2 FUNCTIONS AND THEIR GRAPHS

2.1 The Cartesian Coordinate System and Straight Lines

Concept Questions

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1. a. $a < 0$ and $b > 0$. **b.** $a < 0$ and $b < 0$. **c.** $a > 0$ and $b < 0$.

2. The slope of a nonvertical line is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $P(x_1, y_1)$ and $P(x_2, y_2)$ are any two distinct points on the line. The slope of a vertical line is undefined.

Exercises

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1. The coordinates of A are $(3, 3)$ and it is located in Quadrant I.

2. The coordinates of B are $(-5, 2)$ and it is located in Quadrant II.

3. The coordinates of C are $(2, -2)$ and it is located in Quadrant IV.

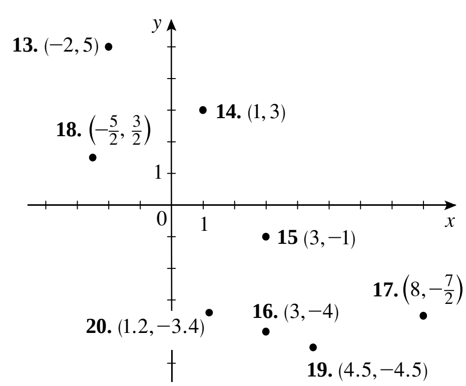
4. The coordinates of D are $(-2, 5)$ and it is located in Quadrant II.

5. The coordinates of E are $(-4, -6)$ and it is located in Quadrant III.

6. The coordinates of F are $(8, -2)$ and it is located in Quadrant IV.

7. A **8.** $(-5, 4)$ **9.** E, F, and G **10.** E **11.** F **12.** D

For Exercises 13–20, refer to the following figure.



21. Referring to the figure shown in the text, we see that $m = \dfrac{2 - 0}{0 - (-4)} = \dfrac{1}{2}$.

22. Referring to the figure shown in the text, we see that $m = \dfrac{4 - 0}{0 - 2} = -2$.

23. This is a vertical line, and hence its slope is undefined.

24. This is a horizontal line, and hence its slope is 0.

25. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{5 - 4} = 5.$

26. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 5}{3 - 4} = \dfrac{3}{-1} = -3.$

27. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{4 - (-2)} = \dfrac{5}{6}.$

28. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{4 - (-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}.$

29. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{d - b}{c - a}$, provided $a \neq c$.

30. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-b - (b - 1)}{a + 1 - (-a + 1)} = -\dfrac{-b - b + 1}{a + 1 + a - 1} = \dfrac{1 - 2b}{2a}.$

31. Because the equation is already in slope-intercept form, we read off the slope $m = 4$.

a. If x increases by 1 unit, then y increases by 4 units.

b. If x decreases by 2 units, then y decreases by $4(-2) = -8$ units.

32. Rewrite the given equation in slope-intercept form: $2x + 3y = 4$, $3y = 4 - 2x$, and so $y = -\frac{2}{3}x + \frac{4}{3}$.

a. Because $m = -\frac{2}{3}$, we conclude that the slope is negative.

b. Because the slope is negative, y decreases as x increases.

c. If x decreases by 2 units, then y increases by $\left(-\frac{2}{3}\right)(-2) = \frac{4}{3}$ units.

33. The slope of the line through A and B is $\dfrac{-10 - (-2)}{-3 - 1} = \dfrac{-8}{-4} = 2$. The slope of the line through C and D is $\dfrac{1 - 5}{-1 - 1} = \dfrac{-4}{-2} = 2$. Because the slopes of these two lines are equal, the lines are parallel.

34. The slope of the line through A and B is $\dfrac{-2 - 3}{2 - 2}$. Because this slope is undefined, we see that the line is vertical. The slope of the line through C and D is $\dfrac{5 - 4}{-2 - (-2)}$. Because this slope is undefined, we see that this line is also vertical. Therefore, the lines are parallel.

35. The slope of the line through the point $(1, a)$ and $(4, -2)$ is $m_1 = \dfrac{-2 - a}{4 - 1}$ and the slope of the line through $(2, 8)$ and $(-7, a + 4)$ is $m_2 = \dfrac{a + 4 - 8}{-7 - 2}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{-2 - a}{3} = \dfrac{a - 4}{-9}$, $-9(-2 - a) = 3(a - 4)$, $18 + 9a = 3a - 12$, and $6a = -30$, so $a = -5$.

36. The slope of the line through the point $(a, 1)$ and $(5, 8)$ is $m_1 = \dfrac{8 - 1}{5 - a}$ and the slope of the line through $(4, 9)$ and $(a + 2, 1)$ is $m_2 = \dfrac{1 - 9}{a + 2 - 4}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{7}{5 - a} = \dfrac{-8}{a - 2}$, $7(a - 2) = -8(5 - a)$, $7a - 14 = -40 + 8a$, and $a = 26$.

37. Yes. A straight line with slope zero ($m = 0$) is a horizontal line, whereas a straight line whose slope does not exist (m cannot be computed) is a vertical line.

2.2 Equations of Lines

Concept Questions

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1. a. $y - y_1 = m(x - x_1)$ **b.** $y = mx + b$

c. $ax + by + c = 0$, where a and b are not both zero.

2. a. $m_1 = m_2$ **b.** $m_2 = -\dfrac{1}{m_1}$

Exercises

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1. (e) **2.** (c) **3.** (a) **4.** (d) **5.** (f) **6.** (b)

7. The slope of the line through A and B is $\dfrac{2-5}{4-(-2)} = -\dfrac{3}{6} = -\dfrac{1}{2}$. The slope of the line through C and D is $\dfrac{6-(-2)}{3-(-1)} = \dfrac{8}{4} = 2$. Because the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

8. The slope of the line through A and B is $\dfrac{-2-0}{1-2} = \dfrac{-2}{-1} = 2$. The slope of the line through C and D is $\dfrac{4-2}{-8-4} = \dfrac{2}{-12} = -\dfrac{1}{6}$. Because the slopes of these two lines are not the negative reciprocals of each other, the lines are not perpendicular.

9. An equation of a horizontal line is of the form $y = b$. In this case $b = -5$, so $y = -5$ is an equation of the line.

10. An equation of a vertical line is of the form $x = a$. In this case $a = 0$, so $x = 0$ is an equation of the line.

11. We use the point-slope form of an equation of a line with the point $(3, -4)$ and slope $m = 2$. Thus $y - y_1 = m(x - x_1)$ becomes $y - (-4) = 2(x - 3)$. Simplifying, we have $y + 4 = 2x - 6$, or $y = 2x - 10$.

12. We use the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = -1$. Thus $y - y_1 = m(x - x_1)$, giving $y - 4 = -1(x - 2)$, $y - 4 = -x + 2$, and finally $y = -x + 6$.

13. Because the slope $m = 0$, we know that the line is a horizontal line of the form $y = b$. Because the line passes through $(-3, 2)$, we see that $b = 2$, and an equation of the line is $y = 2$.

14. We use the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = -\frac{1}{2}$. Thus $y - y_1 = m(x - x_1)$ gives $y - 2 = -\frac{1}{2}(x - 1)$, $2y - 4 = -x + 1$, $2y = -x + 5$, and $y = -\frac{1}{2}x + \frac{5}{2}$.

15. We first compute the slope of the line joining the points $(2, 4)$ and $(3, 7)$, obtaining $m = \dfrac{7-4}{3-2} = 3$. Using the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = 3$, we find $y - 4 = 3(x - 2)$, or $y = 3x - 2$.

16. We first compute the slope of the line joining the points $(2, 1)$ and $(2, 5)$, obtaining $m = \dfrac{5-1}{2-2}$. Because this slope is undefined, we see that the line must be a vertical line of the form $x = a$. Because it passes through $(2, 5)$, we see that $x = 2$ is the equation of the line.

17. We first compute the slope of the line joining the points $(1, 2)$ and $(-3, -2)$, obtaining $m = \dfrac{-2-2}{-3-1} = \dfrac{-4}{-4} = 1$. Using the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = 1$, we find $y - 2 = x - 1$, or $y = x + 1$.

18. We first compute the slope of the line joining the points $(-1, -2)$ and $(3, -4)$, obtaining $m = \dfrac{-4-(-2)}{3-(-1)} = \dfrac{-2}{4} = -\dfrac{1}{2}$. Using the point-slope form of an equation of a line with the point $(-1, -2)$ and slope $m = -\frac{1}{2}$, we find $y - (-2) = -\frac{1}{2}[x - (-1)]$, $y + 2 = -\frac{1}{2}(x + 1)$, and finally $y = -\frac{1}{2}x - \frac{5}{2}$.

19. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 3$ and $b = 5$, the equation is $y = 3x + 5$.

20. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -2$ and $b = -1$, the equation is $y = -2x - 1$.

21. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 0$ and $b = 5$, the equation is $y = 5$.

22. We use the slope-intercept form of an equation of a line:$y = mx + b$. Because $m = -\frac{1}{2}$, and $b = \frac{3}{4}$, the equation is $y = -\frac{1}{2}x + \frac{3}{4}$.

23. We first write the given equation in the slope-intercept form: $x - 2y = 0$, so $-2y = -x$, or $y = \frac{1}{2}x$. From this equation, we see that $m = \frac{1}{2}$ and $b = 0$.

24. We write the equation in slope-intercept form: $y - 2 = 0$, so $y = 2$. From this equation, we see that $m = 0$ and $b = 2$.

25. We write the equation in slope-intercept form: $2x - 3y - 9 = 0$, $-3y = -2x + 9$, and $y = \frac{2}{3}x - 3$. From this equation, we see that $m = \frac{2}{3}$ and $b = -3$.

26. We write the equation in slope-intercept form: $3x - 4y + 8 = 0$, $-4y = -3x - 8$, and $y = \frac{3}{4}x + 2$. From this equation, we see that $m = \frac{3}{4}$ and $b = 2$.

27. We write the equation in slope-intercept form: $2x + 4y = 14$, $4y = -2x + 14$, and $y = -\frac{2}{4}x + \frac{14}{4} = -\frac{1}{2}x + \frac{7}{2}$. From this equation, we see that $m = -\frac{1}{2}$ and $b = \frac{7}{2}$.

28. We write the equation in the slope-intercept form: $5x + 8y - 24 = 0$, $8y = -5x + 24$, and $y = -\frac{5}{8}x + 3$. From this equation, we conclude that $m = -\frac{5}{8}$ and $b = 3$.

29. We first write the equation $2x - 4y - 8 = 0$ in slope-intercept form: $2x - 4y - 8 = 0$, $4y = 2x - 8$, $y = \frac{1}{2}x - 2$. Now the required line is parallel to this line, and hence has the same slope. Using the point-slope form of an equation of a line with $m = \frac{1}{2}$ and the point $(-2, 2)$, we have $y - 2 = \frac{1}{2}[x - (-2)]$ or $y = \frac{1}{2}x + 3$.

30. The slope of the line passing through $(-2, -3)$ and $(2, 5)$ is $m = \dfrac{5-(-3)}{2-(-2)} = \dfrac{8}{4} = 2$. Thus, the required equation is $y - 3 = 2[x - (-1)]$, $y = 2x + 2 + 3$, or $y = 2x + 5$.

31. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form: $3x + 4y - 22 = 0$, so $4y = -3x + 22$ and $y = -\frac{3}{4}x + \frac{11}{2}$ Now the required line is perpendicular to this line, and hence has slope $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$). Using the point-slope form of an equation of a line with $m = \frac{4}{3}$ and the point $(2, 4)$, we have $y - 4 = \frac{4}{3}(x - 2)$, or $y = \frac{4}{3}x + \frac{4}{3}$.

32. The slope of the line passing through $(-2, -1)$ and $(4, 3)$ is given by $m = \dfrac{3 - (-1)}{4 - (-2)} = \dfrac{3 + 1}{4 + 2} = \dfrac{4}{6} = \dfrac{2}{3}$, so the slope of the required line is $m = -\frac{3}{2}$ and its equation is $y - (-2) = -\frac{3}{2}(x - 1)$, $y = -\frac{3}{2}x + \frac{3}{2} - 2$, or $y = -\frac{3}{2}x - \frac{1}{2}$.

33. A line parallel to the x-axis has slope 0 and is of the form $y = b$. Because the line is 6 units below the axis, it passes through $(0, -6)$ and its equation is $y = -6$.

34. Because the required line is parallel to the line joining $(2, 4)$ and $(4, 7)$, it has slope $m = \dfrac{7 - 4}{4 - 2} = \dfrac{3}{2}$. We also know that the required line passes through the origin $(0, 0)$. Using the point-slope form of an equation of a line, we find $y - 0 = \frac{3}{2}(x - 0)$, or $y = \frac{3}{2}x$.

35. We use the point-slope form of an equation of a line to obtain $y - b = 0(x - a)$, or $y = b$.

36. Because the line is parallel to the x-axis, its slope is 0 and its equation has the form $y = b$. We know that the line passes through $(-3, 4)$, so the required equation is $y = 4$.

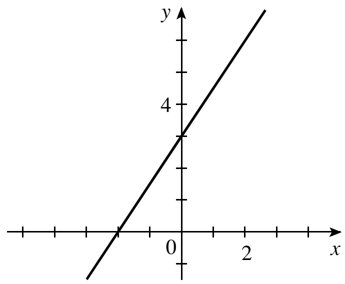
37. Because the required line is parallel to the line joining $(-3, 2)$ and $(6, 8)$, it has slope $m = \dfrac{8 - 2}{6 - (-3)} = \dfrac{6}{9} = \dfrac{2}{3}$. We also know that the required line passes through $(-5, -4)$. Using the point-slope form of an equation of a line, we find $y - (-4) = \frac{2}{3}[x - (-5)]$, $y = \frac{2}{3}x + \frac{10}{3} - 4$, and finally $y = \frac{2}{3}x - \frac{2}{3}$.

38. Because the slope of the line is undefined, it has the form $x = a$. Furthermore, since the line passes through (a, b), the required equation is $x = a$.

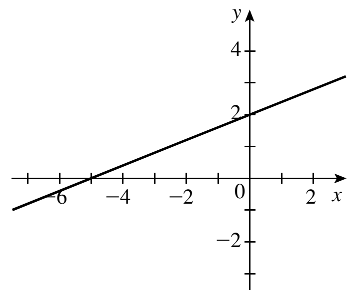
39. Because the point $(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives $-3k + 15 + 9 = 0$, or $k = 8$.

40. Because the point $(2, -3)$ lies on the line $-2x + ky + 10 = 0$, it satisfies the equation. Substituting $x = 2$ and $y = -3$ into the equation gives $-2(2) + (-3)k + 10 = 0$, $-4 - 3k + 10 = 0$, $-3k = -6$, and finally $k = 2$.

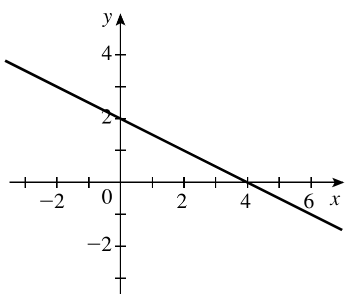
41. $3x - 2y + 6 = 0$. Setting $y = 0$, we have $3x + 6 = 0$ or $x = -2$, so the x-intercept is -2. Setting $x = 0$, we have $-2y + 6 = 0$ or $y = 3$, so the y-intercept is 3.



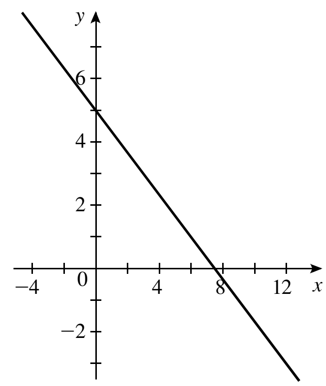
42. $2x - 5y + 10 = 0$. Setting $y = 0$, we have $2x + 10 = 0$ or $x = -5$, so the x-intercept is -5. Setting $x = 0$, we have $-5y + 10 = 0$ or $y = 2$, so the y-intercept is 2.



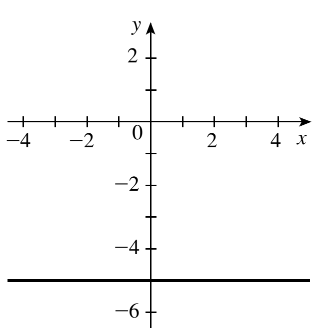
43. $x + 2y - 4 = 0$. Setting $y = 0$, we have $x - 4 = 0$ or $x = 4$, so the x-intercept is 4. Setting $x = 0$, we have $2y - 4 = 0$ or $y = 2$, so the y-intercept is 2.



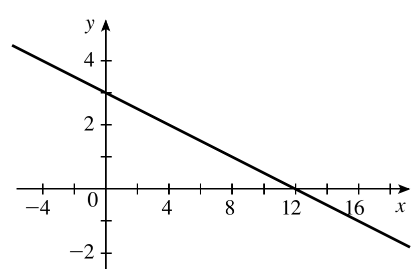
44. $2x + 3y - 15 = 0$. Setting $y = 0$, we have $2x - 15 = 0$, so the x-intercept is $\frac{15}{2}$. Setting $x = 0$, we have $3y - 15 = 0$, so the y-intercept is 5.



45. $y + 5 = 0$. Setting $y = 0$, we have $0 + 5 = 0$, which has no solution, so there is no x-intercept. Setting $x = 0$, we have $y + 5 = 0$ or $y = -5$, so the y-intercept is -5.



46. $-2x - 8y + 24 = 0$. Setting $y = 0$, we have $-2x + 24 = 0$ or $x = 12$, so the x-intercept is 12. Setting $x = 0$, we have $-8y + 24 = 0$ or $y = 3$, so the y-intercept is 3.



47. Because the line passes through the points $(a, 0)$ and $(0, b)$, its slope is $m = \dfrac{b-0}{0-a} = -\dfrac{b}{a}$. Then, using the point-slope form of an equation of a line with the point $(a, 0)$, we have $y - 0 = -\dfrac{b}{a}(x-a)$ or $y = -\dfrac{b}{a}x + b$, which may be written in the form $\dfrac{b}{a}x + y = b$. Multiplying this last equation by $\dfrac{1}{b}$, we have $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

48. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 3$ and $b = 4$, we have $\dfrac{x}{3} + \dfrac{y}{4} = 1$. Then $4x + 3y = 12$, so $3y = 12 - 4x$ and thus $y = -\frac{4}{3}x + 4$.

49. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -2$ and $b = -4$, we have $-\dfrac{x}{2} - \dfrac{y}{4} = 1$. Then $-4x - 2y = 8$, $2y = -8 - 4x$, and finally $y = -2x - 4$.

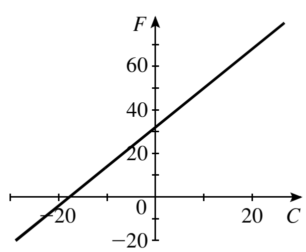
50. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -\frac{1}{2}$ and $b = \frac{3}{4}$, we have $\dfrac{x}{-1/2} + \dfrac{y}{3/4} = 1$, $\frac{3}{4}x - \frac{1}{2}y = \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)$, $-\frac{1}{2}y = -\frac{3}{4}x - \frac{3}{8}$, and finally $y = 2\left(\frac{3}{4}x + \frac{3}{8}\right) = \frac{3}{2}x + \frac{3}{4}$.

51. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 4$ and $b = -\frac{1}{2}$, we have $\dfrac{x}{4} + \dfrac{y}{-1/2} = 1$, $-\frac{1}{4}x + 2y = -1$, $2y = \frac{1}{4}x - 1$, and so $y = \frac{1}{8}x - \frac{1}{2}$.

52. The slope of the line passing through A and B is $m = \dfrac{-2-7}{2-(-1)} = -\dfrac{9}{3} = -3$, and the slope of the line passing through B and C is $m = \dfrac{-9-(-2)}{5-2} = -\dfrac{7}{3}$. Because the slopes are not equal, the points do not lie on the same line.

53. The slope of the line passing through A and B is $m = \dfrac{7-1}{1-(-2)} = \dfrac{6}{3} = 2$, and the slope of the line passing through B and C is $m = \dfrac{13-7}{4-1} = \dfrac{6}{3} = 2$. Because the slopes are equal, the points lie on the same line.

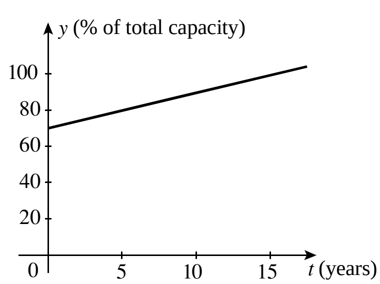
54. a.



b. The slope is $\frac{9}{5}$. It represents the change in °F per unit change in °C.

c. The F-intercept of the line is 32. It corresponds to 0 in °C, so it is the freezing point in °F.

55. a.



b. The slope is 1.9467 and the y-intercept is 70.082.

c. The output is increasing at the rate of 1.9467% per year. The output at the beginning of 1990 was 70.082%.

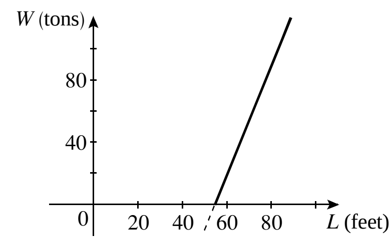
d. We solve the equation $1.9467t + 70.082 = 100$, obtaining $1.9467t = 29.918$ and $t \approx 15.37$. We conclude that the plants were generating at maximum capacity during April 2005.

56. a. $y = 0.0765x$ **b.** \$0.0765 **c.** $0.0765\,(65{,}000) = 4972.50$, or \$4972.50.

57. a. $y = 0.55x$ **b.** Solving the equation $1100 = 0.55x$ for x, we have $x = \dfrac{1100}{0.55} = 2000$.

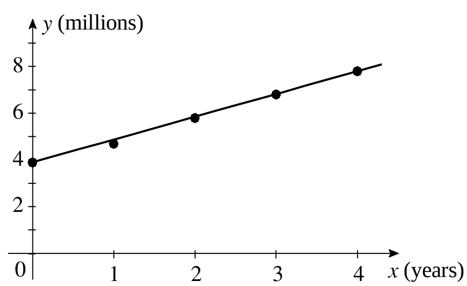
58. a. Substituting $L = 80$ into the given equation, we have $W = 3.51\,(80) - 192 = 280.8 - 192 = 88.8$, or 88.8 British tons.

b.



59. Using the points $(0, 0.68)$ and $(10, 0.80)$, we see that the slope of the required line is $m = \dfrac{0.80 - 0.68}{10 - 0} = \dfrac{0.12}{10} = 0.012$. Next, using the point-slope form of the equation of a line, we have $y - 0.68 = 0.012\,(t - 0)$ or $y = 0.012t + 0.68$. Therefore, when $t = 14$, we have $y = 0.012\,(14) + 0.68 = 0.848$, or 84.8%. That is, in 2004 women's wages were 84.8% of men's wages.

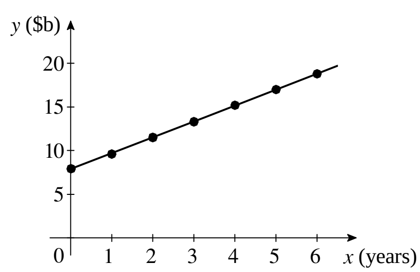
60. a, b.



c. Using $P_1\,(0, 3.9)$ and $P_2\,(4, 7.8)$, we find $m = \dfrac{7.8 - 3.9}{4 - 0} = \dfrac{3.9}{4} = 0.975$. Thus, an equation is $y - 3.9 = 0.975\,(x - 0)$ or $y = 0.975x + 3.9$.

d. If $x = 3$, then $y = 0.975\,(3) + 3.9 = 6.825$. Thus, the number of systems installed in 2005 (when $x = 3$) is 6,825,000 which is close to the projection of 6.8 million.

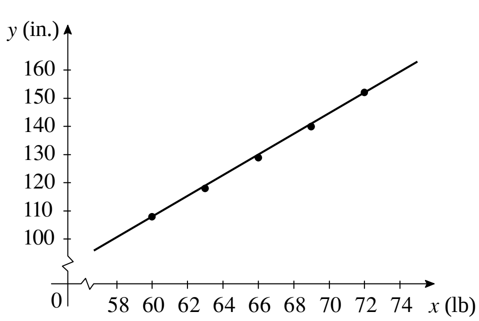
61. a, b.



c. $m = \dfrac{18.8 - 7.9}{6 - 0} \approx 1.82$, so $y - 7.9 = 1.82\,(x - 0)$, or $y = 1.82x + 7.9$.

d. $y = 1.82\,(5) + 7.9 \approx 17$ or \$17 billion. This agrees with the actual data for that year.

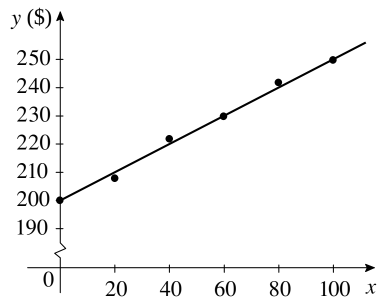
62. a, b.



c. Using the points $(60, 108)$ and $(72, 152)$, we see that the slope of the required line is $m = \dfrac{152 - 108}{72 - 60} = \dfrac{44}{12} = \dfrac{11}{3}$. Therefore, an equation is $y - 108 = \frac{11}{3}\,(x - 60)$, $y = \frac{11}{3}x - \frac{11}{3}\,(60) + 108 = \frac{11}{3}x - 220 + 108$, or $y = \frac{11}{3}x - 112$.

d. Using the equation from part c, we find $y = \frac{11}{3}\,(65) - 112 = 126\frac{1}{3}$, or $126\frac{1}{3}$ pounds.

63. a, b.

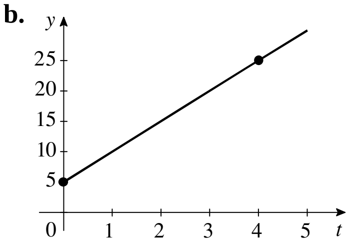


c. Using the points (0, 200) and (100, 250), we see that the slope of the required line is $m = \dfrac{250 - 200}{100} = \dfrac{1}{2}$. Therefore, an equation is $y - 200 = \frac{1}{2}x$ or $y = \frac{1}{2}x + 200$.

d. The approximate cost for producing 54 units of the commodity is $\frac{1}{2}(54) + 200$, or \$227.

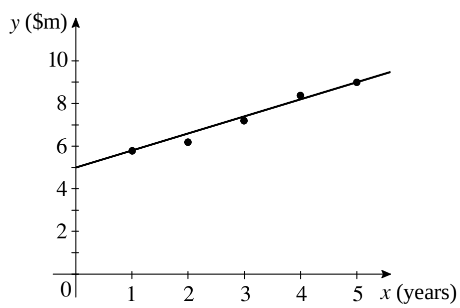
64. a. Using the points (0, 5) and (4, 25), the slope of the required line is $m = \dfrac{25 - 5}{4 - 0} = \dfrac{20}{4} = 5$. Next, using the point-slope form of an equation of a line, we have $y - 5 = 5(t - 0)$ or $y = 5t + 5$.

b.



c. When $t = 2$, $y = 5(2) + 5 = 15$. We conclude that 15 percent of homes had digital TV services at the beginning of 2001.

65. a, b.



c. The slope of L is $m = \dfrac{9.0 - 5.8}{5 - 1} = \dfrac{3.2}{4} = 0.8$. Using the point-slope form of an equation of a line, we have $y - 5.8 = 0.8(x - 1) = 0.8x - 0.8$, or $y = 0.8x + 5$.

d. Using the equation from part c with $x = 9$, we have $y = 0.8(9) + 5 = 12.2$, or \$12.2 million.

66. True. The slope of the line is given by $-\frac{2}{4} = -\frac{1}{2}$.

67. False. Substituting $x = -1$ and $y = 1$ into the equation gives $3(-1) + 7(1) = 4$, and this is not equal to the right-hand side of the equation. Therefore, the equation is not satisfied and so the given point does not lie on the line.

68. True. If $(1, k)$ lies on the line, then $x = 1$, $y = k$ must satisfy the equation. Thus $3 + 4k = 12$, or $k = \frac{9}{4}$. Conversely, if $k = \frac{9}{4}$, then the point $(1, k) = \left(1, \frac{9}{4}\right)$ satisfies the equation. Thus, $3(1) + 4\left(\frac{9}{4}\right) = 12$, and so the point lies on the line.

69. True. The slope of the line $Ax + By + C = 0$ is $-\dfrac{A}{B}$. (Write it in slope-intercept form.) Similarly, the slope of the line $ax + by + c = 0$ is $-\dfrac{a}{b}$. They are parallel if and only if $-\dfrac{A}{B} = -\dfrac{a}{b}$, that is, if $Ab = aB$, or $Ab - aB = 0$.

70. False. Let the slope of L_1 be $m_1 > 0$. Then the slope of L_2 is $m_2 = -\dfrac{1}{m_1} < 0$.

71. True. The slope of the line $ax + by + c_1 = 0$ is $m_1 = -\dfrac{a}{b}$. The slope of the line $bx - ay + c_2 = 0$ is $m_2 = \dfrac{b}{a}$. Because $m_1 m_2 = -1$, the straight lines are indeed perpendicular.

72. True. Set $y = 0$ and we have $Ax + C = 0$ or $x = -C/A$, and this is where the line intersects the x-axis.

73. Writing each equation in the slope-intercept form, we have $y = -\dfrac{a_1}{b_1}x - \dfrac{c_1}{b_1}$ $(b_1 \neq 0)$ and $y = -\dfrac{a_2}{b_2}x - \dfrac{c_2}{b_2}$ $(b_2 \neq 0)$. Because two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $-\dfrac{a_1}{b_1} = -\dfrac{a_2}{b_2}$, or $a_1 b_2 - b_1 a_2 = 0$.

74. The slope of L_1 is $m_1 = \dfrac{b-0}{1-0} = b$. The slope of L_2 is $m_2 = \dfrac{c-0}{1-0} = c$. Applying the Pythagorean theorem to ΔOAC and ΔOCB gives $(OA)^2 = 1^2 + b^2$ and $(OB)^2 = 1^2 + c^2$. Adding these equations and applying the Pythagorean theorem to ΔOBA gives $(AB)^2 = (OA)^2 + (OB)^2 = 1^2 + b^2 + 1^2 + c^2 = 2 + b^2 + c^2$. Also, $(AB)^2 = (b-c)^2$, so $(b-c)^2 = 2 + b^2 + c^2$, $b^2 - 2bc + c^2 = 2 + b^2 + c^2$, and $-2bc = 2$, $1 = -bc$. Finally, $m_1 m_2 = b \cdot c = bc = -1$, as was to be shown.

Technology Exercises

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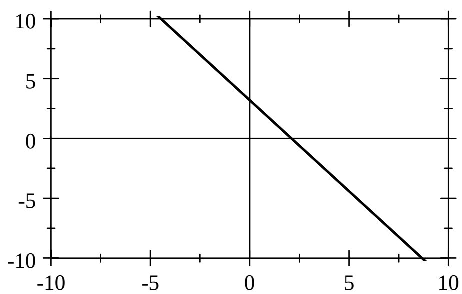
Graphing Utility

1.



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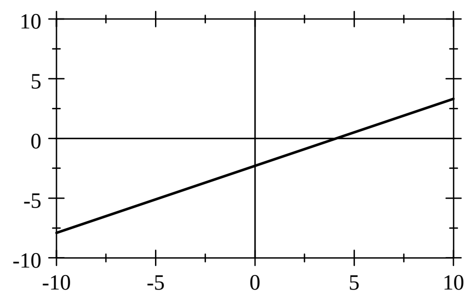

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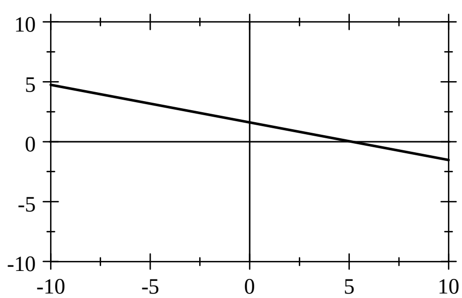

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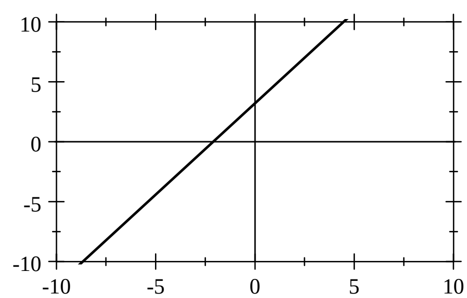
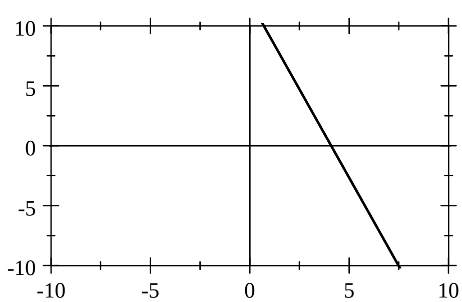
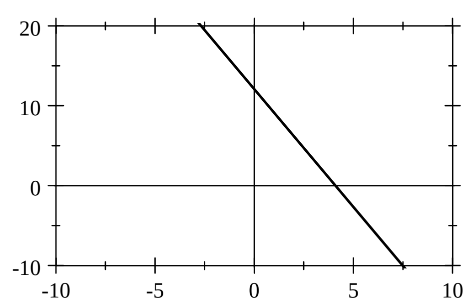


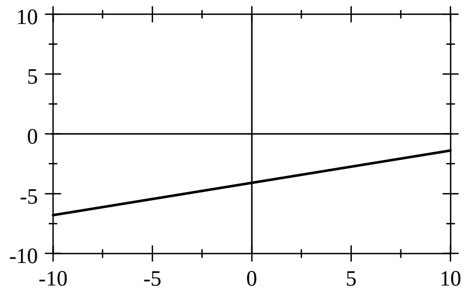
5. a.



b.



6. a.



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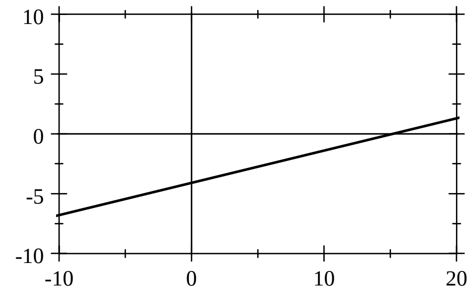
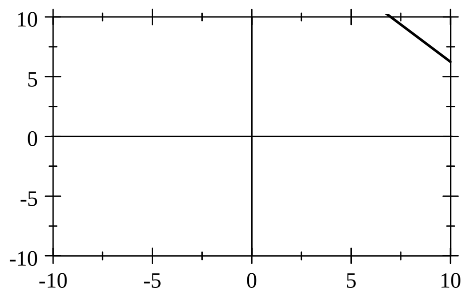


7. a.



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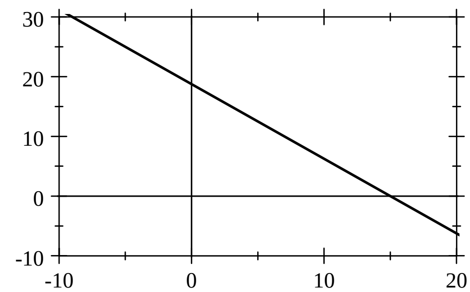
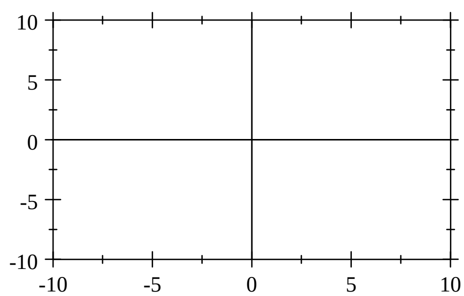


8. a.



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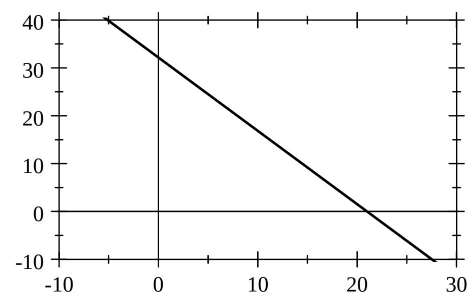

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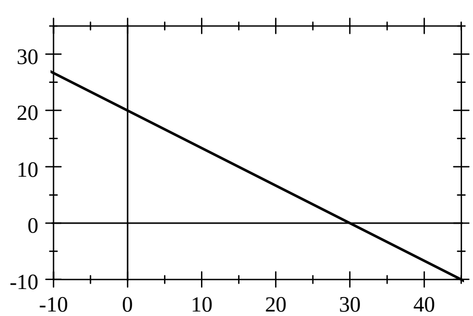

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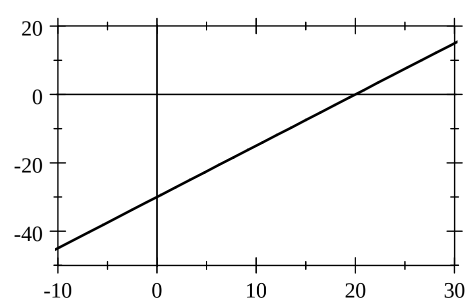

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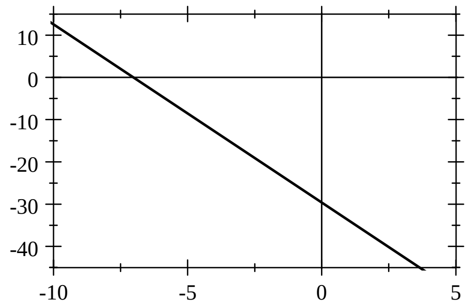

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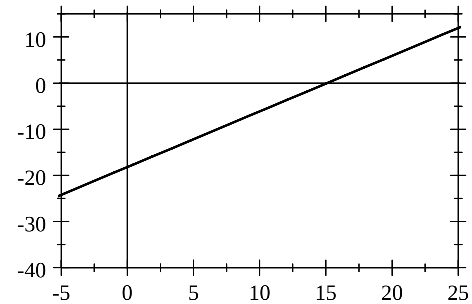
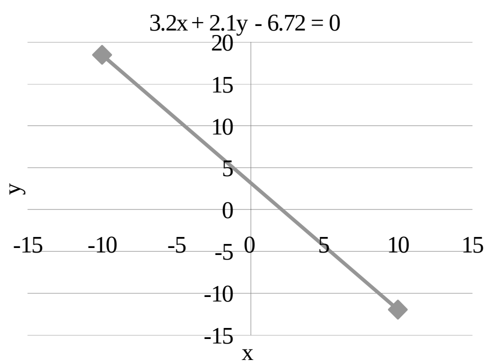


Excel

1.



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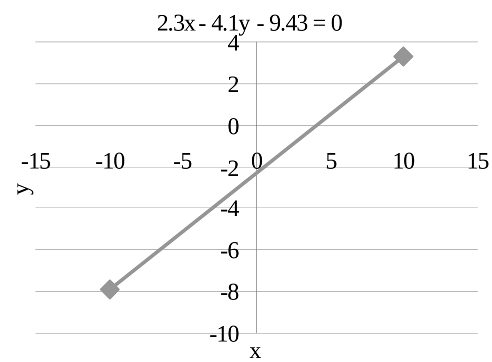

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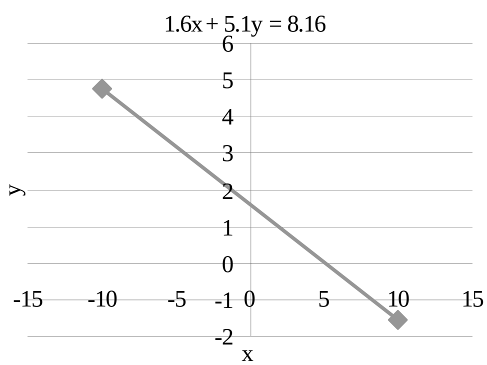

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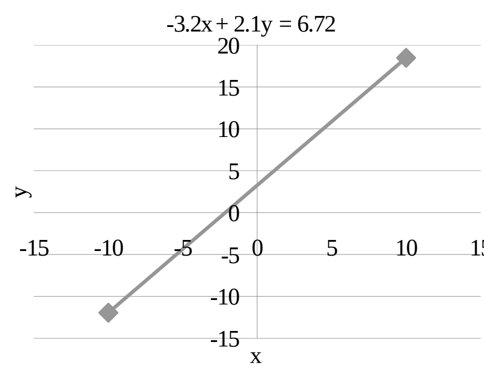

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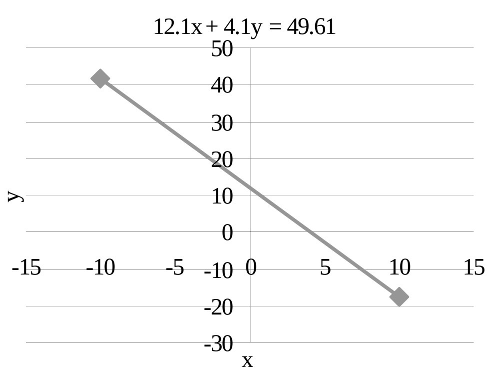

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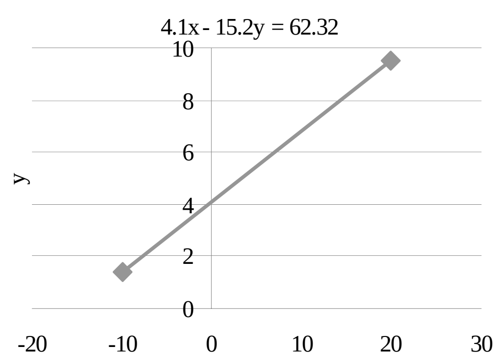

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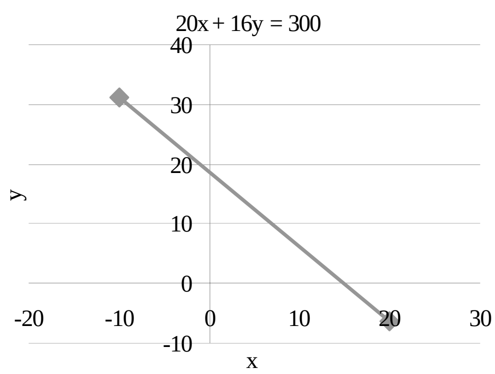

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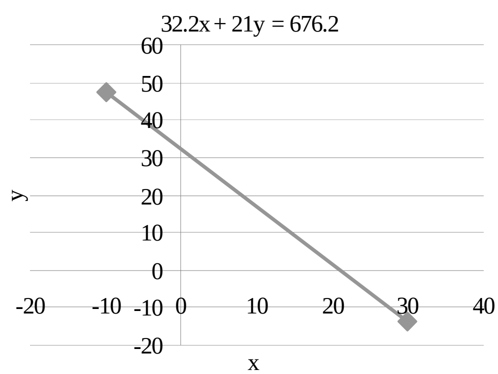

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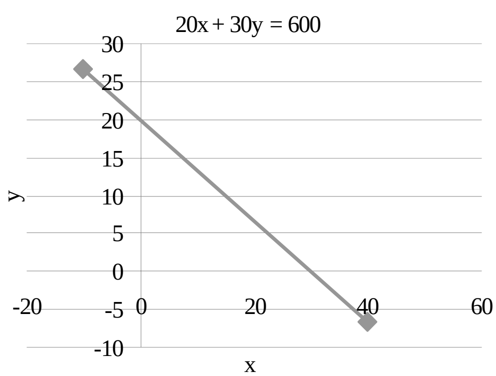

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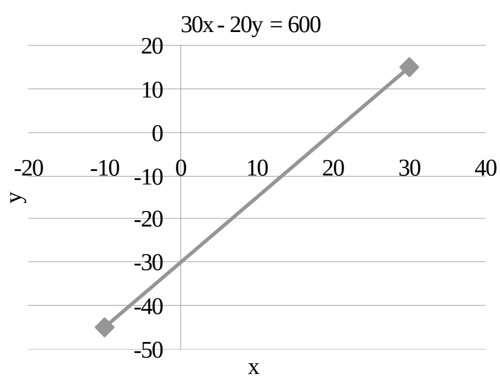

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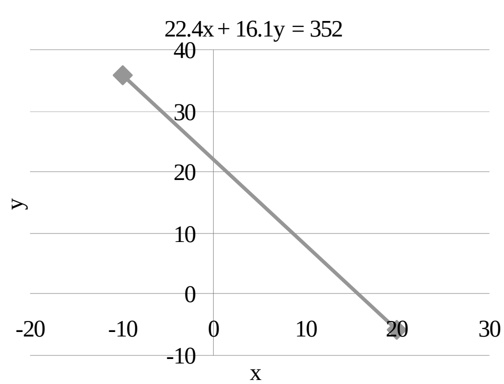

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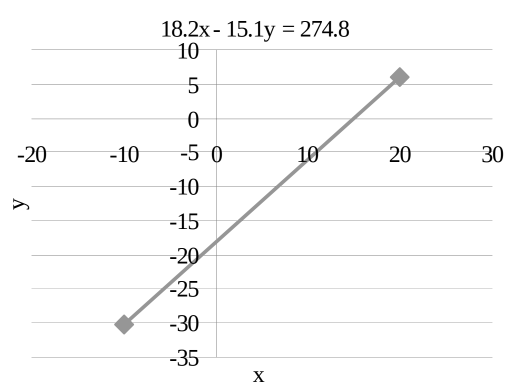

2.3 Functions and Their Graphs

Concept Questions

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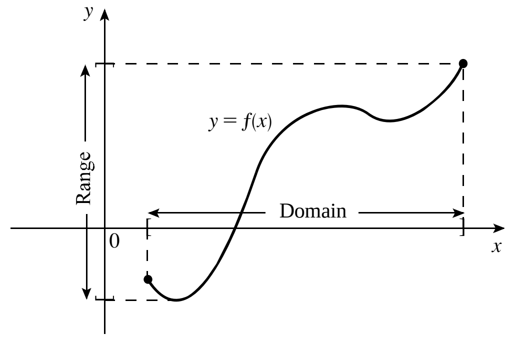
1. **a.** A function is a rule that associates with each element in a set A exactly one element in a set B.

b. The domain of a function f is the set of all elements x in a set A such that $f(x)$ is an element in B. The range of f is the set of all elements $f(x)$ such that x is an element in its domain.

c. An independent variable is a variable in the domain of a function f. The dependent variable is $y = f(x)$.

2. a. The graph of a function f is the set of all ordered pairs (x, y) such that $y = f(x)$, x being an element in the domain of f.



b. Use the vertical line test to determine if every vertical line intersects the curve in at most one point. If so, then the curve is the graph of a function.

3. a. Yes, every vertical line intersects the curve in at most one point.

b. No, a vertical line intersects the curve at more than one point.

c. No, a vertical line intersects the curve at more than one point.

d. Yes, every vertical line intersects the curve in at most one point.

4. The domain is $[1, 5)$ and the range is $\left[\frac{1}{2}, 2\right) \cup (2, 4]$.

Exercises page 97

1. $f(x) = 5x + 6$. Therefore $f(3) = 5(3) + 6 = 21$, $f(-3) = 5(-3) + 6 = -9$, $f(a) = 5(a) + 6 = 5a + 6$, $f(-a) = 5(-a) + 6 = -5a + 6$, and $f(a + 3) = 5(a + 3) + 6 = 5a + 15 + 6 = 5a + 21$.

2. $f(x) = 4x - 3$. Therefore, $f(4) = 4(4) - 3 = 16 - 3 = 13$, $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 3 = 1 - 3 = -2$, $f(0) = 4(0) - 3 = -3$, $f(a) = 4(a) - 3 = 4a - 3$, and $f(a + 1) = 4(a + 1) - 3 = 4a + 1$.

3. $g(x) = 3x^2 - 6x + 3$, so $g(0) = 3(0) - 6(0) + 3 = 3$, $g(-1) = 3(-1)^2 - 6(-1) + 3 = 3 + 6 + 3 = 12$, $g(a) = 3(a)^2 - 6(a) + 3 = 3a^2 - 6a + 3$, $g(-a) = 3(-a)^2 - 6(-a) + 3 = 3a^2 + 6a + 3$, and $g(x + 1) = 3(x + 1)^2 - 6(x + 1) + 3 = 3\left(x^2 + 2x + 1\right) - 6x - 6 + 3 = 3x^2 + 6x + 3 - 6x - 3 = 3x^2$.

4. $h(x) = x^3 - x^2 + x + 1$, so $h(-5) = (-5)^3 - (-5)^2 + (-5) + 1 = -125 - 25 - 5 + 1 = -154$, $h(0) = (0)^3 - (0)^2 + 0 + 1 = 1$, $h(a) = a^3 - (a)^2 + a + 1 = a^3 - a^2 + a + 1$, and $h(-a) = (-a)^3 - (-a)^2 + (-a) + 1 = -a^3 - a^2 - a + 1$.

5. $f(x) = 2x + 5$, so $f(a + h) = 2(a + h) + 5 = 2a + 2h + 5$, $f(-a) = 2(-a) + 5 = -2a + 5$, $f\left(a^2\right) = 2\left(a^2\right) + 5 = 2a^2 + 5$, $f(a - 2h) = 2(a - 2h) + 5 = 2a - 4h + 5$, and $f(2a - h) = 2(2a - h) + 5 = 4a - 2h + 5$

6. $g(x) = -x^2 + 2x$, $g(a + h) = -(a + h)^2 + 2(a + h) = -a^2 - 2ah - h^2 + 2a + 2h$, $g(-a) = -(-a)^2 + 2(-a) = -a^2 - 2a = -a(a + 2)$, $g\left(\sqrt{a}\right) = -\left(\sqrt{a}\right)^2 + 2\left(\sqrt{a}\right) = -a + 2\sqrt{a}$, $a + g(a) = a - a^2 + 2a = -a^2 + 3a = -a(a - 3)$, and $\dfrac{1}{g(a)} = \dfrac{1}{-a^2 + 2a} = -\dfrac{1}{a(a - 2)}$.

7. $s(t) = \dfrac{2t}{t^2-1}$. Therefore, $s(4) = \dfrac{2(4)}{(4)^2-1} = \dfrac{8}{15}$, $s(0) = \dfrac{2(0)}{0^2-1} = 0$,
$s(a) = \dfrac{2(a)}{a^2-1} = \dfrac{2a}{a^2-1}$, $s(2+a) = \dfrac{2(2+a)}{(2+a)^2-1} = \dfrac{2(2+a)}{a^2+4a+4-1} = \dfrac{2(2+a)}{a^2+4a+3}$, and
$s(t+1) = \dfrac{2(t+1)}{(t+1)^2-1} = \dfrac{2(t+1)}{t^2+2t+1-1} = \dfrac{2(t+1)}{t(t+2)}$.

8. $g(u) = (3u-2)^{3/2}$. Therefore, $g(1) = [3(1)-2]^{3/2} = (1)^{3/2} = 1$, $g(6) = [3(6)-2]^{3/2} = 16^{3/2} = 4^3 = 64$,
$g\left(\frac{11}{3}\right) = \left[3\left(\frac{11}{3}\right)-2\right]^{3/2} = (9)^{3/2} = 27$, and $g(u+1) = [3(u+1)-2]^{3/2} = (3u+1)^{3/2}$.

9. $f(t) = \dfrac{2t^2}{\sqrt{t-1}}$. Therefore, $f(2) = \dfrac{2\left(2^2\right)}{\sqrt{2-1}} = 8$, $f(a) = \dfrac{2a^2}{\sqrt{a-1}}$, $f(x+1) = \dfrac{2(x+1)^2}{\sqrt{(x+1)-1}} = \dfrac{2(x+1)^2}{\sqrt{x}}$,
and $f(x-1) = \dfrac{2(x-1)^2}{\sqrt{(x-1)-1}} = \dfrac{2(x-1)^2}{\sqrt{x-2}}$.

10. $f(x) = 2+2\sqrt{5-x}$. Therefore, $f(-4) = 2+2\sqrt{5-(-4)} = 2+2\sqrt{9} = 2+2(3) = 8$,
$f(1) = 2+2\sqrt{5-1} = 2+2\sqrt{4} = 2+4 = 6$, $f\left(\frac{11}{4}\right) = 2+2\left(5-\frac{11}{4}\right)^{1/2} = 2+2\left(\frac{9}{4}\right)^{1/2} = 2+2\left(\frac{3}{2}\right) = 5$,
and $f(x+5) = 2+2\sqrt{5-(x+5)} = 2+2\sqrt{-x}$.

11. For $x = -3 \le 0$, we calculate $f(-3) = (-3)^2+1 = 9+1 = 10$. For $x = 0 \le 0$, we calculate $f(0) = (0)^2+1 = 1$. For $x = 1 > 0$, we calculate $f(1) = \sqrt{1} = 1$.

12. For $x = -2 < 2$, $g(-2) = -\frac{1}{2}(-2)+1 = 1+1 = 2$. For $x = 0 < 2$, $g(0) = -\frac{1}{2}(0)+1 = 0+1 = 1$. For $x = 2 \ge 2$, $g(2) = \sqrt{2-2} = 0$. For $x = 4 \ge 2$, $g(4) = \sqrt{4-2} = \sqrt{2}$.

13. For $x = -1 < 1$, $f(-1) = -\frac{1}{2}(-1)^2+3 = \frac{5}{2}$. For $x = 0 < 1$, $f(0) = -\frac{1}{2}(0)^2+3 = 3$. For $x = 1 \ge 1$, $f(1) = 2\left(1^2\right)+1 = 3$. For $x = 2 \ge 1$, $f(2) = 2\left(2^2\right)+1 = 9$.

14. For $x = 0 \le 1$, $f(0) = 2+\sqrt{1-0} = 2+1 = 3$. For $x = 1 \le 1$, $f(1) = 2+\sqrt{1-1} = 2+0 = 2$. For $x = 2 > 1$, $f(2) = \dfrac{1}{1-2} = \dfrac{1}{-1} = -1$.

15. **a.** $f(0) = -2$.

b. (i) $f(x) = 3$ when $x \approx 2$.
(ii) $f(x) = 0$ when $x = 1$.

c. $[0, 6]$

d. $[-2, 6]$

16. **a.** $f(7) = 3$.

b. $x = 4$ and $x = 6$.

c. $x = 2$; 0.

d. $[-1, 9]$; $[-2, 6]$.

17. $g(2) = \sqrt{2^2-1} = \sqrt{3}$, so the point $\left(2, \sqrt{3}\right)$ lies on the graph of g.

18. $f(3) = \dfrac{3+1}{\sqrt{3^2+7}}+2 = \dfrac{4}{\sqrt{16}}+2 = \dfrac{4}{4}+2 = 3$, so the point $(3, 3)$ lies on the graph of f.

19. $f(-2) = \dfrac{|-2-1|}{-2+1} = \dfrac{|-3|}{-1} = -3$, so the point $(-2, -3)$ does lie on the graph of f.

20. $h(-3) = \dfrac{|-3+1|}{(-3)^3+1} = \dfrac{2}{-27+1} = -\dfrac{2}{26} = -\dfrac{1}{13}$, so the point $\left(-3, -\frac{1}{13}\right)$ does lie on the graph of h.

21. Because the point $(1, 5)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(1) = 2(1)^2 - 4(1) + c = 5$, or $c = 7$.

22. Because the point $(2, 4)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(2) = 2\sqrt{9-(2)^2} + c = 4$, or $c = 4 - 2\sqrt{5}$.

23. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

24. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

25. $f(x)$ is not defined at $x = 0$ and so the domain of f is $(-\infty, 0) \cup (0, \infty)$.

26. $g(x)$ is not defined at $x = 1$ and so the domain of g is $(-\infty, 1) \cup (1, \infty)$.

27. $f(x)$ is a real number for all values of x. Note that $x^2 + 1 \geq 1$ for all x. Therefore, the domain of f is $(-\infty, \infty)$.

28. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $x - 5 \geq 0$ or $x \geq 5$, and the domain is $[5, \infty)$.

29. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $5 - x \geq 0$, or $-x \geq -5$ and so $x \leq 5$. (Recall that multiplying by -1 reverses the sign of an inequality.) Therefore, the domain of f is $(-\infty, 5]$.

30. Because $2x^2 + 3$ is always greater than zero, the domain of g is $(-\infty, \infty)$.

31. The denominator of f is zero when $x^2 - 4 = 0$, or $x = \pm 2$. Therefore, the domain of f is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

32. The denominator of f is equal to zero when $x^2 + x - 2 = (x+2)(x-1) = 0$; that is, when $x = -2$ or $x = 1$. Therefore, the domain of f is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

33. f is defined when $x + 3 \geq 0$, that is, when $x \geq -3$. Therefore, the domain of f is $[-3, \infty)$.

34. g is defined when $x - 1 \geq 0$; that is when $x \geq 1$. Therefore, the domain of g is $[1, \infty)$.

35. The numerator is defined when $1 - x \geq 0$, $-x \geq -1$ or $x \leq 1$. Furthermore, the denominator is zero when $x = \pm 2$. Therefore, the domain is the set of all real numbers in $(-\infty, -2) \cup (-2, 1]$.

36. The numerator is defined when $x - 1 \geq 0$, or $x \geq 1$, and the denominator is zero when $x = -2$ and when $x = 3$. So the domain is $[1, 3) \cup (3, \infty)$.

37. **a.** The domain of f is the set of all real numbers.

b. $f(x) = x^2 - x - 6$, so

$f(-3) = (-3)^2 - (-3) - 6 = 9 + 3 - 6 = 6$,

$f(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$,

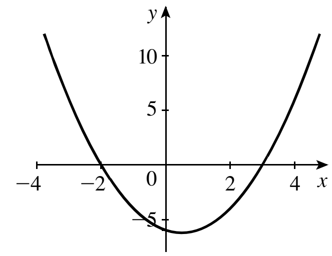
$f(-1) = (-1)^2 - (-1) - 6 = 1 + 1 - 6 = -4$,

$f(0) = (0)^2 - (0) - 6 = -6$,

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{2}{4} - \frac{24}{4} = -\frac{25}{4}$, $f(1) = (1)^2 - 1 - 6 = -6$,

$f(2) = (2)^2 - 2 - 6 = 4 - 2 - 6 = -4$, and $f(3) = (3)^2 - 3 - 6 = 9 - 3 - 6 = 0$.

c.



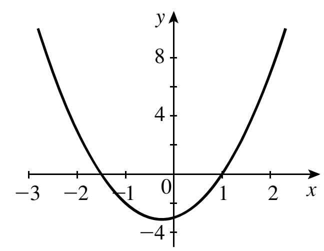
38. $f(x) = 2x^2 + x - 3$.

a. Because $f(x)$ is a real number for all values of x, the domain of f is $(-\infty, \infty)$.

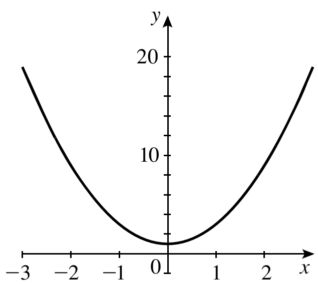
b.

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2	3
y	12	3	-2	-3	-3	0	7	18

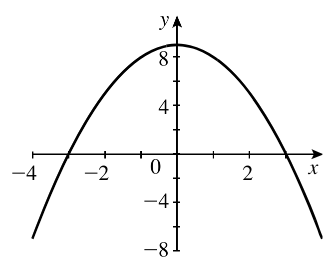
c.



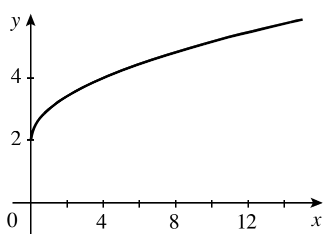
39. $f(x) = 2x^2 + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$.



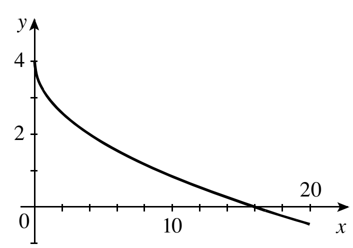
40. $f(x) = 9 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 9]$.



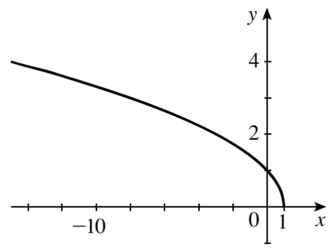
41. $f(x) = 2 + \sqrt{x}$ has domain $[0, \infty)$ and range $[2, \infty)$.



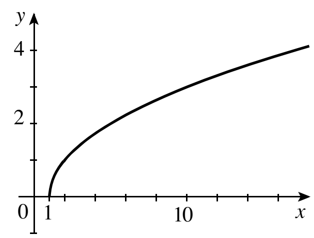
42. $g(x) = 4 - \sqrt{x}$ has domain $[0, \infty)$ and range $(-\infty, 4]$.



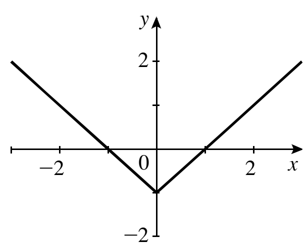
43. $f(x) = \sqrt{1-x}$ has domain $(-\infty, 1]$ and range $[0, \infty)$



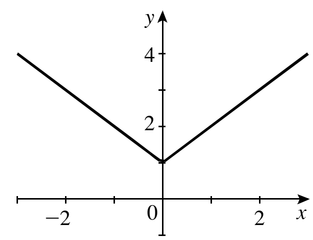
44. $f(x) = \sqrt{x-1}$ has domain $(1, \infty)$ and range $[0, \infty)$.



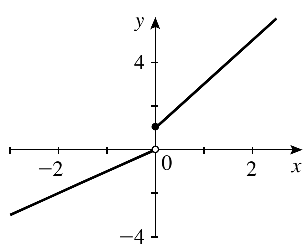
45. $f(x) = |x| - 1$ has domain $(-\infty, \infty)$ and range $[-1, \infty)$.



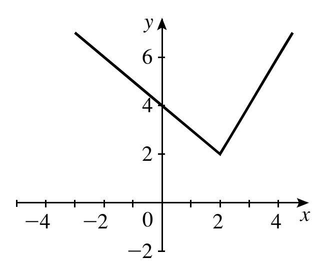
46. $f(x) = |x| + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$



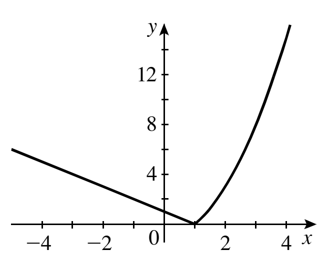
47. $f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x+1 & \text{if } x \geq 0 \end{cases}$ has domain $(-\infty, \infty)$ and range $(-\infty, 0) \cup [1, \infty)$.



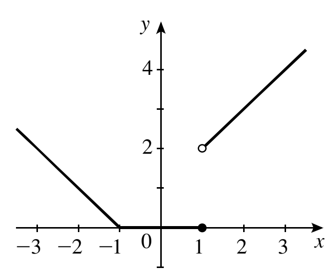
48. For $x < 2$, the graph of f is the half-line $y = 4 - x$. For $x \geq 2$, the graph of f is the half-line $y = 2x - 2$. f has domain $(-\infty, \infty)$ and range $[2, \infty)$.



49. If $x \leq 1$, the graph of f is the half-line $y = -x + 1$. For $x > 1$, we calculate a few points: $f(2) = 3$, $f(3) = 8$, and $f(4) = 15$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



50. If $x < -1$ the graph of f is the half-line $y = -x - 1$. For $-1 \leq x \leq 1$, the graph consists of the line segment $y = 0$. For $x > 1$, the graph is the half-line $y = x + 1$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



51. Each vertical line cuts the given graph at exactly one point, and so the graph represents y as a function of x.

52. Because the y-axis, which is a vertical line, intersects the graph at two points, the graph does not represent y as a function of x.

53. Because there is a vertical line that intersects the graph at three points, the graph does not represent y as a function of x.

54. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

55. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

56. The y-axis intersects the circle at *two* points, and this shows that the circle is not the graph of a function of x.

57. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

58. A vertical line containing a line segment comprising the graph cuts it at infinitely many points and so the graph does not define y as a function of x.

59. The circumference of a circle with a 5-inch radius is given by $C(5) = 2\pi(5) = 10\pi$, or 10π inches.

60. $V(2.1) = \frac{4}{3}\pi(2.1)^3 \approx 38.79$, $V(2) = \frac{4}{3}\pi(8) \approx 33.51$, and so $V(2.1) - V(2) = 38.79 - 33.51 = 5.28$ is the amount by which the volume of a sphere of radius 2.1 exceeds the volume of a sphere of radius 2.

61. $\frac{4}{3}(\pi)(2r)^3 = \frac{4}{3}\pi 8r^3 = 8\left(\frac{4}{3}\pi r^3\right)$. Therefore, the volume of the tumor is increased by a factor of 8.

62. **a.** The rate of change of the average life expectancy after age 65 is given by the slope of the linear equation L, or 0.056 yr/yr.

b. The average life expectancy after age 65 in 2010 is $L(7) = 0.056(7) + 18.1 = 18.492$, or approximately 18.5 yr.

63. **a.** The slope of the straight line passing through the points $(0, 0.58)$ and $(20, 0.95)$ is $m_1 = \dfrac{0.95 - 0.58}{20 - 0} = 0.0185$, so an equation of the straight line passing through these two points is $y - 0.58 = 0.0185(t - 0)$ or $y = 0.0185t + 0.58$. Next, the slope of the straight line passing through the points $(20, 0.95)$ and $(30, 1.1)$ is $m_2 = \dfrac{1.1 - 0.95}{30 - 20} = 0.015$, so an equation of the straight line passing through the two points is $y - 0.95 = 0.015(t - 20)$ or $y = 0.015t + 0.65$. Therefore, a rule for f is

$$f(t) = \begin{cases} 0.0185t + 0.58 & \text{if } 0 \le t \le 20 \\ 0.015t + 0.65 & \text{if } 20 < t \le 30 \end{cases}$$

b. The ratios were changing at the rates of 0.0185/yr from 1960 through 1980 and 0.015/yr from 1980 through 1990.

c. The ratio was 1 when $t \approx 20.3$. This shows that the number of bachelor's degrees earned by women equaled the number earned by men for the first time around 1983.

64. a. The slope of the straight line passing through $(0, 0.61)$ and $(10, 0.59)$ is $m_1 = \dfrac{0.59 - 0.61}{10 - 0} = -0.002$.
Therefore, an equation of the straight line passing through the two points is $y - 0.61 = -0.002\,(t - 0)$ or $y = -0.002t + 0.61$. Next, the slope of the straight line passing through $(10, 0.59)$ and $(20, 0.60)$ is $m_2 = \dfrac{0.60 - 0.59}{20 - 10} = 0.001$, and so an equation of the straight line passing through the two points is $y - 0.59 = 0.001\,(t - 10)$ or $y = 0.001t + 0.58$. The slope of the straight line passing through $(20, 0.60)$ and $(30, 0.66)$ is $m_3 = \dfrac{0.66 - 0.60}{30 - 20} = 0.006$, and so an equation of the straight line passing through the two points is $y - 0.60 = 0.006\,(t - 20)$ or $y = 0.006t + 0.48$. The slope of the straight line passing through $(30, 0.66)$ and $(40.0, 0.78)$ is $m_4 = \dfrac{0.78 - 0.66}{40 - 30} = 0.012$, and so an equation of the straight line passing through the two points is $y = 0.012t + 0.30$. Therefore, a rule for f is $f(t) = \begin{cases} -0.002t + 0.61 & \text{if } 0 \le t \le 10 \\ 0.001t + 0.58 & \text{if } 10 < t \le 20 \\ 0.006t + 0.48 & \text{if } 20 < t \le 30 \\ 0.012t + 0.30 & \text{if } 30 < t \le 40 \end{cases}$

b. The gender gap was expanding between 1960 and 1970 and shrinking between 1970 and 2000.

c. The gender gap was expanding at the rate of 0.002/yr between 1960 and 1970, shrinking at the rate of 0.001/yr between 1970 and 1980, shrinking at the rate of 0.006/yr between 1980 and 1990, and shrinking at the rate of 0.012/yr between 1990 and 2000.

65. $N(t) = -t^3 + 6t^2 + 15t$. Between 8 a.m. and 9 a.m., the average worker can be expected to assemble $N(1) - N(0) = (-1 + 6 + 15) - 0 = 20$, or 20 walkie-talkies. Between 9 a.m. and 10 a.m., we expect that $N(2) - N(1) = \left[-2^3 + 6\left(2^2\right) + 15\,(2)\right] - (-1 + 6 + 15) = 46 - 20 = 26$, or 26 walkie-talkies can be assembled by the average worker.

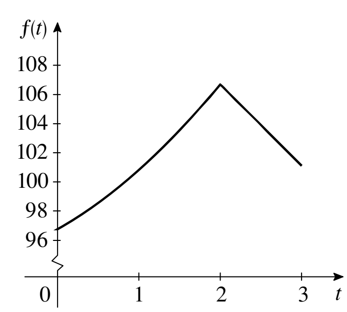
66. When the proportion of popular votes won by the Democratic presidential candidate is 0.60, the proportion of seats in the House of Representatives won by Democratic candidates is given by
$$s(0.6) = \frac{(0.6)^3}{(0.6)^3 + (1 - 0.6)^3} = \frac{0.216}{0.216 + 0.064} = \frac{0.216}{0.280} \approx 0.77.$$

67. The amount spent in 2004 was $S(0) = 5.6$, or \$5.6 billion. The amount spent in 2008 was $S(4) = -0.03\,(4)^3 + 0.2\,(4)^2 + 0.23\,(4) + 5.6 = 7.8$, or \$7.8 billion.

68. a.

Year t	0	1	2
Rate	96.75	100.84	106.69

b.

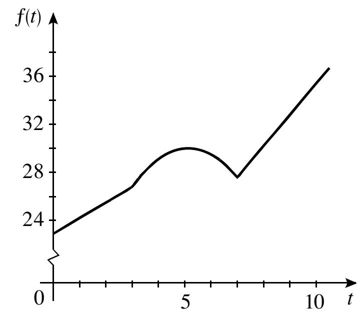


69. a. The assets at the beginning of 2002 were \$0.6 trillion. At the beginning of 2003, they were $f(1) = 0.6$, or \$0.6 trillion.

b. The assets at the beginning of 2005 were $f(3) \approx 0.96$, or \$0.96 trillion. At the beginning of 2007, they were $f(5) \approx 1.20$, or \$1.2 trillion.

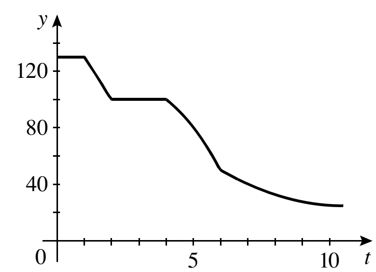
70. a. The median age of the U.S. population at the beginning of 1900 was $f(0) = 22.9$, or 22.9 years; at the beginning of 1950 it was $f(5) = -0.7(5)^2 + 7.2(5) + 11.5 = 30$, or 30 years; and at the beginning of 1990 it was $f(9) = 2.6(9) + 9.4 = 32.8$, or 32.8 years.

b.



71. a. The amount of solids discharged in 1989 ($t = 0$) was 130 tons/day; in 1992 ($t = 3$), it was 100 tons/day; and in 1996 ($t = 7$), it was $f(7) = 1.25(7)^2 - 26.25(7) + 162.5 = 40$, or 40 tons/day.

b.



72. True, by definition of a function (page 89).

73. False. Take $f(x) = x^2$, $a = 1$, and $b = -1$. Then $f(1) = 1 = f(-1)$, but $a \neq b$.

74. False. Let $f(x) = x^2$, then take $a = 1$ and $b = 2$. Then $f(a) = f(1) = 1$, $f(b) = f(2) = 4$, and $f(a) + f(b) = 1 + 4 \neq f(a+b) = f(3) = 9$.

75. False. It intersects the graph of a function in at most one point.

76. True. We have $x + 2 \geq 0$ and $2 - x \geq 0$ simultaneously; that is $x \geq -2$ and $x \leq 2$. These inequalities are satisfied if $-2 \leq x \leq 2$.

Technology Exercises

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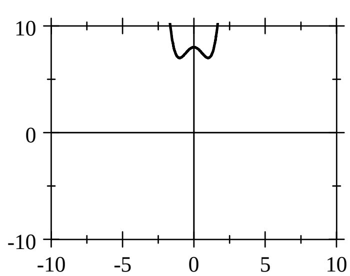
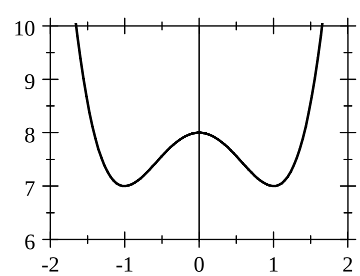
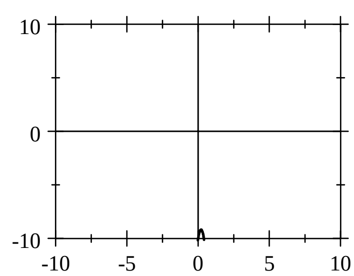
1. a.



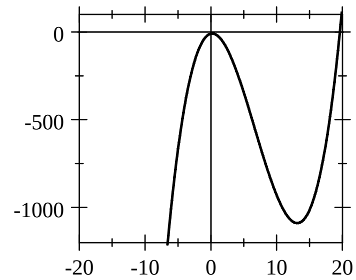
b.



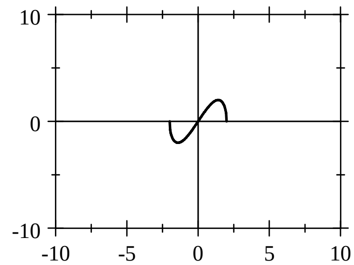
2. a.



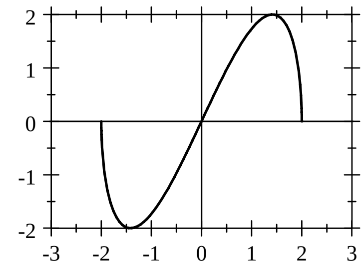
b.



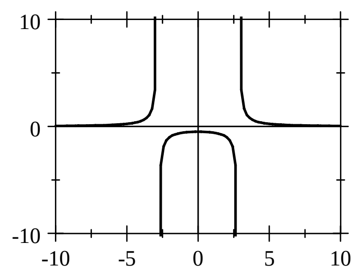
3. a.



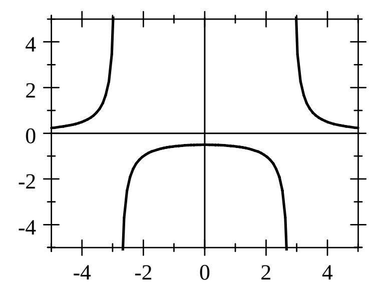
b.



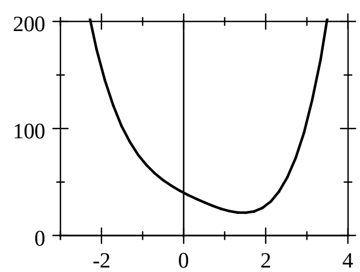
4. a.



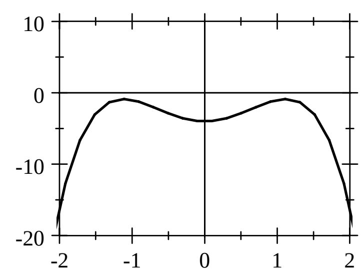
b.



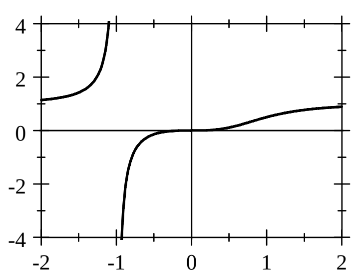
5.



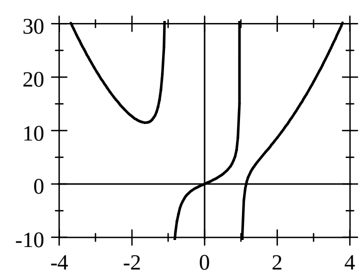
6.



7.



8.



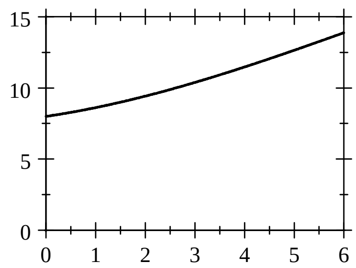
9. $f(2.145) \approx 18.5505$.

10. $f(1.28) \approx 17.3850$.

11. $f(2.41) \approx 4.1616$.

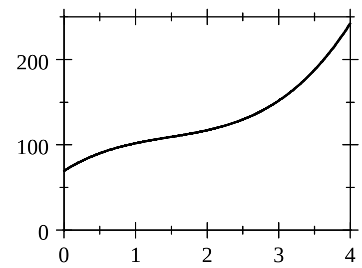
12. $f(0.62) \approx 1.7214$.

13. a.



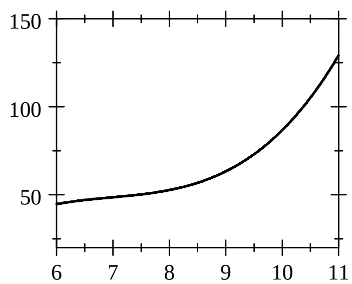
b. The amount spent in the year 2005 was $f(2) \approx 9.42$, or approximately \$9.4 billion. In 2009, it was $f(6) \approx 13.88$, or approximately \$13.9 billion.

14. a.



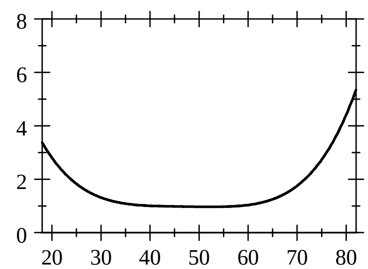
b. The number using surveillance cameras in 2004 was $N(1) = 101.85$, or approximately 102. The number in 2006 was $N(3) = 151.73$, or approximately 152.

15. a.



b. $f(6) = 44.7$, $f(8) = 52.7$, and $f(11) = 129.2$.

16. a.



b. $f(18) = 3.3709$, $f(50) = 0.971$, and $f(80) = 4.4078$.

2.4 The Algebra of Functions

Concept Questions

page 109

1. a. $P(x_1) = R(x_1) - C(x_1)$ gives the profit if x_1 units are sold.

b. $P(x_2) = R(x_2) - C(x_2)$. Because $P(x_2) < 0$, $|R(x_2) - C(x_2)| = -[R(x_2) - C(x_2)]$ gives the loss sustained if x_2 units are sold.

2. a. $(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, and $(fg)(x) = f(x)g(x)$; all have domain $A \cap B$. $(f/g)(x) = \dfrac{f(x)}{g(x)}$ has domain $A \cap B$ excluding $x \in A \cap B$ such that $g(x) = 0$.

b. $(f+g)(2) = f(2) + g(2) = 3 + (-2) = 1$, $(f-g)(2) = f(2) - g(2) = 3 - (-2) = 5$, $(fg)(2) = f(2)g(2) = 3(-2) = -6$, and $(f/g)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{-2} = -\dfrac{3}{2}$

3. a. $y = (f+g)(x) = f(x) + g(x)$

b. $y = (f-g)(x) = f(x) - g(x)$

c. $y = (fg)(x) = f(x)g(x)$

d. $y = \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

4. a. The composition of f and g is $(f \circ g)(x) = f(g(x))$. Its domain is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

The composition of g and f is $(g \circ f)(x) = g(f(x))$. Its domain is the set of all x in the domain of f such that $f(x)$ is in the domain of g.

b. $(g \circ f)(2) = g(f(2)) = g(3) = 8$. We cannot calculate $(f \circ g)(3)$ because $(f \circ g)(3) = f(g(3)) = f(8)$, and we don't know the value of $f(8)$.

5. No. Let $A = (-\infty, \infty)$, $f(x) = x$, and $g(x) = \sqrt{x}$. Then $a = -1$ is in A, but $(g \circ f)(-1) = g(f(-1)) = g(-1) = \sqrt{-1}$ is not defined.

Exercises page 110

1. $(f+g)(x) = f(x) + g(x) = \left(x^3+5\right) + \left(x^2-2\right) = x^3 + x^2 + 3$.

2. $(f-g)(x) = f(x) - g(x) = \left(x^3+5\right) - \left(x^2-2\right) = x^3 - x^2 + 7$.

3. $fg(x) = f(x)g(x) = \left(x^3+5\right)\left(x^2-2\right) = x^5 - 2x^3 + 5x^2 - 10$.

4. $gf(x) = g(x)f(x) = \left(x^2-2\right)\left(x^3+5\right) = x^5 - 2x^3 + 5x^2 - 10$.

5. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3+5}{x^2-2}$.

6. $\dfrac{f-g}{h}(x) = \dfrac{f(x)-g(x)}{h(x)} = \dfrac{x^3+5-\left(x^2-2\right)}{2x+4} = \dfrac{x^3-x^2+7}{2x+4}$.

7. $\dfrac{fg}{h}(x) = \dfrac{f(x)g(x)}{h(x)} = \dfrac{\left(x^3+5\right)\left(x^2-2\right)}{2x+4} = \dfrac{x^5-2x^3+5x^2-10}{2x+4}$.

8. $$fgh(x) = f(x)g(x)h(x) = \left(x^3+5\right)\left(x^2-2\right)(2x+4) = \left(x^5-2x^3+5x^2-10\right)(2x+4)$$
$$= 2x^6 - 4x^4 + 10x^3 - 20x + 4x^5 - 8x^3 + 20x^2 - 40 = 2x^6 + 4x^5 - 4x^4 + 2x^3 + 20x^2 - 20x - 40.$$

9. $(f+g)(x) = f(x) + g(x) = x - 1 + \sqrt{x+1}$.

10. $(g-f)(x) = g(x) - f(x) = \sqrt{x+1} - (x-1) = \sqrt{x+1} - x + 1$.

11. $(fg)(x) = f(x)g(x) = (x-1)\sqrt{x+1}$.

12. $(gf)(x) = g(x)f(x) = \sqrt{x+1}(x-1)$.

13. $\dfrac{g}{h}(x) = \dfrac{g(x)}{h(x)} = \dfrac{\sqrt{x+1}}{2x^3-1}$.

14. $\dfrac{h}{g}(x) = \dfrac{h(x)}{g(x)} = \dfrac{2x^3-1}{\sqrt{x+1}}$.

15. $\dfrac{fg}{h}(x) = \dfrac{(x-1)\left(\sqrt{x+1}\right)}{2x^3-1}$.

16. $\dfrac{fh}{g}(x) = \dfrac{(x-1)\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{2x^4-2x^3-x+1}{\sqrt{x+1}}$.

17. $\dfrac{f-h}{g}(x) = \dfrac{x-1-\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{x-2x^3}{\sqrt{x+1}}$.

18. $\dfrac{gh}{g-f}(x) = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-(x-1)} = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-x+1}$.

19. $(f+g)(x) = x^2+5+\sqrt{x}-2 = x^2+\sqrt{x}+3$, $(f-g)(x) = x^2+5-\left(\sqrt{x}-2\right) = x^2-\sqrt{x}+7$, $(fg)(x) = \left(x^2+5\right)\left(\sqrt{x}-2\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+5}{\sqrt{x}-2}$.

20. $(f+g)(x) = \sqrt{x-1}+x^3+1$, $(f-g)(x) = \sqrt{x-1}-x^3-1$, $(fg)(x) = \sqrt{x-1}\left(x^3+1\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{x^3+1}$.

21. $(f+g)(x) = \sqrt{x+3}+\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}+1}{x-1}$, $(f-g)(x) = \sqrt{x+3}-\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}-1}{x-1}$, $(fg)(x) = \sqrt{x+3}\left(\dfrac{1}{x-1}\right) = \dfrac{\sqrt{x+3}}{x-1}$, and $\left(\dfrac{f}{g}\right) = \sqrt{x+3}\,(x-1)$.

22. $(f+g)(x) = \dfrac{1}{x^2+1}+\dfrac{1}{x^2-1} = \dfrac{x^2-1+x^2+1}{\left(x^2+1\right)\left(x^2-1\right)} = \dfrac{2x^2}{\left(x^2+1\right)\left(x^2-1\right)}$, $(f-g)(x) = \dfrac{1}{x^2+1}-\dfrac{1}{x^2-1} = \dfrac{x^2-1-x^2-1}{\left(x^2+1\right)\left(x^2-1\right)} = -\dfrac{2}{\left(x^2+1\right)\left(x^2-1\right)}$, $(fg)(x) = \dfrac{1}{\left(x^2+1\right)\left(x^2-1\right)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2-1}{x^2+1}$.

23.
$$\begin{aligned}(f+g)(x) &= \frac{x+1}{x-1}+\frac{x+2}{x-2} = \frac{(x+1)(x-2)+(x+2)(x-1)}{(x-1)(x-2)} = \frac{x^2-x-2+x^2+x-2}{(x-1)(x-2)}\\ &= \frac{2x^2-4}{(x-1)(x-2)} = \frac{2\left(x^2-2\right)}{(x-1)(x-2)},\\ (f-g)(x) &= \frac{x+1}{x-1}-\frac{x+2}{x-2} = \frac{(x+1)(x-2)-(x+2)(x-1)}{(x-1)(x-2)} = \frac{x^2-x-2-x^2-x+2}{(x-1)(x-2)}\\ &= \frac{-2x}{(x-1)(x-2)},\end{aligned}$$
$(fg)(x) = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{(x+1)(x-2)}{(x-1)(x+2)}$.

24. $(f+g)(x) = x^2+1+\sqrt{x+1}$, $(f-g)(x) = x^2+1-\sqrt{x+1}$, $(fg)(x) = \left(x^2+1\right)\sqrt{x+1}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+1}{\sqrt{x+1}}$.

25. $(f\circ g)(x) = f(g(x)) = f\left(x^2+1\right) = \left(x^2+1\right)^2+\left(x^2+1\right)+1 = \left(x^4+2x^2+1\right)+x^2+2 = x^4+3x^2+3$ and $(g\circ f)(x) = g(f(x)) = g\left(x^2+x+1\right) = \left(x^2+x+1\right)^2+1$.

26. $(f\circ g)(x) = f(g(x)) = 3\left[g(x)\right]^2+2g(x)+1 = 3(x+3)^2+2(x+3)+1 = 3x^2+20x+34$ and $(g\circ f)(x) = g(f(x)) = f(x)+3 = 3x^2+2x+1+3 = 3x^2+2x+4$.

27. $(f\circ g)(x) = f(g(x)) = f\left(x^2-1\right) = \sqrt{x^2-1}+1$ and $(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x}+1\right) = \left(\sqrt{x}+1\right)^2-1 = x+2\sqrt{x}+1-1 = x+2\sqrt{x}$.

28. $(f\circ g)(x) = f(g(x)) = 2\sqrt{g(x)}+3 = 2\sqrt{x^2+1}+3$ and $(g\circ f)(x) = g(f(x)) = \left[f(x)\right]^2+1 = \left(2\sqrt{x}+3\right)^2+1 = 4x+12\sqrt{x}+10$.

29. $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} \div \left(\dfrac{1}{x^2}+1\right) = \dfrac{1}{x} \cdot \dfrac{x^2}{x^2+1} = \dfrac{x}{x^2+1}$ and

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x}{x^2+1}\right) = \dfrac{x^2+1}{x}$.

30. $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x-1}\right) = \sqrt{\dfrac{1}{x-1}+1} = \sqrt{\dfrac{x}{x-1}}$ and

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+1}\right) = \dfrac{1}{\sqrt{x+1}-1} \cdot \dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \dfrac{\sqrt{x+1}+1}{x}$.

31. $h(2) = g(f(2))$. But $f(2) = 2^2+2+1 = 7$, so $h(2) = g(7) = 49$.

32. $h(2) = g(f(2))$. But $f(2) = \left(2^2-1\right)^{1/3} = 3^{1/3}$, so $h(2) = g\left(3^{1/3}\right) = 3\left(3^{1/3}\right)^3 + 1 = 3(3)+1 = 10$.

33. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2(2)+1} = \dfrac{1}{5}$, so $h(2) = g\left(\dfrac{1}{5}\right) = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.

34. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2-1} = 1$, so $g(1) = 1^2+1 = 2$.

35. $f(x) = 2x^3+x^2+1$, $g(x) = x^5$.

36. $f(x) = 3x^2-4$, $g(x) = x^{-3}$.

37. $f(x) = x^2-1$, $g(x) = \sqrt{x}$.

38. $f(x) = (2x-3)$, $g(x) = x^{3/2}$.

39. $f(x) = x^2-1$, $g(x) = \dfrac{1}{x}$.

40. $f(x) = x^2-4$, $g(x) = \dfrac{1}{\sqrt{x}}$.

41. $f(x) = 3x^2+2$, $g(x) = \dfrac{1}{x^{3/2}}$.

42. $f(x) = \sqrt{2x+1}$, $g(x) = \dfrac{1}{x}+x$.

43. $f(a+h) - f(a) = [3(a+h)+4] - (3a+4) = 3a+3h+4-3a-4 = 3h$.

44. $f(a+h) - f(a) = -\frac{1}{2}(a+h)+3 - \left(-\frac{1}{2}a+3\right) = -\frac{1}{2}a - \frac{1}{2}h + 3 + \frac{1}{2}a - 3 = -\frac{1}{2}h$.

45. $f(a+h) - f(a) = 4-(a+h)^2 - \left(4-a^2\right) = 4-a^2-2ah-h^2-4+a^2 = -2ah-h^2 = -h(2a+h)$.

46. $f(a+h) - f(a) = \left[(a+h)^2 - 2(a+h)+1\right] - \left(a^2-2a+1\right)$

$= a^2+2ah+h^2-2a-2h+1-a^2+2a-1 = h(2a+h-2)$.

47. $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\left[2(a+h)^2+1\right] - \left(2a^2+1\right)}{h} = \dfrac{2a^2+4ah+2h^2+1-2a^2-1}{h} = \dfrac{4ah+2h^2}{h}$

$= \dfrac{2h(2a+h)}{h} = 2(2a+h)$.

48. $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\left[2(a+h)^2-(a+h)+1\right] - \left(2a^2-a+1\right)}{h}$

$= \dfrac{2a^2+4ah+2h^2-a-h+1-2a^2+a-1}{h} = \dfrac{4ah+2h^2-h}{h} = 4a+2h-1$.

49. $$\frac{f(a+h)-f(a)}{h}=\frac{\left[(a+h)^3-(a+h)\right]-\left(a^3-a\right)}{h}=\frac{a^3+3a^2h+3ah^2+h^3-a-h-a^3+a}{h}$$
$$=\frac{3a^2h+3ah^2+h^3-h}{h}=3a^2+3ah+h^2-1.$$

50. $$\frac{f(a+h)-f(a)}{h}=\frac{\left[2(a+h)^3-(a+h)^2+1\right]-\left(2a^3-a^2+1\right)}{h}$$
$$=\frac{2a^3+6a^2h+6ah^2+2h^3-a^2-2ah-h^2+1-2a^3+a^2-1}{h}$$
$$=\frac{6a^2h+6ah^2+2h^3-2ah-h^2}{h}=6a^2+6ah+2h^2-2a-h.$$

51. $$\frac{f(a+h)-f(a)}{h}=\frac{\dfrac{1}{a+h}-\dfrac{1}{a}}{h}=\frac{\dfrac{a-(a+h)}{a(a+h)}}{h}=-\frac{1}{a(a+h)}.$$

52. $$\frac{f(a+h)-f(a)}{h}=\frac{\sqrt{a+h}-\sqrt{a}}{h}\cdot\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}=\frac{(a+h)-a}{h\left(\sqrt{a+h}+\sqrt{a}\right)}=\frac{1}{\sqrt{a+h}+\sqrt{a}}.$$

53. $F(t)$ represents the total revenue for the two restaurants at time t.

54. $F(t)$ represents the net rate of growth of the species of whales in year t.

55. $f(t)g(t)$ represents the dollar value of Nancy's holdings at time t.

56. $f(t)/g(t)$ represents the unit cost of the commodity at time t.

57. $g\circ f$ is the function giving the amount of carbon monoxide pollution from cars in parts per million at time t.

58. $f\circ g$ is the function giving the revenue at time t.

59. $C(x)=0.6x+12{,}100.$

60. a. $D(t) = (f - g)(t) = f(t) - g(t) = \left(1.54t^2 + 7.1t + 31.4\right) - \left(1.21t^2 + 6t + 14.5\right) = 0.33t^2 + 1.1t + 16.9$, so $D(4) = 0.33(4)^2 + 1.1(4) + 16.9 = 26.58$. This gives the total number of non-spam email messages per day as approximately 26.6 billion.

b. $P(t) = \left(\dfrac{f}{g}\right)(t) = \dfrac{f(t)}{g(t)} = \dfrac{1.54t^2 + 7.1t + 31.4}{1.21t^2 + 6t + 14.5}$, so $P(4) = \dfrac{1.54(4)^2 + 7.1(4) + 31.4}{1.21(4)^2 + 6(4) + 14.5} \approx 1.459$. This says that the ratio of all email messages to spam messages is approximately 1.5.

61. $D(t) = (D_2 - D_1)(t) = D_2(t) - D_1(t) = \left(0.035t^2 + 0.21t + 0.24\right) - \left(0.0275t^2 + 0.081t + 0.07\right)$

$= 0.0075t^2 + 0.129t + 0.17$.

The function D gives the difference in year t between the deficit without the \$160 million rescue package and the deficit with the rescue package.

62. a. $(g \circ f)(0) = g(f(0)) = g(0.64) = 26$, so the mortality rate of motorcyclists in the year 2000 was 26 per 100 million miles traveled.

b. $(g \circ f)(6) = g(f(6)) = g(0.51) = 42$, so the mortality rate of motorcyclists in 2006 was 42 per 100 million miles traveled.

c. Between 2000 and 2006, the percentage of motorcyclists wearing helmets had dropped from 64 to 51, and as a consequence, the mortality rate of motorcyclists had increased from 26 million miles traveled to 42 million miles traveled.

63. a. $(g \circ f)(1) = g(f(1)) = g(406) = 23$. So in 2002, the percentage of reported serious crimes that end in arrests or in the identification of suspects was 23.

b. $(g \circ f)(6) = g(f(6)) = g(326) = 18$. In 2007, 18% of reported serious crimes ended in arrests or in the identification of suspects.

c. Between 2002 and 2007, the total number of detectives had dropped from 406 to 326 and as a result, the percentage of reported serious crimes that ended in arrests or in the identification of suspects dropped from 23 to 18.

64. $C(x) = V(x) + 100{,}000 = 0.000003x^3 - 0.03x^2 + 200x + 100{,}000$, so $C(2000) = 0.000003(2000)^3 - 0.03(2000)^2 + 200(2000) + 100{,}000 = 404{,}000$, or \$404,000.

65. a. $P(x) = R(x) - C(x) = -0.1x^2 + 500x - \left(0.000003x^3 - 0.03x^2 + 200x + 100{,}000\right)$

$= -0.000003x^3 - 0.07x^2 + 300x - 100{,}000$.

b. $P(1500) = -0.000003(1500)^3 - 0.07(1500)^2 + 300(1500) - 100{,}000 = 182{,}375$, or \$182,375.

66. a. $C(x) = V(x) + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20{,}000$.

b. $P(x) = R(x) - C(x) = -0.02x^2 + 150x - 0.000001x^3 + 0.01x^2 - 50x - 20{,}000$

$= -0.000001x^3 - 0.01x^2 + 100x - 20{,}000$.

c. $P(2000) = -0.000001(2000)^3 - 0.01(2000)^2 + 100(2000) - 20{,}000 = 132{,}000$, or \$132,000.

67. a. The gap is $N(t) - C(t) = \left(3.5t^2 + 26.7t + 436.2\right) - (24.3t + 365) = 3.5t^2 + 2.4t + 71.2$.

b. At the beginning of 1983, the gap was $G(0) = 3.5(0)^2 + 2.4(0) + 71.2 = 71.2$, or 71,200.
At the beginning of 1986, the gap was $G(3) = 3.5(3)^2 + 2.4(3) + 71.2 = 109.9$, or 109,900.

68. **a.** $N(r(t)) = \dfrac{7}{1 + 0.02\left(\dfrac{5t + 75}{t + 10}\right)^2}$.

b. $N(r(0)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 0 + 75}{0 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{75}{10}\right)^2} \approx 3.29$, or 3.29 million units.

$N(r(12)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 12 + 75}{12 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{135}{22}\right)^2} \approx 3.99$, or 3.99 million units.

$N(r(18)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 18 + 75}{18 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{165}{28}\right)^2} \approx 4.13$, or 4.13 million units.

69. **a.** The occupancy rate at the beginning of January is $r(0) = \frac{10}{81}(0)^3 - \frac{10}{3}(0)^2 + \frac{200}{9}(0) + 55 = 55$, or 55%.
$r(5) = \frac{10}{81}(5)^3 - \frac{10}{3}(5)^2 + \frac{200}{9}(5) + 55 \approx 98.2$, or approximately 98.2%.

b. The monthly revenue at the beginning of January is $R(r(0)) = R(55) = -\frac{3}{5000}(55)^3 + \frac{9}{50}(55)^2 \approx 444.68$, or approximately \$444,700.
The monthly revenue at the beginning of June is $R(r(5)) \approx R(98.2) = -\frac{3}{5000}(98.2)^3 + \frac{9}{50}(98.2)^2 \approx 1167.6$, or approximately \$1,167,600.

70. $N(t) = N(x(t)) = 1.42 \cdot x(t) = \dfrac{1.42 \cdot 7(t + 10)^2}{(t + 10)^2 + 2(t + 15)^2} = \dfrac{9.94(t + 10)^2}{(t + 10)^2 + 2(t + 15)^2}$. The number of jobs created 6 months from now will be $N(6) = \dfrac{9.94(16)^2}{(16)^2 + 2(21)^2} \approx 2.24$, or approximately 2.24 million jobs. The number of jobs created 12 months from now will be $N(12) = \dfrac{9.94(22)^2}{(22)^2 + 2(27)^2} \approx 2.48$, or approximately 2.48 million jobs.

71. **a.** $s = f + g + h = (f + g) + h = f + (g + h)$. This suggests we define the sum s by $s(x) = (f + g + h)(x) = f(x) + g(x) + h(x)$.

b. Let f, g, and h define the revenue (in dollars) in week t of three branches of a store. Then its total revenue (in dollars) in week t is $s(t) = (f + g + h)(t) = f(t) + g(t) + h(t)$.

72. **a.** $(h \circ g \circ f)(x) = h(g(f(x)))$

b. Let t denote time. Suppose f gives the number of people at time t in a town, g gives the number of cars as a function of the number of people in the town, and H gives the amount of carbon monoxide in the atmosphere. Then $(h \circ g \circ f)(t) = h(g(f(t)))$ gives the amount of carbon monoxide in the atmosphere at time t.

73. True. $(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x)$.

74. False. Let $f(x) = x + 2$ and $g(x) = \sqrt{x}$. Then $(g \circ f)(x) = \sqrt{x + 2}$ is defined at $x = -1$, But $(f \circ g)(x) = \sqrt{x} + 2$ is not defined at $x = -1$.

75. False. Take $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Then $(g \circ f)(x) = \sqrt{x} + 1$, but $(f \circ g)(x) = \sqrt{x + 1}$.

76. False. Take $f(x) = x + 1$. Then $(f \circ f)(x) = f(f(x)) = x + 2$, but $f^2(x) = [f(x)]^2 = (x + 1)^2 = x^2 + 2x + 1$.

2.5 Linear Functions

Concept Questions

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1. **a.** A linear function is a function of the form $f(x) = mx + b$, where m and b are constants. For example, $f(x) = 2x + 3$ is a linear function.
   b. The domain and range of a linear function are both $(-\infty, \infty)$.
   c. The graph of a linear function is a straight line.

2. $C(x) = cx + F, R(x) = sx$, and $P(x) = R(x) - C(x) = (s - c)x - F$.

3. **a.** The break-even point $P_0(x_0, p_0)$ is the solution of the simultaneous equations $p = R(x)$ and $p = C(x)$.
   b. The number x_0 is called the break-even quantity.
   c. The number p_0 is called the break-even revenue.

Exercises

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1. Yes. Solving for y in terms of x, we find $3y = -2x + 6$, or $y = -\frac{2}{3}x + 2$.

2. Yes. Solving for y in terms of x, we find $4y = 2x + 7$, or $y = \frac{1}{2}x + \frac{7}{4}$.

3. Yes. Solving for y in terms of x, we find $2y = x + 4$, or $y = \frac{1}{2}x + 2$.

4. Yes. Solving for y in terms of x, we have $3y = 2x - 8$, or $y = \frac{2}{3}x - \frac{8}{3}$.

5. Yes. Solving for y in terms of x, we have $4y = 2x + 9$, or $y = \frac{1}{2}x + \frac{9}{4}$.

6. Yes. Solving for y in terms of x, we find $6y = 3x + 7$, or $y = \frac{1}{2}x + \frac{7}{6}$.

7. y is not a linear function of x because of the quadratic term $2x^2$.

8. y is not a linear function of x because of the nonlinear term $3\sqrt{x}$.

9. y is not a linear function of x because of the nonlinear term $-3y^2$.

10. y is not a linear function of x because of the nonlinear term $\sqrt{y}$.

11. **a.** $C(x) = 8x + 40{,}000$, where x is the number of units produced.
    b. $R(x) = 12x$, where x is the number of units sold.
    c. $P(x) = R(x) - C(x) = 12x - (8x + 40{,}000) = 4x - 40{,}000$.
    d. $P(8000) = 4(8000) - 40{,}000 = -8000$, or a loss of \$8,000. $P(12{,}000) = 4(12{,}000) - 40{,}000 = 8000$, or a profit of \$8000.

12. **a.** $C(x) = 14x + 100{,}000$.
    b. $R(x) = 20x$.
    c. $P(x) = R(x) - C(x) = 20x - (14x + 100{,}000) = 6x - 100{,}000$.
    d. $P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$, or a loss of \$28,000.
    $P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$, or a profit of \$20,000.

13. $f(0) = 4$ gives $m(0) + b = 4$, or $b = 4$. Thus, $f(x) = mx + 4$. Next, $f(3) = -2$ gives $m(3) + 4 = -2$, or $m = -2$.

14. The fact that the straight line represented by $f(x) = mx + b$ has slope -1 tells us that $m = -1$ and so $f(x) = -x + b$. Next, the condition $f(2) = 4$ gives $f(2) = -1(2) + b = 4$, or $b = 6$.

15. We solve the system $y = 3x + 4$, $y = -2x + 19$. Substituting the first equation into the second yields $3x + 4 = -2x + 19$, $5x = 15$, and $x = 3$. Substituting this value of x into the first equation yields $y = 3(3) + 4$, so $y = 13$. Thus, the point of intersection is $(3, 13)$.

16. We solve the system $y = -4x - 7$, $-y = 5x + 10$. Substituting the first equation into the second yields $-(-4x - 7) = 5x + 10$, $4x + 7 = 5x + 10$, and $x = -3$. Substituting this value of x into the first equation, we obtain $y = -4(-3) - 7 = 12 - 7 = 5$. Therefore, the point of intersection is $(-3, 5)$.

17. We solve the system $2x - 3y = 6$, $3x + 6y = 16$. Solving the first equation for y, we obtain $3y = 2x - 6$, so $y = \frac{2}{3}x - 2$ .Substituting this value of y into the second equation, we obtain $3x + 6\left(\frac{2}{3}x - 2\right) = 16$, $3x + 4x - 12 = 16$, $7x = 28$, and $x = 4$. Then $y = \frac{2}{3}(4) - 2 = \frac{2}{3}$, so the point of intersection is $\left(4, \frac{2}{3}\right)$.

18. We solve the system $2x + 4y = 11$, $-5x + 3y = 5$. Solving the first equation for x, we find $x = -2y + \frac{11}{2}$. Substituting this value into the second equation of the system, we have $-5\left(-2y + \frac{11}{2}\right) + 3y = 5$, so $10y - \frac{55}{2} + 3y = 5$, $20y - 55 + 6y = 10$, $26y = 65$, and $y = \frac{5}{2}$. Substituting this value of y into the first equation, we have $2x + 4\left(\frac{5}{2}\right) = 11$, so $2x = 1$ and $x = \frac{1}{2}$. Thus, the point of intersection is $\left(\frac{1}{2}, \frac{5}{2}\right)$.

19. We solve the system $y = \frac{1}{4}x - 5$, $2x - \frac{3}{2}y = 1$. Substituting the value of y given in the first equation into the second equation, we obtain $2x - \frac{3}{2}\left(\frac{1}{4}x - 5\right) = 1$, so $2x - \frac{3}{8}x + \frac{15}{2} = 1$, $16x - 3x + 60 = 8$, $13x = -52$, and $x = -4$. Substituting this value of x into the first equation, we have $y = \frac{1}{4}(-4) - 5 = -1 - 5$, so $y = -6$. Therefore, the point of intersection is $(-4, -6)$.

20. We solve the system $y = \frac{2}{3}x - 4$, $x + 3y + 3 = 0$. Substituting the first equation into the second equation, we obtain $x + 3\left(\frac{2}{3}x - 4\right) + 3 = 0$, so $x + 2x - 12 + 3 = 0$, $3x = 9$, and $x = 3$. Substituting this value of x into the first equation, we have $y = \frac{2}{3}(3) - 4 = -2$. Therefore, the point of intersection is $(3, -2)$.

21. We solve the equation $R(x) = C(x)$, or $15x = 5x + 10{,}000$, obtaining $10x = 10{,}000$, or $x = 1000$. Substituting this value of x into the equation $R(x) = 15x$, we find $R(1000) = 15{,}000$. Therefore, the break-even point is $(1000, 15000)$.

22. We solve the equation $R(x) = C(x)$, or $21x = 15x + 12{,}000$, obtaining $6x = 12{,}000$, or $x = 2000$. Substituting this value of x into the equation $R(x) = 21x$, we find $R(2000) = 42{,}000$. Therefore, the break-even point is $(2000, 42000)$.

23. We solve the equation $R(x) = C(x)$, or $0.4x = 0.2x + 120$, obtaining $0.2x = 120$, or $x = 600$. Substituting this value of x into the equation $R(x) = 0.4x$, we find $R(600) = 240$. Therefore, the break-even point is $(600, 240)$.

24. We solve the equation $R(x) = C(x)$ or $270x = 150x + 20{,}000$, obtaining $120x = 20{,}000$ or $x = \frac{500}{3} \approx 167$. Substituting this value of x into the equation $R(x) = 270x$, we find $R(167) = 45{,}090$. Therefore, the break-even point is $(167, 45090)$.

25. Let V be the book value of the office building after 2005. Since $V = 1{,}000{,}000$ when $t = 0$, the line passes through $(0, 1000000)$. Similarly, when $t = 50$, $V = 0$, so the line passes through $(50, 0)$. Then the slope of the line is given by $m = \dfrac{0 - 1{,}000{,}000}{50 - 0} = -20{,}000$. Using the point-slope form of the equation of a line with the point $(0, 1000000)$, we have $V - 1{,}000{,}000 = -20{,}000(t - 0)$, or $V = -20{,}000t + 1{,}000{,}000$.
In 2010, $t = 5$ and $V = -20{,}000(5) + 1{,}000{,}000 = 900{,}000$, or \$900,000.
In 2015, $t = 10$ and $V = -20{,}000(10) + 1{,}000{,}000 = 800{,}000$, or \$800,000.

26. Let V be the book value of the automobile after 5 years. Since $V = 24{,}000$ when $t = 0$, and $V = 0$ when $t = 5$, the slope of the line L is $m = \dfrac{0 - 24{,}000}{5 - 0} = -4800$. Using the point-slope form of an equation of a line with the point $(0, 5)$, we have $V - 0 = -4800(t - 5)$, or $V = -4800t + 24{,}000$. If $t = 3$, $V = -4800(3) + 24{,}000 = 9600$. Therefore, the book value of the automobile at the end of three years will be \$9600.

27. a. $y = I(x) = 1.033x$, where x is the monthly benefit before adjustment and y is the adjusted monthly benefit.

b. His adjusted monthly benefit is $I(1520) = 1.033(1520) = 1570.16$, or \$1570.16.

28. $C(x) = 8x + 48{,}000$.

b. $R(x) = 14x$.

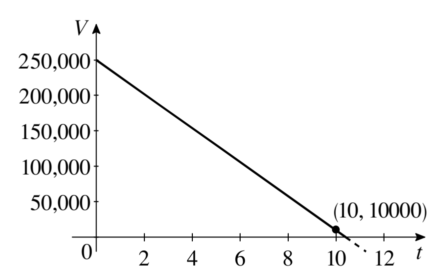
c. $P(x) = R(x) - C(x) = 14x - (8x + 48{,}000) = 6x - 48{,}000$.

d. $P(4000) = 6(4000) - 48{,}000 = -24{,}000$, a loss of \$24,000.
$P(6000) = 6(6000) - 48{,}000 = -12{,}000$, a loss of \$12,000.
$P(10{,}000) = 6(10{,}000) - 48{,}000 = 12{,}000$, a profit of \$12,000.

29. Let the number of tapes produced and sold be x. Then $C(x) = 12{,}100 + 0.60x$, $R(x) = 1.15x$, and $P(x) = R(x) - C(x) = 1.15x - (12{,}100 + 0.60x) = 0.55x - 12{,}100$.

30. a. Let V denote the book value of the machine after t years. Since $V = 250{,}000$ when $t = 0$ and $V = 10{,}000$ when $t = 10$, the line passes through the points $(0, 250000)$ and $(10, 10000)$. The slope of the line through these points is given by $m = \dfrac{10{,}000 - 250{,}000}{10 - 0} = -\dfrac{240{,}000}{10} = -24{,}000$. Using the point-slope form of an equation of a line with the point $(10, 10000)$, we have $V - 10{,}000 = -24{,}000(t - 10)$, or $V = -24{,}000t + 250{,}000$.

b.



c. In 2011, $t = 4$ and $V = -24{,}000(4) + 250{,}000 = 154{,}000$, or \$154,000.

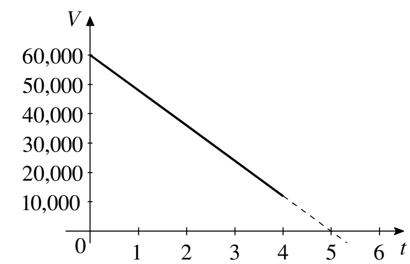
d. The rate of depreciation is given by $-m$, or \$24,000/yr.

31. Let the value of the workcenter system after t years be V. When $t = 0$, $V = 60{,}000$ and when $t = 4$, $V = 12{,}000$.

a. Since $m = \frac{12{,}000 - 60{,}000}{4} = -\frac{48{,}000}{4} = -12{,}000$, the rate of depreciation $(-m)$ is \$12,000/yr.

b. Using the point-slope form of the equation of a line with the point (4, 12000), we have $V - 12{,}000 = -12{,}000\,(t - 4)$, or $V = -12{,}000t + 60{,}000$.

c.



d. When $t = 3$, $V = -12{,}000\,(3) + 60{,}000 = 24{,}000$, or \$24,000.

32. The slope of the line passing through the points $(0, C)$ and (N, S) is $m = \frac{S - C}{N - 0} = \frac{S - C}{N} = -\frac{C - S}{N}$. Using the point-slope form of an equation of a line with the point $(0, C)$, we have $V - C = -\frac{C - S}{N}t$, or $V = C - \frac{C - S}{N}t$.

33. The formula given in Exercise 32 is $V = C - \frac{C - S}{N}t$. When $C = 1{,}000{,}000$, $N = 50$, and $S = 0$, we have $V = 1{,}000{,}000 - \frac{1{,}000{,}000 - 0}{50}t$, or $V = 1{,}000{,}000 - 20{,}000t$. In 2010, $t = 5$ and $V = 1{,}000{,}000 - 20{,}000\,(5) = 900{,}000$, or \$900,000. In 2015, $t = 10$ and $V = 1{,}000{,}000 - 20{,}000\,(10) = 800{,}000$, or \$800,000.

34. The formula given in Exercise 32 is $V = C - \frac{C - S}{N}t$. When $C = 24{,}000$, $N = 5$, and $S = 0$, we have $V = 24{,}000 - \frac{24{,}000 - 0}{5}t = 24{,}000 - 4800t$. When $t = 3$, $V = 24{,}000 - 4800\,(3) = 9600$, or \$9600.

35. a. $D\,(S) = \frac{Sa}{1.7}$. If we think of D as having the form $D\,(S) = mS + b$, then $m = \frac{a}{1.7}$, $b = 0$, and D is a linear function of S.

b. $D\,(0.4) = \frac{500\,(0.4)}{1.7} \approx 117.647$, or approximately 117.65 mg.

36. a. $D\,(t) = \frac{(t + 1)}{24}a = \frac{a}{24}t + \frac{a}{24}$. If we think of D as having the form $D\,(t) = mt + b$, then $m = \frac{a}{24}$, $b = \frac{a}{24}$, and D is a linear function of t.

b. If $a = 500$ and $t = 4$, $D\,(4) = \frac{4 + 1}{24}\,(500) = 104.167$, or approximately 104.2 mg.

37. a. $f\,(t) = 6.5t + 20$, where $0 \le t \le 8$.

b. $f\,(8) = 6.5\,(8) + 20 = 72$, or 72 million.

38. a. Let y denote the number of households in millions. Then $y = 42.5$ when $t = 0$. The rate of decline is the slope of the linear equation in t and y. Thus, with $m = -3.9$, we find $y = -3.9t + 42.5$.

b. The projected number of households at the beginning of 2010 (when $t = 6$) is $y = -3.9\,(6) + 42.5 = 19.1$, or 19.1 million.

39. a. Since the relationship is linear, we can write $F = mC + b$, where m and b are constants. Using the condition $C = 0$ when $F = 32$, we have $32 = b$, and so $F = mC + 32$. Next, using the condition $C = 100$ when $F = 212$, we have $212 = 100m + 32$, or $m = \frac{9}{5}$. Therefore, $F = \frac{9}{5}C + 32$.

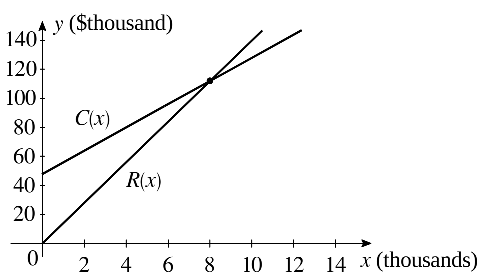
b. From part a, we have $F = \frac{9}{5}C + 32$. When $C = 20$, $F = \frac{9}{5}(20) + 32 = 68$, and so the temperature equivalent to 20° C is 68° F.

c. Solving for C in terms of F, we find $\frac{9}{5}C = F - 32$, or $C = \frac{5}{9}F - \frac{160}{9}$. When $F = 70$, $C = \frac{5}{9}(70) - \frac{160}{9} = \frac{190}{9}$, or approximately 21.1° C.

40. a. Since the relationship between T and N is linear, we can write $N = mT + b$, where m and b are constants.

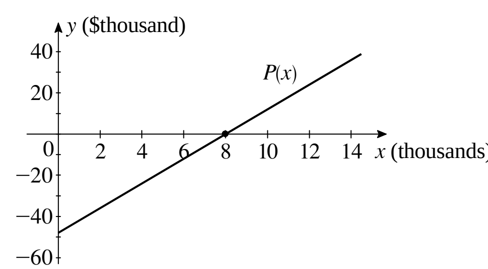
b. Using the points (70, 120) and (80, 160), we find that the slope of the line joining these points is $\dfrac{160 - 120}{80 - 70} = \dfrac{40}{10} = 4$. If $T = 70$, then $N = 120$, and this gives $120 = 70(4) + b$, or $b = -160$. Therefore, $N = 4T - 160$. If $T = 102$, we find $N = 4(102) - 160 = 248$, or 248 chirps per minute.

41. a. $C(x) = 8x + 48{,}000$ and $R(x) = 14x$.



c. $P(x) = R(x) - C(x) = 14x - 8x - 48{,}000$

$= 6x - 48{,}000.$



b. We solve the equation $R(x) = C(x)$ or $14x = 8x + 48{,}000$, obtaining $6x = 48{,}000$, so $x = 8000$. Substituting this value of x into the equation $R(x) = 14x$, we find $R(8000) = 14(8000) = 112{,}000$. Therefore, the break-even point is (8000, 112000).

d. The graph of the profit function crosses the x-axis when $P(x) = 0$, or $6x = 48{,}000$ and $x = 8000$. This means that the revenue is equal to the cost when 8000 units are produced and consequently the company breaks even at this point.

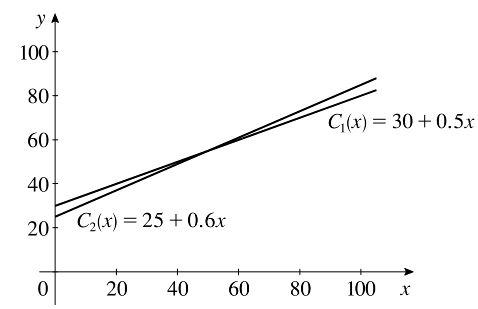
42. a. $R(x) = 8x$ and $C(x) = 25{,}000 + 3x$, so $P(x) = R(x) - C(x) = 5x - 25{,}000$. The break-even point occurs when $P(x) = 0$, that is, $5x - 25{,}000 = 0$, or $x = 5000$. Then $R(5000) = 40{,}000$, so the break-even point is (5000, 40000).

b. If the division realizes a 15% profit over the cost of making the diaries, then $P(x) = 0.15\,C(x)$, so $5x - 25{,}000 = 0.15(25{,}000 + 3x)$, $4.55x = 28{,}750$, and $x \approx 6318.68$, or approximately 6319 diaries.

43. Let x denote the number of units sold. Then, the revenue function R is given by $R(x) = 9x$. Since the variable cost is 40% of the selling price and the monthly fixed costs are \$50,000, the cost function C is given by $C(x) = 0.4(9x) + 50{,}000 = 3.6x + 50{,}000$. To find the break-even point, we set $R(x) = C(x)$, obtaining $9x = 3.6x + 50{,}000$, $5.4x = 50{,}000$, and $x \approx 9259$, or 9259 units. Substituting this value of x into the equation $R(x) = 9x$ gives $R(9259) = 9(9259) = 83{,}331$. Thus, for a break-even operation, the firm should manufacture 9259 bicycle pumps, resulting in a break-even revenue of \$83,331.

44. a. The cost function associated with renting a truck from the Ace Truck Leasing Company is $C_1(x) = 30 + 0.50x$. The cost function associated with renting a truck from the Acme Truck Leasing Company is $C_2(x) = 25 + 0.60x$.

b.

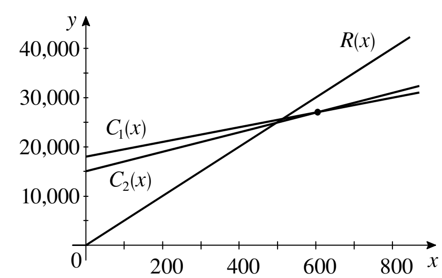


c. The cost of renting a truck from the Ace Truck Leasing Company for one day and driving 70 miles is $C_1(70) = 30 + 0.50(70) = 65$, or \$65.

The cost of renting a truck from the Acme Truck Leasing Company for one day and driving it 70 miles is $C_2(70) = 25 + 0.60(70) = 67$, or \$67. Thus, the customer should rent the car from Ace Truck Leasing Company. This answer may also be obtained by inspecting the graph of the two functions and noting that the graph of $C_1(x)$ lies below that of $C_2(x)$ for $x \le 50$.

45. a. The cost function associated with using machine I is $C_1(x) = 18{,}000 + 15x$. The cost function associated with using machine II is $C_2(x) = 15{,}000 + 20x$.

b.



c. Comparing the cost of producing 450 units on each machine, we find $C_1(450) = 18{,}000 + 15(450) = 24{,}750$ or \$24,750 on machine I, and $C_2(450) = 15{,}000 + 20(450) = 24{,}000$ or \$24,000 on machine II. Therefore, machine II should be used in this case. Next, comparing the costs of producing 550 units on each machine, we find $C_1(550) = 18{,}000 + 15(550) = 26{,}250$ or \$26,250 on machine I, and $C_2(550) = 15{,}000 + 20(550) = 26{,}000$, or \$26,000 on machine II. Therefore, machine II should be used in this instance. Once again, we compare the cost of producing 650 units on each machine and find that $C_1(650) = 18{,}000 + 15(650) = 27{,}750$, or \$27,750 on machine I and $C_2(650) = 15{,}000 + 20(650) = 28{,}000$, or \$28,000 on machine II. Therefore, machine I should be used in this case.

d. We use the equation $P(x) = R(x) - C(x)$ and find $P(450) = 50(450) - 24{,}000 = -1500$, indicating a loss of \$1500 when machine II is used to produce 450 units. Similarly, $P(550) = 50(550) - 26{,}000 = 1500$, indicating a profit of \$1500 when machine II is used to produce 550 units. Finally, $P(650) = 50(650) - 27{,}750 = 4750$, for a profit of \$4750 when machine I is used to produce 650 units.

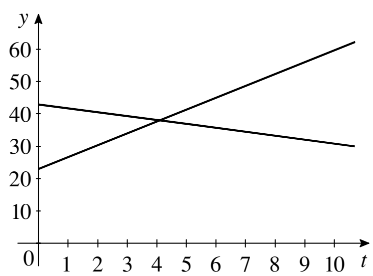
46. First, we find the point of intersection of the two straight lines. (This gives the time when the sales of both companies are the same). Substituting the first equation into the second gives $2.3 + 0.4t = 1.2 + 0.6t$, so $1.1 = 0.2t$ and $t = \frac{1.1}{0.2} = 5.5$. From the observation that the sales of Cambridge Drug Store are increasing at a faster rate than that of the Crimson Drug Store (its trend line has the greater slope), we conclude that the sales of the Cambridge Drug Store will surpass the annual sales of the Crimson Drug Store in $5\frac{1}{2}$ years.

47. We solve the two equations simultaneously, obtaining $18t + 13.4 = -12t + 88$, $30t = 74.6$, and $t \approx 2.487$, or approximately 2.5 years. So shipments of LCDs will first overtake shipments of CRTs just before mid-2003.

48. a. The number of digital cameras sold in 2001 is given by $f(0) = 3.05(0) + 6.85 = 6.85$, or 6.85 million. The number of film cameras sold in 2001 is given by $g(0) = -1.85(0) + 16.58$, or 16.58 million. Therefore, more film cameras than digital cameras were sold in 2001.

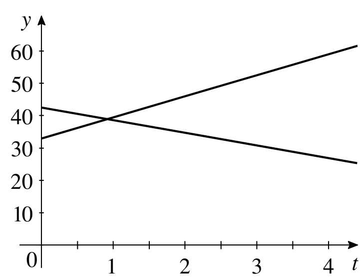
b. The sales are equal when $3.05t + 6.85 = -1.85t + 16.58$, $4.9t = 9.73$, or $t = \frac{9.73}{4.9} \approx 1.986$, approximately 2 years. Therefore, digital camera sales surpassed film camera sales near the end of 2003.

49. a.



b. We solve the two equations simultaneously, obtaining $\frac{11}{3}t + 23 = -\frac{11}{9}t + 43$, $\frac{44}{9}t = 20$, and $t \approx 4.09$. Thus, electronic transactions first exceeded check transactions in early 2005.

50. a.



b. $6.5t + 33 = -3.9t + 42.5$, $10.4t = 9.5$, $t \approx 0.91$, and so $f(0.91) = g(0.91) \approx 38.9$. The number of U.S. broadband Internet households was the same as the number of dial-up Internet households (39 million each) around November of 2004. Since then, the former has exceeded the latter.

51. True. The slope of the line is $-a$.

52. True. $P(x) = R(x) - C(x) = sx - (cx + F) = (s - c)x - F$. Therefore, the firm is making a profit if $P(x) = (s - c)x - F > 0$; that is, if $x > \dfrac{F}{s - c}$ $(s \neq c)$.

Technology Exercises page 126

1. 2.2875 **2.** 3.0125 **3.** 2.880952381 **4.** 0.7875

5. 7.2851648352 **6.** −26.82928836 **7.** 2.4680851064 **8.** 1.24375

2.6 Quadratic Functions

Concept Questions page 132

1. a. $(-\infty, \infty)$. **b.** It opens upward. **c.** $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$. **d.** $-\dfrac{b}{2a}$.

2. a. A demand function defined by $p = f(x)$ expresses the relationship between the unit price p and the quantity demanded x. It is a decreasing function of x.

A supply function defined by $p = f(x)$ expresses the relation between the unit price p and the quantity supplied x. It is an increasing function of x.

b. Market equilibrium occurs when the quantity produced is equal to the quantity demanded.

c. The equilibrium quantity is the quantity produced at market equilibrium. The equilibrium price is the price corresponding to the equilibrium quantity. These quantities are found by finding the point at which the demand curve and the supply curve intersect.

Exercises

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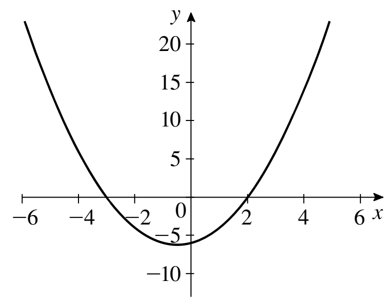
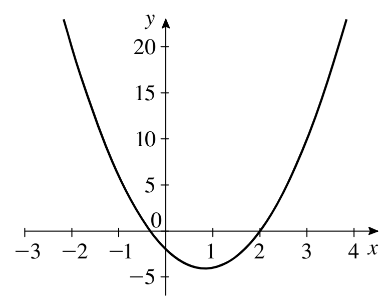
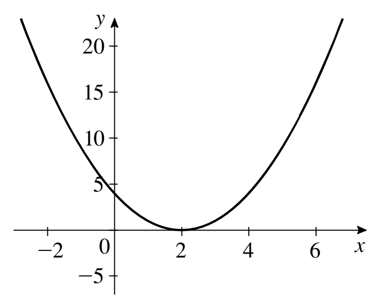
1. $f(x) = x^2 + x - 6$; $a = 1$, $b = 1$, and $c = -6$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{1}{2(1)} = -\frac{1}{2}$ and the y-coordinate is $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 6 = -\frac{25}{4}$. Therefore, the vertex is $\left(-\frac{1}{2}, -\frac{25}{4}\right)$. Setting $x^2 + x - 6 = (x+3)(x-2) = 0$ gives -3 and 2 as the x-intercepts.



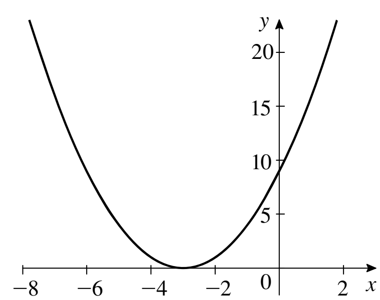
2. $f(x) = 3x^2 - 5x - 2$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{(-5)}{6} = \frac{5}{6}$ and the y-coordinate is $f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 2 = -\frac{49}{12}$. Therefore, the vertex is $\left(\frac{5}{6}, -\frac{49}{12}\right)$. Setting $3x^2 - 5x - 2 = (3x+1)(x-2) = 0$ gives $-1/3$ and 2 as the x-intercepts.



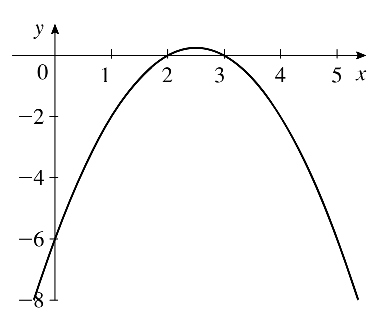
3. $f(x) = x^2 - 4x + 4$; $a = 1$, $b = -4$, and $c = 4$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{(-4)}{2} = 2$ and the y-coordinate is $f(2) = 2^2 - 4(2) + 4 = 0$.Therefore, the vertex is $(2, 0)$. Setting $x^2 - 4x + 4 = (x-2)^2 = 0$ gives 2 as the x-intercept.



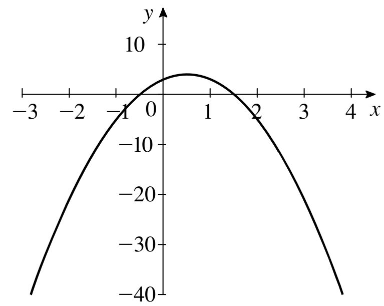
4. $f(x) = x^2 + 6x + 9$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{6}{2} = -3$ and the y-coordinate is $f(-3) = (-3)^2 + 6(-3) + 9 = 0$. Therefore, the vertex is $(-3, 0)$. Setting $x^2 + 6x + 9 = (x+3)^2 = 0$ gives -3 as the x-intercept.



5. $f(x) = -x^2 + 5x - 6$; $a = -1$, $b = 5$, and $c = -6$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{5}{2(-1)} = \frac{5}{2}$ and the y-coordinate is $f\left(\frac{5}{2}\right) = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - 6 = \frac{1}{4}$.Therefore, the vertex is $\left(\frac{5}{2}, \frac{1}{4}\right)$. Setting $-x^2 + 5x - 6 = 0$ or $x^2 - 5x + 6 = (x-3)(x-2) = 0$ gives 2 and 3 as the x-intercepts.

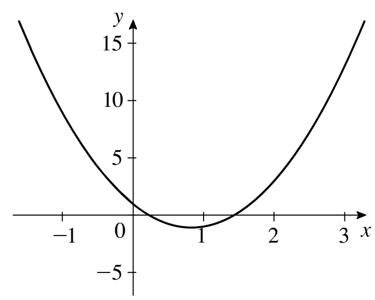


6. $f(x) = -4x^2 + 4x + 3$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{4}{2(-4)} = \frac{1}{2}$ and the y-coordinate is $f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 3 = 4$. Therefore, the vertex is $\left(\frac{1}{2}, 4\right)$. Setting $-4x^2 + 4x + 3 = 0$, or equivalently, $4x^2 - 4x - 3 = (2x - 3)(2x + 1) = 0$ giving $-\frac{1}{2}$ and $\frac{3}{2}$ as the x-intercepts.



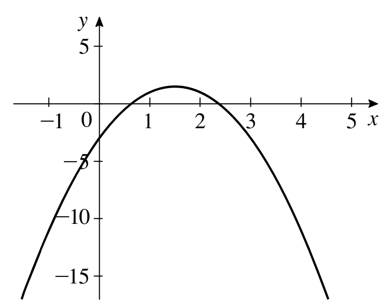
7. $f(x) = 3x^2 - 5x + 1$; $a = 3$, $b = -5$, and $c = 1$; The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{(-5)}{2(3)} = \frac{5}{6}$ and the y-coordinate is $f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 1 = -\frac{13}{12}$. Therefore, the vertex is $\left(\frac{5}{6}, -\frac{13}{12}\right)$. Next, solving $3x^2 - 5x + 1 = 0$, we use the quadratic formula and obtain

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{13}}{6}$$ and so the x-intercepts are 0.23241 and 1.43426.

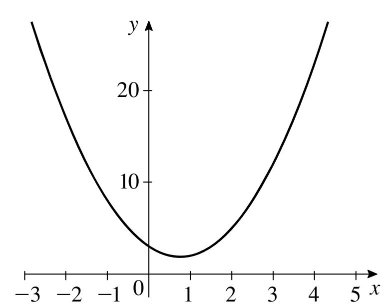


8. $f(x) = -2x^2 + 6x - 3$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{6}{2(-2)} = \frac{3}{2}$ and the y-coordinate is $f\left(\frac{3}{2}\right) = -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) - 3 = \frac{3}{2}$.Therefore, the vertex is $\left(\frac{3}{2}, \frac{3}{2}\right)$. Next, solving $-2x^2 + 6x - 3 = 0$ using the quadratic formula, we find

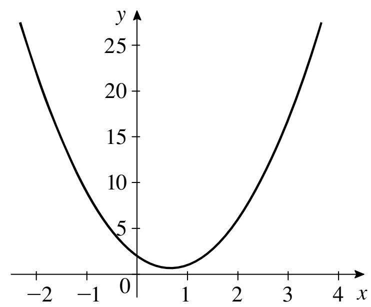
$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)(-3)}}{2(-2)} = \frac{-6 \pm \sqrt{12}}{-4} = \frac{3}{2} \pm \frac{\sqrt{3}}{2}$$ and so the x-intercepts are 0.63397 and 2.36603.



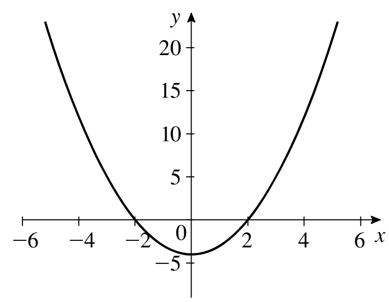
9. $f(x) = 2x^2 - 3x + 3$; $a = 2$, $b = -3$, and $c = 3$.The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{(-3)}{2(2)} = \frac{3}{4}$ and the y-coordinate is $f\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 3 = \frac{15}{8}$. Therefore, the vertex is $\left(\frac{3}{4}, \frac{15}{8}\right)$. Next, observe that the discriminant of the quadratic equation $2x^2 - 3x + 3 = 0$ is $(-3)^2 - 4(2)(3) = 9 - 24 = -15 < 0$ and so it has no real roots. In other words, there are no x-intercepts.



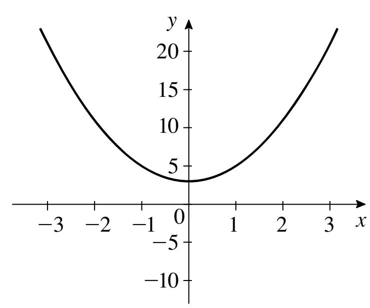
10. $f(x) = 3x^2 - 4x + 2$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-4}{2(3)} = \frac{2}{3}$ and the y-coordinate is $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 2 = \frac{2}{3}$. Therefore, the vertex is $\left(\frac{2}{3}, \frac{2}{3}\right)$. Next, observe that the discriminant of the quadratic equation $3x^2 - 4x + 2 = 0$ is $(-4)^2 - 4(3)(2) = 16 - 24 = -8 < 0$ and so it has no real roots. Therefore, the parabola has no x-intercepts.



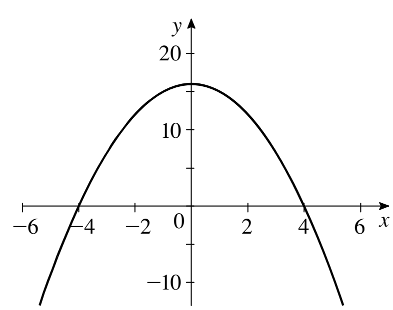
11. $f(x) = x^2 - 4$; $a = 1$, $b = 0$, and $c = -4$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{0}{2(1)} = 0$ and the y-coordinate is $f(0) = -4$. Therefore, the vertex is $(0, -4)$. The x-intercepts are found by solving $x^2 - 4 = (x + 2)(x - 2) = 0$ giving $x = -2$ or $x = 2$.



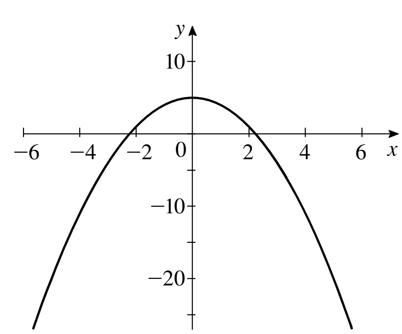
12. $f(x) = 2x^2 + 3$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{0}{2(2)} = 0$ and the y-coordinate is $f(0) = 3$. Therefore, the vertex is $(0, 3)$. Since $2x^2 + 3 \geq 3 > 0$, we see that there are no x-intercepts.



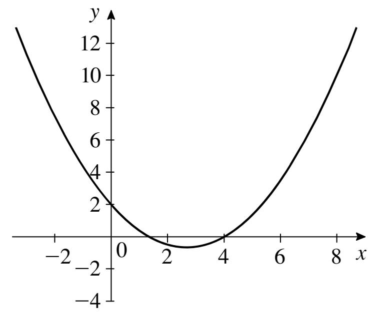
13. $f(x) = 16 - x^2$; $a = -1$, $b = 0$, and $c = 16$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{0}{2(-1)} = 0$ and the y-coordinate is $f(0) = 16$. Therefore, the vertex is $(0, 16)$. The x-intercepts are found by solving $16 - x^2 = 0$, giving $x = -4$ or $x = 4$.



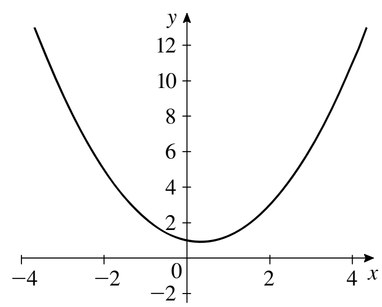
14. $f(x) = 5 - x^2$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{0}{2(-1)} = 0$ and the y-coordinate is $f(0) = 5$. Therefore, the vertex is $(0, 5)$. The x-intercepts are found by solving $5 - x^2 = 0$, giving $x = \pm\sqrt{5} \approx \pm 2.23607$.



15. $f(x) = \frac{3}{8}x^2 - 2x + 2$; $a = \frac{3}{8}$, $b = -2$, and $c = 2$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{(-2)}{2\left(\frac{3}{8}\right)} = \frac{8}{3}$ and the y-coordinate is $f\left(\frac{8}{3}\right) = \frac{3}{8}\left(\frac{8}{3}\right)^2 - 2\left(\frac{8}{3}\right) + 2 = -\frac{2}{3}$. Therefore, the vertex is $\left(\frac{8}{3}, -\frac{2}{3}\right)$. The equation $f(x) = 0$ can be written $3x^2 - 16x + 16 = (3x-4)(x-4) = 0$ giving $x = \frac{4}{3}$ or $x = 4$ and so the x-intercepts are $\frac{4}{3}$ and 4.



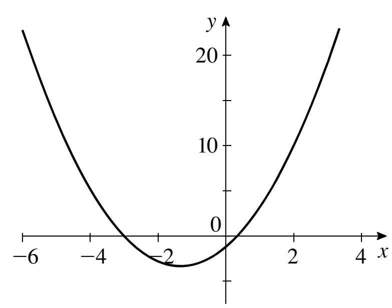
16. $f(x) = \frac{3}{4}x^2 - \frac{1}{2}x + 1$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-\frac{1}{2}}{2\left(\frac{3}{4}\right)} = \frac{1}{3}$, and the y-coordinate is $f\left(\frac{1}{3}\right) = \frac{3}{4}\left(\frac{1}{3}\right)^2 - \frac{1}{2}\left(\frac{1}{3}\right) + 1 = \frac{11}{12}$. Therefore, the vertex is $\left(\frac{1}{3}, \frac{11}{12}\right)$. The discriminant of the equation $f(x) = 0$ is $\left(-\frac{1}{2}\right)^2 - 4\left(\frac{3}{4}\right)(1) = -\frac{11}{4} < 0$ and this shows that there are no x-intercepts.



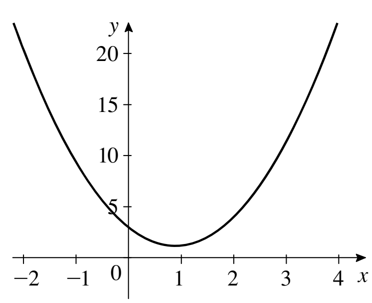
17. $f(x) = 1.2x^2 + 3.2x - 1.2$, so $a = 1.2$, $b = 3.2$, and $c = -1.2$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{3.2}{2(1.2)} = -\frac{4}{3}$ and the y-coordinate is $f\left(-\frac{4}{3}\right) = 1.2\left(-\frac{4}{3}\right)^2 + 3.2\left(-\frac{4}{3}\right)(1) - 1.2 = -\frac{10}{3}$. Therefore, the vertex is $\left(-\frac{4}{3}, -\frac{10}{3}\right)$. Next, we solve $f(x) = 0$ using the quadratic formula, obtaining

$$x = \frac{-3.2 \pm \sqrt{(3.2)^2 - 4(1.2)(-1.2)}}{2(1.2)} = \frac{-3.2 \pm \sqrt{16}}{2(1.2)} = \frac{-3.2 \pm 4}{2(1.2)} = -3$$

or $\frac{1}{3}$. Therefore, the x-intercepts are -3 and $\frac{1}{3}$.



18. $f(x) = 2.3x^2 - 4.1x + 3$. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-4.1}{2(2.3)} = 0.891304$ and the y-coordinate is $f(0.891304) = 2.3(0.891304)^2 - 4.1(0.891304) + 3 = 1.172826$. Therefore, the vertex is $(0.8913, 1.1728)$. The discriminant of the equation $f(x) = 0$ is $(-4.1)^2 - 4(2.3)(3) = -10.79 < 0$ and so it has no real roots. Therefore, there are no x-intercepts.



19. We solve the equation $-x^2 + 4 = x - 2$. Rewriting, we have $x^2 + x - 6 = (x+3)(x-2) = 0$, giving $x = -3$ or $x = 2$. Therefore, the points of intersection are $(-3, -5)$ and $(2, 0)$.

20. We solve $x^2 - 5x + 6 = \frac{1}{2}x + \frac{3}{2}$ or $x^2 - \frac{11}{2}x + \frac{9}{2} = 0$. Rewriting, we obtain $2x^2 - 11x + 9 = (2x - 9)(x - 1) = 0$ giving $x = 1$ or $\frac{9}{2}$. Therefore, the points of intersection are $(1, 2)$ and $\left(\frac{9}{2}, \frac{15}{4}\right)$.

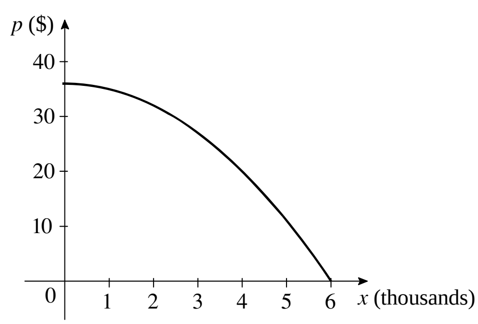
21. We solve $-x^2 + 2x + 6 = x^2 - 6$, or $2x^2 - 2x - 12 = 0$. Rewriting, we have $x^2 - x - 6 = (x - 3)(x + 2) = 0$, giving $x = -2$ or 3. Therefore, the points of intersection are $(-2, -2)$ and $(3, 3)$.

22. We solve $x^2 - 2x - 2 = -x^2 - x + 1$, or $2x^2 - x - 3 = (2x - 3)(x + 1) = 0$ giving $x = -1$ or $\frac{3}{2}$. Therefore, the points of intersection are $(-1, 1)$ and $\left(\frac{3}{2}, -\frac{11}{4}\right)$.

23. We solve $2x^2 - 5x - 8 = -3x^2 + x + 5$, or $5x^2 - 6x - 13 = 0$. Using the quadratic formula, we obtain $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-13)}}{2(5)} = \dfrac{6 \pm \sqrt{296}}{10} \approx -1.12047$ or 2.32047. Next, we find $f(-1.12047) = 2(-1.12047)^2 - 5(-1.12047) - 8 \approx 0.11326$ and $f(2.32047) = 2(2.32047)^2 - 5(2.32047) - 8 \approx -8.8332$. Therefore, the points of intersection are $(-1.1205, 0.1133)$ and $(2.3205, -8.8332)$.

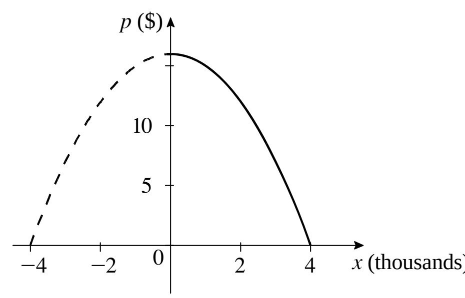
24. We solve $0.2x^2 - 1.2x - 4 = -0.3x^2 + 0.7x + 8.2$, or $0.5x^2 - 1.9x - 12.2 = 0$. Using the quadratic formula, we find $x = \dfrac{-(-1.9) \pm \sqrt{(-1.9)^2 - 4(0.5)(-12.2)}}{2(0.5)} = 1.9 \pm \sqrt{28.01} \approx -3.39245$ or 7.19245. Next, we find $f(-3.39245) = 0.2(-3.39245)^2 - 1.2(-3.39245) - 4 \approx 2.37268$ and and $f(7.19245) = 0.2(7.19245)^2 - 1.2(7.19245) - 4 \approx -2.28467$. Therefore, the points of intersection are $(-3.3925, 2.3727)$ and $(7.1925, -2.2847)$.

25. a.



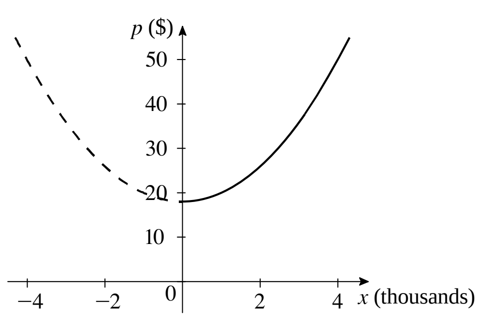
b. If $p = 11$, we have $11 = -x^2 + 36$, or $x^2 = 25$, so that $x = \pm 5$. Therefore, the quantity demanded when the unit price is \$11 is 5000 units.

26. a.



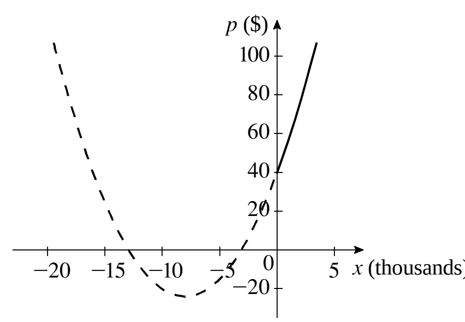
b. If $p = 7$, we have $7 = -x^2 + 16$, or $x^2 = 9$, so that $x = \pm 3$. Therefore, the quantity demanded when the unit price is \$7 is 3000 units.

27. a.



b. If $x = 2$, then $p = 2(2)^2 + 18 = 26$, or \$26.

28. a.



b. If $x = 2$, then $p = 2^2 + 16(2) + 40 = 76$, or \$76.

29. We solve the equation $-2x^2 + 80 = 15x + 30$, or $-2x^2 + 80 = 15x + 30$, or $2x^2 + 15x - 50 = 0$, for x. Thus, $(2x - 5)(x + 10) = 0$, so $x = \frac{5}{2}$ or $x = -10$. Rejecting the negative root, we have $x = \frac{5}{2}$. The corresponding value of p is $p = -2\left(\frac{5}{2}\right)^2 + 80 = 67.5$. We conclude that the equilibrium quantity is 2500 and the equilibrium price is \$67.50.

30. We solve the system of equations $\begin{cases} p = -x^2 - 2x + 100 \\ p = 8x + 25 \end{cases}$ Thus, $-x^2 - 2x + 100 = 8x + 25$, or $x^2 + 10x - 75 = 0$. Factoring the left-hand side, we have $(x + 15)(x - 5) = 0$, so $x = -15$ or $x = 5$. We reject the negative root, so $x = 5$ and the corresponding value of p is $p = 8(5) + 25 = 65$. We conclude that the equilibrium quantity is 5000 and the equilibrium price is \$65.

31. Solving both equations for x, we have $x = -\frac{11}{3}p + 22$ and $x = 2p^2 + p - 10$. Equating the right-hand sides of these two equations, we have $-\frac{11}{3}p + 22 = 2p^2 + p - 10$, $-11p + 66 = 6p^2 + 3p - 30$, and $6p^2 + 14p - 96 = 0$. Dividing this last equation by 2 and then factoring, we have $(3p + 16)(p - 3) = 0$, so discarding the negative root $p = -\frac{16}{3}$, we conclude that $p = 3$. The corresponding value of x is $2(3)^2 + 3 - 10 = 11$. Thus, the equilibrium quantity is 11,000 and the equilibrium price is \$3.

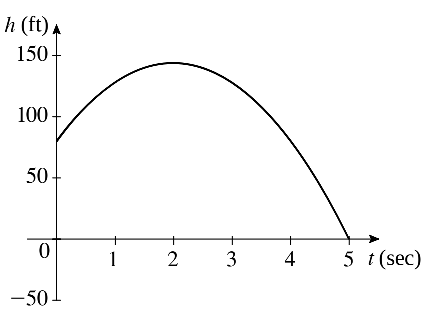
32. We solve the system $\begin{cases} p = 60 - 2x^2 \\ p = x^2 + 9x + 30 \end{cases}$ Equating the right-hand-sides of the two equations, we have $x^2 + 9x + 30 = 60 - 2x^2$, so $3x^2 + 9x - 30 = 0$, $x^2 + 3x - 10 = 0$, and $(x + 5)(x - 2) = 0$. Thus, $x = -5$ (which we discard) or $x = 2$. The corresponding value of p is 52. Therefore, the equilibrium quantity is 2000 and the equilibrium price is \$52.

33. a. $N(0) = 3.6$, or 3.6 million people; $N(25) = 0.0031(25)^2 + 0.16(25) + 3.6 = 9.5375$, or approximately 9.5 million people.

b. $N(30) = 0.0031(30)^2 + 0.16(30) + 3.6 = 11.19$, or approximately 11.2 million people.

34. The percentage of the U.S. population expected to have Alzheimer's disease at age 65 is given by $P(0) = 0.0726(0)^2 + 0.7902(0) + 4.9623 = 4.9623$, or 4.96%. The percentage of the U.S. population expected to have Alzheimer's disease at age 90 is given by $P(25) = 0.0726(25)^2 + 0.7902(25) + 4.9623 = 70.09$, or 70.09%.

35. a.

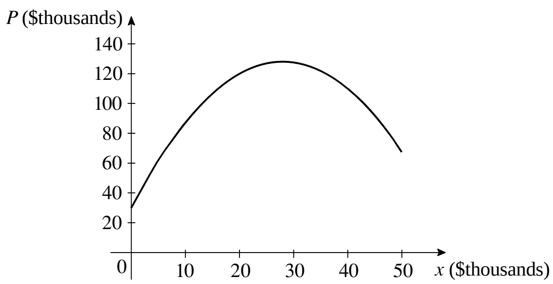


b. The time at which the stone reaches the highest point is given by the t-coordinate of the vertex of the parabola. This is $\frac{-b}{2a} = -\frac{64}{2(-16)} = 2$, so the stone it reaches its maximum height 2 seconds after it was thrown. Its maximum height is given by $h(2) = -16(2)^2 + 64(2) + 80 = 144$, or 144 ft.

36. The optimal number of units to be rented out is given by the x-coordinate of the vertex of the parabola; that is, by $\frac{-b}{2a} = \frac{-1760}{2(-10)} = 88$, or 88 units. The maximum profit is given by $P(88) = -10(88)^2 + 1760(88) - 50{,}000 = 27{,}440$, or \$27,440 per month.

37. $P(x) = -0.04x^2 + 240x - 10{,}000$. The optimal production level is given by the x-coordinate of the vertex of parabola; that is, by $\frac{-b}{2a} = -\frac{240}{2(-0.04)} = 3000$, or 3000 cameras.

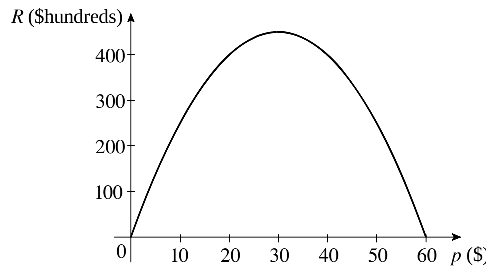
38. a. $P(x) = -\frac{1}{8}x^2 + 7x + 30$



b. The required advertising expenditure is given by the x-coordinate of the vertex of the parabola; that is by

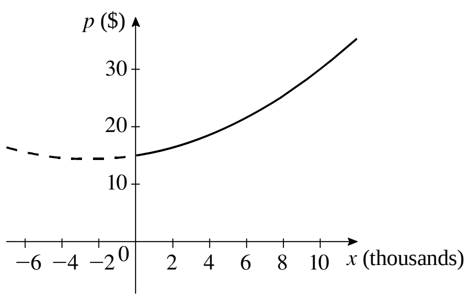
$$\frac{-b}{2a} = -\frac{7}{2\left(-\frac{1}{8}\right)} = 28, \text{ or } \$28{,}000 \text{ per quarter.}$$

39. a. $R(p) = -\frac{1}{2}p^2 + 30p$.



b. The monthly revenue is maximized when $p = -\frac{30}{2\left(-\frac{1}{2}\right)} = 30$; that is, when the unit price is \$30.

40. $p = 0.1x^2 + 0.5x + 15$

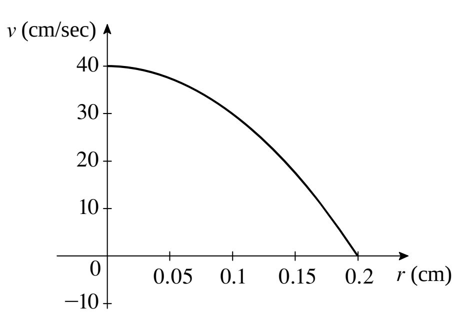


If $x = 5$, then $p = 0.1(5)^2 + 0.5(5) + 15 = 20$, or \$20.

41. Equating the right-hand sides of the two equations, we have $0.1x^2 + 2x + 20 = -0.1x^2 - x + 40$, so $0.2x^2 + 3x - 20 = 0$, $2x^2 + 30x - 200 = 0$, $x^2 + 15x - 100 = 0$, and $(x + 20)(x - 5) = 0$. Thus, $x = -20$ or $x = 5$. Discarding the negative root and substituting $x = 5$ into the first equation, we obtain $p = -0.1(25) - 5 + 40 = 32.5$. Therefore, the equilibrium quantity is 500 tents and the equilibrium price is \$32.50.

42. Equating the right-hand sides of the two equations, we have $144 - x^2 = 48 + \frac{1}{2}x^2$, so $288 - 2x^2 = 96 + x^2$, $3x^2 = 192$, and $x^2 = 64$. We discard the negative root and take $x = 8$. The corresponding value of p is $144 - 8^2 = 80$. We conclude that the equilibrium quantity is 8000 tires and the equilibrium price is \$80.

43. a.



b. $v(r) = -1000r^2 + 40$. Its graph is a parabola, as shown in part a. $v(r)$ has a maximum value at $r = -\frac{0}{2(-1000)} = 0$ and a minimum value at $r = 0.2$ (r must be nonnegative). Thus the velocity of blood is greatest along the central artery (where $r = 0$) and smallest along the wall of the artery (where $r = 0.2$). The maximum velocity is $v(0) = 40$ cm/sec and the minimum velocity is $v(0.2) = 0$ cm/sec.

44. The graph of $s(t) = -16t^2 + 128t + 4$ is a parabola that opens downward. The vertex of the parabola is is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. Here $a = -16$ and $b = 128$. Therefore, the t-coordinate of the vertex is $t = -\frac{128}{2(-16)} = 4$ and the s-coordinate is $s(4) = -16(4)^2 + 128(4) + 4 = 260$. So the ball reaches the maximum height after 4 seconds; its maximum height is 260 ft.

45. We want the window to have the largest possible area given the constraints. The area of the window is $A = 2xy + \frac{1}{2}\pi x^2$. The constraint on the perimeter dictates that $2x + 2y + \pi x = 28$. Solving for y gives $y = \dfrac{28 - 2x - \pi x}{2}$. Therefore, $A = 2x\left(\dfrac{28 - 2x - \pi x}{2}\right) + \frac{1}{2}\pi x^2 = \dfrac{56x - 4x^2 - 2\pi x^2 + \pi x^2}{2} = \dfrac{-(\pi + 4)x^2 + 56x}{2}$. A is maximized at $x = -\dfrac{b}{2a} = -\dfrac{56}{-2(\pi + 4)} = \dfrac{28}{\pi + 4}$ and $y = \dfrac{28 - \frac{56}{\pi+4} - \frac{28\pi}{\pi+4}}{2} = \dfrac{28\pi + 112 - 56 - 28\pi}{2(\pi + 4)} = \dfrac{28}{\pi + 4}$, or $\dfrac{28}{\pi + 4}$ ft.

46. $x^2 = \left(2\sqrt{y(h - y)}\right)^2 = 4y(h - y) = -4y^2 + 4hy$. The maximum of $f(y) = -4y^2 + 4hy$ is attained when $y = -\frac{b}{2a} = -\frac{4h}{2(-4)} = \frac{h}{2}$. So the hole should be located halfway up the tank. The maximum value of x is $x = 2\sqrt{\left(\frac{h}{2}\right)\left(h - \frac{h}{2}\right)} = 2\sqrt{\frac{h^2}{4}} = h$.

47. True. $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is a root of the equation $ax^2 + bx + c = 0$, and therefore $f\left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right) = 0$.

48. False. It has two roots if $b^2 - 4ac > 0$.

49. True. If a and c have opposite signs then $b^2 - 4ac > 0$ and the equation has 2 roots.

50. True. If $b^2 = 4ac$, then $x = -\frac{b}{2a}$ is the only root of the equation $ax^2 + bx + c = 0$, and the graph of the function f touches the x-axis at exactly one point.

51. True. The maximum occurs at the vertex of the parabola.

52. $$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left[x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right] = a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right]$$
$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}.$$

Technology Exercises

page 136

1. $(-3.0414, 0.1503)$, $(3.0414, 7.4497)$.

2. $(-5.3852, 9.8007)$, $(5.3852, -4.2007)$.

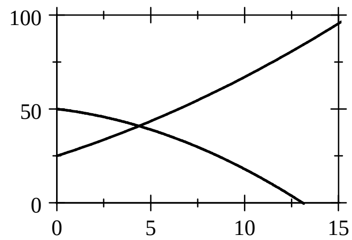
3. $(-2.3371, 2.4117)$, $(6.0514, -2.5015)$.

4. $(-2.5863, -0.3586)$, $(6.1863, -4.5694)$.

5. $(-1.1055, -6.5216)$ and $(1.1055, -1.8784)$

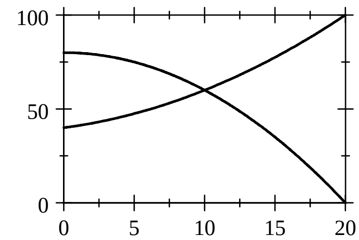
6. $(-0.0484, 2.0608)$ and $(1.4769, 2.8453)$.

7. a.



b. 438 wall clocks; \$40.92.

8. a.



b. 1000 cameras; \$60.00.

2.7 Functions and Mathematical Models

Concept Questions

page 144

1. See page 137 of the text. Answers will vary.

2. a. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where $a_n \neq 0$ and n is a positive integer. An example is $P(x) = 4x^3 - 3x^2 + 2$.

b. $R(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials with $Q(x) \neq 0$. An example is $R(x) = \dfrac{3x^4 - 2x^2 + 1}{x^2 + 3x + 5}$.

Exercises

page 144

1. f is a polynomial function in x of degree 6.

2. f is a rational function.

3. Expanding $G(x) = 2\left(x^2 - 3\right)^3$, we have $G(x) = 2x^6 - 18x^4 + 54x^2 - 54$, and we see that G is a polynomial function in x of degree 6.

4. We can write $H(x) = \dfrac{2}{x^3} + \dfrac{5}{x^2} + 6 = \dfrac{2 + 5x + 6x^3}{x^3}$, and we see that H is a rational function.

5. f is neither a polynomial nor a rational function.

6. f is a rational function.

7. a. The number of enterprise IM accounts in 2006 is given by $N(0) = 59.7$, or 59.7 million.

b. The number of enterprise IM accounts in 2010, assuming a continuing trend, is given by $N(4) = 2.96(4)^2 + 11.37(4) + 59.7 = 152.54$ million.

8. $S(6) = 0.73(6)^2 + 15.8(6) + 2.7 = 123.78$ million kilowatt-hr.
$S(8) = 0.73(8)^2 + 15.8(8) + 2.7 = 175.82$ million kilowatt-hr.

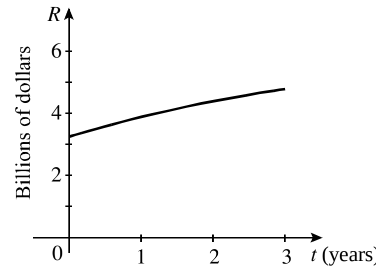
9. a. The property tax in 1997 is given by $N(0) = 2360$, or \$2360.

b. The property tax in 2010, assuming a continuing trend, is given by $N(13) = 7.26(13)^2 + 91.7(13) + 2360 = 4779.04$, or \$4779.04.

10. a. The revenue of the company in 2005 is given by $R(0) = 3.25$, or \$3.25 billion.

b. $R(1) = -0.06(1)^2 + 0.69(1) + 3.25 = 3.88$. This says that the revenue of the company in 2006 was \$3.88 billion. The revenue in 2007 was $R(2) = -0.06(2)^2 + 0.69(2) + 3.25 = 4.39$, or \$4.39 billion. The revenue in 2008 was $R(3) = -0.06(3)^2 + 0.69(3) + 3.25 = 4.78$, or \$4.78 billion.

c.



11. a. The amount of benefits paid out in 2010 is $S(0) = 0.72$, or \$720 million.

b. The amount of benefits projected to be paid out in 2040 is $S(3) = 0.1375(3)^2 + 0.5185(3) + 0.72 = 3.513$, or \$3.513 trillion.

12. a. The amount of Medicare benefits paid out in 2010 is $B(0) = 0.25$, or \$250 billion.

b. The amount of Medicare benefits projected to be paid out in 2040 is $B(3) = 0.09(3)^2 + (0.102)(3) + 0.25 = 1.366$, or \$1.366 trillion.

13. a. The given data imply that $R(40) = 50$, that is, $\dfrac{100(40)}{b+40} = 50$, so $50(b+40) = 4000$, or $b = 40$. Therefore, the required response function is $R(x) = \dfrac{100x}{40+x}$.

b. The response will be $R(60) = \dfrac{100(60)}{40+60} = 60$, or approximately 60 percent.

14. $N(0) = 0.7$ per 100 million vehicle miles driven. $N(7) = 0.0336(7)^3 - 0.118(7)^2 + 0.215(7) + 0.7 = 7.9478$ per 100 million vehicle miles driven.

15. a. Total global mobile data traffic in 2009 was $f(0) = 0.06$, or 60,000 terabytes.

b. The total in 2012 will be $f(3) = 0.021(3)^3 + 0.015(3)^2 + 0.12(3) + 0.06 = 1.122$, or 1,122,000 terabytes.

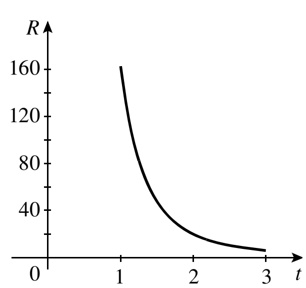
16. a. Sales in 2003 are given by $S(0) = 47.4$, or \$47.4 billion.

b. Sales in 2008 are given by $S(5) = -0.6204(5)^3 + 4.671(5)^2 + 3.354(5) + 47.4 = 103.395$, or approximately \$103.40 billion.

17. a. $N(0) = 0.32$ or 320,000.

b. $N(4) = -0.0675(4)^4 + 0.5083(4)^3 - 0.893(4)^2 + 0.66(4) + 0.32 = 3.9232$, or 3,923,200.

18. a.



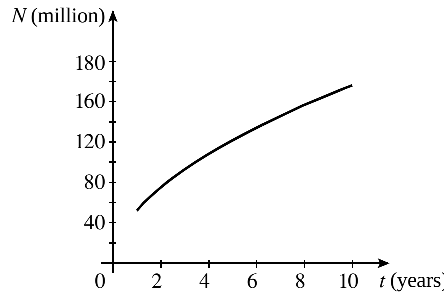
$R(1) = 162.8(1)^{-3.025} = 162.8$, $R(2) = 162.8(2)^{-3.025} \approx 20.0$, and $R(3) = 162.8(3)^{-3.025} \approx 5.9$.

b. The infant mortality rates in 1900, 1950, and 2000 are 162.8, 20.0, and 5.9 per 1000 live births, respectively.

19. a. We first construct a table.

t	$N(t)$
1	52
2	75
3	93
4	109
5	122

t	$N(t)$
6	135
7	146
8	157
9	167
10	177



b. The number of viewers in 2012 is given by $N(10) = 52(10)^{0.531} \approx 176.61$, or approximately 177 million viewers.

20. a. $S(0) = 4.3(0+2)^{0.94} \approx 8.24967$, or approximately \$8.25 billion.

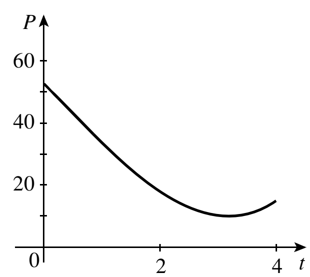
b. $S(8) = 4.3(8+2)^{0.94} \approx 37.45$, or approximately \$37.45 billion.

21. $N(5) = 0.0018425(10)^{2.5} \approx 0.58265$, or approximately 582,650. $N(10) = 0.0018425(15)^{2.5} \approx 1.60559$, or approximately 1,605,590.

22. a.

t	$P(t)$
0	52.7
1	33.7
2	18.0
3	10.2
4	14.9

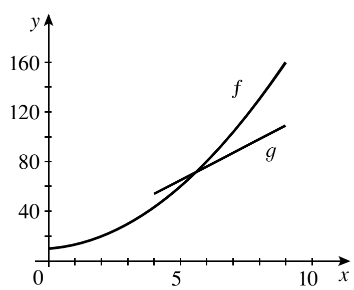
b.



c. Greatest in the 1950s; smallest in the 1980s.

23. $A(0) = \dfrac{699}{1^{0.94}} = 699$ or \$699. $A(5) = \dfrac{699}{6^{0.94}} \approx 129.722$, or approximately \$130.

24. a.



b. $5x^2 + 5x + 30 = 33x + 30$, so $5x^2 - 28x = 0$, $x(5x - 28) = 0$, and $x = 0$ or $x = \frac{28}{5} = 5.6$, representing 5.6 mi/h.
$g(x) = 11(5.6) + 10 = 71.6$, or 71.6 mL/lb/min.

c. The oxygen consumption of the walker is greater than that of the runner.

25. $h(t) = f(t) - g(t) = \dfrac{110}{\frac{1}{2}t+1} - 26\left(\frac{1}{4}t^2 - 1\right)^2 - 52.$

$h(0) = f(0) - g(0) = \dfrac{110}{\frac{1}{2}(0)+1} - 26\left[\frac{1}{4}(0)^2 - 1\right]^2 - 52 = 110 - 26 - 52 = 32$, or \$32.

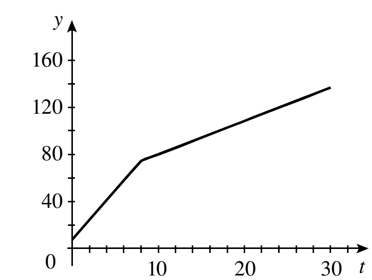
$h(1) = f(1) - g(1) = \dfrac{110}{\frac{1}{2}(1)+1} - 26\left[\frac{1}{4}(1)^2 - 1\right]^2 - 52 \approx 6.71$, or approximately \$6.71.

$h(2) = f(2) - g(2) = \dfrac{110}{\frac{1}{2}(2)+1} - 26\left[\frac{1}{4}(2)^2 - 1\right]^2 - 52 = 3$, or \$3. We conclude that the price gap was narrowing.

26. $P(0) = 4.6$ or 4.6%. $P(15) = -0.01005(15)^2 + 0.945(15) - 3.4 \approx 8.51$, or approximately 8.5%. $P(30) = -0.01005(30)^2 + 0.945(30) - 3.4 = 15.905$, or approximately 15.9%.

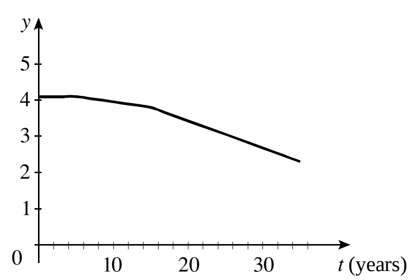
27. The total cost by 2011 is given by $f(1) = 5$, or \$5 billion. The total cost by 2015 is given by $f(5) = -0.5278(5^3) + 3.012(5^2) + 49.23(5) - 103.29 = 152.185$, or approximately \$152 billion.

28. a.



b. $f(0) = 8.37(0) + 7.44 = 7.44$, or \$7.44/kilo. $f(20) = 2.84(20) + 51.68 = 108.48$, or \$108.48/kilo.

29. a.

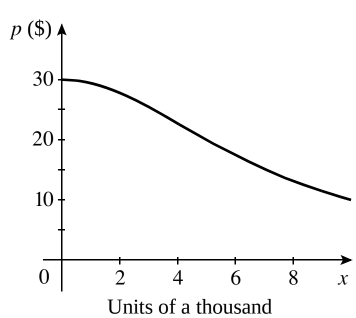


b. At the beginning of 2005, the ratio will be $f(10) = -0.03(10) + 4.25 = 3.95$. At the beginning of 2020, the ratio will be $f(25) = -0.075(25) + 4.925 = 3.05$.

c. The ratio is constant from 1995 to 2000.

d. The decline of the ratio is greatest from 2010 through 2030. It is $\dfrac{f(35) - f(15)}{35 - 15} = \dfrac{2.3 - 3.8}{20} = -0.075.$

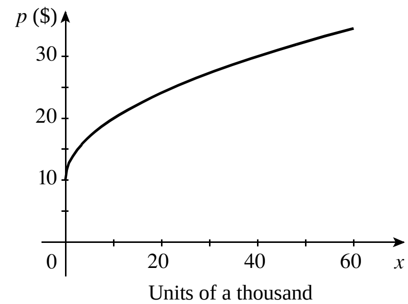
30. a.



b. Substituting $x = 10$ into the demand function, we have $p = \dfrac{30}{0.02(10)^2 + 1} = \dfrac{30}{3} = 10$, or \$10.

31. Substituting $x = 6$ and $p = 8$ into the given equation gives $8 = \sqrt{-36a + b}$, or $-36a + b = 64$. Next, substituting $x = 8$ and $p = 6$ into the equation gives $6 = \sqrt{-64a + b}$, or $-64a + b = 36$. Solving the system $\begin{cases} -36a + b = 64 \\ -64a + b = 36 \end{cases}$ for a and b, we find $a = 1$ and $b = 100$. Therefore the demand equation is $p = \sqrt{-x^2 + 100}$. When the unit price is set at \$7.50, we have $7.5 = \sqrt{-x^2 + 100}$, or $56.25 = -x^2 + 100$ from which we deduce that $x \approx \pm 6.614$. Thus, the quantity demanded is approximately 6614 units.

32. Substituting $x = 10{,}000$ and $p = 20$ into the given equation yields $20 = a\sqrt{10{,}000} + b = 100a + b$. Next, substituting $x = 62{,}500$ and $p = 35$ into the equation yields $35 = a\sqrt{62{,}500} + b = 250a + b$. Subtracting the first equation from the second yields $15 = 150a$, or $a = \frac{1}{10}$. Substituting this value of a into the first equation gives $b = 10$. Therefore, the required equation is $p = \frac{1}{10}\sqrt{x} + 10$. Substituting $x = 40{,}000$ into the supply equation yields $p = \frac{1}{10}\sqrt{40{,}000} + 10 = 30$, or \$30.



33. **a.** We solve the system of equations $p = cx + d$ and $p = ax + b$. Substituting the first equation into the second gives $cx + d = ax + b$, so $(c - a)x = b - d$ and $x = \dfrac{b - d}{c - a}$. Because $a < 0$ and $c > 0$, $c - a \neq 0$ and x is well-defined. Substituting this value of x into the second equation, we obtain $p = a\left(\dfrac{b - d}{c - a}\right) + b = \dfrac{ab - ad + bc - ab}{c - a} = \dfrac{bc - ad}{c - a}$. Therefore, the equilibrium quantity is $\dfrac{b - d}{c - a}$ and the equilibrium price is $\dfrac{bc - ad}{c - a}$.

b. If c is increased, the denominator in the expression for x increases and so x gets smaller. At the same time, the first term in the first equation for p decreases and so p gets larger. This analysis shows that if the unit price for producing the product is increased then the equilibrium quantity decreases while the equilibrium price increases.

c. If b is decreased, the numerator of the expression for x decreases while the denominator stays the same. Therefore, x decreases. The expression for p also shows that p decreases. This analysis shows that if the (theoretical) upper bound for the unit price of a commodity is lowered, then both the equilibrium quantity and the equilibrium price drop.

34. Because there is 80 feet of fencing available, $2x + 2y = 80$, so $x + y = 40$ and $y = 40 - x$. Then the area of the garden is given by $f = xy = x(40 - x) = 40x - x^2$. The domain of f is $[0, 40]$.

35. The area of Juanita's garden is 250 ft^2. Therefore $xy = 250$ and $y = \dfrac{250}{x}$. The amount of fencing needed is given by $2x + 2y$. Therefore, $f = 2x + 2\left(\dfrac{250}{x}\right) = 2x + \dfrac{500}{x}$. The domain of f is $x > 0$.

36. The volume of the box is given by area of the base times the height of the box. Thus, $V = f(x) = (15 - 2x)(8 - 2x)x$.

37. Because the volume of the box is the area of the base times the height of the box, we have $V = x^2 y = 20$. Thus, we have $y = \dfrac{20}{x^2}$. Next, the amount of material used in constructing the box is given by the area of the base of the box, plus the area of the four sides, plus the area of the top of the box; that is, $A = x^2 + 4xy + x^2$. Then, the cost of constructing the box is given by $f(x) = 0.30x^2 + 0.40x \cdot \dfrac{20}{x^2} + 0.20x^2 = 0.5x^2 + \dfrac{8}{x}$, where $f(x)$ is measured in dollars and $x > 0$.

38. **a.** $f(r) = \pi r^2$.

b. $g(t) = 2t$.

c. $h(t) = (f \circ g)(t) = f(g(t)) = \pi [g(t)]^2 = 4\pi t^2$.

d. $h(30) = 4\pi (30^2) = 3600\pi$, or 3600π ft^2.

39. The average yield of the apple orchard is 36 bushels/tree when the density is 22 trees/acre. Let x be the unit increase in tree density beyond 22. Then the yield of the apple orchard in bushels/acre is given by $(22 + x)(36 - 2x)$.

40. $xy = 50$ and so $y = \dfrac{50}{x}$. The area of the printed page is $A = (x - 1)(y - 2) = (x - 1)\left(\dfrac{50}{x} - 2\right) = -2x + 52 - \dfrac{50}{x}$, so the required function is $f(x) = -2x + 52 - \dfrac{50}{x}$. We must have $x > 0$, $x - 1 \geq 0$, and and $\dfrac{50}{x} - 2 \geq 0$. The last inequality is solved as follows: $\dfrac{50}{x} \geq 2$, so $\dfrac{x}{50} \leq \dfrac{1}{2}$ and $x \leq \frac{50}{2} = 25$. Thus, the domain is $[1, 25]$.

41. **a.** Let x denote the number of bottles sold beyond 10,000 bottles. Then $P(x) = (10{,}000 + x)(5 - 0.0002x) = -0.0002x^2 + 3x + 50{,}000$.

b. He can expect a profit of $P(6000) = -0.0002(6000^2) + 3(6000) + 50{,}000 = 60{,}800$, or \$60,800.

42. **a.** Let x denote the number of people beyond 20 who sign up for the cruise. Then the revenue is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12{,}000$.

b. $R(40) = -4(40^2) + 520(40) + 12{,}000 = 26{,}400$, or \$26,400.

c. $R(60) = -4(60^2) + 520(60) + 12{,}000 = 28{,}800$, or \$28,800.

43. False. $f(x) = 3x^{3/4} + x^{1/2} + 1$ is not a polynomial function. The powers of x must be nonnegative integers.

44. True. If $P(x)$ is a polynomial function, then $P(x) = \dfrac{P(x)}{1}$ and so it is a rational function. The converse is false. For example, $R(x) = \dfrac{x + 1}{x - 1}$ is a rational function that is not a polynomial.

45. False. $f(x) = x^{1/2}$ is not defined for negative values of x.

46. False. A power function has the form x^r, where r is a real number.

Technology Exercises

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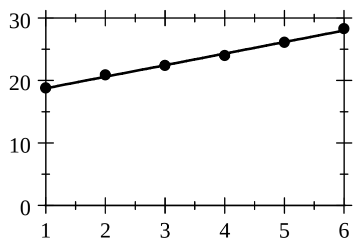
1. a. $f(t) = 1.85t + 16.9$.

b.



c.

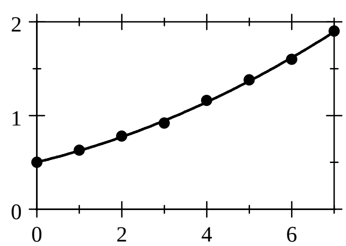
t	y
1	18.8
2	20.6
3	22.5
4	24.3
5	26.2
6	28.0

These values are close to the given data.

d. $f(8) = 1.85(8) + 16.9 = 31.7$ gallons.

2. a. $f(t) = 0.0128t^2 + 0.109t + 0.50$.

b.



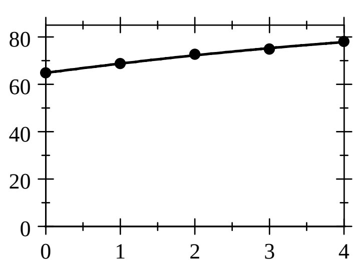
c.

t	y
0	0.50
3	0.94
6	1.61
7	1.89

These values are close to the given data.

3. a. $f(t) = -0.221t^2 + 4.14t + 64.8$.

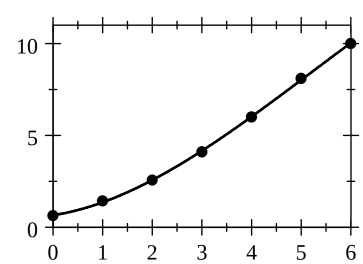
b.



c. 77.8 million

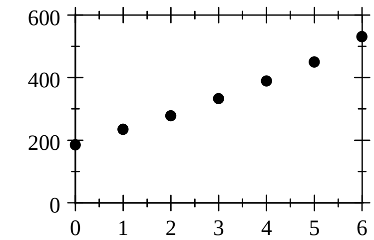
4. a. $y = -0.02028t^3 + 0.31393t^2 + 0.40873t + 0.66024$

b.



c. 0.66, 1.36, 2.57, 4.16, 6.02, 8.02, and 10.03.

5. a.

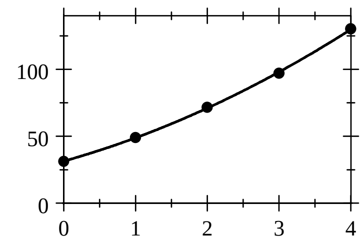


b. $f(t) = 2.94t^2 + 38.75t + 188.5$

c. The spending in 2010 was
$f(t) = 2.94\left(10^2\right) + 38.75(10) + 188.5 = 870$,
or approximately \$870 billion.

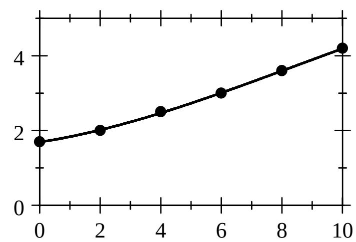
6. a. $f(t) = 2.4t^2 + 15t + 31.4$.

b.



7. a. $f(t) = -0.00081t^3 + 0.0206t^2 + 0.125t + 1.69$.

b.



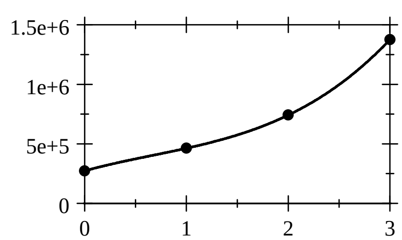
c.

t	y
1	1.8
5	2.7
10	4.2

The revenues were \$1.8 trillion in 2001, \$2.7 trillion in 2005, and \$4.2 trillion in 2010.

8. a. $y = 44{,}560t^3 - 89{,}394t^2 + 234{,}633t + 273{,}288$.

b.

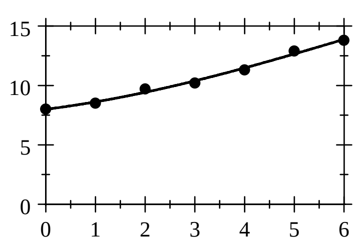


c.

t	$f(t)$
0	273,288
1	463,087
2	741,458
3	1,375,761

9. a. $f(t) = -0.0056t^3 + 0.112t^2 + 0.51t + 8$.

b.

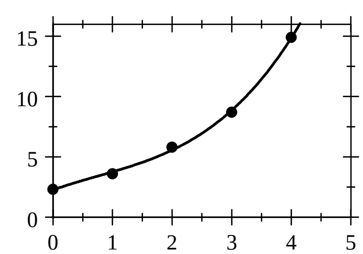


c.

t	0	3	6
$f(t)$	8	10.4	13.9

10. a. $f(t) = 0.2t^3 - 0.45t^2 + 1.75t + 2.26$.

b.

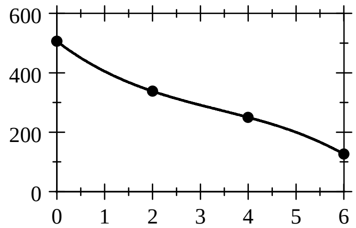


c.

t	0	1	2	3	4
$f(t)$	2.3	3.8	5.6	8.9	14.9

11. a. $f(t) = -2.4167t^3 + 24.5t^2 - 123.33t + 506.$

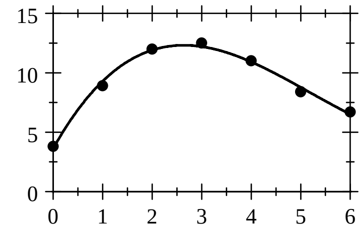
b.



c. $f(0) = 506$, or 506,000; $f(2) = 338$, or 338,000; and $f(6) = 126$, or 126,000.

12. a. $f(t) = 0.1222t^3 - 1.892t^2 + 7.434t + 3.63.$

b.

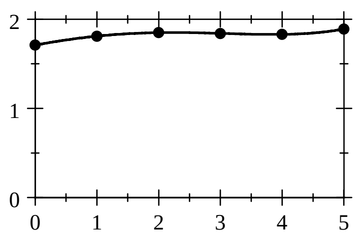


c.

t	1	2	3	4	5	6
$f(t)$	9.3	11.9	12.2	10.9	8.8	6.5

13. a. $f(t) = 0.00125t^4 - 0.0051t^3 - 0.0243t^2 + 0.129t + 1.71.$

b.



c.

t	0	1	2	3	4	5
$f(t)$	1.71	1.81	1.85	1.84	1.83	1.89

d. The average amount of nicotine in 2005 is $f(6) = 2.128$, or approximately 2.13 mg/cigarette.

14. $A(t) = 0.0000263t^4 - 0.0017501t^3 + 0.0206t^2 + 0.0999t + 2.7.$

CHAPTER 2 Concept Review Questions page 153

1. ordered, abscissa (x-coordinate), ordinate (y-coordinate)

2. a. x-, y-

b. third

3. a. $\dfrac{y_2 - y_1}{x_2 - x_1}$ **b.** undefined **c.** zero **d.** positive

4. $m_1 = m_2$, $m_1 = -\dfrac{1}{m_2}$

5. a. $y - y_1 = m(x - x_1)$, point-slope form

b. $y = mx + b$, slope-intercept

6. a. $Ax + By + C = 0$, where A and B are not both zero

b. $-a/b$

7. domain, range, B

8. domain, $f(x)$, vertical, point

9. $f(x) \pm g(x)$, $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$, $A \cap B$, $A \cap B$, 0

10. $g(f(x))$, f, $f(x)$, g

11. $ax^2 + bx + c$, parabola, upward, downward, vertex, $-\frac{b}{2a}$, $x = -\frac{b}{2a}$.

12. **a.** $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$ and n is a positive integer

b. linear, quadratic

c. quotient, polynomials

d. x^r, where r is a real number

CHAPTER 2 Review Exercises page 154

1. An equation is $x = -2$.

2. An equation is $y = 4$.

3. The slope of L is $m = \dfrac{\frac{7}{2} - 4}{3 - (-2)} = \dfrac{\frac{7-8}{2}}{5} = -\dfrac{1}{10}$ and an equation of L is $y - 4 = -\frac{1}{10}[x - (-2)] = -\frac{1}{10}x - \frac{1}{5}$, or $y = -\frac{1}{10}x + \frac{19}{5}$.

4. The line passes through the points $(3, 0)$ and $(-2, 4)$, so its slope is $m = \dfrac{4 - 0}{-2 - 3} = -\dfrac{4}{5}$. An equation is $y - 0 = -\frac{4}{5}(x - 3)$, or $y = -\frac{4}{5}x + \frac{12}{5}$.

5. Writing the given equation in the form $y = \frac{5}{2}x - 3$, we see that the slope of the given line is $\frac{5}{2}$. Thus, an equation is $y - 4 = \frac{5}{2}(x + 2)$, or $y = \frac{5}{2}x + 9$.

6. Writing the given equation in the form $y = -\frac{4}{3}x + 2$, we see that the slope of the given line is $-\frac{4}{3}$. Therefore, the slope of the required line is $\frac{3}{4}$ and an equation of the line is $y - 4 = \frac{3}{4}(x + 2)$, or $y = \frac{3}{4}x + \frac{11}{2}$.

7. Using the slope-intercept form of the equation of a line, we have $y = -\frac{1}{2}x - 3$.

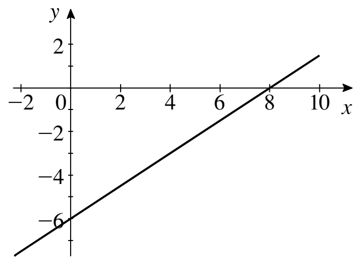
8. Rewriting the given equation in slope-intercept form, we have $-5y = -3x + 6$, or $y = \frac{3}{5}x - \frac{6}{5}$. From this equation, we see that the slope of the line is $\frac{3}{5}$ and its y-intercept is $-\frac{6}{5}$.

9. Rewriting the given equation in slope-intercept form, we have $4y = -3x + 8$, or $y = -\frac{3}{4}x + 2$, and we conclude that the slope of the required line is $-\frac{3}{4}$. Using the point-slope form of the equation of a line with the point $(2, 3)$ and slope $-\frac{3}{4}$, we obtain $y - 3 = -\frac{3}{4}(x - 2)$, so $y = -\frac{3}{4}x + \frac{6}{4} + 3 = -\frac{3}{4}x + \frac{9}{2}$.

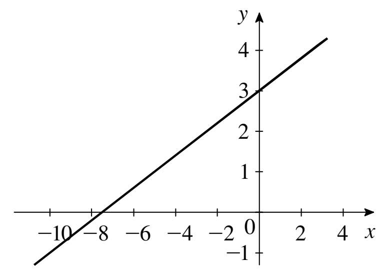
10. The slope of the line joining the points $(-3, 4)$ and $(2, 1)$ is $m = \dfrac{1 - 4}{2 - (-3)} = -\dfrac{3}{5}$. Using the point-slope form of the equation of a line with the point $(-1, 3)$ and slope $-\frac{3}{5}$, we have $y - 3 = -\frac{3}{5}[x - (-1)]$, so $y = -\frac{3}{5}(x + 1) + 3 = -\frac{3}{5}x + \frac{12}{5}$.

11. Rewriting the given equation in the slope-intercept form $y = \frac{2}{3}x - 8$, we see that the slope of the line with this equation is $\frac{2}{3}$. The slope of the required line is $-\frac{3}{2}$. Using the point-slope form of the equation of a line with the point $(-2, -4)$ and slope $-\frac{3}{2}$, we have $y - (-4) = -\frac{3}{2}[x - (-2)]$, or $y = -\frac{3}{2}x - 7$.

12. $3x - 4y = 24$. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept.



13. $-2x + 5y = 15$. Setting $x = 0$ gives $5y = 15$, or $y = 3$. Setting $y = 0$ gives $-2x = 15$, or $x = -\frac{15}{2}$.



14. $9 - x \geq 0$ gives $x \leq 9$, and the domain is $(-\infty, 9]$.

15. $2x^2 - x - 3 = (2x - 3)(x + 1)$, and $x = \frac{3}{2}$ or -1. Because the denominator of the given expression is zero at these points, we see that the domain of f cannot include these points and so the domain of f is $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

16. a. $f(-2) = 3(-2)^2 + 5(-2) - 2 = 0$.

b. $f(a+2) = 3(a+2)^2 + 5(a+2) - 2 = 3a^2 + 12a + 12 + 5a + 10 - 2 = 3a^2 + 17a + 20$.

c. $f(2a) = 3(2a)^2 + 5(2a) - 2 = 12a^2 + 10a - 2$.

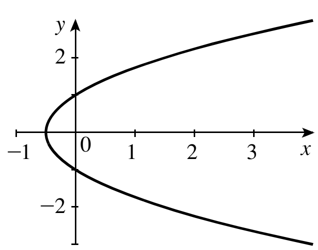
d. $f(a+h) = 3(a+h)^2 + 5(a+h) - 2 = 3a^2 + 6ah + 3h^2 + 5a + 5h - 2$.

17. a. From $t = 0$ to $t = 5$, the graph for cassettes lies above that for CDs, so from 1985 to 1990, the value of prerecorded cassettes sold was greater than that of CDs.

b. Sales of prerecorded CDs were greater than those of prerecorded cassettes from 1990 onward.

c. The graphs intersect at the point with coordinates $x = 5$ and $y \approx 3.5$, and this tells us that the sales of the two formats were the same in 1990 at the sales level of approximately \$3.5 billion.

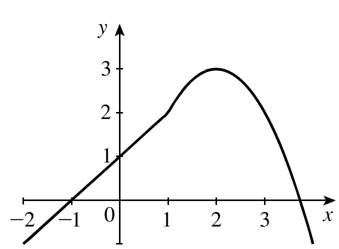
18. a.



b. For each value of $x > 0$, there are two values of y. We conclude that y is not a function of x. (We could also note that the function fails the vertical line test.)

c. Yes. For each value of y, there is only one value of x.

19.



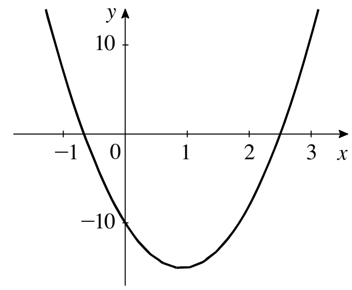
20. a. $f(x)\,g(x) = \dfrac{2x+3}{x}$.

b. $\dfrac{f(x)}{g(x)} = \dfrac{1}{x(2x+3)}$.

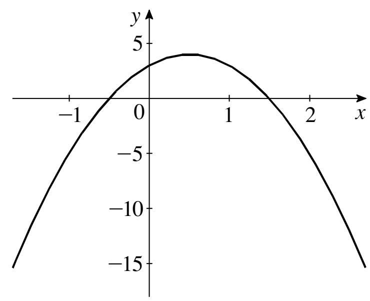
c. $f(g(x)) = \dfrac{1}{2x+3}$.

d. $g(f(x)) = 2\left(\dfrac{1}{x}\right) + 3 = \dfrac{2}{x} + 3$.

21. $y = 6x^2 - 11x - 10$. The x-coordinate of the vertex is $-\frac{-11}{2(6)} = \frac{11}{12}$ and the y-coordinate is $6\left(\frac{11}{12}\right)^2 - 11\left(\frac{11}{12}\right) - 10 = -\frac{361}{24}$. Therefore, the vertex is $\left(\frac{11}{12}, -\frac{361}{24}\right)$. Next, solving $6x^2 - 11x - 10 = (3x + 2)(2x - 5) = 0$ gives $-\frac{2}{3}$ and $\frac{5}{2}$ as the x-intercepts.



22. $y = -4x^2 + 4x + 3$. The x-coordinate of the vertex is $-\frac{4}{2(-4)} = \frac{1}{2}$ and the y-coordinate is $-4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 3 = 4$. Therefore, the vertex is $\left(\frac{1}{2}, 4\right)$. Next, solving $-4x^2 + 4x + 3 = 0$, we find $4x^2 - 4x - 3 = (2x - 3)(2x + 1) = 0$, so the x-intercepts are $-\frac{1}{2}$ and $\frac{3}{2}$.



23. We solve the system $3x + 4y = -6$, $2x + 5y = -11$. Solving the first equation for x, we have $3x = -4y - 6$ and $x = -\frac{4}{3}y - 2$. Substituting this value of x into the second equation yields $2\left(-\frac{4}{3}y - 2\right) + 5y = -11$, so $-\frac{8}{3}y - 4 + 5y = -11$, $\frac{7}{3}y = -7$, and $y = -3$. Thus, $x = -\frac{4}{3}(-3) - 2 = 4 - 2 = 2$, so the point of intersection is $(2, -3)$.

24. We solve the system $y = \frac{3}{4}x + 6$, $3x - 2y = -3$. Substituting the first equation into the second equation, we have $3x - 2\left(\frac{3}{4}x + 6\right) = -3$, $3x - \frac{3}{2}x - 12 = -3$, $\frac{3}{2}x = 9$, and $x = 6$. Substituting this value of x into the first equation, we have $y = \frac{3}{4}(6) + 6 = \frac{21}{2}$. Therefore, the point of intersection is $\left(6, \frac{21}{2}\right)$.

25. We solve the system $7x + 9y = -11$, $3x = 6y - 8$. Multiplying the second equation by $\frac{1}{3}$, we have $x = 2y - \frac{8}{3}$. Substituting this value of x into the first equation, we have $7\left(2y - \frac{8}{3}\right) + 9y = -11$. Solving this equation for y, we have $14y - \frac{56}{3} + 9y = -11$, $69y = -33 + 56$, and $y = \frac{23}{69} = \frac{1}{3}$. Thus, $x = 2\left(\frac{1}{3}\right) - \frac{8}{3} = -2$. The lines intersect at $\left(-2, \frac{1}{3}\right)$.

26. Setting $C(x) = R(x)$, we have $12x + 20{,}000 = 20x$, $8x = 20{,}000$, and $x = 2500$. Next, $R(2500) = 20(2500) = 50{,}000$, and we conclude that the break-even point is $(2500, 50000)$.

27. The slope of L_2 is greater than that of L_1. This tells us that if the manufacturer lowers the unit price for each model clock radio by the same amount, the additional demand for model B radios will be greater than that for model A radios.

28. The slope of L_2 is greater than that of L_1. This tells us that if the unit price for each model clock radio is raised by the same amount, the manufacturer will make more model B than model A radios available in the market.

29. In 2012 (when $x = 5$), we have $S(5) = 6000(5) + 30{,}000 = 60{,}000$.

30. Let x denote the time in years. Since the function is linear, we know that it has the form $f(x) = mx + b$.

a. The slope of the line passing through $(0, 2.4)$ and $(5, 7.4)$ is $m = \dfrac{7.4 - 2.4}{5} = 1$. Since the line passes through $(0, 2.4)$, we know that the y-intercept is 2.4. Therefore, the required function is $f(x) = x + 2.4$.

b. In 2009 (when $x = 3$), the sales were $f(3) = 3 + 2.4 = 5.4$, or \$5.4 million.

31. Let x denote the number of units produced and sold.

a. The cost function is $C(x) = 6x + 30{,}000$.

b. The revenue function is $R(x) = 10x$.

c. The profit function is $P(x) = R(x) - C(x) = 10x - (30{,}000 + 6x) = 4x - 30{,}000$.

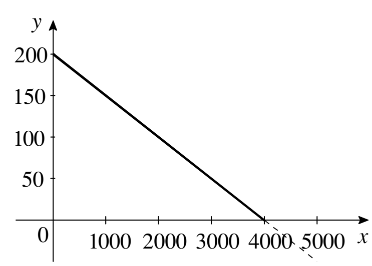
d. $P(6000) = 4(6000) - 30{,}000 = -6{,}000$, a loss of \$6000; $P(8000) = 4(8000) - 30{,}000 = 2{,}000$, a profit of \$2000; and $P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, a profit of \$18,000.

32. Let V denote the value of the building after t years.

a. The rate of depreciation is $-\dfrac{\Delta V}{\Delta t} = \dfrac{6{,}000{,}000}{30} = 200{,}000$, or \$200,000/year.

b. From part a, we know that the slope of the line is $-200{,}000$. Using the point-slope form of the equation of a line, we have $V - 0 = -200{,}000(t - 30)$, or $V = -200{,}000t + 6{,}000{,}000$. In the year 2018 (when $t = 10$), we have $V = -200{,}000(10) + 6{,}000{,}000 = 4{,}000{,}000$, or \$4,000,000.

33. The slope of the demand curve is $\dfrac{\Delta p}{\Delta x} = -\dfrac{10}{200} = -0.05$. Using the point-slope form of the equation of a line with the point $(0, 200)$, we have $p - 200 = -0.05(x)$, or $p = -0.05x + 200$.



34. The slope of the supply curve is $\dfrac{\Delta p}{\Delta x} = \dfrac{100 - 50}{2000 - 200} = \dfrac{50}{1800} = \dfrac{1}{36}$. Using the point-slope form of the equation of a line with the point $(200, 50)$, we have $p - 50 = \frac{1}{36}(x - 200)$, so $p = \frac{1}{36}x - \frac{200}{36} + 50 = \frac{1}{36}x + \frac{1600}{36} = \frac{1}{36}x + \frac{400}{9}$.

35. $D(w) = \dfrac{a}{150}w$. The given equation can be expressed in the form $y = mx + b$, where $m = \dfrac{a}{150}$ and $b = 0$. If $a = 500$ and $w = 35$, $D(35) = \frac{500}{150}(35) = 116\frac{2}{3}$, or approximately 117 mg.

36. a. The slope of the line is $m = \frac{1-0.5}{4-2} = 0.25$. Using the point-slope form of an equation of a line, we have $y - 1 = 0.25(x - 4)$, or $y = 0.25x$.

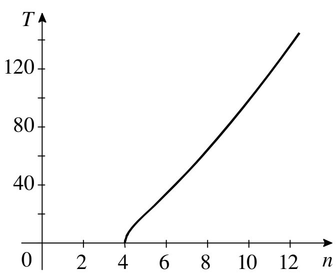
b. $y = 0.25(6.4) = 1.6$, or 1600 applications.

37. $R(30) = -\frac{1}{2}(30)^2 + 30(30) = 450$, or \$45,000.

38. $N(0) = 200(4 + 0)^{1/2} = 400$, and so there are 400 members initially. $N(12) = 200(4 + 12)^{1/2} = 800$, and so there are 800 members after one year.

39. The population will increase by $P(9) - P(0) = \left[50{,}000 + 30(9)^{3/2} + 20(9)\right] - 50{,}000$, or 990, during the next 9 months. The population will increase by $P(16) - P(0) = \left[50{,}000 + 30(16)^{3/2} + 20(16)\right] - 50{,}000$, or 2240 during the next 16 months.

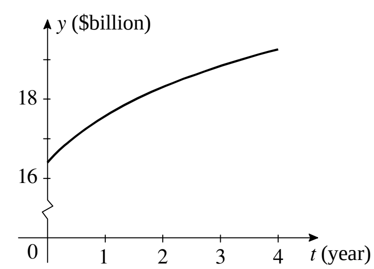
40. $T = f(n) = 4n\sqrt{n-4}$. $f(4) = 0$, $f(5) = 20\sqrt{1} = 20$, $f(6) = 24\sqrt{2} \approx 33.9$, $f(7) = 28\sqrt{3} \approx 48.5$, $f(8) = 32\sqrt{4} = 64$, $f(9) = 36\sqrt{5} \approx 80.5$, $f(10) = 40\sqrt{6} \approx 98$, $f(11) = 44\sqrt{7} \approx 116$, and $f(12) = 48\sqrt{8} \approx 135.8$.



41. $N(0) = 648$, or 648,000, $N(1) = -35.8 + 202 + 87.7 + 648 = 901.9$, or 901,900, $N(2) = -35.8(2)^3 + 202(2)^2 + 87.8(2) + 648 = 1345.2$ or 1,345,200, and $N(3) = -35.8(3)^3 + 202(3)^2 + 87.8(3) + 648 = 1762.8$ or 1,762,800.

42. a. $A(0) = 16.4$, or \$16.4 billion; $A(1) = 16.4(1+1)^{0.1} \approx 17.58$, or \$17.58 billion; $A(2) = 16.4(2+1)^{0.1} \approx 18.30$, or \$18.3 billion; $A(3) = 16.4(3+1)^{0.1} \approx 18.84$, or \$18.84 billion; and $A(4) = 16.4(4+1)^{0.1} \approx 19.26$, or \$19.26 billion. The nutritional market grew over the years 1999 to 2003.

b.



43. a. $f(t) = 267$ and $g(t) = 2t^2 + 46t + 733$.

b. $h(t) = (f+g)(t) = f(t) + g(t) = 267 + \left(2t^2 + 46t + 733\right) = 2t^2 + 46t + 1000$.

c. $h(13) = 2(13)^2 + 46(13) + 1000 = 1936$, or 1936 tons.

44. We solve $-1.1x^2 + 1.5x + 40 = 0.1x^2 + 0.5x + 15$, obtaining $1.2x^2 - x - 25 = 0$, $12x^2 - 10x - 250 = 0$, $6x^2 - 5x - 125 = 0$, and $(x-5)(6x+25) = 0$. Therefore, $x = 5$. Substituting this value of x into the second supply equation, we have $p = 0.1(5)^2 + 0.5(5) + 15 = 20$. So the equilibrium quantity is 5000 and the equilibrium price is \$20.

45. a. $V = \frac{4}{3}\pi r^3$, so $r^3 = \dfrac{3V}{4\pi}$ and $r = f(V) = \sqrt[3]{\dfrac{3V}{4\pi}}$.

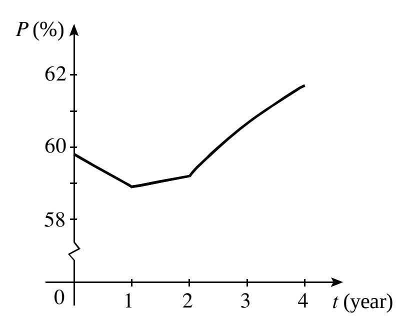
b. $g(t) = \frac{9}{2}\pi t$.

c. $h(t) = (f \circ g)(t) = f(g(t)) = \left[\dfrac{3g(t)}{4\pi}\right]^{1/3} = \left[\dfrac{3(9)\pi t}{4\pi(2)}\right]^{1/3} = \frac{3}{2}\sqrt[3]{t}$.

d. $h(8) = \frac{3}{2}\sqrt[3]{8} = 3$, or 3 ft.

46. a. $P(0) = 59.8$, $P(1) = 0.3(1) + 58.6 = 58.9$, $P(2) = 56.79(2)^{0.06} \approx 59.2$, $P(3) = 56.79(3)^{0.06} \approx 60.7$, and $P(4) = 56.79(4)^{0.06} \approx 61.7$.

b.



c. $P(3) \approx 60.7$, or 60.7%.

47. Measured in inches, the sides of the resulting box have length $20 - 2x$ and its height is x, so its volume is $V = x(20 - 2x)^2$ in^3.

48. Let h denote the height of the box. Then its volume is $V = (x)(2x)h = 30$, so that $h = \dfrac{15}{x^2}$. Thus, the cost is

$$\begin{aligned} C(x) &= 30(x)(2x) + 15[2xh + 2(2x)h] + 20(x)(2x) \\ &= 60x^2 + 15(6xh) + 40x^2 = 100x^2 + (15)(6)x\left(\frac{15}{x^2}\right) \\ &= 100x^2 + \frac{1350}{x}. \end{aligned}$$

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1. $m = \dfrac{5-(-2)}{4-(-1)} = \dfrac{7}{5}$, so an equation is $y - (-2) = \frac{7}{5}[x - (-1)]$. Simplifying, $y = \frac{7}{5}x + \frac{7}{5} - 2$, or $y = \frac{7}{5}x - \frac{3}{5}$.

2. $m = -\frac{1}{3}$ and $b = \frac{4}{3}$, so an equation is $y = -\frac{1}{3}x + \frac{4}{3}$.

3. a. $f(-1) = -2(-1) + 1 = 3$. **b.** $f(0) = 2$. **c.** $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 2 = \frac{17}{4}$.

4. a. $(f+g)(x) = f(x) + g(x) = \dfrac{1}{x+1} + x^2 + 1$. **b.** $(fg)(x) = f(x)g(x) = \dfrac{x^2+1}{x+1}$.

c. $(f \circ g)(x) = f(g(x)) = \dfrac{1}{g(x)+1} = \dfrac{1}{x^2+2}$. **d.** $(g \circ f)(x) = g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{(x+1)^2} + 1$.

5. $4x + h = 108$, so $h = 108 - 4x$. The volume is $V = x^2h = x^2(108 - 4x) = 108x^2 - 4x^3$.

CHAPTER 2 Explore & Discuss

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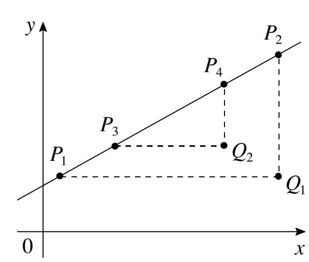
Refer to the accompanying figure. Observe that triangles $\Delta P_1Q_1P_2$ and $\Delta P_3Q_2P_4$ are similar. From this we conclude that $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3}$. Because P_3 and P_4 are arbitrary, the conclusion follows.



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In Example 7, we are told that the object is expected to appreciate in value at a given rate for the next five years, and the equation obtained in that example is based on this fact. Thus, the equation may not be used to predict the value of the object very much beyond five years from the date of purchase.

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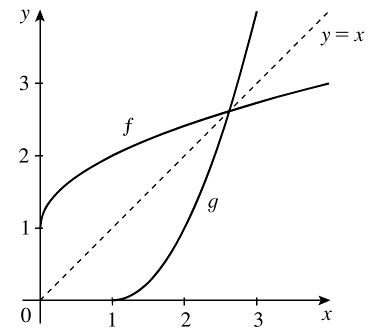
1. $(g \circ f)(x) = g(f(x)) = [f(x) - 1]^2 = [(\sqrt{x} + 1) - 1]^2 = (\sqrt{x})^2 = x$ and $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} + 1 = \sqrt{(x-1)^2} + 1 = (x-1) + 1 = x$.

2. From the figure, we see that the graph of one is the mirror reflection of the other if we place a mirror along the line $y = x$.



CHAPTER 2 Exploring with Technology

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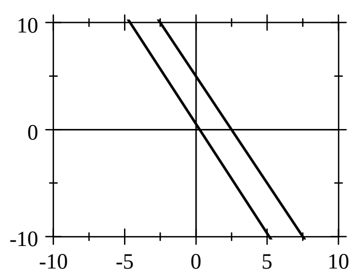
1.

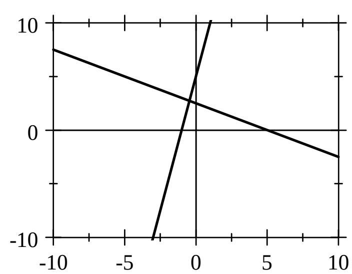


The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be parallel.

b. Writing each equation in the slope-intercept form gives $y = -2x + 5$ and $y = -\frac{41}{20}x + \frac{11}{20}$, from which we see that the slopes of L_1 and L_2 are -2 and $-\frac{41}{20} = -2.05$, respectively. This shows that L_1 and L_2 are not parallel.

2.



The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be perpendicular.

b. The slopes of L_1 and L_2 are $m_1 = -\frac{1}{2}$ and $m_2 = 5$, respectively. Because $m_1 = -\dfrac{1}{2} \neq -\dfrac{1}{5} = -\dfrac{1}{m_2}$, we see that L_1 and L_2 are not perpendicular.

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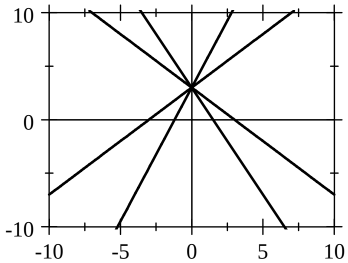
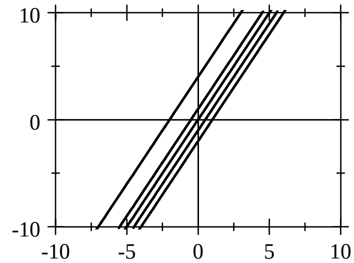
1.



The straight lines with the given equations are shown in the figure. Changing the value of m in the equation $y = mx + b$ changes the slope of the line and thus rotates it.

2.



The straight lines of interest are shown in the figure. Changing the value of b in the equation $y = mx + b$ changes the y-intercept of the line and thus translates it (upward if $b > 0$ and downward if $b < 0$).

3. Changing both m and b in the equation $y = mx + b$ both rotates and translates the line.

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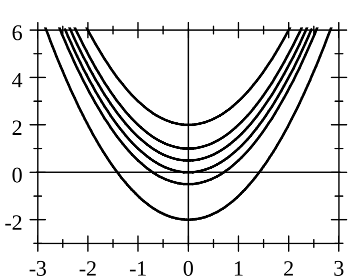
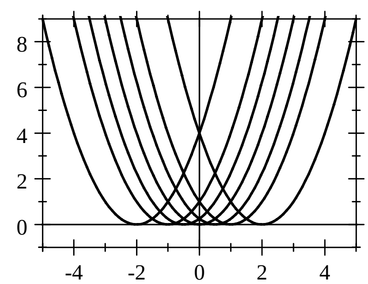
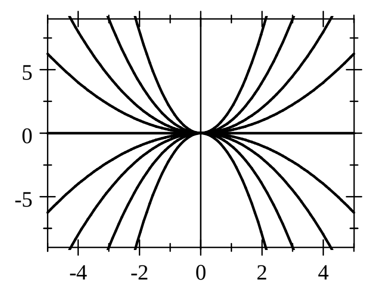
1.



2.



3.



4. The graph of $f(x) + c$ is obtained by translating the graph of f along the y-axis by c units. The graph of $f(x + c)$ is obtained by translating the graph of f along the x-axis by c units. Finally, the graph of cf is obtained from that of f by "expanding"(if $c > 1$) or "contracting"(if $0 < c < 1$) that of f. If $c < 0$, the graph of cf is obtained from that of f by reflecting it with respect to the x-axis as well as expanding or contracting it.

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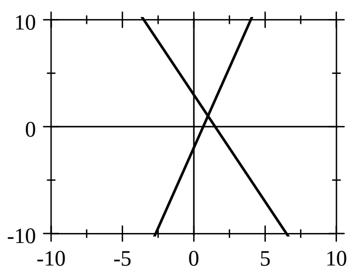
1.



Plotting the straight lines L_1 and L_2 and using TRACE and ZOOM repeatedly, you will see that the iterations approach the answer $(1, 1)$. Using the intersection feature of the graphing utility gives the result $x = 1$ and $y = 1$ immediately.

2. Substituting the first equation into the second yields $3x - 2 = -2x + 3$, so $5x = 5$ and $x = 1$. Substituting this value of x into either equation gives $y = 1$.

3. The iterations obtained using TRACE and ZOOM converge to the solution $(1, 1)$. The use of the intersection feature is clearly superior to the first method. The algebraic method also yields the desired result easily.

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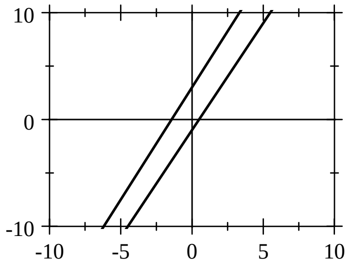
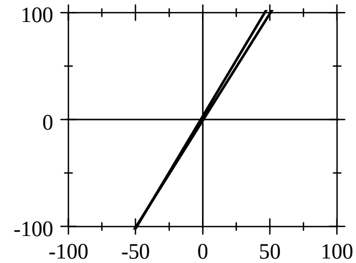
1.



The lines seem to be parallel to each other and do not appear to intersect.

2.



They appear to intersect. But finding the point of intersection using TRACE and ZOOM with any degree of accuracy seems to be an impossible task. Using the intersection feature of the graphing utility yields the point of intersection $(-40, -81)$ immediately.

3. Substituting the first equation into the second gives $2x - 1 = 2.1x + 3$, $-4 = 0.1x$, and thus $x = -40$. The corresponding y-value is -81.

4. Using TRACE and ZOOM is not effective. The intersection feature gives the desired result immediately. The algebraic method also yields the answer with little effort and without the use of a graphing utility.