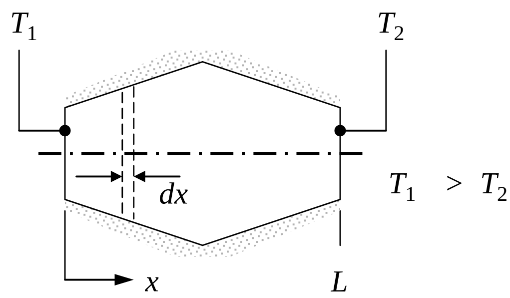
PROBLEM 2.1

KNOWN: Axisymmetric object with varying cross-sectional area and different temperatures at its two ends, insulated on its sides.

FIND: Shapes of heat flux distribution and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Adiabatic sides, (5) No internal heat generation. (6) Surface temperatures T_1 and T_2 are fixed.

ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.12c, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

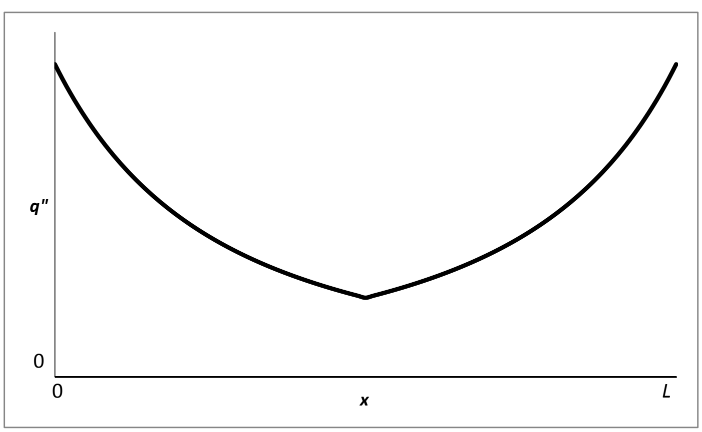
$$q_x \text{ is independent of x.}$$

Therefore

$$q_x = q''_x A_c = \text{constant} \qquad (1)$$

where A_c is the cross-sectional area perpendicular to the x-direction. Therefore the heat flux must be inversely proportional to the cross-sectional area. The radius of the object first increases and then decreases linearly with x, so the cross-sectional area increases and then decreases as x^2.

The resulting heat flux distribution is sketched below. <



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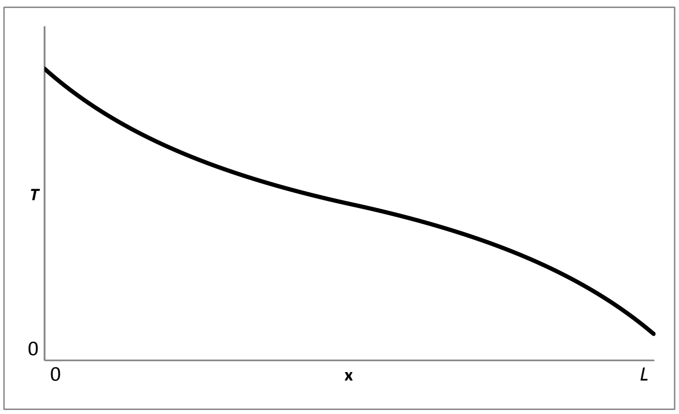
PROBLEM 2.1 (Cont.)

To find the temperature distribution, we can use Fourier's law:

$$q''_x = -k \frac{dT}{dx} \tag{2}$$

Therefore the temperature gradient is negative and its magnitude is proportional to the heat flux. The temperature decreases most rapidly where the heat flux is largest and more slowly where the heat flux is smaller.

Based on the heat flux plot above we can prepare the sketch of the temperature distribution below.



<

The temperature distribution is independent of the thermal conductivity. The heat rate and local heat fluxes are both proportional to the thermal conductivity of the material.

<

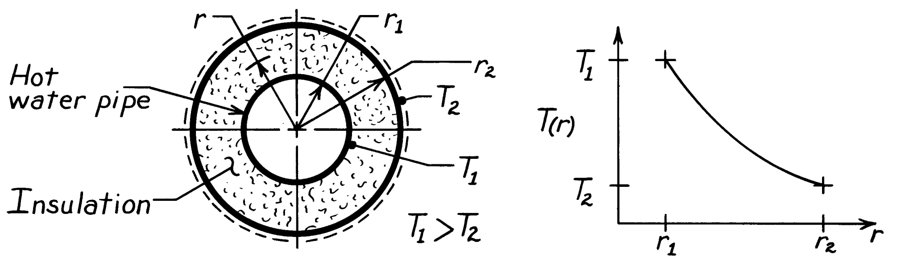
COMMENTS: If the heat rate was fixed the temperature difference, T_1 - T_2, would be inversely proportional to the thermal conductivity. The temperature distribution would be of the same shape, but local temperatures T(x) would vary as the thermal conductivity is adjusted.

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi r\ell)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0.$ Hence

$$q_r = \text{Constant}.$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant}.$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch. <

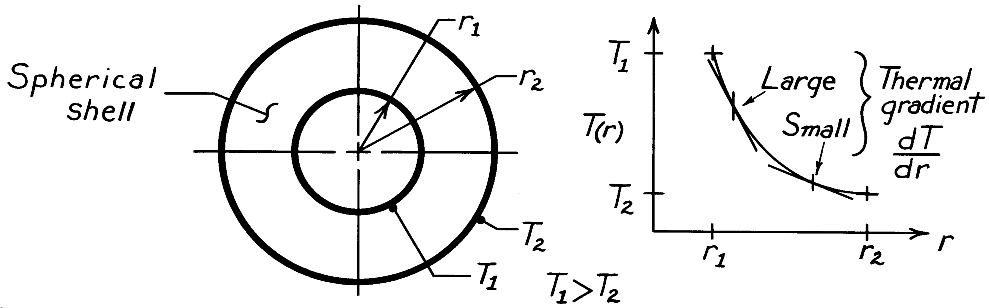
COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

PROBLEM 2.3

KNOWN: A spherical shell with prescribed geometry and surface temperatures.

FIND: Sketch temperature distribution and explain shape of the curve.

SCHEMATIC:



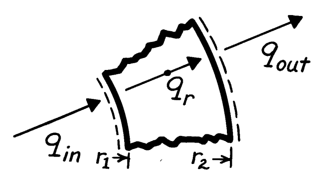
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

$$q_r = -k\,A_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$

where A_r is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields $\dot{E}_{in} = \dot{E}_{out}$, since $\dot{E}_g = \dot{E}_{st} = 0$. Hence,

$$q_{in} = q_{out} = q_r \neq q_r(r).$$



That is, q_r is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

$$r^2\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius squared, r^2, remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch. <

COMMENTS: Note that, for the above conditions, $q_r \neq q_r(r)$; that is, q_r is everywhere constant. How does q_r'' vary as a function of radius?

PROBLEM 2.4

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\,\frac{dT}{dx} = -\left(k_o + aT\right)\frac{dT}{dx}. \qquad (1)$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[\left(k_o + aT\right)\frac{dT}{dx}\right] = 0$$

$$-\left(k_o + aT\right)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

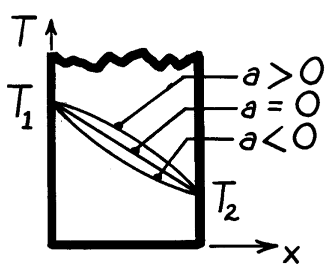
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where} \begin{cases} k_o + aT = k > 0 \\ \left[\dfrac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$a > 0$: $d^2T/dx^2 < 0$

$a = 0$: $d^2T/dx^2 = 0$

$a < 0$: $d^2T/dx^2 > 0$.



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COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

$a > 0$: k decreases with increasing x = > | dT/dx | increases with increasing x

$a = 0$: $k = k_o$ = > dT/dx is constant

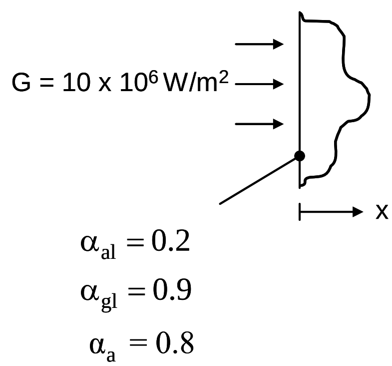
$a < 0$: k increases with increasing x = > | dT/dx | decreases with increasing x.

PROBLEM 2.5

KNOWN: Irradiation and absorptivity of aluminum, glass and aerogel.

FIND: Ability of the protective barrier to withstand the irradiation in terms of the temperature gradients that develop in response to the irradiation.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Constant properties, (c) Negligible emission and convection from the exposed surface.

PROPERTIES: Table A.1, pure aluminum (300 K): k_{al} = 238 W/m·K. Table A.3, glass (300 K): k_{gl} = 1.4 W/m·K.

ANALYSIS: From Eqs. 1.6 and 2.32

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s'' = G_{abs} = \alpha G$$

or

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = -\frac{\alpha G}{k}$$

The temperature gradients at x = 0 for the three materials are: <

Material	$\partial T/\partial x\|_{x=0}$ (K/m)
aluminum	8.4×10^3
glass	6.4×10^6
aerogel	1.6×10^9

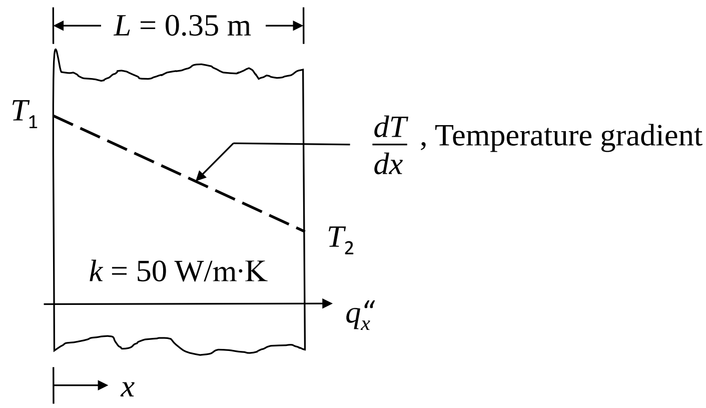
COMMENT: It is unlikely that the aerogel barrier can sustain the thermal stresses associated with the large temperature gradient. Low thermal conductivity solids are prone to large temperature gradients, and are often brittle.

PROBLEM 2.6

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

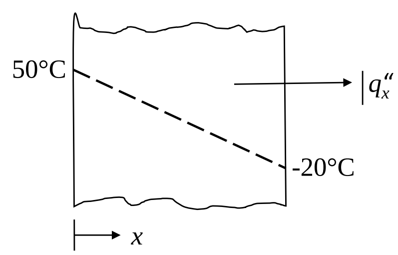
$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \tag{1,2}$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a)

$$\frac{dT}{dx} = \frac{(-20-50)\,K}{0.35m} = -200\ K/m$$ <

$$q''_x = -50\frac{W}{m\cdot K}\times\left[-200\frac{K}{m}\right] = 10.0\ kW/m^2.$$ <



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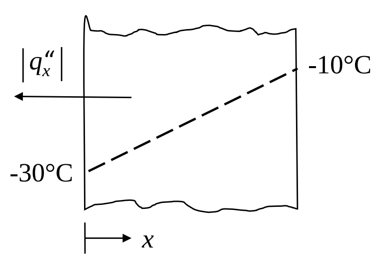
(b)

$$\frac{dT}{dx} = \frac{(-10-(-30))\,K}{0.35m} = 57\ K/m$$ <

$$q''_x = -50\frac{W}{m\cdot K}\times\left[57\frac{K}{m}\right] = -2.86\ kW/m^2.$$ <



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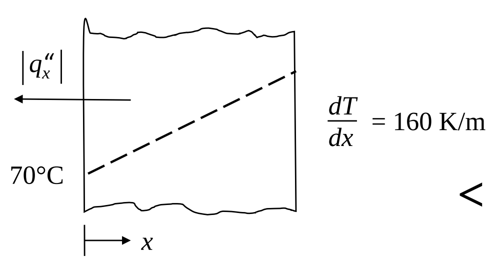
(c)

$$q''_x = -50\frac{W}{m\cdot K}\times\left[160\frac{K}{m}\right] = -8.0\ kW/m^2$$ <

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.35m\times\left[160\frac{K}{m}\right] + 70^\circ C.$$

$$T_2 = 126^\circ C.$$ <



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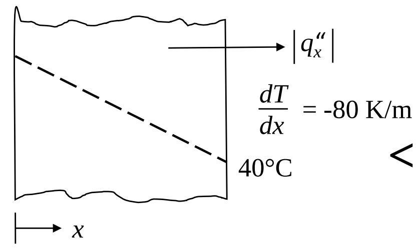
PROBLEM 2.6 (Cont.)

(d) $q''_x = -50 \dfrac{W}{m \cdot K} \times \left[-80 \dfrac{K}{m}\right] = 4.0 \text{ kW/m}^2$ <

$$T_1 = T_2 - L \cdot \frac{dT}{dx} = 40^\circ C - 0.35m \left[-80 \frac{K}{m}\right]$$

$T_1 = 68^\circ C.$ <



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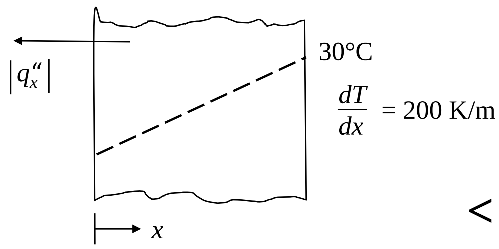
(e) $q''_x = -50 \dfrac{W}{m \cdot K} \times \left[200 \dfrac{K}{m}\right] = -10.0 \text{ kW/m}^2$ <

$T_1 = T_2 - L \cdot \dfrac{dT}{dx} = 30^\circ C - 0.35m \left[200 \dfrac{K}{m}\right] = -40^\circ C.$ <



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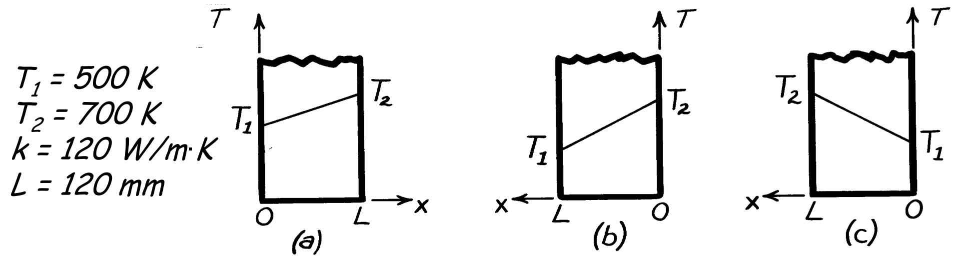
PROBLEM 2.7

KNOWN: Plane wall with prescribed thermal conductivity, thickness, and surface temperatures.

FIND: Heat flux, q_x'', and temperature gradient, dT/dx, for the three different coordinate systems shown.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat flow, (2) Steady-state conditions, (3) No internal generation, (4) Constant properties.

ANALYSIS: The rate equation for conduction heat transfer is

$$q_x'' = -k\frac{dT}{dx}, \tag{1}$$

where the temperature gradient is constant throughout the wall and of the form

$$\frac{dT}{dx} = \frac{T(L) - T(0)}{L}. \tag{2}$$

Substituting numerical values, find the temperature gradients,

(a) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(700-500)\,K}{0.120m} = 1667\ K/m$$ <

(b) $$\frac{dT}{dx} = \frac{T_1 - T_2}{L} = \frac{(500-700)\,K}{0.120m} = -1667\ K/m$$ <

(c) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(700-500)\,K}{0.120m} = 1667\ K/m.$$ <

The heat rates, using Eq. (1) with k = 120 W/m·K, are

(a) $$q_x'' = -120\frac{W}{m\cdot K}\times 1667\ K/m = -200\ kW/m^2$$ <

(b) $$q_x'' = -120\frac{W}{m\cdot K}(-1667\ K/m) = +200\ kW/m^2$$ <

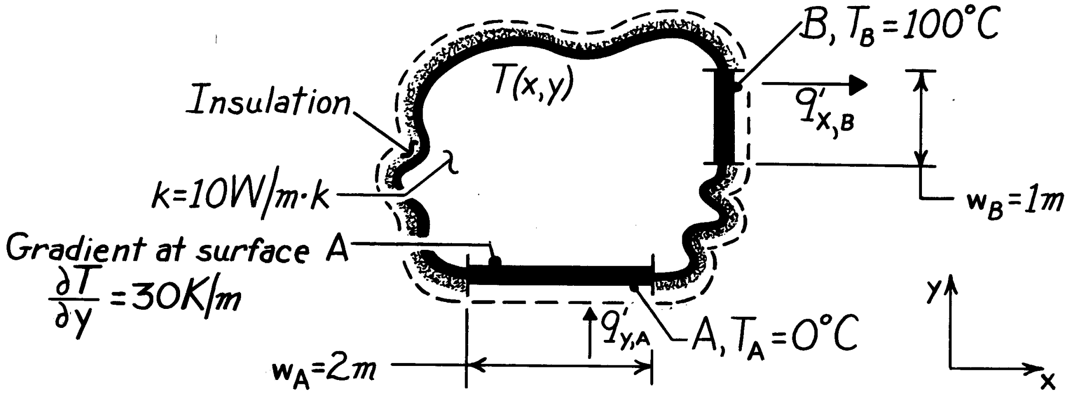
(c) $$q_x'' = -120\frac{W}{m\cdot K}(1667\ K/m) = -200\ kW/m^2$$ <

PROBLEM 2.8

KNOWN: Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

FIND: Temperature gradients, $\partial T/\partial x$ and $\partial T/\partial y$, at the surface B.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

ANALYSIS: At the surface A, the temperature gradient in the x-direction must be zero. That is, $(\partial T/\partial x)_A = 0$. This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$q'_{y,A} = -k \cdot w_A \left.\frac{\partial T}{\partial y}\right]_A = -10\frac{W}{m \cdot K} \times 2m \times 30\frac{K}{m} = -600 W/m.$$

On the surface B, it follows that

$$(\partial T/\partial y)_B = 0 \qquad <$$

in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.12c, on the body, find

$$q'_{y,A} - q'_{x,B} = 0 \qquad \text{or} \qquad q'_{x,B} = q'_{y,A}.$$

Note that,

$$q'_{x,B} = -k \cdot w_B \left.\frac{\partial T}{\partial x}\right]_B$$

and hence

$$(\partial T/\partial x)_B = \frac{-q'_{y,A}}{k \cdot w_B} = \frac{-(-600 \text{ W/m})}{10 \text{ W/m} \cdot \text{K} \times 1\text{m}} = 60 \text{ K/m}. \qquad <$$

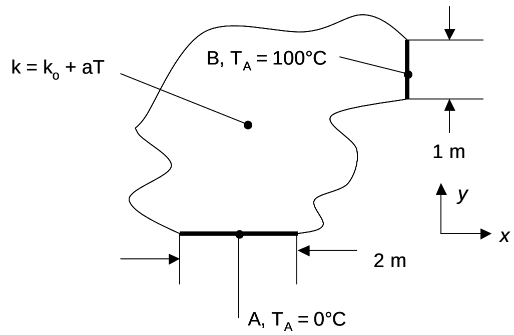
COMMENTS: Note that, in using the conservation requirement, $q'_{in} = +q'_{y,A}$ and $q'_{out} = +q'_{x,B}$.

PROBLEM 2.9

KNOWN: Temperature, size and orientation of Surfaces A and B in a two-dimensional geometry. Thermal conductivity dependence on temperature.

FIND: Temperature gradient $\partial T/\partial y$ at surface A.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No volumetric generation, (3) Two-dimensional conduction.

ANALYSIS: At Surface A, $k_A = k_o + aT_A$ = 10 W/m·K – 10^{-3} W/m·K^2 × 273 K = 9.73 W/m·K while at Surface B, $k_B = k_o + aT_B$ = 10 W/m·K – 10^{-3} W/m·K^2 × 373 K = 9.63 W/m·K. For steady-state conditions, $\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$ which may be written in terms of Fourier's law as

$$-k_B \left.\frac{\partial T}{\partial x}\right|_B A_B = -k_A \left.\frac{\partial T}{\partial y}\right|_A A_A$$

or

$$\left.\frac{\partial T}{\partial y}\right|_A = \left.\frac{\partial T}{\partial x}\right|_B \frac{k_B A_B}{k_A A_A} = 30\text{K/m} \times \frac{9.63}{9.73} \times \frac{1}{2} = 14.85 \text{ K/m}$$

<

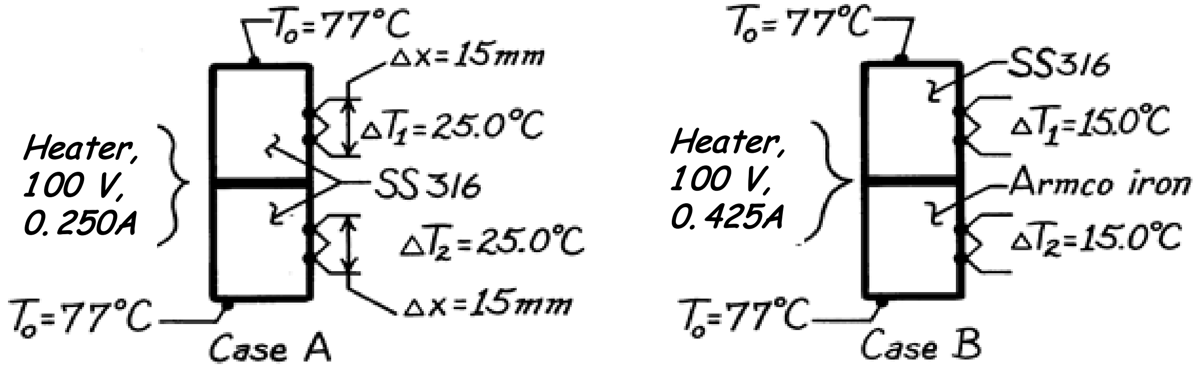
COMMENTS: (1) If the thermal conductivity is not temperature-dependent, then the temperature gradient at A is 15 K/m. (2) Surfaces A and B are both isothermal. Hence, $\partial T/\partial x|_A = \partial T/\partial y|_B = 0$.

PROBLEM 2.10

KNOWN: Electrical heater sandwiched between two identical cylindrical (25 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2*, Stainless steel 316 $\left(\bar{T}=400\text{ K}\right)$: $k_{ss} = 15.2\text{ W/m}\cdot\text{K}$; Armco iron $\left(\bar{T}=380\text{ K}\right)$: $k_{iron} = 67.2\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c \frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c \Delta T} = \frac{0.5\left(100\text{V} \times 0.250\text{A}\right) \times 0.015\text{ m}}{\pi\left(0.025\text{ m}\right)^2 / 4 \times 25.0^\circ\text{C}} = 15.3\text{ W/m}\cdot\text{K}. \quad <$$

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\bar{T} = \left(T_o + T_h\right)/2 = 127^\circ\text{C} = 400\text{ K}. \quad <$$

We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

Continued …

$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.425\text{A} - 15.3\ \text{W/m}\cdot\text{K} \times \frac{\pi(0.025\ \text{m})^2}{4} \times \frac{15.0^\circ\text{C}}{0.015\ \text{m}}$$

$$q_{iron} = (42.5 - 7.51)\,\text{W} = 35.0\ \text{W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{35.0\ \text{W} \times 0.015\ \text{m}}{\pi(0.025\ \text{m})^2 / 4 \times 15.0^\circ\text{C}} = 71.3\ \text{W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\bar{T} = (137 + 77)^\circ\,\text{C}/2 = 107^\circ\text{C} = 380\ \text{K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metals, but must be used with caution on nonmetallic materials.

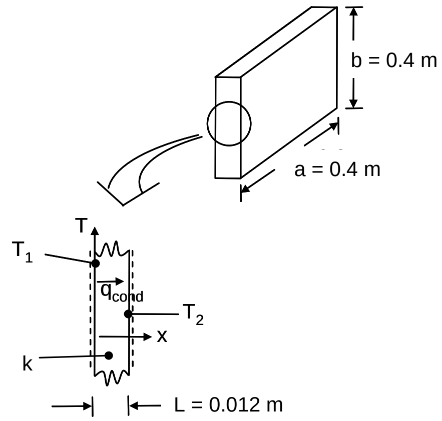
For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.11

KNOWN: Dimensions of and temperature difference across an aircraft window. Window materials and cost of energy.

FIND: Heat loss through one window and cost of heating for 130 windows on 8-hour trip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the x-direction, (3) Constant properties.

PROPERTIES: Table A.3, soda lime glass (300 K): k_{gl} = 1.4 W/m·K.

ANALYSIS: From Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = k\,a\,b\frac{(T_1 - T_2)}{L}$$

For glass,

$$q_{x,g} = 1.4\ \frac{W}{m \cdot K} \times 0.4\ m \times 0.4\ m \times \left[\frac{90°C}{0.012m}\right] = 1680\ W \qquad <$$

The cost associated with heat loss through N windows at a rate of R = \$1/kW·h over a t = 8 h flight time is

$$C_g = Nq_{x,g}Rt = 130 \times 1680\ W \times 1\ \frac{\$}{kW \cdot h} \times 8\ h \times \frac{1kW}{1000W} = \$1750 \qquad <$$

Repeating the calculation for the polycarbonate yields

$$q_{x,p} = 252\ W,\ C_p = \$262 \qquad <$$

while for aerogel,

$$q_{x,a} = 16.8\ W,\ C_a = \$17.5 \qquad <$$

COMMENT: Polycarbonate provides significant savings relative to glass. It is also lighter (ρ_p = 1200 kg/m^3) relative to glass (ρ_g = 2500 kg/m^3). The aerogel offers the best thermal performance and is very light (ρ_a = 2 kg/m^3) but would be relatively expensive.

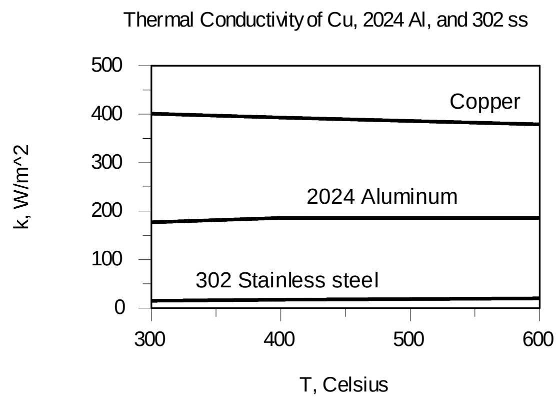
PROBLEM 2.12

KNOWN: Temperatures of various materials.

FIND: (a) Graph of thermal conductivity, k, versus temperature, T, for pure copper, 2024 aluminum and AISI 302 stainless steel for $300 \le T \le 600$ K, (b) Graph of thermal conductivity, k, for helium and air over the range $300 \le T \le 800$ K, (c) Graph of kinematic viscosity, ν, for engine oil, ethylene glycol, and liquid water for $300 \le T \le 360$ K, (d) Graph of thermal conductivity, k, versus volume fraction, φ, of a water-Al_2O_3 nanofluid for $0 \le \varphi \le 0.08$ and $T = 300$ K. Comment on the trends for each case.

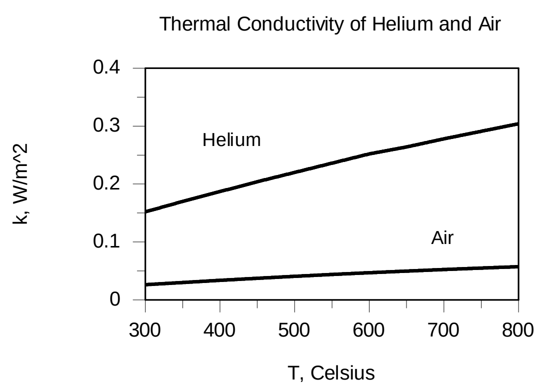
ASSUMPTION: (1) Constant nanoparticle properties.

ANALYSIS: (a) Using the *IHT* workspace of Comment 1 yields



Note the large difference between the thermal conductivities of these metals. Copper conducts thermal energy effectively, while stainless steels are relatively poor thermal conductors. Also note that, depending on the metal, the thermal conductivity increases (2024 Aluminum and 302 Stainless Steel) or decreases (Copper) with temperature.

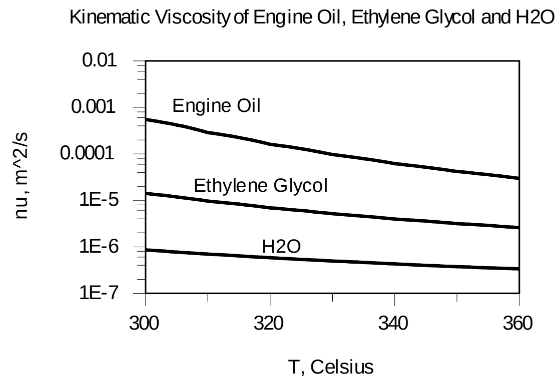
(b) Using the *IHT* workspace of Comment 2 yields



Note the high thermal conductivity of helium relative to that of air. As such, He is sometimes used as a coolant. The thermal conductivity of both gases increases with temperature, as expected from inspection of Figure 2.8.

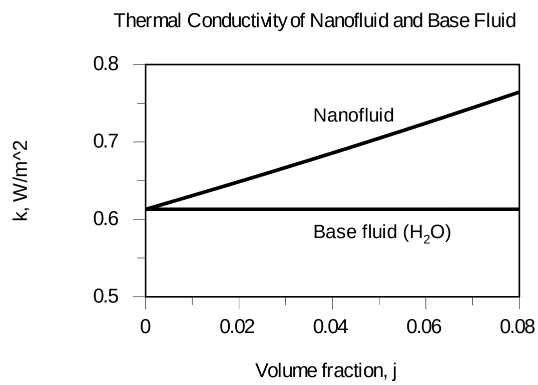
Continued…

(c) Using the *IHT* workspace of Comment 3 yields



The kinematic viscosities vary by three orders of magnitude between the various liquids. For each case the kinematic viscosity decreases with temperature.

(d) Using the *IHT* workspace of Comment 4 yields



Note the increase in the thermal conductivity of the nanofluid with addition of more nanoparticles. The solid phase usually has a higher thermal conductivity than the liquid phase, as noted in Figures 2.5 and 2.9, respectively.

COMMENTS: (1) The *IHT* workspace for part (a) is as follows.

```
// Copper (pure) property functions : From Table A.1
// Units: T(K)
kCu = k_T("Copper",T)                          // Thermal conductivity,W/m·K

// Aluminum 2024 property functions : From Table A.1
// Units: T(K)
kAl = k_T("Aluminum 2024",T)                   // Thermal conductivity,W/m·K

// Stainless steel-AISI 302 property functions : From Table A.1
// Units: T(K)
kss = k_T("Stainless Steel-AISI 302",T)        // Thermal conductivity,W/m·K

T = 300                                        // Temperature, K
```

Continued…

PROBLEM 2.12 (Cont.)

(2) The *IHT* workspace for part (b) follows.

```
// Helium property functions : From Table A.4
// Units: T(K)
kHe = k_T("Helium",T)                       // Thermal conductivity, W/m·K

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
kAir = k_T("Air",T)                         // Thermal conductivity, W/m·K

T = 300                                     // Temperature, K
```

(3) The *IHT* workspace for part (c) follows.

```
// Engine Oil property functions : From Table A.5
// Units: T(K)
nuOil = nu_T("Engine Oil",T)                // Kinematic viscosity, m^2/s

// Ethylene glycol property functions : From Table A.5
// Units: T(K)
nuEG = nu_T("Ethylene Glycol",T)            // Kinematic viscosity, m^2/s

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xH2O =0                                     // Quality (0=sat liquid or 1=sat vapor)
nuH2O = nu_Tx("Water",T,xH2O)               // Kinematic viscosity, m^2/s

T = 300                                     // Temperature, K
```

(4) The *IHT* workspace for part (d) follows.

```
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xH2O =0                                     // Quality (0=sat liquid or 1=sat vapor)
kH2O = k_Tx("Water",T,xH2O)                 // Thermal conductivity, W/m·K
kbf = kH2O
T = 300

j = 0.01                                    // Volume fraction of nanoparticles

//Particle Properties

kp = 36                                     // Thermal conductivity, W/mK

knf = (num/den)*kbf
num = kp + 2*kbf-2*j*(kbf - kp)
den = kp + 2*kbf + j*(kbf - kp)
```

PROBLEM 2.13

KNOWN: Ideal gas behavior for air, hydrogen and carbon dioxide.

FIND: The thermal conductivity of each gas at 300 K. Compare calculated values to values from Table A.4.

ASSUMPTIONS: (1) Ideal gas behavior.

PROPERTIES: Table A.4 (T = 300 K): Air; c_p = 1007 J/kg·K, k = 0.0263 W/m·K, Hydrogen; c_p = 14,310 J/kg·K, k = 0.183 W/m·K, Carbon dioxide; c_p = 851 J/kg·K, k = 0.0166 W/m·K. Figure 2.8: Air; $\mathcal{M}$ = 28.97 kg/kmol, $d = 0.372 \times 10^{-9}$ m, Hydrogen; $\mathcal{M}$ = 2.018 kg/kmol, $d = 0.274 \times 10^{-9}$ m, Carbon Dioxide; $\mathcal{M}$ = 44.01 kg/kmol, $d = 0.464 \times 10^{-9}$ m.

ANALYSIS: For air, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{28.97\ \text{kg/kmol}} = 0.287\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 1.007\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.720\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{1.007}{0.720} = 1.399$$

From Equation 2.12

$$k = \frac{9\gamma - 5}{4}\frac{c_v}{\pi d^2}\sqrt{\frac{\mathcal{M}k_B T}{\mathcal{N}\pi}}$$

$$= \frac{9\times 1.399 - 5}{4}\times\frac{720\ \text{J/kg}\cdot\text{K}}{\pi\left(0.372\times 10^{-9}\text{m}\right)^2}\sqrt{\frac{28.97\ \text{kg/kmol}\times 1.381\times 10^{-23}\ \text{J/K}\ \times\ 300\ \text{K}}{\pi\ \times\ 6.024\ \times\ 10^{23}\ \text{mol}^{-1}\times 1000\ \text{mol/kmol}}}$$

$$= 0.025\frac{\text{W}}{\text{m}\cdot\text{K}}$$ <

The thermal conductivity of air at T = 300 K is 0.0263 W/m·K. Hence, the computed value is within 5 % of the reported value.

For hydrogen, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{2.018\ \text{kg/kmol}} = 4.120\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 14.31\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 4.120\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 10.19\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{14.31}{10.19} = 1.404$$

Equation 2.12 may be used to calculate

$$k = 0.173\ \frac{\text{W}}{\text{m}\cdot\text{K}}$$ <

Continued...

PROBLEM 2.13 (Cont.)

The thermal conductivity of hydrogen at T = 300 K is 0.183 W/m·K. Hence, the computed value is within 6 % of the reported value.

For <u>carbon dioxide</u>, the ideal gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315 \text{ kJ/kmol} \cdot \text{K}}{44.01 \text{ kg/kmol}} = 0.189 \ \frac{\text{kJ}}{\text{kg} \cdot \text{K}};$$

$$c_v = c_p - R = 0.851 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - 0.189 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.662 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}; \quad \gamma = \frac{c_p}{c_v} = \frac{0.851}{0.662} = 1.285$$

Equation 2.12 may be used to calculate

$$k = 0.0158 \ \frac{\text{W}}{\text{m} \cdot \text{K}}$$ <

The thermal conductivity of carbon dioxide at T = 300 K is 0.0166 W/m·K. Hence, the computed value is within 5 % of the reported value.

COMMENTS: The preceding analysis may be used to *estimate* the thermal conductivity at various temperatures. However, the analysis is not valid for extreme temperatures or pressures. For example, (1) the thermal conductivity is predicted to be independent of the pressure of the gas. As pure vacuum conditions are approached, the thermal conductivity will suddenly drop to zero, and the preceding analysis is no longer valid. Also, (2) for temperatures considerably higher or lower than normally-encountered room temperatures, the agreement between the predicted and actual thermal conductivities can be poor. For example, for carbon dioxide at T = 600 K, the predicted thermal conductivity is k = 0.0223 W/m·K, while the actual (tabular) value is k = 0.0407 W/m·K. For extreme temperatures, thermal correction factors must be included in the predictions of the thermal conductivity.

PROBLEM 2.14

KNOWN: Thermal conductivity of helium.

FIND: The helium temperature. Compare to value from Table A.4.

ASSUMPTIONS: (1) Ideal gas behavior.

PROPERTIES: Table A.4: Helium; $\mathcal{M}$ = 4.003 kg/kmol, c_p = 5.193 kJ/kg·K (independent of temperature). Figure 2.8: Helium, d = 0.219 nm.

ANALYSIS: For helium, the gas constant, specific heat at constant volume, and ratio of specific heats are:

$$R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.315\ \text{kJ/kmol}\cdot\text{K}}{4.003\ \text{kg/kmol}} = 2.077\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}};$$

$$c_v = c_p - R = 5.193\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 2.077\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 3.116\frac{\text{kJ}}{\text{kg}\cdot\text{K}};\quad \gamma = \frac{c_p}{c_v} = \frac{5.193}{3.166} = 1.667$$

From Equation 2.12

$$k = \frac{9\gamma - 5}{4}\frac{c_v}{\pi d^2}\sqrt{\frac{\mathcal{M}k_B T}{\mathcal{N}\pi}}$$

$$T = \frac{\mathcal{N}\pi}{\mathcal{M}k_B}\left[\frac{4k\pi d^2}{(9\gamma - 5)c_v}\right]^2$$

$$= \frac{6.024\times 10^{23}\ \text{mol}^{-1}\times 1000\ \text{mol/kmol}\times\pi}{4.003\ \text{kg/kmol}\times 1.381\times 10^{-23}\ \text{J/K}}\times\left[\frac{4\times 0.15\ \text{W/m}\cdot\text{K}\times\pi\times\left(0.219\times 10^{-9}\text{m}\right)^2}{(9\times 1.667 - 5)\times 3166\ \text{J/kg}\cdot\text{K}}\right]^2$$

$= 288$ K <

From Table A.4, the thermal conductivity of helium is 0.15 W/m·K when T = 294 K. <
The computed value of 288 K is within 2% of the reported value.

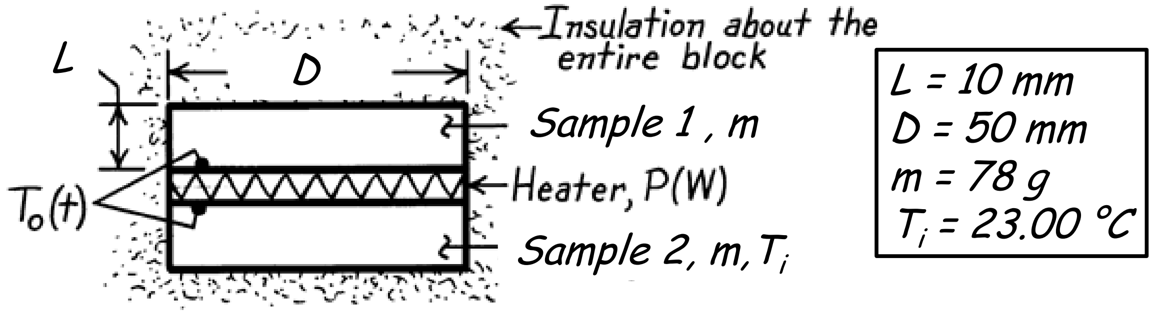
COMMENTS: The preceding analysis may be used to *estimate* the thermal conductivity at various temperatures. However, the analysis is not valid for extreme temperatures or pressures. For example, (1) the thermal conductivity is predicted to be independent of the pressure of the gas. As pure vacuum conditions are approached, the thermal conductivity will suddenly drop to zero, and the preceding analysis is no longer valid. Also, (2) for temperatures considerably higher or lower than normally-encountered room temperatures, the agreement between the predicted and actual thermal conductivities can be poor. For example, for carbon dioxide at T = 600 K, the predicted thermal conductivity is k = 0.0223 W/m·K, while the actual (tabular) value is k = 0.0407 W/m·K. For extreme temperatures, thermal correction factors must be included in the predictions of the thermal conductivity.

PROBLEM 2.15

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q''_o for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

FIND: Specific heat and thermal conductivity of the test sample material. From the properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:

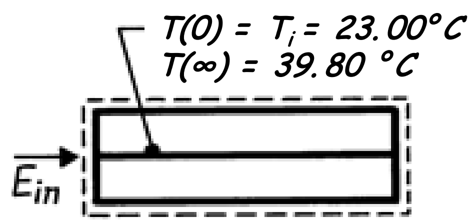


ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: The density of the sample is

$$\rho = \frac{m}{\pi D^2 L/4} = \frac{0.078\text{ kg}}{\pi \times (0.05\text{ m})^2 \times 0.01\text{ m}/4} = 3970\text{ kg/m}^3$$

Now consider a control volume about the two samples (of mass 2m) and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = 2mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the power condition and the final temperature T_f is known. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{2m\left[T(\infty) - T_i\right]} = \frac{20\text{ W} \times 100\text{ s}}{2 \times 0.078\text{ kg}\left[39.80\text{-}23.00\right]^\circ\text{C}}$$

$$c_p = 763\text{ J/kg}\cdot\text{K}$$ <

The transient thermal response of the heater is given by

Continued …

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{60\text{ s}}{\pi \times 3970\text{ kg/m}^3 \times 764\text{ J/kg}\cdot\text{K}}\left[\frac{2\times 5093\text{ W/m}^2}{(26.77\text{ - }23.00)^\circ\text{ C}}\right]^2 = 46.0\text{ W/m}\cdot\text{K} \qquad <$$

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2\left(\pi D^2/4\right)} = \frac{20\text{ W}}{2\left(\pi\times 0.050^2/4\right)\text{m}^2} = 5093\text{ W/m}^2.$$

With the following properties now known,

ρ = 3970 kg/m^3 $\qquad$ c_p = 763 J/kg·K $\qquad$ k = 46 W/m·K

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

From Table A.2, the first entry, sapphire, has properties at 300 K corresponding to those found for the samples. $<$

PROBLEM 2.16

KNOWN: Five materials at 300 K.

FIND: Heat capacity, ρc_p. Which material has highest thermal energy storage per unit volume. Which has lowest cost per unit heat capacity.

ASSUMPTIONS: Constant properties.

PROPERTIES: *Table A.3*, Common brick (T = 300 K): ρ = 1920 kg/m^3, c_p = 835 J/kg·K. *Table A.1*, Plain carbon steel (T = 300 K): ρ = 7854 kg/m^3, c_p = 434 J/kg·K. *Table A.5*, Engine oil (T = 300 K): ρ = 884.1 kg/m^3, c_p = 1909 J/kg·K. *Table A.6*, Water (T = 300 K): $\rho = 1/v_f$ = 997 kg/m^3, c_p = 4179 J/kg·K. *Table A.3*, Soil (T = 300 K): ρ = 2050 kg/m^3, c_p = 1840 J/kg·K.

ANALYSIS: The values of heat capacity, ρc_p, are tabulated below.

Material	Common brick	Plain carbon steel	Engine oil	Water	Soil
Heat Capacity (kJ/m^3·K)	1603	3409	1688	4166	3772

<

Thermal energy storage refers to either sensible or latent energy. The change in sensible energy per unit volume due to a temperature change ΔT is equal to $\rho c_p \Delta T$. Thus, for a given temperature change, the heat capacity values in the table above indicate the relative amount of sensible energy that can be stored in the material.

Of the materials considered, water has the largest capacity for sensible energy storage. <

Various materials also have the potential for latent energy storage due to either a solid-liquid or liquid-vapor phase change. Taking water as an example, the latent heat of fusion is 333.7 kJ/kg. With a density of $\rho \approx$ 1000 kg/m^3 at 0°C, the latent energy per unit volume associated with the solid-liquid phase transition is 333,700 kJ/m^3. This corresponds to an 80°C temperature change in the liquid phase. The latent heat of vaporization for water is very large, 2257 kJ/kg, but it is generally inconvenient to use a liquid-vapor phase change for thermal energy storage because of the large volume change.

The two materials with the largest heat capacity are also inexpensive. The consumer price of soil is around \$15 per cubic meter, or around \$4 per MJ/K. The consumer price of water is around \$0.40 per cubic meter, or around \$0.10 per MJ/K. In a commercial application, soil could probably be obtained much more inexpensively.

Therefore we conclude that water has the lowest cost per unit heat capacity of the materials considered. <

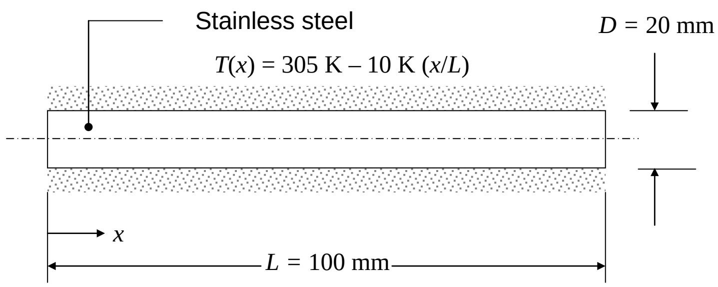
COMMENTS: (1) Many materials used for latent thermal energy storage are characterized by relatively low thermal conductivities. Therefore, although the materials may be attractive from the thermodynamics point of view, it can be difficult to deliver energy to the solid-liquid or liquid-vapor interface because of the poor thermal conductivity of the material. Hence, many latent thermal energy storage applications are severely hampered by heat transfer limitations. (2) Most liquids and solids have a heat capacity which is in a fairly narrow range of around 1000 – 4000 kJ/m^3·K. Gases have heat capacities that are orders of magnitude smaller.

PROBLEM 2.17

KNOWN: Diameter, length, and mass of stainless steel rod, insulated on its exterior surface other than ends. Temperature distribution.

FIND: Heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) Constant properties.

ANALYSIS: The heat flux can be found from Fourier's law,

$$q_x'' = -k\frac{dT}{dx}$$

Table A.1 gives values for the thermal conductivity of stainless steels, however we are not told which type of stainless steel the rod is made of, and the thermal conductivity varies between them. We do know the mass of the rod, and can use this to calculate its density:

$$\rho = \frac{M}{V} = \frac{M}{\pi D^2 L/4} = \frac{0.248 \text{ kg}}{\pi \times (0.02 \text{ m})^2 \times 0.1 \text{ m}/4} = 7894 \text{ kg/m}^3$$

From Table A.1, it appears that the material is AISI 304 stainless steel. The temperature of the rod varies from 295 K to 305 K. Evaluating the thermal conductivity at 300 K, k = 14.9 W/m·K. Thus,

$$q_x'' = -k\frac{dT}{dx} = -k(-b/L) = 14.9 \text{ W/m}\cdot\text{K} \times 10 \text{ K}/0.1 \text{ m} = 1490 \text{ W/m}^2 \qquad <$$

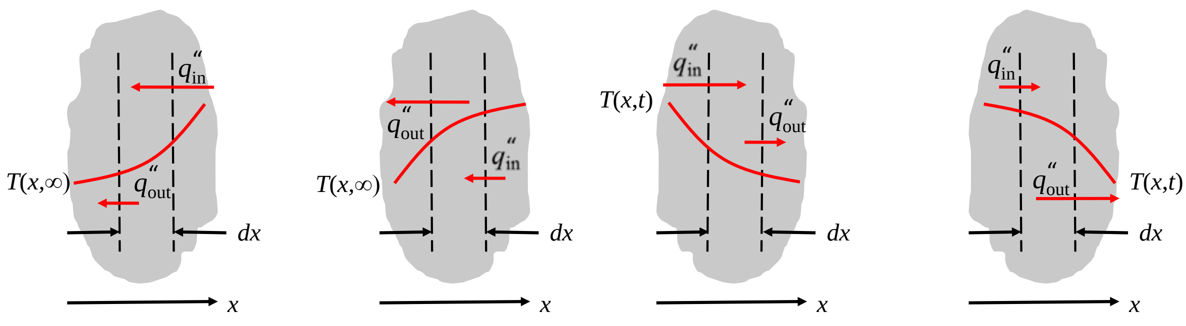
COMMENTS: If the temperature of the rod varies significantly along its length, the thermal conductivity will vary along the rod as much or more than the variation in thermal conductivities between the different stainless steels.

PROBLEM 2.18

KNOWN: Temperature distribution in a plane wall. Whether conditions are steady-state or transient.

FIND: (a) Whether thermal energy is being generated within the wall, and if so, whether it is positive or negative. (b) Whether the volumetric generation rate is positive or negative. (c) and (d) Whether the temperature is increasing or decreasing with time.

SCHEMATIC:



(*a*) (*b*) (*c*) (*d*)

ASSUMPTIONS: (1) One-dimensional conduction in *x*-direction, (2) Constant properties.

ANALYSIS: An energy balance on the differential control volume can be expressed as

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \qquad (1)$$

The heat flux is given by Fourier's law,

$$q''_x = -k\frac{dT}{dx} \qquad (2)$$

Assuming constant thermal conductivity, the slope of the temperature distribution indicates the magnitude and direction of the heat flux according to Eq. (2). A positive slope means heat is flowing from right to left and vice versa. The magnitudes and directions of the heat fluxes are illustrated in the schematic above. With this background we can consider each scenario in turn.

(a) Conditions are steady-state, therefore $dE_{st}/dt = 0$ in Eq. (1). Since the slope of the temperature distribution is positive, heat is flowing from right to left in the schematic. With the slope higher at the right than the left, more heat is entering at the right than leaving at the left. Therefore heat generation must exist and must be negative. <

(b) Conditions are steady-state, therefore $dE_{st}/dt = 0$. The slope of the temperature distribution is positive and is smaller at the right than the left, therefore less heat is entering at the right than leaving at the left. Therefore heat generation must exist and must be positive. <

Continued …

PROBLEM 2.18 (Cont.)

(c) Conditions are transient. There is no heat generation, therefore $\dot{E}_g = 0$ in Eq. (1). Since the slope of the temperature distribution is negative, heat is flowing from left to right in the schematic. With the slope steeper at the left than the right, more heat is entering at the left than leaving at the right. Therefore, there is net heat transfer into the control volume and $dE_{st}/dt > 0$. Thus the temperature is increasing with time. <

(d) Conditions are transient. There is no heat generation. The slope of the temperature distribution is negative and is smaller at the left than the right, therefore less heat is entering at the left than leaving at the right. Therefore, there is net heat transfer out of the control volume and $dE_{st}/dt < 0$. Thus the temperature is decreasing with time. <

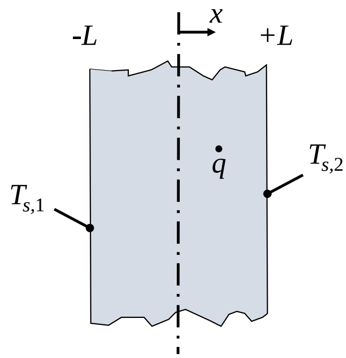
COMMENTS: If the thermal conductivity is not constant, it is not possible to tell whether the heat flux is higher or lower at the two sides of the control volume.

PROBLEM 2.19

KNOWN: Temperature distribution in a plane wall experiencing uniform volumetric heat generation.

FIND: Whether the steady-state form of the heat diffusion equation is satisfied. Expression for the heat flux distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in *x*-direction, (2) Constant properties.

ANALYSIS: The heat diffusion equation with constant properties is given by Eq. 2.21. Under one-dimensional, steady-state conditions this reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0 \tag{1}$$

The temperature distribution is given in the problem statement as

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{2}$$

This temperature distribution can be substituted into Eq. (1) to see if it is satisfied. Taking the derivative of Eq. (2) twice,

$$\frac{\partial T}{\partial x} = \frac{\dot{q}L^2}{2k}\left(-\frac{2x}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{1}{L} \tag{3}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\dot{q}L^2}{2k}\left(-\frac{2}{L^2}\right) \tag{4}$$

Substituting Eq. (4) into the heat diffusion equation, Eq.(1), yields

$$\frac{\dot{q}L^2}{2k}\left(-\frac{2}{L^2}\right) + \frac{\dot{q}}{k} = -\frac{\dot{q}}{k} + \frac{\dot{q}}{k} = 0 \tag{5}$$

Therefore the steady-state form of the heat diffusion equation is satisfied. $<$

Continued …

PROBLEM 2.19 (Cont.)

The heat flux is given by Fourier's Law, with the temperature derivative from Eq. (3). Therefore,

$$q''(x) = -k\frac{\partial T}{\partial x} = -k\left[\frac{\dot{q}L^2}{2k}\left(-\frac{2x}{L^2}\right) + \frac{T_{s,2}-T_{s,1}}{2}\frac{1}{L}\right] = \dot{q}x + \frac{k(T_{s,1}-T_{s,2})}{2L} \qquad (6) \quad <$$

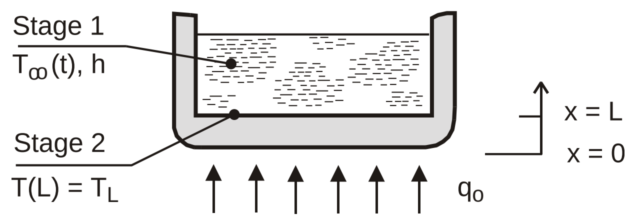
COMMENTS: If there is no heat generation, the temperature distribution in Eq. (2) reduces to the familiar linear form and the heat flux (Eq. (6)) becomes the well-known result from Chapter 1.

PROBLEM 2.20

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions:

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $T(x,0) = T_i$

Stage 2

Heat Equation:

$$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions:

$$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

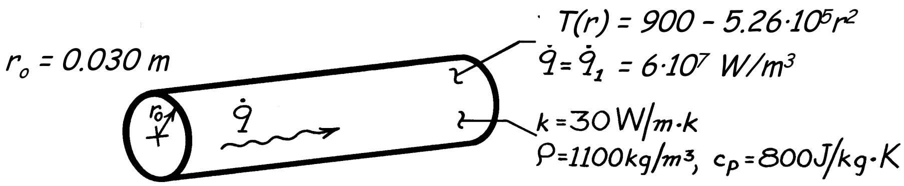
COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx mc_p\, dT_\infty/dt$, where m and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/mc_p)\, t$.

PROBLEM 2.21

KNOWN: Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 6 \times 10^7\ \text{W/m}^3$.

FIND: (a) Steady-state centerline and surface heat transfer rates per unit length, q'_r. (b) Initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $\dot{q}_1$ to $\dot{q}_2 = 10^8\ \text{W/m}^3$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the r direction, (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 6 \times 10^7\ \text{W/m}^3$.

ANALYSIS: (a) From the rate equations for cylindrical coordinates,

$$q''_r = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}.$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

or

$$q'_r = -2\pi kr\frac{\partial T}{\partial r} \tag{1}$$

where $\partial T/\partial r$ may be evaluated from the prescribed temperature distribution, T(r).

At r = 0, the gradient is $(\partial T/\partial r) = 0$. Hence, from Equation (1) the heat rate is

$$q'_r(0) = 0. \qquad <$$

At $r = r_o$, the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -2\left[5.26 \times 10^5 \frac{K}{m^2}\right](r_o) = -2\left(5.26 \times 10^5\right)(0.030\text{m})$$

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -31.6 \times 10^3\ \text{K/m}.$$

Continued ...

Hence, the heat rate at the outer surface ($r = r_o$) per unit length is

$$q'_r(r_o) = -2\pi[30\ \text{W/m}\cdot\text{K}](0.030\text{m})\left[-31.6\times10^3\ \text{K/m}\right]$$

$$q'_r(r_o) = 1.785\times10^5\ \text{W/m}. \qquad <$$

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is determined by the following form of the heat equation, Equation 2.26

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right]+\dot{q}_2 = \rho c_p\frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right]+\dot{q}_2\right].$$

However, initially (at t = 0), the temperature distribution is given by the prescribed form, $T(r) = 800 - 5.26\times10^5 r^2$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\left(-10.52\times10^5\cdot r\right)\right]$$

$$= \frac{k}{r}\left(-21.04\times10^5\cdot r\right)$$

$$= 30\ \text{W/m}\cdot\text{K}\left[-21.04\times10^5\ \text{K/m}^2\right]$$

$$= -6.31\times10^7\ \text{W/m}^3\ \left(\text{the original } \dot{q}=\dot{q}_1\right).$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100\ \text{kg/m}^3\times800\ \text{J/kg}\cdot\text{K}}\left[-6.31\times10^7+10^8\right]\ \text{W/m}^3$$

or

$$\frac{\partial T}{\partial t} = 41.91\ \text{K/s} \qquad <$$

COMMENTS: (1) The value of $(\partial T/\partial t)$ will decrease with increasing time, until a new steady-state condition is reached and once again $(\partial T/\partial t) = 0$. (2) By applying the energy conservation requirement, Equation 1.12c, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - E'_{out} + \dot{E}'_{gen} = 0$.

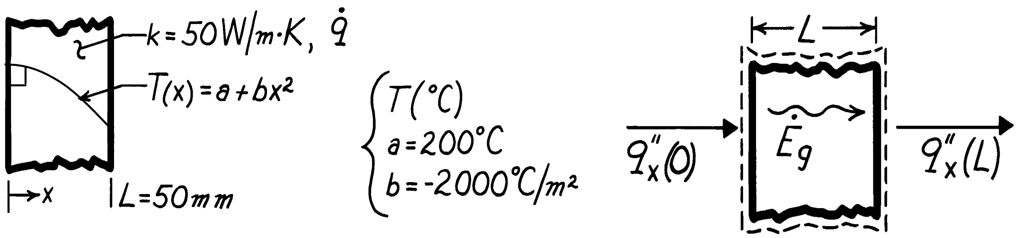
Hence $q'_r(0) - q'_r(r_o) = -\dot{q}_1\left(\pi r_o^2\right)$.

PROBLEM 2.22

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.21 re-written as

$$\dot{q}=-k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q}=-k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right]=-k\frac{d}{dx}[2bx]=-2bk$$

$$\dot{q}=-2\left(-2000^\circ\text{C/m}^2\right)\times 50\ \text{W/m}\cdot\text{K}=2.0\times 10^5\ \text{W/m}^3. \qquad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x)=-k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x)=-k\frac{d}{dx}\left[a+bx^2\right]=-2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0)=0 \qquad <$$

$$q''_x(L)=-2kbL=-2\times 50\text{W/m}\cdot\text{K}\left(-2000^\circ\text{C/m}^2\right)\times 0.050\text{m}$$

$$q''_x(L)=10{,}000\ \text{W/m}^2. \qquad <$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0 \qquad q''_x(0)-q''_x(L)+\dot{q}L=0$$

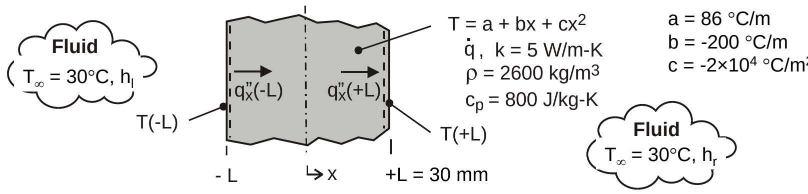
$$\dot{q}=\frac{q''_x(L)-q''_x(0)}{L}=\frac{10{,}000\ \text{W/m}^2-0}{0.050\text{m}}=2.0\times 10^5\,\text{W/m}^3.$$

PROBLEM 2.23

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation $\dot{q}$ while convection occurs at both of its surfaces.

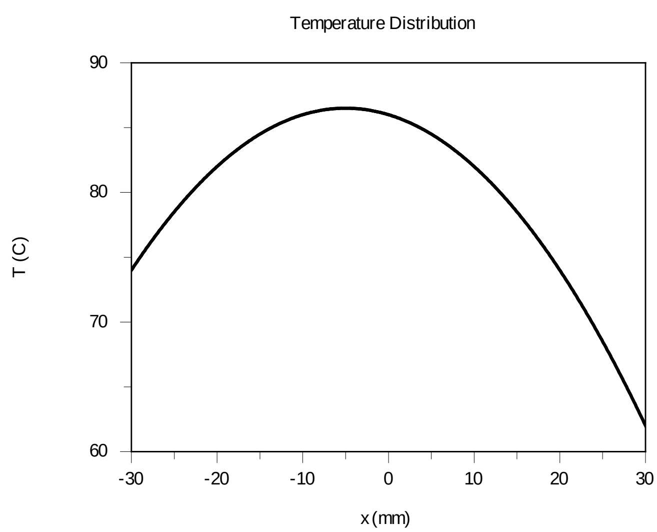
FIND: (a) Sketch the temperature distribution, T(x), and identify significant physical features, (b) Determine $\dot{q}$, (c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces x = L and x = +L, (e) Obtain an expression for the heat flux distribution, $q''_x(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q}$ = 0), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, T(-5 mm) = 86.5°C, (3) the gradient at the x = +L surface is greater than at x = -L. Find also that T(-L) = 74°C and T(+L) = 62°C for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.21, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x)=a+bx+cx^2$$

$$\frac{d}{dx}(0+b+2cx)+\frac{\dot{q}}{k}=(0+2c)+\frac{\dot{q}}{k}=0$$

Continued ...

$$\dot{q} = -2ck = -2\left(-2\times10^4 \,^\circ\mathrm{C/m^2}\right)5\,\mathrm{W/m\cdot K} = 2\times10^5\,\mathrm{W/m^3} \qquad <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k\frac{dT}{dx} \qquad \text{where} \qquad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k\left[0 + b + 2cx\right]_{x=-L} = -\left[b - 2cL\right]k$$

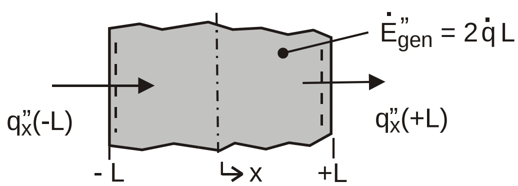
$$q''_x(-L) = -\left[-200^\circ\mathrm{C/m} - 2\left(-2\times10^4 \,^\circ\mathrm{C/m^2}\right)0.030\mathrm{m}\right]\times 5\,\mathrm{W/m\cdot K} = -5000\,\mathrm{W/m^2} \qquad <$$

$$q''_x(+L) = -(b + 2cL)k = +7000\,\mathrm{W/m^2} \qquad <$$

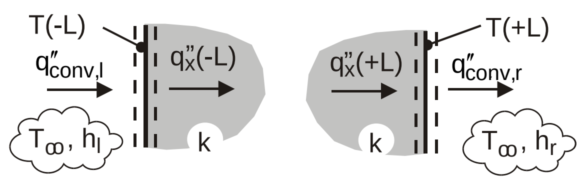
From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L \overset{?}{=} 0 \quad \text{or} \quad -5000\,\mathrm{W/m^2} - 7000\,\mathrm{W/m^2} + 12{,}000\,\mathrm{W/m^2} = 0$$

where $2\dot{q}L = 2\times2\times10^5\,\mathrm{W/m^3}\times0.030\ \mathrm{m} = 12{,}000\,\mathrm{W/m^2}$, so the equality is satisfied



Part (c) Overall energy balance



Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r, for the left- and right-hand boundaries (x = -L and x= +L, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for T(-L) and T(+L).

$$q''_{conv,\ell} = q''_x(-L)$$

$$h_l\left[T_\infty - T(-L)\right] = h_l\left[30 - 74\right]\mathrm{K} = -5000\,\mathrm{W/m^2} \qquad h_l = 114\,\mathrm{W/m^2\cdot K} \qquad <$$

$$q''_{conv,r} = q''_x(+L)$$

$$h_r\left[T(+L) - T_\infty\right] = h_r\left[62 - 30\right]\mathrm{K} = +7000\,\mathrm{W/m^2} \qquad h_r = 219\,\mathrm{W/m^2\cdot K} \qquad <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k\frac{dT}{dx} = -k\left[0 + b + 2cx\right]$$

$$q''_x(x) = -5\,\mathrm{W/m\cdot K}\left[-200^\circ\mathrm{C/m} + 2\left(-2\times10^4 \,^\circ\mathrm{C/m^2}\right)\right]x = 1000 + 2\times10^5 x \qquad <$$

Continued …

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x\left(x_{max}\right) = 0$,

$$x_{max} = -\frac{1000\,W/m^2}{2\times10^5\ W/m^3} = -5.00\times10^{-3}m = -5\,mm \qquad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q}$ = 0, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \rho c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still T(x) = a + bx + cx^2. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}''_{st} = k\frac{\partial}{\partial x}\left[0 + b + 2cx\right] = k\left[0 + 2c\right] = 5\,W/m\cdot K\times 2\left(-2\times10^4{}^\circ C/m^2\right) = -2\times10^5\ W/m^3 \qquad <$$

(g) With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x,\infty) = T_\infty = 30°C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.12b. The "initial" state is that corresponding to the steady-state temperature distribution, T_i, and the "final" state has T_f = 30°C. We've used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \quad E''_{in} = 0.$$

$$E''_{out} = \rho c_p \int_{-L}^{+L}\left(T_i - T_\infty\right)dx$$

$$E''_{out} = \rho c_p \int_{-L}^{+L}\left[a + bx + cx^2 - T_\infty\right]dx = \rho c_p\left[ax + bx^2/2 + cx^3/3 - T_\infty x\right]_{-L}^{+L}$$

$$E''_{out} = \rho c_p\left[2aL + 0 + 2cL^3/3 - 2T_\infty L\right]$$

$$E''_{out} = 2600\,kg/m^3\times 800\,J/kg\cdot K\left[2\times 86°C\times 0.030m + 2\left(-2\times10^4{}^\circ C/m^2\right)\right.$$
$$\left.\left(0.030m\right)^3/3 - 2\left(30°C\right)0.030m\right]$$

$$E''_{out} = 6.24\times10^6\,J/m^2 \qquad <$$

COMMENTS: (1) In part (a), note that the temperature gradient is larger at x = + L than at x = - L. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

Continued …

PROBLEM 2.23 (Cont.)

(2) In evaluating the conduction heat fluxes, $q''_x(x)$, it is important to recognize that this flux is in the positive x-direction. See how this convention is used in formulating the energy balance in part (c).

(3) It is good practice to represent energy balances with a schematic, clearly defining the system or surface, showing the CV or CS with dashed lines, and labeling the processes. Review again the features in the schematics for the energy balances of parts (c & d).

(4) Re-writing the heat diffusion equation introduced in part (b) as

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}\right)+\dot{q}=0$$

recognize that the term in parenthesis is the heat flux. From the differential equation, note that if the differential of this term is a constant $(\dot{q}/k)$, then the term must be a linear function of the x-coordinate. This agrees with the analysis of part (e).

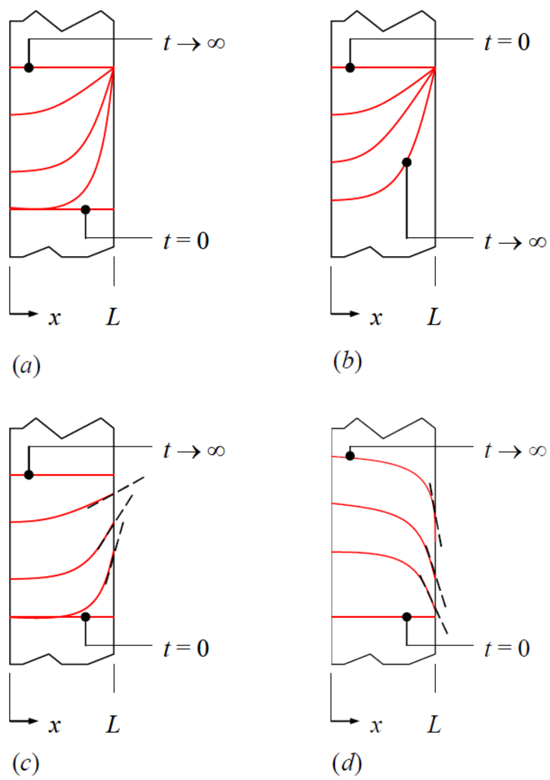
(5) In part (f), we evaluated $\dot{E}_{st}$, the rate of energy change stored in the wall at the instant the volumetric heat generation was deactivated. Did you notice that $\dot{E}_{st} = -2\times10^5\ W/m^3$ is the same value of the deactivated $\dot{q}$? How do you explain this?

PROBLEM 2.24

KNOWN: Transient temperature distributions in a plane wall.

FIND: Appropriate forms of heat equation, initial condition, and boundary conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: The general form of the heat equation in Cartesian coordinates for constant *k* is Equation 2.21. For one-dimensional conduction it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

At steady state this becomes

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0$$

If there is no thermal energy generation the steady-state temperature distribution is linear (or could be constant). If there is uniform thermal energy generation the steady-state temperature distribution must be parabolic.

Continued…

PROBLEM 2.24 (Cont.)

In case (*a*), the steady-state temperature distribution is constant, therefore there must not be any thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad <$$

The initial temperature is uniform throughout the solid, thus the initial condition is

$$T(x,0) = T_i \qquad <$$

At $x = 0$, the slope of the temperature distribution is zero at all times, therefore the heat flux is zero (insulated condition). The boundary condition is

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \qquad <$$

At $x = L$, the temperature is the same for all $t > 0$. Therefore the surface temperature is constant:

$$T(L,t) = T_s \qquad <$$

For case (*b*), the steady-state temperature distribution is not linear and appears to be parabolic, therefore there is thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad <$$

The initial temperature is uniform, the temperature gradient at $x = 0$ is zero, and the temperature at $x = L$ is equal to the initial temperature for all $t > 0$, therefore the initial and boundary conditions are

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad T(L,t) = T_i \qquad <$$

With the left side insulated and the right side maintained at the initial temperature, the cause of the decreasing temperature must be a negative value of thermal energy generation.

In case (*c*), the steady-state temperature distribution is constant, therefore there is no thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad <$$

Continued…

PROBLEM 2.24 (Cont.)

The initial temperature is uniform throughout the solid. At $x = 0$, the slope of the temperature distribution is zero at all times. Therefore the initial condition and boundary condition at $x = 0$ are

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$ <

At $x = L$, neither the temperature nor the temperature gradient are constant for all time. Instead, the temperature gradient is decreasing with time as the temperature approaches the steady-state temperature. This corresponds to a convection heat transfer boundary condition. As the surface temperature approaches the fluid temperature, the heat flux at the surface decreases. The boundary condition is:

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$ <

The fluid temperature, T_∞, must be higher than the initial solid temperature to cause the solid temperature to increase.

For case (*d*), the steady-state temperature distribution is not linear and appears to be parabolic, therefore there is thermal energy generation. The heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

Since the temperature is increasing with time and it is *not* due to heat conduction due to a high surface temperature, the energy generation must be positive.

The initial temperature is uniform and the temperature gradient at $x = 0$ is zero. The boundary condition at $x = L$ is convection. The temperature gradient and heat flux at the surface are *increasing* with time as the thermal energy generation causes the temperature to rise further and further above the fluid temperature. The initial and boundary conditions are:

$$T(x,0) = T_i, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad -k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$ <

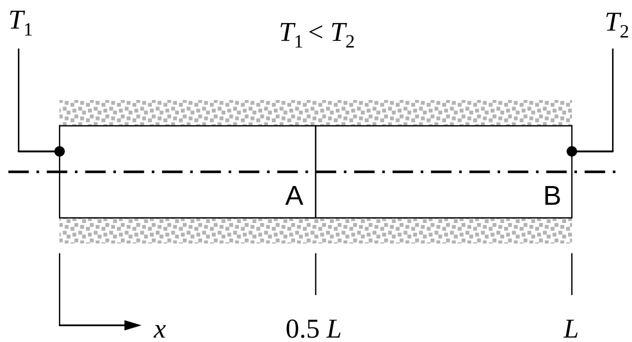
COMMENTS: 1. You will learn to solve for the temperature distribution in transient conduction in Chapter 5. 2. Case (b) might correspond to a situation involving a spatially-uniform endothermic chemical reaction. Such situations, although they can occur, are not common.

PROBLEM 2.25

KNOWN: Rod consisting of two materials with same lengths. Ratio of thermal conductivities.

FIND: Sketch temperature and heat flux distributions.

SCHEMATIC:

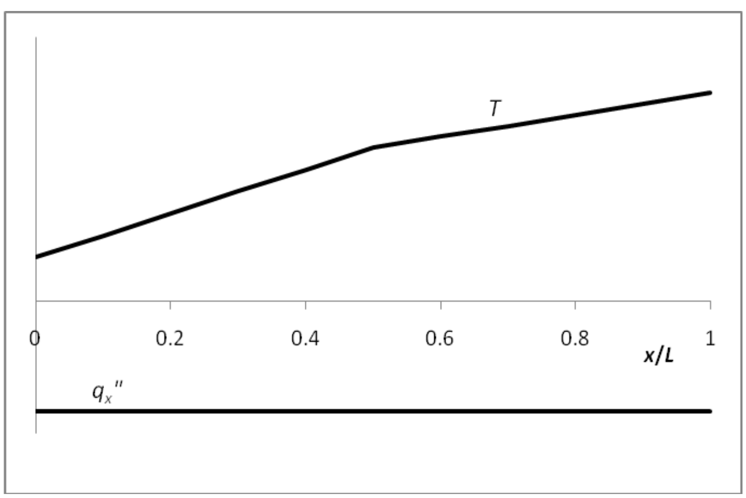


ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (3) No internal generation.

ANALYSIS: From Equation 2.19 for steady-state, one-dimensional conduction with constant properties and no internal heat generation,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0 \quad \text{or} \quad \frac{\partial q_x''}{\partial x}=0$$

From these equations we know that heat flux is constant and the temperature gradient is inversely proportional to k. Thus, with $k_A = 0.5k_B$, we can sketch the temperature and heat flux distributions as shown below:



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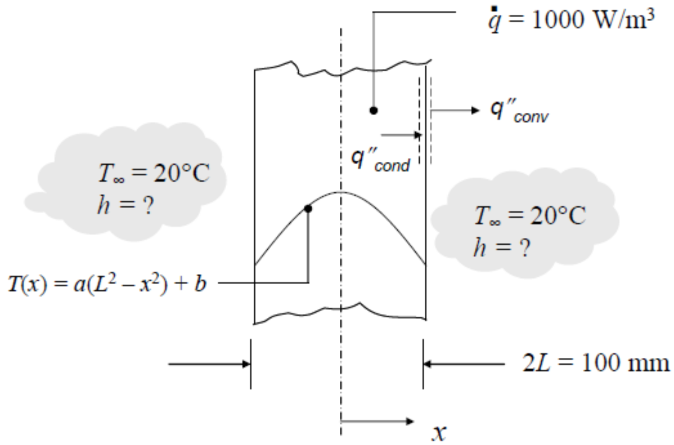
COMMENTS: (1) Note the discontinuity in the slope of the temperature distribution at $x/L = 0.5$. The constant heat flux is in the negative x-direction. (2) A discontinuity in the temperature distribution may occur at x/L = 0.5 due the joining of dissimilar materials. We shall address *thermal contact resistances* in Chapter 3.

PROBLEM 2.26

KNOWN: Wall thickness. Thermal energy generation rate. Temperature distribution. Ambient fluid temperature.

FIND: Thermal conductivity. Convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Under the specified conditions, the heat equation, Equation 2.21, reduces to

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0$$

With the given temperature distribution, $d^2T/dx^2 = -2a$. Therefore, solving for k gives

$$k=\frac{\dot{q}}{2a}=\frac{1000\ \text{W/m}^3}{2\times 15°\text{C/m}^2}=33.3\ \text{W/m}\cdot\text{K} \qquad <$$

The convection heat transfer coefficient can be found by applying the boundary condition at $x = L$ (or at $x = -L$),

$$-k\left.\frac{dT}{dx}\right|_{x=L}=h\left[T(L)-T_\infty\right]$$

Therefore

$$h=\frac{-k\left.\dfrac{dT}{dx}\right|_{x=L}}{\left[T(L)-T_\infty\right]}=\frac{2kaL}{b-T_\infty}=\frac{2\times 33.3\ \text{W/m}\cdot\text{K}\times 15°\text{C/m}^2\times 0.04\ \text{m}}{40°\text{C}-30°\text{C}}=4\ \text{W/m}^2\cdot\text{K} \qquad <$$

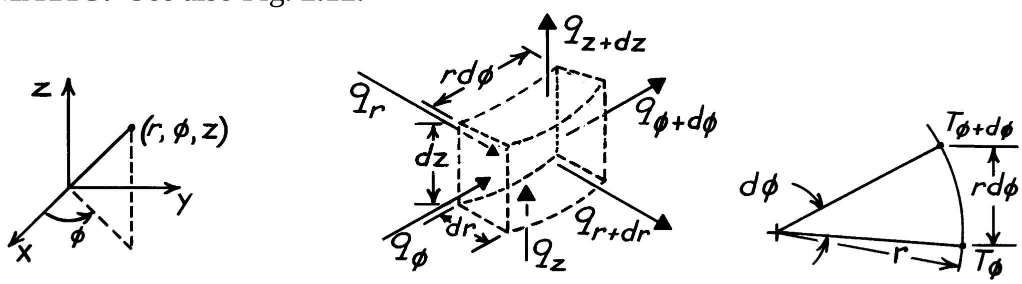
COMMENTS: (1) In Chapter 3, you will learn how to determine the temperature distribution. (2) The heat transfer coefficient could also have been found from an energy balance on the wall. With $\dot{E}_{\text{in}}-\dot{E}_{\text{out}}+\dot{E}_g=0$, we find $-2hA[T(L) - T_\infty] + 2\dot{q}LA = 0$. This yields the same result for h.

PROBLEM 2.27

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,z) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See also Fig. 2.12.



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Consider the differential control volume identified above having a volume given as V = dr·rdϕ·dz. From the conservation of energy requirement,

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_z - q_{z+dz} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}\left(dr \cdot rd\phi \cdot dz\right) \qquad \dot{E}_g = \rho Vc\partial T/\partial t = \rho\left(dr \cdot rd\phi \cdot dz\right)c\,\partial T/\partial t. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}\left(q_r\right)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}\left(q_\phi\right)d\phi, \quad q_{z+dz} = q_z + \frac{\partial}{\partial z}\left(q_z\right)dz. \qquad (4,5,6)$$

Using Fourier's law, the expressions for the conduction heat rates are

$$q_r = -kA_r\partial T/\partial r = -k\left(rd\phi \cdot dz\right)\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\partial\phi = -k\left(dr \cdot dz\right)\partial T/r\partial\phi \qquad (8)$$

$$q_z = -kA_z\partial T/\partial z = -k\left(dr \cdot rd\phi\right)\partial T/\partial z. \qquad (9)$$

Note from the above, right schematic that the gradient in the ϕ-direction is $\partial T/r\partial\phi$ and not $\partial T/\partial\phi$. Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1),

$$-\frac{\partial}{\partial r}\left(q_r\right)dr - \frac{\partial}{\partial\phi}\left(q_\phi\right)d\phi - \frac{\partial}{\partial z}\left(q_z\right)dz + \dot{q}\,dr \cdot rd\phi \cdot dz = \rho\left(dr \cdot rd\phi \cdot dz\right)c\frac{\partial T}{\partial t}. \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k\left(rd\phi \cdot dz\right)\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial \phi}\left[-k\left(drdz\right)\frac{\partial T}{r\partial\phi}\right]d\phi - \frac{\partial}{\partial z}\left[-k\left(dr \cdot rd\phi\right)\frac{\partial T}{\partial z}\right]dz$$

$$+\dot{q}\,dr \cdot rd\phi \cdot dz = \rho\left(dr \cdot rd\phi \cdot dz\right)c\frac{\partial T}{\partial t}. \qquad (11)$$

Dividing Eq. (11) by the volume of the CV, Eq. 2.26 is obtained.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}$$ <

PROBLEM 2.28

KNOWN: Three-dimensional system – described by spherical coordinates (r,ϕ,θ) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See Figure 2.13.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is $V = dr \cdot r\sin\theta d\phi \cdot r d\theta$, and the conduction terms are identified in Figure 2.13. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right] \qquad \dot{E}_{st} = \rho V c\frac{\partial T}{\partial t} = \rho\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right] c\frac{\partial T}{\partial t}. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \qquad (4,5,6)$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k\left[r\sin\theta d\phi \cdot r d\theta\right]\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k\left[dr \cdot r d\theta\right]\partial T/r\sin\theta\partial\phi \qquad (8)$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k\left[dr \cdot r\sin\theta d\phi\right]\partial T/r\partial\theta. \qquad (9)$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right] = \rho\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right]c\frac{\partial T}{\partial t} \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k\left[r\sin\theta d\phi \cdot r d\theta\right]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k\left[dr \cdot r d\theta\right]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k\left[dr \cdot r\sin\theta d\phi\right]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right] = \rho\left[dr \cdot r\sin\theta d\phi \cdot r d\theta\right]c\frac{\partial T}{\partial t} \qquad (11)$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.29 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

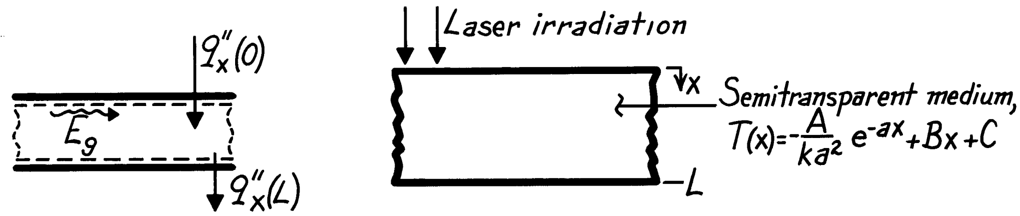
COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

PROBLEM 2.29

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]$$

Front Surface, x=0: $$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right]$$ <

Rear Surface, x=L: $$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right].$$ <

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}.$$ <

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

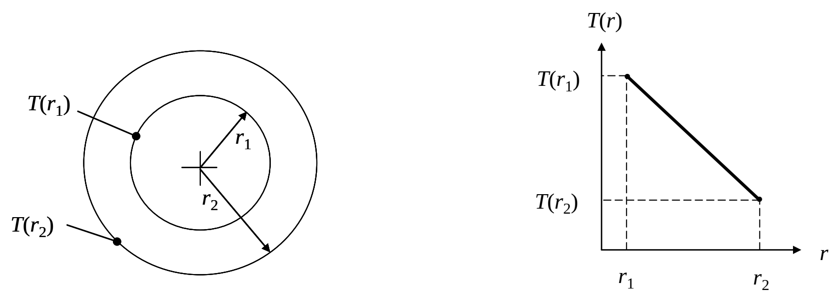
$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$

PROBLEM 2.30

KNOWN: Spherical shell under steady-state conditions with no energy generation.

FIND: Under what conditions is a linear temperature distribution possible.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional, (3) No heat generation.

ANALYSIS: Under the stated conditions, the heat equation in spherical coordinates, Equation 2.29, reduces to

$$\frac{d}{dr}\left(kr^2 \frac{dT}{dr}\right) = 0$$

If the temperature distribution is a linear function of r, then the temperature gradient is constant, and this equation becomes

$$\frac{d}{dr}\left(kr^2\right) = 0$$

which implies kr^2 = constant, or $k \sim 1/r^2$. The only way there could be a linear temperature distribution in the spherical shell is if the thermal conductivity were to vary inversely with r^2. $<$

COMMENTS: It is unlikely to encounter or even create a material for which k varies inversely with the spherical radial coordinate r in the manner necessary to develop a linear temperature distribution. Assuming linear temperature distributions in radial systems is nearly always both fundamentally incorrect and physically implausible.

PROBLEM 2.31

KNOWN: Steady-state temperature distribution in a one-dimensional wall is $T(x) = Ax^2 + Bx + C$, thermal conductivity, thickness.

FIND: Expressions for the heat fluxes at the two wall faces ($x = 0, L$) and the heat generation rate in the wall per unit area.

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

ANALYSIS: The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad \text{or} \qquad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[2Ax + B \ + 0\right]$$

$$\dot{q} = -k\left[2A\right]$$ <

which is constant. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x = -k\frac{dT}{dx} = -k\left[2Ax + B\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0:

$$q''_x(0) = -kB$$ <

Surface x=L:

$$q''_x(L) = -k\left[2AL + B\right].$$ <

COMMENTS: (1) From an overall energy balance on the wall, find

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_g = 0$$
$$q''_x(0) - q''_x(L) + \dot{E}''_g = (-kB) - (-k)\left[2AL + B\right] + \dot{E}''_g = 0$$
$$\dot{E}''_g = -2AkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}''_g$ as

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[2A\right]dx = -k\left[2AL\right]$$

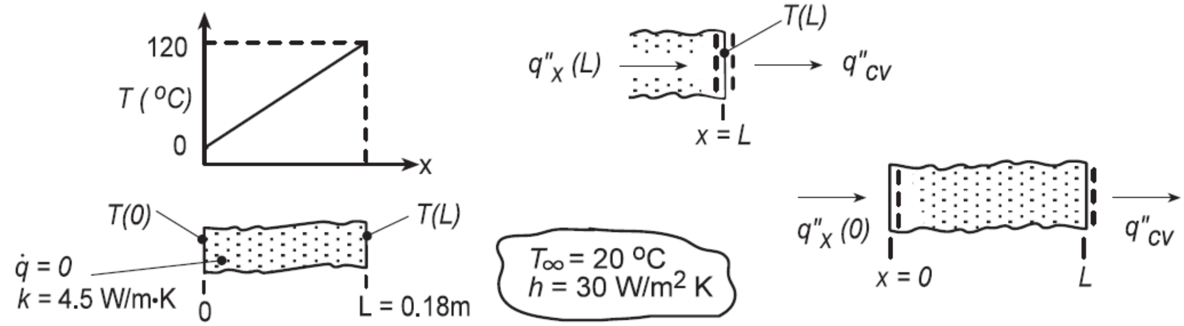
which agrees with the above, as it should.

PROBLEM 2.32

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100\ \text{W/m}^2\cdot\text{K}$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad\qquad q''_x(L) - q''_{cv} = 0 \qquad\qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,\text{W/m}\cdot\text{K}\times(120-0)^\circ\text{C}\big/0.18\,\text{m} = -3000\,\text{W/m}^2$$

$$q''_{cv} = h\left[T(L) - T_\infty\right] = 30\,\text{W/m}^2\cdot\text{K}\times(120-20)^\circ\text{C} = 3000\,\text{W/m}^2$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) ≠ 0 and therefore, the temperature distribution is not possible.

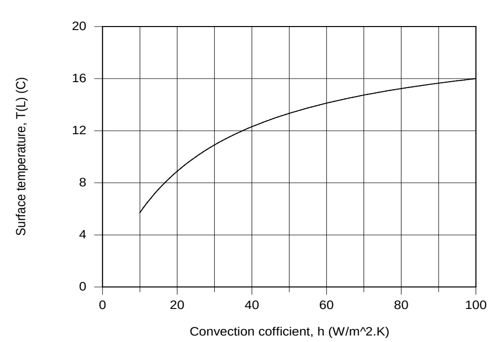
(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L) - T_\infty\right] = 0$$

$$-4.5\,\text{W/m}\cdot\text{K}\left[T(L) - 0^\circ\text{C}\right]\Big/0.18\,\text{m} - 30\,\text{W/m}^2\cdot\text{K}\left[T(L) - 20^\circ\text{C}\right] = 0$$

T(L) = 10.9°C <

Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?

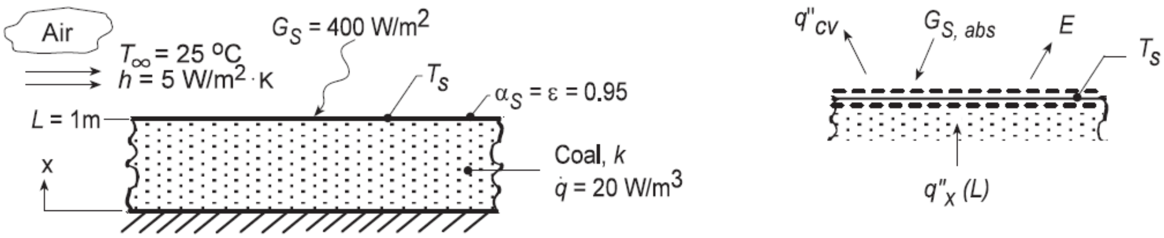


PROBLEM 2.33

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, x = 0; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location x = L; expression for the surface temperature T_s based upon a surface energy balance at x = L; evaluate T_S and T(0) for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and T(0) for (1) h = 5 W/m^2·K with $50 \le G_S \le 500$ W/m^2, (2) G_S = 400 W/m^2 with $5 \le h \le 50$ W/m^2·K.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): k = 0.26 W/m·K

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.22,

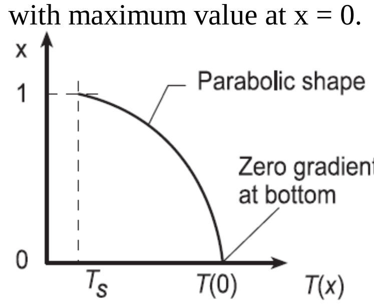
$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad (1)$$ <

Substituting the temperature distribution into the HDE, Eq. (1),

$$T(x)=T_S+\frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]+\frac{\dot{q}}{k}\ ?=?\ 0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at x = 0. At x = 0, the heat flux is



$$q''_x(0)=-k\left.\frac{dT}{dx}\right)_{x=0}=-k\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]_{x=0}=0$$

so that the gradient at x = 0 is zero. Hence, the bottom is insulated.

(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L)=\dot{E}''_g=\dot{q}L$$ <

Continued...

From a surface energy balance per unit area shown in the schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(L) - q''_{conv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h\left(T_S - T_\infty\right) + 0.95G_S - \varepsilon\sigma T_S^4 = 0 \qquad (4)$$

$$10\,W/m^3 \times\ 2\,m - 8\,W/m^2{\cdot}K\left(T_s - 303\,K\right) + 0.95\times 500\,W/m^2 - 0.95\times 5.67\times 10^{-8}\,W/m^2{\cdot}K^4 T_s^4 = 0$$

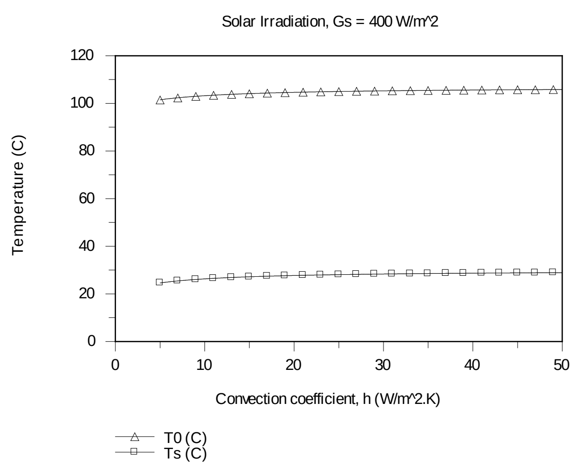
$$T_S = 305.6\ K = 32.6°C$$ <

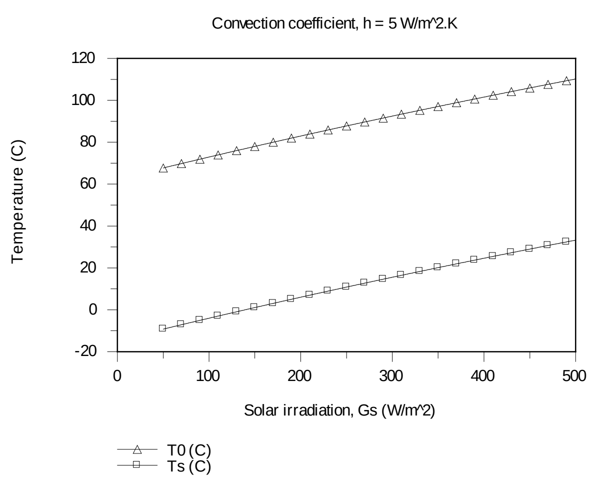
From Eq. (2) with x = 0, find

$$T(0) = T_S + \frac{\dot{q}L^2}{2k} = 32.6^\circ C + \frac{10\,W/m^3 \times (2\,m)^2}{2\times 0.26\,W/m\cdot K} = 109.5^\circ C \qquad (5)$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 W/m^2·K for 50 ≤ G_S ≤ 500 W/m^2 and (2) with G_S = 400 W/m^2 for 5 ≤ h ≤ 50 W/m^2·K.





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PROBLEM 2.33 (Cont.)

From the T vs. h plot with $G_S = 400\ W/m^2$, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with $h = 5\ W/m^2\cdot K$, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

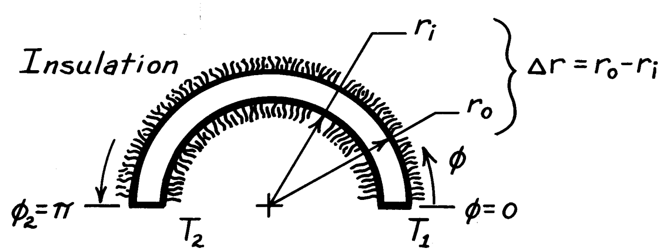
COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where $T_{sky} = -30°C$ for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).

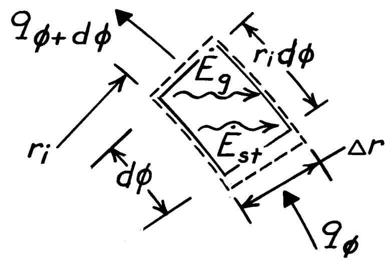
PROBLEM 2.34

KNOWN: Cylindrical system with negligible temperature variation in the r,z directions.

FIND: (a) Heat equation beginning with a properly defined control volume, (b) Temperature distribution $T(\phi)$ for steady-state conditions with no internal heat generation and constant properties, (c) Heat rate for Part (b) conditions.

SCHEMATIC:





ASSUMPTIONS: (1) T is independent of r,z, (2) $\Delta r = (r_o - r_i) << r_i$.

ANALYSIS: (a) Define the control volume as $V = r_i d\phi \cdot \Delta r \cdot L$ where L is length normal to page. Apply the conservation of energy requirement, Eq. 1.12c,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_\phi - q_{\phi+d\phi} + \dot{q}V = \rho V c \frac{\partial T}{\partial t} \qquad (1,2)$$

where
$$q_\phi = -k(\Delta r \cdot L)\frac{\partial T}{r_i \partial \phi} \qquad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi) d\phi. \qquad (3,4)$$

Eqs. (3) and (4) follow from Fourier's law, Eq. 2.1, and from Eq. 2.25, respectively. Combining Eqs. (3) and (4) with Eq. (2) and canceling like terms, find

$$\frac{1}{r_i^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t}. \qquad (5) <$$

Since temperature is independent of r and z, this form agrees with Eq. 2.26.

(b) For steady-state conditions with $\dot{q} = 0$, the heat equation, (5), becomes

$$\frac{d}{d\phi}\left[k\frac{dT}{d\phi}\right] = 0. \qquad (6)$$

With constant properties, it follows that $dT/d\phi$ is constant which implies $T(\phi)$ is linear in ϕ. That is,

$$\frac{dT}{d\phi} = \frac{T_2 - T_1}{\phi_2 - \phi_1} = +\frac{1}{\pi}(T_2 - T_1) \qquad \text{or} \qquad T(\phi) = T_1 + \frac{1}{\pi}(T_2 - T_1)\phi. \qquad (7,8) <$$

(c) The heat rate for the conditions of Part (b) follows from Fourier's law, Eq. (3), using the temperature gradient of Eq. (7). That is,

$$q_\phi = -k(\Delta r \cdot L)\frac{1}{r_i}\left[+\frac{1}{\pi}(T_2 - T_1)\right] = -k\left[\frac{r_o - r_i}{\pi r_i}\right] L (T_2 - T_1). \qquad (9) <$$

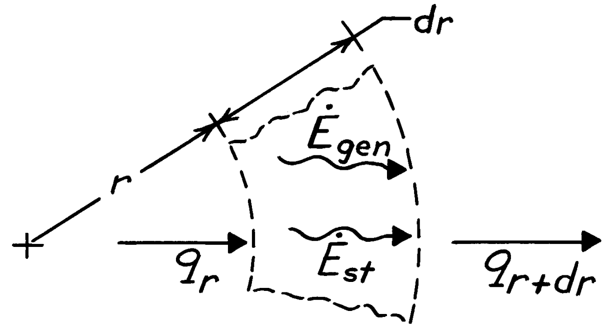
COMMENTS: Note the expression for the temperature gradient in Fourier's law, Eq. (3), is $\partial T / r_i \partial \phi$ not $\partial T / \partial \phi$. For the conditions of Parts (b) and (c), note that q_ϕ is independent of ϕ; this is first indicated by Eq. (6) and confirmed by Eq. (9).

PROBLEM 2.35

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.12c,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, r + dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k2\pi r \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r$ dr, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}. \qquad <$$

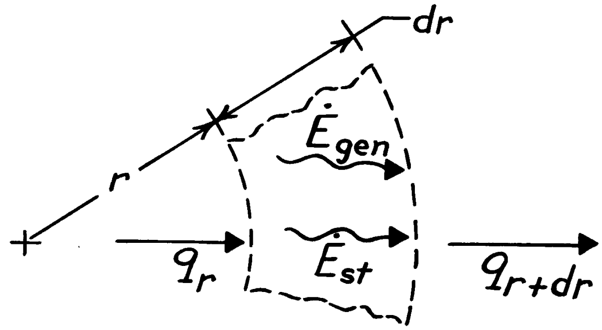
COMMENTS: (1) Note how the result compares with Eq. 2.26 when the terms for the ϕ,z coordinates are eliminated. (2) Recognize that we did not require $\dot{q}$ and k to be independent of r.

PROBLEM 2.36

KNOWN: Heat diffusion with internal heat generation for one-dimensional spherical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has the volume, $V = A_r \cdot dr = 4\pi r^2 dr$. Using the conservation of energy requirement, Eq. 1.12c,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this coordinate system has the form

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}.$$

At the outer surface, r + dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 4\pi r^2 dr = \rho \cdot 4\pi r^2 dr \cdot c_p \frac{\partial T}{\partial t}.$$

Dividing by the factor $4\pi r^2 dr,$ we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2 \frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

COMMENTS: (1) Note how the result compares with Eq. 2.29 when the terms for the θ,φ directions are eliminated.

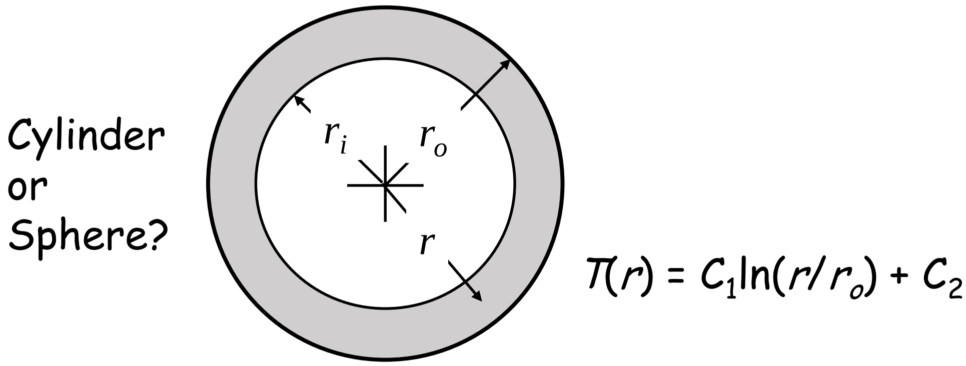
(2) Recognize that we did not require $\dot{q}$ and k to be independent of the coordinate r.

PROBLEM 2.37

KNOWN: Steady-state temperature distribution in a radial wall.

FIND: Whether the wall is that of a cylinder or sphere. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in *r*, (2) Constant properties.

ANALYSIS: From Equation 2.26, the heat equation for a cylinder reduces to

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \tag{1}$$

and for a sphere it reduces to

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0 \tag{2}$$

From the given temperature distribution,

$$\frac{\partial T}{\partial r} = \frac{C_1}{r}, \qquad r\frac{\partial T}{\partial r} = C_1, \qquad r^2\frac{\partial T}{\partial r} = C_1 r \tag{3}$$

Substituting terms into Eqs. (1) and (2), it can be seen that Eq. (1) is satisfied and Eq. (2) is not. Hence, the wall is cylindrical. <

From Equation 2.25, the radial component of the heat flux is

$$q_r'' = -k\frac{\partial T}{\partial r} = -k\frac{C_1}{r} \tag{4}$$

Therefore, the *magnitude* of q_r'' decreases with increasing *r*, $q_r'' \propto 1/r$. <

At any radial location, the heat rate is

$$q_r = -q_r'' A(r) = -k\frac{C_1}{r}2\pi r L = -k 2\pi L C_1 \tag{5}$$

Hence, q_r is independent of *r*. <

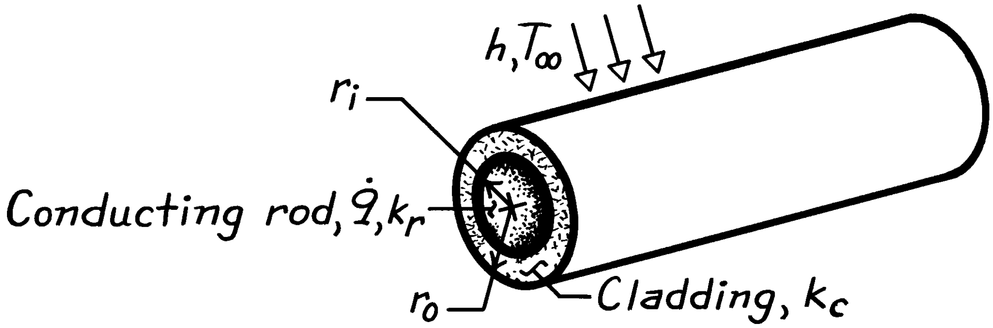
COMMENTS: The result that q_r is invariant with *r* is consistent with the energy conservation requirement. If q_r is constant, the heat flux must vary inversely with the area perpendicular to the direction of heat flow. Thus, q_r'' varies inversely with *r* as seen.

PROBLEM 2.38

KNOWN: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

FIND: Heat equations and boundary conditions for rod and cladding.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: From Equation 2.26, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dr}\right)+\dot{q}=0$$ <

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0.$$ <

Appropriate boundary conditions are:

(a) $dT_r/dr|_{r=0}=0$ <

(b) $T_r(r_i)=T_c(r_i)$ <

(c) $k_r\frac{dT_r}{dr}|_{r_i}=k_c\frac{dT_c}{dr}|_{r_i}$ <

(d) $-k_c\frac{dT_c}{dr}|_{r_o}=h\left[T_c(r_o)-T_\infty\right]$ <

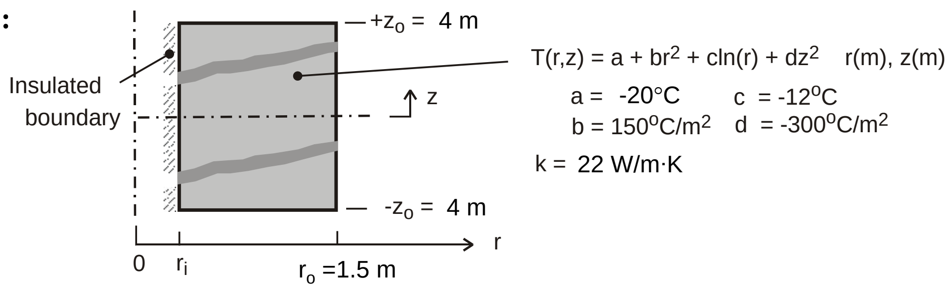
COMMENTS: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r = r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface. Note that contact resistance at the interface between the rod and cladding has been neglected.

PROBLEM 2.39

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q_r''(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q_z''(r, +z_o)$ and $q_z''(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, $r = r_i$, is adiabatic, then $q_r''(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2\times 150°C/m^2}\right)^{1/2} = 0.2\ m$$ <

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.26, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[0 + 2br + c/r + 0\right]\right) + \frac{\partial}{\partial z}\left(0 + 0 + 0 + 2dz\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\left[4br + 0\right] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k\left[4b + 2d\right] = -22\,W/m\cdot K\left[4\times 150°C/m^2 + 2\left(-300°C/m^2\right)\right] = 0\,W/m^3$$ <

(c) The heat flux and the heat rate at the outer surface, $r = r_o$, may be calculated using Fourier's law.

$$q_r''(r_o, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k\left[0 + 2br_o + c/r_o + 0\right]$$

Continued …

$$q''_r\left(r_{o,}z\right) = -22\,\mathrm{W/m\cdot K}\left[2\times 150°\mathrm{C/m^2}\times 1.5\ \mathrm{m} - 12°\mathrm{C}/1.5\ \mathrm{m}\right] = -9724\,\mathrm{W/m^2}$$ <

$$q_r\left(r_o\right) = A_r\,q''_r\left(r_{o,}z\right) \qquad \text{where} \qquad A_r = 2\pi r_o\left(2z_o\right)$$

$$q_r\left(r_o\right) = -4\pi\times 1.5\ \mathrm{m}\times 4.0\ \mathrm{m}\times 9724\,\mathrm{W/m^2} = -733.2\ \mathrm{kW}$$ <

Note that the sign of the heat flux and heat rate in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

(d) The heat fluxes and the heat rates at end faces, $z = +\,z_o$ and $-\,z_o$, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, $z = +\,z_o$: <

$$q''_z\left(r,+z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z\left(r,+z_o\right) = -22\,\mathrm{W/m\cdot K}\times 2\left(-300°\mathrm{C/m^2}\right)4.0\ \mathrm{m} = +52,800\,\mathrm{W/m^2}$$ <

$$q_z\left(+z_o\right) = A_z\,q''_z\left(r,+z_o\right) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z\left(+z_o\right) = \pi\left(1.5^2 - 0.2^2\right)\mathrm{m^2}\times 52,800\,\mathrm{W/m^2} = +366.6\ \mathrm{kW}$$ <

Thus, heat flows out of the cylinder.

At the lower end face, $z = -\,z_o$*:* <

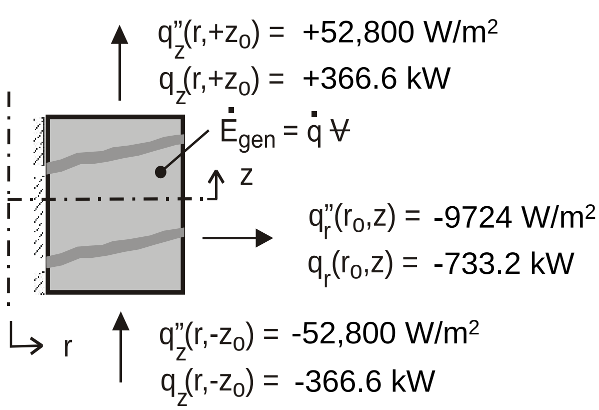
$$q''_z\left(r,-z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2d(-z_o)\right]$$

$$q''_z\left(r,-z_o\right) = -22\,\mathrm{W/m^2\cdot K}\times 2\left(-300°\mathrm{C/m}\right)\left(-4.0\ \mathrm{m}\right) = -52,800\,\mathrm{W/m^2}$$ <

$$q_z\left(-z_o\right) = -366.6\ \mathrm{kW}$$ <

Again, heat flows out of the cylinder.

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.



Continued…

PROBLEM 2.39 (Cont.)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad \text{where} \qquad \dot{E}_{gen} = \dot{q}\forall = 0$$

$$\dot{E}_{in} = -q_r(r_o) = -(-733.2\ \text{kW}) = +733.2\ \text{kW} \qquad <$$

$$\dot{E}_{out} = +q_z(z_o) - q_z(-z_o) = [366.2-(-366.2)]\ \text{kW} = +733.2\ \text{kW} \qquad <$$

The overall energy balance is satisfied.

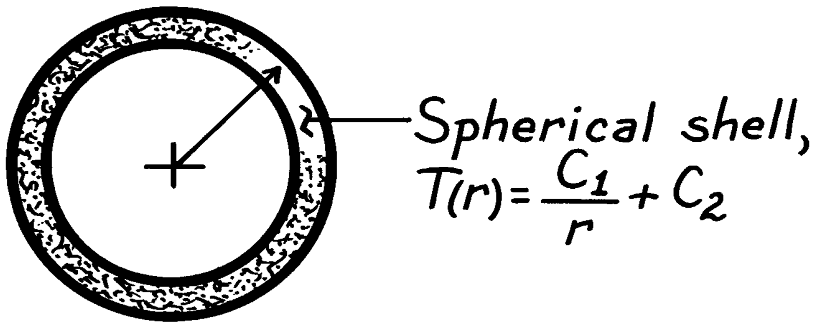
COMMENTS: When using Fourier's law, the heat flux q_z'' denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.

PROBLEM 2.40

KNOWN: Temperature distribution in a spherical shell.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.29, the heat equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{C_1}{r^2}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.28, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=k\frac{C_1}{r^2}.$$

Hence, q_r'' decreases with increasing $r^2\left(q_r''\propto 1/r^2\right)$. <

At any radial location, the heat rate is

$$q_r=4\pi r^2 q_r''=4\pi k C_1.$$

Hence, q_r is independent of r. <

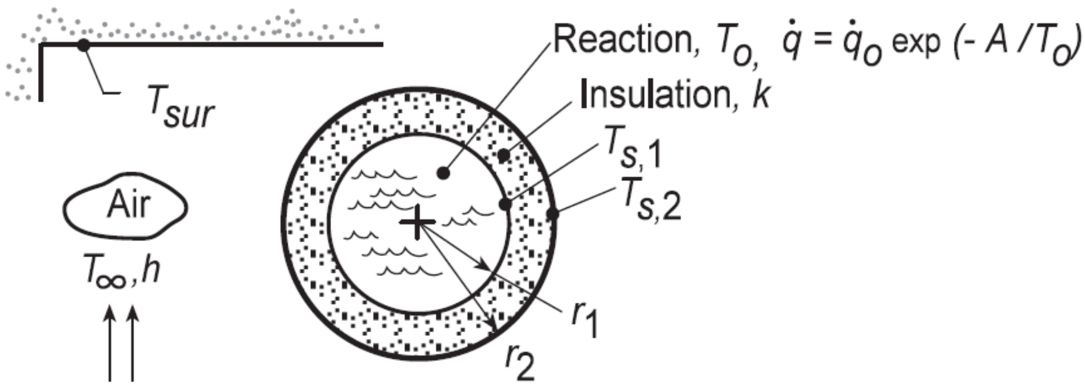
COMMENTS: The fact that q_r is independent of r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r^2.

PROBLEM 2.41

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le$ 45°C to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.29,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1)$$ <

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Continued...

PROBLEM 2.41 (Cont.)

Substitute T(r) into the HDE to see if it is satisfied:

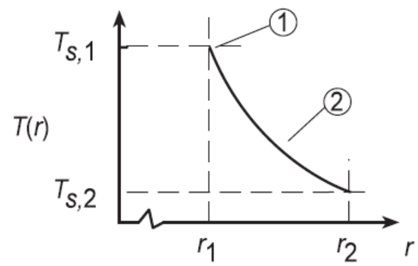
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-\left(r_1/r_2\right)}\right]\right)=0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-\left(r_1/r_2\right)}\right)=0 \qquad <$$

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.



(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r=-kA_r\frac{dT}{dr}=-k\left(4\pi r^2\right)\frac{dT}{dr} \qquad <$$

and substituting for the temperature distribution, Eq. (2),

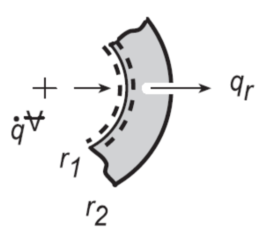
$$q_r=-4k\pi r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-\left(r_1/r_2\right)}\right]$$

$$q_r=\frac{4\pi k\left(T_{s,1}-T_{s,2}\right)}{\left(1/r_1\right)-\left(1/r_2\right)} \qquad (3) \quad <$$

Applying an energy balance to a control surface about the container at r = r_1,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$\dot{q}\forall-q_r=0$$



where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r=\left(4/3\right)\pi r_1^3\dot{q} \qquad (4) \quad <$$

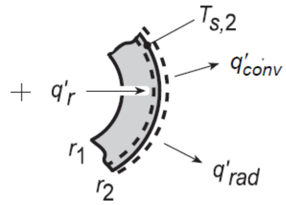
Continued...

PROBLEM 2.41 (Cont.)

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r - q_{conv} - q_{rad} = 0$$



$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s \sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5) <$$

where

$$A_s = 4\pi r_2^2 \qquad (6)$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

r_1 = 200 mm	h = 5 W/m^2·K	ε = 0.9
r_2 = 208 mm	T_∞ = 25°C	T_{sur} = 35°C
k = 0.05 W/m·K		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp\left(-A/T_o\right) \qquad q_o = 5000\,\mathrm{W/m^3} \qquad A = 75\,\mathrm{K} \qquad (7)$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\,\mathrm{W/m\cdot K}\left(T_{s,1} - T_{s,2}\right)}{\left(1/0.200\,\mathrm{m} - 1/0.208\,\mathrm{m}\right)} \qquad (8)$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\pi\left(0.200\,\mathrm{m}\right)^3 \dot{q} \qquad (9)$$

$$\dot{q} = 5000\,\mathrm{W/m^3} \exp\left(-75\,\mathrm{K}/T_{s,1}\right) \qquad (10)$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\,\mathrm{W/m^2\cdot K}\,A_s\left(T_{s,2} - 298\,\mathrm{K}\right) - 0.9 A_s\, 5.67\times10^{-8}\,\mathrm{W/m^2{\cdot}K^4}\left(T_{s,2}^4 - \left(308\,\mathrm{K}\right)^4\right) = 0 \qquad (11)$$

$$A_s = 4\pi\left(0.208\,\mathrm{m}\right)^2 \qquad (12)$$

Continued...

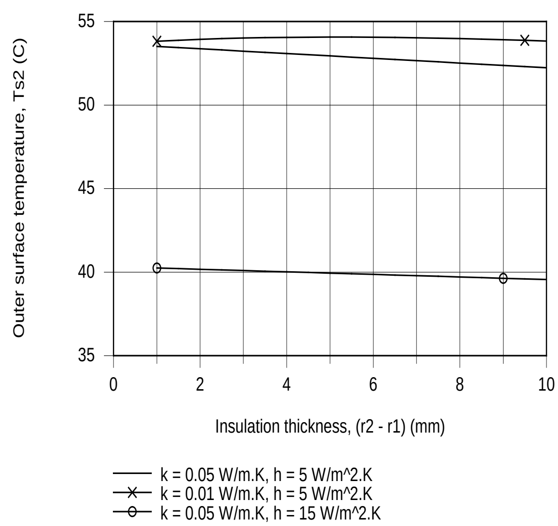
PROBLEM 2.41 (Cont.)

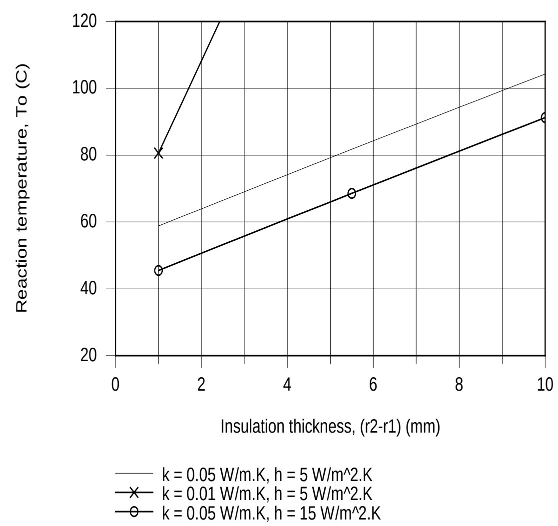
Solving these equations simultaneously, find that

$$T_{s,1} = 94.3^\circ C \qquad T_{s,2} = 52.5^\circ C$$ <

That is, the reactor will be operating at $T_o = T_{s,1} = 94.3°C$, very close to the desired 95°C operating condition.

(e) Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.





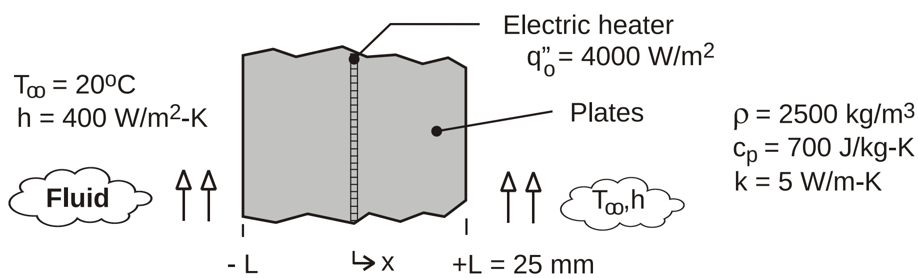
In the $T_{s,2}$ vs. $(r_2 - r_1)$ plot, note that decreasing the thermal conductivity from 0.05 to 0.01 W/m·K slightly increases $T_{s,2}$ while increasing the convection coefficient from 5 to 15 $W/m^2 \cdot K$ markedly decreases $T_{s,2}$. Insulation thickness only has a minor effect on $T_{s,2}$ for either option. In the T_o vs. $(r_2 - r_1)$ plot, note that, for all the options, the effect of increased insulation is to increase the reaction temperature. With k = 0.01 W/m·K, the reaction temperature increases beyond 95°C with less than 2 mm insulation. For the case with h = 15 $W/m^2 \cdot K$, the reaction temperature begins to approach 95°C with insulation thickness around 10 mm. We conclude that by selecting the proper insulation thickness and controlling the convection coefficient, the reaction could be operated around 95°C such that the outer surface temperature would not exceed 45°C.

PROBLEM 2.42

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

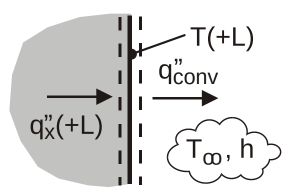
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for -L ≤ × ≤ +L; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual (t → ∞) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,\text{W/m}^2 / \left(2\times 400\,\text{W/m}^2\cdot\text{K}\right) + 20°\text{C} = 25°\text{C} \qquad <$$

From Fourier's law for the conduction flux through the plate, find T(0).

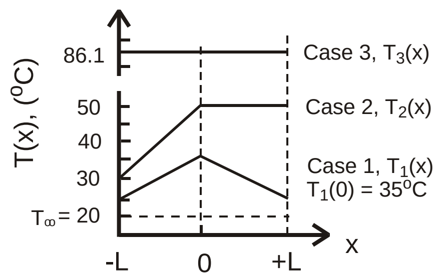
$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°\text{C} + 4000\,\text{W/m}^2\cdot\text{K}\times 0.025\text{m}/\left(2\times 5\,\text{W/m}\cdot\text{K}\right) = 35°\text{C} \quad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

Continued …

PROBLEM 2.42 (Cont.)



(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q''_o / h + T_\infty = 4000\,\mathrm{W/m^2} / 400\,\mathrm{W/m^2\cdot K} + 20°\mathrm{C} = 30°\mathrm{C} \qquad <$$

$$T_2(0) = T_2(-L) + q''_o L / k = 30°\mathrm{C} + 4000\,\mathrm{W/m^2} \times 0.025\ \mathrm{m} / 5\,\mathrm{W/m\cdot K} = 50°\mathrm{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.12b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} + E''_{gen} = \Delta E''_{st} = E''_f - E''_i \qquad (1)$$

Note that $E''_{in} - E''_{out} = 0$, and the dissipated electrical energy is

$$E''_{gen} = q''_o \Delta t_o = 4000\,\mathrm{W/m^2}(15\times 60)\mathrm{s} = 3.600\times 10^6\ \mathrm{J/m^2} \qquad (2)$$

For the final condition,

$$\begin{aligned} E''_f &= \rho c(2L)[T_f - T_\infty] = 2500\,\mathrm{kg/m^3} \times 700\,\mathrm{J/kg\cdot K}(2\times 0.025\mathrm{m})[T_f - 20]°\mathrm{C} \\ E''_f &= 8.75\times 10^4 [T_f - 20]\,\mathrm{J/m^2} \end{aligned} \qquad (3)$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E''_i = \rho c \int_{-L}^{+L} [T_2(x) - T_\infty] dx = \rho c \left\{ \int_{-L}^{0} [T_2(x) - T_\infty] dx + \int_0^{+L} [T_2(0) - T_\infty] dx \right\} \qquad (4)$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)]x/L \qquad -L \le x \le 0$$

$$T_2(x) = 50°\mathrm{C} + [50 - 30]°\mathrm{C}x/0.025\mathrm{m}$$

$$T_2(x) = 50°\mathrm{C} + 800x \qquad (5)$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

Continued …

$$E_i'' = \rho c\left\{\int_{-L}^{0}\left[50 + 800x - T_\infty\right]dx + \left[T_2(0) - T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{\left[50x + 400x^2 - T_\infty x\right]_{-L}^{0} + \left[T_2(0) - T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{-\left[-50L + 400L^2 + T_\infty L\right] + \left[T_2(0) - T_\infty\right]L\right\}$$

$$E_i'' = \rho cL\left\{+50 - 400L - T_\infty + T_2(0) - T_\infty\right\}$$

$$E_i'' = 2500\,\text{kg/m}^3 \times 700\,\text{J/kg}\cdot\text{K} \times 0.025\ \text{m}\left\{+50 - 400\times 0.025 - 20 + 50 - 20\right\}\text{K}$$

$$E_i'' = 2.188\times 10^6\ \text{J/m}^2 \qquad (6)$$

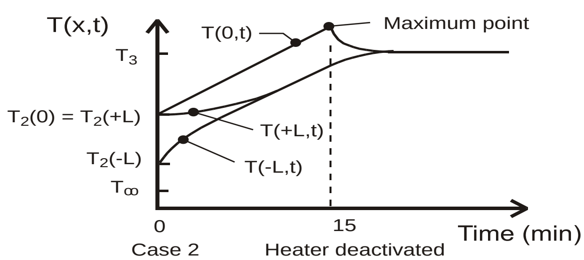
Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600\times 10^6\ \text{J/m}^2 = 8.75\times 10^4\left[T_3 - 20\right] - 2.188\times 10^6\ \text{J/m}^2$$

$$T_3 = (66.1 + 20)°\text{C} = 86.1°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.



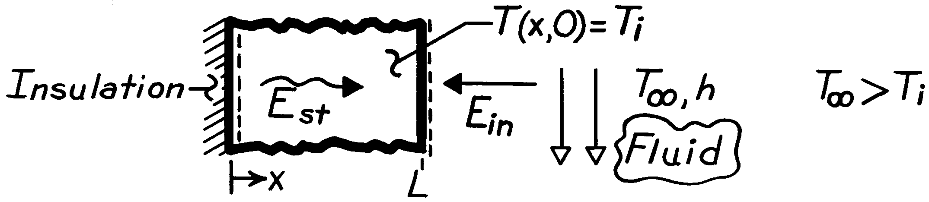
Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.

PROBLEM 2.43

KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for these conditions: initial ($t \le 0$), steady-state, $t \rightarrow \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

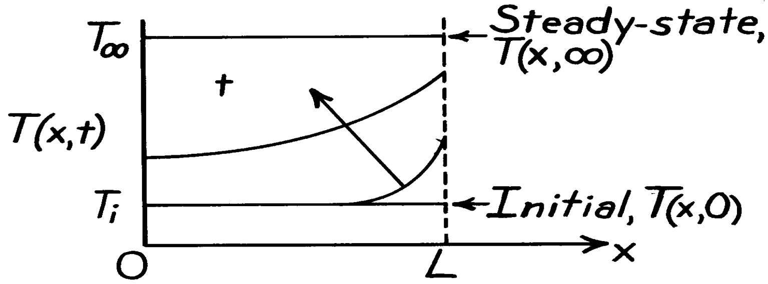
ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \le 0: \quad T(x,0) = T_i & \text{uniform} \\ \text{Boundaries:} \quad x=0 \quad \partial T/\partial x)_0 = 0 & \text{adiabatic} \\ \quad x=L \quad -k\partial T/\partial x)_L = h\left[T(L,t) - T_\infty\right] & \text{convection} \end{cases}$$

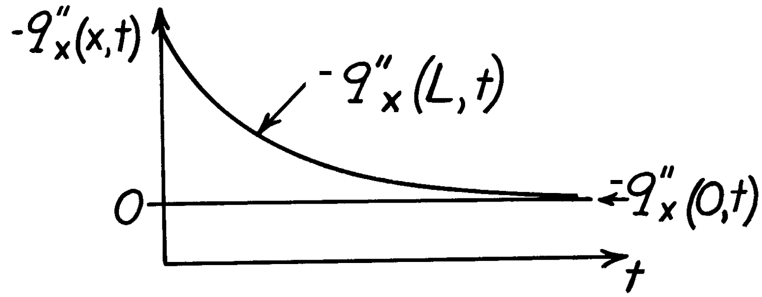
(b) The temperature distributions are shown on the sketch.



Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

(c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.

Continued …



For the surface at $x = 0$, $q''_x(0,t) = 0$ since it is adiabatic. At x = L and t = 0, $q''_x(L,0)$ is a maximum (in magnitude)

$$\left|q''_x(L,0)\right| = h\left|T(L,0) - T_\infty\right|$$

where T(L,0) = T_i. The temperature difference, and hence the flux, decreases with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$

$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right) dt$$

Dividing both sides by $A_s L$, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty \left[T_\infty - T(L,t)\right] dt \qquad \left[\text{J/m}^3\right]$$

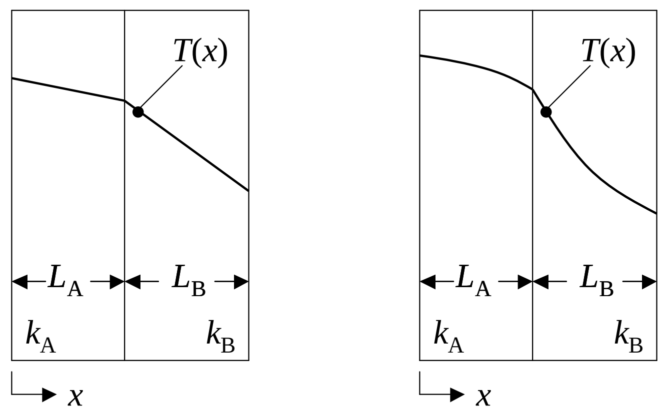
COMMENTS: Note that the heat flux at x = L is into the wall and is hence in the negative x direction.

PROBLEM 2.44

KNOWN: Qualitative temperature distributions in two cases.

FIND: For each of two cases, determine which material (A or B) has the higher thermal conductivity, how the thermal conductivity varies with temperature, description of the heat flux distribution through the composite wall, effect of simultaneously doubling the wall thickness and thermal conductivity.

SCHEMATIC:



Case 1. Case 2.

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible contact resistances, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions with no internal generation, the conservation of energy requirement dictates that the heat flux through the wall must be constant. <

For Materials A and B, Fourier's law is written $q''_A = -k_A \frac{dT_A}{dx} = q''_B = -k_B \frac{dT_B}{dx}$. Therefore,

$$\frac{k_A}{k_B} = \frac{dT_B/dx}{dT_A/dx} > 1 \quad \text{and} \quad k_B < k_A \text{ for both cases.}$$ <

Since the heat flux through the wall is constant, Fourier's law dictates that lower thermal conductivity material must exist where temperature gradients are larger. For Case 1, the temperature distributions are linear. Therefore, the temperature gradient is constant in each material, and the thermal conductivity of each material must not vary significantly with temperature. For Case 2, Material A, the temperature gradient is larger at lower temperatures. Hence, for Material A the thermal conductivity increases with increasing material temperature. For Case 2, Material B, the temperature gradient is smaller at lower temperatures. Hence, for Material B the thermal conductivity decreases with increases in material temperature. <

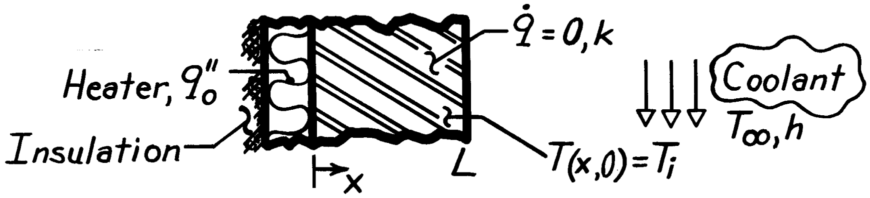
COMMENTS: If you were given information regarding the relative values of the thermal conductivities and how the thermal conductivities vary with temperature in each material, you should be able to sketch the temperature distributions provided in the problem statement.

PROBLEM 2.45

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q''_o.

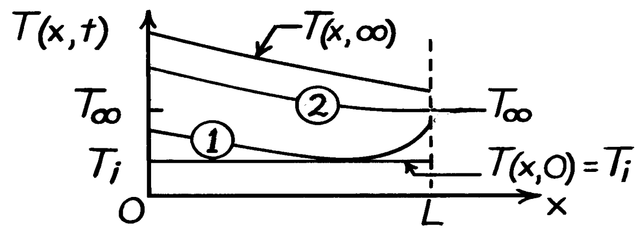
FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q''_x - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, T(0,∞), in terms of q''_o, T_∞, k, h and L.

SCHEMATIC:



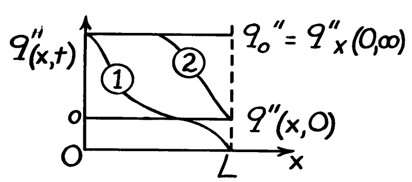
ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

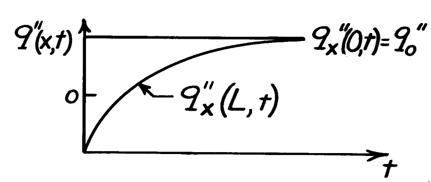
ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are



Note the constant gradient at x = 0 since $q''_x(0) = q''_o$.

(b) The heat flux distribution, $q''_x(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.



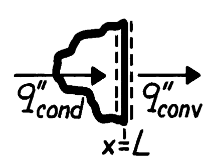


(c) On $q''_x(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q''_o. \quad (2)$$

For the wall, under steady-state conditions, Fourier's law gives



$$q''_o = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$

Combine Eqs. (1), (2), (3) to find:

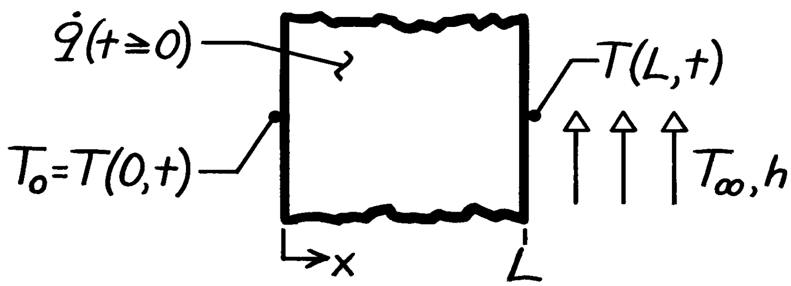
$$T(0,\infty) = T_\infty + q''_o(1/h + L/k).$$

PROBLEM 2.46

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty > T_o$,h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; also show distribution when there is no heat flow at the x = L boundary, (b) Sketch the heat flux $\left(q_x'' \text{ vs. } t\right)$ at the boundaries x = 0 and L.

SCHEMATIC:



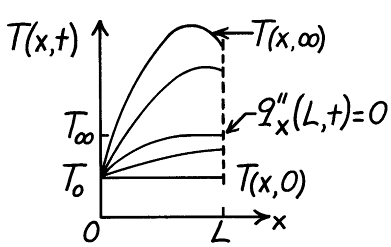
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_o < T_\infty$ and $\dot{q}$ large enough that $T(x,\infty) > T_\infty$ for some x.

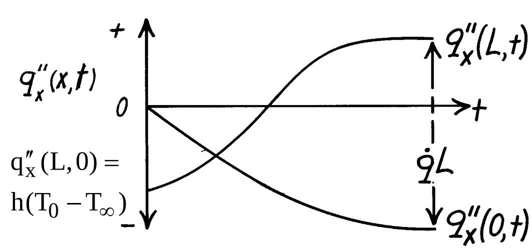
ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	x = 0 $T(0,t) = T_o$	Constant temperature
	x = L $-k\left.\dfrac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note the special condition when the heat flux at (x = L) is zero.

(b) The heat flux as a function of time at the boundaries, $q_x''(0,t)$ and $q_x''(L,t)$, can be inferred from the temperature distributions using Fourier's law.





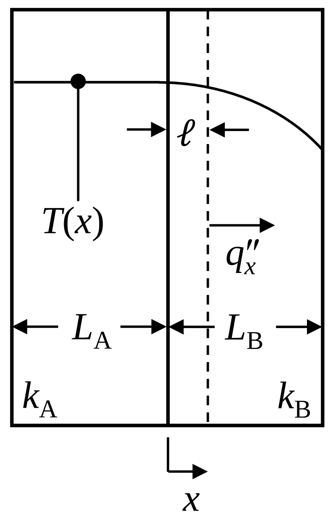
COMMENTS: Since $T(x,\infty) > T_\infty$ for some x and $T_\infty > T_o$, heat transfer at both boundaries must be out of the wall at steady state. From an overall energy balance at steady state, $+q_x''(L,\infty) - q_x''(0,\infty) = \dot{q}L$.

PROBLEM 2.47

KNOWN: Qualitative temperature distribution in a composite wall with one material experiencing uniform volumetric energy generation.

FIND: Which material experiences uniform volumetric generation. The boundary condition at $x = -L_A$. Temperature distribution if the thermal conductivity of Material A is doubled. Temperature distribution if the thermal conductivity of Material B is doubled. Sketch the heat flux distribution $q_x''(x)$ through the composite wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Constant properties.

ANALYSIS: Consider a control volume with the LHS control surface at the interface between the two materials and the RHS control surface located at an arbitrary location within Material B, as shown in the schematic. For this control volume, conservation of energy and Fourier's law may be combined to yield, for uniform volumetric generation in Material B,

$$\dot{q}(\ell) = q_x'' = -k\left.\frac{dT}{dx}\right|_{x=\ell} \quad \text{or} \quad \left.\frac{dT}{dx}\right|_{x=\ell} \propto \ell \qquad (1)$$

The temperature distribution of the problem reflects the preceding proportionality between the temperature gradient and the distance ℓ, and it is appropriate to assume that uniform volumetric generation occurs in Material B but not in Material A. <

The boundary condition at $x = -L_A$ is associated with perfectly insulated conditions,

$$0 = q_x''(x = -L_A) = -k\left.\frac{dT}{dx}\right|_{x=-L_A} \quad \text{or} \quad \left.\frac{dT}{dx}\right|_{x=-L_A} = 0 \qquad <$$

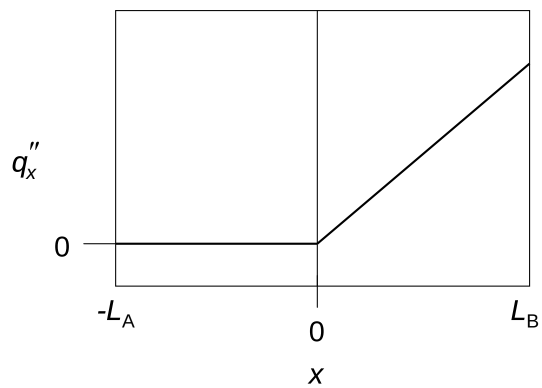
The temperature distribution in Material A corresponds to $q_{x,A}'' = 0$, and is independent of its thermal conductivity. <

Continued...

PROBLEM 2.47 (Cont.)

If the volumetric energy generation rate, $\dot{q}$, is unchanged, Equation (1) requires that the temperature gradient everywhere in Material B will be reduced by half if the thermal conductivity of Material B is doubled. Hence, the difference between the minimum and maximum temperatures in the composite wall would be reduced by half. $<$

Considering Eq. 1, it follows that the heat flux distribution throughout the composite wall is as shown in the sketch below.



$<$

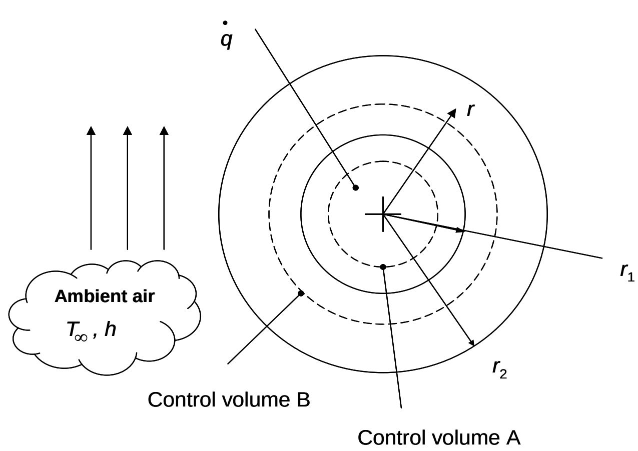
COMMENTS: If you were given information regarding which material experiences internal energy generation, the boundary condition at $x = -L_A$, and the thermal conductivities of both materials, you should be able to sketch the temperature and heat flux distributions.

PROBLEM 2.48

KNOWN: Size and thermal conductivities of a spherical particle encased by a spherical shell.

FIND: (a) Relationship between dT/dr and r for $0 \le r \le r_1$, (b) Relationship between dT/dr and r for $r_1 \le r \le r_2$, (c) Sketch of T(r) over the range $0 \le r \le r_2$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer.

ANALYSIS:

(a) The conservation of energy principle, applied to control volume A, results in

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \qquad (1)$$

where $\dot{E}_g = \dot{q}\forall = \dot{q}\frac{4}{3}\pi r^3 \qquad (2)$

since $\dot{E}_{st} = 0$

$$\dot{E}_{in} - \dot{E}_{out} = q_r''A = -(-k_1\frac{dT}{dr})(4\pi r^2) \qquad (3)$$

Substituting Eqs. (2) and (3) in Eq. (1) yields

$$\dot{q}\frac{4}{3}\pi r^3 + k_1\frac{dT}{dr}(4\pi r^2) = 0$$

or

$$\frac{dT}{dr} = -\frac{\dot{q}}{3}\frac{r}{k_1}$$ <

Continued…

(b) For $r > r_1$, the radial heat rate is constant and is

$$\dot{E}_g = q_r = \dot{q}\forall_1 = \dot{q}\frac{4}{3}\pi r_1^3 \tag{4}$$

$$\dot{E}_{in} - \dot{E}_{out} = q_r'' A = -(-k_2 \frac{dT}{dr})4\pi r^2 \tag{5}$$

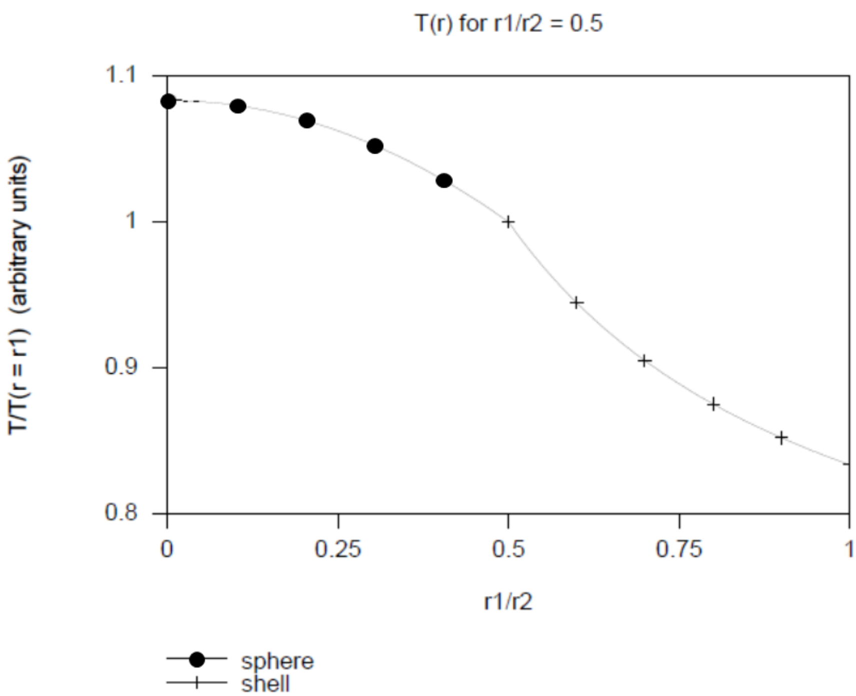
Substituting Eqs. (4) and (5) into Eq. (1) yields

$$k_2 \frac{dT}{dr} 4\pi r^2 + \dot{q}\frac{4}{3}\pi r_1^3 = 0$$

or

$$\frac{dT}{dr} = -\frac{\dot{q} r_1^3}{3k_2 r^2} \quad <$$

(c) The temperature distribution on T-r coordinates is



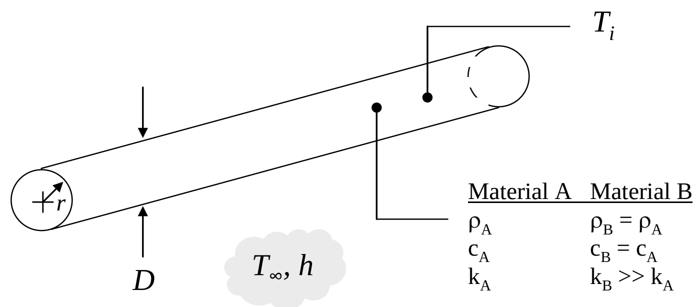
COMMENTS: (1) Note the non-linear temperature distributions in both the particle and the shell. (2) The temperature gradient at r = 0 is zero. (3) The discontinuous slope of T(r) at $r_1/r_2 = 0.5$ is a result of $k_1 = 2k_2$.

PROBLEM 2.49

KNOWN: Long cylindrical rod with uniform initial temperature immersed in liquid at a lower temperature.

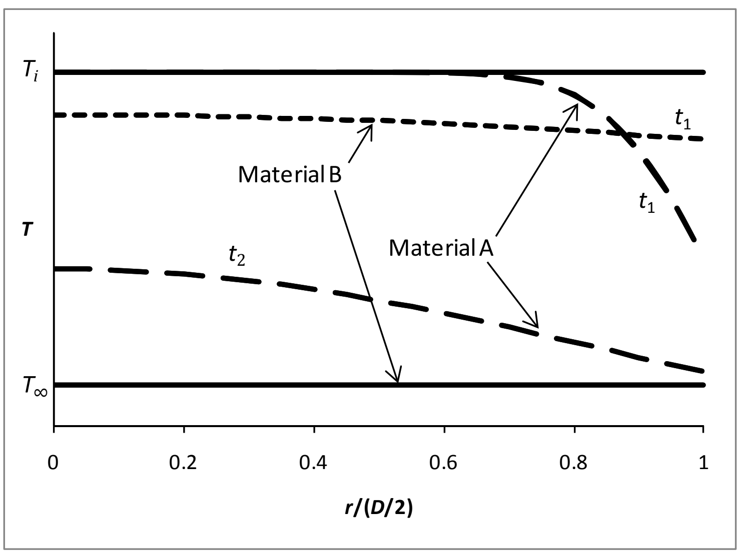
FIND: Sketch temperature distribution at initial time, steady state, and two intermediate times for two rods with different thermal conductivities. State boundary conditions at centerline and surface.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in radial direction, (2) Constant properties, (3) Fluid temperature remains constant, (4) Convection heat transfer coefficient is constant.

ANALYSIS: Referring to the figure below, first consider Material A of moderate thermal conductivity. Initially, the rod temperature is uniform at T_i. When the rod is first exposed to the liquid, heat is transferred from the rod to the fluid due to convection, causing the surface temperature to decrease. The resulting temperature gradient in the rod causes heat to conduct radially outward, and the temperature further inside the rod decreases as well. Toward the beginning of this process, the temperature near the center of the rod is still very close to the initial temperature (see Material A, t_1). As time increases, the temperature everywhere in the rod decreases (see Material A, t_2). Eventually, at steady state, the rod temperature reaches the fluid temperature, T_∞.



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PROBLEM 2.49 (Cont.)

The boundary condition at the rod surface expresses a balance between heat reaching the surface by conduction and heat leaving the surface by convection:

$$-k\left.\frac{\partial T}{\partial r}\right|_{D/2} = h\left[T(D/2,t) - T_\infty\right] \qquad (1)$$ <

From this, it can be seen that the temperature gradient at the surface is negative and its magnitude decreases with time as the surface temperature approaches the fluid temperature. This is shown for the two intermediate times for Material A.

Next compare Material A to Material B having a very large thermal conductivity. At time $t = 0$ when both rods have the same temperature T_i, it can be seen from the right hand side of Equation (1) that the heat flux is the same for both materials. Energy is being removed from both rods at the same rate. However, because of the large thermal conductivity of material B, its temperature gradient is smaller and its temperature tends to be nearly uniform, as shown in the figure for Material B, t_1. Its temperature is higher at the surface and lower in the center as compared to Material A. Because its surface temperature stays higher for longer, the heat flux leaving the rod is larger, and overall it cools faster. At time t_2, when Material A's surface temperature is close to T_∞, but it is still warm in the center, Material B has already reached steady state.

The rod with the higher thermal conductivity reaches steady state sooner. <

The boundary condition at $r = 0$ expresses radial symmetry:

$$\left.\frac{\partial T}{\partial r}\right|_{0} = 0$$ <

The boundary condition at $r = D/2$ was given in Equation (1).

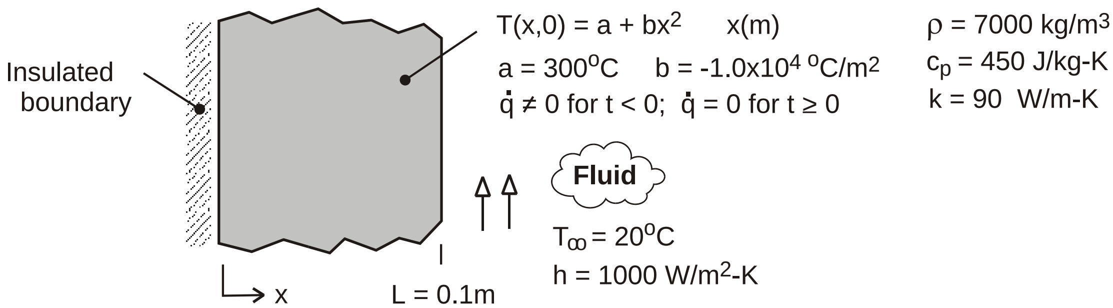
COMMENTS: The problem of transient conduction in a cylinder will be solved in Chapter 5.

PROBLEM 2.50

KNOWN: Temperature distribution in a plane wall of thickness L experiencing uniform volumetric heating $\dot{q}$ having one surface (x = 0) insulated and the other exposed to a convection process characterized by T_∞ and h. Suddenly the volumetric heat generation is deactivated while convection continues to occur.

FIND: (a) Determine the magnitude of the volumetric energy generation rate associated with the initial condition, (b) On T-x coordinates, sketch the temperature distributions for the initial condition ($T \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (c) On q''_x - t coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L,t)$; calculate the corresponding value of the heat flux at t = 0; and (d) Determine the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) Uniform internal volumetric heat generation for t < 0.

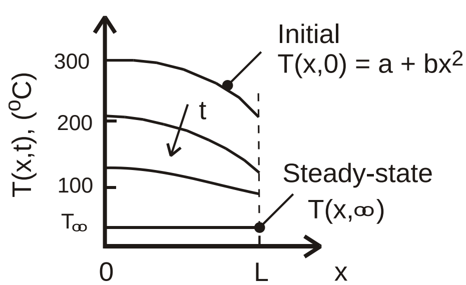
ANALYSIS: (a) The volumetric heating rate can be determined by substituting the temperature distribution for the initial condition into the appropriate form of the heat diffusion equation.

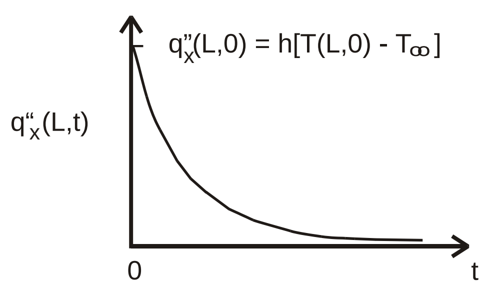
$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x,0)=a+bx^2$$

$$\frac{d}{dx}(0+2bx)+\frac{\dot{q}}{k}=0=2b+\frac{\dot{q}}{k}=0$$

$$\dot{q}=-2kb=-2\times 90\,W/m\cdot K\left(-1.0\times 10^4\,°C/m^2\right)=1.8\times 10^6\,W/m^3$$ <

(b) The temperature distributions are shown in the sketch below.





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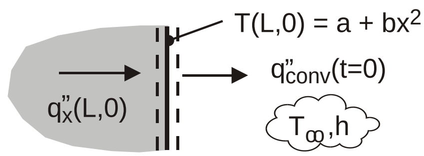
(c) The heat flux at the exposed surface x = L, $q''_x(L,0)$, is initially a maximum value and decreases with increasing time as shown in the sketch above. The heat flux at t = 0 is equal to the convection heat flux with the surface temperature T(L,0). See the surface energy balance represented in the schematic.

$$q''_x(L,0) = q''_{conv}(t=0) = h\left(T(L,0) - T_\infty\right) = 1000\,W/m^2 \cdot K\,(200-20)°C = 1.80 \times 10^5\,W/m^2 \quad <$$

where $\quad T(L,0) = a + bL^2 = 300°C - 1.0 \times 10^4 °C/m^2\,(0.1m)^2 = 200°C.$



(d) The energy removed from the wall to the fluid as it cools from its initial to steady-state condition can be determined from an energy balance on a time interval basis, Eq. 1.12b. For the initial state, the wall has the temperature distribution $T(x,0) = a + bx^2$; for the final state, the wall is at the temperature of the fluid, $T_f = T_\infty$. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[T(x,0) - T_\infty\right] dx$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[a + bx^2 - T_\infty\right] dx = \rho c_p \left[ax + bx^3/3 - T_\infty x\right]_0^L$$

$$E''_{out} = 7000\,kg/m^3 \times 450\,J/kg \cdot K \left[300 \times 0.1 - 1.0 \times 10^4 (0.1)^3/3 - 20 \times 0.1\right] K \cdot m$$

$$E''_{out} = 7.77 \times 10^7\,J/m^2 \quad <$$

COMMENTS: (1) In the temperature distributions of part (a), note these features: initial condition has quadratic form with zero gradient at the adiabatic boundary; for the steady-state condition, the wall has reached the temperature of the fluid; for all distributions, the gradient at the adiabatic boundary is zero; and, the gradient at the exposed boundary decreases with increasing time.

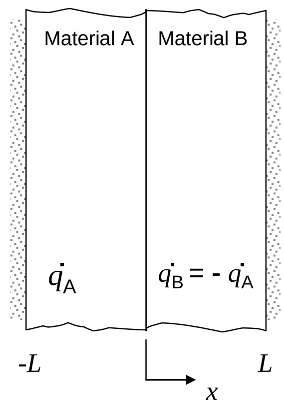
(2) In this thermodynamic analysis, we were able to determine the energy transferred during the cooling process. However, we cannot determine the rate at which cooling of the wall occurs without solving the heat diffusion equation.

PROBLEM 2.51

KNOWN: Thickness of composite plane wall consisting of material A in left half and material B in right half. Exothermic reaction in material A and endothermic reaction in material B, with equal and opposite heat generation rates. External surfaces are insulated.

FIND: Sketch temperature and heat flux distributions for three thermal conductivity ratios, k_A/k_B.

SCHEMATIC:

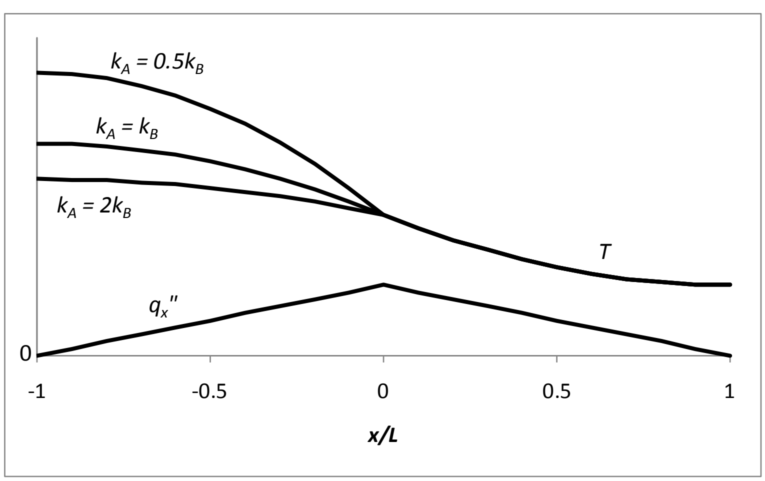


ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: From Equation 2.19 for steady-state, one-dimensional conduction, we find

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -\dot{q} \quad \text{or} \quad \frac{\partial q_x''}{\partial x} = \dot{q}$$

From the second equation, with uniform heat generation rate, we see that q_x'' varies linearly with x, and its slope is $+\dot{q}_A$ in material A and $-\dot{q}_A$ in material B. Furthermore, since the wall is insulated on both exterior surfaces, the heat flux must be zero at $x = \pm L$. Thus, the heat flux is as shown in the graph below and does not depend on the thermal conductivities. The heat generated in the left half is conducting to the right and accumulating as it goes. Once it reaches the centerline, it begins to be consumed by the exothermic reaction and drops to zero at $x = L$.



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Since $q''_x = -k\dfrac{\partial T}{\partial x}$, the temperature gradient is negative everywhere, and its magnitude is greatest where the heat flux is greatest. Thus the slope of the temperature distribution is zero at $x = -L$, it becomes more negative as it reaches the center, and then becomes flatter again until it reaches a slope of zero at $x = L$. When $k_A = k_B$, the temperature distribution has equal and opposite slopes on either side of the centerline. If k_B is held fixed and k_A is varied, the results are as shown in the plot above. Since the temperature gradient is inversely proportional to the thermal conductivity, it is steeper in the region that has the smaller thermal conductivity. Physically, when thermal conductivity is larger, heat conducts more readily and causes the temperature to become more uniform.

If $\dot{q}_B = -2\dot{q}_A$, an energy balance on the wall gives:

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\frac{dE_{st}}{dt} = \dot{E}_g = (\dot{q}_A + \dot{q}_B)V = -\dot{q}_A V$$

where V is the volume. Since dE_{st}/dt is non-zero, the wall cannot be at steady-state. With the exothermic reaction greater than the endothermic reaction, the wall will continuously decrease in temperature. <

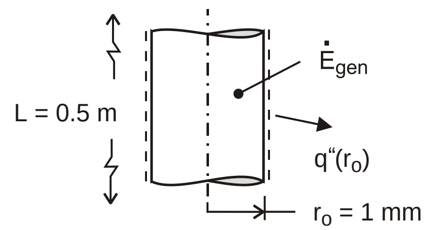
COMMENTS: (1) Given the information in the problem statement, it is not possible to calculate actual temperatures. There are an infinite number of correct solutions regarding temperature *values*, but only one correct solution regarding the *shape* of the temperature distribution. (2) Chemical reactions would cease if the temperature became too small. It would not be possible to continually cool the wall for the case when, initially, $\dot{q}_B = -2\dot{q}_A$.

PROBLEM 2.52

KNOWN: Radius and length of coiled wire in hair dryer. Electric power dissipation in the wire, and temperature and convection coefficient associated with air flow over the wire.

FIND: (a) Form of heat equation and conditions governing transient, thermal behavior of wire during start-up, (b) Volumetric rate of thermal energy generation in the wire, (c) Sketch of temperature distribution at selected times during start-up, (d) Variation with time of heat flux at r = 0 and r = r_o.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties, (3) Uniform volumetric heating, (4) Negligible radiation from surface of wire.

ANALYSIS: (a) The general form of the heat equation for cylindrical coordinates is given by Eq. 2.26. For one-dimensional, radial conduction and constant properties, the equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{k}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial condition is $T(r,0)=T_i$ <

The boundary conditions are: $\partial T/\partial r|_{r=0}=0$ <

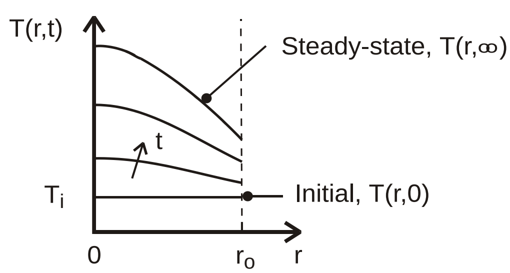
$$-k\frac{\partial T}{\partial r}\bigg|_{r=r_o}=h\left[T(r_o,t)-T_\infty\right]$$ <

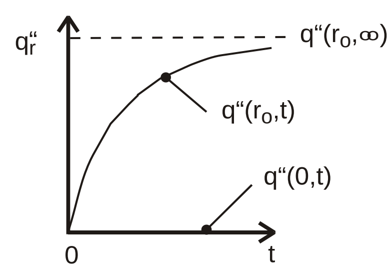
(b) The volumetric rate of thermal energy generation is

$$\dot{q}=\frac{\dot{E}_g}{\forall}=\frac{P_{elec}}{\pi r_o^2 L}=\frac{500\ W}{\pi(0.001m)^2(0.5m)}=3.18\times10^8\ W/m^3$$ <

Under steady-state conditions, all of the thermal energy generated within the wire is transferred to the air by convection. Performing an energy balance for a control surface about the wire, $-\dot{E}_{out}+\dot{E}_g=0$, it follows that $-2\pi r_o L\ q''(r_o, t\to\infty)+P_{elec}=0$. Hence,

$$q''(r_o,t\to\infty)=\frac{P_{elec}}{2\pi r_o L}=\frac{500\ W}{2\pi(0.001m)0.5m}=1.59\times10^5\ W/m^2$$ <





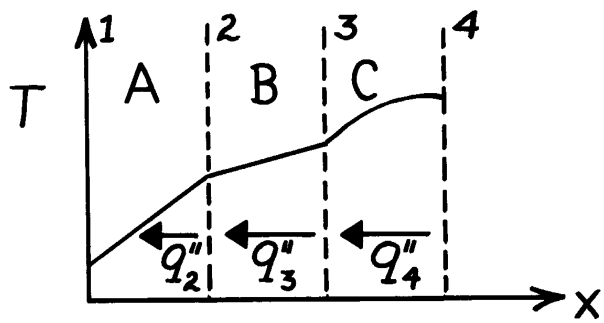
COMMENTS: The symmetry condition at r = 0 imposes the requirement that $\partial T/\partial r|_{r=0}=0$, and hence $q''(0,t)=0$ throughout the process. The temperature at r_o, and hence the convection heat flux, increases steadily during the start-up, and since conduction to the surface must be balanced by convection from the surface at all times, $|\partial T/\partial r|_{r=r_o}$ also increases during the start-up.

PROBLEM 2.53

KNOWN: Temperature distribution in a composite wall.

FIND: (a) Relative magnitudes of interfacial heat fluxes, (b) Relative magnitudes of thermal conductivities, and (c) Heat flux as a function of distance x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) For the prescribed conditions (one-dimensional, steady-state, constant k), the parabolic temperature distribution in C implies the existence of heat generation. Hence, since dT/dx *increases* with *decreasing* x, the heat flux in C *increases* with *decreasing* x. Hence,

$$q''_3 > q''_4$$

However, the linear temperature distributions in A and B indicate no generation, in which case

$$q''_2 = q''_3$$

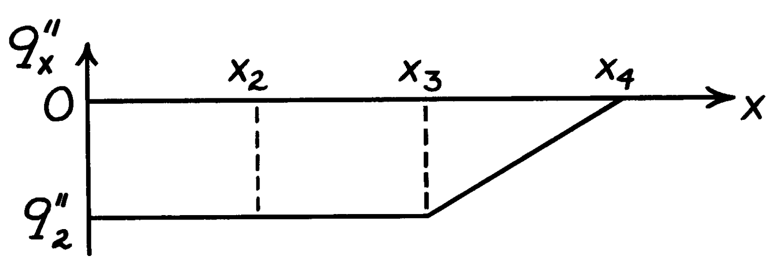
(b) Since conservation of energy requires that $q''_{3,B} = q''_{3,C}$ and $dT/dx)_B < dT/dx)_C$, it follows from Fourier's law that

$$k_B > k_C.$$

Similarly, since $q''_{2,A} = q''_{2,B}$ and $dT/dx)_A > dT/dx)_B$, it follows that

$$k_A < k_B.$$

(c) It follows that the flux distribution appears as shown below.



COMMENTS: Note that, with $dT/dx)_{4,C} = 0$, the interface at 4 is adiabatic.