Problem 2.1

A baseball has a mass of 0.3 lb. What is the kinetic energy relative to home plate of a 94 mile per hour fastball, in Btu?

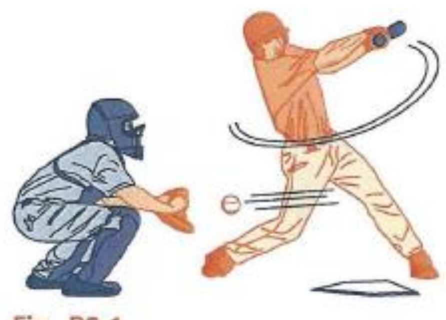


Fig. P2.1

$\mathrm{KE} = \frac{1}{2}\, m\, \mathrm{V}^2$

relative to home plate

$$= \frac{1}{2}\,(0.3\ \mathrm{lb})\left\{\left(44\,\frac{\mathrm{mi}}{\mathrm{h}}\right)\left|\frac{5280\ \mathrm{ft}}{1\ \mathrm{mi}}\right|\left|\frac{1\ \mathrm{h}}{60\ \mathrm{s}}\right|\right\}^2 = 2851.1\ \mathrm{ft \cdot lb/s^2}\left|\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{ft \cdot lb/s^2}}\right|\left|\frac{1\ \mathrm{Btu}}{778\ \mathrm{ft \cdot lbf}}\right|$$

$$= 0.114\ \mathrm{Btu}$$

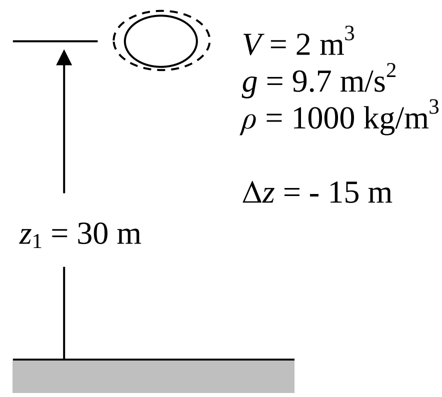
<u>Problem 2.2</u>

Determine the gravitational potential energy, in kJ, of 2 m^3 of liquid water at an elevation of 30 m above the surface of Earth. The acceleration of gravity is constant at 9.7 m/s^2 and the density of the water is uniform at 1000 kg/m^3. Determine the change in gravitational potential energy as the elevation decreased by 15 m.

KNOWN: The elevation of a known quantity of water is decreased from a given initial value by a given amount.

FIND: Determine the initial gravitational potential energy and the change in gravitational potential energy.

SCHEMATIC AND GIVEN DATA:



ENGINERING MODEL:
(1) The water is a closed system. (2) The acceleration of gravity is constant. (3) The density of water is uniform.

ANALYSIS: The initial gravitational potential energy is

$$PE_1 = mgz_1 = (\rho V)gz_1$$

$$= \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)(2\ \text{m}^3)\left(9.7\,\frac{\text{m}}{s^2}\right)(30\ \text{m})\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right|\left|\frac{1\ \text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 582\ \text{kJ} \longleftarrow$$

The change in potential energy is

$$\Delta PE = mg(z_2 - z_1) = mg\Delta z$$

$$= \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.7\,\frac{\text{m}}{s^2}\right)(-15\ \text{m})\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right|\left|\frac{1\ \text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= -145.5\ \text{kJ} \longleftarrow$$

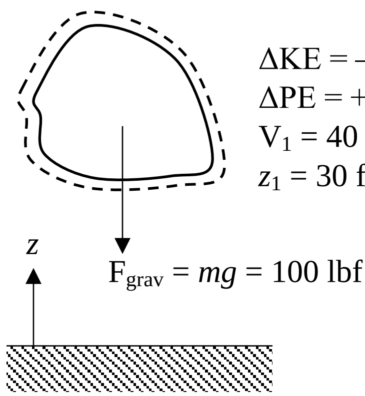
Problem 2.3

An object whose weight is 100 lbf experiences a decrease in kinetic energy of 500 ft·lbf and an increase in potential energy of 1500 ft·lbf. The initial velocity and elevation of the object, each relative to the surface of the earth, are 40 ft/s and 30 ft, respectively. If g = 32.2 ft/s^2, determine
(a) the final velocity, in ft/s.
(b) the final elevation, in ft.

KNOWN: An object experiences specified changes in kinetic and potential energy. The initial velocity and elevation are known.

FIND: Determine the final velocity and the final elevation.

SCHEMATIC AND GIVEN DATA:



$\Delta KE = -\ 500$ ft·lbf
$\Delta PE = +\ 1500$ ft·lbf
$V_1 = 40$ ft/s
$z_1 = 30$ ft

ENGINEERING MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant; g = 32.2 ft/s^2. (3) Velocity and elevation are measured relative to the surface of the earth.

ANALYSIS: (a) The change in kinetic energy is: $\Delta KE = 1/2\ m(V_2^2 - V_1^2)$. Solving for V_2

$$V_2 = \left[\left(\frac{2\Delta KE}{m}\right) + V_1^2\right]^{1/2} \qquad (1)$$

The mass is

$$m = \frac{F_{grav}}{g} = \frac{100 \text{ lbf}}{32.2 \text{ ft/s}^2}\left|\frac{32.2 \text{ ft·lb/s}^2}{1 \text{ lbf}}\right| = 100 \text{ lb}$$

So, inserting values into (1) and converting units

$$V_2 = \left[\frac{2(-500 \text{ ft}-\text{lbf})}{(100 \text{ lb})}\left|\frac{32.2 \text{ ft·lb/s}^2}{i \text{ lbf}}\right| + (40\,\frac{\text{ft}}{\text{s}})^2\right]^{1/2} = 35.75 \text{ ft/s} \longleftarrow$$

(b) The change in potential energy is: $\Delta PE = mg(z_2 - z_1)$. With $mg = F_{grav}$,

$$z_2 = \Delta PE/F_{grav} + z_1 = (1500 \text{ ft·lbf})/(100 \text{ lbf}) + (30 \text{ ft}) = 45 \text{ ft} \longleftarrow$$

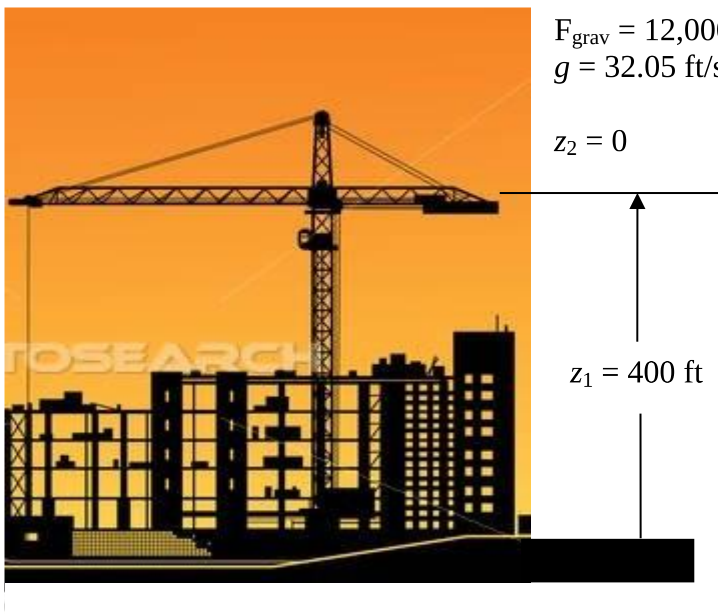
Problem 2.4

A construction crane weighing 12,000 lbf fell from a height of 400 ft to the street below during a severe storm. For g = 32.05 ft/s^2, determine mass, in lb, and the change in gravitational potential energy of the crane, in ft·lbf.

KNOWN: A crane of known weight falls from a known elevation to the street below.

FIND: Determine the change in gravitational potential energy of the crane.

SCHMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The crane is the closed system. (2) The acceleration of gravity is constant.

ANALYSIS:

To get the mass, note that $F_{grrav} = mg$. Thus

$$m = \frac{F_{grav}}{g} = \frac{12000 \text{ lbf}}{32.05 \text{ ft/s}^2}\left|\frac{32.174 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right| = 12{,}046 \text{ lb}$$

The change in gravitational potential energy is

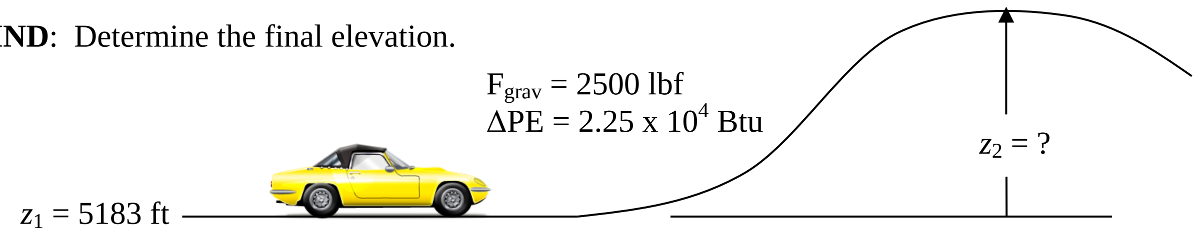
$$\Delta PE = mg(z_2 - z_1) = F_{grav}\Delta z = (12000 \text{ lbf})(-400 \text{ ft}) = 4.8 \times 10^6 \text{ ft}\cdot\text{lbf}$$

Problem 2.5

An automobile weighing 2500-lbf increases its gravitational potential energy by 2.25×10^4 Btu in going from an elevation of 5,183 ft in Denver to the highest elevation on Trail Ridge road in the Rocky Mountains. What is the elevation at the high point of the road, in ft?

KNOWN: An automobile of known weight increases its gravitational potential energy by a given amount. The initial elevation is known.

FIND: Determine the final elevation.



ENGINEERING MODEL: (1) The automobile is the closed system. (2) The acceleration of gravity is constant. (2) Velocity and elevation are each measured relative to a stationary observer on the surface of the earth.

ANALYSIS: The change in gravitational potential energy is: $\Delta PE = mg(z_2 - z_1)$. With $F_{grav} = mg$, we get

$$\Delta PE = F_{grav}(z_2 - z_1)$$

Solving for z_2

$$z_2 = \frac{\Delta PE}{F_{grav}} + z_1 = \frac{(2.25\ x\ 10^4\ \text{Btu})}{(2500\ \text{lbf})}\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| + 5183\ \text{ft} = 12{,}185\ \text{ft} \longleftarrow$$

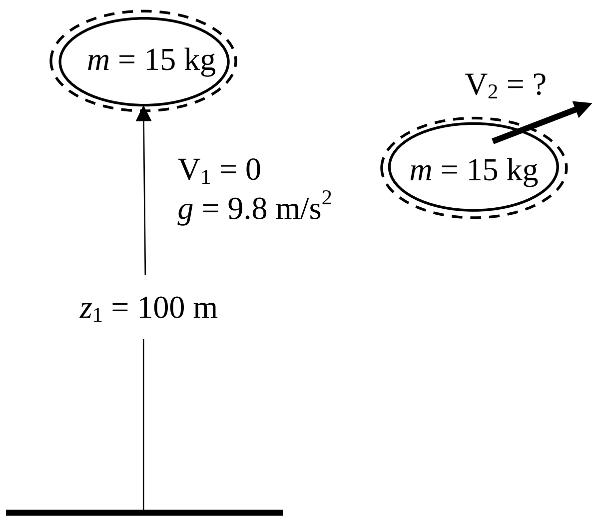
Problem 2.6

An object of mass 15 kg is at an elevation of 100 m relative to the surface of the Earth. What is the potential energy of the object, in kJ? If the object were initially at rest, to what velocity, in m/s, would you have to accelerate it for the kinetic energy to have the same value as the potential energy you calculated above? The acceleration of gravity is 9.8 m/s^2.

KNOWN: An object has known mass and elevation and is initially at rest. The acceleration of gravity is given.

FIND: Determine the initial potential energy and the final velocity if it were accelerated to have kinetic energy equal to the initial potential energy.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant: g = 9.8 m/s^2. (3) The final kinetic energy is equal to the initial gravitational potential energy. (4) Velocity and elevation are each measured relative to a stationary observer on the surface of the earth.

ANALYSIS: First, evaluate the initial potential energy.

$$PE_1 = mgz_1 = (15\ \text{kg})(9.8\ \text{m/s}^2)(100\ \text{m}) \left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 14.7\ \text{kJ} \longleftarrow$$

For $KE_2 = PE_1 \quad \rightarrow \quad \frac{1}{2} m V_2^2 = PE_1$

So

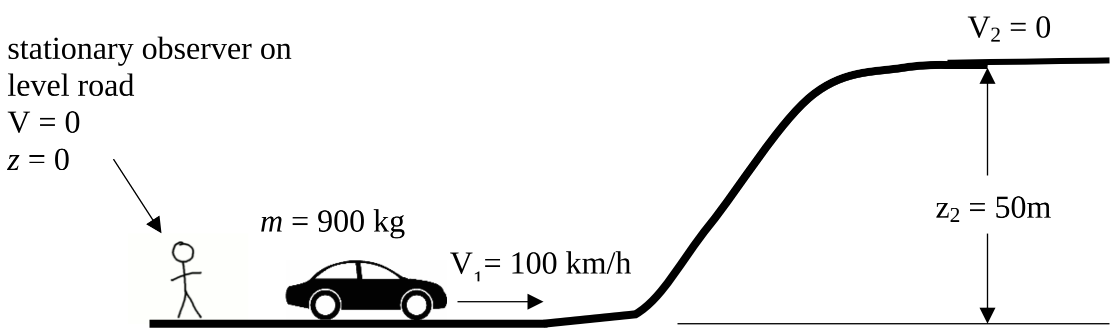
$$V_2 = \sqrt{\frac{2PE_1}{m}} = \sqrt{\frac{2(14.7\ \text{kJ})}{(15\ \text{kg})} \left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| \left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|} = 44.27\ \text{m/s} \longleftarrow$$

2.7 An automobile having a mass of 900 kg initially moves along a level highway at 100 km/h relative to the highway. It then climbs a hill whose crest is 50 m above the level highway and parks at a rest area located there. For the automobile, determine its changes in kinetic and potential energy, each in kJ. For each quantity, kinetic and potential energy, specify your choice of datum and reference value at that datum. Let g = 9.81 m/s^2.

KNOWN: Data are provided for an automobile on the open road.

FIND: Determine the changes in kinetic and potential energy for the automobile and specify an appropriate datum for each.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The automobile is the closed system. (2) The acceleration of gravity is constant; g = 9.81 m/s^2. (3) Velocities and elevations are measured relative to the stationary observer on the level road.

ANALYSIS: The change in kinetic energy is

$$\Delta KE = \tfrac{1}{2} m (\cancel{V_2^2} - V_1^2)$$

$$= \tfrac{1}{2}(900 \text{ kg})[0 - (100 \text{ km/h})^2 \left|\frac{1 \text{ h}}{3600 \text{ s}}\right|^2 \left|\frac{10^3 \text{m}}{1 \text{ km}}\right|^2 \left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right|$$

$$= -347.2 \text{ kJ (decrease)} \longleftarrow$$

The change in potential energy is

$$\Delta PE = mg(z_2 - \cancel{z_1})$$

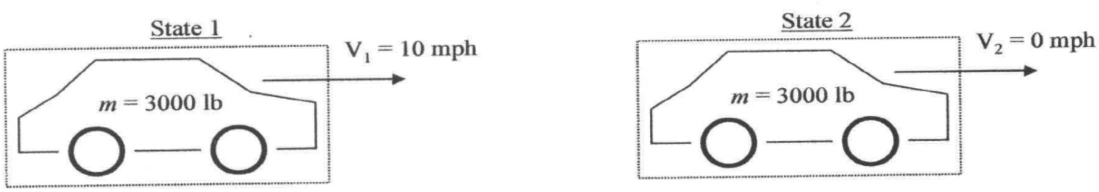
$$= (900 \text{ kg})(9.81 \text{ m/s}^2)(50 \text{ m} - 0) \left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right| = +441.5 \text{ kJ (increase)} \longleftarrow$$

2.8 Vehicle crumple zones are designed to absorb energy during an impact by deforming to reduce transfer of energy to occupants. How much kinetic energy, in Btu, must a crumple zone absorb to fully protect occupants in a 3000 lb vehicle that suddenly decelerates from 10 mph to 0 mph?

KNOWN: Vehicle crumple zone absorbs energy during impact.

FIND: Change in kinetic energy, in Btu, of 3000-lb vehicle decelerating from 10 mph to 0 mph.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The automobile is the system.
2. Crumple zone absorbs all kinetic energy of vehicle upon impact.
3. Weight of occupants can be neglected.

ANALYSIS:
Change in kinetic energy (ΔKE) is determined by

$$\Delta KE = \tfrac{1}{2} m(V_2^2 - V_1^2)$$

where m is mass of the vehicle and V is velocity of the vehicle relative to the roadway.

Substituting and applying appropriate conversion factors yield

$$\Delta KE = \frac{1}{2}(3000\,\text{lb})\left(\left(0\frac{\text{mi}}{\text{h}}\right)^2 - \left(10\frac{\text{mi}}{\text{h}}\right)^2\right)\left|\frac{(5280\,\text{ft})^2}{(\text{mi})^2}\right|\frac{1\,\text{lbf}}{32.2\,\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\left|\frac{\text{h}^2}{(3600\,\text{s})^2}\right|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right|$$

$$\Delta KE = \underline{\mathbf{-12.9\ Btu}}$$

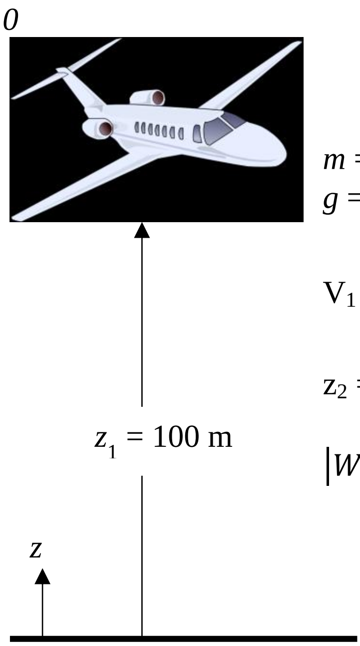
The negative sign indicates the energy of the moving vehicle is reduced and must be absorbed by the crumple zone to protect the occupants.

2.9 In a recent airline disaster, an airliner flying at 30,000 ft, 550 mi/h, lost power and fell to Earth. The mass of the aircraft was 255,000 lb. If the magnitude of the work done by drag force on the plane during the fall was 2.96 x 10^6 Btu, estimate the velocity of the aircraft at the time of impact, in mi/h. Let g = 32.08 ft/s^2.

KNOWN: An airliner falls from a known elevation and velocity to the surface of the Earth. The magnitude of the work done against drag force is specified, and the average acceleration of gravity is given.

FIND: Estimate the velocity at the time of impact with the Earth.

SCHEMATIC AND GIVEN DATA:



m = 255,000 lb
g = 32.08 ft/s^2

$$V_1 = 550 \text{ mi/h}\left|\frac{5280\text{ ft}}{1\text{ mi}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right| = 806.7 \text{ ft/s}$$

$z_2 = 0$

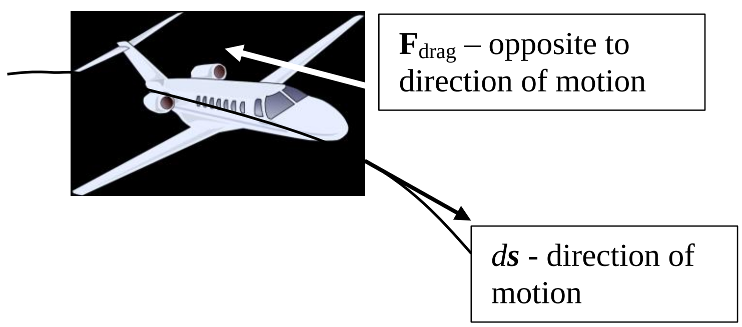
$|W_{\text{drag}}| = 2.96 \times 10^6$ Btu

ENGINEERING MODEL: (1) The airliner is a closed system. (2) The acceleration of gravity is constant: g = 32.08 ft/s^2. (3) The only forces acting on the airliner as it falls are the drag force opposing motion and the force of gravity.

ANALYSIS: The work done against drag is

$$W_{\text{drag}} = \int_{s_1}^{s_2} \mathbf{F}_{\text{drag}} \cdot d\boldsymbol{s}$$

$\mathbf{F}_{\text{drag}} \cdot d\boldsymbol{s}$ is *negative*.



So

$$W_{\text{drag}} = -2.96 \times 10^6 \text{ Btu}$$

The work done against drag equals the change in kinetic energy plus the change in potential energy

$$W_{\text{drag}} = \int_{s_1}^{s_2} \mathbf{F}_{\text{drag}} \cdot d\boldsymbol{s} = \tfrac{1}{2}\, m(V_2^2 - V_1^2) + mg(z_2 - \cancel{z_1})$$

Problem 2.9 (Continued)

Solving for V_2

$$V_2 = \sqrt{\frac{2W_{\text{drag}}}{m} + 2gz_1 + v_1^2}$$

$$= \sqrt{\frac{2(-2.96 \text{ x } 10^6 \text{Btu})}{(255{,}000 \text{ lb})}\left|\frac{778.17 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|\left|\frac{32.174 \text{ ft}\cdot\frac{\text{lb}}{\text{s}^2}}{1 \text{ lbf}}\right| + 2\left(32.08\frac{\text{ft}}{\text{s}^2}\right)(30{,}000 \text{ ft}) + (806.7^2)\frac{\text{ft}^2}{\text{s}^2}}$$

$$= 1412 \text{ ft/s}$$

$$= 1412 \text{ ft/s}\left|\frac{1 \text{ mi}}{5280 \text{ ft}}\right|\left|\frac{3600 \text{ s}}{1 \text{ h}}\right| = 962.7 \text{ mi/h}$$

COMMENTS: The work done against drag is reduces the final kinetic energy, so it is negative in this case. Also, be careful in applying the English system units in this problem.

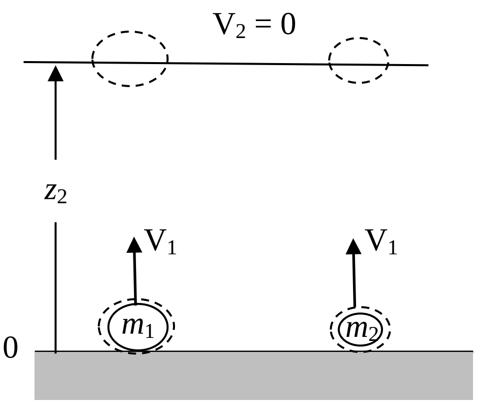
Problem 2.10

Two objects having different masses are propelled vertically from the surface of Earth, each with the same initial velocities. Assuming the objects are acted upon only by the force of gravity, show that they reach zero velocity at the same height.

KNOWN: Two objects are propelled upward from the surface of Earth with the same initial velocities and are acted upon only by the force of gravity.

FIND: Show that they reach zero velocity at the same height.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) Each object is a closed system. (2) The acceleration of gravity is constant. (3) The only force acting is the force of gravity.

ANALYSIS: For an object moving vertically under the influence of gravity only, Eq. 2.11 applies

$$\frac{1}{2}mg(V_2^2 - V_1^2) + mg(z_2 - z_1) = 0$$

For $V_2 = 0$ and $z_1 = 0$

$$-\frac{1}{2}mV_1^2 + mgz_2 = 0$$

Thus

$$z_2 = V_1^2/2g$$

Since the final height doesn’t depend on mass, both objects will reach zero velocity at the same final height.

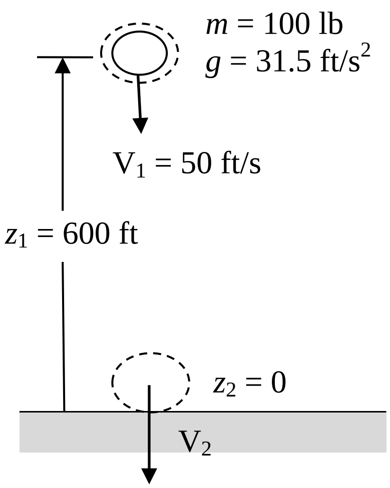
Problem 2.11

An object whose mass is 100 lb falls freely under the influence of gravity from an initial elevation of 600 ft above the surface of Earth. The initial velocity is downward with a magnitude of 50 ft/s. The effect of air resistance is negligible. Determine the velocity, in ft/s, of the object just before it strikes Earth. Assume g = 31.5 ft/s^2.

KNOWN: An object of known mass falls freely from a known elevation and with a given initial velocity. The only force acting is the force of gravity.

FIND: Determine the velocity of the object just before it strikes Earth.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system.
(2) The acceleration of gravity is constant: g = 51.5 ft/s^2.
(3) The only force acting on the object is the force of gravity.

ANALYSIS: Since the only force acting on the object is the force of gravity, Eq. 2.11 applies. Thus

1 $$\frac{1}{2}\cancel{m}(V_2^2 - V_1^2) + \cancel{m}g(\cancelto{0}{z_2} - z_1) = 0$$

Solving for V_2

$$V_2 = \sqrt{V_1^2 + 2gz_1}$$

Inserting values

$$V_2 = \sqrt{50^2\,\frac{\text{ft}^2}{\text{s}^2} + 2\left(31.5\,\frac{\text{ft}}{\text{s}^2}\right)(600\text{ ft})} = 200.7\text{ ft/s}$$

---

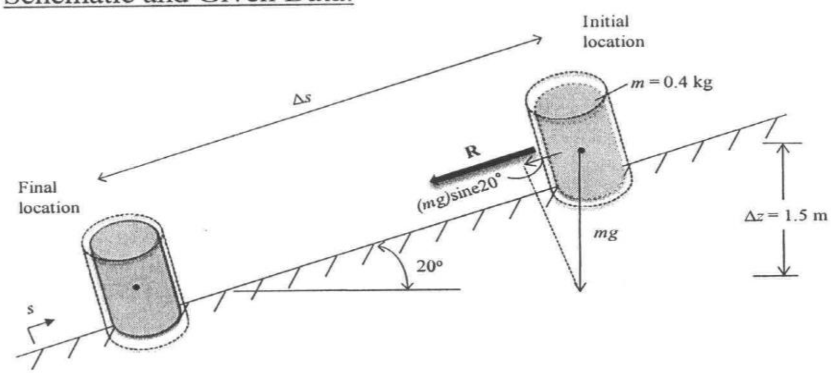
1. Note that the mass cancels out. Any object falling freely under the influence of gravity, with no effects of air resistance, would reach the same final velocity.

2.12 During the packaging process, a can of soda of mass 0.4 kg moves down a surface inclined at 20° relative to the horizontal, as shown in Fig. P2.12. The can is acted upon by a constant force **R** parallel to the incline and by the force of gravity. The magnitude of **R** is 0.05 N. Ignoring friction between the can and the inclined surface, determine the can's change in kinetic energy, in J, and whither it is *increasing* or *decreasing*. If friction between the can and the inclined surface were significant, what would effect would that have on the value of the change in kinetic energy? Let g = 9.8 m/s^2.

Known: Can moves down a surface that is inclined relative to the horizontal. The can is acted upon by a constant force parallel to the incline and by the force of gravity.

Find: Can's change in kinetic energy, in J, and whether it is *increasing* or *decreasing*. If friction between the can and the inclined surface were significant, what effect would that have on the value of the change in kinetic energy?

Schematic and Given Data:



Engineering Model:
(1) The can is a closed system.
(2) The acceleration of gravity is constant.
(3) The applied force **R** is constant.
(4) Ignore friction between the can and inclined surface.

Analysis:

Begin with Eq. 2.6

$$\int_{s_1}^{s_2} \underline{F}\cdot d\underline{s} = \frac{1}{2}m\left(V_2^2 - V_1^2\right) = \Delta KE \qquad (1)$$

From the free body diagram in the schematic, the dot product can be expressed as

$$\underline{F}\cdot d\underline{s} = \left(\mathbf{R} + (mg)\text{sine}20°\right)ds$$

Substituting into Eq. (1)

$$\int_{s_1}^{s_2} \underline{F}\cdot d\underline{s} = \int_{s_1}^{s_2}(\mathbf{R} + (m$$

Since Δz = Δs sine 20°, Eq. (2) becomes

$$\int_{s_1}^{s_2}(\mathbf{R})ds + (mg)\Delta z = (\mathbf{R})\Delta s + (mg)\Delta z = \Delta KE \qquad (3)$$

Evaluate Δs

$$\Delta s = \frac{\Delta z}{\text{sine}\,20°} = \frac{1.5\text{ m}}{0.342} = 4.39\text{ m}$$

Problem 2.12 (Continued)

Subsituting all known and calculated data into Eq. (3)

$$\Delta KE = (0.05N)(4.39m)\left|\frac{1J}{1N\cdot m}\right| + (0.4kg)\left(9.8\frac{m}{s^2}\right)(1.5m)\left|\frac{1N}{1\frac{kg\cdot m}{s^2}}\right|\left|\frac{1J}{1N\cdot m}\right| =$$

#1 $\Delta KE = 0.22\,J + 5.88\,J = 6.1\,J$

Which corresponds to an *increase* in kinetic energy.

If friction were significant, the magnitude of the net force acting in the direction of motion would be less, and thus the kinetic energy change would be less than calculat...

1. Observe that in the absence of the force **R** the can is acted on only by gravity, and the can's change in kinetic energy is 5.88 J.

Problem 2.13

Jack, who weighs 150 lbf, runs 5 miles in 43 minutes on a treadmill set at a one-degree incline (Fig. P2.13). The treadmill display shows he has *burned* 620 kcal. For jack to break even calorie-wise, how much vanilla ice cream, in cups, may he have after his workout?

- Exercise value = 620 kcal
- Calorific value; 1 cup of vanilla ice cream = 264 kcal

To break even calorie-wise, Jack may have

$$\frac{620\ \text{kcal}}{264\ \text{kcal/cup}} = 2.35\ \text{cups} \longleftarrow$$

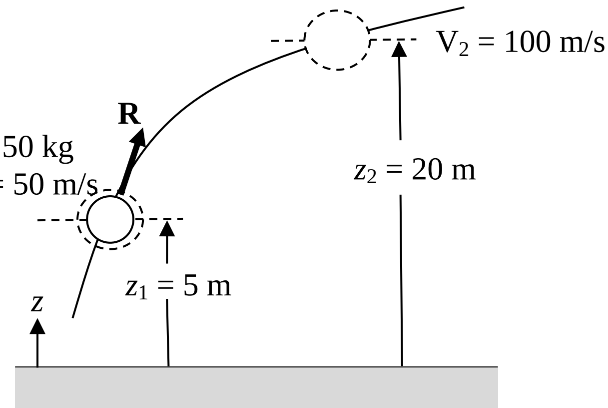
Problem 2.14

An object initially at an elevation of 5 m relative to Earth's surface and with a velocity of 50 m/s is acted on by an applied force **R** and moves along a path. Its final elevation is 20 m and its velocity is 100 m/s. The acceleration of gravity is 9.81 m/s^2. Determine the work done on the object by the applied force, in kJ.

KNOWN: An object moves along a path due to the action of an applied force. The elevation and velocities are known initially and finally.

FIND: Determine the work of the applied force.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system. (2) **R** is the only force acting on the object other than the force of gravity. (3) g = 9.81 m/s^2 and is constant.

ANALYSIS: To find the work of force **R** we use

$$\text{Work} = \int_1^2 \mathbf{R}\cdot d\mathbf{s} = \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1)$$

Inserting values and converting units

$$\text{Work} = \left\{\frac{1}{2}(50\text{ kg})(100^2 - 50^2)\frac{\text{m}^2}{\text{s}^2} + (50\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(20-5)\text{m}\right\}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

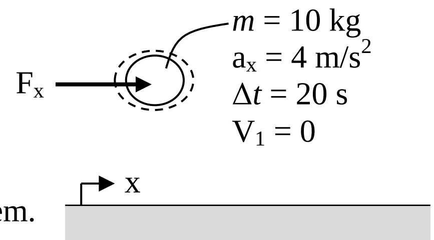
$$= 187.5 + 7.36 = 194.9\text{ kJ}$$

Problem 2.15

An object of mass 10 kg, initially at rest, experiences a constant horizontal acceleration of 4 m/s^2 due to the action of a resultant force applied for 20 s. Determine the total amount of energy transfer by work, in kJ.

KNOWN:
A system of known mass experiences a constant horizontal acceleration due to an applied force for a specified length of time.

FIND: Determine the amount of energy transfer by work.



SCHEMATIC AND GIVEN DATA:

ENGINEERING MODEL: (1) The object is a closed system. (2) The horizontal acceleration is constant.

ANALYSIS: The work of the resultant force is determined using Eq. 2.6

$$\int_{x_1}^{x_2} F_x dx = \frac{1}{2} m(V_2^2 - \cancelto{0}{V_1^2})$$

To find V_2, use the fact that the acceleration is constant

$$a_x = \frac{dV}{dt} \quad \rightarrow dV = a_x dt \quad \rightarrow \quad \int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a_x \, dt$$

or

$$(V_2 - \cancelto{0}{V_1}) = a_x(t_2 - t_1) = a_x \Delta t$$

Thus

$$V_2 = (4 \text{ m/s}^2)(20 \text{ s}) = 80 \text{ m/s}$$

Finally, the work of the resultant force is

$$\int_{x_1}^{x_2} F_x dx = \frac{1}{2} m V_2^2$$

$$= \frac{1}{2}(10 \text{ kg})(80^2) \frac{\text{m}^2}{\text{s}^2} \left| \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 32 \text{ kJ} \longleftarrow$$

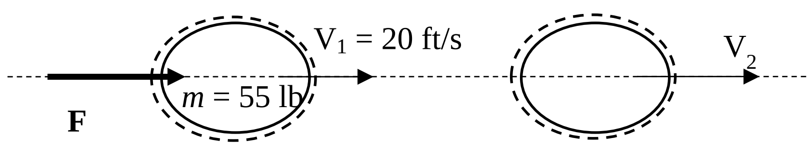
Problem 2.16

An object with an initial horizontal velocity of 20 ft/s experiences a constant horizontal acceleration due to the action of a resultant force applied for 10 s. The work of the resultant force is 10 Btu. The mass of the object is 55 lb. Determine the constant horizontal acceleration, in ft/s^2.

KNOWN: Data are known for an object accelerating horizontally under the action of a constant resultant force for a specified amount of time. The work of the force is given.

FIND: Determine the constant horizontal acceleration.

SCHEMATIC AND GIVEN DATA:



Initially

Finally
Δt = 10 s
Work of force **F** is 10 Btu

ENGINEERING MODEL: (1) The object is a closed system. (2) The resultant force is constant over the time interval. (3) All forces and motions are horizontal.

ANALYSIS: The constant horizontal acceleration is $a = \mathrm{d}V/\mathrm{d}t = (V_2 - V_1)/\Delta t$. To find V_2 = use Eq. 2.6

$$\underbrace{\int_{s_1}^{s_2} \mathbf{F}\cdot d\mathbf{s}}_{\boxed{\text{Work, } W\text{, of the resultant force } \mathbf{F}}} = \tfrac{1}{2}\, m(V_2^2 - V_1^2)$$

So

$$V_2 = \sqrt{\frac{2W}{m} + V_1^2} = \sqrt{\frac{2(10\ \text{Btu})}{(55\ \text{lb})}\left|\frac{778\ \text{ft·lbf}}{1\ \text{Btu}}\right|\left|\frac{32.2\ \text{ft·lb/s}^2}{1\ \text{lbf}}\right| + \left(20\,\frac{\text{ft}}{\text{s}}\right)^2} = 97.52\ \text{ft/s}$$

The acceleration is

$$a = (V_2 - V_1)/\Delta t = [(97.52 - 20)\text{ft/s}](10\text{s}) = 7.752\ \text{ft/s}^2 \longleftarrow$$

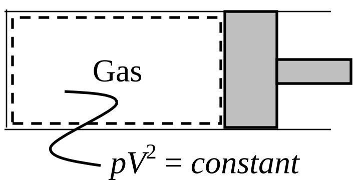
Problem 2.17

A gas in a piston-cylinder assembly undergoes a process for which the relationship between pressure and volume is $pV^2 = constant$. The initial pressure is 1 bar, the initial volume is 0.1 m^3, and the final pressure is 9 bar. Determine (a) the final volume, in m^3, and (b) the work for the process, in kJ.

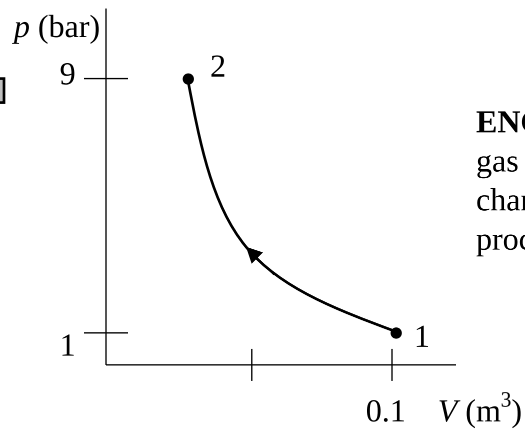
KNOWN: A gas in a piston-cylinder assembly undergoes a process during which $pV^2 = constant$. State data are provided.

FIND: Determine the final volume occupied by the gas and the work for the process.

SCHEMATIC AND GIVEN DATA:



$pV^2 = constant$
$p_1 = 1$ bar
$V_1 = 0.1$ m^3
$p_2 = 9$ bar



ENGINEERING MODEL: (1) The gas is a closed system. (2) Volume change is the only work mode. (3) the process of the obeys $pV^2 = constant$.

ANALYSIS:

(a) We have $pV^2 = constant$. Thus $p_1V_1^2 = p_2V_2^2$. Solving for V_2

$$V_2 = \left[\frac{p_1}{p_2}\right]^{1/2} V_1 = \left[\frac{1}{9}\right]^{1/2} (0.1 \text{ m}^3) = 0.0333 \text{ m}^3 \longleftarrow$$

(b) Using Eq. 2.17 to determine the work

$$W = \int_1^2 p dV = \frac{p_2V_2 - p_1V_1}{(1-2)} \qquad \text{(See Example 2.1(a) for the integration)}$$

Inserting values and converting units

$$W = \frac{(9 \text{ bar})(0.033 \text{ m}^3) - (1 \text{ bar})(0.1 \text{ m}^3)}{(-1)} \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right|$$

$$= -20 \text{ kJ} \longleftarrow$$

Negative sign denotes energy transfer *to* the gas by work during the compression process.

Problem 2.18

Carbon dioxide (CO_2) gas within a piston-cylinder assembly undergoes a process from a state where p_1 = 5 lbf/in.^{2}, V_1 = 2.5 ft^3 to a state where p_2 = 20 lbf/in.^{2}, V_2 = 0.5 ft^3. The relationship between pressure and volume during the process is given by $p = 23.75 - 7.5V$, where V is in ft^3 and p is in lbf/in.^{2} Determine the work for the process, in Btu.

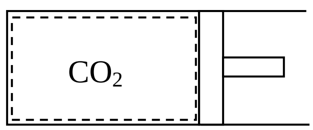
KNOWN: CO_2 gas within a piston-cylinder assembly undergoes a process where the *p*-*V* relation is given. The initial and final states are specified.

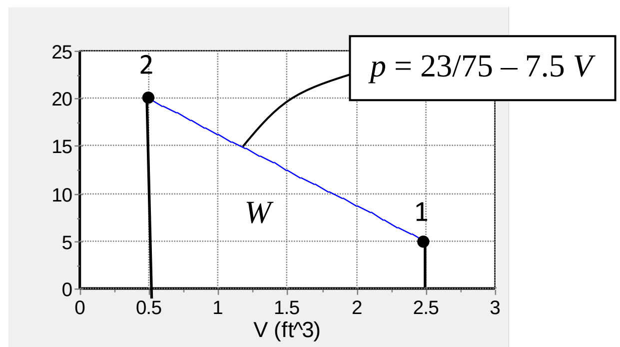
FIND: Determine the work for the process.

SCHEMATIC AND GIVEN DATA:

p_1 = 5 lbf/in.^{2}
V_1 = 2.5 ft^3

p_2 = 20 lbf/in.^{2}
V_2 = 0.5 ft^3





ENGINEERING MODEL: (1) The CO_2 is the closed system. (2) The *p*-*V* relation during the process is linear. (3) Volume change is the only work mode.

ANALYSIS: The given *p*-*V* relation can be used with Eq. 2.17 as follows:

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} [23.75 - 7.5V]\,dV = \left[23.75V - \frac{7.5V^2}{2}\right]_{V_1}^{V_2}$$

$$= 23.75[V_2 - V_1] - \frac{7.5}{2}[V_2^2 - V_1^2]$$

$$W = \left(23.75\ \frac{\text{lbf}}{\text{in.}^2}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|[0.5 - 2.5]\text{ft}^3 - \left(\frac{7.5}{2}\frac{\text{lbf/in.}^2}{\text{ft}^3}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|[0.5^2 - 2.5^2](\text{ft}^3)^2$$

= -3600 ft·lbf

$= (-3600\ \text{ft}\cdot\text{lbf})\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = -4.63$ Btu (negative sign denotes energy transfer *in*.)

Alternative Solution

Since the *p*-*V* relation is linear, *W* can also be evaluated geometrically as the area under the process line:

$$W = p_{\text{ave}}(V_2 - V_1) = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) = \left(\frac{20+5}{2}\right)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|(0.5 - 2.5)\text{ft}^3\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

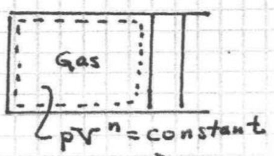
= -4.63 Btu

PROBLEM 2.19

KNOWN: A gas in a piston-cylinder assembly undergoes a compression process for which pV^n = constant. State data is provided.

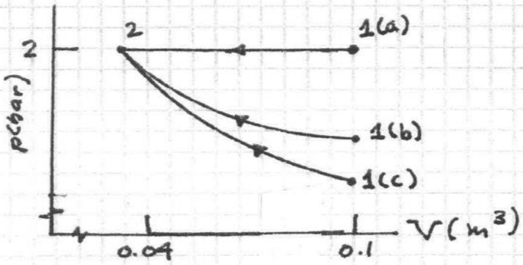
FIND: For each of $n = 0$, $n = 1$, and $n = 1.3$, determine the initial pressure, in bar, and the work, in kJ.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.1\ m^3$

$V_2 = 0.04\ m^3$, $P_2 = 2$ bar



ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. Volume change is the only work mode.
3. The process of the gas obeys pV^n = constant, where (a) $n=0$, (b) $n=1$, (c) $n=1.3$.

ANALYSIS:

(a) $\underline{n=0}$: Thus, $pV^0 = \text{constant} \Rightarrow p = \text{constant}$. So, $P_1 = 2$ bar ←

Using Eq. 2.17 with p = constant,

① $$W = \int_1^2 p\,dV = p[V_2 - V_1] = 2\text{bar}[0.04 - 0.1]\,m^3 \left|\frac{10^5 N/m^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right| = -12\text{ kJ}$$ ←

(b) $\underline{n=1}$: Thus, $pV = \text{constant} \Rightarrow P_1V_1 = P_2V_2 \Rightarrow P_1 = P_2\left[\frac{V_2}{V_1}\right] = 2\text{bar}\left[\frac{0.04}{0.10}\right] = 0.8\text{bar}$ ←

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V}\,dV = c \ln\frac{V_2}{V_1} = P_2V_2 \ln\frac{V_2}{V_1} = \left(2\times10^5 \frac{N}{m^2}\right)(0.04\,m^3)\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right| \ln\frac{0.04}{0.10}$$

$c = P_1V_1 = P_2V_2$

$$= -7.33\text{ kJ}$$ ←

(c) $\underline{n=1.3}$: Thus, $P_1V_1^n = P_2V_2^n$, where $n=1.3$. $\Rightarrow P_1 = P_2\left[\frac{V_2}{V_1}\right]^{1.3} = 2\text{bar}\left[\frac{0.04}{0.10}\right]^{1.3}$

$$= 0.608\text{ bar}$$ ←

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V^n}\,dV = \frac{P_2V_2 - P_1V_1}{(1-n)}$$. See Example 2.1(a) for the integration.

$$\therefore W = \frac{(2\times10^5 N/m^2)(0.04m^3) - (0.608\times10^5 N/m^2)(0.1\,m^3)}{(1-1.3)}\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right|$$

$$= -6.4\text{ kJ}$$ ←

1. The negative sign for W denotes work done $\underline{\text{on}}$ the gas during compression.

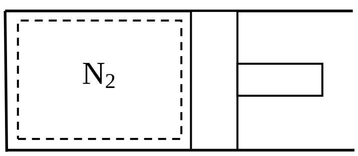
Problem 2.20

Nitrogen (N_2) gas within a piston-cylinder assembly undergoes a process from p_1 = 20 bar, V_1 = 0.5 m^3 to a state where V_2 = 2.75 m^3. The relationship between pressure and volume during the process is $pV^{1.35}$ = *constant*. For the N_2, determine (a) the pressure at state 2, in bar, and (b) the work, in kJ.

KNOWN: N_2 gas within a piston-cylinder assembly undergoes a process where the *p*-*V* relation is $pV^{1.35}$ = *constant*. Data are given at the initial and final states.

FIND: Determine the pressure at the final state and the work.

SCHEMATIC AND GIVEN DATA:



$pV^{1.35}$ = *constant*
p_1 = 20 bar, V_1 = 0.5 m^3
V_2 = 2.75 m^3

ENGINEERING MODEL: (1) The N_2 is the closed system. (2) The *p*-*V* relation is specified for the process. (3) Volume change is the only work mode.

ANALYSIS: (a) $p_1V_1^n = p_2V_2^n \quad \rightarrow \quad p_2 = p_1\left(\frac{V_1}{V_2}\right)^n$; n = 1.35. Thus

$$p_2 = (20\text{ bar})\left(\frac{0.5\text{ m}^3}{2.75\text{ m}^3}\right)^{1.35} = 2\text{ bar}$$

(b) Since volume change is the only work mode, Eq. 2.17 applies. Following the procedure of part (a) of Example 2.1, we have

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{(2\text{ bar})(2.75\text{ m}^3) - (20)(0.5)}{1-1.35}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

$$= 1285.7\text{ kJ}$$

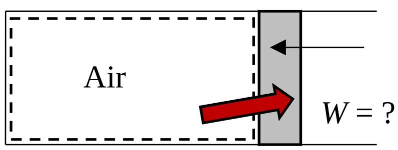
Problem 2.21

Air is compressed slowly in a piston-cylinder assembly from an initial state where p_1 = 1.4 bar, V_1 = 4.25 m^3, to a final state where p_2 = 6.8 bar. During the process, the relation between pressure and volume follows pV = *constant*. For the air as the closed system, determine the work, in Btu.

KNOWN: Air is compressed in a piston-cylinder assembly from a known initial state to a known final pressure. The pressure-volume relation during the process is specified.

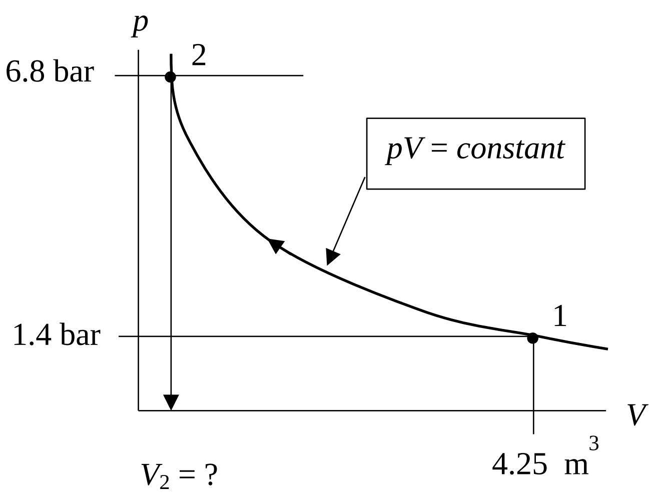
FIND: Determine the work for the air as the closed system.

SCHEMATIC AND GIVEN DATA:



p_1 = 1.4
V_1 = 4.25 m^3

p_2 = 6.8 bar



ENGINEERING MODEL: (1) The air is a closed system. (2) The process is polytropic, with pV = *constant*.

ANALYSIS: The work for the polytropic process can be determined by integrating $W = \int_{V_1}^{V_2} p\,dV$. The pressure-volume relation is pV = *constant*.

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{constant}{V}\,dV = (constant)\ \ln(V_2/V_1)$$

The constant can be evaluated from the given data as *constant* = p_1V_1, and V_2 can be determined using $p_1V_1 = p_2V_2$. Thus

$$V_2 = (p_1/p_2)V_1 = (1.4/6.8)(4.25\ \text{m}^3) = 0.875\ \text{m}^3$$

And

$$W = (p_1V_1)\ \ln(V_2/V_1)$$

$$W = (1.4\ \text{bar})(4.25\ \text{m}^3)\ \ln(0.875/4.25)\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = -\,940.4\ \text{kJ}$$

Negative – energy transfer by work is in, as expected

Problem 2.22

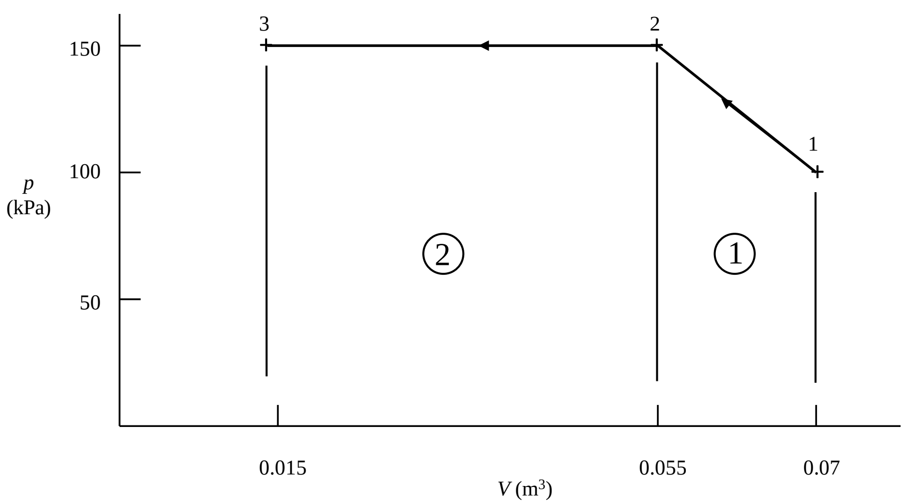
Air contained within a piston-cylinder assembly is slowly compressed. As shown in Fig P2.32, during this first process the pressure first varies linearly with volume and then remains constant. Determine the total work, in kJ.

KNOWN: Air within a piston-cylinder assembly undergoes two processes in series.

FIND: Determine the total work.

SCEMATIC AND GIVEN DATA:

ENGINEERING MODEL: (1) The air within the piston-cylinder assembly is the closed system. (2) The two-step *p*-*V* relation is specified graphically. (3) Volume change is the only work mode.



ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies. Furthermore, the integral can be evaluated geometrically in terms of the total area under process lines:

$$W = \int_{V_1}^{V_2} p\,dV = \overbrace{p_{\text{ave}}(V_2 - V_1)}^{1} + \overbrace{p_2(V_3 - V_2)}^{2} = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) + p_2(V_3 - V_2)$$

$$= \left[\left(\frac{100+150}{2}\right)\text{kPa}(0.055 - 0.07)\text{m}^3 + (150)(0.015 - 0.055)\right]\left|\frac{10^3\,\text{N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

= (-1.875 kJ) + (-6 kJ) = -7.875 kJ (in)

Problem 2.23

A gas contained within a piston-cylinder assembly undergoes three processes in series:

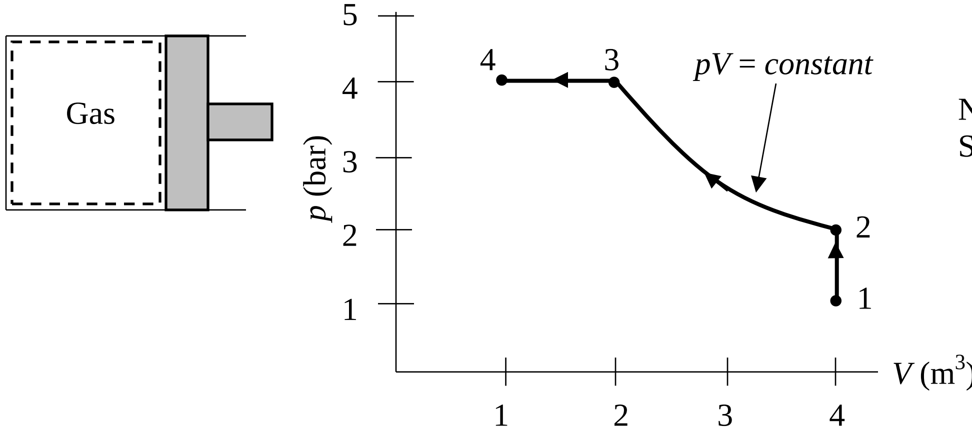
Process 1-2: Constant volume from p_1 = 1 bar, V_1 = 4 m^3 to state 2, where p_2 = 2 bar.
Process 2-3: Compression to V_3 = 2 m^3, during which the pressure-volume relationship is $pV = constant$.
Process 3-4: Constant pressure to state 4, where V_4 = 1 m^3.

Sketch the processes in series on *p-V* coordinates and evaluate the work for each process, in kJ.

KNOWN: A gas contained within a piston-cylinder assembly undergoes three processes in series. State data are provided.

FIND: Sketch the processes on *p–V* coordinates and evaluate the work for each process.

SCHEMATIC AND GIVEN DATA:



Note: $p_2V_2 = p_3V_3$
So $p_3 = (V_2/V_3)p_2$
= (4/3)(2 bar)
= 4 bar

ENGINEERING MODEL: (1) The gas in the piston-cylinder assembly is a closed system. (2) The gas undergoes three processes in series, as illustrated. (3) Volume change is the only work mode.

ANALYSIS: The work is evaluated using Eq. 2.17: $W = \int pdV$

Process 1-2: The volume is constant, so $W_{12} = 0$.

Process 2-3: $W_{23} = \int_{V_2}^{V_3} \frac{constant}{V} dV = constant \ln (V_3/V_2) = (p_2V_2) \ln (V_3/V_2)$

$$= (2 \text{ bar})(4 \text{ m}^3) \ln(2/4) \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| = -554.5 \text{ kJ (in)}$$

Process 3-4: For the constant-pressure process

$$W_{34} = p(V_4 - V_3) = (4 \text{ bar})(1 - 2)\text{m}^3 \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| = -400 \text{ k J (in)}$$

Problem 2.24

Carbon dioxide (CO_2) gas in a piston-cylinder assembly undergoes three processes in series that begin and end at the same state (a cycle).

Process 1-2: Expansion from State 1 where p_1 = 10 bar, V_1 = 1 m^3, to State 2 where V_2 = 4 m^3. During the process, pressure and volume are related by $pV^{1.5}$ = *constant*.

Process 2-3: Constant volume heating to State 3 where p_3 = 10 bar.

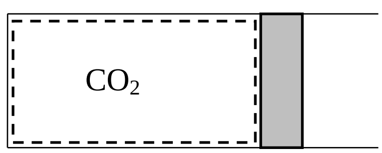
Process 3-1: Constant pressure compression to State 1.

Sketch the processes on *p-V* coordinates and evaluate the work for each process, in kJ. What is *net* work for the cycle, in kJ?

KNOWN: Carbon dioxide gas undergoes a cycle in a piston-cylinder assembly. Data are provided for the key states and the different process that make up the cycle.

FIND: Sketch the processes on *p-V* coordinates, and determine the work for each process and the *net* work for the cycle overall.

SCHEMATIC AND GIVEN DATA:



Process 1-2: Expansion from State 1 where p_1 = 10 bar, V_1 = 1 m^3, to State 2 where V_2 = 4 m^3. During the process, pressure and volume are related by $pV^{1.5}$ = *constant*.

Process 2-3: Constant volume heating to State 3 where p_3 = 10 bar.

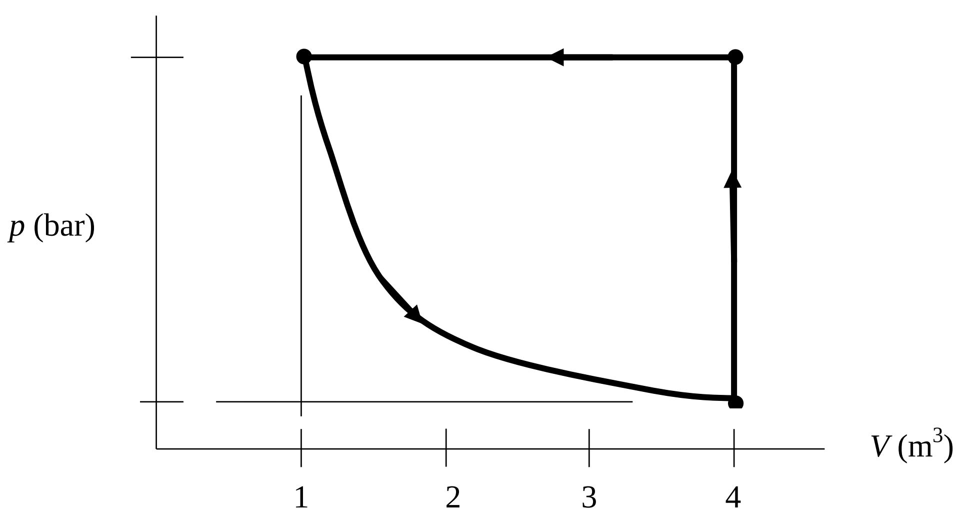
Process 3-1: Constant pressure compression to State 1.

ENGINEERING MODEL: (1) The CO_2 is a closed system. (2) Processes 1-2 and 3-1 are quasi-equilibrium processes. (3) Process 2-3 is at constant volume and Process 3-1 is at constant pressure.

ANALYSIS: First, for Process 1-2, $pV^{1.5}$ = *constant*. Thus

$$p_1 V_1^{1.5} = p_2 V_2^{1.5} \quad \rightarrow \quad p_2 = (V_1/V_2)^{1.5}\, p_1 = (1/4)^{1.5}\,(10 \text{ bar}) = 1.25 \text{ bar}$$

With $p_3 = p_2$ = 10 bar, and $V_3 = V_2$ = 4 m^3, all states are known and the *p-V* diagram is



Problem 2.24 (Continued)

Process 1-2 The work is $W = \int_{V_1}^{V_2} p dV$. For a polytropic process with $pV^{1.5} = constant$, the integral becomes

$$W_{12} = \frac{p_2V_2 - p_1V_1}{1-1.5} = \left[\frac{(1.25\text{ bar})(4\text{ m}^3) - (10\text{ bar})(1\text{ m}^3)}{(1-1.5)}\right]\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

$$= 1000\text{ kJ (out)} \longleftarrow$$

Process 2-3 Since there is no volume change, $W_{23} = 0$ $\longleftarrow$

Process 3-1 The work is $W_{31} = \int_{V_3}^{V_1} p dV$. Since the pressure is constant, the integral becomes

$$W_{31} = p_3(V_1 - V_3)$$

$$= (10\text{ bar})(1-4)\text{ m}^3 \left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = -3000\text{ kJ (in)} \longleftarrow$$

The *net* work for the cycle is:

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = (1000) + (0) + (-3000) = -2000\text{ kJ (in)} \longleftarrow$$

Problem 2.25

Air contained within a piston-cylinder assembly undergoes three processes in series:

Process 1-2: Compression during which the pressure-volume relationship is pV = *constant* from p_1 = 10 lbf/in.^{2}, V_1 = 4 ft^3 to p_2 = 50 lbf/in.^{2}

Process 2-3: Constant volume from state 2 to state 3 where p = 10 lbf/in.^{2}

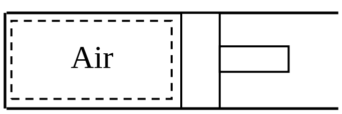
Process 3-1: Constant pressure expansion to the initial state.

Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, in ft^3, and (b) the work for each process, in Btu.

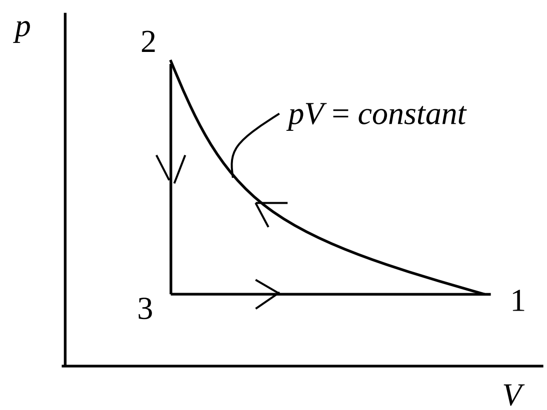
KNOWN: Air within a piston-cylinder assembly undergoes three processes in series.

FIND: Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, and (b) the work for each process.

SCHEMATIC AND GIVEN DATA:



Process 1-2: Compression during which the pressure-volume relationship is pV = *constant* from p_1 = 10 lbf/in.^{2}, V_1 = 4 ft^3 to p_2 = 50 lbf/in.^{2}
Process 2-3: Constant volume from state 2 to state 3 where p = 10 lbf/in.^{2}
Process 3-1: Constant pressure expansion to the initial state.



ENGINEERING MODEL: (1) The gas is the closed system. (2) Volume change is the only work mode. (3) Each of the three processes is specified.

ANALYSIS: (a) For process 1-2; pV = *constant*. Thus $p_1V_1 = p_2V_2$, and

$$V_2 = \left(\frac{p_1}{p_2}\right)V_1 = \left(\frac{10\text{lbf/in.}^2}{50\text{lbf/in.}^2}\right)(4\text{ ft}^3) = 0.8\text{ ft}^3$$

(b) Since volume change is the only work mode, Eq. 2.17 applies.

Process 1-2: For process 1-2, pV = *constant* = p_1V_1. Thus

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{C}{V}\,dV = C\ln\left(\frac{V_2}{V_1}\right) = (p_1V_1)\ln\left(\frac{V_2}{V_1}\right)$$

Problem 2.25 (Continued)

Inserting values and converting units

$$W_{12} = \left(10\ \frac{\text{lbf}}{\text{in.}^2}\right)(4\ \text{ft}^3)\ \ln\ \left(\frac{0.8\ \text{ft}^3}{4\ \text{ft}^3}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = \text{-11.92 Btu (in)} \longleftarrow$$

Process 2-3: Constant volume (piston does not move). Thus $W_{23} = 0$ $\longleftarrow$

Process 3-1: Constant pressure processes ($p_3 = p_1$): $W_{31} = \int_{V_3}^{V_1} pdV = p_1(V_1 - V_3)$

Noting that $V_3 = V_2$

$$W_{31} = \left(10\,\frac{\text{lbf}}{\text{in}^2.}\right)(4 - 0.8)\text{ft}^3\ \left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = \text{5.92 Btu (out)} \longleftarrow$$

---

1. The *net* work for the three process is

$W_{\text{net}} = W_{12} + W_{23} + W_{31}$ = (-11.92) + 0 + (5.92)) = - 6 kJ (*net* work is negative - in)

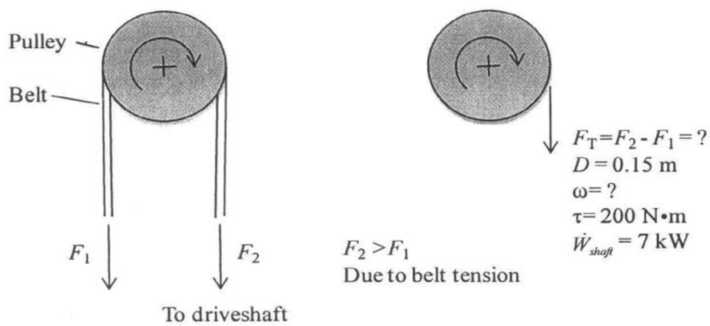
PROBLEM 2.26

A 0.15-m-diameter pulley turns a belt rotating the driveshaft of a power plant pump. The torque applied by the belt on the pulley is 200 N · m, and the power transmitted is 7 kW. Determine the net force applied by the belt on the pulley, in kN, and the rotational speed of the driveshaft, in RPM.

Known: Pulley turns a belt rotating the driveshaft of a power plant pump with known torque and power transmitted.

Find: Determine the net force applied by the belt on the pulley, in kN, and the rotational speed of the driveshaft, in RPM.

Schematic and Given Data:



Engineering Model:
(1) The rotational speed of the pulley and drive shaft are assumed to be equal.
(2) Net tangential force (F_T) on the pulley is due to belt tension (see schematic).

Analysis:

The net force, in kN, applied by the belt on the pulley is calculated using the torque and the diameter of the pulley as follows

$$\tau = F_T\left(\frac{D}{2}\right) \text{ or } F_T = \frac{2\tau}{D} = \frac{2(200\text{N}\cdot\text{m})}{0.15\text{m}}\left|\frac{1\text{ kN}}{1000\text{N}}\right| = 2.67\text{ kN} \quad \longleftarrow$$

Using Eq. 2.20, the rotational speed of the driveshaft, in RPM, is determined using assumption 1, power transmitted, and torque as follows:

$$\dot{W}_{\text{shaft}} = \tau\omega \quad \text{or} \quad \omega = \frac{\dot{W}_{\text{shaft}}}{\tau} = \frac{7\text{kW}}{200\text{N}\cdot\text{m}}\left|\frac{1000\frac{\text{J}}{\text{s}}}{1\text{kW}}\right|\left|\frac{1\text{N}\cdot\text{m}}{1\text{J}}\right|\left|\frac{60\text{s}}{1\text{min}}\right|\left|\frac{\text{rev}}{2\pi\text{ radians}}\right| = 334.2\text{ RPM} \quad \longleftarrow$$

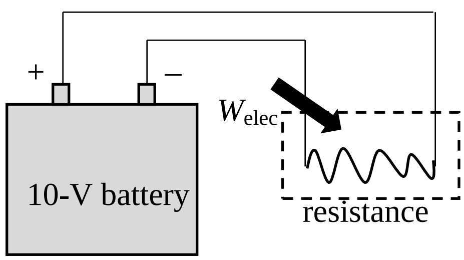
Problem 2.27

A 10-V battery supplies a constant current of 0.5 amp to a resistance for 30 min. (a) Determine the resistance, in ohms. (b) For the battery, determine the amount of energy transfer by work, in kJ.

KNOWN: Operating data are given for a 10-V battery providing current to a resistance.

FIND: Determine the resistance and the amount of energy transfer by work.

SCHEMATIC AND GIVEN DATA:



i = 0.5 amp
Δt = 30 min

ENGINEERING MODEL: (1) The resistor is the closed system. (2) The current is constant with time.

ANALYSIS: For current flow through a resistor: Voltage = Resistance * Current (Ohm's Law) Thus

$$\text{Resistance} = \frac{\text{Voltage}}{\text{Current}} = \frac{10\text{ amps}}{0.5\text{ volts}}\left|\frac{1\text{ ohm}}{1\text{ volt/amp}}\right| = 20\text{ ohm} \longleftarrow$$

With Eq. 2.21applied to the battery which is discharging

$$\left|\dot{W}_{\text{elec}}\right| = (\text{voltage})(\text{current}) = (10\text{ volt})(0.5\text{ amp})\left|\frac{1\text{ watt/amp}}{1\text{ volt}}\right| = 5\text{ watt}$$

So, for 30 min of continuous operation, the energy transfer by work *to* the resistor is

$$W_{\text{elec}} = \int_{t_1}^{t_2}\left|\dot{W}_{\text{elec}}\right|dt = \left|\dot{W}_{\text{elec}}\right|\Delta t$$

$$= (5\text{ watt})(30\text{ min})\left|\frac{60\text{ s}}{1\text{ min}}\right|\left|\frac{1\text{ J/s}}{1\text{ watt}}\right|\left|\frac{1\text{ kJ}}{10^3\text{J}}\right| = 9\text{ kJ} \longleftarrow$$

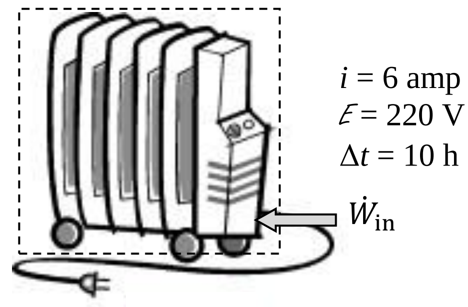
Problem 2.28

An electric heater draws a constant current of 6 amp, with an applied voltage of 220 V, for 24 h. Determine the instantaneous electric power provided to the heater, in kW, and the total amount of energy supplied to the heater by electrical work, in kW·h. If electric power is valued at \$0.08/kW·h, determine the cost of operation for one day.

KNOWN: An electric heater draws a constant current at a specified voltage for a given length of time. The cost of electricity is specified.

FIND: Determine the instantaneous power provided to the heater and the total amount of energy supplied by electrical work. Determine the cost of operation for one day.

SHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The heater is a closed system. (2) The current and voltage are constant.

ANALYSIS: The constant power *input* to the heater is given by Eq. 2.21

$$\dot{W}_{\text{in}} = \mathcal{E} I = (220\text{ V})(6\text{ amp})\left|\frac{1\text{ W/amp}}{1\text{ V}}\right|\left|\frac{1\text{ kW}}{10^3\text{W}}\right| = 1.320\text{ kW} \longleftarrow$$

Thus, the total energy *input* is

$$W_{\text{in}} = \int_{t_1}^{t_2} \dot{W}_{\text{in}} dt = \dot{W}_{\text{in}} \Delta t = (1.320\text{ kW})(24\text{ h}) = 31.68\text{ kW}\cdot\text{h} \longleftarrow$$

Using the specified cost of electricity

Cost per day = (31.68 kW·h) (\$0.08/kW·h) = \$2.53 ←

Problem 2.29

KNOWN: An expression for the power developed by an automobile engine in terms of torque and rotational speed is given.

FIND: For power, in hp, torque, in ft·lbf, and rotational speed, in RPM, evaluate the value and units of the constant appearing in the given expression.

ANALYSIS: The given expression is $\dot{W} = \mathcal{T}\omega / C$. When $\dot{W}$ is in hp, $\mathcal{T}$ is in ft·lbf, and ω is in RPM, by inspection the units of C are $\left[\frac{(\text{ft}\cdot\text{lbf})(\text{rev/min})}{\text{hp}}\right]$ ←

Beginning with $\dot{W} = \mathcal{T}\omega$, Eq. 2.20, and applying unit conversion factors for the product $\mathcal{T}\omega$, we get

$$\underset{\text{in hp}}{\dot{W}} = \mathcal{T}(\text{ft}\cdot\text{lbf})\,\omega\left(\frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right|\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\left|\frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}\right|$$

$$= \mathcal{T}(\text{ft}\cdot\text{lbf})\,\omega\left(\frac{\text{rev}}{\text{min}}\right)\left[\frac{1\,\text{hp}}{5252\,(\text{ft}\cdot\text{lbf})(\text{rev/min})}\right]$$

$$\therefore \quad \underset{\text{in hp}}{\dot{W}} = \frac{\tau(\text{ft}\cdot\text{lbf})\,\omega\left(\frac{\text{rev}}{\text{min}}\right)}{C}$$

where

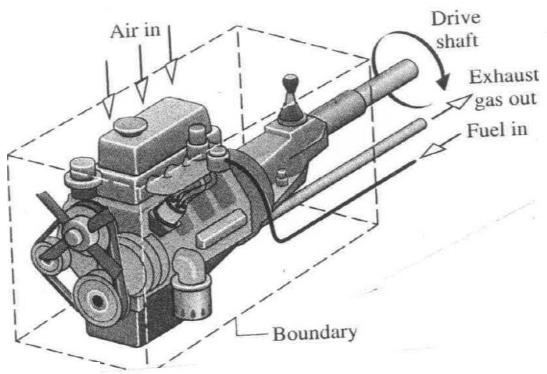
$$C = 5252\,\frac{(\text{ft}\cdot\text{lbf})(\text{rev/min})}{\text{hp}}$$ ←

Problem 2.30

KNOWN: Operating data are provided for a V-6 automobile engine.

FIND: Determine the percentage of the developed power that is transferred to the driveshaft and discuss.

SCHEMATIC & GIVEN DATA:



Driveshaft:
- Rotational speed = 4700 RPM
- Torque = 248 ft·lbf

Engine develops 226 hp

ANALYSIS: Using Eq. 2.20, the power delivered to the drive shaft is

$$\dot{W} = \tau\omega$$
$$= (248\ \text{ft}\cdot\text{lbf})\left(4700\ \frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right|\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lbf/s}}\right|$$
$$= 221.9\ \text{hp}$$

The percentage of the power developed by the engine that is delivered to the driveshaft is

$$\% = \frac{221.9\ \text{hp}}{226\ \text{hp}} = 0.98 \quad (98\%)$$

Frictional and like effects account for the difference.

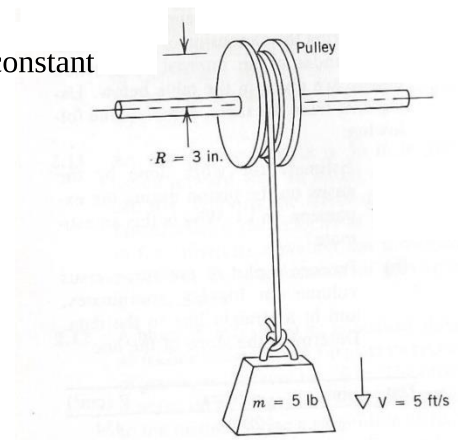
Problem 2.31

Figure P2.31 shows an object whose mass is 5 lb attached to a rope wound around a pulley. The radius of the pulley is 3 in. If the mass falls at a constant velocity of 5 ft/s, determine the power transmitted to the pulley, in hp, and the rotational speed of the shaft, in revolutions per minute (RPM). The acceleration of gravity is 32.2 ft/ s^2.

KNOWN: An object attached to a rope wound around a pulley falls at a constant velocity.

FIND: Find the power transmitted to the pulley and the rotational speed.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object falls at a constant speed. (2) The acceleration of gravity is constant.

ANALYSIS: The power is obtained using Eq. 2.13

$$\dot{W} = \mathbf{F}\cdot\mathbf{V} = (mg)\mathrm{V}$$

$$= (5\ \mathrm{lb})\left(32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}\right)\left(5\ \frac{\mathrm{ft}}{\mathrm{s}}\right)\left|\frac{1\ \mathrm{lbf}}{32.2\ \mathrm{lb\cdot ft}/s^2}\right|$$

$$= 25\ \mathrm{ft\cdot lb/s}$$

Converting to horsepower

$$\dot{W} = \left(25\ \mathrm{ft}\cdot\frac{\mathrm{lbf}}{\mathrm{s}}\right)\left|\frac{1\ \mathrm{hp}}{550\ \mathrm{ft\cdot lbf/s}}\right| = 0.0455\ \mathrm{hp}$$

The rotational speed of the pulley is related to the velocity of the object and the radius by V = Rω. Thus

$$\omega = \frac{\mathrm{V}}{\mathrm{R}} = \left(\frac{5\ \mathrm{ft/s}}{3/12\ \mathrm{ft}}\right)\left|\frac{1\ \mathrm{rev}}{2\pi}\right|\left|\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right| = 191\ \mathrm{rev/min}$$

Problem 2.32

KNOWN: A wire suspended vertically is stretched by an applied force.

FIND: Obtain an expression for the work done on the wire and evaluate the work for a given set of data.

SCHEMATIC & GIVEN DATA:

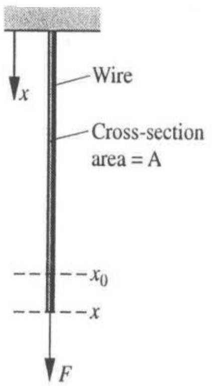


Fig. P2.39

normal stress, strain

$\sigma = C\epsilon$

$\epsilon = \dfrac{x - x_0}{x_0}$

C: Young's modulus

Data set:

$x_0 = 10$ ft

$x = 10.01$ ft

$C = 2.5 \times 10^7 \text{ lbf/in}^2$

$A = 0.1 \text{ in}^2$

ENGR. MODEL:

1. The wire is the closed system.
2. The moving boundary is the only work mode.
3. The change in area A is negligible.
4. The normal stress and thus the applied force varies linearly with strain.

ANALYSIS: (a) The work done on the wire is given by Eq. 2.18

$$W = -\int_{x_0}^{x} \sigma A\, dx$$

From the given stress-strain relation

$$\sigma = C\varepsilon = C\left(\frac{x - x_0}{x_0}\right)$$

Where C is a constant (Young's modulus). From this expression

$$d\varepsilon = \frac{dx}{x_0} \Rightarrow dx = x_0\, d\varepsilon$$

Substituting into the work expression

$$W = -\int_0^{\varepsilon} (C\varepsilon) A (x_0\, d\varepsilon) = -CAx_0 \int_0^{\varepsilon} \varepsilon\, d\varepsilon$$

Finally

$$W = -\frac{CAx_0\varepsilon^2}{2} \longleftarrow \text{work expression}$$

Problem 2.32 (Continued)

(b) Substituting given data into the work expression

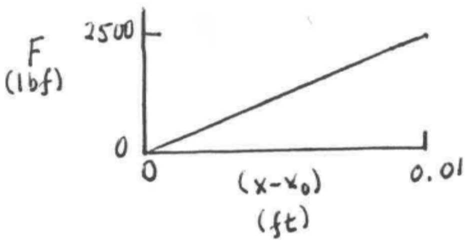
$$W = -\frac{\left(2.5\times10^{7}\ \frac{\text{lbf}}{\text{in}^2}\right)(0.1\ \text{in}^2)(10\ \text{ft})\overbrace{\left[\frac{0.01}{10}\right]^2}^{10^{-6}}}{2} = -12.5\ \text{ft}\cdot\text{lbf}$$

The downward force varies with strain according to

$$F = \sigma A = C\epsilon A = AC\left[\frac{x - x_0}{x_0}\right]$$

When $x = x_0$, $F = 0$. When $x - x_0 = 0.01$ ft,

$$F = (0.1\ \text{in}^2)\left(2.5\times10^{+7}\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{0.01\ \text{ft}}{10\ \text{ft}}\right) = 2500\ \text{lbf}$$



Problem 2.33

KNOWN: Data is provided for a spring stretched by a force applied at its end.

FIND: Obtain an expression for the work done in streching the spring and evaluate the work using given data.

SCHEMATIC & GIVEN DATA:

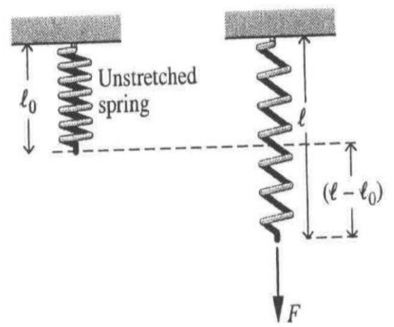


Fig. P2.41

$F = k(\ell - \ell_0)$

$\ell_0 = 3\,cm$

$\ell_1 = 6\,cm$

$\ell_2 = 10\,cm$

$K = 10^4\,N/m$

ENGR. MODEL:

1. The spring is the closed system.
2. The moving boundary is the only work mode.
3. Hooke's law applies.

ANALYSIS: (a) The work done in stretching the spring is given by

$$W = -\int_1^2 F\,d\ell$$

Letting $x = \ell - \ell_0$, this becomes

$$W = -\int_1^2 Kx\,dx = -K\left[\frac{x_2^2 - x_1^2}{2}\right]$$

$$= -\frac{K}{2}\left[(\ell_2 - \ell_0)^2 - (\ell_1 - \ell_0)^2\right] \longleftarrow$$

(b) When $(\ell_1 - \ell_0) = 3\,cm$ and $(\ell_2 - \ell_0) = 7\,cm$,

$$W = \frac{(-10^4\,N/m)}{2}\left[(7\,cm)^2 - (3\,cm)^2\right]\left|\frac{1\,m}{10^2\,cm}\right|^2\left|\frac{1\,J}{1\,N\cdot m}\right|$$

$$= -20\,J \longleftarrow$$

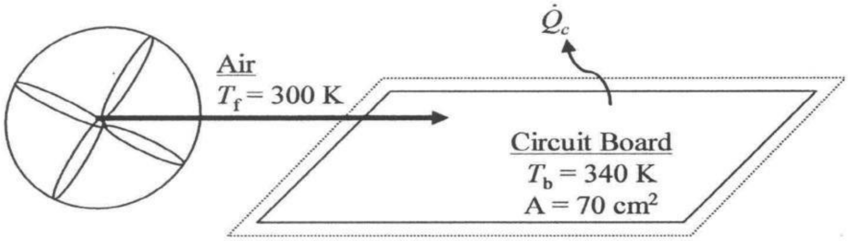
Problem 2.34

A fan forces air over a computer circuit board with a surface area of 70 cm^2 to avoid overheating. The air temperature is 300 K while the circuit board surface temperature is 340 K. Using data from Table 2.1, determine the largest and smallest heat transfer rates, in W, that might be encountered for this forced convection.

KNOWN: A fan forces air to flow over a circuit board to avoid overheating.

FIND: Largest and smallest heat transfer rates associated with this forced convection.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The circuit board is the system.
2. The system is at steady state.

ANALYSIS:
Newton's Law of Cooling is $\dot{Q}_c = hA(T_b - T_f)$, where $\dot{Q}_c$ is the rate of cooling heat transfer, h is the convection heat transfer coefficient, A is area of the surface, T_b is temperature of the surface, and T_f is temperature of the flowing fluid (air).

From Table 2.1 for forced convection using gases, the largest and smallest values for the convection heat transfer coefficient are

(Largest) h = 250 W/(m2·K)
(Smallest) h = 25 W/(m^2·K)

Substituting into Newton's Law of Cooling yields

$$\dot{Q}_c = \left(250 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right)\left(70\ \text{cm}^2\right)\left|\frac{\text{m}^2}{(100\ \text{cm})^2}\right|(340\ \text{K} - 300\ \text{K}) = \underline{\textbf{70 W} \text{ (largest heat transfer rate)}}$$

and

$$\dot{Q}_c = \left(25 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right)\left(70\ \text{cm}^2\right)\left|\frac{\text{m}^2}{(100\ \text{cm})^2}\right|(340\ \text{K} - 300\ \text{K}) = \underline{\textbf{7 W} \text{ (smallest heat transfer rate)}}$$

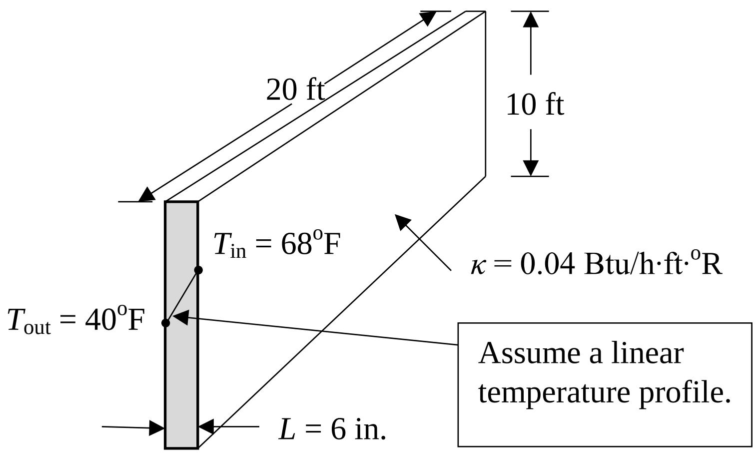
Problem 2.35

A 6-in. insulated frame wall of a house has an average thermal conductivity of 0.04 Btu/h·ft·°R. The inner surface of the wall is at 68°F and the outer surface is at 40°F. Determine at steady state the rate of heat transfer through the wall, in Btu/h. If the wall is 20 ft x 10 ft, determine the total amount of energy transfer in 10 hours, in Btu.

KNOWN: The inner and outer temperatures, thermal conductivity, and thickness of a frame wall are specified. The wall area is also known.

FIND: Determine the rate of energy transfer by heat through the wall and the total amount of energy transferred in 10 hours.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The wall is a closed system at steady state. (2) Conduction follows Fourier's law. (3) The temperature profile through the wall is linear.

ANALYSIS: For conduction through a plane wall with constant thermal conductivity and assuming a linear temperature profile

Note: ΔT (°R) = ΔT (°F)

$$\dot{Q} = \frac{\kappa A(T_{in} - T_{out})}{L} = \frac{\left(0.04\,\frac{\text{Btu}}{\text{h·ft·°R}}\right)(20\ x\ 10\ \text{ft}^2)(68-40)\,°\text{R}}{(6/12\ \text{ft})} = 448\ \text{Btu/h}$$

The total amount of energy transfer by heat is

$$Q = \int \dot{Q}\,dt = \dot{Q}\Delta t = (448\ \text{Btu/h})(10\ \text{h}) = 4480\ \text{Btu}$$

COMMENTS: Note that the temperature *difference* has the same value in °F and °R. Also, since the heat transfer rate is constant for steady state conduction, the integral reduces to $\dot{Q}\Delta t$.

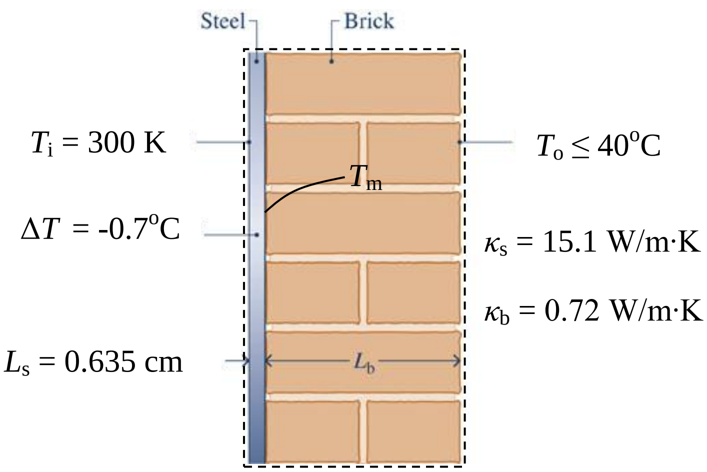
Problem 2.36

As shown in Fig. P2.36, an oven wall consists of a 0.635-cm-thick layer of steel (κ_s = 15.1 W/m·K) and a layer of brick (κ_b = 0.72 W/m·K). At steady state, a temperature decrease of 0.7°C occurs over the steel layer. The inner temperature of the steel layer is 300 K. If the temperature of the outer surface of the brick must be no greater than 40°C, determine the thickness of brick, in cm, that ensures this limit is met. What is the rate of conduction, in kW per m^2 of wall surface area?

KNOWN: Steady-state data are provided for a composite wall formed from a steel layer and a brick layer.

FIND: Determine the minimum thickness of the brick layer to keep the outer surface temperature of the brick at or below a specified value.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: Using Eq. 2.31 together with model assumption 2

$$\left(\frac{\dot{Q}}{A}\right)_{\text{steel}} = -\kappa_s\left[\frac{T_m - T_i}{L_s}\right] \quad \text{and} \quad \left(\frac{\dot{Q}}{A}\right)_{\text{brick}} = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

where T_m denotes the temperature at the steel-brick interface.

At steady state, the rate of conduction *to* the interface through the steel must equal the rate of conduction *from* the interface through the brick: $(\dot{Q}/A)_{\text{steel}} = (\dot{Q}/A)_{\text{brick}}$. Thus

$$-\kappa_s\left[\frac{T_m - T_i}{L_s}\right] = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

And solving for L_b we get

(300 – 0.7) = 299.3 °C

$$L_b = \frac{\kappa_b}{\kappa_s}\left[\frac{T_o - T_m}{\underline{T_m - T_i}}\right] L_s$$

= – 0.7 °C

Problem 2.36 (Continued)

$$L_b = \left(\frac{0.72 \text{ W/m·K}}{15.1 \text{ W/m·K}}\right)\left[\frac{299.3 - T_o}{0.7}\right](0.635 \text{ cm})$$

Since $T_o \leq 40\ ^oC$

$$L_b \geq \left(\frac{0.72}{15.1}\right)\left[\frac{299.3 - 40}{0.7}\right](0.635 \text{ cm})$$

$$L_b \geq 11.22 \text{ cm} \longleftarrow$$

The rate of conduction is

$$\left(\frac{\dot{Q}}{A}\right)_{steel} = -\kappa_s\left[\frac{T_m - T_i}{L_s}\right] = -(15.1 \text{ W/m}\cdot\text{K})\left[\frac{299.3 - 300}{0.635 \text{ cm}}\right]\left|\frac{100 \text{ cm}}{1 \text{ m}}\right|\left|\frac{1 \text{ kW}}{10^3 \text{W}}\right| = 1.665 \text{ kW/m}^2 \longleftarrow$$

or

$$\left(\frac{\dot{Q}}{A}\right)_{brick} = -\kappa_s\left[\frac{T_o - T_m}{L_b}\right] = -(0.72 \text{ W/m}\cdot\text{K})\left[\frac{40 - 299.3}{11.22 \text{ cm}}\right]\left|\frac{100 \text{ cm}}{1 \text{ m}}\right|\left|\frac{1 \text{ kW}}{10^3 \text{W}}\right| = 1.664 \text{ kW/m}^2 \longleftarrow$$

The slight difference is due to round-off.

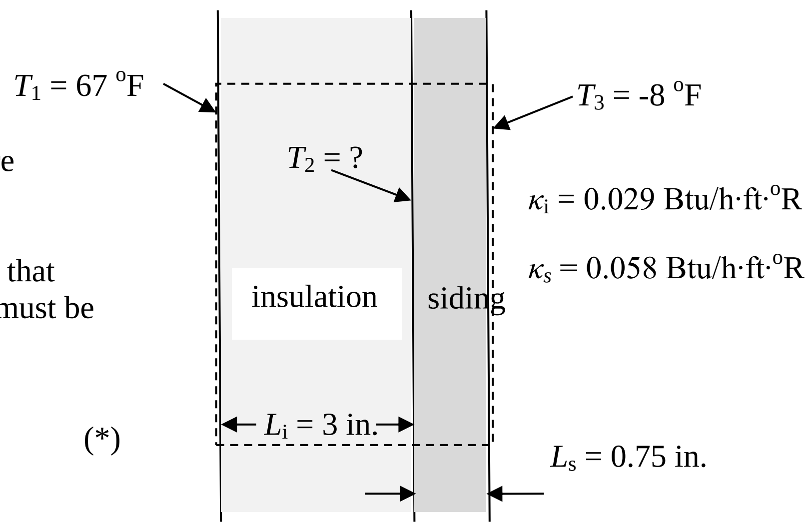
Problem 2.37

A composite plane wall consists of a 3-in.-thick layer of insulation (κ_i = 0.029 Btu/h·ft·°R) and a 0.75-in.-thick layer of siding (κ_s = 0.058 Btu/h·ft·°R). The inner temperature of the insulation is 67°F. The outer temperature of the siding is -8°F. Determine at steady state (a) the temperature at the interface of the two layers, in °F, and (b) the rate of heat transfer through the wall in Btu per ft^2 of surface area.

KNOWN: Energy transfer by conduction occurs through a composite wall consisting of two layers.

FIND: Determine the temperature at the interface between the two layers and the rate of heat transfer per unit area through the wall.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: With Eq. 2.17, and recognizing that at steady state the rates of energy conduction must be equal through each layer

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2-T_1}{L_i}\right] = -\kappa_s\left[\frac{T_3-T_2}{L_s}\right] \qquad (*)$$

Solving for T_2

$$T_2 = \frac{\left(\frac{\kappa_i}{L_i}T_1 + \frac{\kappa_s}{L_s}T_3\right)}{(\kappa_i/L_i)+(\kappa_s/L_s)}$$

$$\frac{\kappa_i}{L_i} = \frac{0.029\ \text{Btu/h·ft·R}}{3\ \text{in.}}\left|\frac{12\ \text{in.}}{1\ \text{ft}}\right| = 0.116\ \text{Btu/h·°R} \qquad \frac{\kappa_s}{L_s} = \frac{0.058\ \text{Btu/h·ft·R}}{.75\ \text{in.}}\left|\frac{12}{1}\right| = 0.928\ \text{Btu/h·°R}$$

Thus

$$T_2 = \frac{(0.116)(527)+(0.928)(452)}{(0.116)+(0.928)} = 460.3\ \text{°R} = 0.33\ \text{°F}$$

Thus, using Eq. (*)

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2-T_1}{L_i}\right] = \left(-0.029\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(0.33-67)\text{R}}{\frac{3}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$

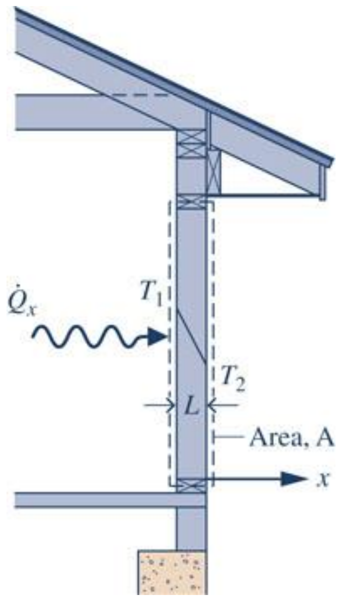
$$\frac{\dot{Q}}{A} = -\kappa_s\left[\frac{T_3-T_2}{L_2}\right] = \left(-0.058\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(-8-0.33)\text{R}}{\frac{0.75}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$

Problem 2.38

Complete the following exercise using heat transfer relations:

(a) Referring to Fig. 2.12, determine the rate of conduction heat transfer, in W, for $\kappa = 0.07$ W/m·K, A = 0.125 m^2, T_1 = 298 K, T_2 = 273 K.
(b) Referring to Fig. 2.14, determine the rate of convection heat transfer from the surface to the air, in W, for h = 10 W/m^2, A = 0.125 m^2, T_b = 305 K, T_f = 298 K.

(a) Referring to Fig. 2.12, determine the rate of conduction heat transfer, in W, for $\kappa = 0.07$ W/m·K, A = 0.125 m^2, T_1 = 298 K, T_2 = 273 K.

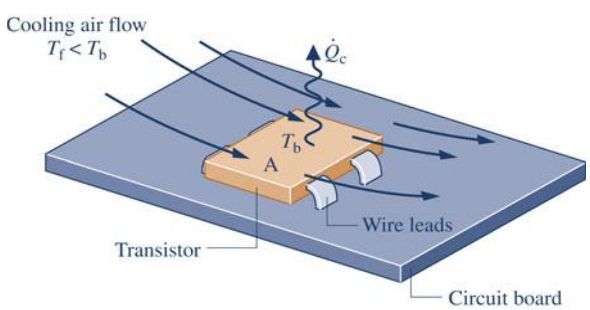


Using Eq. 2.31 and noting that the temperature varies linearly through the wall

$$\dot{Q}_x = -\kappa A\left[\frac{T_2 - T_1}{L}\right]$$

$$= -\left(0.07\,\frac{\text{W}}{\text{m}\cdot\text{K}}\right)(0.125\ \text{m}^2)\left[\frac{(273-298)\text{K}}{(0.127\ \text{m})}\right] = 1.722\ \text{W}$$

(b) Referring to Fig. 2.14, determine the rate of convection heat transfer from the surface to the air, in W, for h = 10 W/m^2, A = 0.125 m^2, T_b = 305 K, T_f = 298 K.



Using Eq. 2.34

$$\dot{Q}_c = hA[T_b - T_f]$$

$$= \left(10\,\frac{\text{W}}{\text{m}^2}\right)(0.125\ \text{m}^2)[305 - 298]\text{K}$$

$$= 8.75\ \text{W}$$

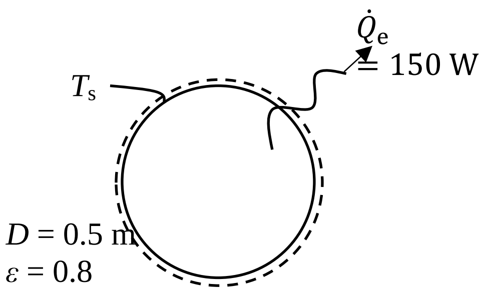
Problem 2.39

At steady state, a spherical interplanetary electronics-laden probe having a diameter of 0.5 m transfers energy by radiation from its outer surface at a rate of 150 W. If the probe does not receive radiation from the sun of deep space, what is the surface temperature, in K? Let $\varepsilon = 0.8$.

KNOWN: Steady-state operating data are provided for a spherical interplanetary probe.

FIND: Determine the surface temperature of the sphere.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The probe is at steady state. (2) The probe emits but does not receive radiation.

ANALYSIS: In this case, Eq. 2.32 applies:

$$\dot{Q}_e = \varepsilon \sigma A T^4,$$

where A = $\pi d^2 = 0.7854\ \text{m}^2$

The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. Thus

$$T_s = \left[\frac{\dot{Q}_e}{\varepsilon \sigma A}\right]^{1/4} = \left[\frac{150\ \text{W}}{(0.8)(5.67 \text{ x} \frac{10^{-8}\text{W}}{\text{m}^2\text{K}^4})(0.7854\ \text{m}^2)}\right]^{1/4} = 254.7\ \text{K}$$

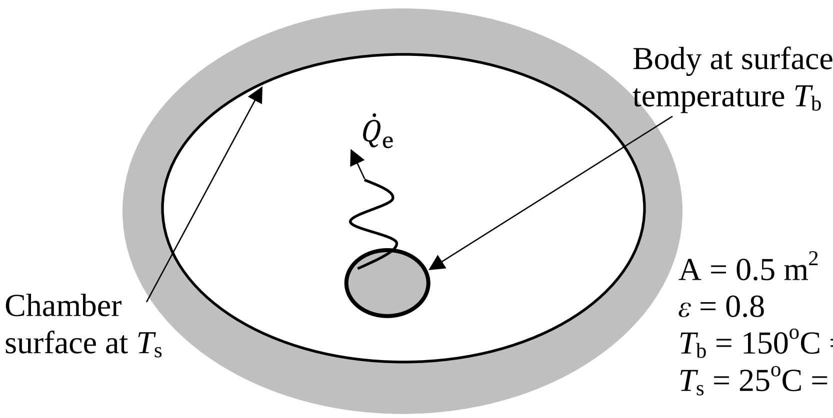
Problem 2.40

A body whose surface area is 0.5 m^2, emissivity is 0.8, and temperature is 150^{o}C is placed in a large, evacuated chamber whose walls are at 25^{o}C. What is the rate at which radiation is *emitted* by the surface, in kW? What is the *net* rate at which radiation is *exchanged* between the body and the walls, in kW?

KNOWN: Data are provided for a body placed in a large, evacuated chamber.

FIND: Determine the rate at which radiation is emitted from the surface and the net rate at which radiation is exchanged between the body and the chamber.

SCHEMATIC AND GIVEN DATA:



A = 0.5 m^2
ε = 0.8
T_b = 150^{o}C = 423 K
T_s = 25^{o}C = 298 K

ENGINEERING MODEL: (1) The area of body is much less than that of the chamber walls. (2) The chamber is evacuated.

ANALYSIS:

The rate radiation is emitted from the surface is given by Eq. 2.32, where σ = 5.67 x 10^{-8} W/m^2·K^4 is the Stefan-Boltzmann constant. That is

$$\dot{Q}_e = \varepsilon\sigma A T_b^4 = (0.8)(5.67 \text{ x } 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(0.5 \text{ m}^2)(423 \text{ K})^4 = 726 \text{ W} \longleftarrow$$

With the assumptions in the Engineering Model, the *net* rate at which energy is exchanged by radiation between the body and the chamber walls is given by Eq. 2.33. Thus

$$(\dot{Q}_e)_{net} = \varepsilon\sigma A(T_b^4 - T_s^4)$$

$$= (0.8)(5.67 \text{ x } 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(0.5 \text{ m}^2)[(423 \text{ K})^4 - (298 \text{ K})^4]$$

$$= 547 \text{ W} \longleftarrow$$

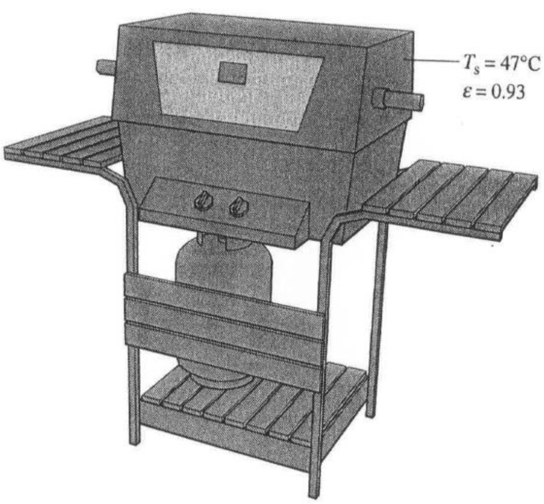
Problem 2.41

The outer surface of the grill shown in Fig. P2.41 is at 47°C and the emissivity is 0.93. The heat transfer coefficient for convection between the hood and the surroundings at 27°C is 10 W/m^2 · K. Determine the rate of heat transfer between the grill hood and the surroundings by convection and radiation, in kW per m^2 of surface area.

KNOWN: Data are provided for a grill hood.

FIND: Determine the net rate of heat transfer between the hood and the surroundings by convection and radiation, per unit area of hood surface.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Radiative heat transfer between the hood and surroundings is modeled as an exchange between a surface at T_s and a much larger surrounding surface at T_0.

ANALYSIS:

Using Eq. 2.34, convection heat transfer is

$$\frac{\dot{Q}_c}{A} = h[T_s - T_0] = 10\frac{W}{m^2K}[20K]\left|\frac{1kW}{10^3W}\right|$$

$$= 0.2\ kW/m^2$$

Using Eq. 2.33, radiative heat transfer is

$$\frac{\dot{Q}_e}{A} = \varepsilon\sigma[T_s^4 - T_0^4]$$

$$= 0.93\left[5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\right]\left[(320K)^4 - (300K)^4\right]$$

$$= 125.8\frac{W}{m^2}\left|\frac{1kW}{10^3W}\right| = 0.13\ kW/m^2$$

$$\text{Total} = (\dot{Q}_c/A) + (\dot{Q}_e/A) = 0.33\ kW/m^2$$

Problem 2.42

Each line of the following table gives data for a process of a closed system. Each entry has the same energy units. Determine the missing entries.

Process	Q	W	E_1	E_2	ΔE
a	+50		-20		+70
b		+20		+50	+30
c		-60	+40	+60	
d	-40		+50		0
e	+50	+150		-80	

Process	Q	W	E_1	E_2	ΔE
a	+50	***-20***	-20	***+50***	+70
b	***+50***	+20	*+20*	+50	+30
c	***-40***	-60	+40	+60	***+20***
d	-90	***-90***	+50	***+50***	0
e	+50	+150	***+20***	-80	***-100***

Process a:

$W = Q - \Delta E = +50 - (+70) = -20$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = +70 + (-20) = +50$

Process b:

$Q = \Delta E + W = +30 + (+20) = +50$

$\Delta E = E_2 - E_1$

$E_1 = E_2 - \Delta E = +50 - (+30) = +20$

Process c:

$\Delta E = E_2 - E_1 = +60 - (+40) = +20$

$Q = \Delta E + W = +20 + (-60) = -40$

Process d:

$W = Q - \Delta E = (-90) - 0 = -90$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = 0 + 50 = +50$

Process e:

$\Delta E = Q - W = +50 - (+150) = -100$

$E_1 = E_2 - \Delta E = (-80) - (-100) = +20$

Problem 2.43

Each line of the following table gives data for a process of a closed system. Each entry has the same energy units. Determine the missing entries.

Process	Q	W	E_1	E_2	ΔE
a		– 25	+15	+30	
b	+27		+7		+15
c	– 4	+10	+6		
d		– 10		+10	+20
e	+3			– 8	0

Process	Q	W	E_1	E_2	ΔE
a	– 10	***– 25***	***+15***	***+ 30***	+15
b	***+27***	– 12	***+7***	+22	***+15***
c	***– 4***	***+10***	***+6***	– 8	– 14
d	+10	***– 10***	– 10	***+10***	***+20***
e	***+3***	+3	– 8	***-8***	***0***

Process a:

$\Delta E = E_2 - E_1 = +30 - (+15) = +15$

$Q = \Delta E + W = +15 + (-25) = -10$

Process b:

$W = \Delta E - Q = +15 - (+27) = -12$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = +15 + (+7) = +22$

Process c:

$\Delta E = Q - W = (-4) - (+10) = -14$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = (-14) + (+6) = -8$

Process d:

$Q = \Delta E + W = (+20) + (-10) = +10$

$\Delta E = E_2 - E_1$

$E_1 = E_2 - \Delta E = (+10) - (+20) = -10$

Process e:

$W = Q - \Delta E = (+3) - (0) = +3$

$\Delta E = E_2 - E_1$

$E_1 = E_2 - \Delta E = (-8) - (0) = -8$

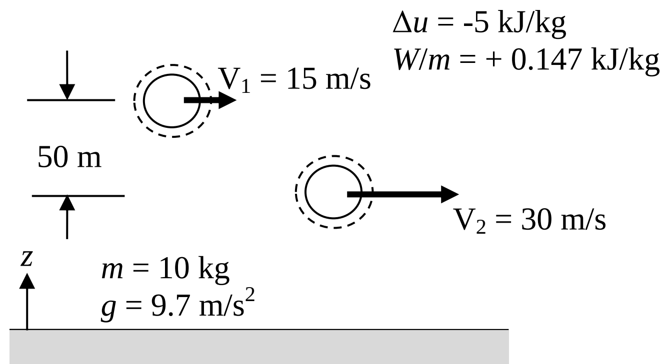
Problem 2.44

A closed system of mass of 10 kg undergoes a process during which there is energy transfer by work from the system of 0.147 kJ per kg, an elevation decrease of 50 m, and an increase in velocity from 15 m/s to 30 m/s. The specific internal energy decreases by 5 kJ/kg and the acceleration of gravity is constant at 9.7 m/s^2. Determine the heat transfer for the process, in kJ.

KNOWN: Data are provided for a closed system undergoing a process involving work, heat transfer, change in elevation, and change in velocity.

FIND: Determine the heat transfer for the process.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The system is a closed system. (2) The acceleration of gravity is constant.

ANALYSIS:

$$\Delta U + \Delta PE + \Delta KE = Q - W \quad \rightarrow \quad Q = \Delta U + \Delta PE + \Delta KE - W$$

$$W = m\,[W/m] = 10\text{ kg } [-0.147\text{ kJ/kg}] = -1.47\text{ kJ}$$

$$\Delta U = m\Delta u = 10\text{ kg } [-\,5\text{ kJ/kg}] = -50\text{ kJ}$$

$$\Delta KE = \frac{m}{2}(V_2^2 - V_1^2) = \frac{10\text{ kg}}{2}\left[\left(30\ \frac{\text{m}}{\text{s}}\right)^2 - \left(15\ \frac{\text{m}}{\text{s}}\right)^2\right]\left|\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right| = +3.38\text{ kJ}$$

$$\Delta PE = mg(z_2 - z_1) = (10\text{ kg})\,(9.7\text{ m/s}^2)(-50\text{ m})\left|\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right| = -\,4.85\text{ kJ}$$

$$Q = (-50) + (-4.85) + (3.38) - (-1.47) = -50\text{ kJ (out)} \longleftarrow$$

Problem 2.45

KNOWN: A gas contained in a piston-cylinder assembly undergoes a constant-pressure expansion while being slowly heated. State data are provided.

FIND: For the gas, evaluate work and heat transfer. For the piston, evaluate work and change in potential energy.

SCHEMATIC & GIVEN DATA:

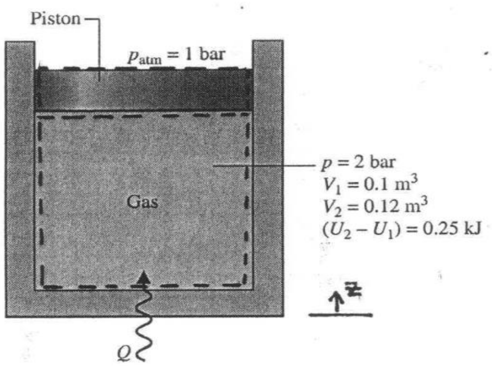


Fig. P2.56

ENGINEERING MODEL:

1. As shown in the schematic, two closed systems are considered: the gas and the piston.
2. The gas undergoes a constant-pressure process.
3. For the gas there is no change in potential energy (see Example 2.3) and no overall change in kinetic energy.
4. For the piston, there is no heat transfer. Also, there is no change in internal energy, no overall change in kinetic energy, and no friction.

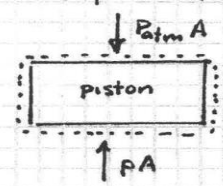
ANALYSIS: (a) Taking the gas as the system, the work is obtained from Eq. 2.17:

$$W = \int_1^2 p\,dV = p[V_2 - V_1] = \left(2\times10^5 \frac{N}{m^2}\right)(0.12 - 0.1)\,m^3 \left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = 4\,kJ$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \Rightarrow Q = W + \Delta U$

$$\Rightarrow Q = 4\,kJ + 0.25\,kJ = 4.25\,kJ$$

(b) Taking the piston as the system, an energy transfer by work occurs on the bottom surface from the gas. At the top surface the piston does work on the atmosphere:



$$W_{piston} = \int F\,dz = (p_{atm}A - pA)\Delta z = (p_{atm} - p)(A\Delta z)$$

$$= (p_{atm} - p)\underbrace{\Delta V}_{gas}$$

$$= (1-2)\left(10^5 \frac{N}{m^2}\right)(0.12 - 0.1)\,m^3 \left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -2\,kJ$$

An energy balance for the piston reduces as follows:

$$[\cancel{\Delta U} + \cancel{\Delta KE} + \Delta PE]_{piston} = \cancel{Q}_{piston} - W_{piston}$$

$$\Rightarrow \Delta PE]_{piston} = -W_{piston}$$

(1)

$$= +2\,kJ$$

1. Overall energy "balance sheet" in terms of magnitudes:

Input: Q = 4.25 kJ

Disposition of the energy input:

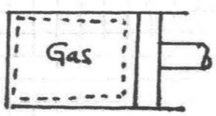
⊙ Stored as ΔU in the gas:	0.25 kJ
⊙ Stored as ΔPE in the piston:	2.00 kJ
⊙ Transfer by work to the atmosphere	2.00 kJ
	4.25 kJ

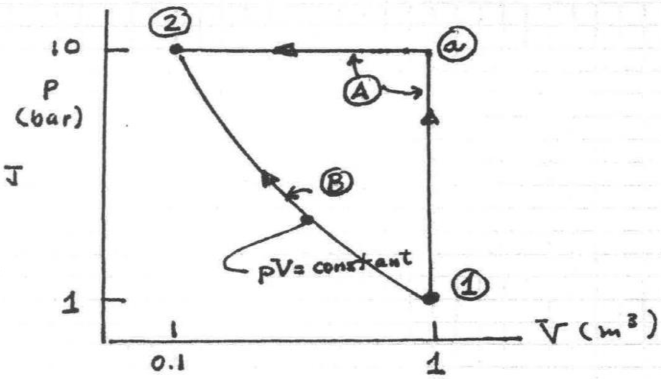
Problem 2.46

KNOWN: A gas contained in a piston-cylinder assembly undergoes two processes, A and B, between the same end states. State data are provided.

FIND: For each process, sketch it on p-V coordinates and evaluate the work and heat transfer, each in kJ.

SCHEMATIC & GIVEN DATA:





$p_1 = 1$ bar, $V_1 = 1\ m^3$, $U_1 = 400$ kJ

$p_2 = 10$ bar, $V_2 = 0.1\ m^3$, $U_2 = 450$ kJ

ENGINEERING MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: Reducing an energy balance for the gas: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow Q = \Delta U + W = (450 - 400)\text{ kJ} + W$$

$$\Rightarrow Q = 50\text{ kJ} + W \qquad (1)$$

Eq. (1) applies to each of the processes, A and B.

Evaluating work from Eq. 2.17, $W = \int_1^2 p\,dV$.

Process A:

$$W_A = \cancelto{0}{W_{1a}} + W_{a2} = p_2 [V_2 - V_1]$$

$$= \left(10 \times 10^5 \frac{N}{m^2}\right)(0.1 - 1)\ m^3 \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = -900\text{ kJ}$$

Then, with Eq. (1)

$$Q_A = 50\text{ kJ} + (-900\text{ kJ}) = -850\text{ kJ}$$

Process B:

$$W_B = \int_1^2 p\,dV = \int_1^2 \frac{C}{V}\,dV = C \ln\frac{V_2}{V_1} = p_1 V_1 \ln\frac{V_2}{V_1}$$

$$= \left(10^5 \frac{N}{m^2}\right)(1\ m^3) \ln\left[\frac{0.1}{1.0}\right] \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = -230.3\text{ kJ}$$

Then, with Eq. (1)

$$Q_B = 50\text{ kJ} + (-230.3\text{ kJ}) = -180.3\text{ kJ}$$

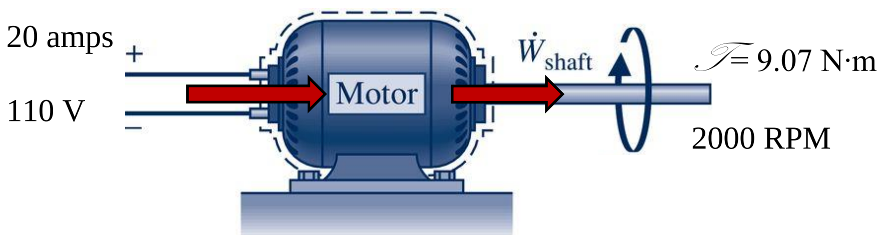
Problem 2.47

An electric motor operating at steady state draws a current of 20 amp at a voltage of 110 V. The output shaft rotates at a constant speed of 2000 RPM and exerts a torque of 9.07 N·m. Determine
(a) the magnitude of the power input, in W.
(b) the output power, in W.
(c) the cost of 24 hours of operation if electricity is valued at $0.09 per kW·h.

KNOWN: An electric motor operates at steady state. The input current and voltage are known and the rotational speed and torque of the exit shaft are specified. The cost of electricity is also specified.

FIND: Determine (a) the magnitude of the power input, (b) the output power, and (c) the cost of 24 hours of operation.

SCHEMCATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The motor operates at steady state.

ANALYSIS: (a) The magnitude of the input power is

$$\left|\dot{W}_{elec}\right| = (20\text{ amp})\,(110\text{ V})\left|\frac{1\text{ W}}{1\text{ volt·amp}}\right|\left|\frac{1\text{ kW}}{10^3\text{W}}\right| = 2.2\text{ kW (input)} \longleftarrow$$

(b) The shaft power is

$$\left|\dot{W}_{shaft}\right| = \mathscr{T}\text{ x }\omega$$

$$= (9.07\text{ N·m})(2000\,\frac{\text{rev}}{\text{min}})\,(2\pi\,\frac{1}{\text{rev}})\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 1.90\text{ kW (out)} \longleftarrow$$

(c) For steady operation (constant with time) the total amount of energy transferred in by electricity is

$$W_{elec} = \int\left|\dot{W}_{elec}\right|dt = \left|\dot{W}_{elec}\right|\,\Delta t = (2.2\text{ Kw})(24\text{ h}) = 52.8\text{ kW·h} \longleftarrow$$

Cost = (52.8 kW·h)($0.09/kW·h) = $4.75 (per day)

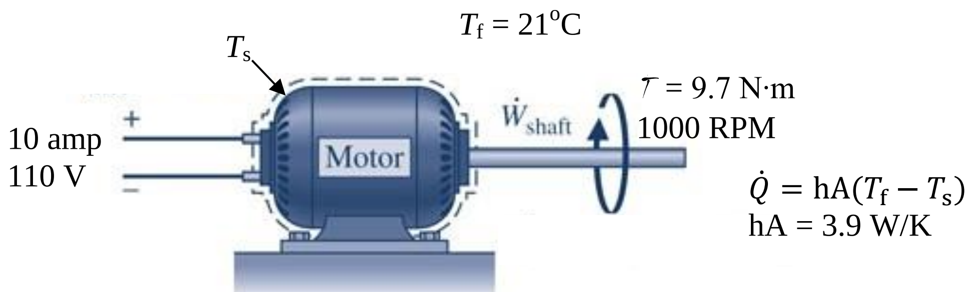
COMMENT: Note, for steady state operation the inputs and outputs are constant with time.

Problem 2.48

An electric motor draws a current of 10 amp with a voltage of 110 V, as shown in Fig. P2.48. The output shaft develops a torque of 9.7 N·m and a rotational speed of 1000 RPM. For operation at steady state, determine for the motor
(a) the electric power required, in kW.
(b) the power developed by the output shaft, in kW.
(c) the average surface temperature, T_s, in ^{o}C, if heat transfer occurs by convection to the surroundings at T_f = 21^{o}C.

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: Determine (a) the electric power required, (b) the power developed by the output shaft, and (c) average the surface temperature.



ENGINEERING MODEL: (1) The motor is the closed system. (2) The system is at steady state.

ANALYSIS: (a) Using Eq. 2.21

$$\dot{W}_{\text{electric}} = \text{- (voltage) (current) = - (110 V)(10 amp)}\left|\frac{1\ \text{W/amp}}{1\ \text{V}}\right|\left|\frac{1\ \text{kW}}{10^3\text{W}}\right| = \text{-1.1 kW (in)} \longleftarrow$$

(b) Using Eq. 2.20

$$\dot{W}_{\text{shaft}} = \text{(torque) (angular velocity)}$$

$$= (9.7\ \text{N}\cdot\text{m})\left(1000\,\frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\ \text{rad}}{\text{rev}}\right|\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{kW}}{10^3\text{N}\cdot\text{m/s}}\right| = \text{1.016 kW (out)} \longleftarrow$$

(c) To determine the surface temperature, first find the rate of energy transfer by heat using the energy balance

$$\cancel{\frac{dE}{dt}}^{\,0} = \dot{Q} - \dot{W} = \dot{Q} - (\dot{W}_{\text{electric}} + \dot{W}_{\text{shaft}})$$

$$\dot{Q} = (\dot{W}_{\text{electric}} + \dot{W}_{\text{shaft}}) = \text{(-1.1 kW) + (1.016 kW) = -0.084 kW}$$

The surface temperature of the motor is

$$T_s = \left(\dot{Q}/\text{hA}\right) + T_f = \text{(-0.084 kW)/(3.9 W/K)}\left|\frac{10^3\ \text{W}}{1\ \text{kW}}\right| + \text{294 K}$$

$$= 315.5\ \text{K} = 42.5\ ^o\text{C} \longleftarrow$$

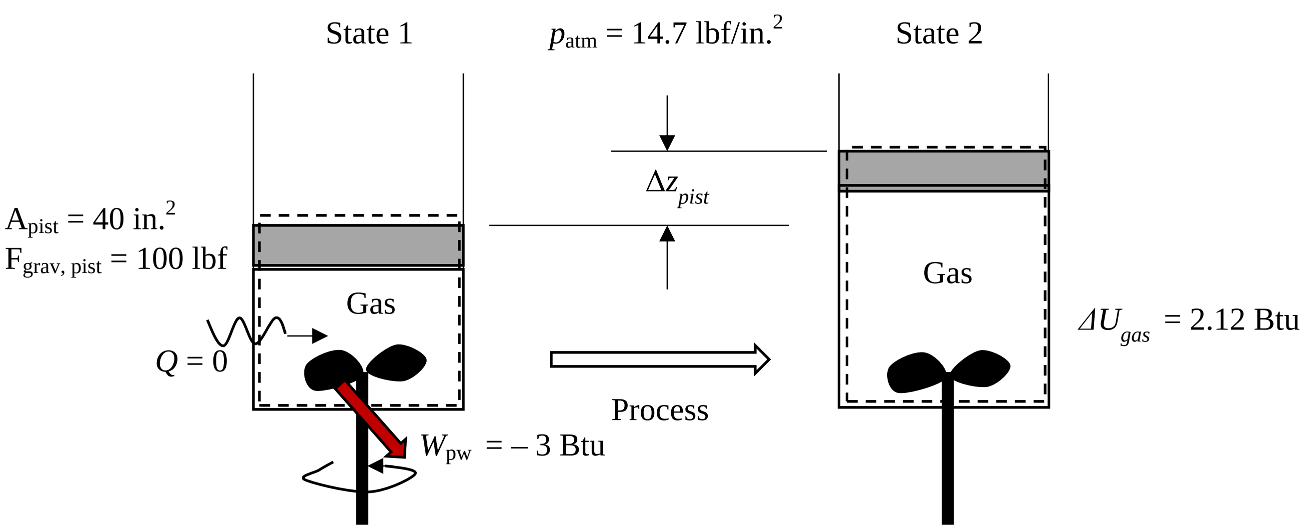
Problem 2.49

A gas is contained is a vertical piston-cylinder assembly by a piston with a face area of 40 in.^{2} and a weight of 100 lbf. The atmosphere exerts a pressure of 14.7 lbf/in.^{2} on top of the piston. A paddle wheel transfers 3 Btu of energy to the gas during a process in which the elevation of the piston increases. The change in internal energy of the gas is 2.12 Btu. The piston and cylinder are poor thermal conductors, and friction between them can be neglected. Determine the elevation increase of the piston, in ft.

KNOWN: A rotating shaft transfers a specified amount of energy to a gas contained in a vertical piston-cylinder assembly and the piton elevation increases. Data are provided for the piston and the change in internal energy of the gas.

FIND: Determine the elevation increase of the piston.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The gas and the piston are the closed system. (2) Energy transfer by heat is negligible; $Q = 0$. (3) Kinetic energy effects are negligible. (4) The potential energy change of the gas is negligible, but the potential energy change of the piston in considered. (5) The change of internal energy of the piston is negligible. (6) Friction between the piton and the cylinder wall is neglected.

ANALYSIS: The change in elevation of the piston is related to its change in potential energy by

$$\Delta PE_{pist} = (m_{pist}\, g)\, \Delta z_{pist} = (F_{grav,\ piston})\, \Delta z_{pist} \qquad (*)$$

To evaluate the change in internal energy of the piston, we apply the energy balance to the system consisting of the gas and the piston:

$$\Delta E_{gas} + \Delta E_{pist} = Q - W$$

Problem 2.49 (Continued)

For the piston: $\Delta E_{pist} = \cancel{\Delta U_{pist}} + \cancel{\Delta KE_{pist}} + \Delta PE_{pist}$

For the gas: $\Delta E_{gas} = \Delta U_{gas} + \cancel{\Delta KE_{gas}} + \cancel{\Delta PE_{gas}}$

There are two mechanisms for mechanical work for the system chosen: the paddle wheel work and the work of the piston against the atmospheric pressure: $W = W_{pw} + W_{atm}$

Since the atmospheric pressure is constant: $W_{atm} = p_{atm}A_{pist}\,\Delta z_{pist}$

Combining these results

$$\Delta U_{gas} + (F_{grav,\ piston})\,\Delta z_{pist} = \cancel{Q} - (W_{pw} + p_{atm}A_{pist}\,\Delta z_{pist})$$

Solving for Δz_{pist}

$$\Delta z_{pist} = \frac{-\,\Delta U_{gas} - W_{pw}}{F_{grav,piston} + p_{atm}A_{pist}}$$

Inserting values and converting units

$$\Delta z_{pist} = \frac{-\,(2.12\text{ Btu}) - (-3\text{ Btu})}{(100\text{ lbf}) + (40\text{ in.}^2)(14.7\text{ lbf/in.}^2)}\left|\frac{778\text{ Btu}}{1\text{ ft·lbf}}\right| = 0.995\text{ ft}$$

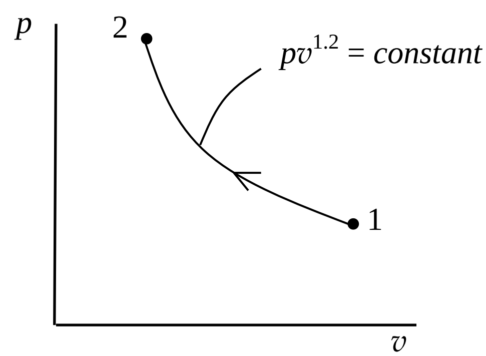
COMMENTS: Note that the gas and the piston can be considered individually as closed systems. With this approach, the work done by the gas on the piston must be considered, which involves evaluating the change in volume of the gas and the gas pressure. The approaches are algebraically equivalent, but the approach chosen leads to a simpler analysis.

Problem 2.50

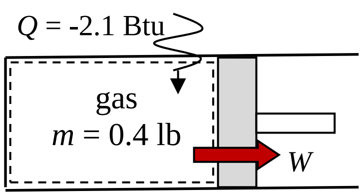
A gas undergoes a process in a piston-cylinder assembly during which the pressure-specific volume relation is $pv^{1.2} = constant$. The mass of the gas is 0.4 lb and the following data are known: $p_1 = 160$ lbf/in.^{2}, $V_1 = 1$ ft^3, and $p_2 = 390$ lbf/in.^{2} During the process, heat transfer *from* the gas is 2.1 Btu. Kinetic and potential energy effects are negligible. Determine the change in specific internal energy of the gas, in Btu/lb.

KNOWN: A gas is compressed in a piston-cylinder assembly. The pressure-specific volume relation is specified.

FIND: Determine the change in specific internal energy.



SCHEMATIC AND GIVEN DATA:



$p_1 = 160$ lbf/in.^{2}
$V_1 = 1$ ft^3
$p_2 = 390$ lbf/in.^{2}

ENGINEERING MODEL: (1) The gas is a closed system. (2) The process follows $pv^{1.2} = constant$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The change in specific internal energy will be found from an energy balance. First, determine the work. Since volume change is the only work mode, Eq. 2.17 applies:

$$W = \int_{V_1}^{V_2} p dV = = \int_{V_1}^{V_2} \frac{const}{V^{1.2}} dV = \frac{(p_2 V_2 - p_1 V_1)}{1-1.2}$$

Evaluating V_2

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.2} V_1 = \left(\frac{160 \text{ lbf/in.}^2}{390 \text{ lbf/in.}^2}\right)^{1/1.2} (1 \text{ ft}^3) = 0.4759 \text{ ft}^3$$

Thus

$$W = \left[\frac{(390 \text{ lbf/in.}^2)(0.4759 \text{ ft}^3) - (160)(1)}{1-1.2}\right] \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = -23.69 \text{ Btu (in)}$$

Now, writing the energy balance: $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = Q - W$

With $\Delta U = m\Delta u$

1 $$\Delta u = \frac{Q-W}{m} = \frac{(-2.1 \text{ Btu}) - (-23.69 \text{ Btu})}{0.4 \text{ lb}} = 54.0 \text{ Btu}$$

---

1. The amount of energy transfer in by work exceeds the amount of energy transfer out by heat, resulting in a net increase in internal energy.

Problem 2.51

Four kilograms of carbon monoxide (CO) is contained in a rigid tank with a volume of 1 m^3. The tank is fitted with a paddle wheel that transfers energy to the CO at a constant rate of 14 W for 1 h. During the process, the specific internal energy of the carbon monoxide increases by 10 kJ/kg. If no overall changes in kinetic or potential energy occur, determine (a) the specific volume at the final state, in m^3/kg. (b)the energy transfer by work, in kJ. (c) the energy transfer by heat, in kJ, and the direction of the heat transfer.

Known: Carbon monoxide (CO) is contained in a rigid tank with a paddle wheel that transfers energy to the air at a constant rate of 14 W for 1 h. During the process, the specific internal energy of the carbon monoxide increases.

Find: Determine the specific volume at the final state, in m^3/kg; the energy transfer by work, in kJ; and the energy transfer by heat transfer, in kJ, with direction.

Schematic and Given Data:

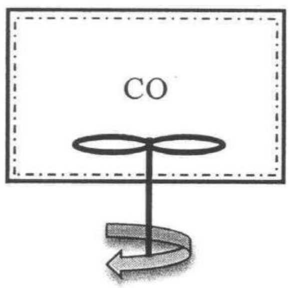
$V = 1\ \text{m}^3$

$m = 4\ \text{kg}$

$\Delta u = 10\ \text{kJ/kg}$

$\dot{W} = -14\ \text{W}$

$\Delta t = 1\ \text{h}$



Engineering Model:
(1) The carbon monoxide within the tank is the closed system.
(2) The tank is rigid, therefore $V_1 = V_2$.
(3) The system experiences no change in potential and kinetic energy.

Analysis:

(a) The mass and volume remain constant in the process due to assumptions (1) and (2), therefore

$$v = \frac{V}{m} = \frac{1\ \text{m}^3}{4\ \text{kg}} = 0.25\frac{\text{m}^3}{\text{kg}} \longleftarrow$$

(b) To evaluate W, in kJ, integrate the following

$$\int_0^{1\,\text{h}} \dot{W}dt = \int_0^{1\,\text{h}} (-14\ \text{W})dt = (-14\ \text{W})(1\ \text{h})\left|\frac{3600\ \text{s}}{1\ \text{h}}\right|\left|\frac{1\ \frac{\text{J}}{\text{s}}}{1\ \text{W}}\right|\left|\frac{1\ \text{kJ}}{1000\ \text{J}}\right| = -50.4\ \text{kJ} \longleftarrow$$

The minus sign for W indicates that energy is added to the system by work, as expected.

(c) To evaluate Q, in kJ, use the closed system energy balance

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

$$Q = \Delta U + W = m\Delta u + W = (4\ \text{kg})\left(10\ \frac{\text{kJ}}{\text{kg}}\right) + (-50.4\ \text{kJ}) = -10.4\ \text{kJ} \longleftarrow$$

Energy is removed from the system through heat transfer. $\longleftarrow$

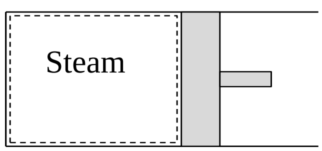
Problem 2.52

Steam in a piston-cylinder assembly undergoes a polytropic process. Data for the initial and final states are given in the accompanying table. Kinetic and potential energy effects are negligible. For the process, determine the work and heat transfer, each in Btu per lb of steam.

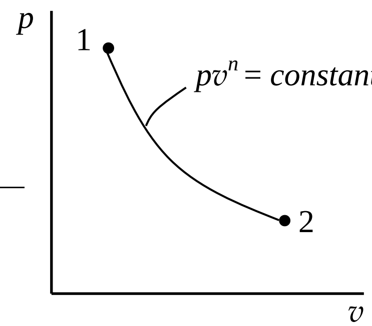
State	p (lbf/in.^{2})	v (ft^3/lb)	u (Btu/lb)
1	100	4.934	1136.2
2	40	11.04	1124.2

KNOWN: Steam undergoes a polytropic process in a piston-cylinder assembly. Data are known at the initial and final states.

FIND: Determine the work and heat transfer, each per unit mass of steam.



State	p (lbf/in.^{2})	v (ft^3/lb)	u (Btu/lb)
1	100	4.934	1136.2
2	40	11.04	1124.2



ENGINEERING MODEL: (1) The steam is a closed system. (2) The process is polytropic, and volume change is the only work mode. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: Since the process is polytropic, Eq 2.17 applies for the work:

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{const}{v^n}\,dv = \frac{(p_2 v_2 - p_1 v_1)}{1-n}$$

The pressures and specific volumes are known at each state, but n is unknown. To find n, $pv^n = constant$, as follows:

$$p_1 v_1^n = p_2 v_2^n \quad \rightarrow \quad \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n \quad \rightarrow \quad n = \frac{\ln(p_1/p_2)}{\ln(v_2/v_1)} = \frac{\ln(100/40)}{\ln(11.04/4.934)} = 1.1377$$

Thus

$$W/m = \frac{(40\text{ lbf/in.}^2)(11.04\text{ ft}^3/\text{lb}) - (100)(4.934)}{1 - 1.1377}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 69.63\text{ Btu/lb (out)} \longleftarrow$$

The heat transfer is obtained using the energy balance.

Problem 2.52 (Continued)

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \quad \rightarrow \quad Q = \Delta U + W$$

With $\Delta U = m\, \Delta u = m(u_2 - u_1)$

$$Q/m = (u_2 - u_1) + (W/m) = (1124.2 \ - 1136.2) \text{ Btu/lb} + (69.63 \text{ Btu/lb})$$

$$= \ 57.63 \text{ Btu/lb} \ \ \text{(in)}$$

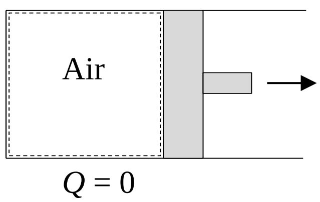
Problem 2.53

Air expands adiabatically in a piston-cylinder assembly from an initial state where p_1 = 100 lbf/in.^{2}, v_1 = 3.704 ft^3/lb, and T_1 = 1000 ^{o}R, to a final state where p_2 = 50 lbf/in.^{2} The process is polytropic with n = 1.4. The change in specific internal energy, in Btu/lb, can be expressed in terms of temperature change as $\Delta u = (0.171)(T_2 - T_1)$. Determine the final temperature, in ^{o}R. Kinetic and potential energy effects can be neglected.

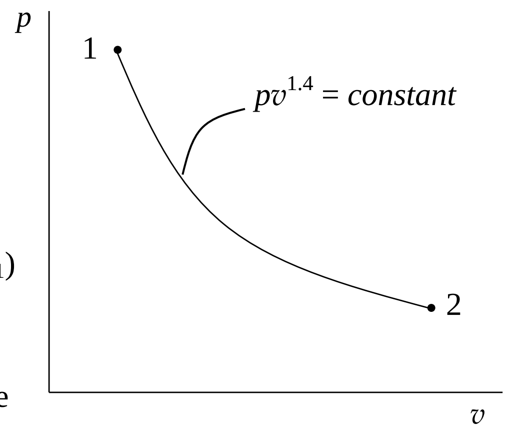
KNOWN: Air undergoes a polytropic process with known n in a piston-cylinder assembly. Data are known at the initial and final states, and the change in specific internal energy is expressed as a function of temperature change.

FIND: Determine the final temperature.

SCHEMATIC AND GIVEN DATA:



p_1 = 100 lbf/in.^{2}
v_1 = 3.704 ft^3/lb
T_1 = 1000 ^{o}R
p_2 = 50 lbf/in.^{2}
Δu = (0.171Btu/lb·^{o}R)($T_2 - T_1$)



ENGINEERING MODEL: (1) The air is a closed system.
(2) The process is polytropic with n = 1.4 and volume change is the only work mode. (3) The process is adiabatic: Q = 0.
(4) Kinetic and potential energy effects are negligible.

AN[illegible]emperature, we will use the energy balance with the given expression for change in specific internal energy as a function of temperature change. First, determine the work using Eq. 2.17

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{const}{v^{1.4}}\,dv = \frac{(p_2 v_2 - p_1 v_1)}{1-1.4}$$

For the polytropic proess, $p_1 v_1^{1.4} = p_2 v_2^{1.4}$. Thus

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.4}} v_1 = \left(\frac{100\text{ lbf/in.}^2}{50\text{ lbf/in.}^2}\right)^{\frac{1}{1.4}} (3.704\text{ ft}^3/\text{lb}) = 6.077\text{ ft}^3/\text{lb}$$

So, the work is

$$W/m = \frac{(50\text{ lbf/in.}^2)(6.077\text{ ft}^3/\text{lb}) - (100\ )(3.704)}{1-1.4}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right| = 30.794\text{ Btu/lb}$$

The energy balance is: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$. With $\Delta U = m(u_2 - u_1)$

Problem 2.53 (Continued)

$$(u_2 - u_1) = -W/m$$

Inserting values

$$(0.171\ \text{Btu/lb}\cdot{}^{\circ}\text{R})(T_2 - 1000\ {}^{\circ}\text{R}) = -(30.794\ \text{Btu/lb})$$

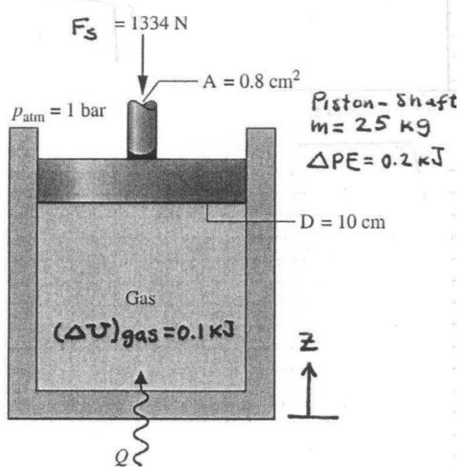
Solving; $T_2 = (-30.794\ \text{Btu/lb})/(0.171\ \text{Btu/lb}\cdot{}^{\circ}\text{R}) + 1000 = 819.9\ {}^{\circ}\text{R}$

Problem 2.54

KNOWN: A gas contained in a piston-cylinder assembly is slowly heated. State data and operating data are provided.

FIND: Determine the work done by the shaft mounted on the top of the piston and work done in displacing the atmosphere, each in kJ. Also, determine the heat tranfer to the gas, in kJ, and develop an accounting of the heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the gas plus the piston and attached shaft.
2. There is no overall change in kinetic energy. For the piston-shaft, $\Delta U = 0$. For the gas, $\Delta PE = 0$.
3. $g = 9.81\ m/s^2$

ANALYSIS: The work can be evaluated using $F\Delta z$, where Δz is the change in elevation of the piston-shaft found as follows:

$$\Delta PE = mg\Delta z$$

$$\Rightarrow \Delta z = \frac{\Delta PE}{mg} = \frac{0.2\ kJ}{(25\ kg)(9.81\ m/s^2)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|$$

$$= 0.82\ m$$

Thus, the work done by the shaft is

$$W_s = F_s\Delta z = (1334\ N)(0.82\ m)\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 1.094\ kJ$$

The work done in displacing the atmosphere is $W_{atm} = (P_{atm} A_{net})\Delta z$, where A_{net} is the net area: Area of piston face less area of the shaft. That is,

$$A_{net} = \left[\frac{\pi D^2}{4} - A\right] = \left[\frac{\pi(10\ cm)^2}{4} - 0.8\ cm^2\right] = 77.74\ cm^2. \quad \text{Thus}$$

$$W_{atm} = \left(10^5\frac{N}{m^2}\right)(77.74\ cm^2)\left|\frac{1\ m}{100\ cm}\right|^2\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|(0.82\ m) = 0.637\ kJ$$

An energy balance for the system reads

$$\left\{[\cancel{\Delta KE} + \Delta PE + \cancel{\Delta U}]_{\text{piston-shaft}} + [\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U]_{gas}\right\} = Q - W$$

$$\Rightarrow Q = (\Delta PE)_{\text{piston-shaft}} + (\Delta U)_{gas} + W$$

$$= (0.2\ kJ) + (0.1\ kJ) + [1.094\ kJ + 0.637\ kJ] = 2.031\ kJ$$

ENERGY "balance sheet":

ENERGY IN:

$Q = 2.031\ kJ$

DISPOSITION OF THE ENERGY IN:

ENERGY STORED: $(\Delta U)_{gas}$	0.10 kJ	(4.92%)
ENERGY STORED: $(\Delta PE)_{\text{piston-shaft}}$	0.20 kJ	(9.85%)
ENERGY OUT BY WORK: ✓ SHAFT	1.094 kJ	(53.87%)
✓ ATM	0.637 kJ	(31.36%)
	2.031 kJ	

Problem 2.55

KNOWN: A system undergoes a power cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:

Process	ΔE	Q	W
1-2	-1200	0	
2-3		800	
3-4		-200	-200
4-1	400		400

ANALYSIS:

(a) Process 1-2: $\Delta E = Q_{12} - W_{12} \Rightarrow W_{12} = Q_{12} - \Delta E = 0 - (-1200) = +1200$

Process 3-4: $\Delta E = Q_{34} - W_{34} \Rightarrow \Delta E = (-200) - (-200) = 0$

Process 4-1: $\Delta E = Q_{41} - W_{41} \Rightarrow Q_{41} = \Delta E + W_{41} = 400 + 600 = 1000$

Process 2-3:

Method 1: use $\Sigma(\Delta E) = 0$ to get

$$(-1200) + (E_3 - E_2) + (0) + (400) = 0$$

$$\Rightarrow (E_3 - E_2) = 800$$

Then, $\Delta E = Q_{23} - W_{23} \Rightarrow W_{23} = Q_{23} - \Delta E = 800 - 800 = 0$

Method 2: Use $\Sigma(Q) = \Sigma(W)$ to get

$$\underbrace{0 + 800 + (-200) + (1000)}_{1600} = 1200 + W_{23} + (-200) + (600)$$

$$= 1600 + W_{23} \Rightarrow W_{23} = 0$$

Then $(E_3 - E_2) = Q_{23} - \cancel{W_{23}} = 800$

(b) Thermal efficiency: $\eta = \dfrac{W_{cycle}}{Q_{in}}$

$$W_{cycle} = (+1200) + (0) + (-200) + (600) = 1600$$

$$Q_{in} = (+800) + (+1000) = 1800$$

$$\therefore \quad \eta = \frac{1600}{1800} = 0.889 \ (88.9\%)$$

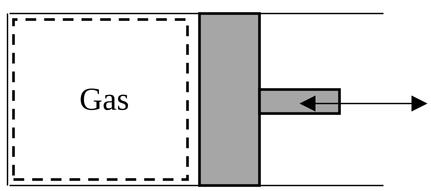
Problem 2.56

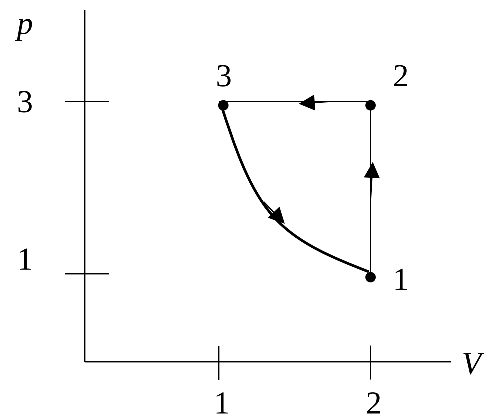
A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes:
Process 1-2: Constant volume $V_1 = 2\ \text{m}^3$, $p_1 = 1$ bar, to $p_2 = 3$ bar, $U_2 - U_1 = 400$ kJ.
Process 2-3: Constant pressure compression to $V_3 = 1\ \text{m}^3$.
Process 3-1: Adiabatic expansion, with $W_{31} = 150$ kJ.
There are no significant changes in kinetic or potential energy. Determine the net work for the cycle, in kJ, and the heat transfers for Processes 1-2 and 2-3, in kJ. Is this a power cycle or refrigeration cycle? Explain.

KNOWN: Data are provided for a gas undergoing a thermodynamic cycle consisting of three processes.

FIND: Determine the net work for the cycle and the heat transfers for processes 1-2 and 2-3. Explain whether it is a power cycle or a refrigeration cycle.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are neglected. (3) Volume change is the only work mode. (4) Process 3-1 is adiabatic, so $Q_{31} = 0$.

ANALYSIS:
Beginning with Process 1-2, the volume is constant, so $W_{12} = 0$. The energy balance reduces to $\Delta U_{12} = Q_{12} - W_{12}$. Thus

$$Q_{12} = \Delta U_{12} = 400 \text{ kJ (in)}$$

Process 2-3 is at constant pressure. So, with Eq. 2.17

$$W_{23} = \int_{V_2}^{V_3} p\,dV = p_2(V_3 - V_2) = (3 \text{ bar})(1 - 2)\text{m}^3 \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right| = -300 \text{ kJ (in)}$$

The energy balance becomes $\Delta U_{23} = Q_{23} - W_{23}$. So

$$Q_{23} = \Delta U_{23} + W_{23}$$

To get ΔU_{23}, let's consider that for a cycle, $\Delta U_{\text{cycle}} = 0$. So

$$\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0 \quad \rightarrow \quad \Delta U_{23} = -\Delta U_{12} - \Delta U_{31}$$

Problem 2.56 (Continued)

Now, for Process 3-1: $Q_{31} = 0$, so $\Delta U_{31} = - W_{31} = - 150$ kJ
And

$$\Delta U_{23} = - \Delta U_{12} - \Delta U_{31} = -(400 \text{ kJ}) - (-150 \text{ kJ}) = -250 \text{ kJ}$$

Finally

$$Q_{23} = \Delta U_{23} + W_{23} = (-250 \text{ kJ}) + (-300 \text{ kJ}) = -550 \text{ kJ} \longleftarrow$$

The net work for the cycle is

$$W_{cycle} = W_{12} + W_{23} + W_{31} = 0 + (-300 \text{ kJ}) + (150 \text{ kJ}) = -150 \text{ kJ (in)} \longleftarrow$$

From the p-V diagram, we see that the cycle is executed in a counter clockwise fashion. And, the net work is in. So, the cycle is a **refrigeration** cycle. ←

Problem 2.57

A gas undergoes a cycle in a piston-cylinder assembly consisting of the following three processes:

Process 1-2: Constant pressure, p = 1.4 bar, V_1 = 0.028 m^3, W_{12} = 10.5 kJ

Process 2-3: Compression with pV = *constant*, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3$ = -26.4 kJ

There are no significant changes in kinetic or potential energy.

(a) Sketch the cycle on a p-V diagram.
(b) Calculate the net work for the cycle, in kJ.
(c) Calculate the heat transfer for process 1-2, in kJ

KNOWN: A gas undergoes a cycle consisting of three processes.

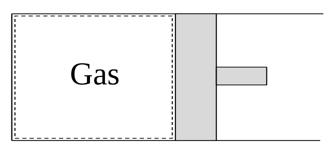
FIND: Sketch the cycle on a p-V diagram and determine the net work for the cycle and the heat transfer for process 1-2.

SCHEMATIC AND GIVEN DATA:

Process 1-2: Constant pressure, p = 1.4 bar, V_1 = 0.028 m^3, W_{12} = 10.5 kJ

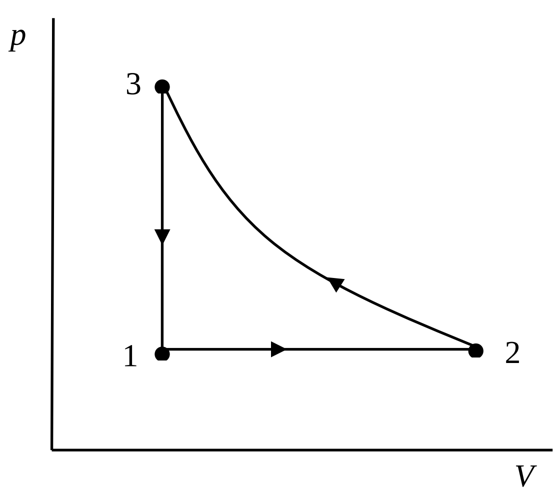
Process 2-3: Compression with pV = *constant*, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3$ = -26.4 kJ



ENGINEERING MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The compression from state 2 to 3 is a polytropic process.

ANALYSIS: (a) Since $W_{12} > 0$, the process is an expansion. Thus



Problem 2.57 (Continued)

(b) The net work for the cycle is $W_{cycle} = W_{12} + W_{23} + \cancelto{0}{W_{31}}$. $W_{12} = 10.5$ kJ, so we need W_{23}.

$$W_{23} = \int_{V_2}^{V_3} p\,dV = \int_{V_2}^{V_3} \frac{const}{V}\,dV = (p_2V_2)\ln\left(\frac{V_3}{V_2}\right) = (p_2V_2)\ln\left(\frac{V_1}{V_2}\right) \qquad (*)$$

where $V_3 = V_1$ has been incorporated. But, we still need to evaluate V_2. For Process 1-2 at constant pressure

$$W_{12} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

or

$$V_2 = \frac{W_{12}}{p} + V_1 = \frac{(10.5\text{ kJ})}{(1.4\text{ bar})}\left|\frac{10^3\text{N·m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right| + 0.028\text{ m}^3 = 0.103\text{ m}^3$$

Thus, with Eq. (*)

$$W_{23} = (1.4\text{ bar})(0.103\text{ m}^3)\ln\left(\frac{0.028}{0.103}\right)\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right| = -18.78\text{ kJ}$$

Thus

$$W_{cycle} = 10.5\text{ kJ} + (-18.78\text{ kJ}) + 0 = -8.28\text{ kJ} \longleftarrow$$

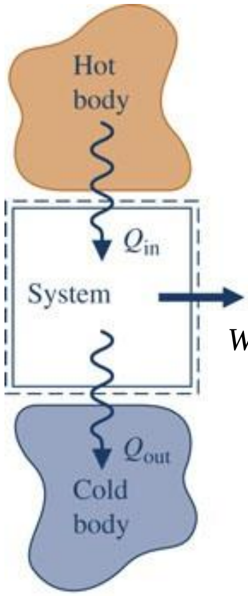
(c) To get Q_{12}, we apply the energy balance to process 1-2: $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + (U_2 - U_1) = Q_{12} - W_{12}$

With $U_2 = U_3$,

$$Q_{12} = (U_3 - U_1) + W_{12} = (+26.4\text{ kJ}) + (10.5\text{ kJ}) = 36.9\text{ kJ} \longleftarrow$$

Problem 2.58

The net work of a power cycle operating as in Fig. 2.17a is 10,000 kJ, and the thermal efficiency is 0.4. Determine the heat transfers Q_{in} and Q_{out}, each in kJ.



W_{cycle} = 10,000 kJ

η = 0.4

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} \quad \rightarrow \quad Q_{in} = \frac{W_{\text{cycle}}}{\eta}$$

Q_{in} = (10,000 kJ) / (0.4) = 25,000 kJ

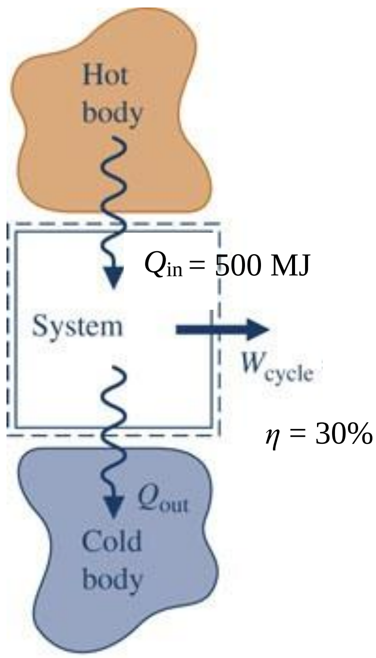
$W_{cycle} = Q_{cycle} = Q_{in} - Q_{out}$

Thus

$Q_{out} = Q_{in} - W_{cycle}$ = 25,000 – 10,000 = 15,000 kJ

Problem 2.59

For a power cycle operating as shown in Fig. 2.17a, the energy transfer by heat into the cycle, Q_{in}, is 500 MJ. What is the net work developed, in MJ, if the cycle thermal efficiency is 30%? What is the value of Q_{out}, in MJ?



$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}}$$

$W_{\text{cycle}} = \eta Q_{\text{in}} = (0.3)(500 \text{ MJ}) = 150 \text{ MJ}$ ⟵

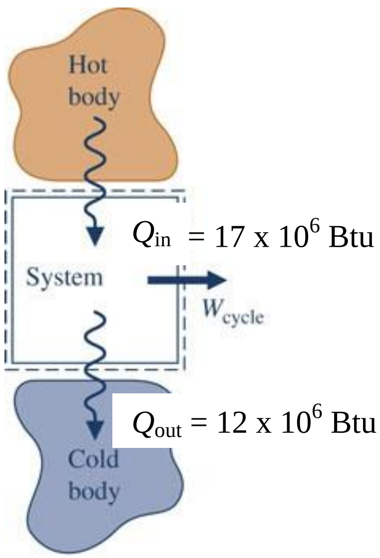
$W_{\text{cycle}} = Q_{\text{cycle}} = Q_{\text{in}} - Q_{\text{out}}$

Thus

$Q_{\text{out}} = Q_{\text{in}} - w_{\text{cycle}} = 500 \text{ MJ} - 150 \text{ MJ} = 350 \text{ MJ}$ ⟵

Problem 2.60

For a power cycle operating as in fig. 2.17a, Q_{in} = 17 x 10^6 Btu and Q_{out} = 12 x 10^6 Btu. Determine W_{cycle}, in Btu, and η.



$W_{cycle} = Q_{cycle} = Q_{in} - Q_{out}$

$= (17 \text{ x } 10^6) - (12 \text{ x } 10^6) = 5 \text{ x } 10^6$ Btu

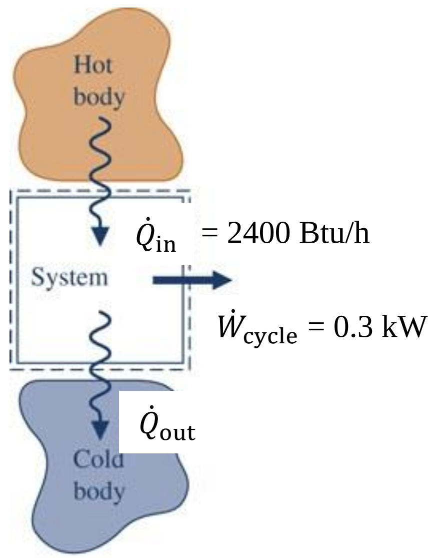
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{5 \text{ x } 10^6 \text{Btu}}{17 \text{ x } 10^6 \text{Btu}} = 0.294\ (29.4\%)$$

Alternatively

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{12 \text{ x } 10^6}{17 \text{ x } 10^6} = 0.294$$

Problem 2.61

A system undergoing a power cycle develops a steady power output of 0.3 kW while receiving energy input by heat transfer of 2400 Btu/h. Determine the thermal efficiency and the total amount of energy developed by work, in kW · h, for one full year of operation.



The thermal efficiency, based on constant rates of energy transfer, is $\eta = \dot{W}_{\text{cycle}}/\dot{Q}_{\text{in}}$

$$\eta = [(0.3\ \text{kW})/(2400\ \text{Btu/h})]\left|\frac{3.413\ \text{Btu/h}}{1\ \text{W}}\right|\left|\frac{10^3\ \text{W}}{1\ \text{kW}}\right|$$
$$= 0.427\ (42.7\ \%) \longleftarrow$$

For one year of steady operation

$$W = \int \dot{W}_{\text{cycle}}\, dt = \dot{W}_{\text{cycle}}\, \Delta t$$

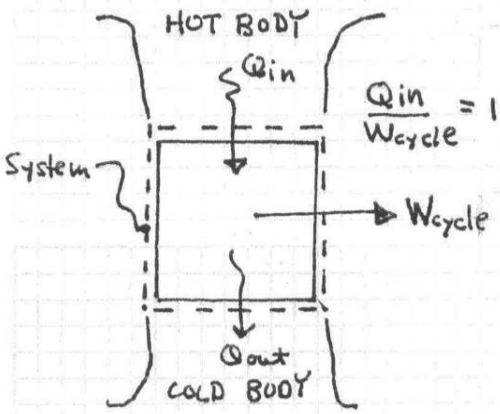
$$= (0.3\ \text{k W})\ (365\ \text{days})\ (24\ \text{h/day}) = 2628\ \text{kW} \cdot \text{h} \longleftarrow$$

Problem 2.62

KNOWN: Operating data are provided for a system undergoing a power cycle.

FIND: Determine the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$$\frac{Q_{in}}{W_{cycle}} = 10^4 \frac{\text{Btu}}{\text{kW}\cdot\text{h}}$$

ENGINEERING MODEL:

1. The system undergoes a power cycle.
2. Energy transfers are positive in the direction of arrows on the schematic.

ANALYSIS: The thermal efficiency is $\eta = \dfrac{W_{cycle}}{Q_{in}}$.

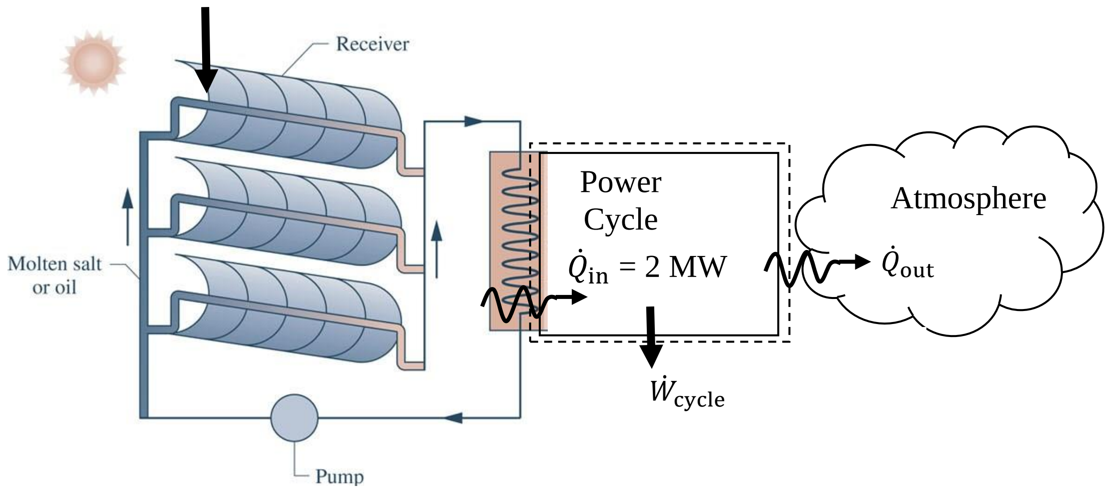
With given data,

(See unit conversions: $1\text{ W} = 3.413\text{ Btu/h}$)

$$\eta = \left[\frac{1}{10^4 \frac{\text{Btu}}{\text{kW}\cdot\text{h}}}\right]\left|\frac{3413\text{ Btu}}{1\text{ kW}\cdot\text{h}}\right| = 0.3413 \quad (34.13\%)$$

Problem 2.63

A concentrating solar collector system, as shown in Fig. P2.63, provides energy by heat transfer to a power cycle at a rate of 2 MW. The cycle thermal efficiency is 36%. Determine the power developed by the cycle, in MW. What is the work output, in MW·h, for 4380 hours of steady-state operation? If the work is valued at \$0.08/kW·h, what is the total dollar value of the work output?



The power developed is

$$\dot{W}_{cycle} = \eta \dot{Q}_{in} = (0.36)\,(2\text{ MW}) = 0.72\text{ MW}$$

For 4380 hours of steady-state operation

$$W_{cycle} = \dot{W}_{cycle}\Delta t = (0.72\text{ MW})(4380\text{ h}) = 3153.6\text{ MW·h} \longleftarrow$$

The total dollar value is

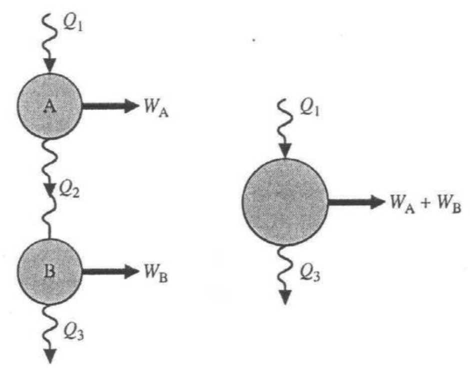
$$\$\text{ Value} = (3153.6\text{ MW·h})(\$0.08/\text{kW·h})\left|\frac{10^3\text{kW}}{1\text{ MW}}\right| = \$252{,}300 \longleftarrow$$

Problem 2.64

KNOWN: Power cycles A and B operate in series.

FIND: Determine an expression for the thermal efficiency of an overall cycle consisting of A and B together in terms of η_A and η_B.

SCHEMATIC & GIVEN DATA:



(a) A and B in series (b) Overall cycle

ENGINEERING MODEL:

1. Cycles A, B and the overall cycle are power cycles.
2. Energy transfer is positive in the directions of the arrows on the schematic.

ANALYSIS:

$$\eta_A = \frac{W_A}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow Q_2 = Q_1(1-\eta_A) \quad (1)$$

$$\eta_B = \frac{W_B}{Q_2} = 1 - \frac{Q_3}{Q_2} \Rightarrow Q_3 = Q_2(1-\eta_B) \quad (2)$$

$$= Q_1(1-\eta_A)(1-\eta_B) \quad (3)$$

$$\eta = \frac{(W_A + W_B)}{Q_1} = 1 - \frac{Q_3}{Q_1} \quad (4)$$

Introducing (3) into (4),

$$\eta = 1 - \frac{Q_1(1-\eta_A)(1-\eta_B)}{Q_1}$$

$$= 1 - (1-\eta_A)(1-\eta_B)$$

$$= 1 - (1 - \eta_A - \eta_B + \eta_A\eta_B)$$

① $\therefore \ \eta = \eta_A + \eta_B - \eta_A\eta_B$

---

1. Sample calculation: $\eta_A = 0.25$, $\eta_B = 0.32$

$$\eta = 0.25 + 0.32 - (0.25)(0.32)$$

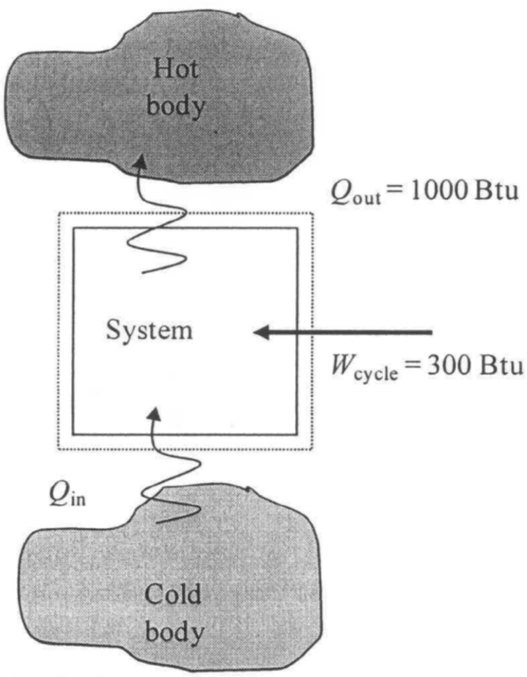
$$= 0.49 \ (49\%)$$

Problem 2.65

A refrigeration cycle operating as shown in Fig. 2.17*b* has $Q_{out} = 1000$ Btu and $W_{cycle} =$ 300 Btu. Determine the coefficient of performance for the cycle.

Solution:

Schematic and Given Data:



Analysis:

Using the following, determine β

$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$W_{cycle} = Q_{out} - Q_{in}$$

$$Q_{in} = Q_{out} - W_{cycle} = (1000 - 300)\,\text{Btu} = 700\,\text{Btu}$$

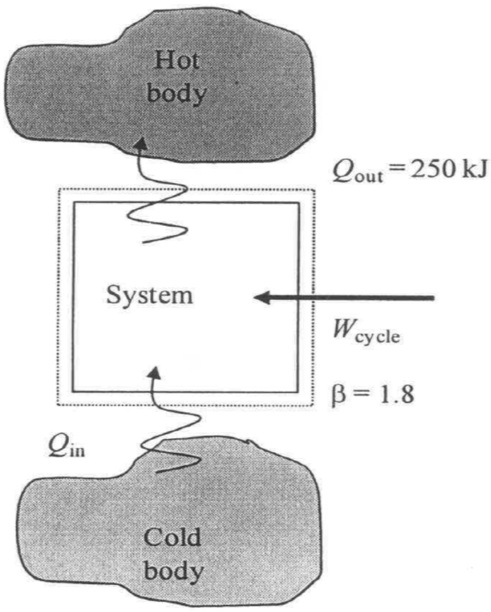
$$\beta = \frac{700\,\text{Btu}}{300\,\text{Btu}} = 2.33$$

Problem 2.66

A refrigeration cycle operating as shown in Fig. 2.17*b* has a coefficient of performance $\beta = 1.8$. For the cycle, $Q_{out} = 250$ kJ. Determine Q_{in} and W_{cycle}, each in kJ.

Solution:

Schematic and Given Data:



Analysis:

Using the following, determine Q_{out} and W_{cycle}, each in kJ

$$\beta = \frac{Q_{in}}{W_{cycle}} \text{ and } W_{cycle} = Q_{out} - Q_{in}$$

$$\beta = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

$$Q_{in} = \beta(Q_{out} - Q_{in}) = Q_{out}\left(\frac{\beta}{1+\beta}\right) = 250\text{ kJ}\left(\frac{1.8}{1+1.8}\right) = 161\text{ kJ} \longleftarrow$$

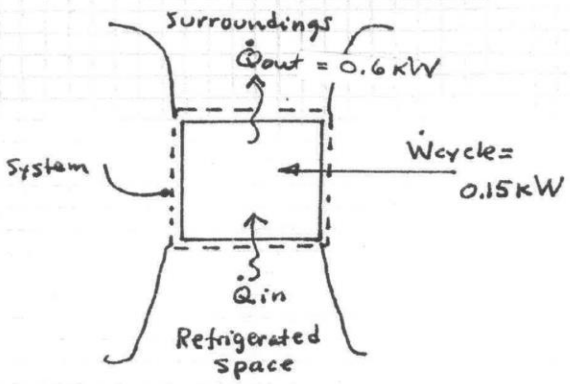
$$W_{cycle} = \frac{Q_{in}}{\beta} = \frac{161\text{ kJ}}{1.8} = 89\text{ kJ} \longleftarrow$$

Problem 2.67

KNOWN: Steady-state operating data are provided for refrigerator.

FIND: Determine the rate energy is removed by heat transfer from the refrigerated space, in kW, and the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system undergoes a refrigeration cycle.
2. Energy transfers are positive in the direction of the arrows on the schematic.
3. The cycle operates steadily.

ANALYSIS:

Applying Eq. 2.44 on a time rate basis:

$$\dot{W}_{cycle} = \dot{Q}_{out} - \dot{Q}_{in}$$

$$\Rightarrow \quad \dot{Q}_{in} = \dot{Q}_{out} - \dot{W}_{cycle}$$

$$= 0.6\text{ kW} - 0.15\text{ kW}$$

$$= 0.45\text{ kW} \quad \longleftarrow \dot{Q}_{in}$$

Then, with Eq. 2.45 on a time rate basis,

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}}$$

$$= \frac{0.45\text{ kW}}{0.15\text{ kW}}$$

$$= 3 \quad \longleftarrow \beta$$

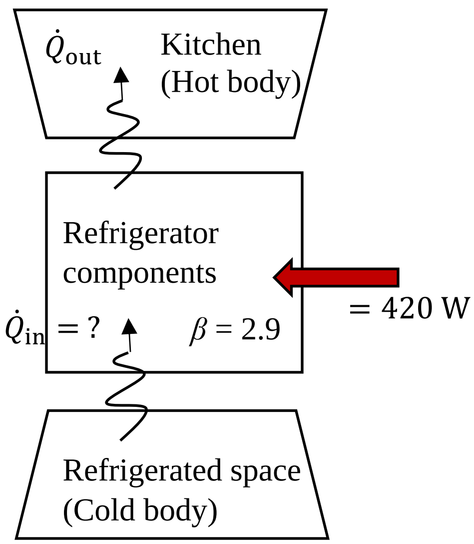
Problem 2.68

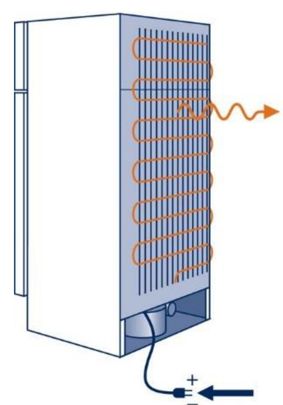
For a refrigerator with automatic defrost and a top-mounted freezer, the electric power required is approximately 420 watts to operate. If the coefficient of performance is 2.9, determine the rate that energy is removed from its refrigerated space, in watts. Evaluating electricity at \$0.10/kW · h, and assuming the units runs 60% of the time, estimate the cost of one month's operation, in \$.

KNOWN: Data are provided for steady operation of a refrigerator-freezer.

FIND: Determine the rate that energy is removed from the refrigerated space, and estimate the cost of one month's operation.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
(1) The refrigeration components execute a refrigeration cycle.
(2) Data are given for steady operation.

ANALYSIS: The coefficient of performance for steady operation is $\beta = \dfrac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}}$. Thus

$$\dot{Q}_{\text{in}} = \beta\,\dot{W}_{\text{cycle}} = (2.9)(420\text{ W}) = 1218\text{ W}$$

The total amount of electric energy input for one month's operation is determined for steady operation as

$$W_{\text{elec}} = \dot{W}_{\text{cycle}}\Delta t$$

Where

$$\Delta t = (0.6)(30\text{ days/month})(24\text{ h/day}) = 432\text{ h/month}$$

And

$$W_{\text{elec}} = (420\text{ W})(432\text{ h/month})\left|\frac{1\text{ kW}}{10^3\text{W}}\right| = 181.4\text{ kW}\cdot\text{h/month}$$

$$\text{Cost} = (\$0.1/\text{ kW}\cdot\text{h})\ (181.4\text{ kW}\cdot\text{h/month}) = \$18.14\text{ / month}$$

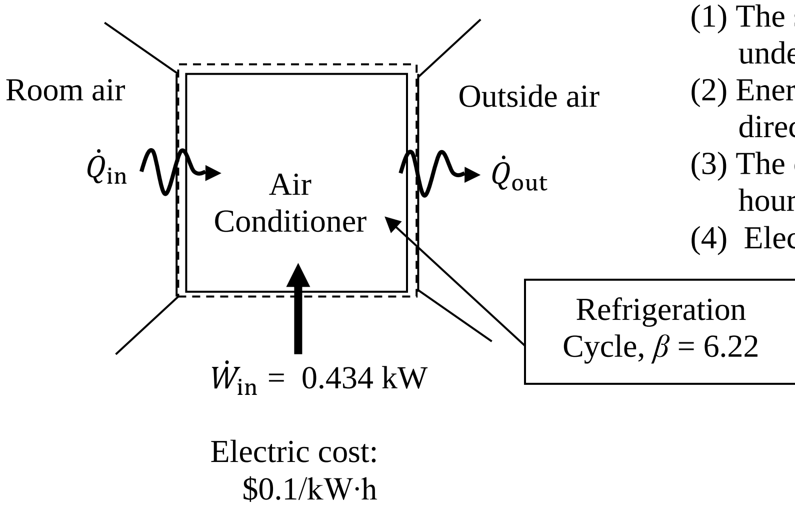
Problem 2.69

A window-mounted room air conditioner removes energy by heat transfer from a room and rejects energy by heat transfer to the outside air. For steady operation, the air conditioner cycle requires a power input of 0.434 kW and has a coefficient of performance of 6.22. Determine the rate that energy is removed from the room air, in kW. If electricity is valued at \$0.1/kW·h, determine the cost of operation for 24 hours of operation.

KNOWN: Steady-state operating data are provided for an air conditioner.

FIND: Determine the rate energy is removed from the room and air the cost of 24 hours of operation.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The system shown in the schematic undergoes a refrigeration cycle.

(2) Energy transfers are positive in the directions of the arrows.

(3) The cycle operates steadily for 24 hours.

(4) Electricity is valued at \$0.1/kW·h.

ANALYSIS: Using Eq. 2.45 on a time rate basis

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}} \quad \rightarrow \quad \dot{Q}_{\text{in}} = \beta \dot{W}_{\text{cycle}} = (6.22)(0.434 \text{ kW}) = 2.70 \text{ kW} \longleftarrow$$

The total amount of electric energy input by work for 24 h of operation is

$$W_{\text{cycle}} = \dot{W}_{\text{cycle}} \Delta t = (0.434 \text{ kW})(24 \text{ h}) = 10.42 \text{ kW·h}$$

Thus, the total cost is

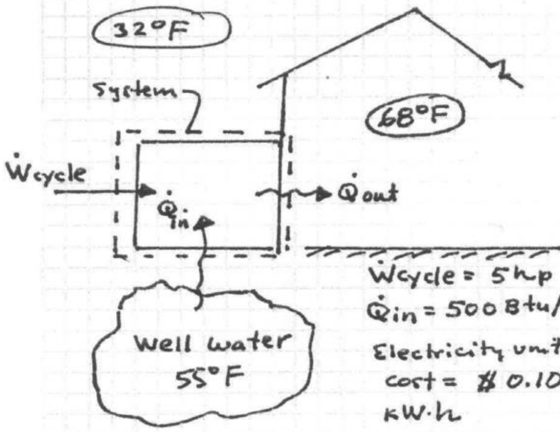
$$\text{Total cost} = (10.42 \text{ kW·h})(\$0.1/\text{kW·h}) = \$1.04 \text{ (for 24 hours)} \longleftarrow$$

Problem 2.70

KNOWN: Operating and cost data are provided for a heat pump.

FIND: Determine the coefficient of performance for the heat pump and the cost of electricity in a month when the heat pump operates for 300 hours.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 5$ hp
$\dot{Q}_{in} = 500$ Btu/min
Electricity unit cost = \$ 0.10 per kW·h

ENGINEERING MODEL:

1. The system shown in the schematic operates in heat pump cycle.
2. Energy transfers are positive in the direction of the arrows.
3. The heat pump operates steadily.
4. Electricity is valued at \$0.10 per kW·h.

ANALYSIS:

(a) Using Eq. 2.47 on a time rate basis,

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \qquad (1)$$

together with the cycle energy balance, Eq. 2.44, on a time rate basis: $\dot{W}_{cycle} = \dot{Q}_{out} - \dot{Q}_{in}$, we get:

$$\dot{Q}_{out} = \dot{W}_{cycle} + \dot{Q}_{in} = \underbrace{(5\,\text{hp})\left|\frac{2545\ \text{Btu/h}}{1\,\text{hp}}\right|\left|\frac{1\,\text{h}}{60\,\text{min}}\right|}_{212.1\ \text{Btu/min}} + 500\ \frac{\text{Btu}}{\text{min}}$$

$$\therefore\ \dot{Q}_{out} = 712.1\ \text{Btu/min}$$

Eq. (1) gives,

$$\gamma = \frac{712.1\ \text{Btu/min}}{212.1\ \text{Btu/min}} = 3.36$$

Unit conversions: 1 W = 3.413 Btu/h

(b)

$$\$ = \left(212.1\ \frac{\text{Btu}}{\text{min}}\right)\left[\frac{300\,\text{h}}{\text{month}}\right]\left[\frac{\$\ 0.10}{\text{kW}\cdot\text{h}}\right]\left|\frac{60\,\text{min}}{1\,\text{h}}\right|\left|\frac{1\,\text{kW}}{3413\ \text{Btu/h}}\right|$$

$$= \$111.86/\text{month}$$

Problem 2.71

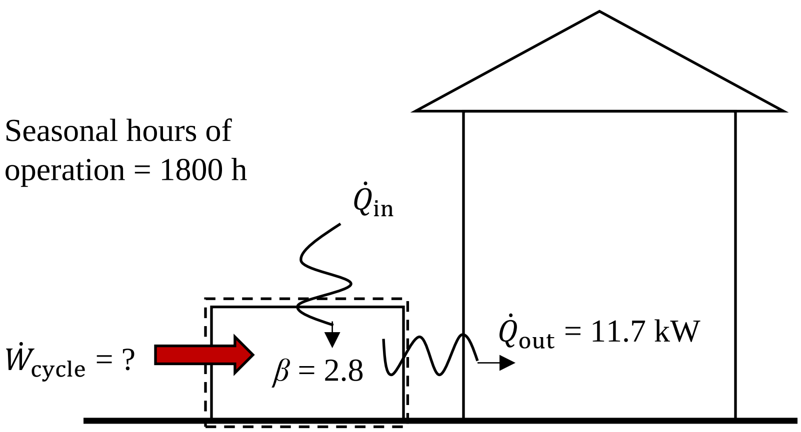
A heat pump delivers energy by heat transfer to a dwelling at a rate of 11.7 kW. The coefficient of performance is 2.8.

(a) Determine the power input to the cycle, in kW.
(b) Evaluating electricity at \$0.1/kW · h, determine the cost of electricity during the heating season when the heat pump operates for 1800 hours.

KNOWN: Operating data are provided for a residential heat pump.

FIND: Determine the power input to the cycle, and the seasonal operating cost.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The closed system undergoes a heat pump cycle. (2) The cycle operates steadily for 1800 h during the heating season. (3) Electricity is valued at \$0.1/kW · h.

ANALYSIS:
(a) The coefficient of performance for steady operation of the heat pump cycle is: $\gamma = \dot{Q}_{\text{out}}/\dot{W}_{\text{cycle}}$. Thus

$$\dot{W}_{\text{cycle}} = \frac{\dot{Q}_{\text{out}}}{\gamma} = \frac{11.7 \text{ kW}}{2.8} = 4.179 \text{ kW} \longleftarrow$$

(b) Based upon modeling assumptions, the cost to operate the heat pump is estimated to be

Cost = (4.179 kW) (1800 h/season) (\$0.1/kW · h)

= \$752.22/season ⟵