CHAPTER 2
LINEAR PROGRAMMING: BASIC CONCEPTS
SOLUTION TO SOLVED PROBLEMS

2.S1 Back Savers Production Problem

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. *Formulate and solve a linear programming model for this problem on a spreadsheet.*

To build a spreadsheet model for this problem, start by entering the data. The data for this problem are the unit profit of each type of backpack, the resource requirements (square feet of nylon and labor hours required), the availability of each resource, 5400 square feet of nylon and (35 laborers)(40 hours/laborer) = 1400 labor hours, and the sales forecast for each type of backpack (1000 Collegiates and 1200 Minis). In order to keep the units consistent in row 8 (hours), the labor required for each backpack (in cells C8 and D8) are converted from minutes to hours (0.75 hours = 45 minutes, 0.667 hours = 40 minutes). The range names UnitProfit (C4:D4), Available (G7:G8), and SalesForecast (C13:D13) are added for these data.

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5						
6		Resource Used per Unit Produced				Available
7	Nylon (sq. ft.)	3	2			5400
8	Labor (hours)	0.75	0.667			1400
9						
10						
11						
12						
13	Sales Forecast	1000	1200			

The decision to be made in this problem is how many of each type of backpack to make. Therefore, we add two changing cells with range name UnitsProduced (C11:D11). The values in CallsPlaced will eventually be determined by the Solver. For now, arbitrary values of 10 and 10 are entered.

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5						
6		Resource Used per Unit Produced				Available
7	Nylon (sq. ft.)	3	2			5400
8	Labor (hours)	0.75	0.667			1400
9						
10						
11	Units Produced	10	10			
12						
13	Sales Forecast	1000	1200			

The goal is to produce backpacks so as to achieve the highest total profit. Thus, the objective cell should calculate the total profit, where the objective will be to maximize this objective cell. In this case, the total profit will be

Total Profit = ($32)(# of Collegiates) + ($24)(# of Minis)

or

Total Cost = SUMPRODUCT(UnitProfit, UnitsProduced).

This formula is entered into cell G11 and given a range name of TotalProfit. With 10 Collegiates and 10 Minis produced, the total profit would be ($32)(10) + ($24)(10) = $560.

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5						
6		Resource Used per Unit Produced				Available
7	Nylon (sq. ft.)	3	2			5400
8	Labor (hours)	0.75	0.667			1400
9						
10						Total Profit
11	Units Produced	10	10			$560
12						
13	Sales Forecast	1000	1200			

	G
10	Total Profit
11	=SUMPRODUCT(UnitProfit,UnitsProduced)

The first set of constraints in this problem involve the limited available resources (nylon and labor hours). Given the number of units produced (UnitsProduced in C11:D11), we calculate the total resources required. For nylon, this will be =SUMPRODUCT(C7:D7, UnitsProduced) in cell E7. By using a range name or an absolute reference for the units produced, this formula can be copied into cell E8 to calculate the labor hours required. The total resources used (TotalResources in E7:E8) must be <= Available (in cells G7:G8), as indicated by the <= in F7:F8.

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5				Total		
6		Resource Used per Unit Produced		Required		Available
7	Nylon (sq. ft.)	3	2	50	<=	5400
8	Labor (hours)	0.75	0.667	14.166667	<=	1400
9						
10						Total Profit
11	Units Produced	10	10			$560

	E
5	Total
6	Required
7	=SUMPRODUCT(C7:D7,UnitsProduced)
8	=SUMPRODUCT(C8:D8,UnitsProduced)

The final constraint is that it does not make sense to produce more backpacks than can be sold (as predicted by the sales forecast). Therefore UnitsProduced (C11:D11) should be less-than-or-equal-to the SalesForecast (C13:D13), as indicated by the <= in C12:D12

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5				Total		
6		Resource Used per Unit Produced		Required		Available
7	Nylon (sq. ft.)	3	2	50	<=	5400
8	Labor (hours)	0.75	0.667	14.16666667	<=	1400
9						
10						Total Profit
11	Units Produced	10	10			$560
12		<=	<=			
13	Sales Forecast	1000	1200			

The Solver information and solved spreadsheet are shown below.

	B	C	D	E	F	G
3		Collegiate	Mini			
4	Unit Profit	$32	$24			
5				Total		
6		Resource Used per Unit Produced		Required		Available
7	Nylon (sq. ft.)	3	2	4950	<=	5400
8	Labor (hours)	0.75	0.667	1400	<=	1400
9						
10						Total Profit
11	Units Produced	1000	975			$55,400
12		<=	<=			
13	Sales Forecast	1000	1200			

Solver Parameters

Set Objective Cell: TotalProfit
To: Max
By Changing Variable Cells:
UnitsProduced
Subject to the Constraints:
TotalRequired <= Available
UnitsProduced <= SalesForecast

Solver Options:
Make Variables Nonnegative
Solving Method: Simplex LP

Range Name	Cells
Available	G7:G8
SalesForecast	C13:D13
TotalProfit	G11
TotalRequired	E7:E8
UnitProfit	C4:D4
UnitsProduced	C11:D11

	E
5	Total
6	Required
7	=SUMPRODUCT(C7:D7,UnitsProduced)
8	=SUMPRODUCT(C8:D8,UnitsProduced)

	G
10	Total Profit
11	=SUMPRODUCT(UnitProfit,UnitsProduced)

Thus, they should produce 1000 Collegiates and 975 Minis to achieve the maximum total profit of $55,400.

b. *Formulate this same model algebraically.*

To build an algebraic model for this problem, start by defining the decision variables. In this case, the two decisions are how many Collegiates to produce and how many Minis to produce. These variables are defined below:

Let C = Number of Collegiates to produce,
M = Number of Minis to produce.

Next determine the goal of the problem. In this case, the goal is to produce the number of each type of backpack to achieve the highest possible total profit. Each Collegiate yields a unit profit of $32 while each Mini yields a unit profit of $24. The objective function is therefore

Maximize Total Profit = $\$32C + \$24M$.

The first set of constraints in this problem involve the limited resources (nylon and labor hours). Given the number of backpacks produced, C and M, and the required nylon and labor hours for each, the total resources used can be calculated. These total resources used need to be less than or equal to the amount available. Since the labor available is in units of hours, the labor required for each backpack needs to be in units of hours (3/4 hour and 2/3 hour) rather than minutes (45 minutes and 40 minutes). These constraints are as follows:

Nylon: $3C + 2M \leq 5400$ square feet,
Labor Hours: $(3/4)C + (2/3)M \leq 1400$ hours.

The final constraint is that they should not produce more of each backpack than the sales forecast. Therefore,

Sales Forecast: $C \leq 1000$
$M \leq 1200$.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let C = Number of Collegiates to produce,
M = Number of Minis to produce.

Maximize Total Profit = $\$32C + \$24M$,

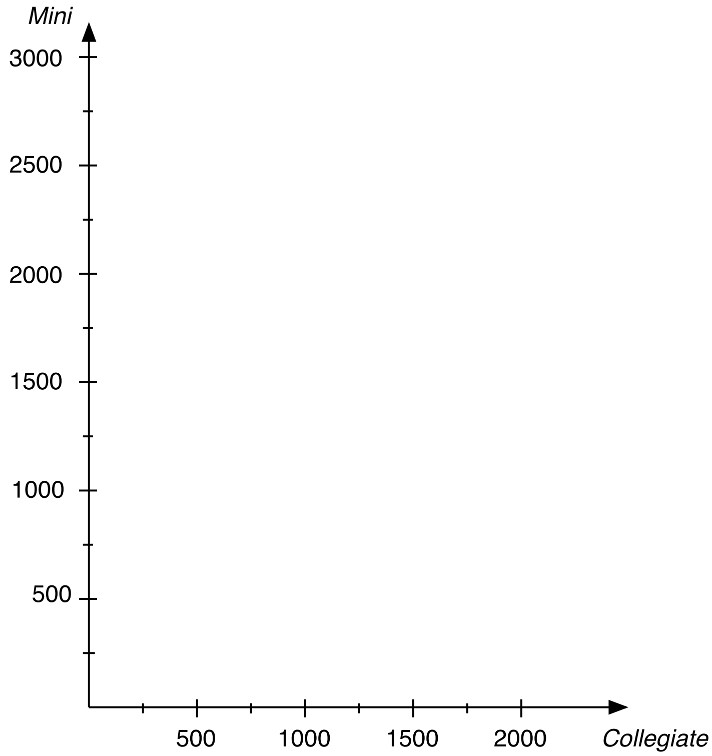
subject to

Nylon: $3C + 2M \leq 5400$ square feet,
Labor Hours: $(3/4)C + (2/3)M \leq 1400$ hours,
Sales Forecast: $C \leq 1000$
$M \leq 1200$.

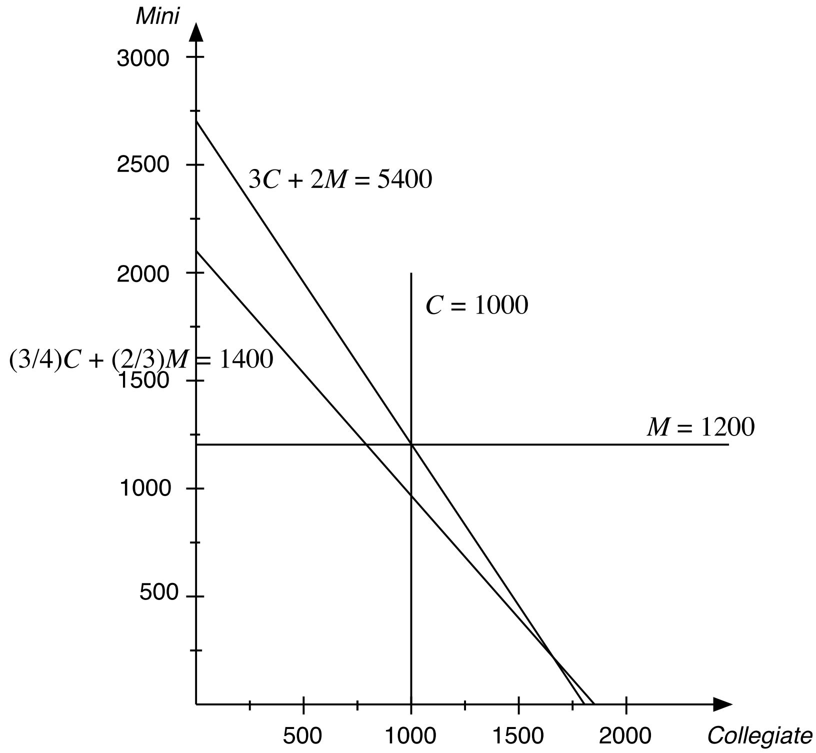
and $C \geq 0$, $M \geq 0$.

c. Use the graphical method by hand to solve this model.

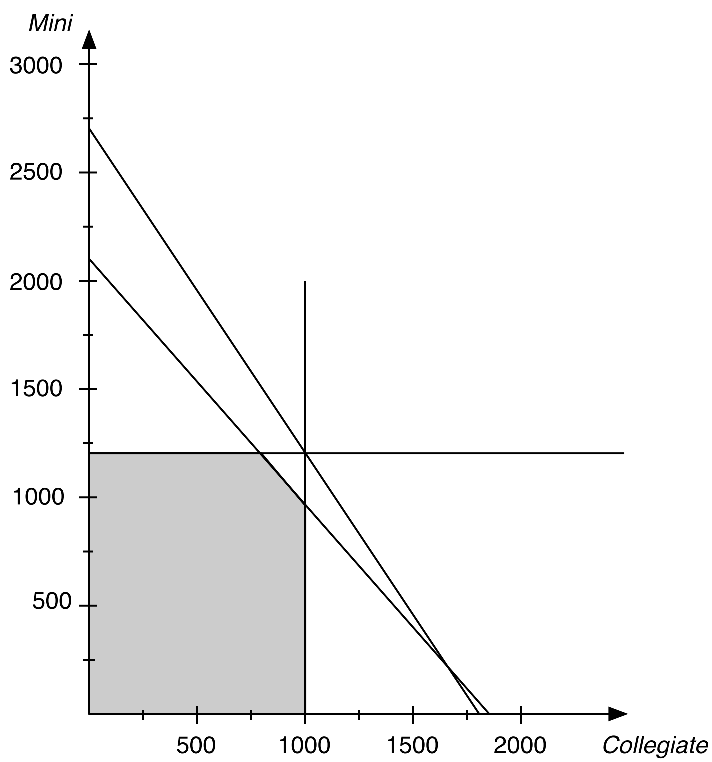
Start by plotting a graph with Collegiates (C) on the horizontal axis and Minis (M) on the vertical axis, as shown below.



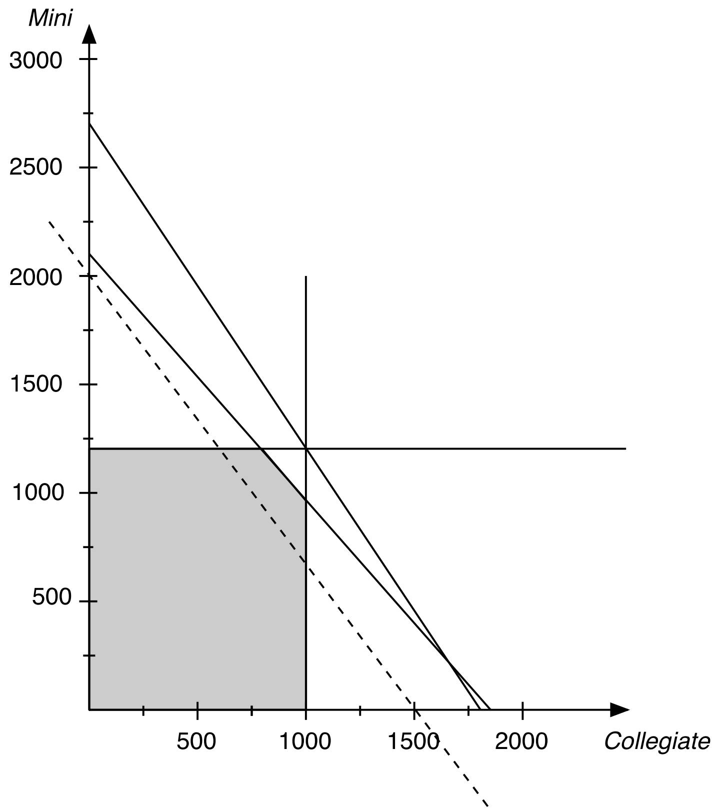
Next, the four constraint boundary lines (where the left-hand-side of the constraint exactly equals the right-hand-side) need to be plotted. The easiest way to do this is by determining where these lines intercepts the two axes. For the Nylon constraint boundary line ($3C + 2M = 5400$), setting $M = 0$ yields a C-intercept of 1800 while setting $C = 0$ yields an M-intercept of 2700. For the Labor constraint boundary line ($(3/4)C + (2/3)M = 1400$), setting $M = 0$ yields a C-intercept of 1866.67 while setting $C = 0$ yields an M-intercept of 2100. The sales forecast constraints are a horizontal line at $M = 1200$ and a vertical line at $C = 1000$. These constraint boundary lines are plotted below.



A feasible solution must be below and/or to the left of all four of these constraints while being above the Collegiate axis (since $C \geq 0$) and to the right of the Mini axis (since $M \geq 0$). This yields the feasible region shown below.

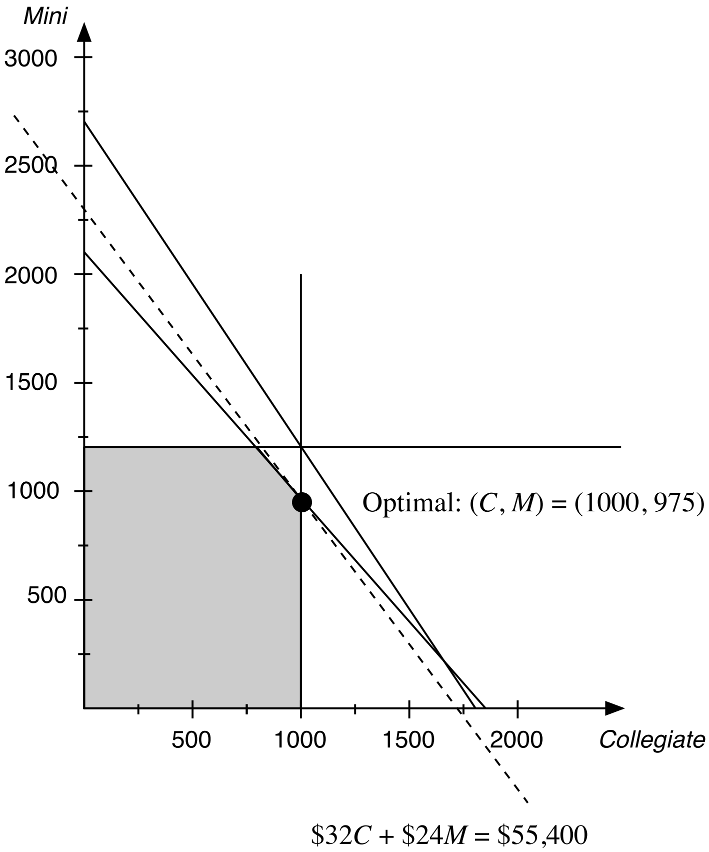


To find the optimal solution, an objective function line is plotted by setting the objective function equal to a value. For example, the objective function line when the value of the objective function is $48,000 is plotted as a dashed line below.



$\$32C + \$24M = \$48{,}000$

All objective function lines will be parallel to this one. To find the feasible solution that maximizes profit, slide this line out as far as possible while still touching the feasible region. This occurs when the profit is $55,400, and the objective function line intersect the feasible region at the single point with (C, M) = (1000, 975) as shown below.



Therefore, the optimal solution is to produce 1000 Collegiates and 975 Minis, yielding a total profit of $55,400.

2.S2 Conducting a Marketing Survey

The marketing group for a cell phone manufacturer plans to conduct a telephone survey to determine consumer attitudes toward a new cell phone that is currently under development. In order to have a sufficient sample size to conduct the analysis, they need to contact at least 100 young males (under age 40), 150 older males (over age 40), 120 young females (under age 40), and 200 older females (over age 40). It costs $1 to make a daytime phone call and $1.50 to make an evening phone call (due to higher labor costs). This cost is incurred whether or not anyone answers the phone. The table below shows the likelihood of a given customer type answering each phone call. Assume the survey is conducted with whoever first answers the phone. Also, because of limited evening staffing, at most one-third of phone calls placed can be evening phone calls. How should the marketing group conduct the telephone survey so as to meet the sample size requirements at the lowest possible cost?

Who Answers?	*Daytime Calls*	*Evening Calls*
Young Male	*10%*	*20%*
Older Male	*15%*	*30%*
Young Female	*20%*	*20%*
Older Female	*35%*	*25%*
No Answer	*20%*	*5%*

a. *Formulate and solve a linear programming model for this problem on a spreadsheet.*

To build a spreadsheet model for this problem, start by entering the data. The data for this problem are the cost of each type of phone call, the percentages of each customer type answering each type of phone call, and the total number of each customer type needed for the survey.

	B	C	D	E	F	G
3		Daytime Call	Evening Call			
4	Unit Cost	$1.00	$1.50			
5						Responses
6	**Respondent**					Needed
7	Young Male	10%	20%			100
8	Older Male	15%	30%			150
9	Young Female	20%	20%			120
10	Older Female	35%	25%			200

The decision to be made in this problem is how many of each type of phone call to make. Therefore, we add two changing cells with range name CallsPlaced (C13:D13). The values in CallsPlaced will eventually be determined by the Solver. For now, arbitrary values of 10 and 5 are entered.

	B	C	D	E	F	G
3		Daytime Call	Evening Call			
4	Unit Cost	$1.00	$1.50			
5						Responses
6	**Respondent**					Needed
7	Young Male	10%	20%			100
8	Older Male	15%	30%			150
9	Young Female	20%	20%			120
10	Older Female	35%	25%			200
11						
12						
13	Calls Placed	10	5			

The goal of the marketing group is to conduct the survey at the lowest possible cost. Thus, the objective cell should calculate the total cost, where the objective will be to minimize this objective cell. In this case, the total cost will be

Total Cost = ($1)(# of daytime calls) + ($1.50)(# of evening calls)

or

Total Cost = SUMPRODUCT(UnitCost, CallsPlaced).

This formula is entered into cell G13 and given a range name of TotalCost. With 10 daytime phone calls and 5 evening calls, the total cost would be ($1)(10) + ($1.50)(5) = $17.50.

	B	C	D	E	F	G
3		Daytime Call	Evening Call			
4	Unit Cost	$1.00	$1.50			
5						Responses
6	**Respondent**					Needed
7	Young Male	10%	20%			100
8	Older Male	15%	30%			150
9	Young Female	20%	20%			120
10	Older Female	35%	25%			200
11						
12						Total Cost
13	Calls Placed	10	5			$17.50

	G
12	Total Cost
13	=SUMPRODUCT(UnitCost,CallsPlaced)

The first set of constraints in this problem involve the minimum responses required from each customer group. Given the number of calls placed (CallsPlaced in C13:D13), we calculate the total responses by each customer type. For young males, this will be =SUMPRODUCT(C7:D7, CallsPlaced). By using a range name or an absolute reference for the calls placed, this formula can be copied into cells E8-E10 to calculate the number of older males, young females, and older females reached. The total responses of each customer type (Total Responses in E7:E10) must be >= ResponsesNeeded (in cells G7:G10), as indicated by the >= in F7:F10.

	B	C	D	E	F	G
3		Daytime Call	Evening Call			
4	Unit Cost	$1.00	$1.50			
5				Total		Responses
6	**Respondent**			Responses		Needed
7	Young Male	10%	20%	2	>=	100
8	Older Male	15%	30%	3	>=	150
9	Young Female	20%	20%	3	>=	120
10	Older Female	35%	25%	4.75	>=	200
11						
12						Total Cost
13	Calls Placed	10	5			$17.50

	E
5	Total
6	Responses
7	=SUMPRODUCT(C7:D7,CallsPlaced)
8	=SUMPRODUCT(C8:D8,CallsPlaced)
9	=SUMPRODUCT(C9:D9,CallsPlaced)
10	=SUMPRODUCT(C10:D10,CallsPlaced)

The final constraint is that at most one third of the total calls placed can be evening calls. In other words:

Evening Calls <= (1/3)(Total Calls Placed)

The two sides of this constraint (i.e., evening calls and 1/3 of total calls placed) are calculated in cells C15 and E15. Enter <= in D15 to show that C15 <= E15.

	B	C	D	E	F	G
3		Daytime Call	Evening Call			
4	Unit Cost	$1.00	$1.50			
5				Total		Responses
6	**Respondent**			Responses		Needed
7	Young Male	10%	20%	2	>=	100
8	Older Male	15%	30%	3	>=	150
9	Young Female	20%	20%	3	>=	120
10	Older Female	35%	25%	4.75	>=	200
11						
12						Total Cost
13	Calls Placed	10	5			$17.50
14						
15	Evening Calls	5	<=	5	33.33%	of Total Calls

	B	C	D	E	F	G
15	Evening Calls	=D13	<=	=F15*(C13+D13)	0.333333	of Total Calls

The Solver information and solved spreadsheet are shown below.

	A	B	C	D	E	F	G
1	**Conducting a Marketing Survey**						
2							
3			Daytime Call	Evening Call			
4		Unit Cost	$1.00	$1.50			
5					Total		Responses
6		**Respondent**			Responses		Needed
7		Young Male	10%	20%	100	>=	100
8		Older Male	15%	30%	150	>=	150
9		Young Female	20%	20%	150	>=	120
10		Older Female	35%	25%	237.5	>=	200
11							
12							Total Cost
13		Calls Placed	500	250			$875.00
14							
15		Evening Calls	250	<=	250	33.33%	of Total Calls

Solver Parameters
Set Objective Cell: TotalCost
To: Min
By Changing Variable Cells:
CallsPlaced
Subject to the Constraints:
EveningCalls <= E15
TotalResponses >= ResponsesNeeded
Solver Options:
Make Variables Nonnegative
Solving Method: Simplex LP

Range Name	Cells
CallsPlaced	C13:D13
EveningCalls	C15
ResponsesNeeded	G7:G10
TotalCost	G13
TotalResponses	E7:E10
UnitCost	C4:D4

	E
5	Total
6	Responses
7	=SUMPRODUCT(C7:D7,CallsPlaced)
8	=SUMPRODUCT(C8:D8,CallsPlaced)
9	=SUMPRODUCT(C9:D9,CallsPlaced)
10	=SUMPRODUCT(C10:D10,CallsPlaced)

	G
12	Total Cost
13	=SUMPRODUCT(UnitCost,CallsPlaced)

	B	C	D	E	F	G
15	Evening Calls	=D13	<=	=F15*(C13+D13)	0.333333	of Total Calls

Thus, the marketing group should place 500 daytime calls and 250 evening calls at a total cost of $875.

b. Formulate this same model algebraically.

To build an algebraic model for this problem, start by defining the decision variables. In this case, the two decisions are how many daytime calls and how many evening calls to place. These variables are defined below:

Let D = Number of daytime calls to place
E = Number of evening calls to place.

Next determine the goal of the problem. In this case, the goal is to conduct the marketing survey at the lowest possible cost. Each daytime call costs \$1 while each evening call costs \$1.50. The objective function is therefore

Minimize Total Cost = $\$1D + \$1.50E$.

The first set of constraints in this problem involve the minimum responses required from each customer group. Given the number of calls place, D and E, and the percentage of calls answered by each customer group, the total responses for each customer group is calculated. These total responses need to be greater than or equal to the minimum responses required. These constraints are as follows:

Young Males: $(10\%)D + (20\%)E \geq 100$
Older Males: $(15\%)D + (30\%)E \geq 150$
Young Females: $(20\%)D + (20\%)E \geq 120$
Older Females: $(35\%)D + (25\%)E \geq 200$.

The final constraint is that at most one third of the total calls placed can be evening calls. In other words:

Evening Calls <= (1/3)(Total Calls Placed)

Substituting E for Evening Calls, and $D + E$ for Total Calls Placed yields the following constraint:

$E \leq (1/3)(D + E)$.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let D = Number of daytime calls to place
E = Number of evening calls to place.

Minimize Total Cost = $\$1D + \$1.50E$.

subject to

Young Males: $(10\%)D + (20\%)E \geq 100$
Older Males: $(15\%)D + (30\%)E \geq 150$
Young Females: $(20\%)D + (20\%)E \geq 120$
Older Females: $(35\%)D + (25\%)E \geq 200$
Evening Call Ratio: $E \leq (1/3)(D + E)$

and $D \geq 0$, $E \geq 0$.