Chapter 2. Forecasting Demands

I. Summary

This chapter focuses on introducing the concept of customer demands forecasting in the manufacturing environment to the readers. Forecasting customer demands is the process of estimating future demands based on the past or historical data and other factors such as price increase or new product introduction to the market that the managers think would affect the competitive environment the firm is positioned in. Having the ability to generate the most accurate predictions of future demand is essential since this information drives all of the key business functions' plans. This is truer for firms that utilize push manufacturing or have a make to stock (MTS) policy. These firms operate their manufacturing activities based on the plans formulated using forecasted information. As the actual visibility between the activities and the customers is low compared to pull manufacturing, the activities are almost wholly dependent on the manufacturing plans. Thus, it could be said that the efficiency of the manufacturing processes is dependent on the firm's ability to generate accurate forecasts.

Forecasts are made using both statistic techniques to process historical data and the management's intuitive predictions or judgments in regard to other factors not contain in the statistical data. The former is called Quantitative forecasts while the latter is called Qualitative forecasts. The chapter introduces the conceptual framework for the process of forecasting. It starts with determining the forecast purpose, the forecast interval, and choosing the forecasting technique most appropriate. Once the foundation for developing a forecast model has been laid and the appropriate data has been collected and sorted, the oldest portion of the data is used to initialize the forecast model's parameters. The remaining data is then used to generate a time series forecast model using the most current data. Once the forecast model has been completed and adjusted based on management's judgments, it can be used to predict the future customer demands.

The chapter provides a brief introduction to the readers concerning the individual statistical forecast techniques. This includes the following techniques: linear regression, simple moving average, weighted moving average, and exponential smoothing.

II. Chapter Outline

- Introduction to Forecasting
- The Conceptual Forecasting Framework
- Measures of Forecast Accuracy
    - *Mean error (BIAS)*
    - *Mean absolute deviation (MAD)*
- Forecasting Approaches
    - *Quantitative forecasts*
    - *Qualitative forecasts*
- Components of time series forecast
    - *Trend*
    - *Seasonality*
    - *Cyclical*
    - *Irregular variations*
    - *Random fluctuation*
- Quantitative Forecasting Techniques
    - *Linear regression*
    - *Simple moving average*
    - *Weighted moving average*
    - *Simple exponential smoothing*
    - *Trend-enhanced exponential smoothing*
    - *Seasonality-enhanced exponential smoothing*
    - *Trend and seasonality exponential smoothing*

III. Suggested Teaching Strategy

For this chapter, the focus should be on two things. The first is about understanding the demand patterns and identifying the pattern components. After examining the historical demand data gathered, it should be evident what forecasting technique should be used. In addition, it should be evident how the data should be partitioned to initialize and build the model. The second is getting comfortable with the practical working of the various forecasting techniques especially the exponential forecasting techniques. The exponential forecasting techniques have many components that are timing sensitive regarding which period data should be used. Thus, repeated practice with problems and detailed step by step examples are recommended.

IV. Solutions to Discussion Questions

1. *Grey's Audiovisual rents audio and visual equipment to musicians for local concerts. The company is interested in forecasting the rentals for the d2-console so that it can make sure the d2-console is on hand when needed. The d2-console data for the last 10 months are shown here.*

MONTH	DEMAND	MONTH	DEMAND
Jan.	23	June	28
Feb	24	July	32
March	32	Aug.	35
April	26	Sept.	26
May	31	Oct.	24

a. *Prepare a forecast for months 6 through 10 by using three- and five-month moving averages. What is the forecast for month 11?*

The simple moving average techniques treats the data equally by weighting all data the same.

3-month moving average forecast for month 6:

$$\frac{Demand\ sum\ for\ month\ 3,4, and\ 5}{3} = \frac{32 + 26 + 31}{3} = 29.67$$

3-month moving average forecast for month 11:

$$\frac{35 + 26 + 24}{3} = 28.33$$

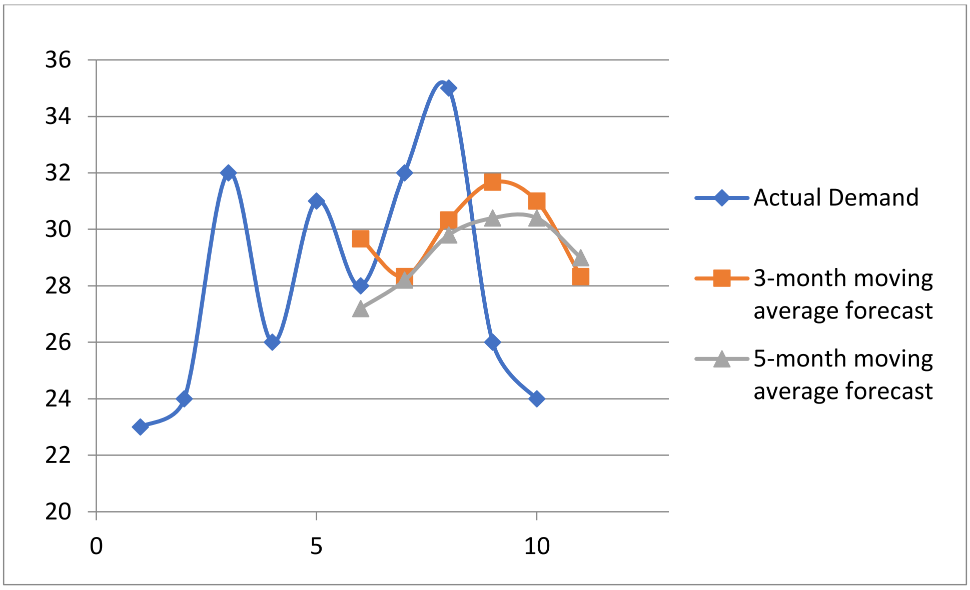
5-month moving average forecast for month 6:

$$\frac{Demand\ sum\ for\ month\ 1,2,3,4, and\ 5}{5} = \frac{23 + 24 + 32 + 26 + 31}{5} = 27.20$$

5-month moving average forecast for month 11:

$$\frac{28 + 32 + 35 + 26 + 24}{5} = 29.00$$

Month	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
Demand	**23**	**24**	**32**	**26**	**31**	**28**	**32**	**35**	**26**	**24**	
3-month moving average forecast						**29.67**	**28.33**	**30.33**	**31.67**	**31.00**	**28.33**
5-month moving average forecast						**27.20**	**28.20**	**29.80**	**30.40**	**30.40**	**29.00**



b. Calculate the mean absolute deviation (for periods 6–10) for each forecasting method.

MAD for 3-month moving average forecast:

$$\frac{1}{5}\sum_{6}^{10}(|D_i - F_i|) = \frac{(1.67 + 3.67 + 4.67 + 5.67 + 7)}{5} = 4.53$$

MAD for 5-month moving average forecast:

$$\frac{1}{5}\sum_{6}^{10}(|D_i - F_i|) = \frac{(0.8 + 3.8 + 5.2 + 4.4 + 6.4)}{5} = 4.12$$

c. What are your recommendations for Gray's?

The MAD value of the 5-month moving average forecast is slightly smaller than the 3-month moving average forecast's MAD. Considering that a smaller MAD value equates to closer to the actual demand, 5-month moving average forecast method is a better choice.

Difficulty Level: Moderate

2. *Sales for the past 12 months at Bell, Inc. are given below.*

MONTH	SALES ($ MILLIONS)	MONTH	SALES ($ MILLIONS)
January	10	July	26
February	12	August	31
March	14	September	27
April	16	October	18
May	18	November	16
June	23	December	14

a. *Use a three-month moving average to forecast the sales for the months May through December.*

3-month moving average forecast for month 5:

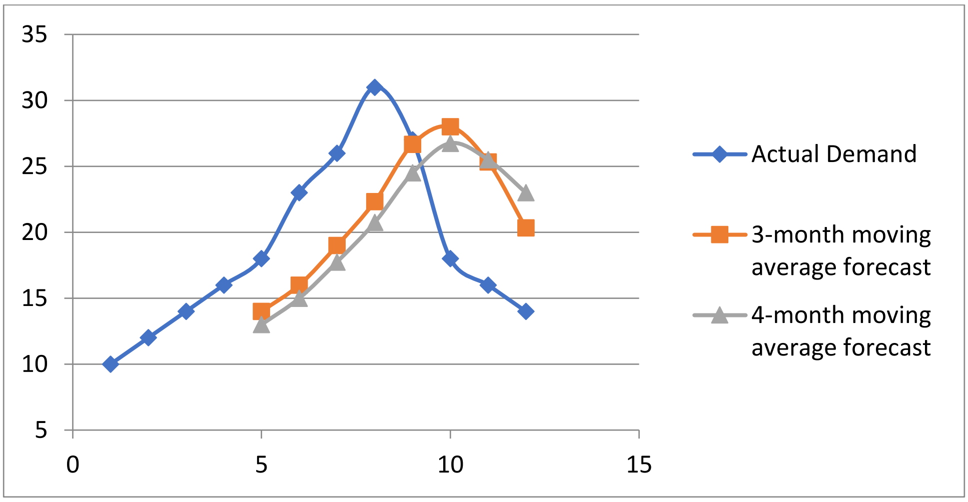
$$\frac{Demand\ sum\ for\ month\ 2,3, and\ 4}{3} = \frac{12+14+16}{3} = 14$$

b. *Use a four-month moving average to forecast the sales for the months May through December.*

4-month moving average forecast for month 5:

$$\frac{Demand\ sum\ for\ month\ 1,2,3, and\ 4}{3} = \frac{10+12+14+16}{3} = 13$$

Month	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Demand	**10**	**12**	**14**	**16**	**18**	**23**	**26**	**31**	**27**	**18**	**16**	**14**
3-month moving average forecast					**14.00**	**16.00**	**19.00**	**22.33**	**26.67**	**28.00**	**25.33**	**20.33**
4-month moving average forecast					**13.00**	**15.00**	**17.75**	**20.75**	**24.50**	**26.75**	**25.50**	**23.00**



c. *Compare the performance of the two methods by using the bias and MAD as the performance criterion. Which method do you recommend?*

Bias for 3-month moving average forecast:

$$\frac{1}{8}\sum_{5}^{12}(D_i - F_i) = \frac{(4 + 7 + 7 + 8.67 + 0.33 - 10 - 9.3 - 6.3)}{8} = 0.167$$

MAD for 3-month moving average forecast:

$$\frac{1}{8}\sum_{5}^{12}(|D_i - F_i|) = \frac{(4 + 7 + 7 + 8.67 + 0.33 + 10 + 9.3 + 6.3)}{8} = 6.58$$

Bias for 4-month moving average forecast:

$$\frac{1}{8}\sum_{5}^{12}(D_i - F_i) = \frac{(5 + 8 + 8.25 + 10.25 + 2.5 - 8.75 - 9.5 - 9)}{8} = 0.844$$

MAD for 4-month moving average forecast:

$$\frac{1}{8}\sum_{5}^{12}(|D_i - F_i|) = \frac{(5 + 8 + 8.25 + 10.25 + 2.5 + 8.75 + 9.5 + 9)}{8} = 7.66$$

Comparing 3-month and 4-month moving average forecast techniques, the 3-month moving average forecast result in both lower Bias and MAD than the 4-month moving average forecast.

Bias: 0.167<0.6844
MAD: 6.58<7.66

Thus, the 3-month moving average forecast technique should be used for forecasting.

Difficulty Level: Moderate

3. Nelson Fabricators sells a portable EKG machine. The sales manager requires a weekly forecast of the portable EKG machine so that he can schedule production. The manager uses exponential smoothing with a = 0.30.

WEEK	ACTUAL PRODUCTION
1	535
2	689
3	601
4	768
5	433

a. *Forecast the number of machines for week 6 made at the end of week 4.*

Using a Simple Exponential Smoothing Model, forecasts can only be made one time period ahead of the time the forecast is being made with any accuracy since the most recent information is not incorporated.
Thus, you cannot forecast for week 6 made at the end of week 4 using Simple Exponential Smoothing.

b. *Forecast the number of machines at the end of week 5.*

Since week 1 is the first data point, the forecast for week 1 (ESF_0) should be initialized to be the same as the actual demand for week 1.

- ESF_0=535
- $ESF_1 = 0.3 \times D_1 + 0.7 \times ESF_0 = 535$
- $ESF_2 = 0.3 \times D_2 + 0.7 \times ESF_1 = 581.2$
- $ESF_3 = 0.3 \times D_3 + 0.7 \times ESF_2 = 587.14$
- $ESF_4 = 0.3 \times D_4 + 0.7 \times ESF_3 = 641.398$
- $ESF_5 = 0.3 \times D_5 + 0.7 \times ESF_4 = 578.89$

Thus, the forecast for week 6 made at the end of week 5 is 578.89 or 579 units.

c. *Calculate the bias and MAD for parts a and b. Why is one of the forecasts more accurate than the other?*

The bias of the forecast for week 6 made at the end of week 5 is 29.25
The MAD the forecast for week 6 made at the end of week 5 is 112.61
Forecasting at week 6 is more accurate than making one at the end of week 4 is more accurate because it incorporates more recent information which is especially important for simple exponential smoothing.

Difficulty Level: Easy

4. *Consider the sales data for Bell, Inc. given in Problem 2.*
   a. *Use a three-month weighted moving average to forecast the sales for the months April through December. Use weights of (3/6), (2/6), and (1/6), giving more weight to more recent data.*

   3-month weighted moving average forecast for month 4:

$$\frac{Demand\ \ for\ month\ 1*\frac{1}{6}+Demand\ \ for\ month\ 2*\frac{2}{6}+Demand\ \ for\ month\ \ 3*\frac{3}{6}}{3}$$

$$=\frac{10*\frac{1}{6}+12*\frac{2}{6}+14*\frac{3}{6}}{3}=16.67$$

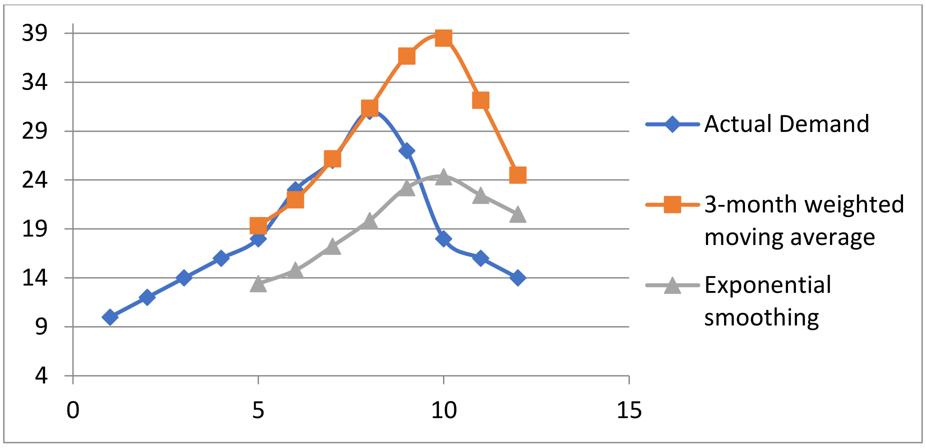
   3-month weighted moving average forecast for month 12:

$$\frac{Demand\ \ for\ month\ 9*\frac{1}{6}+Demand\ \ for\ month\ 10*\frac{2}{6}+Demand\ \ for\ month\ \ 11*\frac{3}{6}}{3}$$

$$=\frac{27*\frac{1}{6}+18*\frac{2}{6}+16*\frac{3}{6}}{3}=24.5$$

   b. *Use exponential smoothing with a = 0.30 to forecast the sales for the months April through December. The forecast for January was $12 million.*
      - ESF_0=12
      - $ESF_1 = 0.3 \times D_1 + 0.7 \times ESF_0 = 11.4$
      - $ESF_2 = 0.3 \times D_2 + 0.7 \times ESF_1 = 11.58$
      - $ESF_3 = 0.3 \times D_3 + 0.7 \times ESF_2 = 12.306$
      - $ESF_4 = 0.3 \times D_4 + 0.7 \times ESF_3 = 13.41$
      - $ESF_5 = 0.3 \times D_5 + 0.7 \times ESF_4 = 14.78994$

Month	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
Demand	**10**	**12**	**14**	**16**	**18**	**23**	**26**	**31**	**27**	**18**	**16**	**14**
3-month weighted moving average				**16.67**	**19.33**	**22.00**	**26.17**	**31.33**	**36.67**	**38.50**	**32.17**	**24.50**
Exponential smoothing(ESF_i)				**12.31**	**13.41**	**14.79**	**17.25**	**19.88**	**23.21**	**24.35**	**22.45**	**20.51**



c. Compare the performance of the two methods by using the bias and MAD as the performance criteria. Which performance method is the most reasonable? Why?

Bias of the 3-month weighted moving average forecast:

$$\frac{1}{9}\sum_{4}^{12}(D_i - F_i) = \frac{(-0.67 - 1.33 + 1 - 0.17 - 0.33 - 9.67 - 20.5 - 16.17 - 10.5)}{8}$$

$$= -6.48$$

MAD of 3-month moving average forecast:

$$\frac{1}{9}\sum_{45}^{12}(|D_i - F_i|) = \frac{(0.67 + 1.33 + 1 + 0.17 + 0.33 + 9.67 + 20.5 + 16.17 + 10.5)}{8}$$

$$= 6.7$$

Bias of exponential smoothing

$$\frac{1}{9}\sum_{4}^{12}(D_i - F_i)$$

$$= \frac{(3.69 + 4.59 + 8.21 + 8.75 + 11.12 + 3.79 - 6.35 - 6.44 - 6.51)}{8} = 2.32$$

MAD of exponential smoothing

$$\frac{1}{9}\sum_{4}^{12}(D_i - F_i)$$

$$= \frac{(3.69 + 4.59 + 8.21 + 8.75 + 11.12 + 3.79 + 6.35 + 6.44 + 6.51)}{8} = 6.61$$

The exponential smoothing generates better forecasts as the MAD values are better (6.61<6.7) and the bias value is smaller and positive. Unlike the 3-month moving average method, exponential smoothing under-estimates the demand. The 3-month moving average method over-estimates the demand.

Difficulty Level: Moderate

5. *A toy manufacturer recently introduced a new computer game. Management is interested in estimating future sales volume to determine whether it should continue to carry the new game or replace it with another game. At the end of April, the average monthly sales volume of the new game was 500 games and the trend was +50 games per month. The actual sales volume figures for May, June, and July are 560, 600, and 620, respectively. Use trend-adjusted exponential smoothing with α= 0.2 and β= 0.1 to forecast usage for June, July, and August.*

Forecast for May:

- Step 1: $\bar{F}_4 = 500$ (initialization)
- Step 2: $T_4 = 50$ (initialization)
- Step 3: $F_5 = \bar{F}_4 + T_4$=550

Forecast for June:

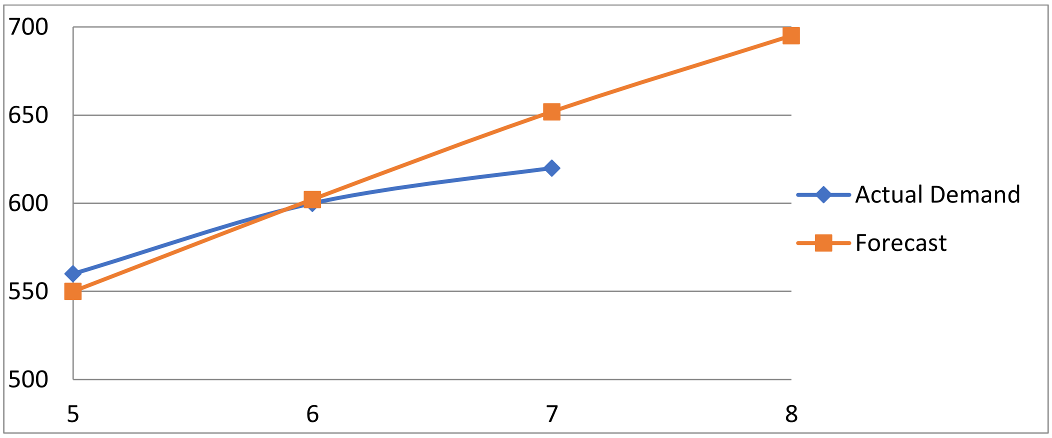
- Step 1: $\bar{F}_5 = \alpha \times D_5 + (1-\alpha)(\bar{F}_4 + T_4) = \alpha \times D_5 + (1-\alpha)F_5 = 552$
- Step 2: $T_5 = \beta(\bar{F}_5 - \bar{F}_4) + (1-\beta)T_4 = 50.2$
- Step 3: $F_6 = \bar{F}_5 + T_5$=602.2

Forecast for July:

- Step 1: $\bar{F}_6 = \alpha \times D_6 + (1-\alpha)(\bar{F}_5 + T_5) = \alpha \times D_6 + (1-\alpha)F_6 = 601.76$
- Step 2: $T_6 = \beta(\bar{F}_6 - \bar{F}_5) + (1-\beta)T_5 = 50.156$
- Step 3: $F_7 = \bar{F}_6 + T_6$= 651.916

Forecast for August:

- Step 1: $\bar{F}_7 = \alpha \times D_7 + (1-\alpha)(\bar{F}_6 + T_6) = \alpha \times D_7 + (1-\alpha)F_7 =$ 645.5328
- Step 2: $T_7 = \beta(\bar{F}_7 - \bar{F}_6) + (1-\beta)T_6 = 49.51768$
- Step 3: $F_8 = \bar{F}_7 + T_7$= 695.05048



Difficulty Level: Moderate

6. *At the end of April, the average monthly use of an ATM was 320 customers and the trend was +100 customers per month. The actual use rates for May, June, and July are 880, 910, and 990, respectively. Use trend-adjusted exponential smoothing with a =0.3 and β = 0.2 to forecast the ATM usage for June, July, and August.*

Forecast for May:

- Step 1: $\bar{F}_4 = 320$ (initialization)
- Step 2: $T_4 = 100$ (initialization)
- Step 3: $F_5 = \bar{F}_4 + T_4$=420

Forecast for June:

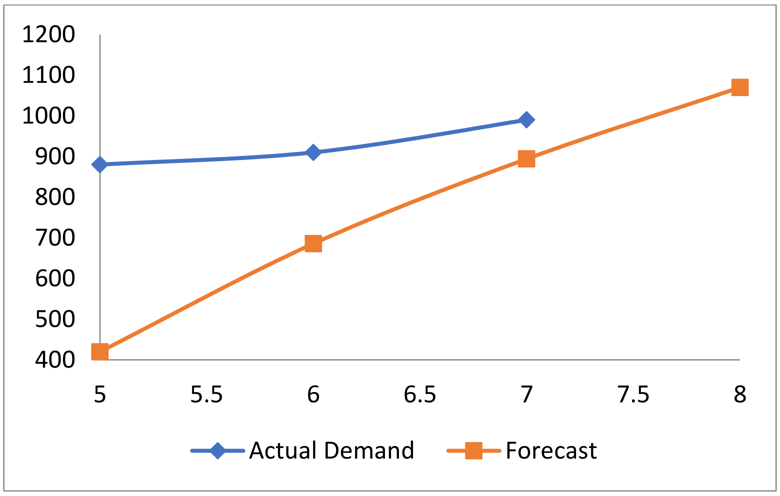
- Step 1: $\bar{F}_5 = \alpha \times D_5 + (1-\alpha)(\bar{F}_4 + T_4) = \alpha \times D_5 + (1-\alpha)F_5 = 558$
- Step 2: $T_5 = \beta(\bar{F}_5 - \bar{F}_4) + (1-\beta)T_4 = 127.6$
- Step 3: $F_6 = \bar{F}_5 + T_5$= 685.6

Forecast for July:

- Step 1: $\bar{F}_6 = \alpha \times D_6 + (1-\alpha)(\bar{F}_5 + T_5) = \alpha \times D_6 + (1-\alpha)F_6 = 752.92$
- Step 2: $T_6 = \beta(\bar{F}_6 - \bar{F}_5) + (1-\beta)T_5 = 141.064$
- Step 3: $F_7 = \bar{F}_6 + T_6$= 893.984

Forecast for August:

- Step 1: $\bar{F}_7 = \alpha \times D_7 + (1-\alpha)(\bar{F}_6 + T_6) = \alpha \times D_7 + (1-\alpha)F_7 = 922.7888$
- Step 2: $T_7 = \beta(\bar{F}_7 - \bar{F}_6) + (1-\beta)T_6 = 146.82496$
- Step 3: $F_8 = \bar{F}_7 + T_7$= 1069.61376



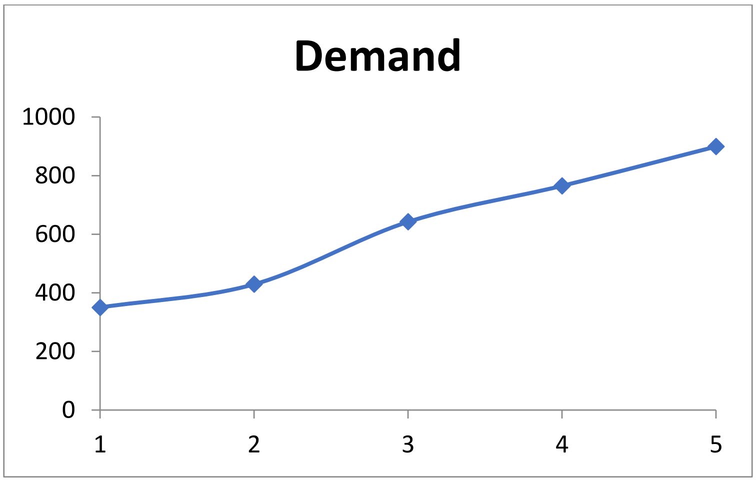
Difficulty Level: Moderate

7. *The number of knee surgeries performed at New Albany Surgical Hospitals (NASH) has increased steadily over the past five years. The hospital's administration is seeking the best method to forecast the demand for such surgeries in years 6 through 9. The data for the past five years are shown below.*

YEAR	DEMAND
1	350
2	429
3	642
4	765
5	899

NASH's administration is considering alternative exponential forecasting methods. Use the first three years to initialize your methods. All methods must be tested and compared using the same years. Use the bias and MAD as the performance criteria to determine the best knee surgery forecasting method for periods 6 through 9.

Demand Graph:



It is evident from the demand pattern that there is a strong trend component to the demand. At this point, an exponential smoothing model with trend or regression model could be used. However, with the current data point limitations of only 6 points, the regression model is not suitable. Thus, an exponential smoothing model with trend should be used.

Since the model is built using month 4 and 5's data, the initialization starts with the base forecast made at month 3 ($\bar{F}_3$) and the trend at month 3 (T_3).

The smoothing constants are set as the following for a MAD value of 95.73.

$\alpha = 0.4 \quad \beta = 0.5$

Forecast for Month 4

- Step 1: $\bar{F}_3 = \frac{1}{3}\sum_{i=1}^{3} d_i = 473.6666667$ (initialization)
- Step 2: $T_3 = \frac{1}{3}\sum_{i=1}^{3}(d_{i+1} - d_i) = 146$ (initialization)
- Step 3: $F_4 = \bar{F}_3 + T_3$=619.67

Forecast for Month 5:

- Step 1: $\bar{F}_4 = \alpha \times D_4 + (1-\alpha)(\bar{F}_3 + T_3) = 677.8$
- Step 2: $T_4 = \beta(\bar{F}_4 - \bar{F}_3) + (1-\beta)T_3 = 175.067$
- Step 3: $F_5 = \bar{F}_4 + T_4$= 852.87

Forecast for Month 6:

- Step 1: $\bar{F}_5 = \alpha \times D_5 + (1-\alpha)(\bar{F}_4 + T_4) = \alpha \times D_5 + (1-\alpha)F_5 = 871.32$
- Step 2: $T_5 = \beta(\bar{F}_5 - \bar{F}_4) + (1-\beta)T_4 = 184.29$
- Step 3: $F_6 = \bar{F}_5 + T_5$= 1055.61

Forecast for Month 7:

Since there is no more historical data, steps 1 and 2 can be skipped over.

- Step 3: $F_7 = \bar{F}_5 + 2 \times T_5$= 1239.906667

Forecast for Month 8:

Since there is no more historical data, steps 1 and 2 can be skipped over.

- Step 3: $F_8 = \bar{F}_5 + 3 \times T_5$= 1424.2

Forecast for Month 9:

Since there is no more historical data, steps 1 and 2 can be skipped over.

- Step 3: $F_9 = \bar{F}_5 + 4 \times T_5$= 1608.49

Difficulty Level: Moderate

8. *The following data are for computer sales in units at an electronics store over the past five weeks:*

YWEEK	DEMAND
1	56
2	59
3	53
4	60
5	63

Use trend-adjusted exponential smoothing with a = 0.2 and β= 0.2 to forecast sales for weeks 3 through 6. Assume that the smoothed average is 55 units and that the average trend was +6 units per week just before week 1.

Forecast for week 1:

- Step 1: $\bar{F}_0 = 55$ (initialization)
- Step 2: $T_0 = 6$ (initialization)
- Step 3: $F_1 = \bar{F}_0 + T_0$=61

Forecast for week 2:

- Step 1: $\bar{F}_1 = \alpha \times D_1 + (1-\alpha)(\bar{F}_0 + T_0) = 60$
- Step 2: $T_1 = \beta(\bar{F}_1 - \bar{F}_0) + (1-\beta)T_0 = 5.8$
- Step 3: $F_2 = \bar{F}_1 + T_1$= 65.8

Forecast for week 3:

- Step 1: $\bar{F}_2 = \alpha \times D_2 + (1-\alpha)(\bar{F}_1 + T_1) = 64.44$
- Step 2: $T_2 = \beta(\bar{F}_2 - \bar{F}_1) + (1-\beta)T_1 = 5.528$
- Step 3: $F_3 = \bar{F}_2 + T_2$= 69.968

Forecast for week 4:

- Step 1: $\bar{F}_3 = \alpha \times D_3 + (1-\alpha)(\bar{F}_2 + T_2) = 66.5744$
- Step 2: $T_3 = \beta(\bar{F}_3 - \bar{F}_2) + (1-\beta)T_2 = 4.84928$
- Step 3: $F_4 = \bar{F}_3 + T_3$= 71.42368

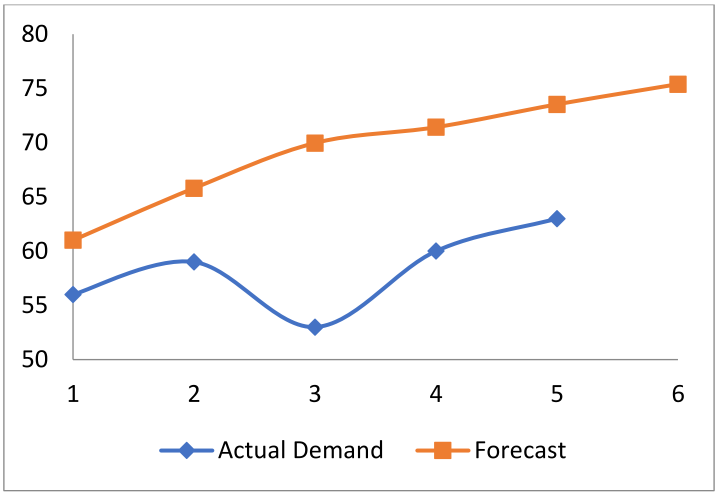
Forecast for week 5:

- Step 1: $\bar{F}_4 = \alpha \times D_4 + (1-\alpha)(\bar{F}_3 + T_3) = 69.138944$
- Step 2: $T_4 = \beta(\bar{F}_4 - \bar{F}_3) + (1-\beta)T_3 = 4.3923328$
- Step 3: $F_5 = \bar{F}_4 + T_4$= 73.53

Forecast for week 6:

- Step 1: $\bar{F}_5 = \alpha \times D_5 + (1-\alpha)(\bar{F}_4 + T_4) = 71.42502144$
- Step 2: $T_5 = \beta(\bar{F}_5 - \bar{F}_4) + (1-\beta)T_4 = 3.971081728$

- Step 3: $F_6 = \bar{F}_5 + T_5 = 75.39610317$



Difficulty Level: Moderate

9. *The demand for refurbished cell phones is experiencing a decline. The company wants to monitor cell phone demand closely. The trend-adjusted exponential smoothing method is used with a =0.1 and β = 0.2. At the end of December, the updated estimate for the average number of cell phones sold in January was 340 and the updated trend for January was 52 per month. The following table shows the actual sales history for January, February, and March. Generate forecasts for February, March, and April.*

MONTH	UNITS SOLD
January	168
February	198
March	230

Forecast for Month 1:

- Step 1: $\bar{F}_0 = 340$ (initialization)
- Step 2: $T_0 = 52$ (initialization)
- Step 3: $F_1 = \bar{F}_0 + T_0 = 392$

Forecast for Month 2:

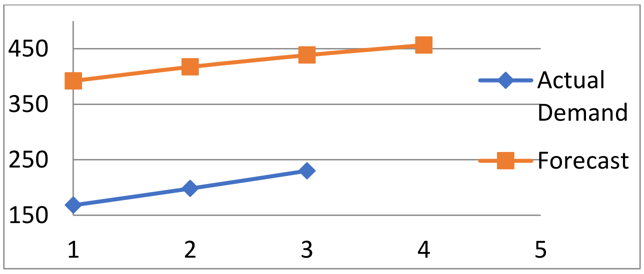
- Step 1: $\bar{F}_1 = \alpha \times D_1 + (1-\alpha)(\bar{F}_0 + T_0) = 369.6$
- Step 2: $T_1 = \beta(\bar{F}_1 - \bar{F}_0) + (1-\beta)T_0 = 47.52$
- Step 3: $F_2 = \bar{F}_1 + T_1 = 417.12$

Forecast for Month 3:

- Step 1: $\bar{F}_2 = \alpha \times D_2 + (1-\alpha)(\bar{F}_1 + T_1) = 395.208$
- Step 2: $T_2 = \beta(\bar{F}_2 - \bar{F}_1) + (1-\beta)T_1 = 43.1376$
- Step 3: $F_3 = \bar{F}_2 + T_2 = 438.3456$

Forecast for Month 4:

- Step 1: $\bar{F}_3 = \alpha \times D_3 + (1-\alpha)(\bar{F}_2 + T_2) = 417.51104$
- Step 2: $T_3 = \beta(\bar{F}_3 - \bar{F}_2) + (1-\beta)T_2 = 38.970688$
- Step 3: $F_4 = \bar{F}_3 + T_3 = 456.481728$

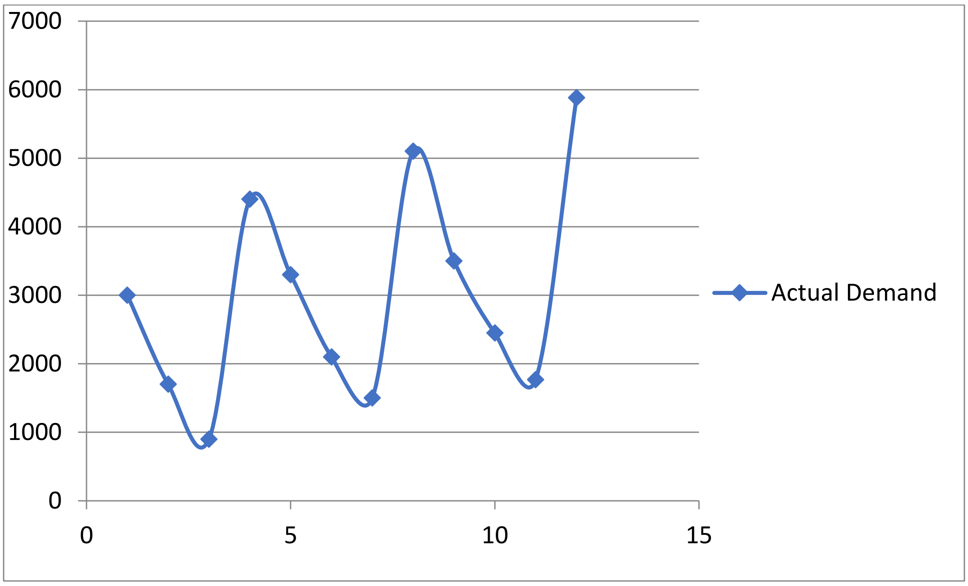


Difficulty Level: Moderate

10. *Consider the quarterly data below:*

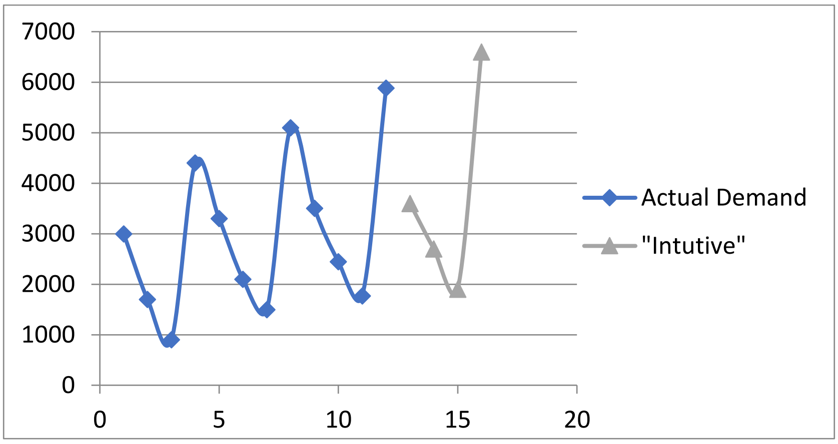
QUARTER	YEAR 1	YEAR 2	YEAR 3
1	3,000	3,300	3,502
2	1,700	2,100	2,448
3	900	1,500	1,768
4	4,400	5,100	5,882
Total	10,000	12,000	13,600

a. Use intuition and judgment to estimate quarterly demand for the fourth year.



The data shows seasonality and trend

- Quarter 1 year 4: 3,600
- Quarter 2 year 4: 2,700
- Quarter 3 year 4: 1,900
- Quarter 4 year 4: 6,600



b. Now use a three-period moving average to forecast year 4. Are any of the quarterly forecasts different from what you thought you would get in part (a)?

Considering there is a seasonality component, treat each quarter for each year as a series of data points.

E.g. Quarter 1 year 1, Quarter 1 year 2, Quarter 1 year 3, Quarter 1 year 4.

3-month moving average forecast for month Quarter 1 year 4:

$$\frac{Demand\ sum\ for\ all\ quarter\ 1s}{3} = \frac{3000 + 3300 + 3501}{3} = 3267.33$$

3-month moving average forecast for month Quarter 2 year 4:

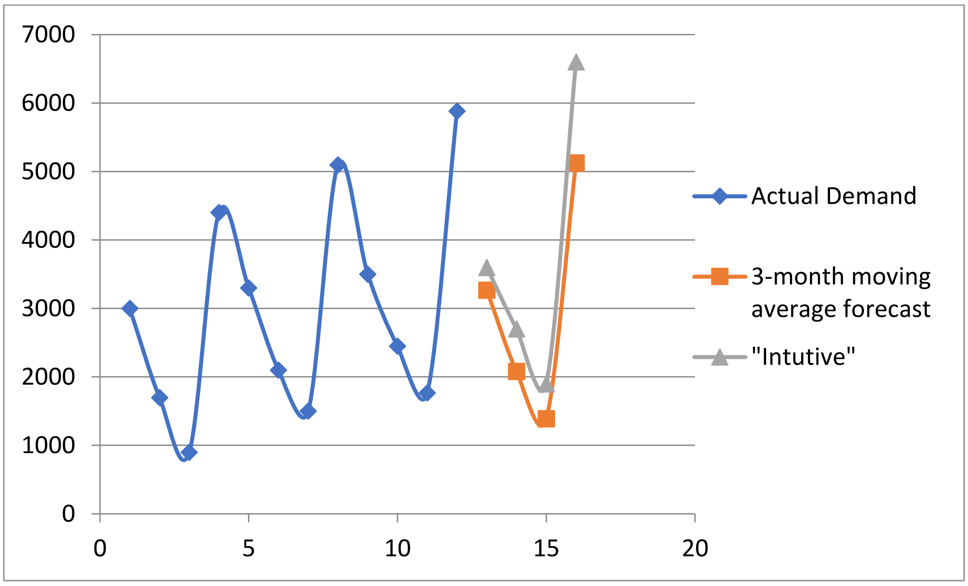
$$\frac{Demand\ sum\ for\ all\ quarter\ 2s}{3} = \frac{1700 + 2100 + 2448}{3} = 2082.67$$

3-month moving average forecast for month Quarter 3 year 4:

$$\frac{Demand\ sum\ for\ all\ quarter\ 3s}{3} = \frac{900 + 1500 + 1768}{3} = 1389.33$$

3-month moving average forecast for month Quarter 4 year 4:

$$\frac{Demand\ sum\ for\ all\ quarter\ 4s}{3} = \frac{4400 + 5100 + 5882}{3} = 5127.33$$



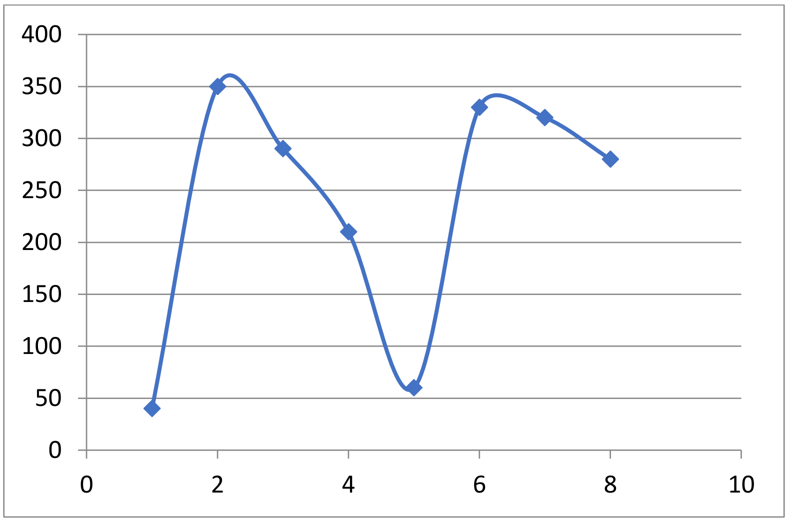
The forecast tend to be lower than the intuitive judgment. The difference is larger for Quarter 4 year 4 than the other predictions.

Difficulty Level: Easy

11. *The manager of Pat's Organic Farm must make the annual purchasing plans for rakes, gloves, and other gardening items. One of the items the company stocks is Lean-feed for chickens. The sales of this item are seasonal, with peaks in the spring, summer, and fall months. Quarterly demand (in cases) for the past two years is as follows:*

QUARTER	YEAR 1	YEAR 2
1	40	60
2	350	440
3	290	320
4	210	280
Total	890	1,100

If the expected sales for Lean-feed are 1,150 cases for year 3, use the adjusted exponential smoothing model to prepare a forecast for each quarter of the year. Make the appropriate initialization assumptions.



Use a seasonality adjusted exponential smoothing model. As two years' worth of data is available and the seasonality repeats annually, the first year can be used to initialize the model. The second year's data can be used to build the model.

Initialization

Base value:

$$\bar{F}_4 = ESF_4 = \frac{1}{4}\sum_{i=1}^{4} D_i = 222.5$$

Seasonality Index:

$$S_i = \frac{D_i}{\bar{F}_4} \quad S_1 = 0.18 \;\; S_2 = 1.573 \;\;\; S_3 = 1.303 \;\;\; S_4 = 0.943820225$$

Forecast for quarter 1 year 2:

$$F_{5,2} = \bar{F}_4 \times S_1 = 40$$

<u>Building</u> with constant $\alpha = 0.2$ $\gamma = 0.2$ MAD=34.81

Base value at quarter 1 year 2:

$$\bar{F}_5 = \alpha\left(\frac{D_5}{S_1}\right) + (1-\alpha)\bar{F}_4 = 244.75$$

Seasonality Index at quarter 1 year 2:

$$S_5 = \gamma\left(\frac{D_5}{\bar{F}_5}\right) + (1-\gamma)S_1 = 0.193$$

Forecast for quarter 2 year 2:

$$F_{6,2} = \bar{F}_5 \times S_2 = 385$$

Base value at quarter 2 year 2:

$$\bar{F}_6 = \alpha\left(\frac{D_6}{S_2}\right) + (1-\alpha)\bar{F}_5 = 237.76$$

Seasonality Index at quarter 2 year 2:

$$S_6 = \gamma\left(\frac{D_6}{\bar{F}_6}\right) + (1-\gamma)S_2 = 1.53602115$$

Forecast for quarter 3 year 2:

$$F_{7,3} = \bar{F}_6 \times S_3 = 309.89$$

Base value at quarter 3 year 2: $\bar{F}_7 = 239.31$

Seasonality Index at quarter 3 year 2: $S_7 = 1.31$

Forecast for quarter 4 year 2: $F_{8,4} = 225.87$

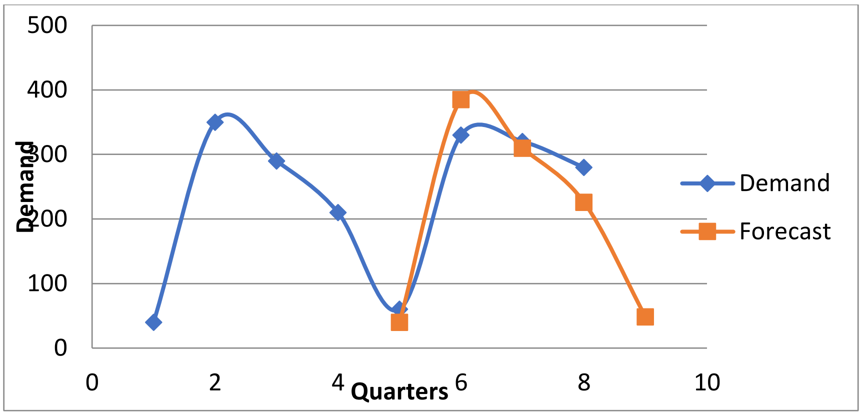
Base value at quarter 4 year 2: $\bar{F}_8 = 250.78$

Seasonality Index at quarter 7 year 2: $S_8 = 0.98$

<u>Forecasting</u> at the end of the data

Forecast for quarter 1 year 3: $F_{9,1} = 48.36$

Beyond quarter 1 year 3, it is difficult for the model to forecast with accuracy as there is no trend to increase the base value after $\bar{F}_8$.



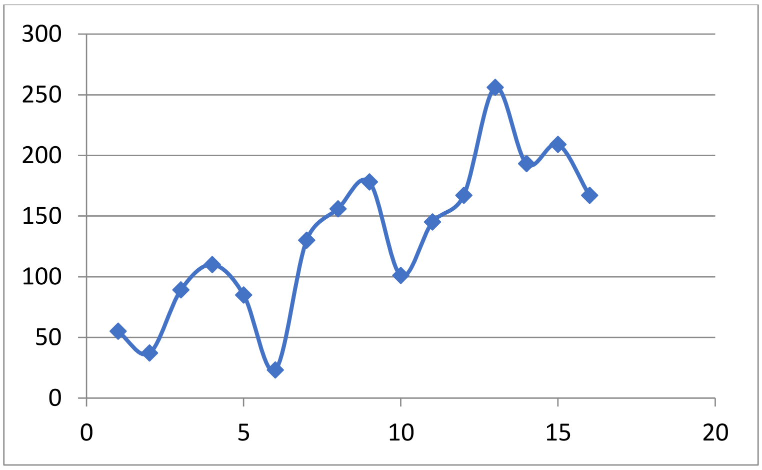
Difficulty Level: Difficult

12. *The Textron company in Mt. Vernon, Texas, asked you to develop quarterly forecasts of combine sales for next year. Combine sales are seasonal, and the data on the quarterly sales for the last four years are as follows:*

QUARTER	YEAR 1	YEAR 2	YEAR 3	YEAR 4
1	55	85	178	256
2	37	23	101	193
3	89	130	145	209
4	110	156	167	167

Lisa Williams estimates the total demand for the next year at Textron. Use the seasonally adjusted exponential smoothing model to develop the forecast for each quarter. Use the appropriate assumptions to initialize the model.

Demand Graph:



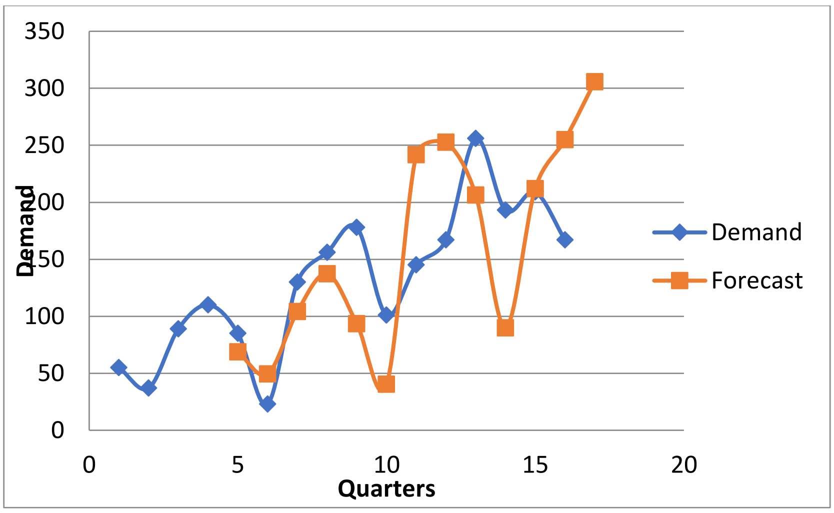
From the graph, it can be seen that the demand shows both trend and seasonality. Thus the exponential smoothing model will include both.

The first year is used to initialize the model and the remainder is used to build the model.

The constants are set to $\alpha = 0.2$ $\gamma = 0.8$ $\beta = 0.2$ MAD=54.845 Bias=4.926.

The following tables show the calculations.

Periods	Years	Quarters	Demand	Base value	Seasonality Index	Trend	Forecast
1	1	1	55		0.7560137	-18	
2		2	37		0.5085911	52	
3		3	89		1.2233677	21	
4		4	110	72.75	1.5120275	18.3333	
5	2	1	85	80.6864	0.9939722	16.2539	68.8603
6		2	23	73.5937	0.3517397	11.5846	49.303
7		3	130	80.1278	1.5426008	10.5745	104.204
8		4	156	84.7368	1.7752021	9.3814	137.144
9	3	1	178	103.605	1.5732416	11.2788	93.5508
10		2	101	140.313	0.646203	16.3646	40.4093
11		3	145	131.05	1.1936793	11.2391	241.691
12		4	167	123.655	1.4354688	7.5122	252.592
13	4	1	256	131.468	1.8724419	7.57243	206.357
14		2	193	164.908	1.0655205	12.7459	89.8483
15		3	209	166.944	1.2402685	10.604	212.062
16		4	167	156.823	1.1390098	6.45895	254.865
17	5	1					305.736
18		2					180.862
19		3					218.535
20		4					208.05

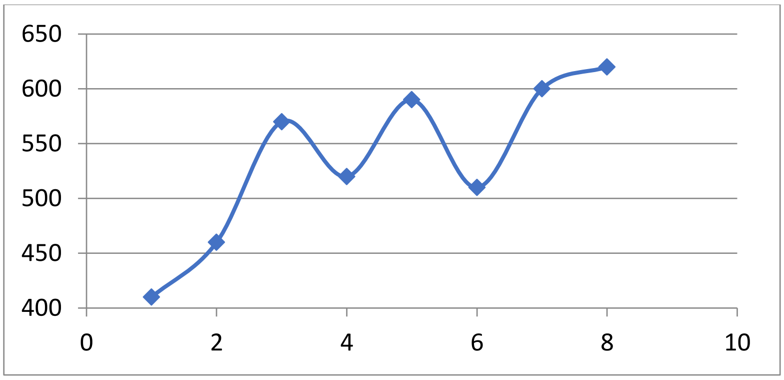


Difficulty Level: Difficult

13. *Demand for car washes at Jenny's Car Wash has been as follows:*

MONTH	NUMBER OF WASHES
January	410
February	460
March	570
April	520
May	590
June	510
July	600
August	620

Compare the performance of at least three exponential smoothing models using the bias and MAD as the performance criteria. Make the appropriate initiating assumptions. Which method do you recommend?



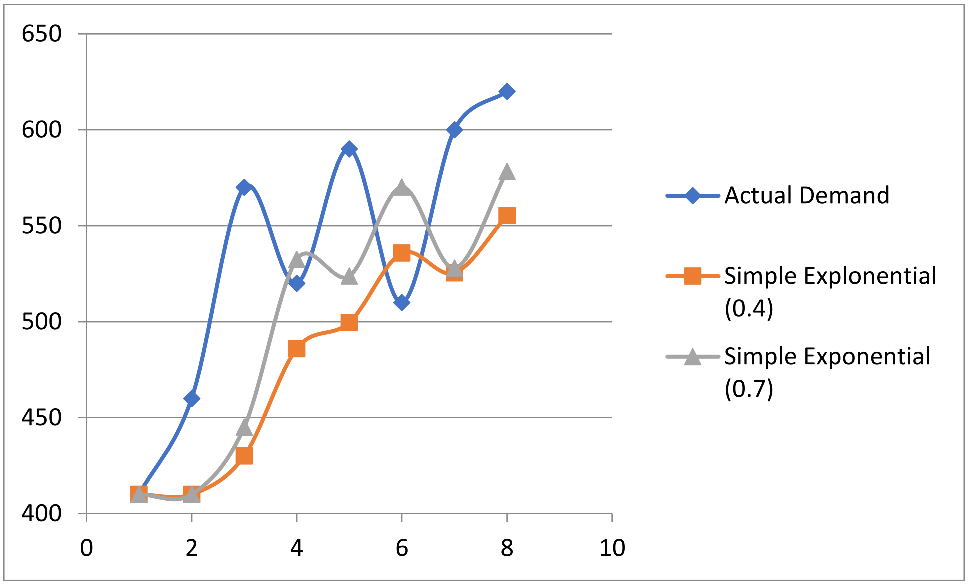
Initializing assumptions: $\bar{F}_0 = D_1 = 410$

1) Simple Exponential smoothing (α=0.4)

   Bias= 53.49 MAD= 59.93

2) Simple Exponential smoothing (α=0.7)

   Bias= 35.27 MAD= 53.43



3) Exponential smoothing with Trend

Initializing assumptions

Use the first 3 weeks to initialize the model.

$$\alpha = 0.5 \qquad \beta = 0.7$$

- Step 1:

$$\bar{F}_3 = \frac{1}{3}\sum_{i=1}^{3} D_i = 480$$

- Step 2:

$$T_3 = \frac{1}{2}\sum_{i=2}^{3}(D_i - D_{i-1})$$

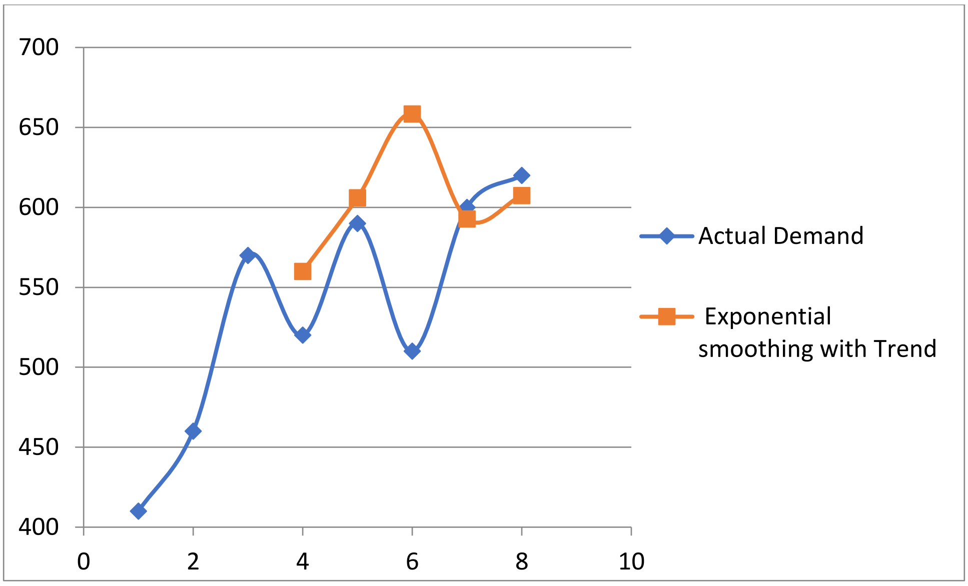
- Step 3: $F_4 = \bar{F}_3 + T_0$=560

Forecast for week 5:

- Step 1: $\bar{F}_4 = \alpha \times D_4 + (1-\alpha)(\bar{F}_3 + T_3) = 540$
- Step 2: $T_4 = \beta(\bar{F}_4 - \bar{F}_3) + (1-\beta)T_3 = 66$
- Step 3: $F_5 = \bar{F}_4 + T_4$= 606

Forecast for week 6:

- Step 1: $\bar{F}_5 = \alpha \times D_5 + (1-\alpha)(\bar{F}_4 + T_4) = 598$
- Step 2: $T_5 = \beta(\bar{F}_5 - \bar{F}_4) + (1-\beta)T_4 = 60.4$
- Step 3: $F_6 = \bar{F}_5 + T_5 = 658.4$



Bias= -26.35 MAD= 32.054

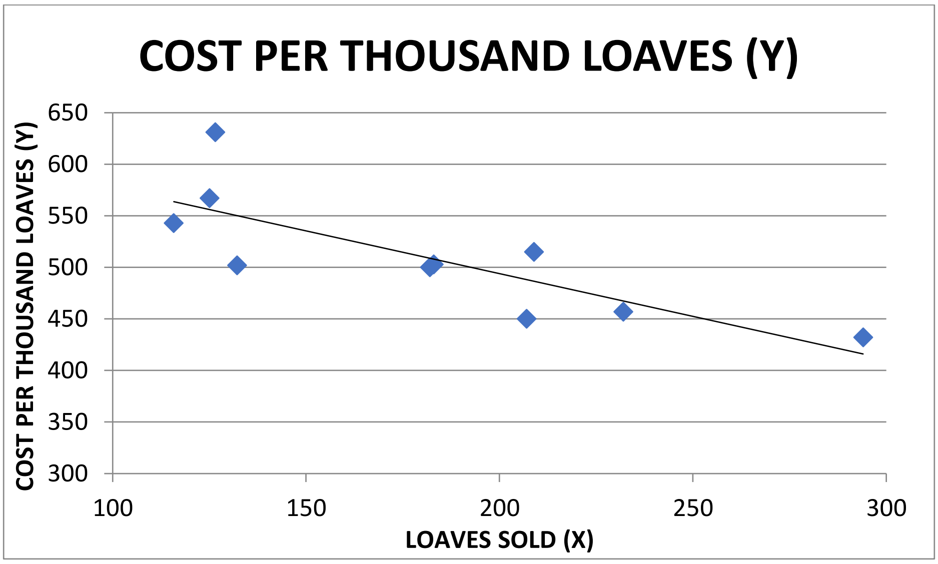
4) As the performance measures are better than the others, Exponential smoothing with Trend could result in more accurate forecasts.

Difficulty Level: Moderate

14. *Mrs. Cole's Bakery bakes and distributes bread throughout central Indiana. The company wants to expand operations by locating another plant in Ohio. The size of the new plant will be a function of the expected demand for baked goods within the area served by the plant. The company wants to estimate the relationship between the manufacturing cost per loaf and the number of loaves sold in a year to determine the demand for bread and, thus, the size of the new plant. The following data have been collected:*

PLANT	COST PER THOUSAND LOAVES (Y)	THOUSANDS OF LOAVES SOLD (X)
1	$515	208.9
2	457	232
3	567	125
4	500	182
5	543	115.8
6	631	126.5
7	432	294
8	450	207.0
9	502	132.2
10	503	183.0
Total		

a. *Develop a regression equation to forecast the cost per loaf as a function of the number of loaves produced.*
x=THOUSANDS OF LOAVES SOLD (X)
y=COST PER THOUSAND LOAVES (Y)
y = -0.8289x + 659.74



b. *What are the correlation coefficient and the coefficient of determination?*

Correlation coefficient= -0.800439533
Coefficient of determination: $R^2 = 0.6407$

c. *Comment on your regression equation in light of these measures.*

64% of the time, an increase of 1,000 loaves sold will decrease the cost per thousand loaves by 0.8289.

d. *Estimate the manufacturing cost per loaf for a plant producing 160,000 loaves per year.*

As x is THOUSANDS OF LOAVES SOLD, x becomes 160.

$$y = -0.8289x + 659.74 = -0.8289 \times (160) + 659.74 = \$527.08$$

The cost per loaf is \$0.517 (\$527.08/1,000).