

## Solution 2.CS

### THE AMAZING IMPACT OF COMPOUND INTEREST

#### Background on Five Situations

1. The first mass produced automobile was the Ford Model T, initially manufactured and sold in 1909 for \$825. The rate of inflation in the United States over the period 1909 to 2015 has averaged 3.10% per year. You just purchased a new car for \$28,000. You wonder what the cost of this same car might be 50 years from now when your son may be purchasing a similar car for his daughter (your granddaughter) so she can attend college. Also, you wonder what the value of the Model T will be 50 years in the future, that is, in the year 2065.
2. The purchase price of Manhattan Island, where much of the city of New York is concentrated, in the year 1626 was \$24. After 391 years in 2017, you wonder what the value of the land might be, provided it has appreciated in value at a rate of 6% per year, every year.
3. Last week, your friend Jeremy borrowed \$200 from a pawn shop operator because he was totally broke. He was to pay the operator \$230 after 1 week, but missed the payment. At first, you thought this was “no big deal,” but then started to realize that the interest was \$30 the first week alone and would increase, compounded at the same rate until the total debt was repaid. When your friend told you he would pay off the loan in a year (provided the operator didn’t come after him), you gave him the results of your analysis. He was shocked and paid the total amount immediately.
4. In 1939, two people teamed up to manufacture and market electronic test equipment. By 1957, the initial capital investment from themselves and a few friends that amounted to only \$80,805.12 in 1939 had increased in value to an equivalent of \$1 million. After this, the

company skyrocketed to become a world leader in electronic equipment, computers, and a wide range of other products. If the net cash flow averaged \$150,000 per year from 1957 to 2017 (60 years) at the same rate, these two individuals would be quite wealthy.

5. Assume that when your great-grandmother was 25 years old, she received an engagement ring from her husband-to-be. He paid \$50 for the ring containing a single, high-quality diamond. When she passed away at the age of 90, the ring went to your grandmother, who kept it for 60 years and then gave it to your mother. After 30 years of keeping the ring in a safe place, she gave it to you on your 24th birthday. Today is your 48th birthday and you have just discovered the ring in a desk drawer, forgotten for all these years. If this high-grade diamond has been now appraised as a collector’s grade stone, which has appreciated in value at an average rate of 4% per year, every year, since it was first purchased, you wonder what the ring might be valued at today.

#### Team Exercises

For each situation described, do the following using a five-person team.

- (a) Determine the annual compound interest or inflation rate and discuss the differences in these rates from one situation to another.
- (b) First, each member of the team should make an estimate (a guess) of the beginning and ending amounts of money involved *for each situation*. Each team member should now calculate the two amounts for one selected situation. Between team members, discuss the accuracy of their first estimates compared to the actual amounts calculated.

*Solution continued on next page...*

*Solution:*

1. Ford Model T and a New Car

(a) Inflation rate is substituted for  $i = 3.10\%$  per year

(b) Model T: Beginning cost in 1909:  $P = \$825$

Ending cost:  $n = 1909$  to  $2015 + 50$  years = 156 years;  $F = \$96,562$

$$\begin{aligned} F &= P(1+i)^n = 825(1.031)^{156} \\ &= 825(117.0447) \\ &= \$96,562 \end{aligned}$$

New car: Beginning cost:  $P = \$28,000$

Ending cost:  $n = 50$  years;  $F = \$128,853$

$$\begin{aligned} F &= P(1+i)^n = 28,000(1.031)^{50} \\ &= 28,000(4.6019) \\ &= \$128,853 \end{aligned}$$

2. Manhattan Island

(a)  $i = 6.0\%$  per year

(b) Beginning amount in 1626:  $P = \$24$

Ending value:  $n = 391$ ;  $F = \$188.3$  billion

*Solution continued on the next page...*

$$\begin{aligned}
F &= 24(1.06)^{391} \\
&= 24(7,845,006.7) \\
&= \$188,280,161 \quad (\$188.3 \text{ billion})
\end{aligned}$$

### 3. Pawn Shop Loan

(a)  $i$  per week =  $(30/200) * 100 = 15\%$  per week

$$i \text{ per year} = [(1.15)^{52} - 1] * 100 = 143,214\% \text{ per year}$$

Subtraction of 1 considers repayment of the original loan of \$200 when the interest rate is calculated (see Chapter 4 for details.)

- (b) Beginning amount:  $P = \$200$   
Ending owed: 1 year later,  $F = \$286,627$

$$\begin{aligned}
F &= P(F/P, 15\%, 52) \\
&= 200(1.15)^{52} \\
&= 200(1433.1370) \\
&= \$286,627
\end{aligned}$$

### 4. Capital Investment

(a)  $i = 15\%$  per year

*Solution continued on the next page...*

$$1,000,000 = 150,000(P/A, i\%, 60)$$

$$(P/A, i\%, 60) = 6.6667$$

$$i = 15\%$$

- (b) Beginning amount:  $P = \$1,000,000$  invested  
Ending total amount over 60 years:  $150,000(60) = \$9$  million

$$\begin{aligned}\text{Value: } F_{60} &= 150,000(F/A, 15\%, 60) \\ &= 150,000(29220.0) \\ &= \$4,383,000,000 \quad (\$4.38 \text{ billion})\end{aligned}$$

## 5. Diamond Ring

- (a)  $i = 4\%$  per year

- (b) Beginning price:  $P = \$50$   
Ending value after 179 years:  $F = \$55,968$

$$\begin{aligned}n &= \text{great grandmother} + \text{grandmother} + \text{mother} + \text{girl} \\ &= 65 + 60 + 30 + 24 \\ &= 179 \text{ years}\end{aligned}$$

$$\begin{aligned}F &= 50(F/P, 4\%, 179) \\ &= 50(1119.35) \\ &= \$55,968\end{aligned}$$

## Solution 2.1

Look up the numerical value for the following factors from the compound interest factor tables.

1.  $(F/P, 10\%, 7)$
2.  $(A/P, 12\%, 10)$
3.  $(P/G, 15\%, 20)$
4.  $(F/A, 2\%, 50)$
5.  $(P/G, 35\%, 15)$

*Solution:*

$$(1) (F/P, 10\%, 7) = 1.9487$$

$$(2) (A/P, 12\%, 10) = 0.17698$$

$$(3) (P/G, 15\%, 20) = 33.5822$$

$$(4) (F/A, 2\%, 50) = 84.5794$$

$$(5) (A/G, 35\%, 15) = 2.6889$$

## Solution 2.2

The Department of Traffic Security of a city is considering the purchase of a new drone for aerial surveillance of traffic on its most congested streets. A similar purchase 4 years ago cost \$1,200,000. At an interest rate of 7% per year, what is the equivalent value today of the previous \$1,200,000 expenditure?

*Solution:*

$$\begin{aligned}2 F &= 1,200,000(F/P, 7\%, 4) \\ &= 1,200,000(1.3108) \\ &= \$1,572,960\end{aligned}$$

### **Solution 2.3**

Pressure Systems, Inc. manufactures high-accuracy liquid level transducers. It is investigating whether it should update in-place equipment now or wait and do it later. If the cost now is \$200,000, what will be the equivalent amount 3 years from now at an interest rate of 10% per year?

*Solution:*

$$\begin{aligned} F &= 200,000(F/P, 10\%, 3) \\ &= 200,000(1.3310) \\ &= \$266,200 \end{aligned}$$

## Solution 2.4

How much money should a bank be willing to loan a real estate developer who will repay the loan by selling seven lakefront lots at \$120,000 *each* 2 years from now? Assume the bank's interest rate is 10% per year.

*Solution:*

$$\begin{aligned} P &= 7(120,000)(P/F, 10\%, 2) \\ &= 840,000(0.8264) \\ &= \$694,176 \end{aligned}$$



## Solution 2.5

A rotary engine is the essence of a vertical takeoff and landing (VTOL) personal aircraft known as the Moller Skycar M400. It is a flying car known as a personal air vehicle (PAV) and it is expected to make its first untethered flight in 2020. The PAV has been under development for 30 years at a total cost of \$100 million. Assuming the \$100 million was spent in an equal amount each year, determine the future worth at the end of the 30-year period at an interest rate of 10% per year.

*Solution:*

$$\begin{aligned} F &= 100,000,000/30(F/A,10\%,30) \\ &= 3,333,333(164.4940) \\ &= \$548,313,333 \end{aligned}$$

## Solution 2.6

What is the present worth of an expenditure of \$25,000 in year 8 if the interest rate is 10% per year?

*Solution:*

$$\begin{aligned} P &= 25,000(P/F, 10\%, 8) \\ &= 25,000(0.4665) \\ &= \$11,662.50 \end{aligned}$$

### **Solution 2.7**

Calculate the present worth of 10 uniform payments of \$8000 that begin 1 year from now at an interest rate of 10% per year. ear?

*Solution:*

$$\begin{aligned} P &= 8000(P/A, 10\%, 10) \\ &= 8000(6.1446) \\ &= \$49,156.80 \end{aligned}$$

## Solution 2.8

Atlas Long-Haul Transportation is considering installing Valutemp temperature loggers in all of its refrigerated trucks for monitoring temperatures during transit. If the systems will reduce insurance claims by \$100,000 in each of the next 2 years, how much should the company be willing to spend now if it uses an interest rate of 12% per year?

*Solution:*

$$\begin{aligned} P &= 100,000((P/A, 12\%, 2)) \\ &= 100,000(1.6901) \\ &= \$169,010 \end{aligned}$$

### **Solution 2.9**

Determine the size of your investment account 30 years from now (when you plan to retire) if you deposit \$12,000 each year, beginning 1 year from now, and the account earns interest at a rate of 10% per year.

*Solution:*

$$\begin{aligned} F &= 12,000(F/A, 10\%, 30) \\ &= 12,000(164.4940) \\ &= \$1,973,928 \end{aligned}$$

### **Solution 2.10**

How much could BTU Oil & Gas Fracking afford to spend on new equipment each year for the next 3 years if it expects a profit of \$50 million 3 years from now? Assume the company's MARR is 20% per year.

*Solution:*

$$\begin{aligned} A &= 50,000,000(A/F, 20\%, 3) \\ &= 50,000,000(0.27473) \\ &= \$13,736,500 \end{aligned}$$

### **Solution 2.11**

Thompson Mechanical Products is planning to set aside \$150,000 now for possible replacement of large synchronous refiner motors when it becomes necessary. If the replacement isn't needed for 5 years, how much will the company have in its investment set-aside account? Assume a rate of return of 18% per year.

*Solution:*

$$\begin{aligned} F &= 150,000(F/P, 18\%, 5) \\ &= 150,000(2.2878) \\ &= \$343,170 \end{aligned}$$

### **Solution 2.12**

Electric car maker Gentech signed a \$75 million contract with Power Systems, Inc. to automate a major part of its assembly line system. If Power Systems will be paid 2 years from now, when the systems are ready, determine the present worth of the contract at 18% per year interest.

*Solution:*

$$\begin{aligned} P &= 75(P/F, 18\%, 2) \\ &= 75(0.7182) \\ &= \$53.865 \text{ million} \end{aligned}$$



### **Solution 2.13**

Labco Scientific sells high-purity chemicals to universities, research laboratories, and pharmaceutical companies. The company wants to invest in new equipment that will reduce shipping costs by better matching the size of the completed products with the size of the shipping container. The new equipment is estimated to cost \$450,000 to purchase and install. How much must Labco save each year for 3 years in order to justify the investment at an interest rate of 10% per year?

*Solution:*

$$\begin{aligned}A &= 450,000(A/P, 10\%, 3) \\ &= 450,000(0.40211) \\ &= \$180,950\end{aligned}$$

### **Solution 2.14**

Loadstar Sensors is a company that makes load/ force sensors based on capacitive sensing technology. For a major plant expansion project, the company wants to have \$30 million 5 years from now. If the company already has \$15 million in an investment account for the expansion, how much more must the company add to the account now so that it will have the \$30 million 5 years from now? The funds earn interest at the rate of 10% per year.

*Solution:*

$$\begin{aligned} P &= 30,000,000(P/F, 10\%, 5) - 15,000,000 \\ &= 30,000,000(0.6209) - 15,000,000 \\ &= \$3,627,000 \end{aligned}$$

### **Solution 2.15**

Meggitt Systems, a company that specializes in extreme-high-temperature accelerometers, is investigating whether it should update certain equipment now or wait and do it later. If the cost now is \$280,000, what is the equivalent amount 2 years from now at an interest rate of 12% per year?

*Solution:*

$$\begin{aligned} F &= 280,000(F/P, 12\%, 2) \\ &= 280,000(1.2544) \\ &= \$351,232 \end{aligned}$$

### Solution 2.16

Henry Mueller Supply Co. sells vibration control equipment for wind turbines exposed to harsh environmental factors. Annual cash flows for an 8-year period are shown in the table. Determine the future worth of the cash flows at an interest rate of 10% per year.

Year	1	2	3	4	5	6	7	8
Revenue, \$1000	200	200	200	200	200	200	200	200
Expenses, \$1000	90	90	90	90	90	90	90	90

*Solution:*

$$\begin{aligned} F &= (200 - 90)(F/A, 10\%, 8) \\ &= 110(11.4359) \\ &= \$1,257,949 \end{aligned}$$

**Solution 2.17**

Stanley, Inc. makes self-clinching fasteners for stainless steel applications. It expects to acquire new time-saving punching equipment 4 years from now. If the company sets aside \$125,000 each year, determine the amount available in 4 years at an earning rate of 10% per year.

*Solution:*

$$\begin{aligned} F &= 125,000(F/A, 10\%, 4) \\ &= 125,000(4.6410) \\ &= \$580,125 \end{aligned}$$

## Solution 2.18

China spends an estimated \$100,000 per year on cloud seeding efforts which include using anti-aircraft guns and rocket launchers to fill the sky with silver iodide. In the United States, utilities that run hydroelectric dams are among the most active cloud seeders because they believe it is a cost-effective way to increase limited water supplies by 10% or more. If the yields of cash crops will increase by 4% per year for the next 3 years because of extra irrigation water captured behind dams during cloud seeding, what is the future worth (in year 3) of the *extra value* of the cash crops? Assume the interest rate is 10% per year and the value of the cash crops without the extra irrigation water would be \$600,000 per year.

*Solution:*

$$\begin{aligned} F &= 600,000(0.04)(F/A, 10\%, 3) \\ &= 24,000(3.3100) \\ &= \$79,440 \end{aligned}$$

### **Solution 2.19**

American Gas Products manufactures a device called a Can-Emitor that empties the contents of old aerosol cans in 2 to 3 seconds. This eliminates the need to dispose of the cans as hazardous waste. If a paint manufacturing company can save \$90,000 per year in waste disposal costs, how much could the company afford to spend now on the Can-Emitor if it wants to recover its investment in 3 years at an interest rate of 20% per year?

*Solution:*

$$\begin{aligned} P &= 90,000(P/A, 20\%, 3) \\ &= 90,000(2.1065) \\ &= \$189,585 \end{aligned}$$

**Solution 2.20**

Durban Moving and Storage wants to have enough money available 5 years from now to purchase a new tractor-trailer. If the estimated cost is \$250,000, how much should the company set aside each year if the funds earn 9% per year?

*Solution:*

$$\begin{aligned}A &= 250,000(A/F, 9\%, 5) \\ &= 250,000(0.16709) \\ &= \$41,772.50\end{aligned}$$



### **Solution 2.21**

The Public Service Board (PSB) awarded two contracts worth a combined \$3.07 million to increase the depth of a retention basin and reconstruct a spillway that was severely damaged in a flood 2 years ago. The PSB president stated that, surprisingly, the bids were \$1,150,000 lower than PSB engineers estimated. If the projects are assumed to have a 20-year life, what is the annual worth of the savings at an interest rate of 5% per year?

*Solution:*

$$\begin{aligned} A &= 1,150,000(A/P, 5\%, 20) \\ &= 1,150,000(0.08024) \\ &= \$92,276 \end{aligned}$$

**Solution 2.22**

Syringe pumps oftentimes fail because reagents adhere to the ceramic piston and deteriorate the seal. Trident Chemical developed an integrated polymer dynamic seal that provides a higher sealing force on the sealing lip, resulting in extended seal life. One of Trident's customers expects to reduce down time by 30% as a result of the new seal design. If lost production would have cost the company \$110,000 per year for each of the next 4 years, how much could the company afford to spend now on the new seals? Use a MARR of 12% per year.

*Solution:*

$$\begin{aligned} P &= (110,000 * 0.3)(P/A, 12\%, 4) \\ &= (33,000)(3.0373) \\ &= \$100,231 \end{aligned}$$

**Solution 2.23**

The cost of a fence that can detect poacher intrusion into a National Wildlife Preserve is \$3 million per mile. If the effective life of the fence is 10 years, determine the equivalent annual cost of a 10-mile long fence at an interest rate of 8% per year.

*Solution:*

$$\begin{aligned}A &= 3,000,000(10)(A/P, 8\%, 10) \\ &= 30,000,000(0.14903) \\ &= \$4,470,900\end{aligned}$$

### **Solution 2.24**

A small oil company wants to replace its Micro Motion Coriolis flowmeters with nickel-based steel alloy flowmeters from the Emerson F-Series. The replacement process will cost the company \$50,000 three years from now. How much money must the company set aside each year beginning 1 year from now in order to have the total amount in 3 years? Assume the company earns a generous 20% per year on investment funds.

*Solution:*

$$\begin{aligned} A &= 50,000(A/F, 20\%, 3) \\ &= 50,000(0.27473) \\ &= \$13,736 \end{aligned}$$

### Solution 2.25

Determine the numerical value of the following factors using (a) linear interpolation, (b) the formula, and (c) the spreadsheet function from Figure 2-9.

1. (P/F, 8.4%, 15)    2. (A/F, 17%, 10)

	A	B	C	D
1	i =	n =		
2	3.25%	25.00	Enter requested i and n	
3			<b>Value obtained</b>	
4	<b>Factor</b>	<b>Value</b>	<b>with this function</b>	
5	P/F	0.44952	= -PV(\$A\$2,\$B\$2,,1)	
6	P/A	16.93786	= -PV(\$A\$2,\$B\$2,1)	
7				
8	F/P	2.22460	= -FV(\$A\$2,\$B\$2,,1)	
9	F/A	37.67993	= -FV(\$A\$2,\$B\$2,1)	
10				
11	A/F	0.02654	= -PMT(\$A\$2,\$B\$2,,1)	
12	A/P	0.05904	= -PMT(\$A\$2,\$B\$2,1)	
13				

**Figure 2-9**

Use of Excel functions to display factor values for any  $i$  and  $n$  values.

*Solution:*

- (a) 1. Interpolate between  $i = 8\%$  and  $i = 9\%$  at  $n = 15$ :

$$0.4/1 = x/(0.3152 - 0.2745)$$

$$x = 0.0163$$

$$(P/F, 8.4\%, 15) = 0.3152 - 0.0163$$

$$= 0.2989$$

2. Interpolate between  $i = 16\%$  and  $i = 18\%$  at  $n = 10$ :

$$1/2 = x/(0.04690 - 0.04251)$$

$$x = 0.00220$$

$$(A/F, 17\%, 10) = 0.04690 - 0.00220$$

$$= 0.04470$$

*Solution continued on the next page....*

- (b)
1.  $(P/F, 8.4\%, 15) = 1/(1 + 0.084)^{15}$   
 $= 0.2982$
  2.  $(A/F, 17\%, 10) = 0.17/[(1 + 0.17)^{10} - 1]$   
 $= 0.04466$
- (c)
1.  $= -PV(8.4\%, 15, 1)$  displays 0.29824
  2.  $= -PMT(17\%, 10, 1)$  displays 0.04466

## Solution 2.26

Find the numerical value of the following factors by (a) linear interpolation, (b) using the appropriate formula, and (c) a spreadsheet function.

1.  $(F/A, 19\%, 20)$

2.  $(P/A, 26\%, 15)$

*Solution:*

- (a) 1. Interpolate between  $i = 18\%$  and  $i = 20\%$  at  $n = 20$ :

$$1/2 = x/40.06$$

$$x = 20.03$$

$$\begin{aligned}(F/A, 19\%, 20) &= 146.6280 + 20.03 \\ &= 166.658\end{aligned}$$

2. Interpolate between  $i = 25\%$  and  $i = 30\%$  at  $n = 15$ :

$$1/5 = x/0.5911$$

$$x = 0.11822$$

$$\begin{aligned}(P/A, 26\%, 15) &= 3.8593 - 0.11822 \\ &= 3.7411\end{aligned}$$

(b) 1.  $(F/A, 19\%, 20) = [(1 + 0.19)^{20} - 1]/0.19$   
 $= 165.418$

2.  $(P/A, 26\%, 15) = [(1 + 0.26)^{15} - 1]/[0.26(1 + 0.26)^{15}]$   
 $= 3.7261$

- (c) 1.  $= -FV(19\%, 20, 1)$  displays 165.41802  
2.  $= -PV(26\%, 15, 1)$  displays 3.72607

### Solution 2.27

Find the numerical value of the following factors using (a) linear interpolation and (b) the appropriate formula.

1.  $(F/P, 18\%, 33)$

2.  $(A/G, 12\%, 54)$

*Solution:*

(a) 1. Interpolate between  $n = 32$  and  $n = 34$ :

$$1/2 = x/78.3345$$

$$x = 39.1673$$

$$\begin{aligned}(F/P, 18\%, 33) &= 199.6293 + 39.1673 \\ &= 238.7966\end{aligned}$$

2. Interpolate between  $n = 50$  and  $n = 55$ :

$$4/5 = x/0.0654$$

$$x = 0.05232$$

$$\begin{aligned}(A/G, 12\%, 54) &= 8.1597 + 0.05232 \\ &= 8.2120\end{aligned}$$

(b) 1.  $(F/P, 18\%, 33) = (1+0.18)^{33}$   
 $= 235.5625$

2.  $(A/G, 12\%, 54) = \{(1/0.12) - 54/(1+0.12)^{54} - 1\}$   
 $= 8.2143$



### Solution 2.28

For the factor  $(F/P, 10\%, 43)$ , find the percent difference between the interpolated and formula-calculated values, assuming the formula-calculated value is the correct one.

*Solution:*

Interpolated value: Interpolate between  $n = 40$  and  $n = 45$ :

$$3/5 = x/(72.8905 - 45.2593)$$

$$x = 16.5787$$

$$\begin{aligned}(F/P, 10\%, 43) &= 45.2593 + 16.5787 \\ &= 61.8380\end{aligned}$$

$$\begin{aligned}\text{Formula value: } (F/P, 10\%, 43) &= (1 + 0.10)^{43} \\ &= 60.2401\end{aligned}$$

$$\begin{aligned}\% \text{ difference} &= [(61.8380 - 60.2401) / 60.2401] * 100 \\ &= 2.65\%\end{aligned}$$

**Solution 2.29**

A cash flow sequence starts in year 1 at \$4000 and decreases by \$300 each year through year 9. Determine (a) the value of the gradient  $G$ ; (b) the amount of cash flow in year 5; and (c) the value of  $n$  for the  $(P/G, i\%, n)$  factor.

*Solution:*

(a)  $G = \$-300$  (b)  $CF_5 = \$2800$  (c)  $n = 9$

**Solution 2.30**

An arithmetic cash flow gradient series equals \$500 in year 1, \$600 in year 2, and amounts increasing by \$100 per year through year 9. At  $i = 10\%$  per year, determine the present worth of the cash flow series in year 0.

*Solution:*

$$\begin{aligned}P_0 &= 500(P/A, 10\%, 9) + 100(P/G, 10\%, 9) \\ &= 500(5.7590) + 100(19.4215) \\ &= 2879.50 + 1942.15 \\ &= \$4821.65\end{aligned}$$

### Solution 2.31

NMTeX Oil owns several gas wells in Carlsbad, NM. Revenue from the wells has been increasing according to an arithmetic gradient for the past 5 years. The revenue in year 1 from well no. 24 was \$390,000 and it increased by \$15,000 each year thereafter. Determine (a) the revenue in year 3, and (b) the equivalent annual worth of the revenue through year 5. Assume an interest rate of 10% per year.

*Solution:*

$$\begin{aligned} \text{(a) Revenue} &= 390,000 + 2(15,000) \\ &= \$420,000 \end{aligned}$$

$$\begin{aligned} \text{(b) A} &= 390,000 + 15,000(A/G, 10\%, 5) \\ &= 390,000 + 15,000(1.8101) \\ &= \$417,151.50 \end{aligned}$$

### **Solution 2.32**

Solar Hydro manufactures a revolutionary aeration system that combines coarse and fine bubble aeration components. This year (year 1) the cost for check valve components is \$9,000. Based on closure of a new contract with a distributor in China and volume discounts, the company expects this cost to decrease. If the cost in year 2 and each year thereafter decreases by \$560, what is the equivalent annual cost for a 5-year period at an interest rate of 10% per year?

*Solution:*

$$\begin{aligned} A &= 9000 - 560(A/G, 10\%, 5) \\ &= 9000 - 560(1.8101) \\ &= \$7986 \end{aligned}$$

**Solution 2.33**

For the cash flow revenues shown below, find the value of  $G$  that makes the equivalent annual worth in years 1 through 7 equal to \$500. The interest rate is 10% per year.

Year	Cash Flow, \$	Year	Cash Flow, \$
0		4	$200 + 3G$
1	200	5	$200 + 4G$
2	$200 + G$	6	$200 + 5G$
3	$200 + 2G$	7	$200 + 6G$

*Solution:*

$$500 = 200 + G(A/G, 10\%, 7)$$

$$500 = 200 + G(2.6216)$$

$$G = \$114.43$$

### **Solution 2.34**

A low-cost noncontact temperature measuring tool may be able to identify railroad car wheels that are in need of repair long before a costly structural failure occurs. If the BNF railroad saves \$100,000 in year 1, \$110,000 in year 2, and amounts increasing by \$10,000 each year for 5 years, what is the future worth of the savings in year 5 at an interest rate of 10% per year?

*Solution:*

$$\begin{aligned} A &= 100,000 + 10,000(A/G, 10\%, 5) \\ &= 100,000 + 10,000(1.8101) \\ &= \$118,101 \end{aligned}$$

$$\begin{aligned} F &= 118,101(F/A, 10\%, 5) \\ &= 118,101(6.1051) \\ &= \$721,018 \end{aligned}$$

**Solution 2.35**

For the cash flows below determine the amount in year 1, if the annual worth in years 1 through 9 is \$3500 and the interest rate is 10% per year.

Year	1	2	3	4	5	6	7	8	9
Cost, \$1000	A	A+40	A+80	A+120	A+160	A+200	A+240	A+280	A+320

*Solution:*

$$3500 = A + 40(A/G, 10\%, 9)$$

$$3500 = A + 40(3.3724)$$

$$A = \$3365.10$$



### Solution 2.36

Apple Computer wants to have \$2.1 billion available 5 years from now in order to finance initial production of a device that, based on your behavior, will learn how to monitor and control nearly all of the electronic devices in your home, such as thermostat, coffee pot, TV, sprinkler system, etc. using Internet of Things (IOT) technology. The company expects to set aside uniformly increasing amounts of money each year to meet its goal. If the amount set aside at the end of year 1 is \$100 million, how much will the uniform increase,  $G$ , have to be each year? Assume the investment funds grow at a rate of 18% per year.

*Solution:*

In \$ billion units,

$$\begin{aligned}P &= 2.1(P/F, 18\%, 5) \\ &= 2.1(0.4371) \\ &= 0.91791 = \$917,910,000\end{aligned}$$

$$917,910,000 = 100,000,000(P/A, 18\%, 5) + G(P/G, 18\%, 5)$$

$$917,910,000 = 100,000,000(3.1272) + G(5.2312)$$

$$G = \$115,688,561$$

**Solution 2.37**

Tacoza Electric, which manufactures brush dc servo motors, budgeted \$95,000 per year to pay for specific components over the next 5 years. If the company expects to spend \$55,000 in year 1, how much of a uniform (arithmetic) increase each year is the company expecting in the cost of this part? Assume the company uses an interest rate of 10% per year.

*Solution:*

$$95,000 = 55,000 + G(A/G, 10\%, 5)$$

$$95,000 = 55,000 + G(1.8101)$$

$$G = \$22,098$$

### **Solution 2.38**

The future worth in year 10 of an arithmetic gradient cash flow series for years 1 through 10 is \$500,000. If the gradient increase each year,  $G$ , is \$3,000, determine the cash flow in year 1 at an interest rate of 10% per year.

*Solution:*

$$\begin{aligned} P \text{ in year 0} &= 500,000(P/F, 10\%, 10) \\ &= 500,000(0.3855) \\ &= \$192,750 \end{aligned}$$

$$192,750 = A + 3000(P/G, 10\%, 10)$$

$$192,750 = A + 3000(22.8913)$$

$$A = \$124,076$$

### Solution 2.39

Assume you were asked to prepare a table of compound interest factor values (like those in the back of this book) used in calculating the present worth of a geometric gradient series. Determine the two values for  $n = 1$  and 2 for an interest rate of 10% per year and a rate of change  $g$  of 5% per year.

*Solution:*

Find  $(P/A, g, i, n)$  using Equation [2.32] and  $A_1 = 1$

$$\begin{aligned}\text{For } n = 1: P_{\frac{g}{i}} &= 1 * \{ 1 - [(1 + 0.05)/(1 + 0.10)]^1 \} / (0.10 - 0.05) \\ &= 0.90909\end{aligned}$$

$$\begin{aligned}\text{For } n = 2: P_{\frac{g}{i}} &= 1 * \{ 1 - [(1 + 0.05)/(1 + 0.10)]^2 \} / (0.10 - 0.05) \\ &= 1.77686\end{aligned}$$

**Solution 2.40**

A company that manufactures purgeable hydrogen sulfide monitors will make deposits such that each one is 7% larger than the preceding one. How large must the first deposit at the end of year 1 be if the deposits extend through year 10 and the fourth deposit is \$5550? Use an interest rate of 10% per year.

*Solution:*

Decrease deposit in year 4 by 7% per year for three years to get back to year 1.

$$\begin{aligned}\text{First deposit} &= 5550/(1 + 0.07)^3 \\ &= \$4530.45\end{aligned}$$

**Solution 2.41**

Calculate the present worth of a geometric gradient series with a cash flow of \$35,000 in year 1 and increases of 5% each year through year 6. The interest rate is 10% per year.

*Solution:*

$$\begin{aligned}P_g &= 35,000\{1 - [(1 + 0.05)/(1 + 0.10)]^6\}/(0.10 - 0.05) \\ &= \$170,486\end{aligned}$$

**Solution 2.42**

To improve crack detection in military aircraft, the Air Force combined ultrasonic inspection procedures with laser heating to identify fatigue cracks. Early detection of cracks may reduce repair costs by as much as \$200,000 per year. If the savings start at the end of year 1 and increase by 3% each year through year 5, calculate the present worth of these savings at an interest rate of 10% per year.

*Solution:*

$$\begin{aligned} P_g &= 200,000 \{ 1 - [(1 + 0.03)/(1 + 0.10)]^5 \} / (0.10 - 0.03) \\ &= \$800,520 \end{aligned}$$

### Solution 2.43

A civil engineer planning for her retirement places 10% of her salary each year into a high-technology stock fund. If her salary this year (end of year 1) is \$160,000 and she expects her salary to increase by 3% each year, what will be the future worth of her retirement fund after 15 years provided it earns 7% per year?

*Solution:*

First find  $P_g$  and then convert to F in year 15

$$\begin{aligned} P_g &= (0.10)(160,000)\{1 - [(1 + 0.03)/(1 + 0.07)]^{15} / (0.07 - 0.03)\} \\ &= 16,000(10.883) = \$174,128.36 \end{aligned}$$

$$\begin{aligned} F &= 174,128.36(F/P, 7\%, 15) \\ &= 174,128.36 (2.7590) \\ &= \$480,420.15 \end{aligned}$$



### Solution 2.44

El Paso Water (EPW) purchases surface water for treatment and distribution to EPW customers from the County Water Improvement District during the irrigation season. A new contract between the two entities resulted in a reduction in future price increases in the cost of the water from 8% per year to 4% per year for the next 20 years. The cost of water next year (which is year 1 of the new contract) will be \$260 per acre-ft. Using an interest rate of 6% per year,

- (a) Determine the present worth of the savings (in terms of \$/acre-ft) to EPW between the old and the new contracts.
- (b) Determine the total present worth of the savings over the life of the contract if EPW uses 51,000 acre-ft per year.

*Solution:*

$$\begin{aligned} \text{(a) } P_g &= 260 \left\{ 1 - \left[ \frac{1 + 0.04}{1 + 0.06} \right]^{20} \right\} / (0.06 - 0.04) \\ &= 260(15.8399) \\ &= \$4119.37 \end{aligned}$$

$$\begin{aligned} \text{(b) } P_{\text{Total}} &= (4119.37)(51,000) \\ &= \$210,087,870 \end{aligned}$$

### Solution 2.45

Toselli Animation plans to offer its employees a salary enhancement package that has revenue sharing as its main component. Specifically, the company will set aside 1% of total sales revenue for year-end bonuses. The sales are expected to be \$5 million the first year, \$5.5 million the second year, and amounts increasing by 10% each year for the next 5 years. At an interest rate of 8% per year, what is the equivalent annual worth in years 1 through 5 of the bonus package?

*Solution:*

Solve for  $P_g$  in geometric gradient equation and then convert to A

$$A_1 = 5,000,000(0.01) = 50,000$$

$$\begin{aligned} P_g &= 50,000[1 - (1.10/1.08)^5]/(0.08 - 0.10) \\ &= \$240,215 \end{aligned}$$

$$\begin{aligned} A &= 240,215(A/P, 8\%, 5) \\ &= 240,215(0.25046) \\ &= \$60,164 \end{aligned}$$

### Solution 2.46

A northern California consulting firm wants to start saving money for replacement of network servers. If the company invests \$5000 at the end of year 1 but decreases the amount invested by 5% each year, how much will be available 5 years from now at an earning rate of 8% per year?

*Solution:*

First find  $P_g$  and then convert to F

$$\begin{aligned}P_g &= 5000[1 - (0.95/1.08)^5]/(0.08 + 0.05) \\ &= \$18,207\end{aligned}$$

$$\begin{aligned}F &= 18,207(F/P, 8\%, 5) \\ &= 18,207(1.4693) \\ &= \$26,751\end{aligned}$$

### **Solution 2.47**

A start-up company that makes robotic hardware for CIM (computer integrated manufacturing) systems borrowed \$1 million to expand its packaging and shipping facility. The contract required the company to repay the lender through an innovative mechanism called “faux dividends,” a series of uniform annual payments over a fixed period of time. If the company paid \$290,000 per year for 5 years, what was the interest rate on the loan?

*Solution:*

$$1,000,000 = 290,000(P/A, i, 5)$$

$$(P/A, i, 5) = 3.44828$$

Interpolate between 12% and 14% interest tables or use Excel's RATE function

By RATE,  $i = 13.8\%$

**Solution 2.48**

Your grandmother deposited \$10,000 in an investment account on the day you were born to help pay the tuition when you go to college. If the account was worth \$50,000 seventeen years after she made the deposit, what was the rate of return on the account?

*Solution:*

$$50,000 = 10,000(F/P, i, 17)$$

$$5.0000 = (F/P, i, 17)$$

$$5.0000 = (1 + i)^{17}$$

$$i = 9.93\%$$

### Solution 2.49

An A&E firm planning for a future expansion deposited \$40,000 each year for 5 years into a sinking (investment) fund that was to pay an unknown rate of return. If the account had a total of \$451,000 immediately after the fifth deposit, what rate of return did the company make on these deposits?

*Solution:*

$$F = A(F/A, i\%, 5)$$

$$451,000 = 40,000(F/A, i\%, 5)$$

$$(F/A, i\%, 5) = 11.2750$$

Interpolate between 40% and 50% interest tables or use Excel's RATE function

By RATE,  $i = 41.6\%$

### **Solution 2.50**

Parkhill, Smith, and Cooper, a consulting engineering firm, pays a bonus to each engineer at the end of the year based on the company's profit for that year. If the company's initial investment was \$1.2 million, what rate of return has it made if each engineer's bonus has been \$3000 per year for the past 10 years? Assume the company has six engineers and that the bonus money represents 5% of the company's profit.

*Solution:*

$$\text{Bonus/year} = 6(3000)/0.05 = \$360,000$$

$$1,200,000 = 360,000(P/A, i, 10)$$

$$(P/A, i, 10) = 3.3333$$

$$i = 27.3\%$$

### Solution 2.51

For a 5-year period, determine the compound interest rate per year that is equivalent to a simple interest rate of 15% per year.

*Solution:*

Set future values equal to each other

$$\begin{aligned}\text{Simple: } F &= P + Pni \\ &= P(1 + 5 \cdot 0.15) \\ &= 1.75P\end{aligned}$$

$$\begin{aligned}\text{Compound: } F &= P(1 + i)^n \\ &= P(1 + i)^5\end{aligned}$$

$$\begin{aligned}1.75P &= P(1 + i)^5 \\ i &= 11.84\%\end{aligned}$$



### **Solution 2.52**

A person's credit score is important in determining the interest rate they have to pay on a home mortgage. According to Consumer Credit Counseling Service, a homeowner with a \$100,000 mortgage and a 580 credit score will pay \$90,325 more in interest charges over the life of a 30-year loan than a homeowner with the same mortgage and a credit score of 720. How much higher would the interest rate per year have to be in order to account for this much difference in interest charges, if the \$100,000 loan is repaid in a single lump sum payment at the end of 30 years?

*Solution:*

$$100,000 = 190,325(P/F, i, 30)$$

$$(P/F, i, 30) = 0.52542$$

Find  $i$  by interpolation between 2% and 3%, or by solving P/F equation, or by Excel

By RATE function,  $i = 2.17\%$

### **Solution 2.53**

RKE & Associates is considering the purchase of a building it currently leases for \$30,000 per year. The owner of the building put it up for sale at a price of \$170,000, but because the firm has been a good tenant, the owner offered to sell it to RKE for a cash price of \$160,000 now. If purchased now, how long will it be before the company recovers its investment at an interest rate of 15% per year? Solve by spreadsheet function or factor.

*Solution:*

$$400,000 = 320,000 + 50,000(A/G, i, 5)$$

$$(A/G, i, 5) = 1.6000$$

Interpolate between  $i = 22\%$  and  $i = 24\%$

$$i = 22.6\%$$

### **Solution 2.54**

RKE & Associates is considering the purchase of a building it currently leases for \$30,000 per year. The owner of the building put it up for sale at a price of \$170,000, but because the firm has been a good tenant, the owner offered to sell it to RKE for a cash price of \$160,000 now. If purchased now, how long will it be before the company recovers its investment at an interest rate of 15% per year? Solve by spreadsheet function or factor.

*Solution:*

$$160,000 = 30,000(P/A, 15\%, n)$$

$$(P/A, 15\%, n) = 5.3333$$

From 15% table, n is between 11 and 12 years; therefore, n = 12 years

By NPER, n = 11.5 years

## Solution 2.55

A systems engineer who invested wisely can retire now because she has \$2,000,000 in her self-directed retirement account. Determine how many years she can withdraw (a) \$100,000 per year, or (b) \$150,000 per year (beginning 1 year from now) provided her account earns at a rate of 5% per year. (c) Explain why the increased annual withdrawal from \$100,000 to \$150,000 per year is important.

*Solution:*

$$(a) \quad 2,000,000 = 100,000(P/A, 5\%, n)$$
$$(P/A, 5\%, n) = 20.000$$

From 5% table,  $n$  is  $> 100$  years. In fact, at 5% per year, her account earns \$100,000 per year. Therefore, she will be able to withdraw \$100,000 forever; actually,  $n$  is  $\infty$ .

$$(b) \quad 2,000,000 = 150,000(P/A, 5\%, n)$$
$$(P/A, 5\%, n) = 13.333$$

By NPER,  $n = 22.5$  years

(c) The reduction is impressive from forever ( $n$  is infinity) to  $n = 22.5$  years for a 50% increase in annual withdrawal. It is important to know how much can be withdrawn annually when a fixed amount and a specific rate of return are involved.

### **Solution 2.56**

How many years will it take for a uniform annual deposit of size  $A$  to accumulate to 10 times the size of a single deposit at a rate of return of 10% per year?

*Solution:*

$$10A = A(F/A, 10\%, n)$$

$$(F/A, 10\%, n) = 10.000$$

From 10% factor table,  $n$  is between 7 and 8 years; therefore,  $n = 8$  years

### Solution 2.57

Demco Products, a company that manufactures stainless steel control valves, has a fund for equipment replacement that contains \$500,000. The company plans to spend \$85,000 each year on new equipment. (a) Estimate the number of years it will take to reduce the fund to no more than \$85,000 at an interest rate of 10% per year. (b) Use the NPER function to determine the exact number of years.

*Solution:*

$$\begin{aligned} \text{(a)} \quad 500,000 &= 85,000(P/A, 10\%, n) \\ (P/A, 10\%, n) &= 5.8824 \end{aligned}$$

From 10% table, n is between 9 and 10 years.

(b) Using the function = NPER(10%, -85000, 500000), the displayed n = 9.3 years.

### **Solution 2.58**

A company that manufactures ultrasonic wind sensors invested \$1.5 million 2 years ago to acquire part-ownership in an innovative chip-making company. How long will it take from the date of their initial investment for their share of the chip-making company to be worth \$6 million, if that company is growing at a rate of 25% per year?

*Solution:*

$$1,500,000 = 6,000,000(P/F, 25\%, n)$$
$$(P/F, 25\%, n) = 0.2500$$

From 25% table, n is between 6 and 7 years; therefore, n = 7 years

### **Solution 2.59**

A trusted friend told you that a cash flow sequence that started at \$3000 in year 1 and increased by \$2000 each year would be worth \$15,000 in 12 years at a rate of return of 10% per year. Is she correct?

*Solution:*

$$15,000 = 3000 + 2000(A/G,10\%,n)$$

$$(A/G,10\%,n) = 6.0000$$

From 10% table, n is between 17 and 18 years; therefore, n = 18 years. She is not correct; it takes longer.



### **Solution 2.60**

You are a well-paid engineer with a well-established international corporation. In planning for your retirement, you are optimistic and expect to make an investment of \$10,000 in year 1 and increase this amount by 10% each year. How long will it take for your account to have a future worth of \$2,000,000 at a rate of return of 7% per year?

*Solution:*

First set up equation to find present worth of \$2,000,000 and set that equal to P in the geometric gradient equation. Then, solve for n.

$$P = 2,000,000(P/F, 7\%, n)$$

$$2,000,000(P/F, 7\%, n) = 10,000 \{ 1 - [(1+0.10)/(1+0.07)]^n \} / (0.07 - 0.10)$$

Solve for n using Goal Seek or trial and error.

By trial and error, n = is between 25 and 26; therefore, n = 26 years

## Solution 2.61

*This is an introductory spreadsheet problem to allow you to become familiar with using Excel functions to solve single-factor problems. The development of correct functions is the primary goal of working it. The statement and question may be similar to or an extension of previous problems.*

A solar-powered personal aircraft with VTOL capability has been under development for the past 30 years by a group of engineers and physicists. SPPAV, as the plane will be termed, is expected to be available for its final test flight in exactly 3 years from now. Over the previous 30 years, a total of \$100 million has been spent in its development. Assuming the \$100 million was spent in an equal amount each year, and assuming an interest rate of 10%, determine the following:

- (a) The value of the total investment now, after the 30 years.
- (b) The value of the total investment at the expected time of the final test flight in three more years, assuming the same amount is spent for each of the next three years.
- (c) The value of the total investment at the expected time of the final test flight in three more years, assuming that *twice* the amount is spent for each of the *next 3 years* than that for the previous 30 years.

*Solution:*

Part	Function	Answer
a	= -FV(10%,30,100000000/30)	\$548,313,409
b	= -FV(10%,33,100000000/30)	\$740,838,481
c	= -FV(10%,33,100000000/30) + FV(10%,3,(100000000/30)*2)	\$718,771,814

## Solution 2.62

*This is an introductory spreadsheet problem to allow you to become familiar with using Excel functions to solve single-factor problems. The development of correct functions is the primary goal of working it. The statement and question may be similar to or an extension of previous problems.*

You expect to contribute to an investment fund for your retirement over the next 30 years with an annual deposit of a yet-to-be determined amount. Assume your goal is to have \$2 million available when you stop the annual deposits and that the fund is able to return 10% per year every year.

(a) Determine if you will reach your goal for either of the following two deposit scenarios:

(1) \$12,000 each and every year; (2) \$8000 at the end of next year for 15 years, followed by \$15,000 deposits in each of years 16 through 30.

(b) Determine the exact number of years necessary to accumulate the \$2 million if \$12,000 is deposited each year until the goal is achieved.

(c) For a little more of a challenge, use only the FV function to determine the number of years necessary to attain the \$2 million goal for the second deposit scenario, assuming the \$15,000 is deposited annually until the goal is achieved.

*Solution:*

	A	B	C	D	E	F
1	<b>Part</b>		<b>Function</b>	<b>Result</b>	<b>Conclusion</b>	
2	(a) \$12,000 for 30 years		= - FV(10%,30,12000)	\$1,973,928.27	Not quite reached	
3						
4	(a) \$8000 for 15; \$15,000 for 15 years		= - FV(10%,30,8000) - FV(10%,15,7000)	\$ 1,538,359.55	Not reached	
5						
6	(b) \$12,000 for n years		= NPER(10%,-12000,,2000000)	30.13	Years	
7						
8	(c) \$8000 for 15; \$15000 for 15 years					
9	One solution: Continue the deposits beyond year 30 and determine the future worth year by year.	<b>Year</b>	<b>Function</b>	<b>Accumulated</b>	<b>Conclusion</b>	
10		31	= -FV(10%,\$B10,8000) - FV(10%,\$B10-15,7000)	\$ 1,707,195.51		
11		32	= -FV(10%,\$B11,8000) - FV(10%,\$B11-15,7000)	\$ 1,892,915.06		
12		33	= -FV(10%,\$B12,8000) - FV(10%,\$B12-15,7000)	\$ 2,097,206.57	33 years	
13		34	= -FV(10%,\$B13,8000) - FV(10%,\$B13-15,7000)	\$ 2,321,927.22		
14		35	= -FV(10%,\$B14,8000) - FV(10%,\$B14-15,7000)	\$ 2,569,119.94		

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## Solution 2.63

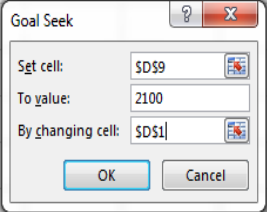
*This is an introductory spreadsheet problem to allow you to become familiar with using Excel functions to solve single-factor problems. The development of correct functions is the primary goal of working it. The statement and question may be similar to or an extension of previous problems.*

(This is a restatement of Problem 2.36.) Apple Computer wants to have \$2.1 billion available 5 years from now in order to finance initial production of a device that applies IOT technology for home use. The company expects to set aside uniformly increasing amounts of money each year to meet its goal, starting with \$100 million at the end of year 1. How much will the constant increase,  $G$ , have to be each year at a rate of return of 18% per year? Try your skill by using the Goal Seek tool to find the required gradient. Start the evaluation with  $G = \$50$  million per year.

*Solution:*

Goal Seek template before and result after with solution for  $G = \$115.69$  million

	A	B	C	D	E	F	G	H	I
1	Gradient amount is (\$1000)			\$ 50.00					
2									
3	Year	Deposit	PV in year 0	FV in year 5					
4	0								
5	1	100.00	\$84.75						
6	2	150.00	\$192.47						
7	3	200.00	\$314.20						
8	4	250.00	\$443.15						
9	5	300.00	\$574.28	\$1,313.81					
10									



The image shows an Excel spreadsheet with a Goal Seek dialog box overlaid. The spreadsheet has columns A through I and rows 1 through 10. Row 1, column D contains the value \$ 50.00. Row 3, column A is 'Year', column B is 'Deposit', column C is 'PV in year 0', and column D is 'FV in year 5'. Rows 4 through 9 contain data for years 0 through 5. The Goal Seek dialog box is open, showing 'Set cell:' as SDS9, 'To value:' as 2100, and 'By changing cell:' as SDS1. The dialog box has 'OK' and 'Cancel' buttons.

*Solution continued on the next page...*

	A	B	C	D	E
1	Gradient amount is (\$1000)			\$ 115.69	
2					
3	<b>Year</b>	<b>Deposit</b>	<b>PV in year 0</b>	<b>FV in year 5</b>	
4	0				
5	1	100.00	\$84.75		
6	2	215.69	\$239.65		
7	3	331.38	\$441.34		
8	4	447.08	\$671.94		
9	5	562.77	\$917.93	\$2,100.00	

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## **Solution 2.64**

*This is an introductory spreadsheet problem to allow you to become familiar with using Excel functions to solve single-factor problems. The development of correct functions is the primary goal of working it. The statement and question may be similar to or an extension of previous problems.*

For the data in Problem 2.60, find the year that your retirement account first exceeds (a) \$2 million, and (b) then \$3 million. In setting up the spreadsheet you wish to know the amount that must be deposited each year. Use any spreadsheet functions that you choose.

## **Problem 2.60**

You are a well-paid engineer with a well-established international corporation. In planning for your retirement, you are optimistic and expect to make an investment of \$10,000 in year 1 and increase this amount by 10% each year. How long will it take for your account to have a future worth of \$2,000,000 at a rate of return of 7% per year?

*Solution:*

Here is one approach to the solution using NPV and FV functions with results (left below) and formulas (right below).

*Solution continued on the next page...*

Year, n	Deposit	Present worth in year 0	Future worth in year n
0			
1	10,000	9,346	10,000
2	11,000	18,954	21,700
3	12,100	28,831	35,319
4	13,310	38,985	51,101
5	14,641	49,424	69,319
6	16,105	60,155	90,277
7	17,716	71,188	114,312
8	19,487	82,529	141,801
9	21,436	94,189	173,163
10	23,579	106,176	208,864
11	25,937	118,498	249,422
12	28,531	131,167	295,412
13	31,384	144,190	347,475
14	34,523	157,578	406,321
15	37,975	171,342	472,739
16	41,772	185,492	547,603
17	45,950	200,039	631,885
18	50,545	214,993	726,662
19	55,599	230,367	833,127
20	61,159	246,171	952,605
21	67,275	262,419	1,086,563
22	74,002	279,122	1,236,624
23	81,403	296,294	1,404,591
24	89,543	313,947	1,592,455
25	98,497	332,095	1,802,424
26	108,347	350,752	2,036,941
27	119,182	369,932	2,298,709
28	131,100	389,650	2,590,718
29	144,210	409,920	2,916,279
30	158,631	430,759	3,279,049

Year, n	Deposit	Present worth in year 0	Future worth in year n
0			
= \$A3+1	10000	=NPV(7%,B\$4:\$B4)	= -FV(7%,A4,,C4)
= \$A4+1	= \$B4*1.1	=NPV(7%,B\$4:\$B5)	= -FV(7%,A5,,C5)
= \$A5+1	= \$B5*1.1	=NPV(7%,B\$4:\$B6)	= -FV(7%,A6,,C6)
= \$A6+1	= \$B6*1.1	=NPV(7%,B\$4:\$B7)	= -FV(7%,A7,,C7)
= \$A7+1	= \$B7*1.1	=NPV(7%,B\$4:\$B8)	= -FV(7%,A8,,C8)
= \$A8+1	= \$B8*1.1	=NPV(7%,B\$4:\$B9)	= -FV(7%,A9,,C9)
= \$A9+1	= \$B9*1.1	=NPV(7%,B\$4:\$B10)	= -FV(7%,A10,,C10)
= \$A10+1	= \$B10*1.1	=NPV(7%,B\$4:\$B11)	= -FV(7%,A11,,C11)
= \$A11+1	= \$B11*1.1	=NPV(7%,B\$4:\$B12)	= -FV(7%,A12,,C12)
= \$A12+1	= \$B12*1.1	=NPV(7%,B\$4:\$B13)	= -FV(7%,A13,,C13)
= \$A13+1	= \$B13*1.1	=NPV(7%,B\$4:\$B14)	= -FV(7%,A14,,C14)
= \$A14+1	= \$B14*1.1	=NPV(7%,B\$4:\$B15)	= -FV(7%,A15,,C15)
= \$A15+1	= \$B15*1.1	=NPV(7%,B\$4:\$B16)	= -FV(7%,A16,,C16)
= \$A16+1	= \$B16*1.1	=NPV(7%,B\$4:\$B17)	= -FV(7%,A17,,C17)
= \$A17+1	= \$B17*1.1	=NPV(7%,B\$4:\$B18)	= -FV(7%,A18,,C18)
= \$A18+1	= \$B18*1.1	=NPV(7%,B\$4:\$B19)	= -FV(7%,A19,,C19)
= \$A19+1	= \$B19*1.1	=NPV(7%,B\$4:\$B20)	= -FV(7%,A20,,C20)
= \$A20+1	= \$B20*1.1	=NPV(7%,B\$4:\$B21)	= -FV(7%,A21,,C21)
= \$A21+1	= \$B21*1.1	=NPV(7%,B\$4:\$B22)	= -FV(7%,A22,,C22)
= \$A22+1	= \$B22*1.1	=NPV(7%,B\$4:\$B23)	= -FV(7%,A23,,C23)
= \$A23+1	= \$B23*1.1	=NPV(7%,B\$4:\$B24)	= -FV(7%,A24,,C24)
= \$A24+1	= \$B24*1.1	=NPV(7%,B\$4:\$B25)	= -FV(7%,A25,,C25)
= \$A25+1	= \$B25*1.1	=NPV(7%,B\$4:\$B26)	= -FV(7%,A26,,C26)
= \$A26+1	= \$B26*1.1	=NPV(7%,B\$4:\$B27)	= -FV(7%,A27,,C27)
= \$A27+1	= \$B27*1.1	=NPV(7%,B\$4:\$B28)	= -FV(7%,A28,,C28)
= \$A28+1	= \$B28*1.1	=NPV(7%,B\$4:\$B29)	= -FV(7%,A29,,C29)
= \$A29+1	= \$B29*1.1	=NPV(7%,B\$4:\$B30)	= -FV(7%,A30,,C30)
= \$A30+1	= \$B30*1.1	=NPV(7%,B\$4:\$B31)	= -FV(7%,A31,,C31)
= \$A31+1	= \$B31*1.1	=NPV(7%,B\$4:\$B32)	= -FV(7%,A32,,C32)
= \$A32+1	= \$B32*1.1	=NPV(7%,B\$4:\$B33)	= -FV(7%,A33,,C33)

Answers: (a) 26 years; (b) 30 years, only 4 years more than the \$2 million milestone.

## Solution 2.65

*This is an introductory spreadsheet problem to allow you to become familiar with using Excel functions to solve single-factor problems. The development of correct functions is the primary goal of working it. The statement and question may be similar to or an extension of previous problems.*

The SEWA (Southwestern Electricity and Water Authority) authorized construction projects totaling \$1.07 million to improve desalinization plant efficiency and salinity-reduction technology for reject chemicals. Three bids from potential vendors were received in the amounts of \$1.06, \$1.053, and \$1.045 million. Assume the savings will be realized immediately for each bid, were it accepted. Use an expected life of 10 years and  $i = 6\%$  per year to do the following *for the savings* anticipated from the lower-than authorized bids:

- Determine the equivalent present worth of the savings.
- Determine the equivalent annual worth of the savings.
- Develop a column chart for the equivalent annual worth for the savings for each bid.

*Solution:*

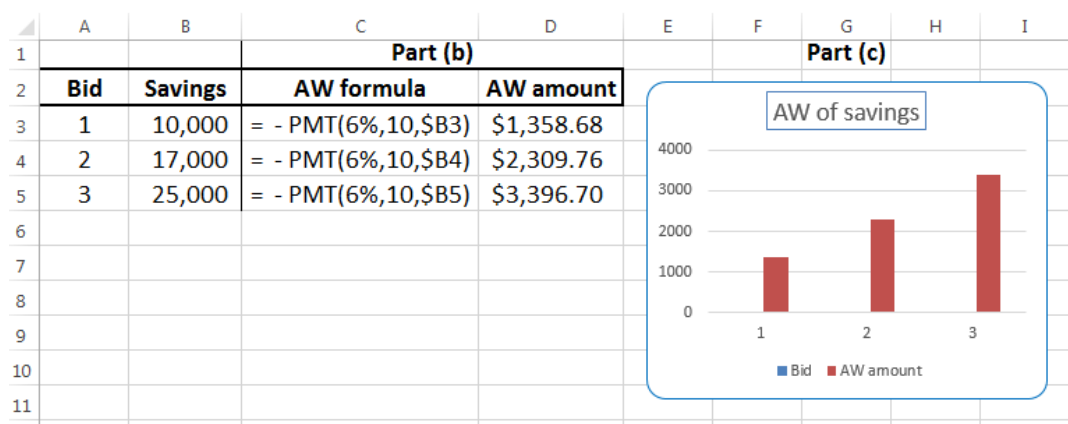
- Present worth is the value of the savings for each bid

Bid 1: Savings = \$10,000

Bid 2: Savings = \$17,000

Bid 3: Savings = \$25,000

- and (c) Spreadsheet for A values and column chart





### **Solution 2.66**

A manufacturer of prototyping equipment wants to have \$3,000,000 available 10 years from now so that a new product line can be initiated. If the company plans to deposit money each year, starting 1 year from now, the equation that represents how much the company is required to deposit each year at 10% per year interest to have the \$3,000,000 immediately after the last deposit is:

- (a)  $3,000,000(A/F, 10\%, 10)$
- (b)  $3,000,000(A/F, 10\%, 11)$
- (c)  $3,000,000 + 3,000,000(A/F, 10\%, 10)$
- (d)  $3,000,000(A/P, 10\%, 10)$

*Solution:*

Answer is (a)

**Solution 2.67**

The amount of money the Teachers Credit Union should be willing to loan a developer who will repay the loan in a lump sum amount of \$840,000 two years from now at the bank's interest rate of 10% per year is:

- (a) \$694,180
- (b) \$99,170
- (c) \$1,106,400
- (d) \$763,650

*Solution:*

$$\begin{aligned} P &= 840,000(P/F, 10\%, 2) \\ &= 840,000(0.8264) \\ &= \$694,176 \end{aligned}$$

Answer is (a)

### Solution 2.68

The cost of updating an outdated production process is expected to be \$81,000 four years from now. The equivalent present worth of the update at 6% per year interest is closest to:

- (a) \$51,230
- (b) \$55,160
- (c) \$60,320
- (d) \$64,160

*Solution:*

$$\begin{aligned} P &= 81,000(P/F, 6\%, 4) \\ &= 81,000(0.7921) \\ &= \$64,160 \end{aligned}$$

Answer is (d)

**Solution 2.69**

A single deposit of \$25,000 was made by your grandparents on the day you were born 25 years ago. The balance in the account today if it grew at 10% per year is closest to:

(a) \$201,667 (b) \$241,224

(c) \$270,870 (d) \$296,454

*Solution:*

$$F = 25,000(F/P, 10\%, 25)$$

$$= 25,000(10.8347)$$

$$= \$270,868$$

Answer is (c)

**Solution 2.70**

A chip manufacturing company wants to have \$10 million available 5 years from now in order to build new warehouse and shipping facilities. If the company can invest money at 10% per year, the amount that it must deposit *each year* in years 1 through 5 to accumulate the \$10 million is closest to:

- (a) \$1,638,000
- (b) \$2,000,000
- (c) \$2,638,000
- (d) \$2,938,000

*Solution:*

$$\begin{aligned}A &= 10,000,000(A/F, 10\%, 5) \\ &= 10,000,000(0.16380) \\ &= \$1,638,000\end{aligned}$$

Answer is (a)

### Solution 2.71

An engineer who believes in “save now; play later” wants to retire in 30 years with \$2.0 million. At 8% per year interest, the amount the engineer will have to invest each year (starting in year 1) to reach the \$2 million goal is closest to:

- (a) \$17,660
- (b) \$28,190
- (c) \$49,350
- (d) \$89,680

*Solution:*

$$\begin{aligned} A &= 2,000,000(A/F, 8\%, 30) \\ &= 2,000,000(0.00883) \\ &= \$17,660 \end{aligned}$$

Answer is (a)

**Solution 2.72**

The cost of tuition at a large public university was \$390 per credit hour 5 years ago. The cost today (exactly 5 years later) is \$585. The annual rate of increase is closest to:

- (a) 5%
- (b) 7%
- (c) 9%
- (d) 11%

*Solution:*

$$390 = 585(P/F, i, 5)$$

$$(P/F, i, 5) = 0.6667$$

From tables,  $i$  is between 8% and 9%

Answer is (c)

**Solution 2.73**

The Gap has some of its jeans stone-washed under a contract with Vietnam Garment Corporation (VGC). If VGC's estimated operating cost per machine is \$26,000 for year 1 and it increases by \$1500 per year through year 5, the equivalent uniform annual cost per machine over years 1 to 5, at an interest rate of 8% per year, is closest to:

- (a) \$30,850
- (b) \$28,770
- (c) \$26,930
- (d) \$23,670

*Solution:*

$$\begin{aligned} AW &= 26,000 + 1500(A/G, 8\%, 5) \\ &= \$28,770 \end{aligned}$$

Answer is (b)



**Solution 2.74**

Adams Manufacturing spent \$30,000 on a new sterilization conveyor belt, which resulted in a cost savings of \$4202 per year. The length of time it should take to recover the investment at 8% per year is closest to:

- (a) Less than 6 years
- (b) 7 years
- (c) 9 years
- (d) 11 years

*Solution:*

$$30,000 = 4202(P/A, 8\%, n)$$

$$(P/A, 8\%, 5) = 7.1395$$

$$n = 11 \text{ years}$$

Answer is (d)

**Solution 2.75**

The present worth of an increasing geometric gradient is \$23,632. The interest rate is 6% per year and the rate of change series is 4% per year. If the cash flow in year 1 is \$3,000, the year in which the gradient ends is:

- (a) 7
- (b) 9
- (c) 11
- (d) 12

*Solution:*

$$23,632 = 3000\{1 - [(1+0.04)^n/(1+0.06)^n]\}/(0.06-0.04)$$

$$[(23,632*0.02)/3000]-1 = (0.98113)^n$$

$$\log 0.84245 = n \log 0.98113$$

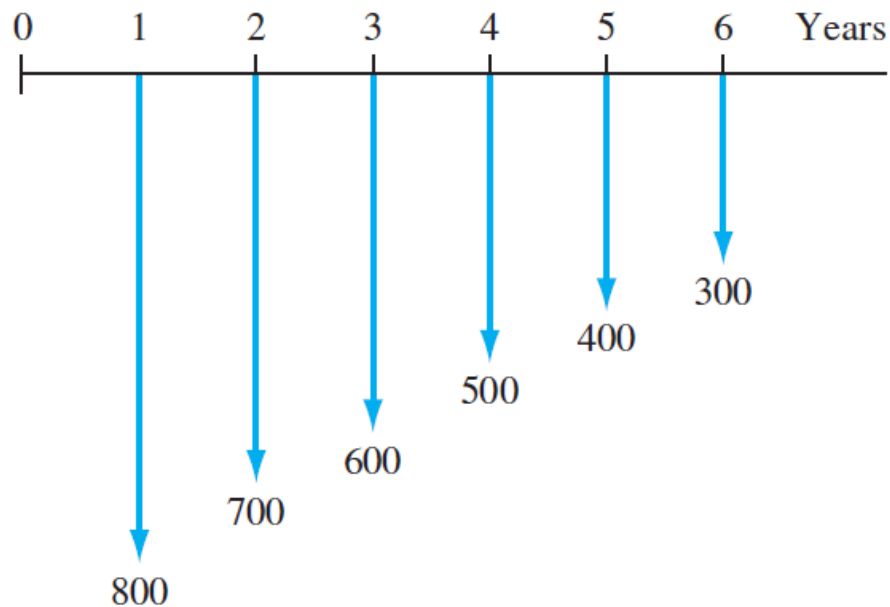
$$n = 9$$

Answer is (b)

**Solution 2.76**

At  $i = 8\%$  per year, the annual worth for years 1 through 6 of the cash flows shown is closest to:

- (a) \$302 (b) \$421  
(c) \$572 (d) \$824



*Solution:*

$$\begin{aligned} A &= 800 - 100(A/G, 8\%, 6) \\ &= 800 - 100(2.2763) \\ &= \$572.37 \end{aligned}$$

Answer is (c)

**Solution 2.77**

Chemical costs associated with a packed-bed flue gas incinerator for odor control have been decreasing uniformly for 5 years because of increases in efficiency. If the cost in year 1 was \$100,000 and it decreased by \$5,000 per year through year 5, the present worth of the costs at 10% per year is closest to:

- (a) \$344,771
- (b) \$402,200
- (c) \$515,400
- (d) \$590,700

*Solution:*

$$\begin{aligned} P &= 100,000(P/A, 10\%, 5) - 5000(P/G, 10\%, 5) \\ &= 100,000(3.7908) - 5000(6.8618) \\ &= \$344,771 \end{aligned}$$

Answer is (a)

**Solution 2.78**

The winner of a multistate mega millions lottery jackpot worth \$175 million was given the option of taking payments of \$7 million per year for 25 years, beginning 1 year from now, or taking \$109.355 million now. The interest rate that renders the two options equivalent to each other is closest to:

- (a) 4%
- (b) 5%
- (c) 6%
- (d) 7%

*Solution:*

$$109.355 = 7(P/A, i, 25)$$

$$(P/A, i, 25) = 15.6221$$

From tables,  $i = 4\%$

Answer is (a)

**Solution 2.79**

Maintenance costs for a regenerative thermal oxidizer increased according to an arithmetic gradient for 5 years. The cost in year 1 was \$7000. If the interest rate is 10% per year and the present worth of the costs for a 5-year period was \$28,800, the amount of the yearly increase,  $G$ , was closest to:

- (a) \$1670
- (b) \$945
- (c) \$620
- (d) \$330

*Solution:*

$$28,800 = 7000(P/A, 10\%, 5) + G(P/G, 10\%, 5)$$

$$28,800 = 7000(3.7908) + G(6.8618)$$

$$G = \$330$$

Answer is (d)

**Solution 2.80**

Aero Serve, Inc. manufactures cleaning nozzles for reverse pulse jet dust collectors. The company spent \$40,000 on a production control system that will increase profits by \$11,096 per year for 5 years. The rate of return per year on the investment is closest to:

- (a) 20%
- (b) 16%
- (c) 12%
- (d) Less than 11%

*Solution:*

$$40,000 = 11,096(P/A, i, 5)$$

$$(P/A, i, 5) = 3.6049$$

$$i = 12 \%$$

Answer is (c)