Chapter 2 How to Calculate Present Values

OVERVIEW

This chapter introduces the concept of present value and shows why a firm should maximize the market value of the stockholders' stake in it. It describes the mechanics of calculating present values of lump sum amounts, perpetuities, annuities, growing perpetuities, growing annuities, and unequal cash flows. Other related topics like simple interest, frequent compounding, continuous compounding, and nominal and effective interest rates are discussed. The net present value rule and the rate of return rule are explained in great detail.

LEARNING OBJECTIVES

- To learn how to calculate present value of lump sum cash flows.
- To understand and use the formulas associated with the present value of perpetuities, growth perpetuities, annuities, and growing annuities.
- To understand more frequent compounding, including continuous compounding.
- To understand the important difference between nominal and effective interest rates.
- To understand the net present value rule and the rate of return rule.

CHAPTER OUTLINE

Future values and present values

The concepts of future value, present value, net present value (NPV), and the opportunity cost of capital (hurdle rate) are introduced. The authors show, using several numerical examples, that simple projects with rates of return exceeding the opportunity cost of capital have positive net present values. The "net present value rule" and the "rate of return rule" are stated here.

This chapter also extends the concept of discounting to assets, which produce a series of cash flows. Using numerical examples, it shows how to calculate the PV and NPV of a series of cash flows over a number of periods (years).

Looking for shortcuts—perpetuities and annuities

This section is devoted to developing formulas for perpetuities and annuities. It explains the difference between an ordinary annuity and an annuity due. It also explains how the future value of an annuity is calculated. The present value of an annuity can be thought of as the difference between two perpetuities beginning at different times. Using this simple idea, the formula for the present value of an annuity is derived. The future value of an annuity formula is also derived. These have numerous applications in pension funds, mortgages, and valuation of financial assets.

More shortcuts—growing perpetuities and annuities

Some applications need the present value of a perpetual cash flow growing at a constant rate, as well as annuities that grow at a constant rate. The formula for the present value of a growing perpetuity is derived. The present value of a growing annuity can be thought of as the difference between two growing perpetuities starting at different times. Using this simple idea, the formula for the present value of a growing annuity is also derived. These formulas have many applications in the valuation of assets.

How interest is paid and quoted

This section explains the differences between compound interest and simple interest, as well as the differences between effective annual rates and annual percentage rates. It deals with how each interest rate is used in the marketplace and the math necessary to move between the two kinds of interest rates.

TEACHING TIPS FOR POWERPOINT SLIDES

Slide 1 – Title slide

Slide 2 – Topics covered

Slide 3 – Present Value and Future Value

Explain the terms "future value" and "present value." The concept must be emphasized at this point. Consequently, it may be necessary to spend some time explaining real-world examples of how present value and future value relate. A good example to use is retirement planning.

Slide 4 – Future Values

$FV = PV \times (1 + r)^t$

Define the terms:
FV = Future value
PV = Present value
r = interest rate
t = number of years (periods)

Explain the time value of money and its importance to financial decision making.

Slide 5 – Future Values Continued

Walk through each step in the math process and show how the value increases. If you plan to have your students use a financial calculator, you can skip the details of the basic math. Be aware that students often stumble when doing simple math calculations.

Slide 6 – Figure 2.1 Future Values with Compounding

The longer the funds are invested, the greater the advantage with compound interest. Discuss the four examples and be sure to use the phrase "power of compounding."

Slide 7 – Present Value

This slide contains the present value formula.

Slide 8 – Present Value Continued

The discount factor (DF) is the present value of $1 expected to be received in the future. Here it is appropriate to introduce the use of the financial calculator to solve these problems.

Slide 9 – Present Value Concluded

Here we reverse the future value process from earlier. Show students how they can easily move between future value and present value with the basic formulas.

Slide 10 – Figure 2.2 Present Values with Compounding

For visual learners, this graph illustrates the reverse of the future value compounding chart shown earlier. It is downward sloping, which can confuse students, so it may be necessary to spend time explaining the concept.

Slide 11 – Valuing an Investment Opportunity

Explain how the present value concept discussed earlier is useful in valuing assets.

Cost of the building = $700,000
Sale price in Year 1 = $800,000
Opportunity cost of capital = 7%

Slide 12 – Valuing an Investment Opportunity Continued

Discount future cash flows at the opportunity cost of capital

PV = 800,000/(1.07) = 747,664

NPV = PV – required investment
NPV = 747,664 – 700,000 = 47,664

Explain the difference between PV and NPV. Explain sign conventions for cash flows.

Slide 13 – Net Present Value

Explain each variable in the equation. It is easy to tell the students that all present values come at a cost. That cost is the initial investment. This may help them easily transition from present value to net present value.

C_0 = initial investment for the project. Normally it is a cash outflow and has a negative sign (–)
C_1 = cash inflow from the project. Normally it has a positive sign (+)
r = opportunity cost of capital

Positive NPVs increase the value of a firm. Negative NPVs lower the value of a firm.

Slide 14 – Figure 2.4 NPV Calculation

This figure illustrates the calculation showing the NPV of the office development example.

Slides 15 and 16 – Risk and Present Value and Risk and Net Present Value

The concept of risk is introduced here. Briefly explain the idea of risk (lottery versus bank deposit). Generally, investors do not like risk. In order to induce the investors to invest in risky projects, a higher rate of return is needed. Higher rate of return causes lower PVs.

Explain the relationship between discount rates and net present values. The higher the discount rate, the lower the net present value.

NPV at 12% : NPV = 714,286 – 700,000 = 14,286
NPV at 7% : NPV = 747,664 – 700,000 = 47,664
NPV at 5% : NPV = 761– 700,000 = 61,904

Slide 17 – Net Present Value Rule

Net Present Value Rule
Accept if NPV > 0: A very powerful financial decision-making rule. It looks simple but can get complicated quickly. This project is acceptable as the NPV > 0. Make sure that students understand this rule clearly.

Slide 18 – Rate of Return Rule

This slide explains the rate of return rule.

Slide 19 – Calculating Present Values When There Are Multiple Cash Flows

Multiple cash flows occurring at different time periods can be evaluated using the DCF formula. It is a simple extension of the NPV formula, but can intimidate students because of the extra equations.

Slide 20 – Figure 2.5 NPV Calculation

The graphic presentation of the net present value of multiple cash flows or sequential cash flows is given here. Here we extend the concept of PV to a series of cash flows by applying the value-additive property of present values. These cash flows can be positive (cash inflows) or negative (cash outflows). We merely add the initial cost to make it NPV.

Slide 21 – How to Value Perpetuities

Depending on the type of cash flow, you can use the formulas to simplify the calculations. There are formulas that can be used for finding the present values for cash flows with a pattern; for example, perpetuities and annuities. Define perpetuity (same cash flow each year forever) and give an example of perpetuity.

Slide 22 – Shortcuts

Introduce the perpetuity concept as one in which you earn money forever. In doing so, you can easily demonstrate the return an investor earns.

Slide 23 – Shortcuts Continued

Now, manipulate the formula to get the value of the infinite cash flow given a discount rate. Provide the formula for calculating the present value perpetuity. This formula is obtained using an algebraic technique; sum of an infinite geometric series.

Slide 24 – Present Values

Present value of $1 billion received forever at 10% is PV = $1/0.1 = $10 billion. Using the formula will simplify the calculations.

Slide 25 – Present Values Continued

The same example is used as in the previous slide, except the modification of time is added. Show the students how the value is reduced if you get the money later. This reinforces the time value of money concepts introduced earlier.

Slide 26 – How to Value Annuities

This slide provides the formula for the present value of an annuity. An annuity can be thought of as the difference between two perpetuities starting at different times. A slight derivation is presented, but it can be ignored if it is beyond the scope of the course, with no harm in understanding the broader concept.

Slide 27 – Perpetuities & Annuities

This is the PVAF formula. Take some time to explain the variables. If a financial calculator is to be used in class, there is no need to cover the use in detail.

Slide 28 – Figure 2.7 Annuity

This slide is a more comprehensive example of an annuity and its relationship to perpetuities.

Slide 29 – Figure 2.8 Costing an Installment Plan

An asset that pays a fixed sum each period for a specified number of periods is called an annuity. As an example, the present value of annual payments of $5,000 per year for five years is presented. In addition to the formula, using a financial calculator, PMT = 5,000; I = 7; N = 5; FV = 0; and compute PV = 20,501.

Slide 30 – Valuing Annuities Due

This is the formula that reverses the PVAF math for future values.

Slide 31 – Example 2.3 Paying off a Bank Loan

This is an example of an annuity. In this case we are determining the payment necessary on a loan.

Slide 32 – Table 2.1 Amortizing Loan Example

An example of an amortizing loan. If you borrow $1,000 at an interest rate of 10%, you would need to make an annual payment of $315.47 over four years to repay that loan with interest.

Slides 33, 34, and 35 – Future Value of an Annuity

As was done for the present value of an annuity earlier, these next few slides present the future value of an annuity.

Slide 36 – Growing Perpetuities

This formula is the present value of a perpetuity that is growing at a constant rate, where *g* is the annual growth rate of the cash flow, and $r > g$. It is useful and should be explained as a formula the students will use often.

Slide 37 – Growth Perpetuity Example

The present value of a $1 billion (first payment starting one year from today) perpetuity that is growing at a constant rate of 4% and requires a rate of return of 10% is $PV_0 = C_1/(r - g) = 1/(0.1 - 0.04) = 16.667$ billion. This is $6.667 billion more than the perpetuity without growth.

Slide 38 – How Interest is Paid and Quoted

Go over each definition: EAR and APR. Students need to know how each is used in the market and why APR is quoted rather than EAR.

Slide 39 – EAR & APR Formulas

The basic formulas for APR and EAR are presented. In reality, students are more likely to use the spreadsheet or financial calculator.

Slide 40 – Effective Interest Rates

Go over the math of the problem. If students are comfortable with the use of a financial calculator, use the following method as a substitute for the formula. Using the financial calculator: PMT = 0; I = 1%; N = 12; PV = 1; FV = –1.1268. By dropping the (–1) students will arrive at the answer of 12.68%.

KEY TERMS AND CONCEPTS

Present value, discount factor, discount rate, hurdle rate, opportunity cost of capital, net present value, net present value rule, rate of return rule, discounted cash flow, perpetuity, growing perpetuity, annuity, growing annuity, compound interest, annual percentage rate, effective annual rate.

CHALLENGE AREAS

The link between financial markets and NPV

ADDITIONAL REFERENCES

Franklin Allen and Douglas Gale, “A Comparative Theory of Corporate Governance,” Working Paper 03-27, Financial Institutions Center, The Wharton School, University of Pennsylvania (December 2002).

M. White, *Financial Analysis with a Calculator*, 5th ed. (Burr Ridge, Ill: McGraw-Hill/Irwin, 2004).

T.E. Copeland, J.F. Weston, and K. Shastri, *Financial Theory and Corporate Policy*, 4th ed. (Boston, MA: Pearson Addison Wesley, 2005).

S. Benninga, *Financial Modeling*. 3rd ed. (Cambridge, MA: The MIT Press, 2008).

WEB LINKS

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