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CHAPTER 1
Functions and Their Graphs

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Section 1.1 Rectangular Coordinates

1. Cartesian

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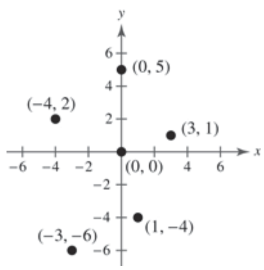
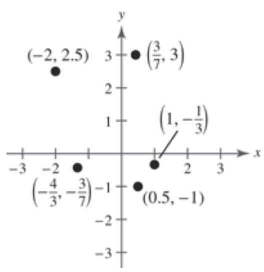
5.



6.



7. $(-3, 4)$

8. $(-12, 0)$

9. $x > 0$ and $y < 0$ in Quadrant IV.

10. $x < 0$ and $y < 0$ in Quadrant III.

11. $x = -4$ and $y > 0$ in Quadrant II.

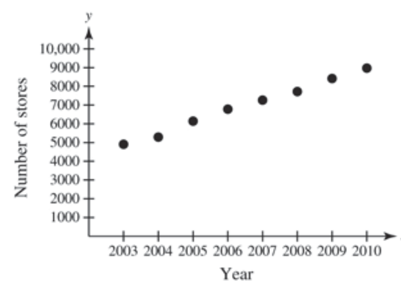
12. $y < -5$ in Quadrant III or IV.

13. $(x, -y)$ is in the second Quadrant means that (x, y) is in Quadrant III.

14. (x, y), $xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

15.

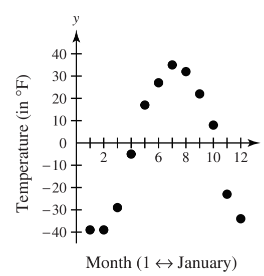
Year, x	Number of Stores, y
2003	4906
2004	5289
2005	6141
2006	6779
2007	7262
2008	7720
2009	8416
2010	8970



Year $(t = 3 \leftrightarrow 2003)$

16.

Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34



17. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(3 - (-2))^2 + (-6 - 6)^2}$
$= \sqrt{(5)^2 + (-12)^2}$
$= \sqrt{25 + 144}$
$= 13$ units

18. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(0 - 8)^2 + (20 - 5)^2}$
$= \sqrt{(-8)^2 + (15)^2}$
$= \sqrt{64 + 225}$
$= \sqrt{289}$
$= 17$ units

19. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-5 - 1)^2 + (-1 - 4)^2}$
$= \sqrt{(-6)^2 + (-5)^2}$
$= \sqrt{36 + 25}$
$= \sqrt{61}$ units

20. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(3 - 1)^2 + (-2 - 3)^2}$
$= \sqrt{(2)^2 + (-5)^2}$
$= \sqrt{4 + 25}$
$= \sqrt{29}$ units

21. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2}$
$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2}$
$= \sqrt{\frac{9}{4} + \frac{49}{9}}$
$= \sqrt{\frac{277}{36}}$
$= \frac{\sqrt{277}}{6}$ units

22. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2}$
$= \sqrt{(-13.4)^2 + (10.8)^2}$
$= \sqrt{179.56 + 116.64}$
$= \sqrt{296.2}$
≈ 17.21 units

23. (a) $(1, 0), (13, 5)$

Distance $= \sqrt{(13 - 1)^2 + (5 - 0)^2}$
$= \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

$(13, 5), (13, 0)$

Distance $= |5 - 0| = |5| = 5$

$(1, 0), (13, 0)$

Distance $= |1 - 13| = |-12| = 12$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

24. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

25. $d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$
$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{25 + 25} = \sqrt{50}$
$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{9 + 36} = \sqrt{45}$
$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

26. $d_1 = \sqrt{(3 - (-1))^2 + (5 - 3)^2} = \sqrt{16 + 4} = \sqrt{20}$
$d_2 = \sqrt{(5 - 3)^2 + (1 - 5)^2} = \sqrt{4 + 16} = \sqrt{20}$
$d_3 = \sqrt{(5 - (-1))^2 + (1 - 3)^2} = \sqrt{36 + 4} = \sqrt{40}$
$\left(\sqrt{20}\right)^2 + \left(\sqrt{20}\right)^2 = \left(\sqrt{40}\right)^2$

27. $d_1 = \sqrt{(1-3)^2 + (-3-2)^2} = \sqrt{4+25} = \sqrt{29}$

$d_2 = \sqrt{(3+2)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$

$d_3 = \sqrt{(1+2)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$

$d_1 = d_2$

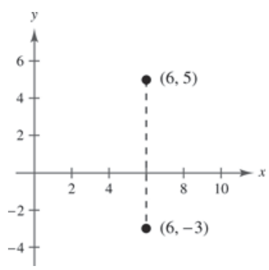
28. $d_1 = \sqrt{(4-2)^2 + (9-3)^2} = \sqrt{4+36} = \sqrt{40}$

$d_2 = \sqrt{(-2-4)^2 + (7-9)^2} = \sqrt{36+4} = \sqrt{40}$

$d_3 = \sqrt{(2-(-2))^2 + (3-7)^2} = \sqrt{16+16} = \sqrt{32}$

$d_1 = d_2$

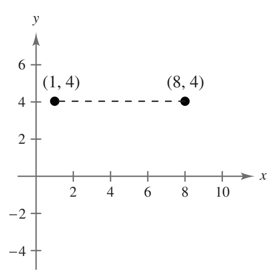
29. (a)



(b) $d = \sqrt{(5-(-3))^2 + (6-6)^2} = \sqrt{64} = 8$

(c) $\left(\dfrac{6+6}{2}, \dfrac{5+(-3)}{2}\right) = (6, 1)$

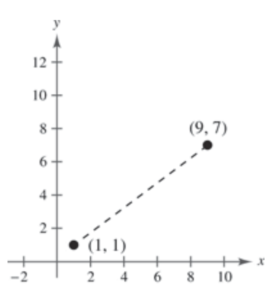
30. (a)



(b) $d = \sqrt{(4-4)^2 + (8-1)^2} = \sqrt{49} = 7$

(c) $\left(\dfrac{1+8}{2}, \dfrac{4+4}{2}\right) = \left(\dfrac{9}{2}, 4\right)$

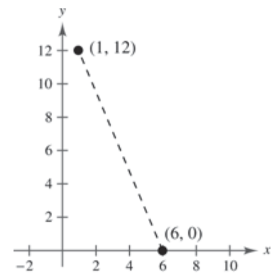
31. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

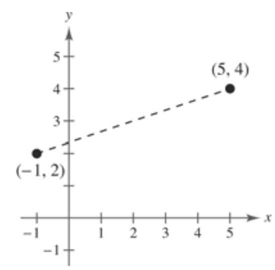
32. (a)



(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

33. (a)

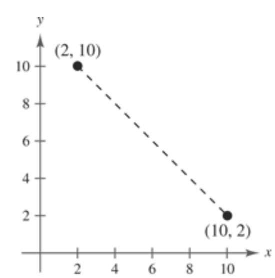


(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$

$= \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

34. (a)

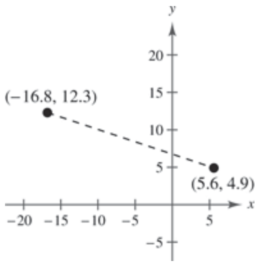


(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

35. (a)

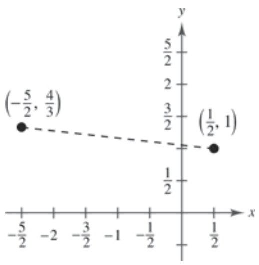


(b) $d = \sqrt{(-16.8 - 5.6)^2 + (12.3 - 4.9)^2}$

$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\dfrac{-16.8 + 5.6}{2}, \dfrac{12.3 + 4.9}{2}\right) = (-5.6, 8.6)$

36. (a)



(b) $d = \sqrt{\left(\dfrac{1}{2} + \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{4}{3}\right)^2}$

$= \sqrt{9 + \dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-(5/2) + (1/2)}{2}, \dfrac{(4/3) + 1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

37. $d = \sqrt{120^2 + 150^2}$

$= \sqrt{36{,}900}$

$= 30\sqrt{41}$

≈ 192.09

The plane flies about 192 kilometers.

38. $d = \sqrt{(42 - 18)^2 + (50 - 12)^2}$

$= \sqrt{24^2 + 38^2}$

$= \sqrt{2020}$

$= 2\sqrt{505}$

≈ 45

The pass is about 45 yards.

39. $\text{midpoint} = \left(\dfrac{2002 + 2010}{2}, \dfrac{19{,}564 + 35{,}123}{2}\right)$

$= (2006, 27{,}343.5)$

In 2006, the sales for the Coca-Cola Company were about $27,343.5 million.

40. $\text{midpoint} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{2008 + 2010}{2}, \dfrac{1.89 + 2.83}{2}\right)$

$= (2009, 2.36)$

In 2009, the earnings per share for Big Lots, Inc. were about $2.36.

41. $(-2 + 2, -4 + 5) = (0, 1)$

$(2 + 2, -3 + 5) = (4, 2)$

$(-1 + 2, -1 + 5) = (1, 4)$

42. $(-3 + 6, 6 - 3) = (3, 3)$

$(-5 + 6, 3 - 3) = (1, 0)$

$(-3 + 6, 0 - 3) = (3, -3)$

$(-1 + 6, 3 - 3) = (5, 0)$

43. $(-7 + 4, -2 + 8) = (-3, 6)$

$(-2 + 4, 2 + 8) = (2, 10)$

$(-2 + 4, -4 + 8) = (2, 4)$

$(-7 + 4, -4 + 8) = (-3, 4)$

44. $(5 - 10, 8 - 6) = (-5, 2)$

$(3 - 10, 6 - 6) = (-7, 0)$

$(7 - 10, 6 - 6) = (-3, 0)$

45. (a) The minimum wage had the greatest increase from 2000 to 2010.

(b) Minimum wage in 1990: \$3.80

Minimum wage in 1995: \$4.25

Percent increase: $\left(\dfrac{4.25 - 3.80}{3.80}\right)(100) \approx 11.8\%$

Minimum wage in 1995: \$4.25

Minimum wage in 2011: \$7.25

Percent increase: $\left(\dfrac{7.25 - 4.25}{4.25}\right)(100) \approx 70.6\%$

So, the minimum wage increased 11.8% from 1990 to 1995 and 70.6% from 1995 to 2011.

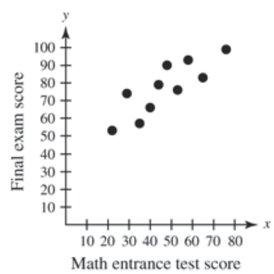
(c) $\begin{pmatrix}\text{Minimum wage}\\ \text{in 2016}\end{pmatrix} = \begin{pmatrix}\text{Minimum wage}\\ \text{in 2011}\end{pmatrix} + \begin{pmatrix}\text{Percent}\\ \text{increase}\end{pmatrix}\begin{pmatrix}\text{Minimum wage}\\ \text{in 2011}\end{pmatrix} \approx \$7.25 + 0.706(\$7.25) \approx \$12.37$

So, the minimum wage will be about \$12.37 in the year 2016.

(d) Answer will vary. *Sample answer:* No, the prediction is too high because it is likely that the percent increase over a 4-year period (2011–2016) will be less than the percent increase over a 16-year period (1995–2011).

46. (a)

x	***y***
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that will affect the final exam score.

47. Because $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$

$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

48. (a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 4 - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 2 - (-5), 2 \cdot 4 - 11) = (9, -3)$

49. The midpoint of the given line segment is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is $\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$.

So, the three points are $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, and $\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$.

50. (a) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3 \cdot 1 + 4}{4}, \frac{3(-2) - 1}{4}\right) = \left(\frac{7}{4}, -\frac{7}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 4}{2}, \frac{-2 - 1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$

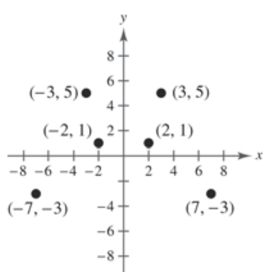
$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{1 + 3 \cdot 4}{4}, \frac{-2 + 3(-1)}{4}\right) = \left(\frac{13}{4}, -\frac{5}{4}\right)$

(b) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3(-2) + 0}{4}, \frac{3(-3) + 0}{4}\right) = \left(-\frac{3}{2}, -\frac{9}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{-3 + 0}{2}\right) = \left(-1, -\frac{3}{2}\right)$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{-2 + 0}{4}, \frac{-3 + 0}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$

51.



(a) The point is reflected through the y-axis.
(b) The point is reflected through the x-axis.
(c) The point is reflected through the origin.

52. (a)

First Set

$d(A, B) = \sqrt{(2 - 2)^2 + (3 - 6)^2} = \sqrt{9} = 3$

$d(B, C) = \sqrt{(2 - 6)^2 + (6 - 3)^2} = \sqrt{16 + 9} = 5$

$d(A, C) = \sqrt{(2 - 6)^2 + (3 - 3)^2} = \sqrt{16} = 4$

Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

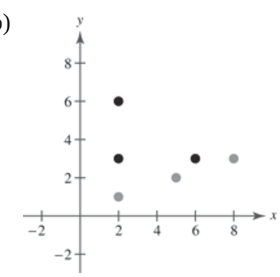
$d(A, B) = \sqrt{(8 - 5)^2 + (3 - 2)^2} = \sqrt{10}$

$d(B, C) = \sqrt{(5 - 2)^2 + (2 - 1)^2} = \sqrt{10}$

$d(A, C) = \sqrt{(8 - 2)^2 + (3 - 1)^2} = \sqrt{40}$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear

Second set: The points are collinear.

(c) If A, B, and C are collinear, then two of the distances will add up to the third distance.

53. No. It depends on the magnitude of the quantities measured.

54. The y-coordinate of a point on the x-axis is 0. The x-coordinates of a point on the y-axis is 0.

55. False, you would have to use the Midpoint Formula 15 times.

56. True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

57. False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

58. (a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (ii).

(b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (iii).

(c) Because (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (iv).

(d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (i).

59. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$$\left(\frac{b + a}{2}, \frac{c + 0}{2}\right) = \left(\frac{a + b}{2}, \frac{c}{2}\right)$$

$$\left(\frac{a + b + 0}{2}, \frac{c + 0}{2}\right) = \left(\frac{a + b}{2}, \frac{c}{2}\right)$$

Section 1.2 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. circle; (h, k); r

6. numerical

7. (a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$

Yes, the point *is* on the graph.

8. (a) $(1, 2)$: $2 \stackrel{?}{=} \sqrt{5 - 1}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 0)$: $0 \stackrel{?}{=} \sqrt{5 - 5}$

$0 = 0$

Yes, the point *is* on the graph.

9. (a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

10. (a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

11. (a) $(2, 3)$: $3 \stackrel{?}{=} |2 - 1| + 2$

$3 \stackrel{?}{=} 1 + 2$

$3 = 3$

Yes, the point *is* on the graph.

(b) $(-1, 0)$: $0 \stackrel{?}{=} |-1 - 1| + 2$

$0 \stackrel{?}{=} 2 + 2$

$0 \neq 4$

No, the point *is not* on the graph.

12. (a) $(1, 2)$: $2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point *is not* on the graph.

(b) $(1, -1)$: $2(1) - (-1) - 3 \stackrel{?}{=} 0$

$2 + 1 - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

13. (a) $(3, -2)$: $(3)^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + (2)^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

14. (a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$

$-\frac{16}{3} = -\frac{16}{3}$

Yes, the point *is* on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$

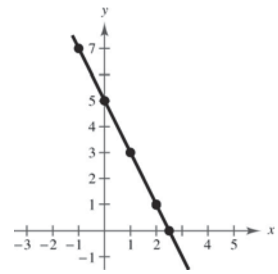
$-9 - 18 \stackrel{?}{=} 9$

$-27 \neq 9$

No, the point *is not* on the graph.

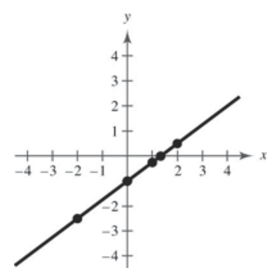
15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



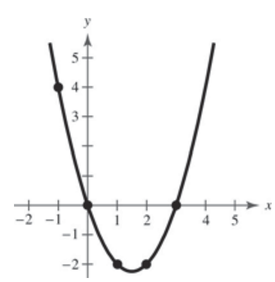
16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



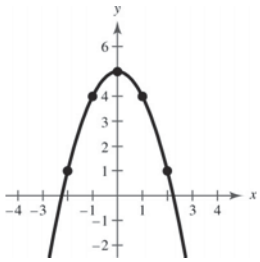
17. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



18. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

22. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

23. x-intercept: $(1, 0)$

y-intercept: $(0, 2)$

24. x-intercepts: $(0, 0), (0, \pm 2)$

y-intercept: $(0, 0)$

25. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

26. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

27. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

28. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

29. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

30. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

31. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

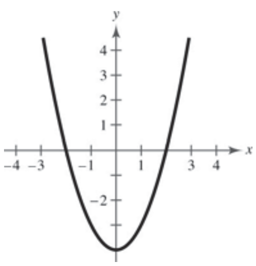
32. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

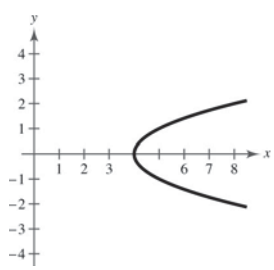
$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

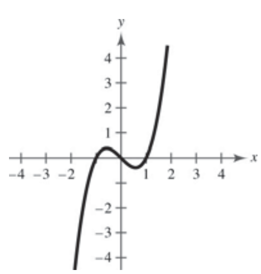
33.



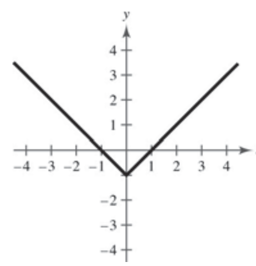
34.



35.



36.

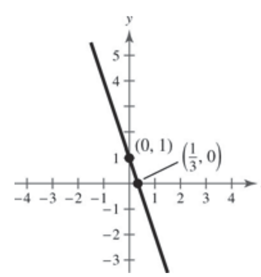


37. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

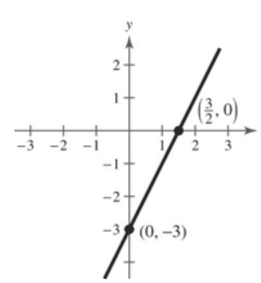


38. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



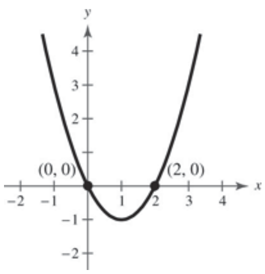
39. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	–1	0	1	2	3
y	3	0	–1	0	3

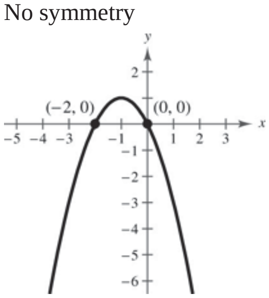


40. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



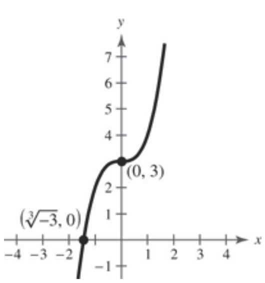
41. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	–2	–1	0	1	2
y	–5	2	3	4	11

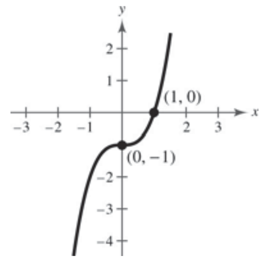


42. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



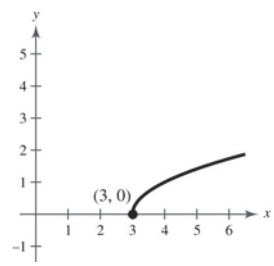
43. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

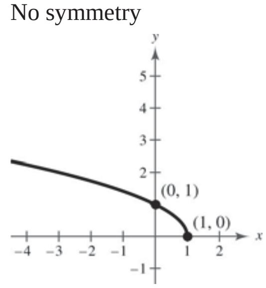


44. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



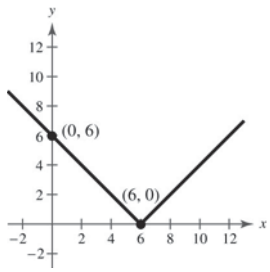
45. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	–2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

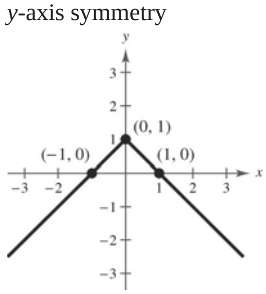


46. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



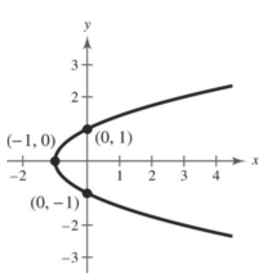
47. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	–1	0	3
y	0	±1	±2

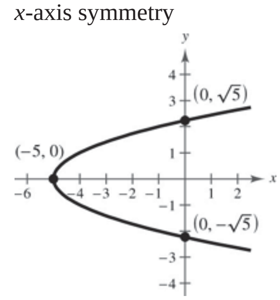


48. $x = y^2 - 5$

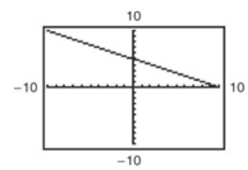
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

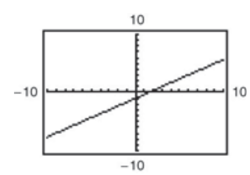


49. $y = 5 - \frac{1}{2}x$



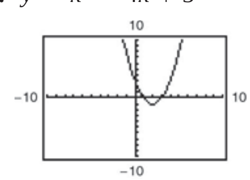
Intercepts: $(10, 0), (0, 5)$

50. $y = \frac{2}{3}x - 1$



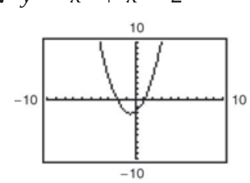
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

51. $y = x^2 - 4x + 3$



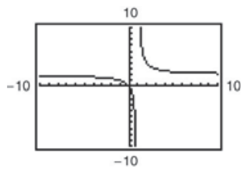
Intercepts: $(3, 0), (1, 0), (0, 3)$

52. $y = x^2 + x - 2$



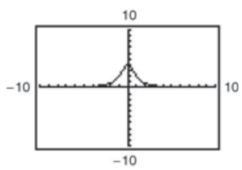
Intercepts: $(-2, 0), (1, 0), (0, -2)$

53. $y = \dfrac{2x}{x - 1}$



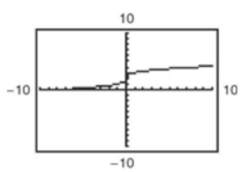
Intercept: $(0, 0)$

54. $y = \dfrac{4}{x^2 + 1}$



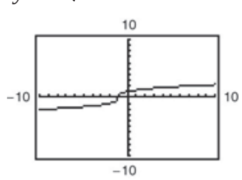
Intercept: $(0, 4)$

55. $y = \sqrt[3]{x} + 2$



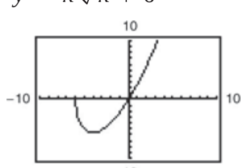
Intercepts: $(-8, 0), (0, 2)$

56. $y = \sqrt[3]{x + 1}$



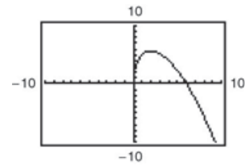
Intercepts: $(-1, 0), (0, 1)$

57. $y = x\sqrt{x + 6}$



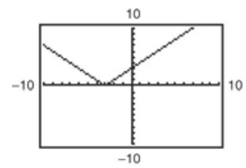
Intercepts: $(0, 0), (-6, 0)$

58. $y = (6 - x)\sqrt{x}$



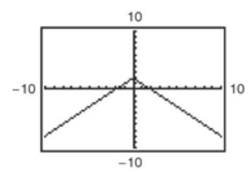
Intercepts: $(0, 0), (6, 0)$

59. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

60. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

61. Center: $(0, 0)$; Radius: 4

$$(x - 0)^2 + (y - 0)^2 = 4^2$$
$$x^2 + y^2 = 16$$

62. Center: $(0, 0)$; Radius: 5

$$(x - 0)^2 + (y - 0)^2 = 5^2$$
$$x^2 + y^2 = 25$$

63. Center: $(2, -1)$; Radius: 4

$$(x - 2)^2 + (y - (-1))^2 = 4^2$$
$$(x - 2)^2 + (y + 1)^2 = 16$$

64. Center: $(-7, -4)$; Radius: 7

$$(x - (-7))^2 + (y - (-4))^2 = 7^2$$
$$(x + 7)^2 + (y + 4)^2 = 49$$

65. Center: $(-1, 2)$; Solution point: $(0, 0)$

$$(x - (-1))^2 + (y - 2)^2 = r^2$$
$$(0 + 1)^2 + (0 - 2)^2 = r^2 \Rightarrow 5 = r^2$$
$$(x + 1)^2 + (y - 2)^2 = 5$$

66. Center: $(3, -2)$; Solution point: $(-1, 1)$

$$r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2}$$
$$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$
$$(x - 3)^2 + (y - (-2))^2 = 5^2$$
$$(x - 3)^2 + (y + 2)^2 = 25$$

67. Endpoints of a diameter: $(0, 0), (6, 8)$

$$\text{Center: } \left(\frac{0 + 6}{2}, \frac{0 + 8}{2}\right) = (3, 4)$$
$$(x - 3)^2 + (y - 4)^2 = r^2$$
$$(0 - 3)^2 + (0 - 4)^2 = r^2 \Rightarrow 25 = r^2$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

68. Endpoints of a diameter: $(-4, -1), (4, 1)$

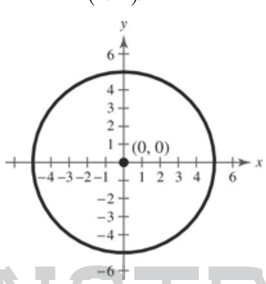
$$r = \frac{1}{2}\sqrt{(-4 - 4)^2 + (-1 - 1)^2}$$
$$= \frac{1}{2}\sqrt{(-8)^2 + (-2)^2}$$
$$= \frac{1}{2}\sqrt{64 + 4}$$
$$= \frac{1}{2}\sqrt{68} = \left(\frac{1}{2}\right)(2)\sqrt{17} = \sqrt{17}$$

Midpoint of diameter (center of circle):

$$\left(\frac{-4 + 4}{2}, \frac{-1 + 1}{2}\right) = (0, 0)$$
$$(x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2$$
$$x^2 + y^2 = 17$$

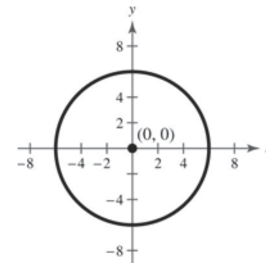
69. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



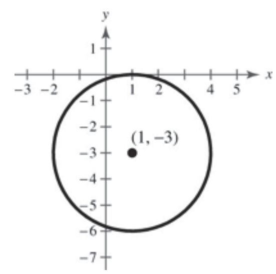
70. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



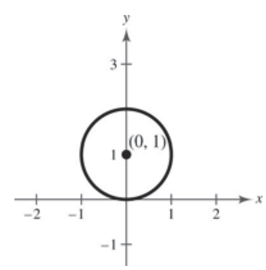
71. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, Radius: 3



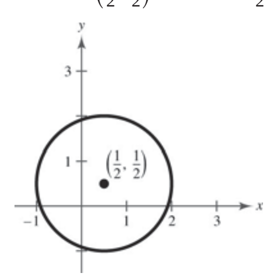
72. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, Radius: 1



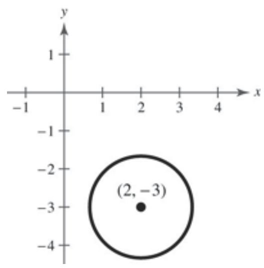
73. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

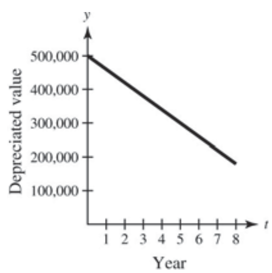


74. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

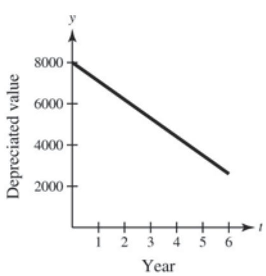
Center: $(2, -3)$, Radius: $\frac{4}{3}$



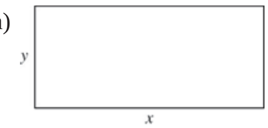
75. $y = 500{,}000 - 40{,}000t,\ 0 \le t \le 8$



76. $y = 8000 - 900t,\ 0 \le t \le 6$



77. (a)



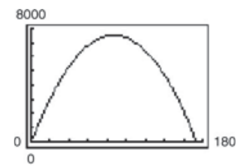
(b) $2x + 2y = \frac{1040}{3}$

$2y = \frac{1040}{3} - 2x$

$y = \frac{520}{3} - x$

$A = xy = x\left(\frac{520}{3} - x\right)$

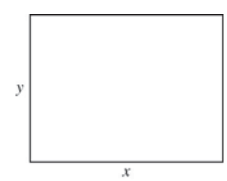
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

78. (a)



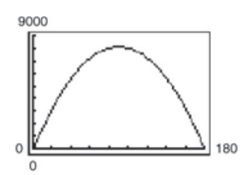
(b) $P = 360$ meters so:

$2x + 2y = 360$

$w = y = 180 - x$

$A = lw = x(180 - x)$

(c)

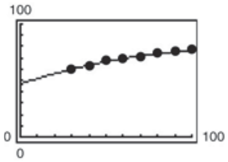


(d) $x = 90$ and $y = 90$

A square will give the maximum area of 8100 square meters.

(e) Answers will vary. *Sample answer:* The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width. A field of length 115 yards and width 75 yards would have an area of 8625 square yards.

79. (a)



Because the line is close to the points, the model fits the data well.

(b) Graphically: The point $(90, 75.4)$ represents a life expectancy of 75.4 years in 1990.

Algebraically:
$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(90)^2 + 0.5(90) + 46.6 \\ &= 75.4 \end{aligned}$$

So, the life expectancy in 1990 was about 75.4 years.

(c) Graphically: The point $(94.6, 76.0)$ represents a life expectancy of 76 years during the year 1994.

Algebraically:
$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ 76.0 &= -0.002t^2 + 0.5t + 46.6 \\ 0 &= -0.002t^2 + 0.5t - 29.4 \end{aligned}$$

Use the quadratic formula to solve.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0.5) \pm \sqrt{(0.5)^2 - 4(-0.002)(-29.4)}}{2(-0.002)} \\ &= \frac{-0.5 \pm \sqrt{0.0148}}{-0.004} \\ &= 125 \pm 30.4 \end{aligned}$$

So, $t = 94.6$ or $t = 155.4$. Since 155.4 is not in the domain, the solution is $t = 94.6$, which is the year 1994.

(d) When $t = 115$:

$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(115)^2 + (0.5)(115) + 46.6 \\ &= 77.65 \end{aligned}$$

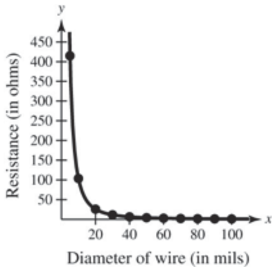
The life expectancy using the model is 77.65 years, which is slightly less than the given projection of 78.9 years.

(e) Answers will vary. *Sample answer:* No. Because the model is quadratic, the life expectancies begin to decrease after a certain point.

80. (a)

x	5	10	20	30	40	50	60	70	80	90	100
y	414.8	103.7	25.9	11.5	6.5	4.1	2.9	2.1	1.6	1.3	1.0

(b)



When $x = 85.5$, the resistance is about 1.4 ohms.

(c) When $x = 85.5$,

$$y = \frac{10{,}370}{(85.5)^2} = 1.4 \text{ ohms.}$$

(d) As the diameter of the copper wire increases, the resistance decreases.

81. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

82. x-axis symmetry:

$x^2 + y^2 = 1$

$x^2 + (-y)^2 = 1$

$x^2 + y^2 = 1$

y-axis symmetry:

$x^2 + y^2 = 1$

$(-x)^2 + y^2 = 1$

$x^2 + y^2 = 1$

Origin symmetry:

$x^2 + y^2 = 1$

$(-x)^2 + (-y)^2 = 1$

$x^2 + y^2 = 1$

So, the graph of the equation is symmetric with respect to the x-axis, the y-axis, and the origin.

Section 1.3 Linear Equations in Two Variables

1. linear

2. slope

3. point-slope

4. parallel

5. perpendicular

6. rate or rate of change

7. linear extrapolation

8. general

9. (a) $m = \frac{2}{3}$. Because the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

(c) $m = -2$. The line falls. Matches L_1.

10. (a) $m = 0$. The line is horizontal. Matches L_2.

(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.