2 Review and Applications of Algebra

Exercise 2.1

1. $(-p) + (-3p) + 4p = -p - 3p + 4p = \underline{0}$

2. $(5s - 2t) - (2s - 4t) = 5s - 2t - 2s + 4t = \underline{3s + 2t}$

3. $4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = \underline{6x^2y}$

4. $1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = \underline{e^3 - 7e^2 - 3e + 6}$

5. $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$
$= \underline{7x^2 + 7xy - 4y^2}$

6. $(7m^3 - m - 6m^2 + 10) - (5m^3 - 9 + 3m - 2m^2)$
$= 7m^3 - m - 6m^2 + 10 - 5m^3 + 9 - 3m + 2m^2$
$= \underline{2m^3 - 4m^2 - 4m + 19}$

7. $2(7x - 3y) - 3(2x - 3y) = 14x - 6y - 6x + 9y = \underline{8x + 3y}$

8. $4(a^2 - 3a - 4) - 2(5a^2 - a - 6) = 4a^2 - 12a - 16 - 10a^2 + 2a + 12$
$= \underline{-6a^2 - 10a - 4}$

9. $15x - [4 - 2(5x - 6)] = 15x - 4 + 10x - 12 = \underline{25x - 16}$

10. $6a - [3a - 2(2b - a)] = 6a - 3a + 4b - 2a = \underline{a + 4b}$

11. $4a(3ab - 5a + 6b) = \underline{12a^2b - 20a^2 + 24ab}$

Exercise 2.1 *(continued)*

12. $9k(4 - 8k + 7k^2) = \underline{36k - 72k^2 + 63k^3}$

13. $-5xy(2x^2 - xy - 3y^2) = \underline{-10x^3y + 5x^2y^2 + 15xy^3}$

14. $-(p^2 - 4pq - 5p)\left(\frac{2q}{p}\right) = \underline{-2pq + 8q^2 + 10q}$

15. $(4r - 3t)(2t + 5r) = 8rt + 20r^2 - 6t^2 - 15rt = \underline{20r^2 - 7rt - 6t^2}$

16. $(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = \underline{-12p^3 + 26p^2 - 10p}$

17. $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$
$= 12a^2 - 21a - 6 - 10a^2 + 55a + 105$
$= \underline{2a^2 + 34a + 99}$

18. $5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$
$= -5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$
$= \underline{24x^2 + 25xy - 5y^2}$

19. $\frac{18x^2}{3x} = \underline{6x}$

20. $\frac{6a^2b}{-2ab^2} = \underline{\underline{-3\frac{a}{b}}}$

21. $\dfrac{x^2y - xy^2}{xy} = \underline{x - y}$

22.. $\dfrac{-4x + 10x^2 - 6x^3}{-0.5x} = \underline{8 - 20x + 12x^2}$

23. $\dfrac{12x^3 - 24x^2 + 36x}{48x} = \underline{\dfrac{x^2 - 2x + 3}{4}}$

24. $\dfrac{32a^2b - 8ab + 14ab^2}{2ab} = \underline{16a - 4 + 7b}$

25. $\dfrac{4a^2b^3 - 6a^3b^2}{2ab^2} = \underline{2ab - 3a^2}$

26. $\dfrac{120(1+i)^2 + 180(1+i)^3}{360(1+i)} = \underline{\dfrac{2(1+i) + 3(1+i)^2}{6}}$

27. $3d^2 - 4d + 15 = 3(2.5)^2 - 4(2.5) + 15$
$= 18.75 - 10 + 15$
$= \underline{23.75}$

28. $15g - 9h + 3 = 15(14) - 9(15) + 3 = \underline{78}$

29. $7x(4y - 8) = 7(3.2)(4 \times 1.5 - 8) = 22.4(6 - 8) = \underline{-44.8}$

30. $I \div Pr = \dfrac{\$13.75}{\$500 \times 0.11} = \underline{0.250}$

31. $\dfrac{I}{rt} = \dfrac{\$23.21}{0.095 \times \frac{283}{365}} = \dfrac{\$23.21}{0.073658} = \underline{\$315.11}$

Exercise 2.1 ***(continued)***

32. $\dfrac{N}{1-d} = \dfrac{\$89.10}{1-0.10} = \underline{\$99.00}$

33. $L(1 - d_1)(1 - d_2)(1 - d_3) = \$490(1 - 0.125)(1 - 0.15)(1 - 0.05) = \underline{\$346.22}$

34. $P(1 + rt) = \$770\left(1 + 0.013 \times \dfrac{223}{365}\right) = \$770(1.0079425) = \underline{\$776.12}$

35. $\dfrac{S}{1+rt} = \dfrac{\$2500}{1+0.085 \times \frac{123}{365}} = \dfrac{\$2500}{1.028644} = \underline{\$2430.38}$

36. $\dfrac{S}{(1+i)^n} = \dfrac{\$850}{(1+0.0075)^6} = \dfrac{\$850}{1.045852} = \underline{\$812.73}$

37. $P(1 + i)^n = \$1280(1 + 0.025)^3 = \underline{\$1378.42}$

38. $\dfrac{x}{2} - x^2 + \dfrac{4}{5} - 0.2x^2 - \dfrac{4}{5}x + \dfrac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$
$= \underline{-1.2x^2 - 0.3x + 1.3}$

39. $\frac{2x+9}{4} - 1.2(x-1) = 0.5x + 2.25 - 1.2x + 1.2 = -\underline{0.7x + 3.45}$

40. $\frac{2x}{1.045} - \frac{2.016x}{3} + \frac{x}{2} = 1.9139x - 0.6720x + 0.5x = \underline{1.7419x}$

41. $\frac{8x}{0.5} + \frac{5.5x}{11} + 0.5(4.6x - 17) = 16x + 0.5x + 2.3x - 8.5 = \underline{18.8x - 8.5}$

42. $y\left(1 - 0.125 \times \frac{213}{365}\right) + \frac{2y}{1 + 0.125 \times \frac{88}{365}} = 0.92705y + 1.94149y = \underline{2.8685y}$

43. $\frac{P}{1 + 0.095 \times \frac{5}{12}} + 2P\left(1 + 0.095 \times \frac{171}{365}\right) = 0.96192P + 2.08901P = \underline{3.0509P}$

44. $\frac{h}{(1 + 0.055)^2} - 3h(1 + 0.055)^3 = 0.89845h - 3.52272h = \underline{-2.6243h}$

45. $k(1 + 0.04)^2 + \frac{2k}{(1 + 0.04)^2} = 1.08160k + 1.84911k = \underline{2.9307k}$

46. $(1 + i)^m - 1 = (1 + 0.0225)^4 - 1 = \underline{0.093083}$

47. $R\left[\frac{(1+i)^n - 1}{i}\right] = \$550\left(\frac{1.085^3 - 1}{0.085}\right) = \$550\left(\frac{0.2772891}{0.085}\right) = \underline{\$1794.22}$

48. $R\left[\frac{(1+i)^n - 1}{i}\right](1+i) = \$910\left(\frac{1.1038129^4 - 1}{0.1038129}\right)(1.1038129)$

$= \$910\left(\frac{0.4845057}{0.1038129}\right)(1.1038129)$

$= \underline{\$4687.97}$

49. $\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{\$630}{0.115}\left(1 - \frac{1}{1.115^2}\right) = \underline{\$1071.77}$

50. $P(1 + rt_1) + \frac{S}{1 + rt_2} = \$470\left(1 + 0.075 \times \frac{104}{365}\right) + \frac{\$390}{1 + 0.075 \times \frac{73}{365}}$

$= \$470(1.021370) + \frac{\$390}{1.01500}$

$= \$480.044 + \$384.236$

$= \underline{\$864.28}$

Exercise 2.2

1. $a^2 \times a^3 = \underline{\underline{a^5}}$
2. $(x^6)(x^{-4}) = \underline{\underline{x^2}}$
3. $b^{10} \div b^6 = b^{10-6} = \underline{\underline{b^4}}$
4. $h^7 \div h^{-4} = h^{7-(-4)} = \underline{\underline{h^{11}}}$
5. $(1+i)^4 \times (1+i)^9 = \underline{\underline{(1+i)^{13}}}$
6. $(1+i) \times (1+i)^n = \underline{\underline{(1+i)^{n+1}}}$
7. $(x^4)^7 = x^{4\times7} = \underline{\underline{x^{28}}}$
8. $(y^3)^3 = \underline{\underline{y^9}}$
9. $\left(t^6\right)^{\frac{1}{3}} = \underline{\underline{t^2}}$
10. $(n^{0.5})^8 = \underline{\underline{n^4}}$
11. $\dfrac{\left(x^5\right)\left(x^6\right)}{x^9} = x^{5+6-9} = \underline{\underline{x^2}}$
12. $\dfrac{\left(x^5\right)^6}{x^9} = x^{5\times6-9} = \underline{\underline{x^{21}}}$
13. $\left[2(1+i)\right]^2 = \underline{\underline{4(1+i)^2}}$
14. $\left(\dfrac{1+i}{3i}\right)^3 = \underline{\underline{\dfrac{(1+i)^3}{27i^3}}}$
15. $\dfrac{4r^5t^6}{\left(2r^2t\right)^3} = \dfrac{4r^5t^6}{8r^6t^3} = \dfrac{r^{5-6}t^{6-3}}{2} = \underline{\underline{\dfrac{t^3}{2r}}}$
16. $\dfrac{\left(-r^3\right)(2r)^4}{\left(2r^{-2}\right)^2} = \dfrac{-r^3\left(16r^4\right)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{\underline{-4r^{11}}}$
17. $8^{4/3} = \left(8^{1/3}\right)^4 = 2^4 = \underline{\underline{16.0000}}$
18. $-27^{2/3} = -\left(27^{1/3}\right)^2 = \underline{\underline{-9.00000}}$
19. $7^{3/2} = 7^{1.5} = \underline{\underline{18.5203}}$
20. $5^{-\frac{3}{4}} = 5^{-0.75} = \underline{\underline{0.299070}}$
21. $(0.001)^{-2} = \underline{\underline{1{,}000{,}000}}$
22. $0.893^{-1/2} = 0.893^{-0.5} = \underline{\underline{1.05822}}$

Exercise 2.2 *(continued)*

23. $(1.0085)^5(1.0085)^3 = 1.0085^8 = \underline{\underline{1.07006}}$

24. $(1.005)^3(1.005)^{-6} = 1.005^{-3} = \underline{\underline{0.985149}}$

25. $\sqrt[3]{103} = 103^{0.\overline{3}} = \underline{\underline{100990}}$

26. $\sqrt[6]{105} = \underline{\underline{100816}}$

27. $\left(4^4\right)\left(3^{-3}\right)\left(-\frac{3}{4}\right)^3 = \frac{4^4}{3^3}\left(-\frac{3^3}{4^3}\right) =$ - $\underline{\underline{4.00000}}$

28. $\left[\left(-\frac{3}{4}\right)^2\right]^{-2} = \left(-\frac{3}{4}\right)^{-4} = \left(-\frac{4}{3}\right)^4 = \frac{256}{81} = \underline{\underline{3.16049}}$

29. $\left(\frac{2}{3}\right)^3\left(-\frac{3}{2}\right)^2\left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3\left(\frac{3}{2}\right)^2\left(-\frac{2}{3}\right)^3 = \frac{2}{3}\left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{\underline{-0.197531}}$

30. $\left(-\frac{2}{3}\right)^3 + \left(\frac{3}{2}\right)^{-2} = \frac{\left(-\frac{2}{3}\right)^3}{\left(\frac{2}{3}\right)^2} = -\frac{2}{3} = \underline{\underline{-0.666667}}$

31. $\frac{103^{16} - 1}{0.03} = \underline{\underline{20.1569}}$

32. $\frac{\left(1008\overline{3}\right)^{30} - 1}{0.008\overline{3}} = \frac{0.2826960}{0.008333333} = \underline{\underline{33.9235}}$

33. $\frac{1 - 10225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \underline{\underline{15.9637}}$

34. $\frac{1 - \left(100\overline{6}\right)^{-32}}{0.00\overline{6}} = \frac{0.1915410}{0.00\overline{6}} = \underline{\underline{28.7312}}$

35. $(1 + 0.0275)^{1/3} =$ $\underline{\underline{1.00908}}$

36. $(1 + 0.055)^{1/6} - 1 =$ $\underline{\underline{0.00896339}}$

Exercise 2.3

1. $10a + 10 = 12 + 9a$
   $10a - 9a = 12 - 10$
   $a = \underline{\underline{2}}$

2. $29 - 4y = 2y - 7$
   $36 = 6y$
   $y = \underline{\underline{6}}$

3. $0.5\,(x - 3) = 20$

$x - 3 = 40$
$x = \underline{\underline{43}}$

Exercise 2.3 ***(continued)***

4. $\frac{1}{3}(x-2) = 4$
$x - 2 = 12$
$x = \underline{\underline{14}}$

5. $y = 192 + 0.04y$
$y - 0.04y = 192$
$y = \frac{192}{0.96} = \underline{\underline{200}}$

6. $x - 0.025x = 341.25$
$0.975x = 341.25$
$x = \frac{341.25}{0.975} = \underline{\underline{350}}$

7. $12x - 4(2x - 1) = 6(x + 1) - 3$
$12x - 8x + 4 = 6x + 6 - 3$
$-2x = -1$
$x = \underline{\underline{0.5}}$

8. $3y - 4 = 3(y + 6) - 2(y + 3)$
$= 3y + 18 - 2y - 6$
$2y = 16$
$y = \underline{\underline{8}}$

9. $8 - 0.5(x + 3) = 0.25(x - 1)$
$8 - 0.5x - 1.5 = 0.25x - 0.25$
$-0.75x = -6.75$
$x = \underline{\underline{9}}$

10. $5(2 - c) = 10(2c - 4) - 6(3c + 1)$
$10 - 5c = 20c - 40 - 18c - 6$
$-7c = -56$
$c = \underline{\underline{8}}$

11. $3.1t + 145 = 10 + 7.6t$
$-4.5t = -135$
$t = \underline{\underline{30}}$

12. $1.25y - 20.5 = 0.5y - 11.5$
$0.75y = 9$
$y = \underline{\underline{12}}$

13. $x - y = 2$ ①
$3x + 4y = 20$ ②
①$\times$ 3: $\underline{3x - 3y} = \underline{\ 6}$
Subtract: $7y = 14$
$y = 2$

Substitute into equation ①:
$x - 2 = 2$
$x = 4$

$(x, y) = \underline{(4, 2)}$

Check: LHS of ② = 3(4) + 4(2) = 20 = RHS of ②

14.

$$\begin{aligned} y - 3x &= 11 \quad ① \\ -4y + 5x &= -30 \quad ② \\ ① \times 4: \quad \underline{4y - 12x} &= \underline{44} \\ \text{Add:} \quad -7x &= 14 \\ x &= -2 \end{aligned}$$

Substitute into equation ①:

$$\begin{aligned} y - 3(-2) &= 11 \\ y &= 11 - 6 = 5 \\ (x, y) &= \underline{(-2, 5)} \end{aligned}$$

Check: LHS of ② = $-4(5) + 5(-2) = -30$ = RHS of ②

Exercise 2.3 ***(continued)***

15.
$$4a - 3b = -3 \quad ①$$
$$5a - b = 10 \quad ②$$
$① \times 1$: $4a - 3b = -3$
$② \times 3$: $\underline{15a - 3b} = \underline{30}$
Subtract: $-11a = -33$
$a = 3$

Substitute into equation ②:
$5(3) - b = 10$
$b = 5$
$(a, b) = \underline{(3, 5)}$

Check: LHS of ① = $4(3) - 3(5) = -3$ = RHS of ①

16.
$$7p - 3q = 23 \quad ①$$
$$\underline{-2p - 3q} = \underline{5} \quad ②$$
Subtract: $9p = 18$
$p = 2$

Substitute into equation ①:
$7(2) - 3q = 23$
$3q = -23 + 14$
$q = -3$
$(p, q) = \underline{(2, -3)}$

Check: LHS of ② = $-2(2) - 3(-3) = 5$ = RHS of ②

17.
$$y = 2x \quad ①$$
$$\underline{7x - y} = \underline{35} \quad ②$$
Add: $7x = 2x + 35$
$5x = 35$
$x = 7$

Substitute into ①:
$y = 2(7) = 14$
$(x, y) = \underline{(7, 14)}$

Check: LHS of ② = $7(7) - 14 = 49 - 14 = 35$ = RHS of ②

18.
$$g - h = 17 \quad ①$$
$$\tfrac{4}{3}g + \tfrac{3}{2}h = 0 \quad ②$$
$$1.\overline{3}g + 1.5h = 0 \quad ②$$
$① \times 1.5$: $\underline{1.5g - 1.5h} = \underline{25.5}$
Add: $2.8\overline{3}g = 25.5$
$g = 9$

Substitute into ②:
$9 - h = 17$
$h = -8$
$(h, g) = \underline{(-8, 9)}$

Check: LHS of ② = $\tfrac{4}{3}(9) + \tfrac{3}{2}(-8) = 12 - 12 = 0$ = RHS of ②

Exercise 2.3 ***(continued)***

19. $d = 3c - 500$ ①

$0.7c + 0.2d = 550$ ②

To eliminate d,

① × 0.2: $-0.6c + 0.2d = -100$

②: $0.7c + 0.2d = 550$

Subtract: $-1.3c + 0 = -650$

$c = 500$

Substitute into ①: $d = 3(500) - 500 = 1000$

$(c, d) = \underline{(500, 1000)}$

Check: LHS of ② = $0.7(500) + 0.2(1000) = 550$ = RHS of ②

20. $0.03x + 0.05y = 51$ ①

$0.8x - 0.7y = 140$ ②

To eliminate y,

① × 0.7: $0.021x + 0.035y = 35.7$

② × 0.05: $0.04x - 0.035y = 7$

Add: $0.061x + 0 = 42.7$

$x = 700$

Substitute into ②:

$0.8(700) - 0.7y = 140$

$-0.7y = -420$

$y = 600$

$(x, y) = \underline{(700, 600)}$

Check: LHS of ① = $0.03(700) + 0.05(600) = 51$ = RHS of ①

21. $2v + 6w = 1$ ①

$10v - 9w = 18$ ②

To eliminate v,

① × 10: $20v + 60w = 10$

② × 2: $20v - 18w = 36$

Subtract: $0 + 78w = -26$

$w = -\frac{1}{3}$

Substitute into ①:

$2v + 6\left(-\frac{1}{3}\right) = 1$

$2v = 1 + 2$

$v = \frac{3}{2}$

$(v, w) = \underline{\underline{\left(\frac{3}{2}, -\frac{1}{3}\right)}}$

Check: LHS of ② = $10\left(\frac{3}{2}\right) - 9\left(-\frac{1}{3}\right) = 18$ = RHS of ②

Exercise 2.3 ***(continued)***

22. $2.5a + 2b = 11$ ①
$8a + 3.5b = 13$ ②

To eliminate b,

① × 3.5: $8.75a + 7b = 38.5$
② × 2: $\underline{16a + 7b = 26}$
Subtract: $-7.25a + 0 = 12.5$
$a = -1.724$

Substitute into ①:

$2.5(-1.724) + 2b = 11$
$2b = 11 + 4.31$
$b = 7.655$
$(a, b) = \underline{(-1.72, 7.66)}$

Check: LHS of ② = $8(-1.724) + 3.5(7.655) = 13.00$ = RHS of ②

23. $37x - 63y = 235$ ①
$18x + 26y = 468$ ②

To eliminate x,

① × 18: $666x - 1134y = 4230$
② × 37: $\underline{666x + 962y = 17{,}316}$
Subtract: $0 - 2096y = -13{,}086$
$y = 6.243$

Substitute into ①:

$37x - 63(6.243) = 235$
$37x = 628.3$
$x = 16.98$
$(x, y) = \underline{(17.0, 6.24)}$

Check: LHS of ② = $18(16.98) + 26(6.243) = 468.0$ = RHS of ②

24. $68.9n - 38.5m = 57$ ①
$45.1n - 79.4m = -658$ ②

To eliminate n,

① × 45.1: $3107n - 1736.4m = 2571$
② × 68.9: $\underline{3107n - 5470.7m = -45{,}336}$
Subtract: $0 + 3734.3m = 47{,}907$
$m = 12.83$

Substitute into ①:

$68.9n - 38.5(12.83) = 57$
$68.9n = 551.0$
$n = 7.996$
$(m, n) = \underline{(12.8, 8.00)}$

Check: LHS of ② = $45.1(7.996) - 79.4(12.83) = -658.1$ = RHS of ②

Exercise 2.3 ***(continued)***

25. $0.33e + 1.67f = 292$ ①
$1.2e + 0.61f = 377$ ②

To eliminate e,

① ÷ 0.33: $e + 5.061f = 884.8$
② ÷ 1.2: $e + 0.508f = 314.2$
Subtract: $0 + 4.552f = 570.6$
$f = 125.4$

Substitute into ①:

$0.33e + 1.67(125.4) = 292$
$0.33e = 82.58$
$e = 250.2$
$(e, f) =$ (250, 125)

Check: LHS of ② = 1.2(250.2) + 0.61(125.4) = 376.7 = RHS of ②

26. $318j - 451k = 7.22$ ①
$-249j + 193k = -18.79$ ②

To eliminate k,

① ÷ 451: $0.7051j - k = 0.01601$
② ÷ 193: $-1.2902j + k = -0.09736$
Add: $-0.5851j + 0 = -0.08135$
$j = 0.1390$

Substitute into ②:

$-249(0.1390) + 193k = -18.79$
$193k = 15.82$
$k = 0.08197$
$(j, k) =$ (0.139, 0.0820)

Check: LHS of ① =318(0.1390) – 451(0.08197) = 7.23 = RHS of ① (within rounding errors.)

27. $\frac{x}{1.1^2} + 2x(1.1)^3 = \$1000$

$0.8264463x + 2.662x = \$1000$
$3.488446x = \$1000$
$x =$ $286.66

28. $\frac{3x}{1.025^6} + x(1.025)^8 = \$2641.35$

$2.586891x + 1.218403x = \$2641.35$
$x =$ $694.13

Exercise 2.3 ***(continued)***

29 $\frac{2x}{1.03^7} + x + x\left(1.03^{10}\right) = \$1000 + \frac{\$2000}{1.03^4}$

$1.626183x + x + 1.343916x = \$1000 + \$1776.974$
$3.970099x = \$2776.974$
$x =$ $699.47

30. $x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$

$1.157625x + 0.7106813x = \$4535.147 - \$1000$

$x = \underline{\underline{\$1892.17}}$

31. $x\left(1 + 0.095 \times \frac{84}{365}\right) + \frac{2x}{1 + 0.095 \times \frac{108}{365}} = \$1160.20$

$1.021863x + 1.945318x = \$1160.20$

$2.967181x = \$1160.20$

$x = \underline{\underline{\$391.01}}$

32. $\frac{x}{1 + 0.115 \times \frac{78}{365}} + 3x\left(1 + 0.115 \times \frac{121}{365}\right) = \$1000\left(1 + 0.115 \times \frac{43}{365}\right)$

$0.9760141x + 3.114370x = \$1013.548$

$x = \underline{\underline{\$247.79}}$

Exercise 2.4

1. $I = Prt$

   $\$6.25 = P(0.05)0.25$

   $\$6.25 = 0.0125P$

   $$P = \frac{\$6.25}{0.0125} = \underline{\underline{\$500.00}}$$

2. $PV = \frac{PMT}{i}$

$\$150{,}000 = \frac{\$900}{i}$

$\$150{,}000i = \$900$

$i = \frac{\$900}{\$150{,}000} = \underline{0.00600}$

3. $S = P(1 + rt)$

$\$3626 = P(1 + 0.004 \times 9)$

$\$3626 = 1.036P$

$P = \frac{\$3626}{1.036} = \underline{\$3500.00}$

4. $N = L(1 - d)$

$\$891 = L(1 - 0.10)$

$\$891 = 0.90L$

$L = \frac{\$891}{0.90} = \underline{\$990.00}$

5. $N = L(1 - d)$

$\$410.85 = \$498(1 - \mathrm{d})$

$\frac{\$410.85}{\$498} = 1 - d$

$0.825 = 1 - d$

$d = 1 - 0.825 = \underline{0.175}$

6. $S = P(1 + rt)$

$\$5100 = \$5000(1 + 0.0025t)$

$\$5100 = \$5000 + \$12.5t$

$\$5100 - \$5000 = \$12.5t$

$t = \frac{\$100}{\$12.5} = \underline{8.00}$

7. $NI = (CM)X - FC$

$\$15{,}000 = CM(5000) - \$60{,}000$

$\$15{,}000 + \$60{,}000 = 5000CM$

$CM = \frac{\$75{,}000}{5000} = \underline{\$15.00}$

8. $NI = (CM)X - FC$

$-\$542.50 = (\$13.50)X - \$18{,}970$

$\$18{,}970 - \$542.50 = (\$13.50)X$

$X = \frac{\$18{,}427.50}{\$13.50} = \underline{1365}$

9. $N = L(1 - d_1)(1 - d_2)(1 - d_3)$

$\$1468.80 = L(1 - 0.20)(1 - 0.15)(1 - 0.10)$

$\$1468.80 = L(0.80)(0.85)(0.90)$

$L = \frac{\$1468.80}{0.6120} = \underline{\$2400.00}$

10. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$70.29 = \$99.99(1 - 0.20)(1 - d_2)(1 - 0.05)$

$\$70.29 = \$75.9924(1 - d_2)$

$\frac{\$70.29}{\$75.9924} = (1 - d_2)$

$d_2 = 1 - 0.92496 = \underline{0.0750}$

11. $FV = PV(1+i_1)(1+i_2)(1+i_3)\cdots(1+i_n)$

$\$1094.83 = \$1000(1 + i_1)(1 + 0.03)(1 + 0.035)$

$\$1094.83 = \$1066.05(1 + i_1)$

$\frac{\$1094.83}{\$1066.05} = 1 + i_1$

$i_1 = 1.02700 - 1 = \underline{0.0270}$

12. $FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$

$\$1508.54 = PMT\left[\frac{(1+0.05)^4 - 1}{0.05}\right]$

$\$1508.54 = PMT\left(\frac{1.21550625 - 1}{0.05}\right)$

$PMT = \$1508.54 \times \frac{0.05}{0.21550625} = \underline{\$350.00}$

13. $PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$

$\$6595.20 = PMT\left[\frac{1-(1+0.06)^{-20}}{0.06}\right]$

$\$6595.20 = PMT\left[\frac{1-0.31180473}{0.06}\right]$

$PMT = \$6595.20 \times \frac{0.06}{0.68819527} = \underline{\$575.00}$

14. $I = Prt$

$\frac{I}{Pr} = \frac{Prt}{Pr}$

$t = \frac{I}{Pr}$

15. $PV = \frac{PMT}{i}$

$i(PV) = PMT$

$i = \frac{PMT}{PV}$

16. $N = L(1 - d)$

$\frac{N}{L} = 1 - d$

$d = 1 - \frac{N}{L}$

17. $NI = (CM)X - FC$

$NI + FC = (CM)X$

$CM = \frac{NI + FC}{X}$

18. $NI = (CM)X - FC$

$NI + FC = (CM)X$

19. $S = P(1 + rt)$

$S = P + Prt$

$$X = \frac{NI + FC}{CM}$$

$$S - P = Prt$$

$$r = (S - P)/Pt$$

20. $S = P(1 + rt)$

$$S = P + Prt$$

$$S - P = Prt$$

$$t = (S - P)/Pr$$

21. $N = L(1 - d_1)(1 - d_2)(1 - d_3)$

$$\frac{N}{L(1 - d_2)(1 - d_3)} = (1 - d_1)$$

$$d_1 = 1 - \frac{N}{L(1 - d_2)(1 - d_3)}$$

22. $N = L(1 - d_1)(1 - d_2)(1 - d_3)$

$$\frac{N}{L(1 - d_1)(1 - d_2)} = (1 - d_3)$$

$$d_3 = 1 - \frac{N}{L(1 - d_1)(1 - d_2)}$$

23. $FV = PV(1 + i)^n$

$$\frac{FV}{(1 + i)^n} = PV$$

$$PV = FV(1 + i)^{-n}$$

24. $FV = PV(1 + i)^n$

$\$9321.91 = \$2000(1 + i)^{20}$

$$\left(\frac{\$9321.91}{\$2000}\right)^{1/20} = 1 + i$$

$1.0800 = 1 + i$

$i = 1.08000 - 1 = \underline{0.0800}$

25. $PV = FV(1+i)^{-n}$

$\$5167.20 = \$10{,}000$

$$\frac{\$5167.20}{\$10{,}000} = \frac{1}{(1+i)^{15}}$$

$$(1+i)^{15} = \frac{\$10{,}000}{\$5167.20}$$

$1 + i = (1.935284)^{1/15} = 1.0450$

$i = \underline{0.0450}$

26. $FV = PV(1+i)^{n}$

$$\left(\frac{FV}{PV}\right)^{1/n} = (1+i)$$

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Exercise 2.5

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase.
   Let the number of hits 1 year ago be n.
   Step 3: Hits last month = Hits 1 year ago $+\frac{2}{7}$ (Hits 1 year ago)
   Step 4: $2655 = n + \frac{2}{7}n$
   Step 5: $2655 = \frac{9}{7}n$
   Multiply both sides by $\frac{7}{9}$.
   $n = 2655 \times \frac{7}{9} = 2065$
   The Web site had <u>2065 hits</u> in the same month 1 year ago.
2. Step 2: Retail price = \$712; Markup = 60% of wholesale of cost.
   Let the wholesale cost be C.
   Step 3: Retail price = Cost + 0.60(Cost)
   Step 4: $\$712 = C + 0.6C$
   Step 5: $\$712 = 1.6C$
   $C = \frac{\$712}{1.6} = \underline{\$445.00}$. The wholesale cost is \$445.00.
3. Step 2: Tag price = \$39.55 (including 13% HST). Let the plant's pretax price be P.
   Step 3: Tag price = Pre-tax price + HST
   Step 4: $\$39.55 = P + 0.13P$
   Step 5: $\$39.55 = 1.13P$
   $P = \frac{\$39.55}{1.13} = \$35.00$
   The amount of HST is \$39.55 – \$35.00 = <u>\$4.55</u>
4. Step 2: Commission rate = 2.5% on the first \$5000 and 1.5% on the remainder
   Commission amount = \$227. Let the transaction amount be x.

Step 3: Commission amount = 0.025($5000) + 0.015(Remainder)

Step 4: $227 = $125.00 + 0.015(x – $5000)

Step 5: $102 = 0.015x – $75.00
$102 + $75 = 0.015x

$$x = \frac{\$177}{0.015} = \underline{\$11{,}800.00}$$

The amount of the transaction was $11,800.00.

5. Step 2: Let the basic price be P. First 20 meals at P.
Next 20 meals at P – $2. Additional meals at P – $3.

Step 3: Total price for 73 meals = $1686

Step 4: 20P + 20(P – $2) + (73 – 40)(P – $3) = $1686

Step 5: 20P + 20P – $40 + 33P – $99 = $1686
73P = $1686 + $99 + $40

$$P = \frac{\$1825}{73} = \underline{\$25.00}$$

The basic price per meal is $25.00.

Exercise 2.5*(continued)*

6. Step 2: Rental Plan 1: \$295 per week + \$0.15 × (Distance in excess of 1000 km)
   Rental Plan 2: \$389 per week
   Let d represent the distance at which the costs of both plans are equal.

   Step 3: Cost of Plan 1 = Cost of Plan 2

   Step 4: $\$295 + \$0.15(d - 1000) = \$389$

   Step 5: $\$295 + \$0.15d - \$150 = \$389$
   $\$0.15d = \$244$
   $d =$ 1627 km

   The unlimited driving plan will be cheaper if you drive more than 1626.7 km in the one-week interval.

7. Step 2: Tax rate = 38%; Overtime hourly rate = 1.5(\$23.50) = \$35.25
   Cost of canoe = \$2750
   Let h represent the hours of overtime Alicia must work.

   Step 3: Gross overtime earnings − Income tax = Cost of the canoe

   Step 4: $\$35.25h - 0.38(\$35.25h) = \$2750$

   Step 5: $\$21.855h = \$2750$
   $h = 125.83$ hours

   Alicia must work 125¾ hours of overtime to earn enough money to buy the canoe.

8. Step 2: Number of two-bedroom homes = 0.4(Number of three-bedroom homes)
   Number of two-bedroom homes = 2(Number of four-bedroom homes)
   Total number of homes = 96
   Let h represent the number of two-bedroom homes

   Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

   Step 4: $h + \frac{h}{0.4} + \frac{h}{2} = 96$

   Step 5: $h + 2.5h + 0.5h = 96$
   $4h = 96$
   $h = 24$

   There should be 24 two-bedroom homes, 2.5(24) = 60 three-bedroom homes, and 0.5(24) = 12 four-bedroom homes.

9. Step 2: Cost of radio advertising = 0.5(Cost of newspaper advertising)
   Cost of TV advertising = 0.6(Cost of radio advertising)
   Total advertising budget = \$160,000
   Let r represent the amount allocated to radio advertising

   Step 3: Radio advertising + TV advertising + Newspaper advertising = \$160,000

   Step 4: $r + 0.6r + \frac{r}{0.5} = \$160{,}000$

   Step 5: $3.6r = \$160{,}000$
   $r = \$44{,}444.44$

   The advertising budget allocations should be:
   \$44,444 to radio advertising,
   0.6(\$44,444.44) = \$26,667 to TV advertising, and
   2(\$44,444.44) = \$88,889 to newspaper advertising.

Exercise 2.5*(continued)*

10. Step 2: By-laws require: 5 parking spaces per 100 square meters,
4% of spaces for physically handicapped
In remaining 96%, # regular spaces = 1.4(# small car spaces)
Total area = 27,500 square meters
Let s represent the number of small car spaces.

Step 3: Total # spaces = # spaces for handicapped + # regular spaces + # small spaces

Step 4: $\frac{27{,}500}{100} \times 5 = 0.04 \times \frac{27{,}500}{100} \times 5 + s + 1.4s$

Step 5: $1375 = 55 + 2.4s$

$s = 550$

The shopping centre must have 55 parking spaces for the physically handicapped, 550 small-car spaces, and 770 regular parking spaces.

11. Step 2: Overall portfolio's rate return = 1.1%, equity fund's rate of return = -3.3%, bond fund's rate of return = 7.7%.
Let e represent the fraction of the portfolio initially invested in the equity fund.

Step 3: Overall rate of return = Weighted average rate of return
= (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

Step 4: $1.1\% = e(-3.3\%) + (1 - e)(7.7\%)$

Step 5: $1.1 = -3.3e + 7.7 - 7.7e$

$-6.6 = -11.0e$

$e = 0.600$

Therefore, 60.0% of Erin's original portfolio was invested in the equity fund.

12. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.
We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.
Let A represent the tonnes of steel required from pile A.

Step 3: Wt. of nickel in 32.5 tonnes of mixture
= Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B
= (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

Step 4: $0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)$

Step 5: $1.34875 = 0.0525A + 0.9230 - 0.0284A$

$0.42575 = 0.0241A$

$A = 17.67$ tonnes

The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

13. Step 2: Total options = 100,000
# of options to an executive = 2000 + # of options to a scientist or engineer
# of options to a scientist or engineer = 1.5(# of options to a technician)
There are 3 executives, 8 scientists and engineers, and 14 technicians.
Let t represent the number of options to each technician.

Step 3: Total options = Total options to scientists and engineers
+ Total options to technicians + Total options to executives

Step 4: $100{,}000 = 8(1.5t) + 14t + 3(2000 + 1.5t)$

Step 5: $= 12t + 14t + 6000 + 4.5t$

$94{,}000 = 30.5t$

$t = 3082$ options

Each technician will receive 3082 options,
each scientist and engineer will receive 1.5(3082) = 4623 options,
and each executive will receive 2000 + 4623 = 6623 options.

Exercise 2.5*(continued)*

14. Step 2: Plan X: 6.5 cents/minute (in business hours) and 4.5 cents/minute (at other times)
Plan Y: 5.3 cents/minute any time
Let b represent the fraction of business-hour usage at which costs are equal.

Step 3: Cost of Plan X = Cost of plan Y

Step 4: Pick any amount of usage in a month—say 1000 minutes.
$b(1000)\$0.065 + (1 - b)(1000)\$0.045 = 1000(\$0.053)$

Step 5:
$$\$65b + \$45 - \$45b = \$53$$
$$\$20b = \$8$$
$$b = 0.40$$

If business-hour usage exceeds 40% of overall usage, plan Y will be cheaper.

15. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.
Cost per kg of ingredients in 50 kg of "trail mix" is to be \$3.20.
Let p represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of $(50 - p)$ kg of raisins

Step 4: $50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)$

Step 5:
$$\$160.00 = \$2.89p + \$187.50 - \$3.75p$$
$$-\$27.50 = -\$0.86p$$
$$p = 31.98 \text{ kg}$$

32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

16. Step 2: Total bill = \$3310. Total hours = 41.
Hourly rate = \$120 for CGA
= \$50 for technician.
Let x represent the CGA's hours.

Step 3: Total bill = (CGA hours x CGA rate) + (Technician hours x Technician rate)

Step 4: $\$3310 = x(\$120) + (41 - x)\$50$

Step 5:
$$\$3310 = \$120x + \$2050 - \$50x$$
$$1260 = 70x$$
$$x = 18$$

The CGA worked 18 hours and the technician worked $41 - 18 = 23$ hours.

17. Step 2: Total investment = \$32,760
Sue's investment = 1.2(Joan's investment)
Joan's investment = 1.2(Stella's investment)
Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

Step 4: Joan's investment = 1.2L
Sue's investment = 1.2L(1.2L) = 1.44L
1.44L + 1.2L + L = \$32,760

Step 5:
$$3.64L = \$32{,}760$$
$$L = \frac{\$32{,}760}{3.64} = \$9000$$

Stella will invest \$9000, Joan will invest 1.2(\$9000) = \$10,800, and
Sue will invest 1.2(\$10,800) = \$12,960

Exercise 2.5*(continued)*

18. Step 2: Sven receives 30% less than George (or 70% of George's share).
Robert receives 25% more than George (or 1.25 times George's share).
Net income = \$88,880
Let G represent George's share.

Step 3: George's share + Robert's share + Sven's share = Net income

Step 4: G + 1.25G + 0.7G = \$88,880

Step 5: 2.95G = \$88,880
G = \$30,128.81
George's share is \$30,128.81, Robert's share is 1.25(\$30,128.81) = \$37,661.02, and Sven's share is 0.7(\$30,128.81) = \$21,090.17.

19. Step 2: Time to make X is 20 minutes.
Time to make Y is 30 minutes.
Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.

Step 3: Total time = (Number of X) $\times$ (Time for X) + (Number of Y) $\times$ (Time for Y)

Step 4: $47 \times 60 = (120 - Y)20 + Y(30)$

Step 5: $2820 = 2400 - 20Y + 30Y$
$420 = 10Y$
$Y = 42$
Forty-two units of product Y were manufactured.

20. Step 2: Price of blue ticket = \$19.00. Price of red ticket = \$25.50.
Total tickets = 4460. Total revenue = \$93,450.
Let the number of tickets in the red section be R.

Step 3: Total revenue = (Number of red $\times$ Price of red) + (Number of blue $\times$ Price of blue)

Step 4: \$93,450 = R(\$25.50) + (4460 – R)\$19.00

Step 5: 93,450 = 25.5R + 84,740 – 19R
6.5R = 8710
R = 1340
1340 seats were sold in the red section and 4460 – 1340 = 3120 seats were sold in the blue section.

21. Let r represent the number of regular members and s the number of student members.

Then	r +	s =	583	①
Total revenue:	\$2140r +	\$856s =	\$942,028	②
①$\times$\$856:	\$856r +	\$856s =	\$499,048	
Subtract:	\$1284r +	0 =	\$442,980	
		r =	345	
Substitute into ①:	345 +	s =	583	
		s =	238	

The club had 238 student members and 345 regular members.

Exercise 2.5*(continued)*

22. Let c represent the number of children and a represent the number of adults.
Then $c + a = 266$ ①
$\$17.90c + \$25.90a = \$6609.40$ ②
① × \$25.90: $\underline{\$25.90c + \$25.90a} = \underline{\$6889.40}$
Subtract: $-\$8c + 0 = -\$280$
$c = 35$
That is, 35 of the 266 customers were children.

23. Let s represent the distance travelled at the lower speed (50 km/h).
Let h represent the distance travelled at the higher speed (100 km/h).
Since the total distance = 1000 km,
then $s + h = 1000$ ①
Since travelling time = $\frac{\text{Distance}}{\text{Speed}}$,
then Time at slower speed = $\frac{s}{50}$ and Time at higher speed = $\frac{h}{100}$
Since the total time = 12.3 hours,
then $\frac{s}{50} + \frac{h}{100} = 12.3$ ②
② × 100: $2s + h = 1230$
Repeat ①: $\underline{s + h} = \underline{1000}$ ①
Subtract: $s + 0 = 230$
Hence, Tina drive 230 km at 50 km/h and 1000 – 230 = 770 km at 100 km/h.

24. Let a represent the adult airfare and c represent the child airfare.
Mrs. Ramsey's cost: $a + 2c = \$610$ ①
Chudnowskis' cost: $2a + 3c = \$1050$ ②
① × 2: $\underline{2a + 4c} = \underline{\$1220}$
Subtract: $0 + -c = -\$170$
Substitute $c = \$170$ into ①: $a + 2(\$170) = \$610$
$a = \$610 - \$340 = \$270$
The airfare is \$270 per adult and \$170 per child.

25. Let h represent the rate per hour and k represent the rate per km.
Vratislav's cost: $2h + 47k = \$54.45$ ①
Bryn's cost: $5h + 93k = \$127.55$ ②

To eliminate x,
① × 5: $10h + 235k = \$272.25$ ①
② × 2: $\underline{10h + 186k} = \underline{\$255.10}$ ②
Subtract: $0 + 49k = \$17.15$
$k = \$0.35$ per km
Substitute into ①:
$2h + 47(\$0.35) = \$54.45$
$2h = \$54.45 - \$16.45$
$= \$38.00$ per hour
$h = \$19.00$ per hour
Budget Truck Rentals charged \$19.00 per hour plus \$0.35 per km.

Exercise 2.5*(continued)*

26. Let s represent the weight of 6% nitrogen fertilizer.
Let t represent the weight of 22% nitrogen fertilizer.

Total weight:	$s + t = 300$	①
Total nitrogen:	$0.06s + 0.22t = 0.16(300)$	
Multiply by 100:	$6s + 22t = 4800$	②
① × 6:	$6s + 6t = 1800$	①
Subtract:	$0 + 16t = 3000$	②
	$t = 187.5$ kg	
	$s = 300 - 187.5 = 112.5$ kg	

Buckerfield's should mix 112.5 kg of 6% fertilizer with 187.5 kg of 22% fertilizer.

27. Let C represent the interest rate on Canada Savings Bonds.
Let O represent the interest rate on Ontario Savings Bonds.

Year 1 interest:	$4(\$1000)C + 6(\$1000)O = \$438$	①
Year 2 interest:	$3(\$1000)C + 4(\$1000)O = \$306$	②
① × 3:	$\$12{,}000C + \$18{,}000O = \$1314$	①
② × 4:	$\$12{,}000C + \$16{,}000O = \$1224$	②
Subtract:	$0 + \$2000O = \$90$	

$$O = \frac{\$90}{\$2000} = 0.045 = 4.5\%$$

Substitute into ②: $\$3000C + \$4000(0.045) = \$306$

$$C = \frac{\$306 - \$180}{\$3000} = 0.042 = 4.2\%$$

The Canada Savings Bonds earn 4.2% per annum and
the Ontario Savings Bonds earn 4.5% per annum.

28. Let r represent the tax rate on residences and
Let f represent the tax rate on land with farm buildings.

LeClair tax:	$\$400{,}000r + \$300{,}000f = \$3870$	①
Bartoli tax:	$\$350{,}000r + \$380{,}000f = \$3774$	②
① × 7:	$\$2{,}800{,}000r + \$2{,}100{,}000f = \$27{,}090$	①
② × 8:	$\$2{,}800{,}000r + \$3{,}040{,}000f = \$30{,}192$	②
Subtract:	$0 - \$940{,}000f = -\$3102$	

$$f = \frac{\$3102}{\$940{,}000} = 0.0033 = 0.33\%$$

Substitute into ①: $\$400{,}000r + \$300{,}000(0.0033) = \$3870$

$$r = \frac{\$3870 - \$990}{\$400{,}000} = 0.0072 = 0.72\%$$

The tax rates are 0.72% on residences and 0.33% on land with farm buildings.

29. Let x represent the number of units of product X and
y represent the number of units of product Y. Then

	$x + y = 93$	①
	$0.5x + 0.75y = 60.5$	②
① × 0.5:	$0.5x + 0.5y = 46.5$	
Subtract:	$0 + 0.25y = 14$	
	$y = 56$	
Substitute into ①:	$x + 56 = 93$	
	$x = 37$	

Therefore, 37 units of X and 56 units of Y were produced last week.

Exercise 2.5*(continued)*

30. Let the price per litre of milk be m and the price per dozen eggs be e. Then

$$5m + 4e = \$19.51 \quad ①$$
$$9m + 3e = \$22.98 \quad ②$$

To eliminate e,

$①\times 3$: $15m + 12e = \$58.53$
$②\times 4$: $36m + 12e = \$91.92$
Subtract: $-21m + 0 = -\$33.39$
$m = \$1.59$

Substitute into ①: $5(\$1.59) + 4e = \$19.51$
$e = \$2.89$

Milk costs $1.59 per litre and eggs cost $2.89 per dozen.

31. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

$$\$1.50M + \$1.30J = \$57.00 \quad ①$$
$$\$1.60M + \$1.37J = \$60.55 \quad ②$$

To eliminate M,

$①\times 1.6$: $\$2.40M + \$2.080J = \$91.200$
$②\times 1.5$: $\$2.40M + \$2.055J = \$90.825$
Subtract: $0 + \$0.025J = \$0.375$
$J = 15$

Substitution of J = 15 into either equation will give M = 25. Hence, 25 litres of milk and 15 cans of orange juice are purchased each week.

32. Let S represent the selling price of a case of beer and R represent the refund per case of empties. Then

$$871S - 637R = \$12{,}632.10 \quad ①$$
$$932S - 805R = \$13{,}331.70 \quad ②$$

To eliminate S,

$①\times 932$: $811{,}772S - 593{,}684R = \$11{,}773{,}117.20$
$②\times 871$: $811{,}772S - 701{,}155R = \$11{,}611{,}910.70$
Subtract: $0 + 107{,}471R = \$161{,}206.50$
$R = \$1.50$

The store paid a refund of $1.50 per case.

33. Let S represent the number of people who bought single tickets and T represent the number of people who bought at three-for-$5. Then

$$S + 3T = 3884 \quad ①$$
$$\$2S + \$5T = \$6925 \quad ②$$

To eliminate S,

$①\times \$2$: $\$2S + \$6T = \$7768$
②: $\$2S + \$5T = \$6925$
Subtract: $0 + \$1T = \$843$
$T = 843$

Hence, 843 people bought tickets at the three-for-$5 discount.

Exercise 2.5*(continued)*

34. Let P represent the number of six-packs and C represent the number of single cans sold.

Then
$$\$4.35P + \$0.90C = \$178.35 \quad ①$$
$$6P + C = 225 \quad ②$$

To eliminate C,

①: $\$4.35P + \$0.90C = \$178.35$

②$\times$\$0.90: $\underline{\$5.40P + \$0.90C} = \underline{\$202.50}$

Subtract: $-\$1.05P + 0 = -\$24.15$

$$P = 23$$

Substitute into ②: $6(23) + C = 225$

$$C = 87$$

The store sold <u>23 six-packs</u> and <u>87 single cans</u>.

35. Let P represent the annual salary of a partner and T represent the annual salary of a technician. Then

$$7P + 12T = \$1{,}629{,}000 \quad ①$$
$$1.05(7P) + 1.08(12T) = \$1{,}734{,}750 \quad ②$$

①$\times$1.05: $\underline{1.05(7P) + 1.05(12T)} = \underline{\$1{,}710{,}450}$

Subtract: $0 + 0.03(12T) = \$24{,}300$

$$T = \$67{,}500$$

Substitute into ①: $7P + 12(\$67{,}500) = \$1{,}629{,}000$

$$P = \$117{,}000$$

The current annual salary of a partner is <u>\$117,000</u> and of a technician is <u>\$67,500</u>.

36. Let P represent the current number of production workers and A the current number of assembly workers. Then

$$\$5100P + \$4200A = \$380{,}700 \quad ①$$
$$\$5100(0.8P) + \$4200(0.75A) = \$297{,}000 \quad ②$$

To eliminate P,

①$\times$0.8: $\$5100(0.8P) + \$4200(0.8A) = \$304{,}560$

②: $\underline{\$5100(0.8P) + \$4200(0.75A)} = \underline{\$297{,}000}$

Subtract: $\$4200(0.05A) = \$7560$

$$A = 36$$

Substitute into ①: $\$5100P + \$4200(36) = \$380{,}700$

$$P = 45$$

Therefore, 0.2P = <u>9 production</u> workers and 0.25A = <u>9 assembly workers</u> will be laid off.

37. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\overline{3}$ (Child's share).

Total distribution = \$759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

Step 4: $\$759{,}000 = w + 4(0.5w) + 13\left(0.\overline{3}\right)(0.5w)$

Step 5: $\$759{,}000 = w + 2w + 2.1\overline{6}w$

$$= 5.1\overline{6}w$$
$$w = \$146{,}903.23$$

Each child will receive 0.5(\$146,903.23) = <u>\$73,451.62</u>

and each grandchild will receive $0.\overline{3}$ (\$73,451.62) = <u>\$24,483.87</u>.

Exercise 2.5*(continued)*

38. Step 2: Stage B workers = 1.6(Stage A workers)
Stage C workers = 0.75(Stage B workers)
Total workers = 114. Let A represent the number of Stage A workers.

Step 3: Total workers = A workers + B workers + C workers

Step 4: 114 = A + 1.6A + 0.75(1.6A)

Step 5: 114 = 3.8A
A = 30
30 workers should be allocated to Stage A, 1.6(30) = 48 workers to Stage B,
and 114 – 30 – 48 = 36 workers to Stage C.

39. Step 2: Hillside charge = 2(Barnett charge) – \$1000
Westside charge = Hillside charge + \$2000
Total charges = \$27,600. Let B represent the Barnett charge.

Step 3: Total charges = Barnett charge + Hillside charge + Westside charge

Step 4: \$27,600 = B + 2B – \$1000 + 2B – \$1000 + \$2000

Step 5: \$27,600 = 5B
B = \$5520
Hence, the Westside charge is 2(\$5520) – \$1000 + \$2000 = \$12,040

40. Step 2: There are 3 managers and 26 production workers. Total distribution = \$100,000.
Manager's share = 1.2 (Production worker's share).
Let p represent a production worker's share.

Step 3: 3(Manager's share) + 26(Production worker's share) = \$100,000

Step 4: 3(1.2p) + 26p = \$100,000
Step 5: 29.6p = \$100,000
p = \$3378.38
Each production worker will receive \$3378.38 and each manager will receive
1.2(\$3378.38) = \$4054.05.

Exercise 2.6

1. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.0175 = \frac{Portion}{\$350}$

$Portion = 0.0175 \times \$350 = \$6.13$

2. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.06\overline{6} = \frac{Portion}{\$666.66}$

$Portion = 0.066667 \times \$666.66 = \$44.44$

3. $Rate = \frac{Portion}{Base} = \frac{\$1.50}{\$11.50} = 0.130 = 13.0\%$

4. $Rate = \frac{Portion}{Base} = \frac{\$0.88}{\$44.00} = 0.0200 = 2.00\%$

5. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.60 = \frac{\$45}{Base}$

$0.60(Base) = \$45$

$$Base = \frac{\$45}{0.60} = \underline{\underline{\$75.00}}$$

6. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.30 = \frac{\$69}{Base}$

$$0.30(Base) = \$69$$

$$Base = \frac{\$69}{0.30} = \underline{\underline{\$230.00}}$$

7. $Rate = \frac{Portion}{Base} \Rightarrow \quad 2.333 = \frac{Portion}{\$75}$

$$Portion = 2.333 \times \$75 = \underline{\underline{\$174.98}}$$

8. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.00075 = \frac{Portion}{\$1650}$

$$Portion = 0.00075 \times \$1650 = \underline{\underline{\$1.24}}$$

9. $Rate = \frac{Portion}{Base} = \frac{\$134}{\$67} = 2.00 = \underline{\underline{200\%}}$

10. $Rate = \frac{Portion}{Base} = \frac{\$1.34}{\$655} = 0.00205 = \underline{\underline{0.205\%}}$

11. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.50 = \frac{Portion}{\$60}$

$$Portion = 1.50 \times \$60 = \underline{\underline{\$90.00}}$$

12. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.00583 = \frac{Portion}{\$1500}$

$$Portion = 0.00583 \times \$1500 = \underline{\underline{\$8.75}}$$

13. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.075 = \frac{\$1.46}{Base}$

$$0.075(Base) = \$1.46$$

$$Base = \frac{\$1.46}{0.075} = \underline{\underline{\$19.47}}$$

14. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.1275 = \frac{\$27.50}{Base}$

$$0.1275(Base) = \$27.50$$

$$Base = \frac{\$27.50}{0.1275} = \underline{\underline{\$215.69}}$$

15. $Rate = \frac{Portion}{Base} = \frac{\$590}{\$950} = 0.621 = \underline{\underline{62.1\%}}$

16. $Rate = \frac{Portion}{Base} = \frac{\$950}{\$590} = 1.61 = \underline{\underline{161\%}}$

17. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.95 = \frac{\$100}{Base}$

$$Base = \frac{\$100}{0.95} = \underline{\$105.26}$$

18. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.08\overline{3} = \frac{\$10}{Base}$

$$Base = \frac{\$10}{0.08\overline{3}} = \underline{\$120.00}$$

19. $Rate = \frac{Portion}{Base} = \frac{30 \text{ metres}}{3000 \text{ metres}} = 0.0100 = \underline{1.00\%}$

20. $Rate = \frac{Portion}{Base} = \frac{500 \text{ grams}}{2800 \text{ grams}} = 0.179 = \underline{17.9\%}$

21. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.005 = \frac{Portion}{\$10.00}$

$$Portion = 0.005 \times \$10.00 = \underline{\$0.05}$$

22. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.0075 = \frac{Portion}{\$100}$

$$Portion = 0.0075 \times \$100 = \underline{\$0.75}$$

Exercise 2.6*(continued)*

23. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.20 = \frac{\$180}{Base}$

$$Base = \frac{\$180}{1.20} = \underline{\$150.00}$$

24. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.13 = \frac{\$559.35}{Base}$

$$Base = \frac{\$559.35}{1.13} = \underline{\$495.00}$$

25. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.305 = \frac{Portion}{\$455}$

$$Portion = 1.305 \times \$455 = \underline{\$593.78}$$

26. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.000505 = \frac{Portion}{\$50{,}000}$

$$Portion = 0.000505 \times \$50{,}000 = \underline{\$25.25}$$

27. $Rate = \frac{Portion}{Base} \Rightarrow \quad 2.25 = \frac{\$281.25}{Base}$

$$Base = \frac{\$281.25}{2.25} = \underline{\$125.00}$$

28. $Rate = \frac{Portion}{Base} \Rightarrow \quad 3.5 = \frac{\$1000}{Base}$

$$Base = \frac{\$1000}{3.5} = \underline{\$285.71}$$

29. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.005 = \frac{\$10}{Base}$

$$Base = \frac{\$10}{0.005} = \underline{\$2000.00}$$

30. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.0075 = \frac{\$1.25}{Base}$

$$Base = \frac{\$1.25}{0.0075} = \underline{\$166.67}$$

31. *a.* $Rate = \frac{Portion}{Base} = \frac{\$14{,}775}{\$8775} = 1.684 = \underline{168\%}$

The trip costs 168% of their gross monthly income.

b. Disposable income = 100% – 72% = 28% of gross income.
Annual disposable income = $12(0.28 \times \$8775) = \$29{,}484$
The cost of the trip is

$$\frac{Portion}{Base} \times 100\% = \frac{\$14{,}775}{\$29{,}484} \times 100\% = \underline{\underline{50.1\%}}$$

of Cecilia's and Nathan's annual disposable income.

Exercise 2.6*(continued)*

32. Total sales per month = \$65,560 + \$36,740 = \$102,300

Gasoline sales are $\frac{\$65{,}560}{\$102{,}300} \times 100\% =$ <u>64.1%</u> of total sales.

33. <u>Given:</u> *Base* = 540 ml

Rate = 100 - (28 + 15.5 + 6) = 50.5% other ingredients

a) $Rate = \frac{Portion}{Base} \Rightarrow 0.505 = \frac{Portion}{540} =$ <u>272.7 ml</u>

b) *Rate* = 28%

Base $= \frac{5}{8}$ x 540 ml = 337.5 ml

$0.28 = \frac{Portion}{337.5} =$ <u>94.5 ml</u>

34. Given: *Rate* = 23.5% and *Portion* = \$2680 million.

$$Rate = \frac{Portion}{Base} \Rightarrow \qquad 0.235 = \frac{\$2680}{\text{Total budget}}$$

$$\text{Total budget} = \frac{\$2680}{0.235} = \$11{,}404 \text{ million} = \underline{\$11.404 \text{ billion}}$$

The province's total budget is \$11.404 billion.

35. The budgeted expenses are the *Base* while the actual expenses are the *Portion.*

$$Rate = \frac{Portion}{Base} \Rightarrow \qquad 1.27 = \frac{\$320{,}200}{\text{Budget}}$$

$$\text{Budget} = \frac{\$320{,}200}{1.27} = \underline{\$252{,}100} \qquad \text{(to the nearest \$100)}$$

Brockton budgeted <u>\$252,100</u> for snow clearance.

36. The gross royalties (\$99,736.41) are 5.7% of total revenue.

That is, $0.057 = \frac{\$99{,}736.41}{\text{Total revenue}}$

$$\text{Total revenue} = \frac{\$99{,}736.41}{0.057} = \$1{,}749{,}761.58$$

Number of downloads at \$0.99 each is

$$\frac{\$1{,}749{,}761.58}{\$0.99} = \underline{1{,}767{,}436}$$

37. Total hours in a year = 52 weeks × 7 days/week × 24 hours/day = 8736 hours
Number of days worked = [(52 – 2) weeks × 5 days/week] – 7 holidays = 243 days
Total hours worked = 243 × 7.5 = 1822.5

Percentage of total hours that are worked = $\frac{1822.5}{8736} \times 100\% = 20.9\%$

38. Total Revenue for December = \$9,820 + \$4,025 + \$1,830 = \$15,675
Returns = 0.17 x \$9,820 + 0.08 x \$4,025 + 0.03 x \$1,830 = \$2046.30

$$Rate = \frac{Portion}{Base} = \frac{2{,}046.30}{15{,}675} = \underline{13.05\%}$$

39. Percentage of impurities = $100\% - 99\frac{44}{100}\% = \frac{56}{100}\% = 0.56\%$

Amount of impurities in a 150-g cake = *Rate*×*Base* = 0.0056 × 150 g = 0.840 g = <u>840 mg</u>

40. Discount broker would charge \$30 + 200(\$0.03) = \$36.00

Full-service broker would charge $0.024(200 \times \$55.40) = \$265.92$
The discount broker charges only

$$\frac{\$36.00}{\$265.92} \times 100\% = \underline{13.5\%}$$

of the amount charged by the full-service broker.

41. Discount broker would charge $\$25 + 800(\$0.05) = \$65.00$
Full-service broker would charge $0.022(800 \times \$21.75) = \$382.80$
The discount broker charges $\$382.80 - \$65.00 = \$317.80$ less
The percent saved is $\frac{\$317.80}{\$382.80} \times 100\% = \underline{83.0\%}$

42. *a.* Income tax = $0.16(\$15{,}000) + 0.26(\$33{,}000 - \$15{,}000)$
$= \$2400 + \$4680$
$= \$7080$
This income tax is $\frac{\$7080}{\$33{,}000} \times 100\% = \underline{21.5\%}$ of taxable income.

b. Income tax = $0.16(\$15{,}000) + 0.26(\$20{,}000) + 0.35(\$66{,}000 - \$35{,}000)$
$= \$2400 + \$5200 + \$10{,}850$
$= \$18{,}450$
This income tax is $\frac{\$18{,}450}{\$66{,}000} \times 100\% = \underline{28.0\%}$ of taxable income.

c. Income tax = $0.16(\$15{,}000) + 0.26(\$20{,}000) + 0.35(\$40{,}000)$
$+ 0.45(\$99{,}000 - \$75{,}000)$
$= \$2400 + \$5200 + \$14{,}000 + \$10{,}800$
$= \$32{,}400$
This income tax is $\frac{\$32{,}400}{\$99{,}000} \times 100\% = \underline{32.7\%}$ of taxable income.

43. Canada's population density = $\frac{33{,}700{,}000 \text{ people}}{9{,}093{,}500 \text{ square km}} = 3.706$ people per square km.

Japan's population density = $\frac{127{,}600{,}000 \text{ people}}{377{,}835 \text{ square km}} = 337.71$ people per square km.

Canada's population density is only $\frac{3.706}{337.71} \times 100\% = \underline{1.10\%}$ of Japan's population density.

44. The selling price is being compared to the original price. Hence, the original price is the *Base* and the selling price is the *Portion*.
Original price = $\frac{Portion}{Rate} = \frac{\$210{,}000}{2.50} = \underline{\$84{,}000.00}$

45. 11,542 seats represent 67.50% of capacity.
That is, $11{,}542 = 0.6750(\text{Capacity})$

$$\text{Capacity} = \frac{11{,}542}{0.6750} = 17{,}099 \text{ seats}$$

Seats not sold to season-ticket holders = 17,099 – 11,542 = 5557
Rounded to the nearest 100, 5600 seats were not sold to season-ticket holders.

46. Waking hours for Males = (24 – 7.5) x 365 x 78 years = 469,755 hours
Waking hours for Females = $(24 - 7.5 + 0.\overline{3})$ x 365 x 82.7 years = 487,998.9167 hours

$$Rate = \frac{Portion}{Base} = \frac{469{,}755}{487{,}998.9167} = \underline{96.26\%}$$

47. Portion of commission retained = $Rate \times Base$ = 0.60 × 4.8% = 2.88%
Income of \$150,480 (*Portion*) is 2.88% of sales (*Base*).
That is, \$150,480 = 0.0288 × Sales
Stan's sales volume was \$5,225,000.

48 The retained commission is $\frac{\$134.55}{\$11{,}500} \times 100\% = 1.17\%$ of the amount of the transaction. This 1.17% (*Portion*) is 45% of the total commission rate (*Base*) charged to clients. Hence,

Rate of total commission = $\frac{Portion}{Rate} = \frac{1.17\%}{0.45} = \underline{2.60\%}$

49 *a.* The expected number of deaths (*Portion*) among 50,000 males (*Base*) is

$$Rate \times Base = 0.0034 \times 50{,}000 = \underline{170}$$

b. The number of 35-year-old males in the city of 1.45 million is
0.0083 × 1,450,000 = 12,035.
The expected number of deaths in this group in a year is 0.0034 × 12,035 = 41.

Exercise 2.7

1. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$95}{\$95} \times 100\% = \underline{5.26\%}$

2. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$95 - \$100}{\$100} \times 100\% = \underline{-5.00\%}$

3. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{135kg - 35kg}{35kg} \times 100\% = \underline{285.71\%}$

4 $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{35kg - 135kg}{135kg} \times 100\% = \underline{-74.07\%}$

5. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{0.13 - 0.11}{0.11} \times 100\% = \underline{18.18\%}$

6. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{0.085 - 0.095}{0.095} \times 100\% = \underline{-10.53\%}$

7. $V_f = V_i(1+c)$ = \$134.39[1 + (–0.12)] = \$134.39(0.88) = \$118.26

8. $V_f = V_i(1+c)$ = 112g(1 + 1.12) = 237.44g

9. $V_f = V_i(1+c)$ = (26.3 cm)(1 + 3.00) = 105.2 cm

10. $V_f = V_i(1+c)$ = 0.043[1 + (–0.30)] = 0.0301

11. $V_i = \frac{V_f}{1+c} = \frac{\$75}{1+2.00} = \underline{\$25.00}$

12. $V_i = \dfrac{V_f}{1+c} = \dfrac{\$75}{1+(-0.50)} = \underline{\underline{\$150.00}}$

13. Given: V_i = \$90, V_f= \$100

$$c = \frac{\$100 - \$90}{\$90} \times 100\% = \underline{\underline{11.11\%}}$$

\$100 is 11.11% more than \$90.

14. Given: V_i = \$110, V_f= \$100

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$110}{\$110} \times 100\% = \underline{\underline{-9.09\%}}$$

\$100 is 9.09% less than \$110.

15. Given: c = 25%, V_f= \$100

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+0.25} = \underline{\underline{\$80.00}}$$

\$80.00 increased by 25% equals \$100.00.

16. Given: c = 7%, V_f= \$52.43

$$V_i = \frac{V_f}{1+c} = \frac{\$52.43}{1+0.07} = \underline{\underline{\$49.00}}$$

\$49.00 increased by 7% equals \$52.43.

17. Given: V_f= \$75, c = 75%

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+0.75} = \underline{\underline{\$42.86}}$$

\$75 is 75% more than \$42.86.

18. Given: V_i= \$56, c = 65%

$$V_f = V_i(1+c) = \$56(165) = \underline{\underline{\$92.40}}$$

\$56 after an increase of 65% is \$92.40.

19. Given: V_i= \$759.00, V_f= \$754.30

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$754.30 - \$759.00}{\$759.00} \times 100\% = \underline{\underline{-0.62\%}}$$

\$754.30 <u>is 0.62% less</u> than \$759.00.

20. Given: V_i= 77,400, V_f = 77,787

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{77{,}787 - 77{,}400}{77{,}400} \times 100\% = \underline{\underline{0.50\%}}$$

77,787 is 0.50% more than 77,400.

21 Given: V_i= \$75, c = 75%

$V_f = V_i(1 + c)$ = \$75(1 + 0.75) = <u>\$131.25</u>

\$75.00 becomes \$131.25 after an increase of 75%.

22. Given: V_f= \$100, c = – 10%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \underline{\underline{\$111.11}}$$

$100.00 is 10% less than $111.11.

23. Given: V_f = $100, c = – 20%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \underline{\underline{\$125.00}}$$

$125 after a reduction of 20% equals $100.

24. Given: V_f = $50, c = – 25%

$$V_i = \frac{V_f}{1+c} = \frac{\$50}{1+(-0.25)} = \underline{\underline{\$66.67}}$$

$66.67 after a reduction of 25% equals $50.

25. Given: V_f = $549, $c = -16.\overline{6}\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$549}{1+\left(-0.1\overline{6}\right)} = \underline{\underline{\$658.80}}$$

$658.80 after a reduction of $16.\overline{6}\%$ equals $549.

26. Given: V_i = $900, c = –90%

$V_f = V_i(1 + c)$ = $900[1 + (–0.9)] = $\underline{\underline{\$90.00}}$

$900 after a decrease of 90% is $90.00.

Exercise 2.7*(continued)*

27. Given: V_i = \$102, c = –2%
$V_f = V_j(1 + c)$ = \$102(1 – 0.02) = \$99.96
\$102 after a decrease of 2% is \$99.96.

28. Given: V_i = \$102, c = –100%
$V_f = V_i(1 + c)$ = \$102[1 + (–1.00)] = \$102(0) = \$0.00
Any positive amount after a decrease of 100% is zero.

29. Given: V_i= \$250, V_f = \$750
$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$750 - \$250}{\$250} \times 100\% = 200.00\%$$
\$750 is 200.00% more than \$250.

30. Given: V_i= \$750, V_f= \$250
$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$250 - \$750}{\$750} \times 100\% = -66.67\%$$
\$250 is 66.67% less than \$750.

31. Given: c = 0.75%, V_i= \$10,000
$V_f = V_i\,(1 + c)$ = \$10,000(1 + 0.0075) = \$10,075.00
\$10,000 after an increase of $\frac{3}{4}$% is \$10,075.00.

32. Given: V_i= \$1045, c = – 0.5%
$V_f = V_i\,(1 + c) = \$1045\left[1 + (-0.005)\right]$ = \$1039.78
\$1045 after an decrease of 0.5% is \$1039.78.

33. Given: c = 150%, V_f = \$575
$$V_i = \frac{V_f}{1 + c} = \frac{\$575}{1 + 1.5} = \$230.00$$
\$230.00 when increased by 150% equals \$575.

34. Given: c = 210%, V_f = \$465
$$V_i = \frac{V_f}{1 + c} = \frac{\$465}{1 + 2.1} = \$150.00$$
\$150.00 after being increased by 210% equals \$465.

35. Given: V_i= \$150, c = 150%
$V_f = V_i\,(1 + c)$ = \$150(1 + 1.5) = \$375.00
\$150 after an increase of 150% is \$375.00.

36. Let the retail price be p. Then
p + 0.13 p = \$281.37
$$p = \frac{\$281.37}{1.13} = \$249.00$$
The coat's sticker price was \$249.00.

Exercise 2.7*(continued)*

37. Let the TV's pre-tax price be p. Then
$p + 0.05p + 0.07p = \$2797.76$

$$p = \frac{\$2797.76}{1.12} = \$2498.00$$

Then, GST = $0.05p = 0.05(\$2498) =$ <u>$124.90</u>
and PST = $0.07p = 0.07(\$2498) =$ <u>$174.86</u>

38. Let the population figure for 1999 be p. Then
$p + 0.1056p = 33{,}710{,}000$

$$p = \frac{\$33{,}710{,}000}{1.1056} = 30{,}490{,}232$$

Rounded to the nearest 10,000, the population in 1999 was <u>30,490,000</u>.

39. *a.* . Given: V_i= 32,400, V_f = 27,450

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27{,}450 - 32{,}400}{32{,}400} \times 100\% = \underline{-15.28\%}$$

The number of hammers sold declined by 15.28%.

b. Given: V_i= \$15.10, V_f= \$15.50

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = \underline{2.65\%}$$

The average selling price increased by 2.65%.

c. Year 1 revenue = 32,400(\$15.10) = \$489,240
Year 2 revenue = 27,450(\$15.50) = \$425,475

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425{,}475 - \$489{,}240}{\$489{,}240} \times 100\% = \underline{-13.03\%}$$

The revenue decreased by 13.03%.

40. *a.* Given: V_i= \$0.55, V_f = \$1.55

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \underline{181.82\%}$$

The share price rose by 181.82% in the first year.

b. Given: V_i= \$1.55, V_f= \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \underline{-51.61\%}$$

The share price declined by 51.61% in the second year.

c. Given: V_i= \$0.55, V_f = \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{36.36\%}$$

The share price rose by 36.36% over 2 years.

Exercise 2.7*(continued)*

41. Pick an arbitrary price, say \$1.00, for a bar of the soap.

The former unit price was $V_i = \frac{\$1.00}{100\text{ g}} = \$0.01$ per gram.

The new unit price is $V_f = \frac{\$1.00}{90\text{ g}} = \$0.011111$ per gram.

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.011111 - \$0.01}{\$0.01} \times 100\% = \underline{11.11\%}$$

42. Initial unit price = $\frac{\$5.49}{1.65\,l}$ = \$3.327 per litre

Final unit price = $\frac{\$7.98}{2.2\,l}$ = \$3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \underline{9.02\%}$$

43. Initial unit price = $\frac{\$7.98}{3.6\text{ kg}}$ = \$2.2167 per kg

Final unit price = $\frac{\$6.98}{3\text{ kg}}$ = \$2.3267 per kg

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$2.3267 - \$2.2167}{\$2.2167} \times 100\% = \underline{4.96\%}$$

44. Initial unit price = $\frac{1098\text{ cents}}{700\text{ g}}$ = 1.5686 cents per g

Final unit price = $\frac{998\text{ cents}}{600\text{ g}}$ = 1.6633 cents per g

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{1.6633 - 1.5686}{1.5686} \times 100\% = \underline{6.04\%}$$

45. Given: V_f = \$338,500, c = 8.7%

$$V_i = \frac{V_f}{1+c} = \frac{\$338{,}500}{1.087} = \underline{\$311{,}400}$$

The average price one year ago was \$311,400.

46. Given: V_f = \$348.60, c = –0.30

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \underline{\$498.00}$$

The regular price of the boots is \$498.00.

47. Canada's exports to US exceeded imports from the US by 23%.

That is, Exports = 1.23(Imports)

Therefore, Imports = $\frac{\text{Exports}}{1.23}$ = 0.8130(Exports)

That is, Canada's imports from US (= US exports to Canada) were

1 – 0.8130 = 0.1870 = 18.70%

less than Canada's exports to US (= US imports from Canada.)

48. Given: For 2013, V_f(Apple) = 55.0 million, c = 35%

$$V_i = \frac{V_f}{1+c} = 40.7401 \text{ million} = 40{,}740{,}100 \text{ Galaxy phones}$$

Rounded to the nearest 10,000, Galaxy sold 40,740,000phones in 2013.

49. Given: For 2012, V_f = 116.4 million, c = 17.2%

$$V_i = \frac{V_f}{1+c} = \frac{116.4 million}{1+0.172} = 99.3174 \text{ million} = 99{,}317{,}400$$

Rounded to the nearest 10,000, Apple sold 99,317,000 iPhones in 2011.

Exercise 2.7*(continued)*

50. The fees to Fund A will be

$$\frac{(\text{Fees to Fund A}) - (\text{Fees to Fund B})}{(\text{Fees to Fund B})} \times 100\% = \frac{2.38\% - 1.65\%}{1.65\%} \times 100\% = 44.24\%$$

more than the fees to Fund B.

51. Percent change in the HST rate

$$= \frac{(\text{Final HST rate}) - (\text{Initial HST rate})}{(\text{Initial HST rate})} \times 100\% = \frac{14\% - 15\%}{15\%} \times 100\% = -6.67\%$$

The GST paid by consumers was reduced by 16.67%.

52. Given: For April of 2013, V_f = 164,130,000 visitors, c = 6.18%

Then, $V_i = \frac{V_f}{1+c} = \frac{164{,}130{,}000}{1+0.0618} = 154{,}577{,}133.2$

That is, Facebook had 154,577,133 unique visitorsin April of 2012

Therefore, the absolute increase from April of 2012 to April of 2013was

164,130,000 – 154,577,133 = 9,553,000 (rounded to the nearest 10,000)

Exercise 2.7*(continued)*

53. Given: V_f = \$0.45, c = 76%

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

Price decline = $V_i - V_f$ = \$1.88 – \$0.45 = <u>\$1.43</u>
The share price dropped by \$1.43.

54. Given: V_f = \$24,300, c = – 55%

$$V_i = \frac{V_f}{1+c} = \frac{\$24{,}300}{1+(-0.55)} = \$54{,}000$$

The amount of depreciation is \$54,000 – \$24,300 = <u>\$29,700</u>.

55. If General Paint's prices are marked down by 30%, then
General Paint's prices = 0.70(Cloverdale Paint's prices)
Hence, Cloverdale's prices = $\frac{\text{General Paint's prices}}{0.70}$ = 1.4286(General Paint's prices)
Therefore, you will pay <u>42.86% more</u> at Cloverdale Paint.

56. Given: January sales were 17.4% less than December sales
Hence, January sales = (1 – 0.174)(December sales) = 0.826(December sales)
Therefore, December sales = $\frac{\text{January sales}}{0.826}$ = 1.2107(January sales)
That is, December sales were <u>121.07%</u> of January sales.

57. Given: Operating expenses = 0.40(Revenue)
Then Revenue = $\frac{\text{Operating expenses}}{0.40}$ = 2.5(Operating expenses)
That is, Revenue is 250% of Operating expenses, or
Revenue exceeds Operating expenses by 250% – 100% = <u>150%</u>.

58. Given: Equity = (100% – 50%) of Debt = 50% of Debt = 0.50(Debt)
Therefore, $\frac{\text{Debt}}{\text{Equity}} = \frac{\text{Debt}}{0.5(\text{Debt})} = \frac{1}{0.5} = 2$
Since Debt is twice (or 200% of) Equity, then debt financing is <u>100% more</u>
than equity financing.

59. Current unit price = $\frac{449 \text{ cents}}{500 \text{ ml}}$ = 0.8980 cents per ml
New unit price = 1.10(0.8980 cents per ml) = 0.9878 cents per ml
Price of a 425-ml container = (425 ml) × (0.9878 cents per ml) = 419.8 cents = <u>\$4.20</u>

60. Current unit price = $\frac{115 \text{ cents}}{100 \text{ g}}$ = 1.15 cents per g
New unit price = 1.075(1.15 cents per g) = 1.23625 cents per g
Price of an 80-g bar = (80 g) × (1.23625 cents per g) = 98.9 cents = <u>\$0.99</u>

61. For Year 1, V_f = \$6 and $V_f - V_i$ = – \$4
Therefore, V_i = V_f + \$4 = \$6 + \$4 = \$10

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{-\$4}{\$10} \times 100\% = \underline{\underline{-40.00\%}}$$

For Year 2, V_i = \$6 and $V_f - V_i$ = \$4

Therefore, $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$4}{\$6} \times 100\% = \underline{\underline{66.67\%}}$

The percent change was -40.00% in Year 1 and 66.67% in Year 2.

62. If the Canadian dollar is worth 6.5% less than the US dollar,

Canadian dollar = (1 − 0.065)(US dollar) = 0.935(US dollar)

Hence, US dollar = $\frac{\text{Canadian dollar}}{0.935}$ = 1.0695(Canadian dollar)

Therefore, the US dollar is worth 6.95% more than the Canadian dollar.

63. Given: For the appreciation, V_i= Purchase price, c = 140%, V_f= List price

For the price reduction, V_i = List price, c = –10%, V_f= \$172,800

List price = $\frac{V_f}{1+c} = \frac{\$172{,}800}{1+(-0.1)} = \$192{,}000$

Original purchase price = $\frac{V_f}{1+c} = \frac{\$192{,}000}{1+1.4} = \$80{,}000$

The owner originally paid \$80,000 for the property.

64. Given: For the markup, V_i= Cost, c = 22%, V_f = List price

For the markdown, V_i = List price, c = –10%, V_f= \$17,568

List price = $\frac{V_f}{1+c} = \frac{\$17{,}568}{1+(-0.10)} = \$19{,}520$

Cost (to dealer) = $\frac{V_f}{1+c} = \frac{\$19{,}520}{1+0.22} = \$16{,}000$

The dealer paid \$16,000 for the car.

Exercise 2.7*(continued)*

65. Suppose the initial ratio is $\frac{x}{y}$.

If the denominator is reduced by 20%, then

$$\text{Final ratio} = \frac{x}{y - 0.20y} = \frac{x}{0.8y} = 1.25\frac{x}{y}$$

That is, the value of the ratio <u>increases by 25%.</u>

66. Next year there must be 15% fewer students per teacher.
With the same number of students,

$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85\left(\frac{\text{Students}}{\text{Teachers now}}\right)$$

Therefore, $\text{Teachers next year} = \frac{\text{Teachers now}}{0.85} = 1.1765(\text{Teachers now})$

That is, if the number of students does not change, the number of teachers must be <u>increased by 17.65%</u>.

67. Use ppm as the abbreviation for "pages per minute".
Given: Lightning printer prints 30% more ppm than the Reliable printer.
That is, the Lightning's printing speed is 1.30 times the Reliable's printing speed.
Therefore, the Reliable's printing speed is

$\frac{1}{1.3}$ = 0.7692 = 76.92% of the Lightning's printing speed

Therefore, the Reliable's printing speed is

100% – 76.92% = 23.08% less than the Lighting's speed.

The Lightning printer will require <u>23.08% less time</u> than the Reliable for a long printing job.

68. Given: Euro is worth 39% more than the Canadian dollar.
That is, Euro = 1.39(Canadian dollar)

Therefore, $\text{Canadian dollar} = \frac{\text{Euro}}{1.39} = 0.7194(\text{Euro}) = 71.94\%$ of a Euro.

That is, the Canadian dollar is worth 100% – 71.94% = <u>28.06% less</u> than the Euro.

Exercise 2.7*(continued)*

69. Let us use OT as an abbreviation for "overtime".
The number of OT hours permitted by this year's budget is

$$\text{OT hours (this year)} = \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

The number of overtime hours permitted by next year's budget is

$$\text{OT hours (next year)} = \frac{\text{OT budget (next year)}}{\text{OT hourly rate (next year)}} = \frac{1.03\,[\text{OT budget (this year)}]}{1.05\,[\text{OT hourly rate (this year)}]}$$

$$= 0.980952\,\frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

= 98.0952% of this year's OT hours

The number of OT hours must be reduced by 100% – 98.0952% = <u>1.90%</u>.

Review Problems

1. $4(3a + 2b)(2b – a) – 5a(2a – b) = 4(6ab – 3a^2 + 4b^2 – 2ab) – 10a^2 + 5ab$

$= \underline{\underline{-22a^2 + 21ab + 16b^2}}$

2. *a.* Given: $c = 17.5\%$, $V_i = \$29.43$
   $V_f = V_i(1 + c) = \$29.43(1.175) = \underline{\underline{\$34.58}}$
   \$34.58 is 17.5% more than \$29.43.

   b. Given: $V_f = \$100$, $c = -80\%$
   $$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.80} = \underline{\underline{\$500.00}}$$
   80% off \$500 leaves \$100.

   c. Given: $V_f = \$100$, $c = -15\%$
   $$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.15} = \underline{\underline{\$117.65}}$$
   \$117.65 reduced by 15% equals \$100.

   d. Given: $V_i = \$47.50$, $c = 320\%$
   $V_f = V_i(1 + c) = \$47.50(1 + 3.2) = \underline{\underline{\$199.50}}$
   \$47.50 after an increase of 320% is \$199.50.

   e. Given: $c = -62\%$, $V_f = \$213.56$
   $$V_i = \frac{V_f}{1+c} = \frac{\$213.56}{1-0.62} = \underline{\underline{\$562.00}}$$
   \$562 decreased by 62% equals \$213.56.

   f. Given: $c = 125\%$, $V_f = \$787.50$
   $$V_i = \frac{V_f}{1+c} = \frac{\$787.50}{1+1.25} = \underline{\underline{\$350.00}}$$
   \$350 increased by 125% equals \$787.50.

   g. Given: $c = -30\%$, $V_i = \$300$
   $$V_f = V_i(1+c) = \$300(1-0.30) = \underline{\underline{\$210.00}}$$
   \$210 is 30% less than \$300.

Review Problems *(continued)*

3. *a.* $\frac{9y-7}{3} - 2.3(y-2) = 3y - 2.\overline{3} - 2.3y + 4.6 = \underline{\underline{0.7y + 2.2\overline{6}}}$

   b. $P\left(1 + 0.095 \times \frac{135}{365}\right) + \frac{2P}{1 + 0.095 \times \frac{75}{365}} = 1.035137P + 1.961706P = \underline{\underline{2.996843P}}$

4. *a.* $6(4y - 3)(2 - 3y) - 3(5 - y)(1 + 4y) = 6(8y - 12y^2 - 6 + 9y) - 3(5 + 20y - y - 4y^2)$
   $= \underline{\underline{-60y^2 + 45y - 51}}$

   b. $\frac{5b-4}{4} - \frac{25-b}{1.25} + \frac{7}{8}b = 1.25b - 1 - 20 + 0.8b + 0.875b = \underline{\underline{2.925b - 21}}$

   c. $\frac{x}{1 + 0.085 \times \frac{63}{365}} + 2x\left(1 + 0.085 \times \frac{151}{365}\right) = 0.985541x + 2.070329x = \underline{\underline{3.05587x}}$

   d. $\frac{96nm^2 - 72n^2m^2}{48n^2m} = \frac{4m - 3nm}{2n}$

5. $P(1+i)^n + \frac{S}{1+rt} = \$2500(1.1025)^2 + \frac{\$1500}{1 + 0.09 \times \frac{93}{365}} = \$3038.766 + \$1466.374 = \underline{\underline{\$4505.14}}$

6. *a.* $L(1 - d_1)(1 - d_2)(1 - d_3) = \$340(1 - 0.15)(1 - 0.08)(1 - 0.05) = \underline{\underline{\$252.59}}$

   b. $\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{\$575}{0.085}\left[1 - \frac{1}{(1+0.085)^3}\right] = \$6764.706(1 - 0.7829081) = \underline{\underline{\$1468.56}}$

7. *a.* $\frac{(-3x^2)^3(2x^{-2})}{6x^5} = \frac{(-27x^6)(2x^{-2})}{6x^5} = \underline{\underline{-\frac{9}{x}}}$

   b. $\frac{(-2a^3)^{-2}(4b^4)^{3/2}}{(-2b^3)(0.5a)^3} = \frac{\left(\frac{1}{4a^6}\right)(8b^6)}{(-2b^3)(0.125a^3)} = \underline{\underline{-\frac{8b^3}{a^9}}}$

8. $\left(-\frac{2x^2}{3}\right)^{-2}\left(\frac{5^2}{6x^3}\right)\left(-\frac{15}{x^5}\right)^{-1} = \left(\frac{3}{2x^2}\right)^2\left(\frac{25}{6x^3}\right)\left(-\frac{x^5}{15}\right) = \underline{\underline{-\frac{5}{8x^2}}}$

9. *a.* $1.0075^{24} = \underline{\underline{1.19641}}$

   b. $(1.05)^{1/6} - 1 = \underline{\underline{0.00816485}}$

   c. $\frac{(1 + 0.0075)^{36} - 1}{0.0075} = \underline{\underline{41.1527}}$

   d. $\frac{1 - (1 + 0.045)^{-12}}{0.045} = \underline{\underline{9.11858}}$

10. *a.* $\frac{(1.00\overline{6})^{240} - 1}{0.00\overline{6}} = \frac{4.926802 - 1}{0.00\overline{6}} = \underline{\underline{589.020}}$

    b. $(1 + 0.025)^{1/3} - 1 = \underline{\underline{0.00826484}}$

Review Problems *(continued)*

11. *a.* $\dfrac{2x}{1+0.13\times\frac{92}{365}}+x\left(1+0.13\times\dfrac{59}{365}\right)=\$831$

$1.936545x + 1.021014x = \$831$

$2.957559x = \$831$

$x = \underline{\$280.97}$

b. $3x(1.03^5) + \dfrac{x}{1.03^3}+x=\dfrac{\$2500}{1.03^2}$

$3.47782x + 0.91514x + x = \$2356.49$

$x = \underline{\$436.96}$

12. *a.* $\dfrac{x}{1.08^3}+\dfrac{x}{2}(1.08)^4=\$850$

$0.793832x + 0.680245x = \$850$

$x = \underline{\$576.63}$

Check: $\dfrac{\$576.63}{1.08^3}+\dfrac{\$576.63}{2}(1.08)^4=\$457.749+\$392.250=\$850.00$

b. $2x\left(1+0.085\times\dfrac{77}{365}\right)+\dfrac{x}{1+0.085\times\frac{132}{365}}=\$1565.70$

$2.03586x + 0.97018x = \$1565.70$

$x = \underline{\$520.85}$

Check:

$2(\$520.85)\left(1+0.085\times\dfrac{77}{365}\right)+\dfrac{\$520.85}{1+0.085\times\frac{132}{365}}=\$1060.38+\$505.32=\$1565.70$

13. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$324.30 = \$498(1 - 0.20)(1 - d_2)(1 - 0.075)$

$\$324.30 = \$368.52(1 - d_2)$

$\dfrac{\$324.30}{\$368.52} = (1 - d_2)$

$d_2 = 1 - 0.8800 = \underline{0.120} = \underline{12.0\%}$

14. $V_f = V_i(1 + c_1)(1 + c_2)(1 + c_3)$

$\$586.64 = \$500(1 + 0.17)(1 + c_2)(1 + 0.09)$

$\$586.64 = \$637.65(1 + c_2)$

$1 + c_2 = \dfrac{\$586.64}{\$637.65}$

$c_2 = 0.9200 - 1 = \underline{-0.0800 = -8.00\%}$

15. $Rate = \dfrac{Portion}{Base} = \dfrac{\$16.39}{\$6.39}\times 100\% = \underline{256.5\%}$

16. $Base = \dfrac{Portion}{Rate} = \dfrac{\$100}{0.80} = \underline{\$125.00}$

17. $Base = \dfrac{Portion}{Rate} = \dfrac{\$1.00}{0.0075} = \underline{\$133.33}$

18. Two hours = 2(60) = 120 minutes

$$Rate = \frac{Portion}{Base} = \frac{15 \text{ minutes}}{120 \text{ minutes}} \times 100\% = \underline{12.5\%}$$

19. $3x + 5y = 11$ ①
$2x - y = 16$ ②

To eliminate y,

①: $3x + 5y = 11$

②× 5: $\underline{10x - 5y} = \underline{80}$

Add: $13x + 0 = 91$

$x = 7$

Substitute into equation ②: $2(7) - y = 16$

$y = -2$

Hence, $\underline{(x, y)} = \underline{(7, -2)}$

Review Problems *(continued)*

20. *a.* $4a - 5b = 30$ ①
$2a - 6b = 22$ ②

To eliminate a,

①×1: $a - 5b = 30$

②× 2: $\underline{4a - 12b} = \underline{44}$

Subtract: $7b = -14$

$b = -2$

Substitute into ①: $4a - 5(-2) = 30$

$4a = 30 - 10$

$a = 5$

Hence, $\underline{(a, b)} = \underline{(5, -2)}$

b. $76x - 29y = 1050$ ①
$-13x - 63y = 250$ ②

To eliminate ①,

①× 13: $988x - 377y = 13{,}650$

②× 76: $-\underline{988x - 4788y} = \underline{19{,}000}$

Add: $-5165y = 32{,}650$

$y = -6.321$

Substitute into ①: $76x - 29(-6.321) = 1050$

$76x = 1050 - 183.31$

$x = 11.40$

Hence, $\underline{(x, y) = (11.40, -6.32)}$

21. $FV = PV(1 + i_1)(1 + i_2)$

$$\frac{FV}{PV(1+i_2)} = (1 + i_1)$$

$$i_1 = \frac{FV}{PV(1+i_2)} - 1$$

22. Given:

	Year 1 value (V_i)	Year 2 *value* (V_f)
Gold produced:	34,300 oz.	23,750 oz.
Average price:	$1160	$1280

a. Percent change in gold production = $\frac{23{,}750 - 34{,}300}{34{,}300} \times 100\% = \underline{\underline{-30.76\%}}$

b. Percent change in price = $\frac{\$1280 - \$1160}{\$1160} \times 100\% = \underline{\underline{10.34\%}}$

c. Year 1 revenue, V_i= 34,300($1160) = $39.788 million
Year 2 revenue, V_f, = 23,750($1280) = $30.400 million
Percent change in revenue = $\frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = \underline{\underline{-23.60\%}}$

23. Given: For the first year, V_i= $3.40, V_f = $11.50.
For the second year, V_i= $11.50, c = – 35%.

a. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = \underline{\underline{238.24\%}}$
The share price increased by 238.24% in the first year.

b. Current share price, $V_f = V_i$ (1 + c) = $11.50(1 – 0.35) = $\underline{\underline{\$7.48}}$.

Review Problems *(continued)*

24. Given: For the first year, c = 150%
For the second year, c = – 40%, V_f= $24
The price at the beginning of the second year was
$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f \text{ for the first year.}$$
The price at the beginning of the first year was
$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \underline{\underline{\$16.00}}$$
Barry bought the stock for $16.00 per share.

25. Given: Last year's revenue = $2,347,000
Last year's expenses = $2,189,000

a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%
Anticipated revenues, $V_f = V_i$(1 + c) = $2,347,000(1.1) = $2,581,700
Anticipated expenses = $2,189,000(1.05) = $\underline{2,298,450}
Anticipated profit = $283,250
Last year's profit = $2,347,000 – $2,189,000 = $158,000
Percent increase in profit = $\frac{\$283{,}250 - \$158{,}000}{\$158{,}000} \times 100\% = \underline{\underline{79.27\%}}$

b. Given: c(revenue) = –10%; c(expenses) = – 5%
Anticipated revenues = $2,347,000(1 – 0.10) = $2,112,300
Anticipated expenses = $2,189,000(1 – 0.05) = $\underline{2,079,550}
Anticipated profit $32,750
Percent change in profit = $\frac{\$32{,}750 - \$158{,}000}{\$158{,}000} \times 100\% = \underline{\underline{-79.27\%}}$
The operating profit will decline by 79.27%.

26. Given: Ken's share = 0.80(Hugh's share) + $15,000; Total distribution = $98,430
Let H represent Hugh's share. Then
Hugh's share + Ken's share = Total distribution
H + 0.8H + $15,000 = $98,430
1.8H = $83,430

$H = \$46{,}350$

Hugh should receive $46,350 and Ken should receive $98,430 – $46,350 = $52,080.

27. Given: Grace's share = 1.2(Kajsa's share); Mary Anne's share = $\frac{5}{8}$ (Grace's share)

Total allocated = $36,000

Let K represent Kajsa's share.

(Kajsa's share) + (Grace's share) + (Mary Anne's share) = $36,000

$$K + 1.2K + \frac{5}{8}(1.2K) = \$36{,}000$$

$$2.95\,K = \$36{,}000$$

$$K = \$12{,}203.39$$

Kajsa's should receive $12,203.39. Grace should receive 1.2K = $14,644.07.

Mary Anne should receive $\frac{5}{8}$ ($14,644.07) = $9152.54.

Review Problems *(continued)*

28. Let R represent the price per kg for red snapper and
let L represent the price per kg for ling cod. Then

$$370R + 264L = \$2454.20 \quad ①$$
$$255R + 304L = \$2124.70 \quad ②$$

To eliminate R,

① ÷ 370: $R + 0.71351L = \$6.6330$

② ÷ 255: $\underline{R + 1.19216L} = \underline{\$8.3322}$

Subtract: $-0.47865L = -\$1.6992$

$$L = \$3.55$$

Substitute into ①: $370R + 264(\$3.55) = \$2454.20$

$$370R = \$1517.00$$
$$R = \$4.10$$

Nguyen was paid $\underline{\$3.55 \text{ per kg for ling cod}}$ and $\underline{\$4.10 \text{ per kg for red snapper}}$.

29. Let b represent the base salary and r represent the commission rate. Then

$$r(\$27{,}000) + b = \$2815.00 \quad ①$$
$$\underline{r(\$35{,}500) + b} = \underline{\$3197.50} \quad ②$$

Subtract: $-\$8500r = \$382.50$

$$r = 0.045$$

Substitute into ①: $0.045(\$27{,}000) + b = \$2815$

$$b = \$1600$$

Deanna's base salary is $\underline{\$1600 \text{ per month}}$ and her commission rate is $\underline{4.5\%}$.

30. Given: Total initial investment = \$7800; Value 1 year later = \$9310
Percent change in ABC portion = 15%
Percent change in XYZ portion = 25%

Let X represent the amount invested in XYZ Inc.

The solution "idea" is:

(Amount invested in ABC)1.15 + (Amount invested in XYZ)1.25 = \$9310

Hence,

$$(\$7800 - X)1.15 + (X)1.25 = \$9310$$
$$\$8970 - 1.15X + 1.25X = \$9310$$
$$0.10X = \$9310 - \$8970$$
$$X = \$3400$$

Rory invested $\underline{\$3400 \text{ in XYZ}}$ Inc. and \$7800 – \$3400 = $\underline{\$4400 \text{ in ABC}}$ Ltd.

31. Let the regular season ticket prices be R for the red section and B for the blue section. Then

$$2500R + 4500B = \$50{,}250 \quad ①$$
$$2500(1.3R) + 4500(1.2B) = \$62{,}400 \quad ②$$

①× 1.2: $\underline{2500(1.2R) + 4500(1.2B)} = \underline{\$60{,}300}$

Subtract: $2500(0.1R) + 0 = \$2100$

$$R = \$8.40$$

Substitute into ①: $2500(\$8.40) + 4500B = \$50{,}250$

$$B = \$6.50$$

The ticket prices for the playoffs cost

$1.3 \times \$8.40 = \underline{\$10.92}$ in the "$\underline{\text{reds}}$"

and $1.2 \times \$6.50 = \underline{\$7.80}$ in the "$\underline{\text{blues}}$".

Review Problems *(continued)*

32. 15(Income exceeding \$68,000) = \$6300

$$\text{Income exceeding \$68,000} = \frac{\$6300}{0.15} = \$42{,}000.00$$

Total net income = \$68,000 + \$42,000 = <u>\$110,000</u>

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