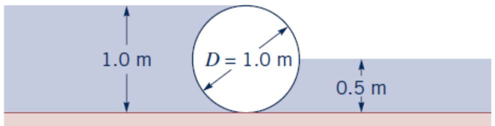
$$\boxed{\bar{x} = 0.525\,D}$$

F_{V2} = weight of water "above" bottom portion of log.

F_{V1} = weight of water above top left portion of log.

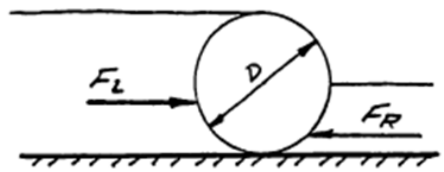
Problem 2.127

Find the net horizontal force on the 4.0-m-long log shown in the figure below.



Solution 2.127

The force F_L on the left side of the log is the horizontal force on the horizontally projected area of the log. This horizontally projected area measures $D = 1.0\,\text{m}$ by $4.0\,\text{m}$ and gives



$$F_L = \rho g h_c A$$

$$= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.5\,\text{m})(1.0\,\text{m}\times 4.0\,\text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 19600\text{ N} = 19.6\text{ kN}$$

The force F_R on the right side of the log is the horizontal force on the horizontally projected area of the lower half of the log. This horizontally projected area measures $\frac{D}{2} = 0.5\text{m}$ by 4.0 m and gives

$$F_R = \rho g h_c A = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.25\,\text{m})(0.5\,\text{m}\times 4.0\,\text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$

$$= 4910\text{ N} = 4.91\text{ kN}$$

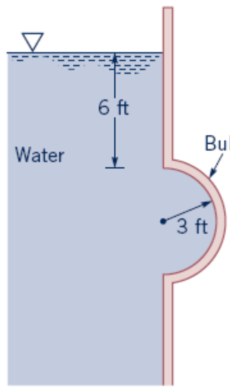
The net horizontal force is

$$F = F_L - F_R = 19.6\,\text{kN} - 4.91\,\text{kN}$$

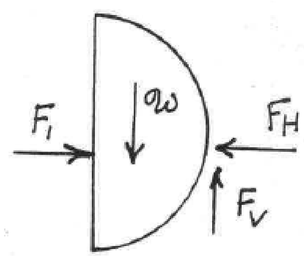
$$\boxed{F = 14.7\,\text{kN},\ \text{acting to right.}}$$

Problem 2.128

An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in the figure below. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.



Solution 2.128



F_H □ horizontal force of wall on fluid

F_V □ vertical force of wall on fluid

$$W = \gamma_{H_2O} V_{vol}$$

$$= \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{\pi (3\,\text{ft})^2}{2}\right)(1\,\text{ft})$$

$$= 882\,\text{lb}$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right)(6\,\text{ft} + 3\,\text{ft})(6\,\text{ft} \times 1\,\text{ft})$$

$$= 3370\,\text{lb}$$

For equilibrium,

$$F_V = W = 882\,\text{lb} \uparrow$$

and $\quad F_H = F_1 = 3370\,\text{lb} \leftarrow$

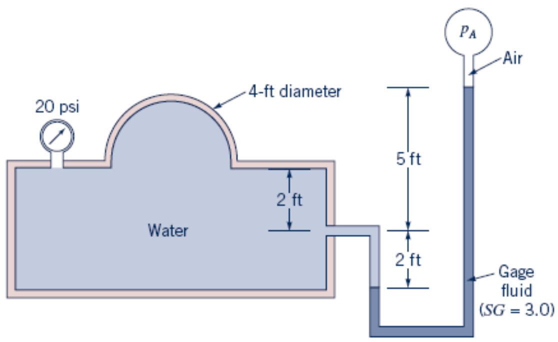
The force the water exerts <u>on</u> the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{\underline{\left(F_H\right)_{wall} = 3370\,\text{lb} \rightarrow}}$$

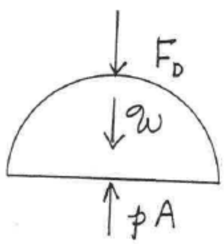
$$\underline{\underline{\left(F_V\right)_{wall} = 882\,\text{lb} \downarrow}}$$

Problem 2.129

A closed tank is filled with water and has a 4-ft-diameter hemispherical dome as shown in the figure below. A U-tube manometer is connected to the tank. Determine the vertical force of the water on the dome if the differential manometer reading is 7 ft and the air pressure at the upper end of the manometer is 12.6 psi.



Solution 2.129



For equilibrium,

$$\sum F_{vertical} = 0$$

So that

$$F_D = pA - W \qquad \text{Eq. (1)}$$

Where F_D is the force the <u>dome exerts on the fluid</u> and p is the water pressure at the base of the dome.

From the manometer,

$$p_A + \gamma_{gf}(7\text{ ft}) - \gamma_{H_2O}(4\text{ ft}) = p$$

So that

$$p = \left(12.6\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) + (3.0)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(7\text{ ft}) - \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ ft})$$

$$= 2880\frac{\text{lb}}{\text{ft}^2}$$

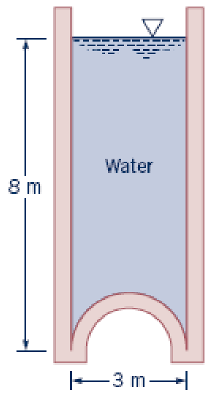
Thus, from Eq.(1) with volume of sphere $= \dfrac{\pi}{6}(\text{diameter})^3$

$$F_D = \left(2880\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{\pi}{4}\right)(4\,\text{ft})^2 - \frac{1}{2}\left[\frac{\pi}{6}(4\,\text{ft})^3\right]\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)$$
$$= 35100\,\text{lb}$$

The force that the vertical force that the water exerts on the dome is $\underline{\underline{35100\,\text{lb}\uparrow}}$.

Problem 2.130

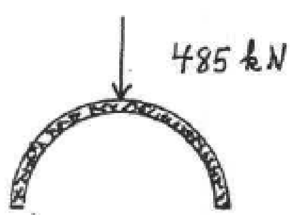
A 3-m-diameter open cylindrical tank contains water and has a hemispherical bottom as shown in the figure below. Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.



Solution 2.130

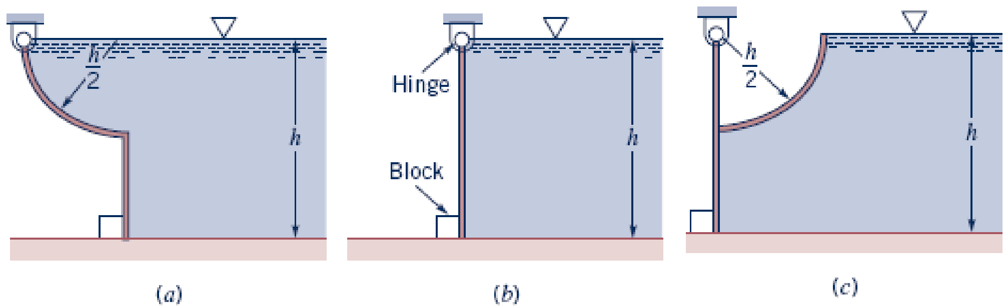
$$
\begin{aligned}
\text{Force} &= \text{weight of water supported by hemispherical bottom} \\
&= \gamma_{H_2O}\left[(\text{volume of cylinder}) - (\text{volume of hemisphere})\right] \\
&= 9.80\frac{\text{kN}}{\text{m}^3}\left[\frac{\pi}{4}(3\,\text{m})^2(8\,\text{m}) - \frac{\pi}{12}(3\,\text{m})^3\right] \\
&= \underline{\underline{485\,\text{kN}}}
\end{aligned}
$$

The force is directed vertically downward, and due to symmetry it acts on the hemisphere along the vertical axis of the cylinder.



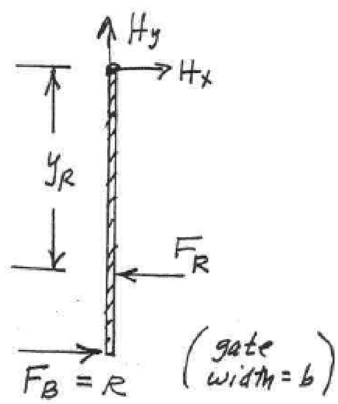
Problem 2.131

Three gates of negligible weight are used to hold back water in a channel of width b as shown in the figure below. The force of the gate against the block for gate (b) is R. Determine (in terms of R) the force against the blocks for the other two gates.



Solution 2.131

For case (b)



$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and

$$y_R = \frac{2}{3}h$$

Thus,

$$\sum M_H = 0$$

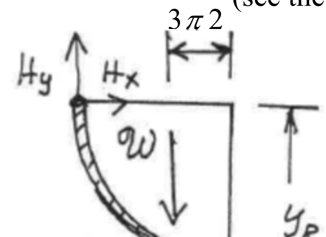
So that

$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$

For case (a) on free-body-diagram shown $\frac{4h}{3\pi 2}$ (see the figure below)



$$F_R = \frac{\gamma h^2 b}{2} \quad \text{(from above) and}$$

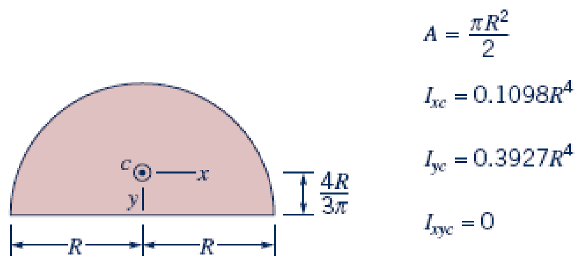
$$y_R = \frac{2}{3}h$$

and

$$W = \gamma \times \cancel{V}_o$$

$$= \gamma\left[\frac{\pi\left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi\gamma h^2 b}{16}$$



Thus,

$$\sum M_H = 0$$

So that

$$W\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B h$$

and

$$\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B h$$

It follows that

$$F_B = \gamma h^2 b(0.390)$$

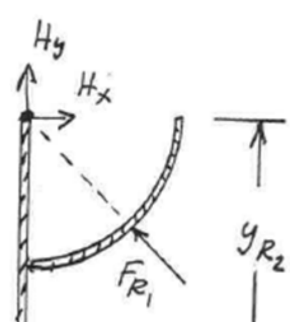
From Eq.(1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$

For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right) = \frac{3}{8}\gamma h^2 b$$

and



$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus,

$$\sum M_H = 0$$

So that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B h$$

or

$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

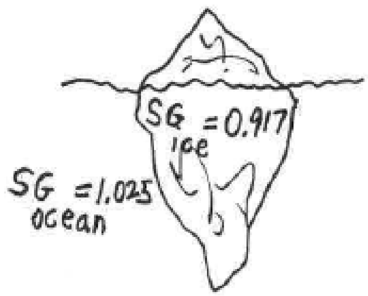
From Eq.(1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

Problem 2.133

An iceberg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?

Solution 2.133



For equilibrium,

W = weight of iceberg = F_B = buoyant force

or

$\forall_{ice}\gamma_{ice} = \forall_{\text{sub}}\gamma_{ocean}$, where $\forall_{sub}$ = volume of ice submerged.

Thus,

$$\frac{\forall_{sub}}{\forall_{ice}} = \frac{\gamma_{ice}}{\gamma_{ocean}} = \frac{SG_{ice}}{SG_{ocean}} = \frac{0.917}{1.025} = 0.895 = \underline{\underline{89.5\%}}$$

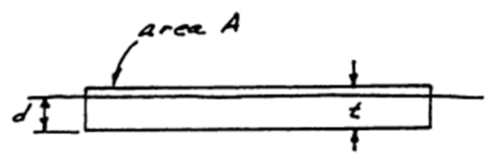
Problem 2.134

A floating 40-in.- thick piece of ice sinks 1 in. with a 500-lb polar bear in the center of the ice. What is the area of the ice in the plane of the water level? For seawater, $S = 1.03$.

Solution 2.134

Without the polar bear on the ice, the submerged depth d of the ice is found by equating the weight of the ice and the buoyant force. Denoting the pure water specific weight by γ and the ice area by A gives

$$F_B = W_{\text{ice}}$$



or

$$W_{\text{ice}} = \gamma SAd\ .$$

The ice sinks an additional depth d' with the bear in the center of the ice. Equating the new buoyant force to the weight of the ice plus bear gives

$$F_B = W_{\text{ice}} + W_{\text{bear}}\ ,$$

$$\gamma SA(d + d') = \gamma SAd + W_{\text{bear}}\ ,$$

or

$$A = \frac{W_{\text{bear}}}{\gamma Sd'} = \frac{500\,\text{lb}}{\left(62.4\,\dfrac{\text{lb}}{\text{ft}^3}\right)(1.03)\left(\dfrac{1}{12}\,\text{ft}\right)} \quad \text{or} \quad \boxed{A = 93.4\,\text{ft}^2}$$

Problem 2.135

A spherical balloon filled with helium at $40\,°\mathrm{F}$ and $20\ \mathrm{psia}$ has a $25-\mathrm{ft}$ diameter. What load can it support in atmospheric air at $40\,°\mathrm{F}$ and $14.696\ \mathrm{psia}$? Neglect the balloon's weight.

Solution 2.135

For static equilibrium, the buoyant force must equal the load. Neglecting the weight of the balloon and assuming air and helium to be ideal gases, the load is

$$L = F_B = \left(\gamma_{air} - \gamma_{He}\right)\cancel{V} = \left(\rho_{air} - \rho_{He}\right) g \cancel{V}$$

$$= \left[\left(\frac{p}{R}\right)_{air} - \left(\frac{p}{R}\right)_{He}\right]\left(\frac{g}{T}\right)\left(\frac{4}{3}\pi R^3\right)$$

Using Table A.4, the numerical values give

$$L = \left[\frac{(14.696\times 144)\dfrac{\mathrm{lb}}{\mathrm{ft}^2}}{\left(53.35\dfrac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{lbm}\cdot{}^{\mathrm{o}}\mathrm{R}}\right)} - \frac{(20\times 144)\dfrac{\mathrm{lb}}{\mathrm{ft}^2}}{\left(386\dfrac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{lbm}\cdot{}^{\mathrm{o}}\mathrm{R}}\right)}\right]\frac{\left(32.2\dfrac{\mathrm{ft}}{\mathrm{sec}^2}\right)\left(\dfrac{4\pi}{3}\right)(12.5\,\mathrm{ft})^3}{\left(500\,{}^{\mathrm{o}}\mathrm{R}\right)\left(\dfrac{32.2\,\mathrm{ft}\cdot\mathrm{lbm}}{\mathrm{lb}\cdot\mathrm{sec}^2}\right)}$$

or

$$\boxed{L = 527\,\mathrm{lb}}$$

Problem 2.136

A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded, its draft (depth of submergence) is 5 ft and with the load of grain the draft is 7 ft. Determine: (**a**) the unloaded weight of the barge, and (**b**) the weight of the grain.

Solution 2.136

(a) For equilibrium,

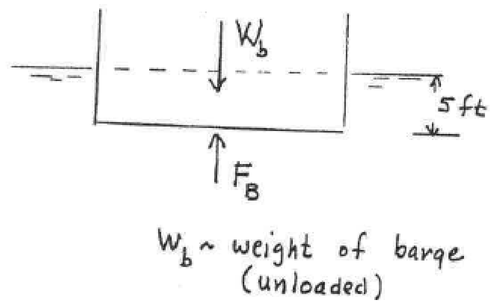
$$\sum F_{vertical} = 0$$

So that

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= (62.4 \frac{\text{lb}}{\text{ft}^3})(5\,\text{ft} \times 28\,\text{ft} \times 90\,\text{ft})$$

$$= \underline{\underline{786000\,\text{lb}}}$$

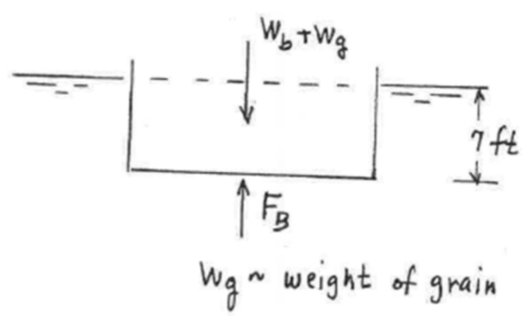


(b) $\sum F_{vertical} = 0$

$$W_B + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = (62.4 \frac{\text{lb}}{\text{ft}^3})(7\,\text{ft} \times 28\,\text{ft} \times 90\,\text{ft}) - 786{,}000\,\text{lb}$$

$$= \underline{\underline{315000\,\text{lb}}}$$



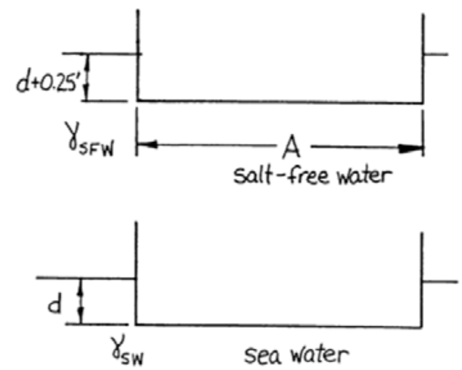
Problem 2.137

A barge is 40 ft wide by 120 ft long. The weight of the barge and its cargo is denoted by W. When in salt-free riverwater, it floats 0.25 ft deeper than when in seawater $\left(\gamma = 64 \text{ lb/ft}^3\right)$. Find the weight W.

Solution 2.137

In both cases, the weight W must equal the weight of the displaced water or

$$W = \gamma_{SFW} A\left(d + 0.25\,\text{ft}\right)$$
$$= \gamma_{SW} A d$$



Soling for d gives

$$\gamma_{SW} A d = \gamma_{SFW} A\left(d + 0.25\,\text{ft}\right)$$

or

$$d = \frac{\left(0.25\,\text{ft}\right)\gamma_{SFW}}{\gamma_{SW} - \gamma_{SFW}} = \frac{\left(0.25\,\text{ft}\right)\left(62.4\dfrac{\text{lb}}{\text{ft}^3}\right)}{\left(64.0 - 62.4\right)\dfrac{\text{lb}}{\text{ft}^3}} = 9.75\,\text{ft}\,.$$

Then

$$W = \gamma_{SW} A d = \left(64.0\frac{\text{lb}}{\text{ft}^3}\right)\left(40 \times 120\right)\text{ft}^2\left(9.75\,\text{ft}\right)$$

$$W = 3.00 \times 10^6\,\text{lb}\left(\frac{\text{short ton}}{2000\,\text{lb}}\right),$$

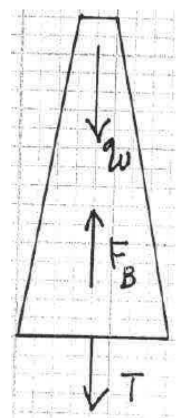
or

$$\boxed{W = 1500\text{ short tons.}}$$

Problem 2.138

When the Tucurui Dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of $8\,\text{ft}$, a top diameter of $2\,\text{ft}$, and a height of $100\,\text{ft}$. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

Solution 2.138



$W \equiv$ weight

$F_B \equiv$ buoyant force

$T \equiv$ tension in ropes

For equilibrium,

$$\sum F_{vertical} = 0$$

So that

$$T = F_B - W \qquad (1)$$

For a truncated cone,

$$\text{Volume} = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

where: r_1 = base radius

r_2 = top radius

h = height

Thus,

$$\cancel{V}_{tree} = \frac{(\pi)(100\,\text{ft})}{3}\left[(4\,\text{ft})^2 + (4\,\text{ft} \times 1\,\text{ft}) + (1\,\text{ft})^2\right]$$

$$= 2200\,\text{ft}^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \cancel{V}_{tree} = (62.4\,\frac{\text{lb}}{\text{ft}^3})\left(2200\,\text{ft}^3\right) = 137000\,\text{lb}$$

For weight,

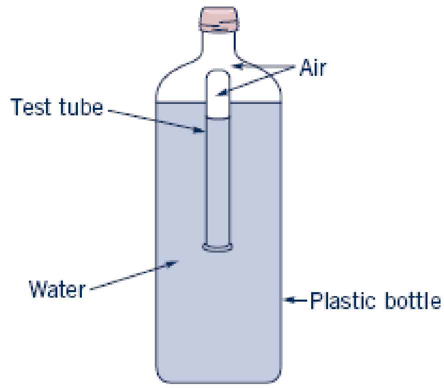
$$W = \gamma_{tree} \times \cancel{V}_{tree} = \left(0.6\right)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(2200\,\text{ft}^3\right) = 82400\,\text{lb}$$

From Eq.(1)

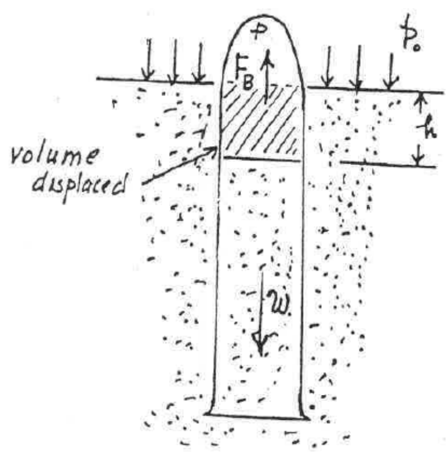
$$T = 137000\,\text{lb} - 82400\,\text{lb} = \underline{\underline{54600\,\text{lb}}}$$

Problem 2.140

An inverted test tube partially filled with air floats in a plastic water-filled soft drink bottle as shown in the figure below. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.



Solution 2.140

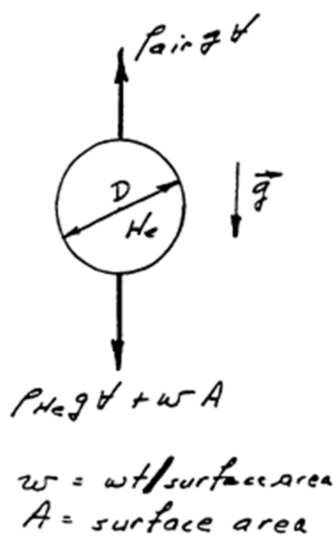


Where the test tube is floating the weight of the tube, W , is balanced by the buoyant force, F_B , as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p , trapped in the tube where $p = p_o + \gamma_{H_2O}h.$ When the bottle is squeezed, the air pressure in the bottle, p_o , is increased slightly and this in turn increases p , the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B . Since W is constant, a decrease in F_B will cause the test tube to sink.

Problem 2.141

A child's balloon is a sphere 1 ft. in diameter. The balloon is filled with helium ($\rho = 0.014\ \text{lbm/ft}^3$). The balloon material weighs $0.008\ \text{lbf/ft}^2$ of surface area. If the child releases the balloon, how high will it rise in the Standard Atmosphere. (Neglect expansion of the balloon as it rises.)

Solution 2.141



A force balance in the vertical direction for the balloon gives

$$+\uparrow \sum F_z = 0 = \rho_{air} g \cancel{V} - \rho_{He} g \cancel{V} - wA$$

for the balloon at rest at its highest elevation. Then

$$\rho_{air} = \frac{\rho_{He} g \cancel{V} + wA}{g \cancel{V}}$$

$$= \rho_{He} + \frac{wA}{g \cancel{V}} = \rho_{He} + \frac{w\left(\pi D^2\right)}{g\left(\dfrac{\pi D^3}{6}\right)}$$

$$= \rho_{He} + \frac{6w}{gD} = 0.014\frac{\text{lbm}}{\text{ft}^3} + \frac{6\left(0.008\dfrac{\text{lb}}{\text{ft}^2}\right)\left(32.2\dfrac{\text{ft}\cdot\text{lbm}}{\text{lb}\cdot\text{sec}^2}\right)}{\left(32.2\dfrac{\text{ft}}{\text{sec}^2}\right)(1.0\,\text{ft})}$$

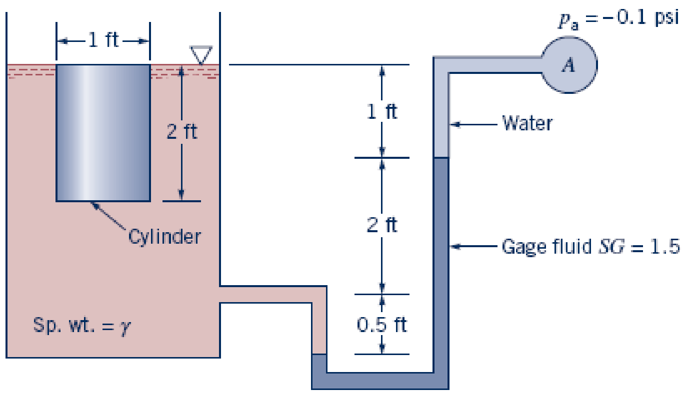
$$= 0.062\frac{\text{lbm}}{\text{ft}^3}.$$

Interpolating Table A.2 for the Standard Atmosphere,

$$z = \text{elevation} = 5000\,\text{ft} + 5000\,\text{ft}\left(\frac{0.06590 - 0.062}{0.06590 - 0.05648}\right)$$

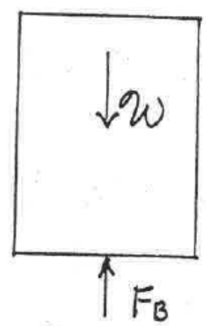
$$\boxed{z = 7070\,\text{ft}}$$

Problem 2.142
A 1-ft-diameter, 2-ft-long cylinder floats in an open tank containing a liquid having a specific weight γ. A U-tube manometer is connected to the tank as shown in the figure below. When the pressure in pipe A is 0.1 psi below atmospheric pressure, the various fluid levels are as shown. Determine the weight of the cylinder. Note that the top of the cylinder is flush with the fluid surface.



Solution 2.142

From a free-body-diagram of the cylinder



$$\sum F_{vertical} = 0$$

So that

$$W = F_B = \gamma\left(\frac{\pi}{4}\right)(1\,\text{ft})^2\,(2\,\text{ft}) \qquad (1)$$
$$= \frac{\pi\gamma}{2}$$

A manometer equation gives,

$$\gamma(3.5\,\text{ft}) - (SG)\left(\gamma_{H_2O}\right)(2.5\,\text{ft}) - \gamma_{H_2O}(1\,\text{ft}) = p_A$$

So that

$$\gamma(3.5\,\text{ft}) - (1.5)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(2.5\,\text{ft}) - \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1\,\text{ft}) = \left(-0.1\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)$$

and

$$\gamma = 80.6\frac{\text{lb}}{\text{ft}^3}$$

Thus, from Eq.(1)

$$W = \left(\frac{\pi}{2}\,\text{ft}^3\right)\left(80.6\,\frac{\text{lb}}{\text{ft}^3}\right) = \underline{\underline{127\,\text{lb}}}$$

Problem 2.143

A not-too-honest citizen is thinking of making bogus gold bars by first making a hollow iridium $(S = 22.5)$ ingot and plating it with a thin layer of gold $(S = 19.3)$ of negligible weight and volume. The bogus bar is to have a mass of $100\,\text{lbm}$. What must be the volumes of the bogus bar and of the air space inside the iridium so that an inspector would conclude it was real gold after weighing it in air and water to determine its density? Could lead $(S = 11.35)$ or platinum $(S = 21.45)$ be used instead of iridium? Would either be a good idea?

Solution 2.143

$$S_x = 22.5\,(\text{iridium})$$
$$S_G = 19.3\,(\text{gold})$$
$$\cancel{V}_{BB} = \cancel{V}_x + \cancel{V}_{AS}$$
$$m_{BB} = m_x = 100\,\text{lbm}$$

Neglect the weight of air in the air space and the buoyant force of air on the bar. The volume of a pure gold bar would be

$$\cancel{V}_{GB} = \frac{W_{GB}}{\gamma_G} \;.$$

The bogus bar must have the same volume and weight as the pure gold bar so it will weigh like a solid gold bar in water. The volume condition gives

$$\cancel{V}_{GB} = \cancel{V}_{BB} = \cancel{V}_{AS} + V_x \,.$$

Since $W_{GB} = W_x$,

$$\cancel{V}_{AS} + \cancel{V}x = \cancel{V}_{GB} = \frac{W_{GB}}{\gamma_G} = \frac{W_x}{\gamma_G}, \quad \cancel{V}_{AS} + \cancel{V}_x = \frac{\gamma_x \cancel{V}_x}{\gamma_G},$$

or

$$\cancel{V}_{AS} = \cancel{V}_x\left(\frac{\gamma_x}{\gamma_G} - 1\right) = \cancel{V}_x\left(\frac{S_x}{S_G} - 1\right).$$

The numerical value of the iridium volume is

$$\cancel{V}_x = \frac{W_x}{\gamma_x} = \frac{W_{GB}}{\gamma_x} = \frac{100\,\text{lb}}{\left(22.5 \times 62.4\,\dfrac{\text{lb}}{\text{ft}^3}\right)} = 0.0712\,\text{ft}^3 \,.$$

The air space volume is

$$\cancel{V}_{AS} = 0.0712\,\text{ft}^3\left(\frac{22.5}{19.3} - 1\right) \quad \text{or} \quad \boxed{\cancel{V}_{AS} = 0.0118\,\text{ft}^3.}$$

The bogus bar volume is

$$\cancel{V}_{BB} = \cancel{V}_{AS} + \cancel{V}_{x} = (0.0118 + 0.0712)\,\text{ft}^3 \quad \text{or} \quad \boxed{\cancel{V}_{BB} = 0.0830\,\text{ft}^3.}$$

Also,

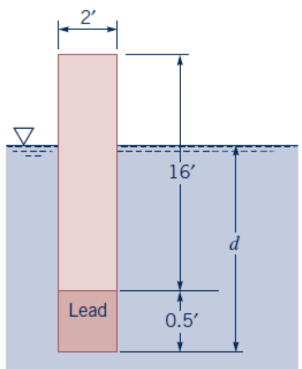
lead will not work since
it is less dense that gold

And

platinum will work since it is
more dense than gold but would
only be used by a not-too-bright
citizen as platinum is more expensive
than gold.

Problem 2.144

A solid cylindrical pine $(S = 0.50)$ spar buoy has a cylindrical lead $(S = 11.3)$ weight attached, as shown in the figure below. Determine the equilibrium position of the spar buoy in seawater (i.e., find d). Is this spar buoy stable or unstable? For seawater, $S = 1.03$.



Solution 2.144

The equilibrium position is found by equating the buoyant force and the body weight (see the sketch below).

$$F_B = W$$

$$\gamma_{sw} dA = \gamma_\ell \ell_\ell A + \gamma_p \ell_p A$$

or

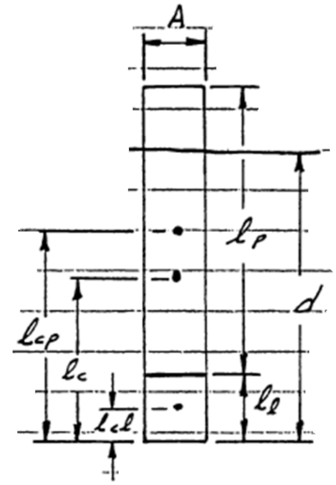
$$d = \frac{\gamma_\ell \ell_\ell + \gamma_p \ell_p}{\gamma_{sw}} = \frac{S_\ell \ell_\ell + S_p \ell_p}{S_{sw}}$$

$$= \frac{11.3(0.5\,\text{ft}) + 0.50(16\,\text{ft})}{1.03} = \boxed{13.3\,\text{ft} = d}$$

Since $d < 13.8\,\text{ft}$ (the total length of the spar buoy), the spar buoy floats. We now have to check the stability of the buoy.

$$I = \frac{\pi}{4}(\text{radius})^4 = \frac{\pi}{4}(1\,\text{ft})^4 = 0.7854\,\text{ft}^4,$$

ℓ_c = distance from bottom of buoy to center of gravity of buoy



$$\ell_c = \frac{\ell_{c\ell}W_\ell + \ell_{cp}W_{cp}}{W_\ell + W_p}$$

$$= \frac{\left(\frac{\ell_\ell}{2}\right)(\gamma_\ell A\ell_\ell) + \left(\ell_\ell + \frac{\ell_p}{2}\right)(\gamma_p A\ell_p)}{\gamma_\ell A\ell_\ell + \gamma_p A\ell_p}$$

$$= \frac{S_\ell \ell_\ell\left(\frac{\ell_\ell}{2}\right) + S_p\ell_p\left(\ell_\ell + \frac{\ell_p}{2}\right)}{S_\ell\ell_\ell + S_p\ell_p}$$

$$\ell_c = \frac{11.3(0.5)(0.25) + 0.5(16)(0.5+8)}{11.3(0.5) + 0.5(16)}\,\text{ft}$$

$\ell_c = 5.09\,\text{ft}$,

$\frac{d}{2} = \frac{13.3\,\text{ft}}{2} = 6.65\,\text{ft}$,

$n = \ell_c - \frac{d}{2} = 5.09\,\text{ft} - 6.65\,\text{ft} = -1.56\,\text{ft}$,

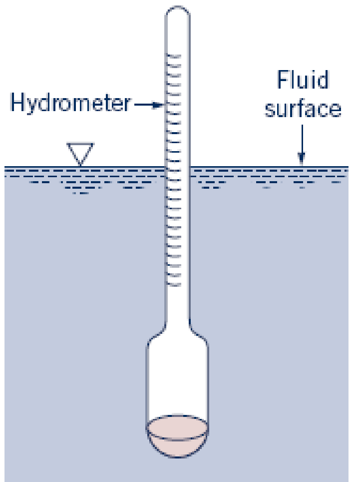
$\cancel{V}_s = Ad = \pi(1\,\text{ft})^2(13.3\,\text{ft}) = 41.8\,\text{ft}^3$,

$m = \frac{I}{\cancel{V}_s} - n = \frac{0.7854\,\text{ft}^4}{41.8\,\text{ft}^3} - (-1.56\,\text{ft}) = 1.58\,\text{ft}$.

Since $m > 0$, the buoy is $\boxed{\text{stable}}$

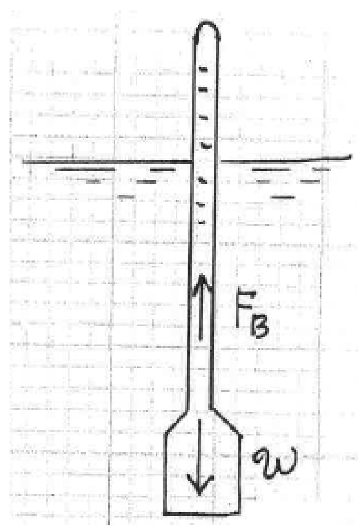
Problem 2.145

When a hydrometer (see the figure below) having a stem diameter of $0.30\,\text{in.}$ is placed in water, the stem protrudes $3.15\,\text{in.}$ above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs $0.042\,\text{lb}$.



Solution 2.145

When the hydrometer is floating its weight, W, is balanced by the buoyant force, F_B. For equilibrium,



$$\sum F_{vertical} = 0$$

Thus, for water

$$F_B = W$$

$$\left(\gamma_{H_2O}\right)\mathcal{V}_1 = W \qquad (1)$$

Where $\mathcal{V}_1$ is the submerged volume. With the new liquid

$$(SG)\left(\gamma_{H_2O}\right)\mathcal{V}_2 = W \quad (2)$$

Combining Eqs.(1) and (2) with W constant

$$\left(\gamma_{H_2O}\right)\mathcal{V}_1 = (SG)\left(\gamma_{H_2O}\right)\mathcal{V}_2$$

And

$$\mathcal{V}_2 = \frac{\mathcal{V}_1}{SG} \qquad (3)$$

From Eq.(1)

$$\mathcal{V}_1 = \frac{W}{\gamma_{H_2O}} = \frac{0.042\,\text{lb}}{62.4\,\dfrac{\text{lb}}{\text{ft}^3}} = 6.73\times10^{-4}\,\text{ft}^3$$

So that from Eq.(3)

$$\mathcal{V}_2 = \frac{6.73\times10^{-4}\,\text{ft}^3}{1.10} = 6.12\times10^{-4}\,\text{ft}^3$$

Thus,

$$\mathcal{V}_1 - \mathcal{V}_2 = (6.73 - 6.12)\times10^{-4}\,\text{ft}^3 = 0.61\times10^{-4}\,\text{ft}^3$$

To obtain this difference the change in length, $\Delta\ell$, is

$$\left(\frac{\pi}{4}\right)(0.30\,\text{in.})^2\,\Delta\ell = \left(0.61\times10^{-4}\,\text{ft}^3\right)\left(1728\,\frac{\text{in.}^3}{\text{ft}^3}\right)$$

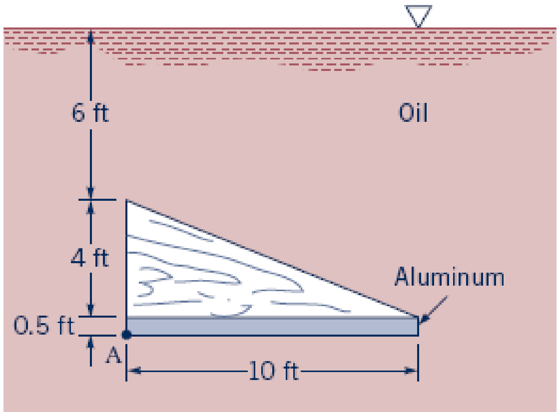
$$\Delta\ell = 1.49\,\text{in.}$$

With the new liquid the stem would protrude

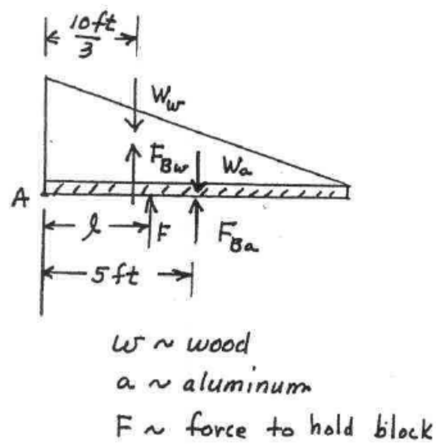
$3.15\,\text{in.} + 1.49\,\text{in.} = \underline{\underline{4.64\,\text{in.}}}$ above the surface.

Problem 2.146

A 2-ft-thick block constructed of wood $(SG = 0.6)$ is submerged in oil $(SG = 0.8)$ and has a 2-ft-thick aluminum (specific weight $= 168\,\text{lb/ft}^3$) plate attached to the bottom as indicated in the figure below. Determine completely the force required to hold the block in the position shown. Locate the force with respect to point *A*.



Solution 2.146



For equilibrium,

$$\sum F_{vertical} = 0$$

So that

$$F = W_w - F_{Bw} + W_a - F_{Ba}$$

where:

$$W_w = (SG_w)(\gamma_{H_2O})\cancel{V_w}$$
$$= (0.6)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(10\,\text{ft}\times 4\,\text{ft}\times 2\,\text{ft}) = 1500\,\text{lb}$$

$$W_a = \left(168\frac{\text{lb}}{\text{ft}^3}\right)(0.5\,\text{ft}\times 10\,\text{ft}\times 2\,\text{ft}) = 1680\,\text{lb}$$

$$F_{Bw} = (SG_{oil})(\gamma_{H_2O})\cancel{V_w} = (0.8)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)(10\,\text{ft}\times 4\,\text{ft}\times 2\,\text{ft}) = 2000\,\text{lb}$$

$$F_{Ba} = (SG_{oil})(\gamma_{H_2O})\cancel{V_a} = (0.8)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(0.5\,\text{ft}\times 10\,\text{ft}\times 2\,\text{ft}) = 499\,\text{lb}$$

Thus,

$$F = 1500\,\text{lb} - 2000\,\text{lb} + 1680\,\text{lb} - 499\,\text{lb} = \underline{\underline{681\,\text{lb upward}}}$$

Also,

$$\sum M_a = 0$$

So that

$$\ell F = \left(\frac{10}{3}\,\text{ft}\right)\left(W_w - F_{Bw}\right) + \left(5\,\text{ft}\right)\left(W_a - F_{Ba}\right)$$

or

$$\ell\left(681\,\text{lb}\right) = \left(\frac{10}{3}\,\text{ft}\right)\left(1500\,\text{lb} - 2000\,\text{lb}\right) + \left(5\,\text{ft}\right)\left(1680\,\text{lb} - 499\,\text{lb}\right)$$

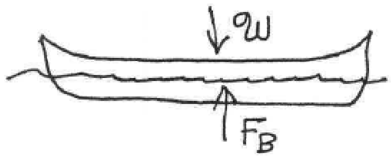
and

$$\underline{\underline{\ell = 6.22\,\text{ft to right of point A}}}$$

Problem 2.147

How much extra water does a $147-\text{lb}$ concrete canoe displace compared to an ultralightweight $38-\text{lb}$ Kevlar canoe of the same size carrying the same load?

Solution 2.147



For equilibrium,

$$\sum F_{vertical} = 0$$

and

$W = F_B = \gamma_{H_2O}\cancel{V}$ and $\cancel{V}$ is displaced volume.

For concrete canoe,

$$147\,\text{lb} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\cancel{V}_c$$

$$\cancel{V}_c = 2.36\,\text{ft}^3$$

For Kevlar canoe,

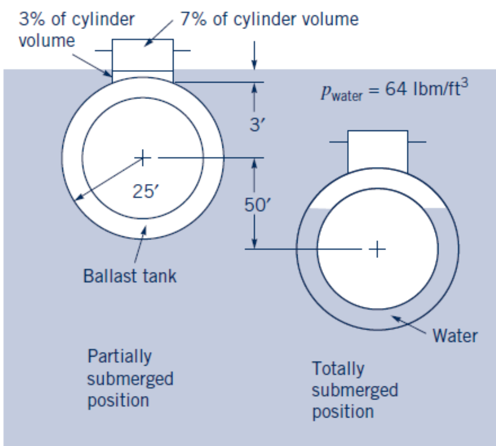
$$38\,\text{lb} = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\cancel{V}_k$$

$$\cancel{V}_k = 0.609\,\text{ft}^3$$

$$\begin{aligned}\text{Extra water displacement} &= 2.36\,\text{ft}^3 - 0.609\,\text{ft}^3 \\ &= \underline{\underline{1.75\,\text{ft}^3}}\end{aligned}$$

Problem 2 .148

A submarine is modeled as a cylinder with a length of 300 ft , a diameter of 50 ft , and a conning tower as shown in the figure below. The submarine can dive a distance of 50 ft from the floating position in about 30 sec . Diving is accomplished by taking water into the ballast tank so the submarine will sink. When the submarine reaches the desired depth, some of the water in the ballast tank is discharged leaving the submarine in "neutral buoyancy" (i.e., it will neither rise nor sink). For the conditions illustrated, find (**a**) the weight of the submarine and (**b**) the volume (or mass) of the water that must be in the ballast tank when the submarine is in neutral buoyancy. For seawater, $S = 1.03$.



Solution 2.148

(a) Denoting the cylinder radius by R , the submarine weight is equal to the buoyant force so

$$W = F_B = \gamma \cancel{V}_{\text{submerged}}$$

$$= \gamma\left(\pi R^2 \ell\right)(1.03)$$

when the submarine is in the partially submerged position. The numerical values give

$$W = \left(64\frac{\text{lb}}{\text{ft}^3}\right)\pi(25\,\text{ft})^2(300\,\text{ft})(1.03) \quad \text{or} \quad \boxed{W = 3.88\times 10^7\ \text{lb}}$$

(b) For neutral buoyancy at the lower depth, the submarine weight W plus the ballast weight W_B must equal the buoyant force so

$$W + W_B = F_B = \gamma\left(\pi R^2 \ell\right)(1.10)$$

or

$$W_B = \gamma\left(\pi R^2 \ell\right)(1.10) - W\,.$$

The ballast volume $\cancel{V}_B = \dfrac{W_B}{\gamma}$ so

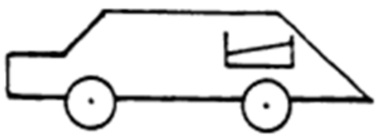
$$\forall_B = \left(\pi R^2 \ell\right)(1.10) - \frac{W}{\gamma} = \pi\left(25\,\text{ft}\right)^2\left(300\,\text{ft}\right)(1.10) - \frac{3.88\times10^7\ \text{lb}}{\left(64\,\dfrac{\text{lb}}{\text{ft}^3}\right)}$$

$$\boxed{\forall_B = 41700\,\text{ft}^3}$$

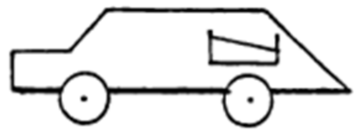
Problem 2.150

When an automobile brakes, the fuel gage indicates a fuller tank than when the automobile is traveling at a constant speed on a level road. Is the sensor for the fuel gage located near the front or rear of the fuel tank? Assume a constant deceleration.

Solution 2.150



accelerating
automobile



decelerating
(braking)
automobile

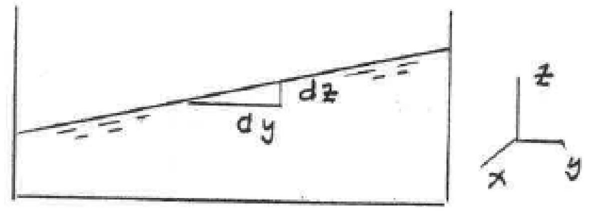
so

$\boxed{\text{sensor located in front of fuel tank.}}$

Problem 2.151

An open container of oil rests on the flatbed of a truck that is traveling along a horizontal road at $55\ \text{mi/hr}$. As the truck slows uniformly to a complete stop in $5\ \text{s}$, what will be the slope of the oil surface during the period of constant deceleration?

Solution 2.151



$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

$$a_y = \frac{\text{final velocity - initial velocity}}{\text{time interval}}$$

$$= \frac{0 - (55\,\text{mph})\left(0.4470\,\frac{\frac{\text{m}}{\text{s}}}{\text{mph}}\right)}{5\,\text{s}} = -4.92\,\frac{\text{m}}{\text{s}^2}$$

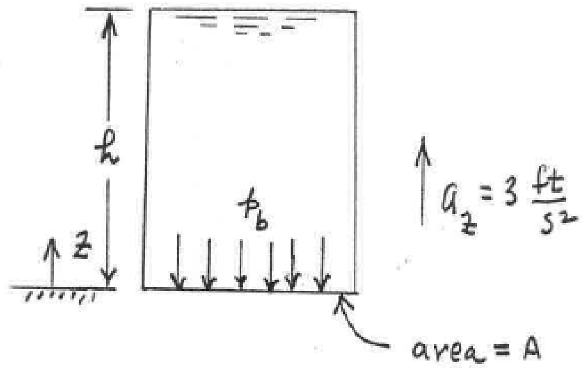
Thus,

$$\frac{dz}{dy} = -\frac{\left(-4.92\,\frac{\text{m}}{\text{s}^2}\right)}{9.81\,\frac{\text{m}}{\text{s}^2} + 0} = \underline{\underline{0.502}}$$

Problem 2.152

A 5-gal, cylindrical open container with a bottom area of 120 in.^2 is filled with glycerin and rests on the floor of an elevator. **(a)** Determine the fluid pressure at the bottom of the container when the elevator has an upward acceleration of 3 ft/s^2. **(b)** What resultant force does the container exert on the floor of the elevator during this acceleration? The weight of the container is negligible. (Note: $1\text{ gal} = 231\text{ in.}^3$)

Solution 2.152



$hA = \text{volume}$

$$h\left(120\text{ in.}^2\right) = (5\text{ gal})\left(\frac{231\text{ in.}^3}{\text{gal}}\right)$$

$h = 9.63\text{ in.}$

(a) $$\frac{\partial p}{\partial z} = -\rho(g + a_z)$$

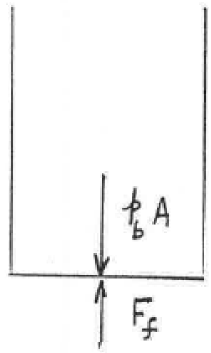
Thus,

$$\int_0^{p_b} dp = -\rho(g + a_z)\int_h^0 dz$$

and

$$\begin{aligned} p_b &= \rho(g + a_z)h \\ &= \left(2.44\frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2\frac{\text{ft}}{\text{s}^2} + 3\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{9.63}{12}\text{ft}\right) \\ &= \underline{\underline{68.9\frac{\text{lb}}{\text{ft}^2}}} \end{aligned}$$

(b)



From free-body-diagram of container,

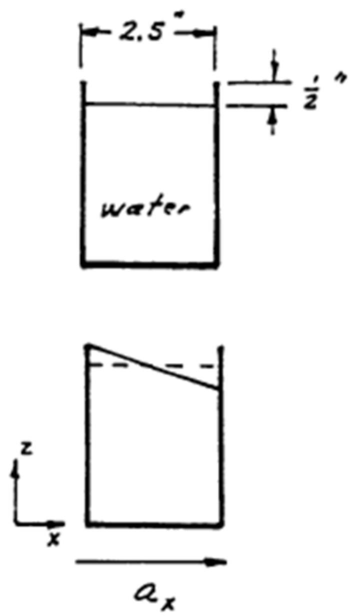
$$
\begin{aligned}
F_f &= p_b A \\
&= \left(68.9\,\frac{\text{lb}}{\text{ft}^2}\right)\left(120\,\text{in.}^2\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right) \\
&= 57.4\,\text{lb}
\end{aligned}
$$

Thus, force of container on floor is $\underline{\underline{57.4\,\text{lb downward}}}$.

Problem 2.153

A plastic glass has a square cross section measuring $2\frac{1}{2}$ in. on a side and is filled to within $\frac{1}{2}$ in. of the top with water. The glass is placed in a level spot in a car with two opposite sides parallel to the direction of travel. How fast can the driver of the car accelerate along a level road without spilling any of the water?

Solution 2.153



Slope of water surface

$$= -\frac{a_{car}}{g}$$

or

$$a_{car} = -g\left(\text{slope}\right)$$

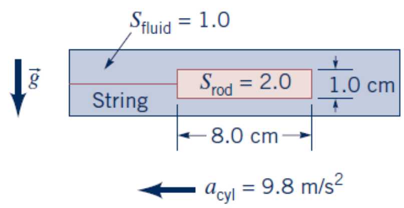
$$= -\left(32.2\frac{\text{ft}}{\text{sec}^2}\right)\left(-\frac{1.0\,\text{in.}}{2.5\,\text{in.}}\right)$$

or

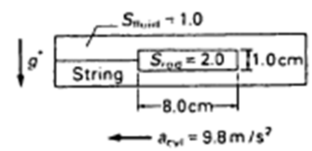
$$\boxed{a_{car} = 12.9\frac{\text{ft}}{\text{sec}^2}}$$

Problem 2.154

The cylinder in the figure below accelerates to the left at the rate of $9.80\,\mathrm{m/s^2}$. Find the tension in the string connecting at rod of circular cross section to the cylinder. The volume between the rod and the cylinder is completely filled with water at $10\,°\mathrm{C}$.



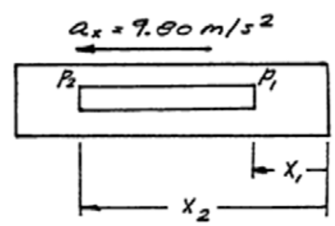
Solution 2.154



FIND Tension in string.

SOLUTION First find the pressure difference in the water over a length $\ell = 8.0\,\mathrm{cm}$. Since gravity is perpendicular to the rod, Eq.(2.41) gives

$$dp = -\rho a_x d_x$$



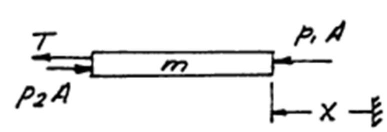
For the x-direction. Integrating gives

$$p_2 - p_1 = -\rho a_x \left(x_2 - x_1\right).$$

For $10\,°\mathrm{C}$ water, Table A.5 gives

$$p_2 - p_1 = -\left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)(8.0\,\mathrm{cm})\left(\frac{\mathrm{m}}{100\,\mathrm{cm}}\right) = -784\frac{\mathrm{N}}{\mathrm{m}^2}$$

We next apply Newton's second law to the rod



$$\overset{\leftarrow}{+}\sum F_x = ma_x,$$

$$T + \left(p_1 - p_2\right)A = \mathrm{ma}_\mathrm{x},$$

or

$$T = \left(p_2 - p_1\right)A + \mathrm{ma}_\mathrm{x}.$$

Assuming the string is not elastic, $a_{x,\mathrm{rod}} = 9.80\frac{\mathrm{m}}{\mathrm{s}^2}$.

Now

$$m = \rho_w S_{\text{rod}} \ell A = \rho_w S_{\text{rod}} \left(\frac{\pi D^2}{4} \right) \ell$$

$$= \left(1000 \frac{\text{kg}}{\text{m}^3} \right) (2.0) \left(\frac{\pi}{4} \right) (1.0\,\text{cm})^2 \,(8.0\,\text{cm}) \left(\frac{= m}{100\,\text{cm}} \right)^3 = 0.0126\,\text{kg}$$

and

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4} (1.0\,\text{cm})^2 \left(\frac{m}{100\,\text{cm}} \right)^2 = 7.854 \times 10^{-5}\,\text{m}^2 .$$

Then

$$T = \left(-784 \frac{\text{N}}{\text{m}^2} \right) \left(7.854 \times 10^{-5}\,\text{m}^2 \right) + (0.0126\,\text{kg}) \left(9.80 \frac{\text{m}}{\text{s}^2} \right)$$

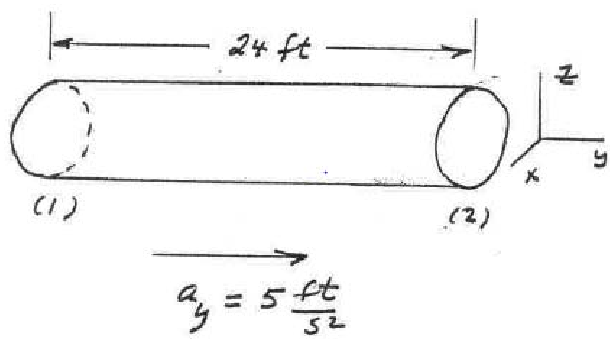
or

$$\boxed{T = 0.062\,\text{N}}$$

Problem 2.155

A closed cylindrical tank that is $8\,\text{ft}$ in diameter and $24\,\text{ft}$ long is completely filled with gasoline. The tank, with its long axis horizontal, is pulled by a truck along a horizontal surface. Determine the pressure difference between the ends (along the long axis of the tank) when the truck undergoes an acceleration of $5\,\text{ft}/\text{s}^2$.

Solution 2.155



$$\frac{\partial p}{\partial y} = -\rho a_y$$

Thus,

$$\int_{p_1}^{p_2} dp = -\rho a_y \int_0^{24} dy$$

Where $p = p_1$ at $y = 0$ and $p = p_2$ at $y = 24\,\text{ft}$,

and

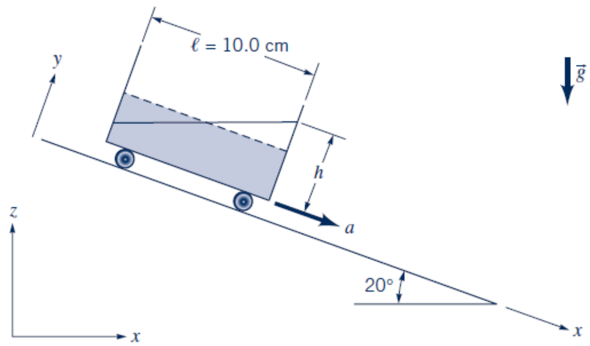
$$\begin{aligned} p_2 - p_1 &= -\rho a_y (24\,\text{ft}) \\ &= -\left(1.32\frac{\text{slugs}}{\text{ft}^3}\right)\left(5\frac{\text{ft}}{\text{s}^2}\right)(24\,\text{ft}) \\ &= -158\frac{\text{lb}}{\text{ft}^2} \end{aligned}$$

or

$$p_1 - p_2 = \underline{\underline{158\frac{\text{lb}}{\text{ft}^2}}}$$

Problem 2.156

The cart shown in the figure below measures 10.0 cm long and 6.0 cm high and has rectangular cross sections. It is half-filled with water and accelerates down a 20° incline plane at $a = 1.0\ \text{m/s}^2$. Find the height h.



Solution 2.156

Unfortunately, there are 2 x-directions in the problem statement.
Noting that the gravity vector is in the negative z-direction, change the label on the axis normal to the z-direction to be "n". Resolving the acceleration along the plane into n,z components:

$$a_z = -a\sin\theta\,,\ \ a_n = a\cos\theta\,,\ \theta = 20^{\text{o}}$$

For rigid-body motion of the fluid in the n,z coordiantes::

$$dp = -\rho a_n dn - \rho\left(g + a_z\right)dz$$
$$dp = -\rho a\cos\theta\ dn - \rho\left(g - a\sin\theta\right)dz = 0 \ \leftarrow \text{along free surface } p = p_{atm}$$

Using trignometirc relationships this equation can be converted into x,y coordinates.

$$dn = dx\cos\theta + dy\sin\theta$$
$$dz = dy\cos\theta - dx\sin\theta$$

$$-\rho a\cos\theta\ dn - \rho\left(g - a\sin\theta\right)dz = 0$$
$$-\rho a\cos\theta\left[dx\cos\theta + dy\sin\theta\right] - \rho\left(g - a\sin\theta\right)\left[dy\cos\theta - dx\sin\theta\right] = 0$$

$$-\left(\rho a\cos^2\theta\right)dx - \left(\rho a\cos\theta\sin\theta\right)dy - \left[\rho\left(g - a\sin\theta\right)\cos\theta\right]dy - \left[\rho\left(g - a\sin\theta\right)\left(-\sin\theta\right)\right]dx = 0$$
$$\left[-\rho a\cos^2\theta + \rho\left(g - a\sin\theta\right)\sin\theta\right]dx + \left[-\rho a\cos\theta\sin\theta - \rho\left(g - a\sin\theta\right)\cos\theta\right]dy = 0$$
$$\left[-\rho a\left(\cos^2\theta + \sin^2\theta\right) + \rho g\sin\theta\right]dx + \left[-\rho g\cos\theta\right]dy = 0$$
$$\left[-\rho a + \rho g\sin\theta\right]dx - \left[\rho g\cos\theta\right]dy = 0$$
$$\left[-a + g\sin\theta\right]dx - \left[g\cos\theta\right]dy = 0$$

Integration yields:

$$\left(-a + g\sin\theta\right)x - \left(g\cos\theta\right)y = -C$$
$$y = \left(-\frac{a}{g\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)x + C$$

The constant of integration can be determined by noting that the container is ½-full:

$$\begin{aligned}\forall_{\text{water}} &= \int_0^{\ell} y\,dx \\ &= \int_0^{\ell}\left[\left(-\frac{a}{g\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)x+C\right]dx \\ &= \left(-\frac{a}{g\cos\theta}+\tan\theta\right)\frac{\ell^2}{2}+C\ell\end{aligned}$$

$$C=\frac{\forall_{\text{water}}}{\ell}+\left(\frac{a}{g\cos\theta}-\tan\theta\right)\frac{\ell}{2}$$

$$C=\frac{(10.0\,\text{cm})(6.0\,\text{cm})/2}{(10.0\,\text{cm})}+\left(\frac{1\frac{\text{m}}{\text{s}^2}}{\left(9.81\frac{\text{m}}{\text{s}^2}\right)\cos 20^{\text{o}}}-\tan 20^{\text{o}}\right)\frac{(10.0\,\text{cm})}{2}$$

$$=1.723\text{ cm}$$

Solving for the the requested length:

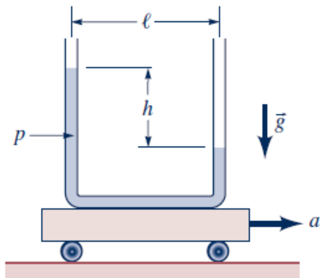
$$y=\left(-\frac{a}{g\cos\theta}+\tan\theta\right)x+C$$

$$h=\left(\frac{-1}{(9.81)\cos 20^{\text{o}}}+\tan 20^{\text{o}}\right)(10\text{ cm})+1.723\text{ cm}=4.277\text{ cm}$$

$$\boxed{h=4.28\text{ cm}}$$

Problem 2.157

The U-tube manometer in the figure below is used to measure the acceleration of the cart on which it sits. Develop an expression for the acceleration of the cart in terms of the liquid height h, the liquid density ρ, the local acceleration of gravity g, and the length ℓ.

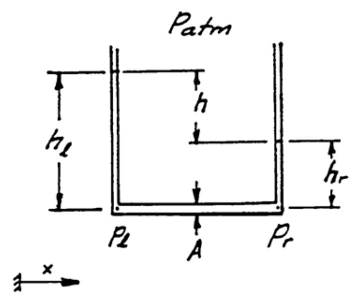


Solution 2.157

Writing Newton's second law in the horizontal direction (x-direction) for the bottom leg of the manometer gives

$$\sum F_x = \mathrm{ma}_\mathrm{x} \,,$$

$$p_\ell A - p_r A = \rho \ell A a \,,$$



or

$$a = \frac{p_\ell - p_r}{\rho \ell}$$

Applying the manometer rule to the two legs of the manometer gives

$$p_\ell = p_{\mathrm{atm}} + \rho g h_\ell$$

and

$$p_r = p_{\mathrm{atm}} + \rho g h_\ell$$

Subtracting gives

$$p_\ell - p_r = \rho g \left(h_\ell - h_r \right) = \rho g h$$

so

$$a = \frac{\rho g h}{\rho \ell} \qquad \text{or} \qquad \boxed{a = g\left(\frac{h}{\ell}\right)}$$

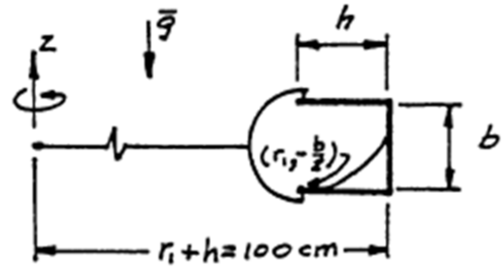
Problem 2.158

A tank has a height of 5.0 cm and a square cross section measuring 5.0 cm on a side. The tank is one third full of water and is rotated in a horizontal plane with the bottom of the tank 100 cm from the center of rotation and two opposite sides parallel to the ground. What is the maximum rotational speed that the tank of water can be rotated with no water coming out of the tank?

Solution 2.158

$$dp = -\rho g dz + \rho\omega^2 r\, dr$$

Since $dp = 0$ along the free surface, the free surface is identified by the equation



$$0 = -\rho g dz + \rho\omega^2 r\, dr$$

or

$$0 = -g dz + \omega^2 r\, dr$$

Integrating gives

$$0 = -g\int_{-\frac{b}{2}}^{z} dz + \omega^2\int_{r_1}^{r} r\, dr\,,$$

$$0 = -g\left(z + \frac{b}{2}\right) + \frac{\omega^2}{2}\left(r^2 - r_1^2\right),$$

or

$$z = -\frac{b}{2} + \frac{\omega^2}{2g}\left(r^2 - r_1^2\right).$$

Recognizing that the volume of water in the rotating tank must equal $\frac{b^2 h}{6}$ gives

$$\frac{b^2 h}{6} = \int_{r_1}^{r_1+h} zb\, dr = b\int_{r_1}^{r_1+h}\left[-\frac{b}{2} + \frac{\omega^2}{2g}\left(r^2 - r_1^2\right)\right] dr\,,$$

$$\frac{b^2 h}{6} = b\left[-\frac{br}{2} + \frac{\omega^2}{2g}\left(\frac{r^3}{3} - r_1^2 r\right)\right]_{r_1}^{r_1+h},$$

$$\frac{b^2h}{6} = b\left[-\frac{bh}{2} + \frac{\omega^2}{2g}\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right)\right],$$

$$\frac{2bh}{3} = \frac{\omega^2}{2g}\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right),$$

or

$$\omega = \sqrt{\frac{4bhg}{3\left(\frac{(r_1+h)^3}{3} - \frac{r_1^3}{3} - r_1^2 h\right)}}.$$

The numerical values give

$$\omega = \sqrt{\frac{4(5\,\text{cm})(5\,\text{cm})\left(981\frac{\text{cm}}{\text{s}^2}\right)}{3\left(\frac{(100\,\text{cm})^3}{3} - \frac{(95\,\text{cm})^3}{3} - (95\,\text{cm})^2(5\,\text{cm})\right)}}$$

$$= \left(3.68\frac{\text{rad}}{\text{s}}\right)\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right) \quad \text{or} \quad \boxed{\omega = 35.1\,\text{rpm}}$$

<u>DISCUSSION</u> Note the that when $r = r_1 + h$,

$$z = -\frac{b}{2} + \frac{\omega^2}{2g}\left((r_1+h)^2 - r_1^2\right) = -\frac{b}{2} + \frac{\omega^2}{2g}\left(2r_1 h + h^2\right).$$

The numerical values give

$$z = \frac{5}{2}\,\text{cm} + \frac{\left(3.68\frac{\text{rad}}{\text{s}}\right)^2}{2\left(981\frac{\text{cm}}{\text{s}^2}\right)}\left[2(95\,\text{cm})(5\,\text{cm}) + (5\,\text{cm})^2\right]$$

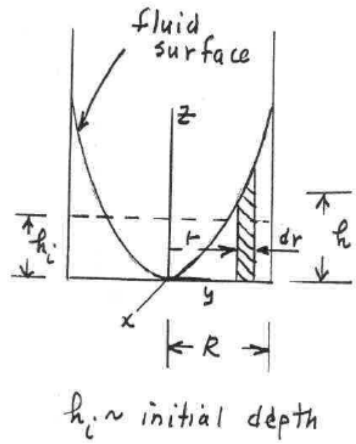
$$= 4.23\,\text{cm}$$

or the assumption indicated in the above figures that the water does not reach the uppermost side of the tank is correct.

Problem 2.159

An open 1-m-diameter tank contains water at a depth of $0.7\,\text{m}$ when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.

Solution 2.159



Equation for surfaces of constant pressure:

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h = 0$ at $r = 0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\cancel{V}_f = \int_0^R 2\pi r h\, dr = \frac{2\pi\omega^2}{2g}\int_0^R r^3\, dr = \frac{\pi\omega^2 R^4}{4g}$$

Since the initial volume, $\cancel{V}_i = \pi R^2 h_i$, must equal the final volume,

$$\cancel{V}_f = \cancel{V}_i$$

So that

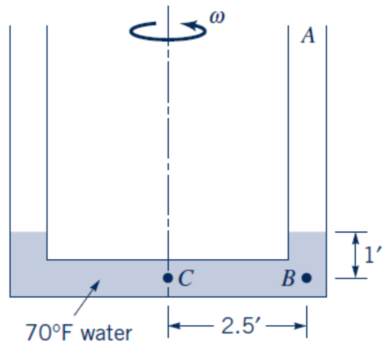
$$\frac{\pi\omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4gh_i}{R^2}} = \sqrt{\frac{4\left(9.81\,\dfrac{\mathrm{m}}{\mathrm{s}^2}\right)(0.7\,\mathrm{m})}{(0.5\,\mathrm{m})^2}} = \underline{\underline{10.5\,\frac{\mathrm{rad}}{\mathrm{s}}}}$$

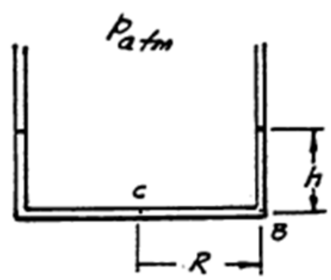
Problem 2.160

The U-tube in the figure below rotates at 2.0 rev/sec. Find the absolute pressures at points C and B if the atmospheric pressure is 14.696 psia. Recall that $70\ °\text{F}$ water evaporates at an absolute pressure of 0.363 psia. Determine the absolute pressures at points C and B if the U-tube rotates at 2.0 rev/sec.



Solution 2.160

Apply the manometer rule to one of the legs to get



$$p_B = p_{\text{atm}} + \rho g h$$

Using Table A.6,

$$p_B = 14.696 \text{ psia} + \frac{\left(62.3 \dfrac{\text{lb}}{\text{ft}^3}\right)(1 \text{ in.})}{\left(1728 \dfrac{\text{in.}^3}{\text{ft}^3}\right)},$$

$$\boxed{p_B = 14.732 \text{ psia}}$$

Section 2.6.2 gives

$$\frac{\partial p}{\partial r} = \rho r \omega^2.$$

Integrating from $r = 0$ to $r = R$ gives

$$\int_{p_C}^{p_B} dp = \rho\omega^2 \int_o^R r\, dr \quad \text{or} \quad p_B - p_C = \frac{\rho\omega^2 R^2}{2}.$$

Then

$$p_c = p_B - \frac{\rho\omega^2 R^2}{2} = 14.732\,\text{psia} = \frac{\left(62.3\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(2.0\,\frac{\text{rev}}{\text{sec}}\right)^2 (2.5\,\text{ft})^2}{2\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right)\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)^2\left(\frac{32.2\,\text{ft}\cdot\text{lbm}}{\text{lb}\cdot\text{sec}^2}\right)}$$

or

$$\boxed{p_c = 8.10\,\text{psia}}$$

Since $p_c > 0.33\,\text{psia}$.

<u>DISCUSSION</u> Note that if p_c were calculated to be less than $0.363\,\text{psia}$, some of the water would vaporize and p_c would be $0.363\,\text{psia}$.

Problem 2.161
A child riding in a car holds a string attached to a floating, helium-filled balloon. As the car decelerates to a stop, the balloon tilts backwards. As the car makes a right-hand turn, the balloon tilts to the right. On the other hand, the child tends to be forced forward as the car decelerates and to the left as the car makes a right-hand turn. Explain these observed effects on the balloon and child.

Solution 2.161

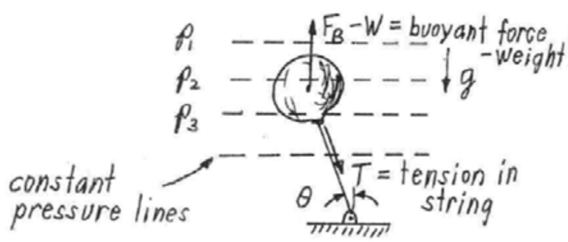


Fig.(1) No acceleration, θ=0 for equilibrium.

A floating balloon attached to a string will align itself so that the string it normal to lines of constant pressure. Thus, if the car is not accelerating, the lines of $p = \text{constant}$ pressure are horizontal (gravity acts vertically down), and the balloon floats "straight up" (i.e. $\theta = 0$). If forced to the side ($\theta \neq 0$), the balloon will return to the vertical ($\theta = 0$) equilibrium position in which the two forces T and F_B-W line up.

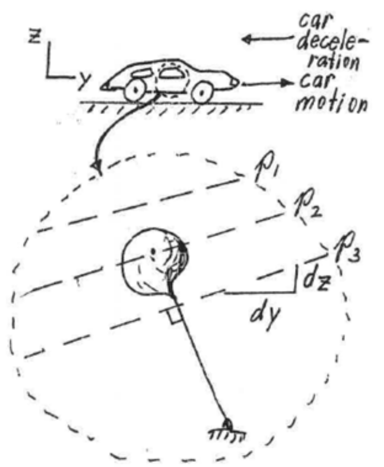


Fig.(2) Balloon aligned so that string is normal to p=constant lines

Consider what happens when the car decelerates with an amount $a_y < 0$. As show by the equation,

$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z},$$

the lines of constant pressure are not horizontal, but have a slope of

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} = -\frac{a_y}{g} > 0 \qquad \text{since } a_z = 0 \text{ and } a_y < 0.$$

Again, the balloon's equilibrium position is with the string normal to $p = \text{const.}$ lines. That is, the balloon tilts back as the car stops.

$a_y = V^2/R$

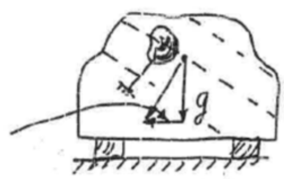


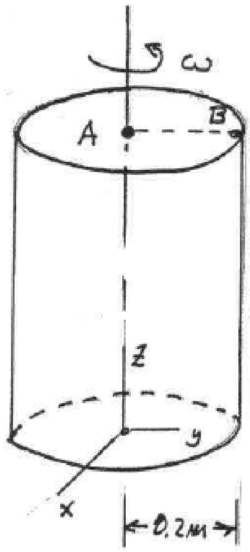
Fig. (3) Left turn; balloon tilts to right

When the car turns, $a_y = \dfrac{V^2}{R}$ (the centrifugal acceleration), the lines of $p = \text{const.}$ are as shown, and the balloon tilts to the outside of the curve.

Problem 2.162

A closed, 0.4-m-diameter cylindrical tank is completely filled with oil $(SG = 0.9)$ and rotates about its vertical longitudinal axis with an angular velocity of $40\ \text{rad/s}$. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.

Solution 2.162



Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant}$$

Since $z_A = z_B$,

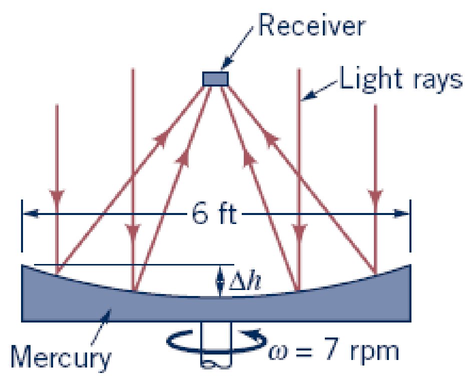
$$p_B - p_A = \frac{\rho\omega^2}{2}\left(r_B^2 - r_A^2\right)$$

$$= \frac{(0.9)\left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(40\frac{\text{rad}}{\text{s}}\right)^2}{2}\left[(0.2\,\text{m})^2 - 0\right]$$

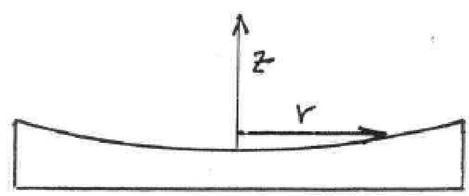
$$= \underline{\underline{28.8\,\text{kPa}}}$$

Problem 2.163

The largest liquid mirror telescope uses a 6-ft-diameter tank of mercury rotating at 7 rpm to produce its parabolic-shaped mirror as shown in the figure below. Determine the difference in elevation of the mercury, Δh, between the edge and the center of the mirror.



Solution 2.163



For free surface of rotating liquid,

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

Let $z = 0$ at $r = 0$ and therefore $\text{constant} = 0$.

Thus, $\Delta h = \Delta z$ for $r = 3\,\text{ft}$ and with

$$\omega = (7\,\text{rpm})\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)$$

$$= 0.733\frac{\text{rad}}{\text{s}}$$

It follows that

$$\Delta h = \frac{\left(0.733\frac{\text{rad}}{\text{s}}\right)^2 (3\,\text{ft})^2}{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)} = \underline{\underline{0.0751\,\text{ft}}}$$