**P2.1** When an axial load is applied to the ends of the two-segment rod shown in Figure P2.1, the total elongation between joints *A* and *C* is 7.5 mm. The segment lengths are *a* = 1.2 m and *b* = 2.8 m. In segment (2), the normal strain is measured as 2,075 μm/m. Determine:
(a) the elongation of segment (2).
(b) the normal strain in segment (1) of the rod.

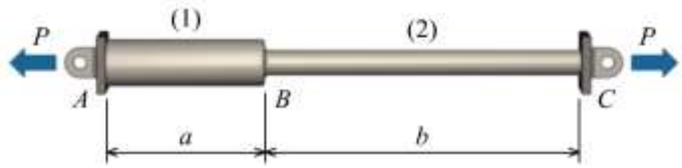



FIGURE P2.1

## Solution

(a) From the definition of normal strain, the elongation in segment (2) can be computed as

$$\delta_2 = \varepsilon_2 L_2 = (2{,}075 \times 10^{-6} \text{ mm/mm})(2{,}800 \text{ mm}) = \boxed{5.81 \text{ mm}}$$ **Ans.**

(b) The combined elongations of segments (1) and (2) is given as 7.5 mm. Therefore, the elongation that occurs in segment (1) must be

$$\delta_1 = \delta_{\text{total}} - \delta_2 = 7.50 \text{ mm} - 5.81 \text{ mm} = 1.69 \text{ mm}$$

The strain in segment (1) can now be computed:

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{1.69 \text{ mm}}{1{,}200 \text{ mm}} = 1{,}408.33 \times 10^{-6} \text{ mm/mm} = \boxed{1{,}408 \ \mu\text{m/m}}$$ **Ans.**

**P2.2** The two bars shown in Figure P2.2 are used to support load *P*. When unloaded, joint *B* has coordinates (0, 0). After load *P* is applied, joint *B* moves to the coordinate position (–0.55 in., –0.15 in.). Assume $a$ = 15 ft, $b$ = 27 ft, $c$ = 11 ft, and $d$ = 21 ft. Determine the normal strain in each bar.

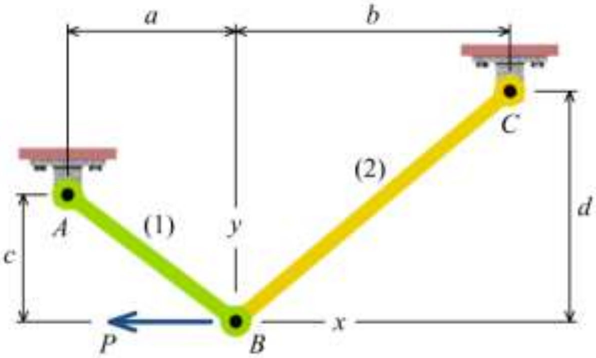



FIGURE P2.2

## Solution

Given

$$a = (15\text{ ft})(12\text{ in./ft}) = 180\text{ in.}$$
$$b = (27\text{ ft})(12\text{ in./ft}) = 324\text{ in.}$$
$$c = (11\text{ ft})(12\text{ in./ft}) = 132\text{ in.}$$
$$d = (21\text{ ft})(12\text{ in./ft}) = 252\text{ in.}$$

The initial length of bar *AB* is

$$L_{AB} = \sqrt{(180\text{ in.})^2 + (132\text{ in.})^2} = 223.2129\text{ in.}$$

and its length after deformation is

$$L'_{AB} = \sqrt{(180\text{ in.} - 0.55\text{ in.})^2 + (132\text{ in.} + 0.15\text{ in.})^2} = 222.8585\text{ in.}$$

The strain in bar *AB* is thus

$$\varepsilon_{AB} = \frac{222.8585\text{ in.} - 223.2129\text{ in.}}{223.2129\text{ in.}} = \frac{-0.3544\text{ in.}}{223.2129\text{ in.}} = -1{,}587.6\times10^{-6}\text{ in./in.} = \boxed{-1{,}588\ \mu\varepsilon}$$ **Ans.**

The initial length of bar *BC* is

$$L_{BC} = \sqrt{(324\text{ in.})^2 + (252\text{ in.})^2} = 410.4632\text{ in.}$$

and its length after deformation is

$$L'_{BC} = \sqrt{(324\text{ in.} + 0.55\text{ in.})^2 + (252\text{ in.} + 0.15\text{ in.})^2} = 410.9895\text{ in.}$$

The strain in bar *BC* is thus

$$\varepsilon_{BC} = \frac{410.9895\text{ in.} - 410.4632\text{ in.}}{410.4632\text{ in.}} = \frac{0.5263\text{ in.}}{410.4632\text{ in.}} = 1{,}282.2\times10^{-6}\text{ in./in.} = \boxed{1{,}282\ \mu\varepsilon}$$ **Ans.**

**P2.3** Pin-connected rigid bars *AB*, *BC*, and *CD* are initially held in the positions shown in Figure P2.3 by taut wires (1) and (2). The bar lengths are $a$ = 24 ft and $b$ = 18 ft. Joint *C* is given a horizontal displacement of 5 in. to the right. (Note that this displacement causes both joints *B* and *C* to move to the right and slightly downward.) What is the change in the average normal strain in wire (1) after the displacement?

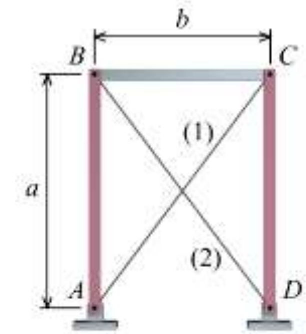


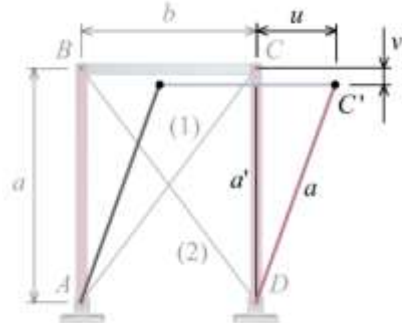

FIGURE P2.3

## Solution

Given

$$a = (24\text{ ft})(12\text{ in./ft}) = 288\text{ in.}$$

$$b = (18\text{ ft})(12\text{ in./ft}) = 216\text{ in.}$$




The initial length of wire (1) is the distance between joints *A* and *C*:

$$L_1 = \sqrt{(288\text{ in.})^2 + (216\text{ in.})^2} = 360\text{ in.}$$

Joint *C* displaces to the right by $u$ = 5 in. Joint *C* also moves down. Use the geometry of the displaced structure to calculate $a'$ using the Pythagorean theorem:

$$a' = \sqrt{a^2 - u^2}$$

$$= \sqrt{(288\text{ in.})^2 - (5.0\text{ in.})^2} = 287.9566\text{ in.}$$

After displacement, the coordinates of joint *C*, relative to joint *A*, are:

$$C' = (b+u,\ a') = (216\text{ in.} + 5\text{ in.},\ 287.9566\text{ in.}) = (221\text{ in.},\ 287.9566\text{ in.})$$

The length of wire (1) after deformation is

$$L_1' = \sqrt{(221\text{ in.})^2 + (287.9566\text{ in.})^2} = 362.9876\text{ in.}$$

The change in strain in wire (1) is thus

$$\varepsilon_{AB} = \frac{362.9876\text{ in.} - 360\text{ in.}}{360\text{ in.}} = \frac{2.9876\text{ in.}}{360\text{ in.}} = 8{,}298.9 \times 10^{-6}\text{ in./in.} = \boxed{8{,}300\ \mu\varepsilon}$$ **Ans.**

For completeness, note that the vertical displacement $v$ is calculated as

$$v = a' - a = 287.9566\text{ in.} - 288\text{ in.} = -0.0434\text{ in.}$$

**P2.4** Bar (1) has a length of $L_1$ = 2.50 m, and bar (2) has a length of $L_2$ = 0.65 m. Initially, there is a gap of Δ = 3.5 mm between the rigid plate at *B* and bar (2). After application of the loads *P* to the rigid plate at *B*, the rigid plate moved to the right, stretching bar (1) and compressing bar (2). The normal strain in bar (1) was measured as 2,740 μm/m after the loads *P* were applied. Determine the normal strain produced in bar (2).

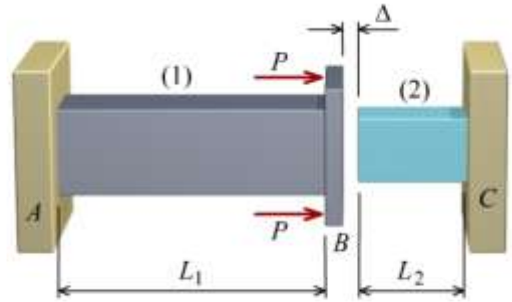


FIGURE P2.4

## Solution

From the measured strain, compute the deformation of bar (1) as:

$$\delta_1 = \varepsilon_1 L_1 = (2{,}740\times10^{-6} \text{ mm/mm})(2.5 \text{ m})(1{,}000 \text{ mm/m}) = 6.8500 \text{ mm}$$

Since it is attached to bar (1), rigid plate *B* displaces to the right by an equal amount.

$$u_B = \delta_1 = 6.8500 \text{ mm} \rightarrow$$

Plate *B* moves 3.5 mm before contacting bar (2). Therefore, the deformation produced in bar (2) is equal to the difference

$$\delta_2 = \delta_1 - \Delta = 6.8500 \text{ mm} - 3.5 \text{ mm} = 3.3500 \text{ mm}$$

However, bar (2) will be contracting; therefore, the deformation in bar (2) is negative.

$$\delta_2 = -3.3500 \text{ mm}$$

The normal strain produced in bar (2) is thus:

$$\varepsilon_2 = \frac{\delta_2}{L_2} = \frac{-3.3500 \text{ mm}}{650 \text{ mm}} = -5{,}153.85\times10^{-6} \text{ mm/mm} = \boxed{-5{,}150 \ \mu\varepsilon}$$ **Ans.**

**P2.5** In Figure P2.5, rigid bar *ABC* is support by a pin at *B* and by post (1) at *A*. However, there is a gap of Δ = 10 mm between the rigid bar at *A* and post (1). After load *P* is applied to the rigid bar, point *C* moves left by 8 mm. If the length of post (1) is $L_1$ = 1.6 m, what is the average normal strain that is produced in post (1)? Use dimensions of *a* = 1.25 m and *b* = 0.85 m.

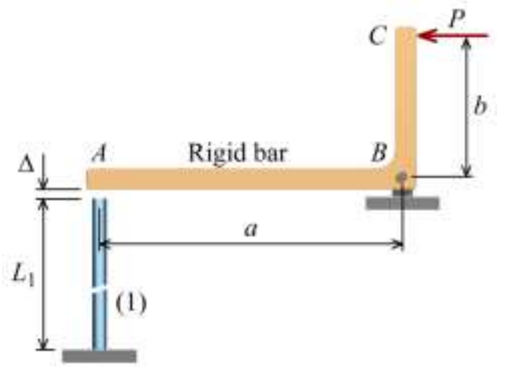
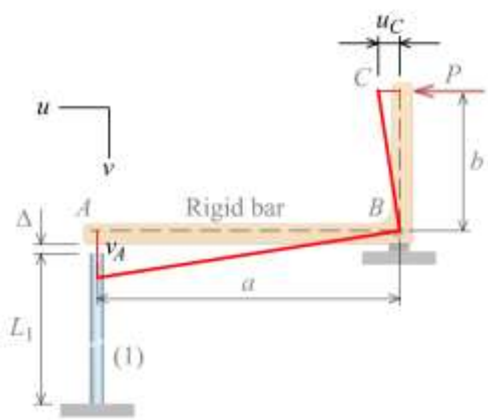



FIGURE P2.5

## Solution

The deflected shape of the rigid bar is shown greatly exaggerated in the figure to the right. The rigid bar rotates about pin *B*. Consequently, a horizontal movement at *C* causes a downward deflection at *A*. Use similar triangles to determine the downward deflection at *A*:




$$\frac{u_C}{b} = \frac{v_A}{a}$$

$$\therefore v_A = \frac{a}{b} u_C = \frac{1.25 \text{ m}}{0.85 \text{ m}}(8 \text{ mm}) = 11.7647 \text{ mm} \downarrow$$

As the rigid bar deflects at *A*, it travels downward Δ = 10 mm before it contacts post (1). The amount of contraction in post (1) is equal to the difference between $v_A$ and Δ. Therefore,

$$\delta_1 = v_A - \Delta$$
$$= 11.7647 \text{ mm} - 10 \text{ mm}$$
$$= 1.7647 \text{ mm}$$

The post becomes shorter after contact with the rigid bar; therefore, the deformation in the post is negative:

$$\delta_1 = -1.7647 \text{ mm}$$

The average normal strain produced in post (1) is thus

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{-1.7647 \text{ mm}}{1{,}600 \text{ mm}} = -1{,}102.9 \times 10^{-6} \text{ mm/mm} = \boxed{-1{,}103 \ \mu\varepsilon}$$ **Ans.**

**P2.6** The rigid bar *ABC* is supported by three bars as shown in Figure P2.6. Bars (1) attached at *A* and *C* are identical, each having a length of $L_1$ = 160 in. Bar (2) has a length of $L_2$ = 110 in.; however, there is a clearance of $c$ = 0.25 in. between bar (2) and the pin in the rigid bar at *B*. There is no strain in the bars before the load *P* is applied, and $a$ = 50 in. After application of load *P*, the tensile normal strain in bar (2) is measured as 960 με. What is the normal strain in bars (1)?

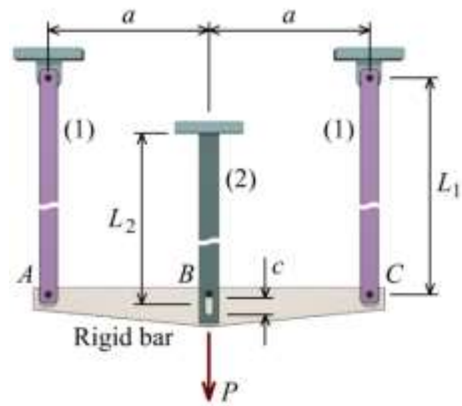



FIGURE P2.6

## Solution

From the strain given for bar (2), we know that the elongation of bar (2) must be:

$$\begin{aligned}\delta_2 &= \varepsilon_2 L_2 \\ &= \left(960\times10^{-6}\text{ in./in.}\right)\left(110\text{ in.}\right) \\ &= 0.1056\text{ in.}\end{aligned}$$

To stretch bar (2) by this amount, the rigid bar must move downward by 0.1056 in. However, there is a clearance in the pin at *B* of 0.25 in., and this clearance means that the rigid bar must deflect downward by 0.25 in. before it even begins to stretch bar (2). Consequently, the rigid bar will have to deflect downward by:

$$\begin{aligned}v_B &= 0.1056\text{ in.} + 0.25\text{ in.} \\ &= 0.3556\text{ in. (downward)}\end{aligned}$$

The deflection of the rigid bar at *A* is equal to the deflection at *B*. We know this because of the symmetry of the assembly…when the rigid bar deflects downward, it remains horizontal. Thus,

$$v_A = v_B = 0.3556\text{ in. (downward)}$$

As joint *A* deflects downward, it stretches bar (1). Since there are no gaps or clearances at *A*, any movement downward of joint *A* will elongate bar (1) by the same amount; thus,

$$\delta_1 = v_A = 0.3556\text{ in.}$$

From the definition of strain,

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{0.3556\text{ in.}}{160\text{ in.}} = 2{,}222.5\times10^{-6}\text{ in./in.} = \boxed{2{,}220\ \mu\varepsilon}$$ **Ans.**

**P2.7** Rigid bar *ABCD* is supported by two bars as shown in Figure P2.7. There is no strain in the vertical bars before load *P* is applied. After load *P* is applied, the normal strain in bar (2) is measured as –3,300 μm/m. Use the dimensions $L_1$ = 1,600 mm, $L_2$ = 1,200 mm, *a* = 240 mm, *b* = 420 mm, and *c* = 180 mm. Determine:

(a) the normal strain in bar (1).
(b) the normal strain in bar (1) if there is a 1 mm gap in the connection at pin *C* before the load is applied.
(c) the normal strain in bar (1) if there is a 1 mm gap in the connection at pin *B* before the load is applied.

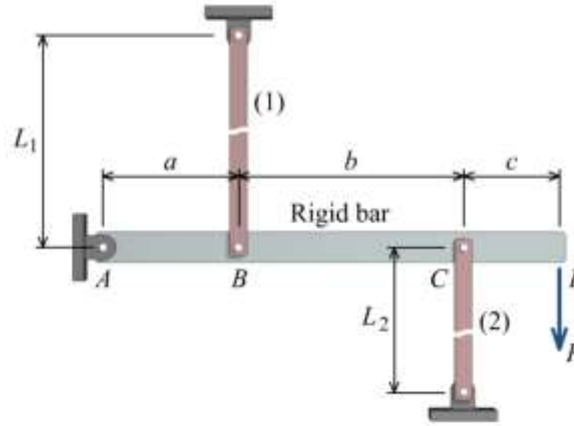
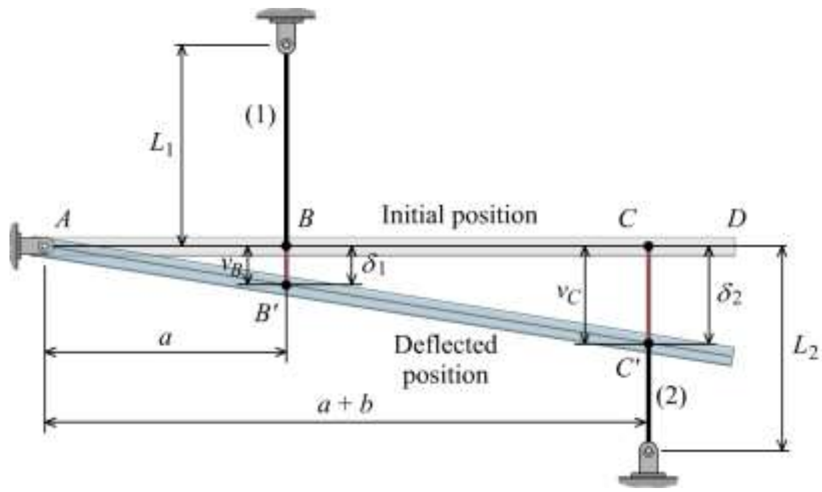



FIGURE P2.7

## Solution

(a) The deformation of bar (2) can be calculated from the specified strain in bar (2)

$$\delta_2 = \varepsilon_2 L_2$$
$$= \left(-3{,}330\times10^{-6}\right)(1{,}200\text{ mm})$$
$$= -3.9600\text{ mm}$$




Since the connection at *C* is perfect (i.e., no gaps or clearances), the contraction of bar (2) corresponds to a downward deflection of the rigid bar at *C* of exactly the same magnitude. Thus, $v_C$ = 3.9600 mm (downward).

From the deformation diagram of rigid bar *ABCD*

$$\frac{v_B}{a} = \frac{v_C}{a+b}$$

$$\therefore v_B = \frac{a}{a+b}v_C = \frac{240\text{ mm}}{240\text{ mm}+420\text{ mm}}(-3.9600\text{ mm}) = -1.4400\text{ mm} = 1.4400\text{ mm}\downarrow$$

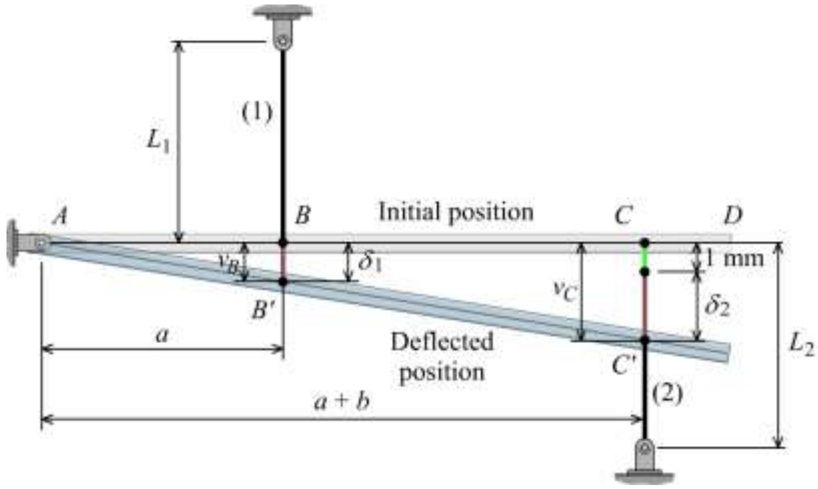
Since the connection at *B* is perfect (i.e., no gaps or clearances), the downward deflection of the rigid bar at *B* will create a deformation of the same magnitude in bar (1). Therefore, $\delta_1$ = 1.4400 mm (elongation), and thus, from the definition of normal strain:

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{1.4400\text{ mm}}{1{,}600\text{ mm}} = 900\times10^{-6}\text{ mm/mm} = \boxed{900\ \mu\varepsilon}$$ **Ans.**

(b) The deformation of bar (2) can be calculated from the specified strain in bar (2)

$$\delta_2 = \varepsilon_2 L_2$$
$$= \left(-3{,}330\times10^{-6}\right)(1{,}200\text{ mm})$$
$$= -3.9600\text{ mm}$$

Here, the connection at $C$ is not perfect—there is a 1 mm gap in the connection at $C$. Therefore, the rigid bar must move downward by 1 mm before it contacts bar (2).




We know that bar (2) has been contracted by −3.9600 mm, and we also know that the rigid bar moves 1 mm downward before even starting to contract bar (2). Thus, joint $C$ on the rigid bar must have moved downward $v_C$ = 1 mm + 3.9600 mm = 4.9600 mm (downward).

From the deformation diagram of rigid bar $ABCD$

$$\frac{v_B}{a} = \frac{v_C}{a+b}$$

$$\therefore v_B = \frac{a}{a+b} v_C = \frac{240\text{ mm}}{240\text{ mm} + 420\text{ mm}}(-4.9600\text{ mm}) = -1.8036\text{ mm} = 1.8036\text{ mm} \downarrow$$

The connection at $B$ is perfect (i.e., no gaps or clearances); thus, the downward deflection of the rigid bar at $B$ will create a deformation of the same magnitude in bar (1). Therefore, $\delta_1$ = 1.8036 mm (elongation), and thus, from the definition of normal strain:

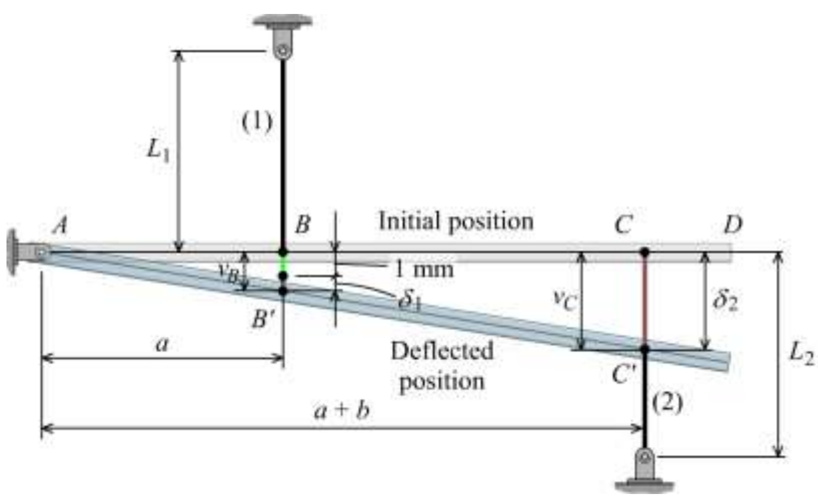
$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{1.8036\text{ mm}}{1{,}600\text{ mm}} = 1{,}127.3\times10^{-6}\text{ mm/mm} = \boxed{1{,}127\ \mu\varepsilon}$$ **Ans.**

---

(c) The deformation of bar (2) can be calculated from the specified strain in bar (2)

$$\delta_2 = \varepsilon_2 L_2$$
$$= \left(-3{,}330\times10^{-6}\right)(1{,}200\text{ mm})$$
$$= -3.9600\text{ mm}$$

Since the connection at $C$ is perfect (i.e., no gaps or clearances), the contraction of bar (2) corresponds to a downward deflection of the rigid bar at $C$ of exactly the same magnitude. Thus, $v_C$ = 3.9600 mm (downward).



From the deformation diagram of rigid bar *ABCD*

$$\frac{v_B}{a} = \frac{v_C}{a+b}$$

$$\therefore v_B = \frac{a}{a+b} v_C = \frac{240 \text{ mm}}{240 \text{ mm} + 420 \text{ mm}} (-3.9600 \text{ mm}) = -1.4400 \text{ mm} = 1.4400 \text{ mm} \downarrow$$

Here, the connection at *B* is not perfect—there is a 1 mm gap at joint *B*. The rigid bar must move downward 1 mm at *B* before it begins to elongate bar (1). Therefore, the elongation of bar (1) is

$$\delta_1 = v_B - 1 \text{ mm}$$
$$= 1.4400 \text{ mm} - 1 \text{ mm}$$
$$= 0.4400 \text{ mm}$$

From the definition of normal strain:

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{0.4400 \text{ mm}}{1{,}600 \text{ mm}} = 275 \times 10^{-6} \text{ mm/mm} = \boxed{275 \ \mu\varepsilon}$$ **Ans.**

**P2.8** The sanding-drum mandrel shown in Figure P2.6 is made for use with a hand drill. The mandrel is made from a rubber-like material that expands when the nut is tightened to secure the sanding sleeve placed over the outside surface. If the diameter $D$ of the mandrel increases from 2.00 in. to 2.15 in. as the nut is tightened, determine

(a) the average normal strain along a diameter of the mandrel.

(b) the circumferential strain at the outside surface of the mandrel.

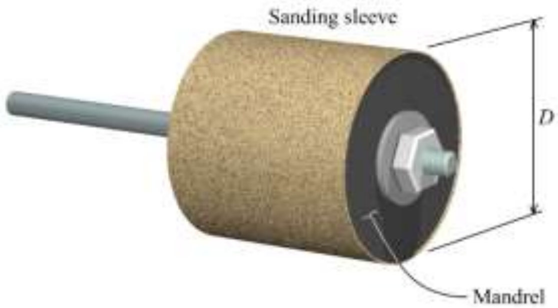



FIGURE P2.8

## Solution

(a) The change in strain along a diameter is found from

$$\varepsilon_D = \frac{\Delta D}{D} = \frac{2.15\text{ in.} - 2.00\text{ in.}}{2.00\text{ in.}} = \boxed{0.075\text{ in./in.}}$$ **Ans.**

(b) Note that the circumference of a circle is given by $\pi D$. The change in strain around the circumference of the mandrel is found from

$$\varepsilon_C = \frac{\Delta C}{C} = \frac{\pi(2.15\text{ in.}) - \pi(2.00\text{ in.})}{\pi(2.00\text{ in.})} = \boxed{0.075\text{ in./in.}}$$ **Ans.**

**P2.9** The normal strain in a suspended bar of material of varying cross section due to its own weight is given by the expression $\gamma y/3E$ where $\gamma$ is the specific weight of the material, $y$ is the distance from the free (i.e., bottom) end of the bar, and $E$ is a material constant. Determine, in terms of $\gamma$, $L$, and $E$,
(a) the change in length of the bar due to its own weight.
(b) the average normal strain over the length $L$ of the bar.
(c) the maximum normal strain in the bar.

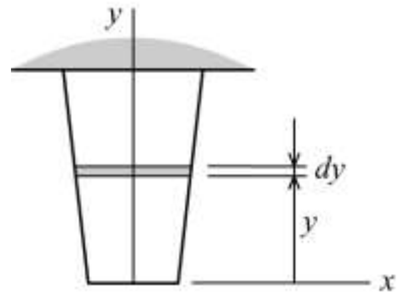
## Solution

(a) The strain of the suspended bar due to its own weight is given as

$$\varepsilon = \frac{\gamma y}{3E}$$

Consider a slice of the bar having length $dy$. In general, $\delta = \varepsilon L$. Applying this definition to the bar slice, the deformation of slice $dy$ is given by

$$d\delta = \varepsilon dy = \frac{\gamma y}{3E} dy$$




Since this strain expression varies with $y$, the total deformation of the bar must be found by integrating $d\delta$ over the bar length:

$$\delta = \int_0^L \frac{\gamma y}{3E} dy = \frac{\gamma}{3E}\left[\frac{y^2}{2}\right]_0^L = \boxed{\frac{\gamma L^2}{6E}}$$ **Ans.**

(b) The average normal strain is found by dividing the expression above for $\delta$ by the bar length $L$

$$\varepsilon_{\text{avg}} = \frac{\gamma L^2 / 6E}{L} = \boxed{\frac{\gamma L}{6E}}$$ **Ans.**

(c) Since the given strain expression varies with $y$, the maximum normal strain occurs at the maximum value of $y$, that is, at $y = L$:

$$\varepsilon_{\max} = \varepsilon\Big|_{y=L} = \frac{\gamma y}{3E}\Big|_{y=L} = \boxed{\frac{\gamma L}{3E}}$$ **Ans.**

**P2.10** A steel cable is used to support an elevator cage at the bottom of a 2,000-ft deep mineshaft. A uniform normal strain of 250 μin./in. is produced in the cable by the weight of the cage. At each point, the weight of the cable produces an additional normal strain that is proportional to the length of the cable below the point. If the total normal strain in the cable at the cable drum (upper end of the cable) is 700 μin./in., determine

(a) the strain in the cable at a depth of 500 ft.

(b) the total elongation of the cable.

## Solution

Call the vertical coordinate *y* and establish the origin of the *y* axis at the lower end of the steel cable. The strain in the cable has a constant term (i.e., $\varepsilon = 250$ με) and a term (we will call it *k*) that varies with the vertical coordinate *y*.

$$\varepsilon = 250\times10^{-6} + k\,y$$

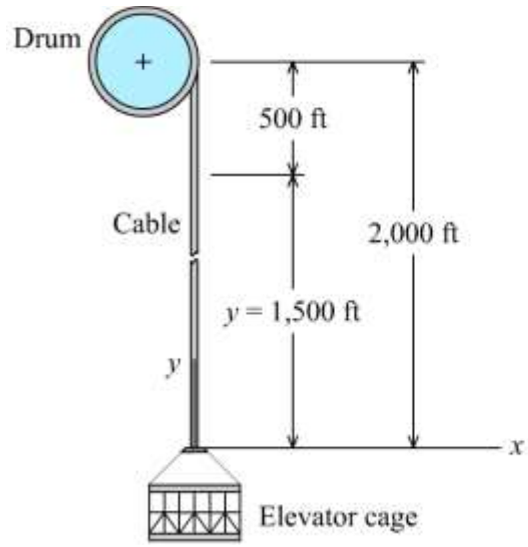
The problem states that the normal strain at the cable drum (i.e., *y* = 2,000 ft) is 700 μin./in. Knowing this value, the constant *k* can be determined

$$700\times10^{-6} = 250\times10^{-6} + k(2{,}000 \text{ ft})$$

$$\therefore k = \frac{700\times10^{-6} - 250\times10^{-6}}{2{,}000 \text{ ft}} = 0.225\times10^{-6}/\text{ft}$$

Substituting this value for *k* in the strain expression gives

$$\varepsilon = 250\times10^{-6} + (0.225\times10^{-6}/\text{ft})\,y$$




(a) At a depth of 500 ft, the *y* coordinate is *y* = 1,500 ft. Therefore, the cable strain at a depth of 500 ft is

$$\varepsilon = 250\times10^{-6} + (0.225\times10^{-6}/\text{ft})(1{,}500 \text{ ft}) = 587.5\times10^{-6} = \boxed{588\ \mu\varepsilon}$$ **Ans.**

(b) The total elongation is found by integrating the strain expression over the cable length; thus,

$$\delta = \int \varepsilon\,dy = \int_0^{2000}\left[250\times10^{-6} + (0.225\times10^{-6}/\text{ft})\,y\right]dy$$

$$= \left[(250\times10^{-6})\,y + \frac{0.225\times10^{-6}}{2}\,y^2\right]_0^{2000}$$

$$= \boxed{0.950 \text{ ft} = 11.40 \text{ in.}}$$ **Ans.**

**P2.11** A thin rectangular polymer plate *PQRS* of width $b$ = 400 mm and height $a$ = 180 mm is shown in Figure P2.11. The plate is deformed so that corner *Q* is displaced upward by $c$ = 3.0 mm and corner *R* is displaced leftward by the same amount. Determine the shear strain at corner *P* after deformation.

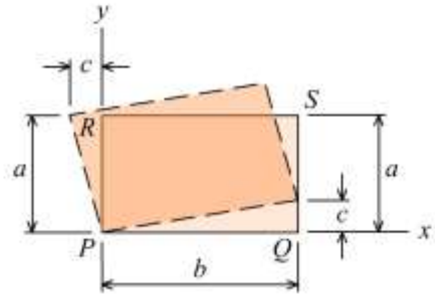
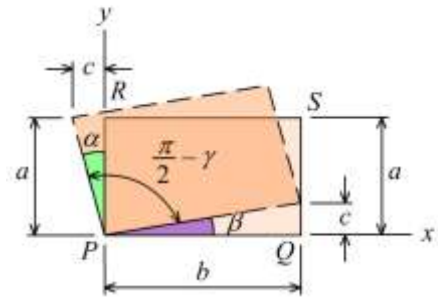


FIGURE P2.10

## Solution

Before deformation, the angle of the plate at *P* was 90° or $\pi/2$ radians. We must now determine the plate angle at *P′* after deformation. The difference between these angles is the shear strain at corner *P*.




After point *Q* displaces upward by 3 mm, we calculate the angle $\beta$ shown in the sketch to the right as

$$\tan\beta = \frac{c}{b} = \frac{3\text{ mm}}{400\text{ mm}} = 0.007500$$

$$\therefore \beta = 0.429710°$$

After point *R* displaces 3 mm to the left, we calculate the angle $\alpha$ as

$$\tan\alpha = \frac{c}{a} = \frac{3\text{ mm}}{180\text{ mm}} = 0.016667$$

$$\therefore \alpha = 0.954841°$$

After deformation, the angle of the plate at *P* is

$$90° + \alpha - \beta = 90° + 0.954841° - 0.429710° = 90.525131° = 1.579962\text{ rad}$$

The difference in the plate angle at *P* before and after deformation is the shear strain:

$$\frac{\pi}{2} - \gamma = 1.579962\text{ rad}$$

$$\therefore \gamma = -0.009165\text{ rad} = \boxed{-9{,}170\ \mu\text{rad}}$$ **Ans.**

**P2.12** A thin triangular plate *PQR* forms a right angle at point *Q*. During deformation, point *Q* moves to the right by $u$ = 0.8 mm and upward by $v$ = 1.3 mm to new position *Q′*, as shown in Figure P2.12. Determine the shear strain $\gamma$ at corner *Q′* after deformation. Use $a$ = 225 mm, $b$ = 455 mm, and $d$ = 319.96 mm.

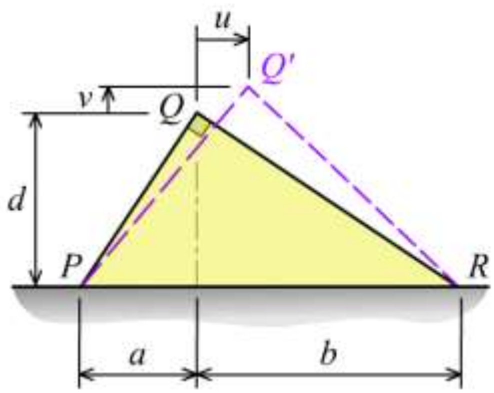
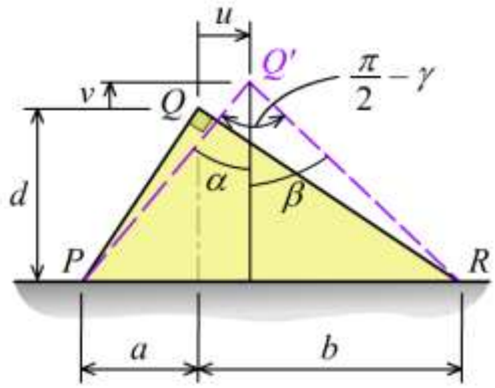


FIGURE P2.12

## Solution

Before deformation, the angle of the plate at *Q* was 90° or $\pi/2$ radians. We must now determine the plate angle at *Q′* after the plate has been deformed. The difference between these angles is the shear strain.




After deformation, the angle $\alpha$ is

$$\tan\alpha = \frac{a+u}{d+v} = \frac{225\text{ mm} + 0.8\text{ mm}}{319.96\text{ mm} + 1.3\text{ mm}} = 0.70286$$

$$\therefore \alpha = 35.10168°$$

and the angle $\beta$ is

$$\tan\beta = \frac{b-u}{d+v} = \frac{455\text{ mm} - 0.8\text{ mm}}{319.96\text{ mm} + 1.3\text{ mm}} = 1.41380$$

$$\therefore \beta = 54.72779°$$

Therefore, after deformation, the angle of the plate at *Q′* is

$$\alpha + \beta = 35.10168° + 54.72779° = 89.82946° = 1.56782\text{ rad}$$

The difference in the angle at *Q* before and after deformation is the shear strain:

$$\frac{\pi}{2} - \gamma = 1.56782\text{ rad}$$

$$\therefore \gamma = 0.002976\text{ rad} = \boxed{2{,}980\ \mu\text{rad}}$$ **Ans.**

**P2.13** A thin triangular plate *PQR* forms a right angle at point *Q*. During deformation, point *Q* moves to the left by $u$ = 2.0 mm and upward by $v$ = 5.0 mm to new position *Q′*, as shown in Figure P2.13. Determine the shear strain $\gamma$ at corner *Q′* after deformation. Use $c$ = 700 mm, $\alpha$ = 28°, and $\beta$ = 62°.

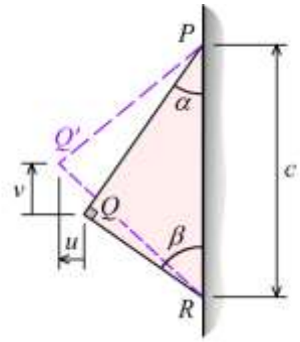
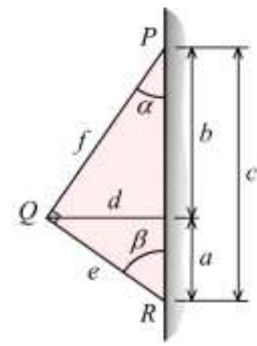



FIGURE P2.13

## Solution

Let us first determine some dimensions of the triangular plate before deformation occurs. Based on the figure to the right and the specified values of $c$ = 700 mm, $\alpha$ = 28°, and $\beta$ = 62°, we calculate




$$\sin\alpha = \frac{e}{c} \qquad \therefore e = c\sin\alpha = (700\text{ mm})\sin(28°) = 328.6301\text{ mm}$$

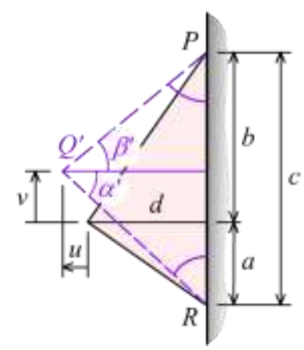
$$\cos\alpha = \frac{f}{c} \qquad \therefore f = c\cos\alpha = (700\text{ mm})\cos(28°) = 618.0633\text{ mm}$$

$$\sin\alpha = \frac{d}{f} \qquad \therefore d = f\sin\alpha = (618.0633\text{ mm})\sin(28°) = 290.1632\text{ mm}$$

$$\cos\alpha = \frac{b}{f} \qquad \therefore b = f\cos\alpha = (618.0633\text{ mm})\cos(28°) = 545.7175\text{ mm}$$

$$a = c - b = 700\text{ mm} - 545.7175\text{ mm} = 154.2825\text{ mm}$$

After deformation, corner *Q* moves to the left by $u$ = 2.0 mm and upward by $v$ = 5.0 mm. Before deformation, the interior angle at *Q* was a right angle. Our next task is to determine the interior angle at *Q′* after deformation. Based on the figure to the right, we calculate




$$\tan\alpha' = \frac{a+v}{d+u} = \frac{154.2825\text{ mm} + 5.0\text{ mm}}{290.1632\text{ mm} + 2.0\text{ mm}} = \frac{159.2825\text{ mm}}{292.1632\text{ mm}} = 0.545183$$

$$\therefore \alpha' = 28.59848° = 0.499138\text{ rad}$$

$$\tan\beta' = \frac{b-v}{d+u} = \frac{545.7175\text{ mm} - 5.0\text{ mm}}{290.1632\text{ mm} + 2.0\text{ mm}} = \frac{540.7175\text{ mm}}{292.1632\text{ mm}} = 1.850738$$

$$\therefore \beta' = 61.61654° = 1.075412\text{ rad}$$

Therefore, after deformation, the angle of the plate at *Q′* is

$$\alpha' + \beta' = 0.499138\text{ rad} + 1.075412\text{ rad} = 1.574550\text{ rad}$$

The difference in the angle at *Q* before and after deformation is the shear strain:

$$\frac{\pi}{2} - \gamma = 1.574550\text{ rad}$$

$$\therefore \gamma = -0.003753\text{ rad} = \boxed{-3{,}750\ \mu\text{rad}}$$ **Ans.**

**P2.14** A thin square polymer plate is deformed into the position shown by the dashed lines in Figure P2.14. Assume that $a$ = 800 mm, $b$ = 85 mm, and $c$ = 960 mm. Determine the shear strain $\gamma_{xy}$ after deformation:
(a) at corner $P$.
(b) at corner Q.

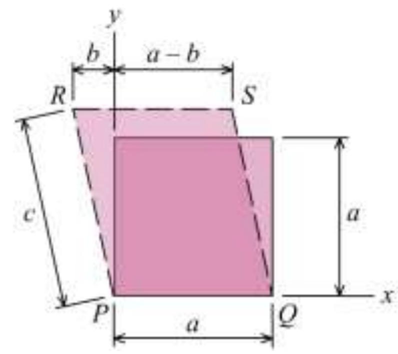



FIGURE P2.14

## Solution

**(a) Shear strain at corner *P*.** After deformation, the angle that side $PR$ makes with the vertical axis can be found from

$$\sin\alpha = \frac{b}{c} = \frac{85 \text{ mm}}{960 \text{ mm}} = 0.088542$$
$$\therefore \alpha = 0.088658 \text{ rad}$$

Before deformation, the interior angle at $P$ was $\pi/2$ radians. After deformation, the interior angle at $P$ is $\pi/2$ + 0.088658 radians. Refer to Figure 2.5$a$ and 2.5$b$ in the text. If we denote the interior angle at $P$ after deformation as

$$\frac{\pi}{2} - \gamma_P$$

then the shear strain at corner $P$ is:

$$\frac{\pi}{2} - \gamma_P = \frac{\pi}{2} + 0.088658 \text{ rad}$$
$$\therefore \gamma_P = -0.088658 \text{ rad} = \boxed{-0.0887 \text{ rad}}$$ **Ans.**

**(b) Shear strain at corner *Q*.** The change of angle at $Q$ has the same magnitude as the angle change at $P$. After deformation, the interior angle at $Q$ is therefore $\pi/2$ – 0.088658 radians. If we denote the interior angle at $Q$ after deformation as

$$\frac{\pi}{2} - \gamma_Q$$

then the shear strain at corner $Q$ is:

$$\frac{\pi}{2} - \gamma_Q = \frac{\pi}{2} - 0.088658 \text{ rad}$$
$$\therefore \gamma_Q = 0.088658 \text{ rad} = \boxed{0.0887 \text{ rad}}$$ **Ans.**

**P2.15** A thin square plate *PQRS* is symmetrically deformed into the shape shown by the dashed lines in Figure P2.15. The initial length of diagonals *PR* and *QS* is $d$ = 295 mm. After deformation, diagonal *PR* has a length of $d_{PR}$ = 295.3 mm, and diagonal *QS* has a length of $d_{QS}$ = 293.7 mm. For the deformed plate, determine:
(a) the normal strain of diagonal *QS*.
(b) the shear strain $\gamma_{xy}$ at corner *P*.

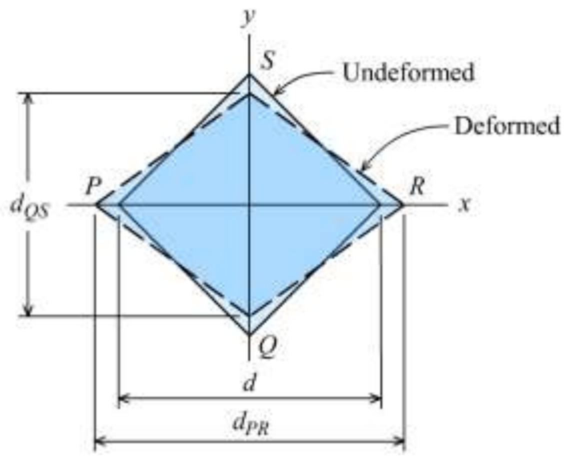



FIGURE P2.15

## Solution

**(a) Normal strain of diagonal *QS*.** The deformation of diagonal *QS* is

$$\delta_{QS} = 293.7 \text{ mm} - 295 \text{ mm} = -1.3 \text{ mm}$$

Thus, the normal strain of diagonal *QS* is

$$\varepsilon_{QS} = \frac{-1.3 \text{ mm}}{295 \text{ mm}} = -0.004407 \text{ mm/mm} = \boxed{-4,410 \ \mu\varepsilon}$$ **Ans.**

**(b) Shear strain at corner *P*.** Before deformation, side *PS* makes an angle of $\pi/4$ radians with the *x* axis. After deformation, the angle that side *PS* makes with the *x* axis can be found from

$$\tan\alpha = \frac{(293.7 \text{ mm}/2)}{(295.3 \text{ mm}/2)} = 0.994582 \text{ rad}$$

$$\therefore \alpha = 0.782682 \text{ rad}$$

Before deformation, the interior angle at *P* (i.e., angle *SPQ*) was $\pi/2$ radians. After deformation, the interior angle at *P* is 2(0.782682 radians) = 1.565364. Refer to Figure 2.5*a* and 2.5*b* in the text. If we denote the interior angle at *P* after deformation as

$$\frac{\pi}{2} - \gamma_P$$

then the shear strain at corner *P* is:

$$\frac{\pi}{2} - \gamma_P = 1.565364 \text{ rad}$$

$$\therefore \gamma_P = \frac{\pi}{2} - 1.565364 \text{ rad} = 0.005433 \text{ rad} = \boxed{5,430 \ \mu\text{rad}}$$ **Ans.**

**P2.16** An airplane has a half-wingspan of 96 ft. Determine the change in length of the aluminum alloy [$\alpha$ = 13.1×10$^{-6}$/°F] wing spar if the plane leaves the ground at a temperature of 59°F and climbs to an altitude where the temperature is –70°F.

## Solution

The change in temperature between the ground and the altitude in flight is

$$\Delta T = T_{\text{final}} - T_{\text{initial}} = -70°\text{F} - 59°\text{F} = -129°\text{F}$$

The thermal strain is given by

$$\varepsilon_T = \alpha\,\Delta T = (13.1\times 10^{-6}/°\text{F})(-129°\text{F}) = -1.6899\times 10^{-3}\ \text{in./in.}$$

and thus the deformation in the 96 ft long wing is

$$\delta = \varepsilon_T L = (-1.6899\times 10^{-3}\ \text{in./in})(96\ \text{ft}) = -0.16223\ \text{ft} = \boxed{-1.947\ \text{in.}}$$ **Ans.**

**P2.17** A square high-density polyethylene [$\alpha = 158\times10^{-6}$/°C] plate has a width of 300 mm. A 180 mm diameter circular hole is located at the center of the plate. If the temperature of the plate increases by 40°C, determine:

(a) the change in width of the plate.
(b) the change in diameter of the hole.

## Solution

The thermal strain is given by

$$\varepsilon_T = \alpha\,\Delta T = (158.0\times10^{-6}/°\text{C})(40°\text{C}) = 6{,}320\times10^{-6}\text{ mm/mm}$$

**(a) Change in width of the plate.** The width *w* of the plate changes by

$$\Delta w = \varepsilon_T w = \left(6{,}320\times10^{-6}\text{ mm/mm}\right)\left(300\text{ mm}\right) = \boxed{1.896\text{ mm}}$$ **Ans.**

**(b) Change in diameter of the hole.** The diameter *d* of the hole changes by

$$\Delta d = \varepsilon_T d = \left(6{,}320\times10^{-6}\text{ mm/mm}\right)\left(180\text{ mm}\right) = \boxed{1.138\text{ mm}}$$ **Ans.**

**P2.18** A circular steel [$\alpha = 6.5\times10^{-6}$/°F] band is to be mounted on a circular steel drum. The outside diameter of the drum is 50 in. The inside diameter of the circular band is 49.95 in. The band will be heated and then slipped over the drum. After the band cools, it will tightly grip the drum. This process is called *shrink fitting*. If the temperature of the band is 72°F before heating, compute the minimum temperature to which the band must be heated so that it can be slipped over the drum. Assume that an extra 0.05 in. in diameter is needed for clearance so that the band can be easily slipped over the drum. Assume that the drum diameter remains constant.

## Solution

To slip over the drum, the inside diameter of the steel band must be increased from 49.95 in. to 50.05 in. (including the 0.05 in. diameter tolerance). Therefore, the deformation required for the inside diameter is

$$\delta_d = 50.05\text{ in.} - 49.95\text{ in.} = 0.10\text{ in.}$$

To produce this deformation, the temperature must be changed by at least

$$\delta_d = \varepsilon_T d = \alpha\,\Delta T d = 0.10\text{ in.}$$

$$\therefore \Delta T = \frac{\delta_d}{\alpha d} = \frac{0.10\text{ in.}}{(6.5\times10^{-6}/°\text{F})(49.95\text{ in.})} = 308.00\ °\text{F}$$

The initial temperature of the band is 72°F; therefore, the minimum final temperature to which the band must be heated is

$$T_{\text{final}} \geq T_{\text{initial}} + \Delta T = 72°\text{F} + 308.0°\text{F} = \boxed{380°\text{F}}$$ **Ans.**

**P2.19** At a temperature of 60°F, a gap of $a$ = 0.125 in. exists between the two polymer bars shown in Figure P2.19. Bar (1) has a length of $L_1$ = 40 in. and a coefficient of thermal expansion of $\alpha_1 = 47\times10^{-6}$/°F. Bar (2) has a length of $L_2$ = 24 in. and a coefficient of thermal expansion of $\alpha_2 = 66\times10^{-6}$/°F, respectively. The supports at $A$ and $D$ are rigid. What is the lowest temperature at which the gap is closed?

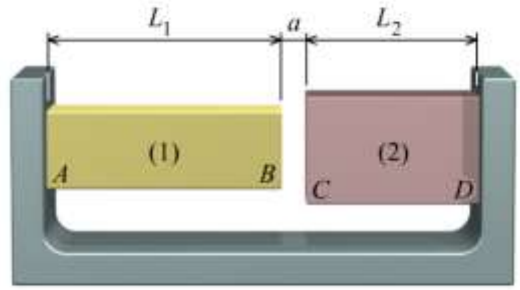



FIGURE P2.19

## Solution

Write expressions for the temperature-induced deformations and set this equal to the gap $a$:

$$\alpha_1 \Delta T\, L_1 + \alpha_2 \Delta T\, L_2 = a$$

$$\Delta T\left[\alpha_1 L_1 + \alpha_2 L_2\right] = a$$

$$\therefore \Delta T = \frac{a}{\alpha_1 L_1 + \alpha_2 L_2} = \frac{0.125 \text{ in.}}{(47\times10^{-6}/°\text{F})(40 \text{ in.}) + (66\times10^{-6}/°\text{F})(24 \text{ in.})} = 36.085°\text{F}$$

Since the initial temperature is 60°F, the temperature at which the gap is closed is 96.1°F. **Ans.**

**P2.20** An aluminum pipe has a length of 60 m at a temperature of 10°C. An adjacent steel pipe at the same temperature is 5 mm longer. At what temperature will the aluminum pipe be 15 mm longer than the steel pipe? Assume that the coefficient of thermal expansion for the aluminum is $22.5\times10^{-6}$/°C and that the coefficient of thermal expansion for the steel is $12.5\times10^{-6}$/°C.

## Solution

The length of the aluminum pipe after a change in temperature can be expressed as

$$\begin{aligned}(L_{\text{final}})_A &= (L_{\text{initial}})_A + (\Delta L)_A \\ &= (L_{\text{initial}})_A + \alpha_A \Delta T (L_{\text{initial}})_A \qquad \text{(a)}\end{aligned}$$

Similarly, the length of the steel pipe after the same change in temperature is given by

$$\begin{aligned}(L_{\text{final}})_S &= (L_{\text{initial}})_S + (\Delta L)_S \\ &= (L_{\text{initial}})_S + \alpha_S \Delta T (L_{\text{initial}})_S \qquad \text{(b)}\end{aligned}$$

From the problem statement, we are trying to determine the temperature change that will cause the final length of the aluminum pipe to be 15 mm longer than the steel pipe. This requirement can be expressed as

$$(L_{\text{final}})_A = (L_{\text{final}})_S + 15 \text{ mm} \qquad \text{(c)}$$

Substitute Eqs. (a) and (b) into Eq. (c) to obtain the following relationship

$$(L_{\text{initial}})_A + \alpha_A \Delta T (L_{\text{initial}})_A = (L_{\text{initial}})_S + \alpha_S \Delta T (L_{\text{initial}})_S + 15 \text{ mm}$$

Collect the terms with $\Delta T$ on the left-hand side of the equation

$$\alpha_A \Delta T (L_{\text{initial}})_A - \alpha_S \Delta T (L_{\text{initial}})_S = (L_{\text{initial}})_S - (L_{\text{initial}})_A + 15 \text{ mm}$$

Factor out $\Delta T$

$$\Delta T \left[ \alpha_A (L_{\text{initial}})_A - \alpha_S (L_{\text{initial}})_S \right] = (L_{\text{initial}})_S - (L_{\text{initial}})_A + 15 \text{ mm}$$

and thus, $\Delta T$ can be expressed as

$$\Delta T = \frac{(L_{\text{initial}})_S - (L_{\text{initial}})_A + 15 \text{ mm}}{\alpha_A (L_{\text{initial}})_A - \alpha_S (L_{\text{initial}})_S}$$

Convert all length dimensions to units of millimeters and solve

$$\begin{aligned}\Delta T &= \frac{60{,}005 \text{ mm} - 60{,}000 \text{ mm} + 15 \text{ mm}}{(22.5\times10^{-6}/°\text{C})(60{,}000 \text{ mm}) - (12.5\times10^{-6}/°\text{C})(60{,}005 \text{ mm})} \\ &= \frac{20 \text{ mm}}{1.35 \text{ mm/°C} - 0.75 \text{ mm/°C}} = \frac{20 \text{ mm}}{0.60 \text{ mm/°C}} \\ &= 33.3°\text{C}\end{aligned}$$

Initially, the pipes were at a temperature of 10°C. With the temperature change determined above, the temperature at which the aluminum pipe is 15 mm longer than the steel pipe is

$$T_{\text{final}} = T_{\text{initial}} + \Delta T = 10°\text{C} + 33.3°\text{C} = \boxed{43.3°\text{C}}$$ **Ans.**

**P2.21** The simple mechanism shown in Figure P2.21 can be calibrated to measure temperature change. Use dimensions of $a$ = 25 mm, $b$ = 90 mm, and $L_1$ = 180 mm. The coefficient of thermal expansion for member (1) is $23.0\times10^{-6}$/°C. Determine the horizontal displacement of pointer tip $D$ of the mechanism shown in response to a temperature increase of 35°C. Assume that pointer $BCD$ is not affected significantly by temperature change.

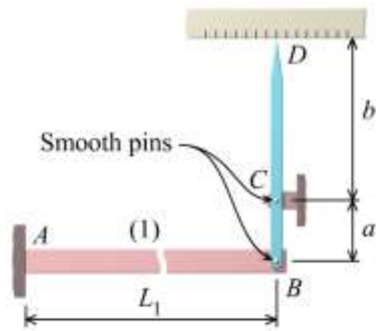



FIGURE P2.21

## Solution

In response to the 35°C temperature increase, member (1) elongates by the amount

$$\delta_1 = \alpha \Delta T\, L_1 = (23.0\times10^{-6}/°\text{C})(35°\text{C})(180\text{ mm}) = 0.1449\text{ mm}$$

The horizontal displacement of joint $B$ is equal to the deformation of member (1):

$$u_B = \delta_1 = 0.1449\text{ mm}$$

The scale reading can be determined from similar triangles

$$\frac{u_D}{b} = \frac{u_B}{a}$$

$$\therefore u_D = \frac{b}{a}u_B = \frac{90\text{ mm}}{25\text{ mm}}(0.1449\text{ mm}) = \boxed{0.522\text{ mm} \leftarrow}$$ **Ans.**

**P2.22** For the assembly shown in Figure P2.22, high-density polyethylene bars (1) and (2) each have coefficients of thermal expansion of $\alpha = 88\times10^{-6}$/°F. If the temperature of the assembly is decreased by 50°F from its initial temperature, determine the resulting displacement of pin *B*. Assume $b$ = 32 in. and $\theta$ = 55°.

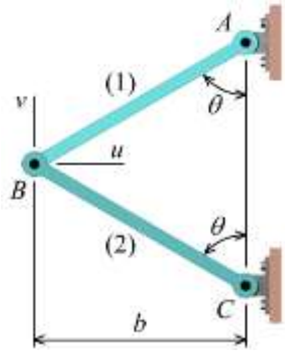
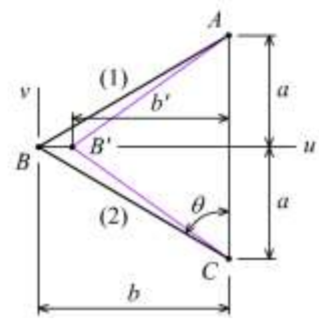



FIGURE P2.22

## Solution

First, we will determine the initial length of member (1). Find the dimension $a$, which is shown in the figure to the right.




$$\tan\theta = \frac{b}{a} \qquad \therefore a = \frac{b}{\tan\theta} = \frac{32 \text{ in.}}{\tan 55°} = 22.40664 \text{ in.}$$

The initial length of member (1) can be found from the Pythagorean theorem:

$$L_1 = \sqrt{a^2 + b^2} = \sqrt{(22.40664 \text{ in.})^2 + (32 \text{ in.})^2} = 39.06479 \text{ in.}$$

In response to the 50°F temperature decrease, member (1) contracts by

$$\delta_1 = \alpha \Delta T\, L_1 = (88\times10^{-6}/°\text{F})(-50°\text{F})(39.06479 \text{ in.}) = -0.17189 \text{ in.}$$

Therefore, the length of member (1) after the temperature decrease is:

$$L_1' = L_1 + \delta_1 = 39.06479 \text{ in.} + (-0.17189 \text{ in.}) = 38.89290 \text{ in.}$$

By symmetry, the dimension $a$ must remain unchanged. The dimension $b'$ is thus

$$L_1' = \sqrt{a^2 + (b')^2}$$

$$\therefore b' = \sqrt{(L_1') - a^2} = \sqrt{(38.89290 \text{ in.})^2 - (22.40664 \text{ in.})^2} = 31.78994 \text{ in.}$$

The displacement of joint $B$ from its original position $B$ to its deflected position $B'$ is

$$u_B = b - b' = 32 \text{ in.} - 31.78994 \text{ in.} = \boxed{0.210 \text{ in.}}$$ **Ans.**