Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
6th Edition
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Chapter 2
HEAT CONDUCTION EQUATION

Introduction

2-1C The term *steady* implies *no change with time* at any point within the medium while *transient* implies *variation with time* or *time dependence*. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location although both quantities may vary from one location to another. During transient heat transfer, the temperature and heat flux may vary with time as well as location. Heat transfer is one-dimensional if it occurs primarily in one direction. It is two-dimensional if heat tranfer in the third dimension is negligible.

2-2C Heat transfer is a *vector* quantity since it has direction as well as magnitude. Therefore, we must specify both direction and magnitude in order to describe heat transfer completely at a point. Temperature, on the other hand, is a scalar quantity.

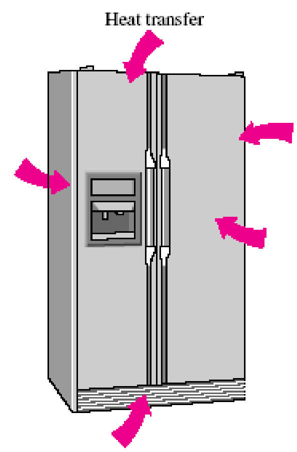
2-3C Yes, the heat flux vector at a point *P* on an isothermal surface of a medium has to be perpendicular to the surface at that point.

2-4C Isotropic materials have the same properties in all directions, and we do not need to be concerned about the variation of properties with direction for such materials. The properties of anisotropic materials such as the fibrous or composite materials, however, may change with direction.

2-5C In heat conduction analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy in solids is called heat generation.

2-6C The phrase “thermal energy generation” is equivalent to “heat generation,” and they are used interchangeably. They imply the conversion of some other form of energy into thermal energy. The phrase “energy generation,” however, is vague since the form of energy generated is not clear.

2-7C The heat transfer process from the kitchen air to the refrigerated space is transient in nature since the thermal conditions in the kitchen and the refrigerator, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the lowest thermostat setting for the refrigerated space, and the anticipated highest temperature in the kitchen (the so-called design conditions). If the compressor is large enough to keep the refrigerated space at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off. Heat transfer into the refrigerated space is three-dimensional in nature since heat will be entering through all six sides of the refrigerator. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer to be onedimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfer at each surface.



2-8C Heat transfer through the walls, door, and the top and bottom sections of an oven is transient in nature since the thermal conditions in the kitchen and the oven, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the highest temperature setting for the oven, and the anticipated lowest temperature in the kitchen (the so called "design" conditions). If the heating element of the oven is large enough to keep the oven at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off.

Heat transfer from the oven is three-dimensional in nature since heat will be entering through all six sides of the oven. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer as being one- dimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfers at each surface.

2-9C Heat transfer to a potato in an oven can be modeled as one-dimensional since temperature differences (and thus heat transfer) will exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the potato will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the potato.

2-10C Assuming the egg to be round, heat transfer to an egg in boiling water can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the egg will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the egg.

2-11C Heat transfer to a hot dog can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction. This would be a transient heat transfer process since the temperature at any point within the hot dog will change with time during cooking. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the hot dog. Heat transfer in a very long hot dog could be considered to be one-dimensional in preliminary calculations.

2-12C Heat transfer to a roast beef in an oven would be transient since the temperature at any point within the roast will change with time during cooking. Also, by approximating the roast as a spherical object, this heat transfer process can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction because of symmetry about the center point.

2-13C Heat loss from a hot water tank in a house to the surrounding medium can be considered to be a steady heat transfer problem. Also, it can be considered to be two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction.)

2-14C Heat transfer to a canned drink can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction. This would be a transient heat transfer process since the temperature at any point within the drink will change with time during heating. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the bottom surface.

2-15 A certain thermopile used for heat flux meters is considered. The minimum heat flux this meter can detect is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The thermal conductivity of kapton is given to be 0.345 W/m·K.

Analysis The minimum heat flux can be determined from

$$\dot{q} = k\frac{\Delta T}{L} = (0.345\ \text{W/m}\cdot{}^\circ\text{C})\frac{0.1^\circ\text{C}}{0.002\ \text{m}} = \mathbf{17.3\ W/m^2}$$

2-16 Diameter, length, and mass of stainless steel rod is given. The rod is insulated on its exterior surface except for the ends. Temperature distribution in the rod is also given. The heat flux along the rod is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional in the x-direction. **2** Thermal conductivity is constant.

Analysis The heat flux can be found from Fourier's law

$$\dot{q} = -k\,\frac{dT}{dx}$$

Table A-3 gives values for the thermal conductivity of stainless steels, however we are not told which type of stainless steel the rod is made of, and the thermal conductivity varies between them. We do know the mass of the rod, and can use this to calculate its density:

$$\rho = \frac{M}{Vol} = \frac{M}{\left(\frac{\pi D^2}{4}\right)L} = \frac{0.221\ \text{kg}}{[\pi(0.018\ \text{m})^2\ (0.11\ \text{m})]/4} = 7895\ \text{kg/m}^3$$

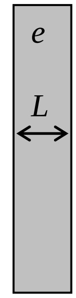
From Table A-3, with$\rho \approx$ 7900 kg/m^3,it appears that the material is AISI 304 stainless steel. The temperature of the rod from the given equation for temperature distribution varies from 310 K at x=0 to290 K at x = L = 110 mm. Evaluating the thermal conductivity at the average temperature of 300 K, from Table A-3, k = 14.9 W/m·K. Thus,

$$\dot{q} = -k\,\frac{dT}{dx} = -k\left(-\frac{20\text{K}}{L}\right) = -14.9\ \text{W/m}\cdot\text{K}\left(-\frac{20\text{K}}{0.11\ \text{m}}\right) = \mathbf{2709\ W/m^2}$$

DiscussionIf the temperature of the rod varies significantly along its length, the thermal conductivity will vary along the rod as much or more than the variation in thermal conductivities between the different stainless steels.

2-17 The rate of heat generation per unit volume in a stainless steel plate is given. The heat flux on the surface of the plate is to be determined.

Assumptions Heat is generated uniformly in steel plate.



Analysis We consider a unit surface area of 1 m^2. The total rate of heat generation in this section of the plate is

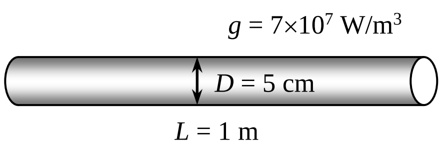
$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{plate}} = \dot{e}_{\text{gen}}(A \times L) = (5\times10^6 \text{ W/m}^3)(1\,\text{m}^2)(0.03\text{ m}) = 1.5\times10^5 \text{ W}$$

Noting that this heat will be dissipated from both sides of the plate, the heat flux on either surface of the plate becomes

$$\dot{q} = \frac{\dot{E}_{\text{gen}}}{A_{\text{plate}}} = \frac{1.5\times10^5 \text{ W}}{2\times1\,\text{m}^2} = 75{,}000 \text{ W/m}^2 = \mathbf{75\,kW/m^2}$$

2-18 The rate of heat generation per unit volume in the uranium rods is given. The total rate of heat generation in each rod is to be determined.

Assumptions Heat is generated uniformly in the uranium rods.



Analysis The total rate of heat generation in the rod is determined by multiplying the rate of heat generation per unit volume by the volume of the rod

$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{rod}} = \dot{e}_{\text{gen}}(\pi D^2/4)L = (7\times10^7 \text{ W/m}^3)[\pi(0.05\text{ m})^2/4](1\,\text{m}) = 1.374\times10^5 \text{ W} = \mathbf{137\,kW}$$

2-19 The variation of the absorption of solar energy in a solar pond with depth is given. A relation for the total rate of heat generation in a water layer at the top of the pond is to be determined.

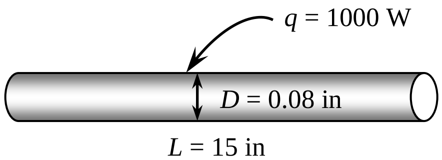
Assumptions Absorption of solar radiation by water is modeled as heat generation.

Analysis The total rate of heat generation in a water layer of surface area A and thickness L at the top of the pond is determined by integration to be

$$\dot{E}_{\text{gen}} = \int_V \dot{e}_{\text{gen}}\, dV = \int_{x=0}^{L} \dot{e}_0 e^{-bx}(A dx) = A\dot{e}_0 \left.\frac{e^{-bx}}{-b}\right|_0^L = \mathbf{\frac{A\dot{e}_0(1-e^{-bL})}{b}}$$

2-20E The power consumed by the resistance wire of an iron is given. The heat generation and the heat flux are to be determined.

Assumptions Heat is generated uniformly in the resistance wire.



Analysis A 1000 W iron will convert electrical energy into heat in the wire at a rate of 1000 W. Therefore, the rate of heat generation in a resistance wire is simply equal to the power rating of a resistance heater. Then the rate of heat generation in the wire per unit volume is determined by dividing the total rate of heat generation by the volume of the wire to be

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{(\pi D^2/4)L} = \frac{1000 \text{ W}}{[\pi(0.08/12 \text{ ft})^2/4](15/12 \text{ ft})}\left(\frac{3.412 \text{ Btu/h}}{1 \text{ W}}\right) = \mathbf{7.820 \times 10^7 \text{ Btu/h} \cdot \text{ft}^3}$$

Similarly, heat flux on the outer surface of the wire as a result of this heat generation is determined by dividing the total rate of heat generation by the surface area of the wire to be

$$\dot{q} = \frac{\dot{E}_{\text{gen}}}{A_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi DL} = \frac{1000 \text{ W}}{\pi(0.08/12 \text{ ft})(15/12 \text{ ft})}\left(\frac{3.412 \text{ Btu/h}}{1 \text{ W}}\right) = \mathbf{1.303 \times 10^5 \text{ Btu/h} \cdot \text{ft}^2}$$

Discussion Note that heat generation is expressed per unit volume in Btu/h·ft^3 whereas heat flux is expressed per unit surface area in Btu/h·ft^2.

Heat Conduction Equation

2-21C The one-dimensional transient heat conduction equation for a plane wall with constant thermal conductivity and heat generation is $\frac{\partial^2 T}{\partial x^2}+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$. Here T is the temperature, x is the space variable, $\dot{e}_{\text{gen}}$ is the heat generation per unit volume, k is the thermal conductivity, α is the thermal diffusivity, and t is the time.

2-22C The one-dimensional transient heat conduction equation for a long cylinder with constant thermal conductivity and heat generation is $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$. Here T is the temperature, r is the space variable, g is the heat generation per unit volume, k is the thermal conductivity, α is the thermal diffusivity, and t is the time.

2-23 We consider a thin element of thickness Δx in a large plane wall (see Fig. 2-12 in the text). The density of the wall is ρ, the specific heat is c, and the area of the wall normal to the direction of heat transfer is A. In the absence of any heat generation, an *energy balance* on this thin element of thickness Δx during a small time interval Δt can be expressed as

$$\dot{Q}_x-\dot{Q}_{x+\Delta x}=\frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}}=E_{t+\Delta t}-E_t=mc(T_{t+\Delta t}-T_t)=\rho cA\Delta x(T_{t+\Delta t}-T_t)$$

Substituting,

$$\dot{Q}_x-\dot{Q}_{x+\Delta x}=\rho cA\Delta x\frac{T_{t+\Delta t}-T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}=\rho c\frac{T_{t+\Delta t}-T_t}{\Delta t}$$

Taking the limit as $\Delta x\to 0$ and $\Delta t\to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right)=\rho c\frac{\partial T}{\partial t}$$

since from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x\to 0}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}=\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$

Noting that the area A of a plane wall is constant, the one-dimensional transient heat conduction equation in a plane wall with constant thermal conductivity k becomes

$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha=k/\rho c$ is the thermal diffusivity of the material.

2-24 We consider a thin cylindrical shell element of thickness Δr in a long cylinder (see Fig. 2-14 in the text). The density of the cylinder is ρ, the specific heat is c, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta r (T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta r$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{\text{gen}} A \Delta r = \rho c A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 2\pi rL$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 2\pi rL$ and the thermal conductivity is constant, the one-dimensional transient heat conduction equation in a cylinder becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k / \rho c$ is the thermal diffusivity of the material.

2-25 We consider a thin spherical shell element of thickness Δr in a sphere (see Fig. 2-16 in the text).. The density of the sphere is ρ, the specific heat is c, and the length is L. The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. When there is no heat generation, an *energy balance* on this thin spherical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta r (T_{t+\Delta t} - T_t)$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} = \rho c A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 4\pi r^2$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 4\pi r^2$ and the thermal conductivity k is constant, the one-dimensional transient heat conduction equation in a sphere becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material.

2-26 For a medium in which the heat conduction equation is given in its simplest by $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$:

(*a*) Heat transfer is transient, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-27 For a medium in which the heat conduction equation is given by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$:

(*a*) Heat transfer is transient, (*b*) it is two-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-28 For a medium in which the heat conduction equation is given in its simplest by $\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right)+\dot{e}_{\text{gen}}=0$:

(*a*) Heat transfer is steady, (*b*) it is one-dimensional, (*c*) there is heat generation, and (*d*) the thermal conductivity is variable.

2-29 For a medium in which the heat conduction equation is given by $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{\text{gen}}=0$:

(*a*) Heat transfer is steady, (*b*) it is two-dimensional, (*c*) there is heat generation, and (*d*) the thermal conductivity is variable.

2-30 For a medium in which the heat conduction equation is given in its simplest by $r\frac{d^2T}{dr^2}+2\frac{dT}{dr}=0$:

(*a*) Heat transfer is steady, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-31 For a medium in which the heat conduction equation is given by $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

(*a*) Heat transfer is transient, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-32 For a medium in which the heat conduction equation is given by $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

(*a*) Heat transfer is transient, (*b*) it is two-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-33 We consider a small rectangular element of length Δx, width Δy, and height $\Delta z = 1$ (similar to the one in Fig. 2-20). The density of the body is ρ and the specific heat is *c*. Noting that heat conduction is two-dimensional and assuming no heat generation, an *energy balance* on this element during a small time interval Δt can be expressed as

$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction at the}\\ \text{surfaces at } x \text{ and } y\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat conduction}\\ \text{at the surfaces at}\\ x+\Delta x \text{ and } y+\Delta y\end{array}\right) = \left(\begin{array}{c}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{array}\right)$$

or $$\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

Noting that the volume of the element is $V_{\text{element}} = \Delta x \Delta y \Delta z = \Delta x \Delta y \times 1$, the change in the energy content of the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c \Delta x \Delta y (T_{t+\Delta t} - T_t)$$

Substituting, $$\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \rho c \Delta x \Delta y \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x \Delta y$ gives

$$-\frac{1}{\Delta y}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the thermal conductivity *k* to be constant and noting that the heat transfer surface areas of the element for heat conduction in the *x* and *y* directions are $A_x = \Delta y \times 1$ and $A_y = \Delta x \times 1,$ respectively, and taking the limit as $\Delta x, \Delta y,$ and $\Delta t \to 0$ yields

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

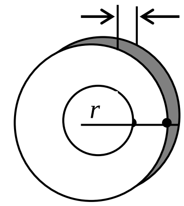
$$\lim_{\Delta x \to 0} \frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z}\frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z}\frac{\partial}{\partial x}\left(-k\Delta y \Delta z \frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -k\frac{\partial^2 T}{\partial x^2}$$

$$\lim_{\Delta y \to 0} \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z}\frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z}\frac{\partial}{\partial y}\left(-k\Delta x \Delta z \frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = -k\frac{\partial^2 T}{\partial y^2}$$

Here the property $\alpha = k/\rho c$ is the thermal diffusivity of the material.

2-34 We consider a thin ring shaped volume element of width Δz and thickness Δr in a cylinder. The density of the cylinder is ρ and the specific heat is c. In general, an *energy balance* on this ring element during a small time interval Δt can be expressed as

$$(\dot{Q}_r - \dot{Q}_{r+\Delta r}) + (\dot{Q}_z - \dot{Q}_{z+\Delta z}) = \frac{\Delta E_{\text{element}}}{\Delta t}$$



But the change in the energy content of the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c(2\pi r\Delta r)\Delta z(T_{t+\Delta t} - T_t)$$

Substituting,

$$(\dot{Q}_r - \dot{Q}_{r+\Delta r}) + (\dot{Q}_z - \dot{Q}_{z+\Delta z}) = \rho c(2\pi r\Delta r)\Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing the equation above by $(2\pi r\Delta r)\Delta z$ gives

$$-\frac{1}{2\pi r\Delta z}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} - \frac{1}{2\pi r\Delta r}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Noting that the heat transfer surface areas of the element for heat conduction in the r and z directions are $A_r = 2\pi r\Delta z$ and $A_z = 2\pi r\Delta r$, respectively, and taking the limit as $\Delta r, \Delta z$ and $\Delta t \to 0$ yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = \rho c\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0}\frac{1}{2\pi r\Delta z}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{1}{2\pi r\Delta z}\frac{\partial Q}{\partial r} = \frac{1}{2\pi r\Delta z}\frac{\partial}{\partial r}\left(-k(2\pi r\Delta z)\frac{\partial T}{\partial r}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)$$

$$\lim_{\Delta z \to 0}\frac{1}{2\pi r\Delta r}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{2\pi r\Delta r}\frac{\partial Q_z}{\partial z} = \frac{1}{2\pi r\Delta r}\frac{\partial}{\partial z}\left(-k(2\pi r\Delta r)\frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

For the case of constant thermal conductivity the equation above reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material. For the case of steady heat conduction with no heat generation it reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = 0$$

2-35 Consider a thin disk element of thickness Δz and diameter D in a long cylinder. The density of the cylinder is ρ, the specific heat is c, and the area of the cylinder normal to the direction of heat transfer is $A = \pi D^2/4$, which is constant. An *energy balance* on this thin element of thickness Δz during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ \text{the surface at } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction at the}\\ \text{surface at } z+\Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation inside}\\ \text{the element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{pmatrix}$$

or,

$$\dot{Q}_z - \dot{Q}_{z+\Delta z} + \dot{E}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

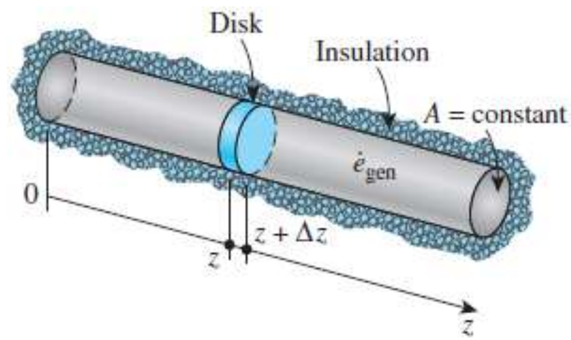
But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta z (T_{t+\Delta t} - T_t)$$

and

$$\dot{E}_{\text{element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta z$$

Substituting,



$$\dot{Q}_z - \dot{Q}_{z+\Delta z} + \dot{e}_{\text{gen}} A \Delta z = \rho c A \Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta z$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta z \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial z}\left(kA\frac{\partial T}{\partial z}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta z \to 0} \frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{\partial Q}{\partial z} = \frac{\partial}{\partial z}\left(-kA\frac{\partial T}{\partial z}\right)$$

Noting that the area A and the thermal conductivity k are constant, the one-dimensional transient heat conduction equation in the axial direction in a long cylinder becomes

$$\frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho c$ is the thermal diffusivity of the material.

Boundary and Initial Conditions; Formulation of Heat Conduction Problems

2-36C The mathematical expressions of the thermal conditions at the boundaries are called the **boundary conditions.** To describe a heat transfer problem completely, *two boundary conditions* must be given for *each direction* of the coordinate system along which heat transfer is significant. Therefore, we need to specify four boundary conditions for two-dimensional problems.

2-37C The mathematical expression for the temperature distribution of the medium initially is called the **initial condition**. We need only one initial condition for a heat conduction problem regardless of the dimension since the conduction equation is first order in time (it involves the first derivative of temperature with respect to time). Therefore, we need only 1 initial condition for a two-dimensional problem.

2-38C A heat transfer problem that is symmetric about a plane, line, or point is said to have thermal symmetry about that plane, line, or point. The thermal symmetry boundary condition is a mathematical expression of this thermal symmetry. It is equivalent to *insulation* or *zero heat flux* boundary condition, and is expressed at a point x_0 as $\partial T(x_0, t)/\partial x = 0$.

2-39C The boundary condition at a perfectly insulated surface (at x = 0, for example) can be expressed as

$$-k\frac{\partial T(0,t)}{\partial x} = 0 \qquad \text{or} \qquad \frac{\partial T(0,t)}{\partial x} = 0$$ which indicates zero heat flux.

2-40C Yes, the temperature profile in a medium must be perpendicular to an insulated surface since the slope $\partial T/\partial x = 0$ at that surface.

2-41C We try to avoid the radiation boundary condition in heat transfer analysis because it is a non-linear expression that causes mathematical difficulties while solving the problem; often making it impossible to obtain analytical solutions.

2-42 Heat conduction through the bottom section of an aluminum pan that is used to cook stew on top of an electric range is considered. Assuming variable thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be variable. **3** There is no heat generation in the medium. **4** The top surface at $x = L$ is subjected to specified temperature and the bottom surface at x = 0 is subjected to uniform heat flux.

Analysis The heat flux at the bottom of the pan is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{\pi D^2/4} = \frac{0.90 \times (900\text{ W})}{\pi (0.18\text{ m})^2/4} = 31{,}831\text{ W/m}^2$$

Then the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$-k\frac{dT(0)}{dx} = \dot{q}_s = 31{,}831\text{ W/m}^2$$

$$T(L) = T_L = 108\,^\circ\text{C}$$

2-43 Heat conduction through the bottom section of a steel pan that is used to boil water on top of an electric range is considered. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The top surface at $x = L$ is subjected to convection and the bottom surface at $x = 0$ is subjected to uniform heat flux.

Analysis The heat flux at the bottom of the pan is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{\pi D^2/4} = \frac{0.85\times(1250\text{ W})}{\pi(0.20\text{ m})^2/4} = 33{,}820\text{ W/m}^2$$

Then the differential equation and the boundary conditions for this heat conduction problem can be expressed as

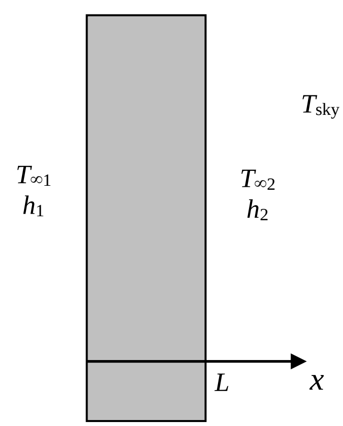
$$\frac{d^2T}{dx^2} = 0$$

$$-k\frac{dT(0)}{dx} = \dot{q}_s = 33{,}280\text{ W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

2-44 The outer surface of the East wall of a house exchanges heat with both convection and radiation., while the interior surface is subjected to convection only. Assuming the heat transfer through the wall to be steady and one-dimensional, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $x = L$ is subjected to convection and radiation while the inner surface at $x = 0$ is subjected to convection only.



Analysis Expressing all the temperatures in Kelvin, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

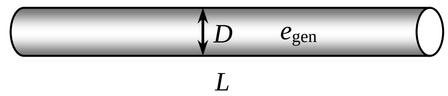
$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)]$$

$$-k\frac{dT(L)}{dx} = h_1[T(L) - T_{\infty 2}] + \varepsilon_2\sigma\left[T(L)^4 - T_{\text{sky}}^4\right]$$

2-45 Heat is generated in a long wire of radius r_o covered with a plastic insulation layer at a constant rate of $\dot{e}_{\text{gen}}$. The heat flux boundary condition at the interface (radius r_o) in terms of the heat generated is to be expressed. The total heat generated in the wire and the heat flux at the interface are

$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{wire}} = \dot{e}_{\text{gen}}(\pi r_o^2 L)$$

$$\dot{q}_s = \frac{\dot{Q}_s}{A} = \frac{\dot{E}_{\text{gen}}}{A} = \frac{\dot{e}_{\text{gen}}(\pi r_o^2 L)}{(2\pi r_o)L} = \frac{\dot{e}_{\text{gen}} r_o}{2}$$

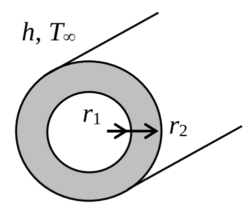


Assuming steady one-dimensional conduction in the radial direction, the heat flux boundary condition can be expressed as

$$-k\frac{dT(r_o)}{dr} = \frac{\dot{e}_{\text{gen}} r_o}{2}$$

2-46 A long pipe of inner radius r_1, outer radius r_2, and thermal conductivity k is considered. The outer surface of the pipe is subjected to convection to a medium at T_∞ with a heat transfer coefficient of h. Assuming steady one-dimensional conduction in the radial direction, the convection boundary condition on the outer surface of the pipe can be expressed as

$$-k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$

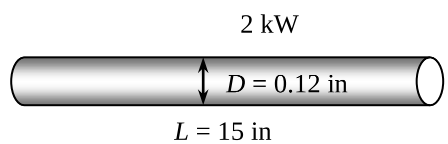


2-47E A 2-kW resistance heater wire is used for space heating. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** Heat is generated uniformly in the wire.

Analysis The heat flux at the surface of the wire is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{2\pi r_o L} = \frac{2000 \text{ W}}{2\pi(0.06 \text{ in})(15 \text{ in})} = 353.7 \text{ W/in}^2$$



Noting that there is thermal symmetry about the center line and there is uniform heat flux at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

$$\frac{dT(0)}{dr} = 0$$

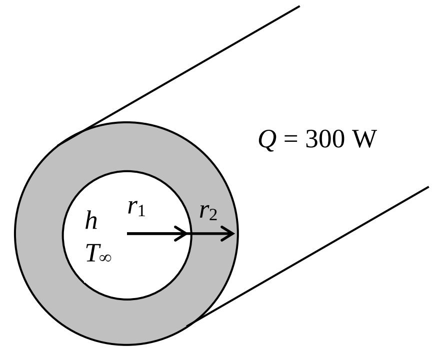
$$-k\frac{dT(r_o)}{dr} = \dot{q}_s = 353.7 \text{ W/in}^2$$

2-48 Water flows through a pipe whose outer surface is wrapped with a thin electric heater that consumes 300 W per m length of the pipe. The exposed surface of the heater is heavily insulated so that the entire heat generated in the heater is transferred to the pipe. Heat is transferred from the inner surface of the pipe to the water by convection. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of the heat conduction in the pipe is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_2$ is subjected to uniform heat flux and the inner surface at $r = r_1$ is subjected to convection.

Analysis The heat flux at the outer surface of the pipe is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{300\text{ W}}{2\pi(0.065\text{ cm})(1\text{ m})} = 734.6\text{ W/m}^2$$



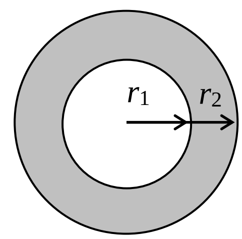
Noting that there is thermal symmetry about the center line and there is uniform heat flux at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$k\frac{dT(r_1)}{dr} = h[T(r_i) - T_\infty] = 85[T(r_i) - 70]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s = 734.6\text{ W/m}^2$$

2-49 A spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k is given. The boundary condition on the inner surface of the container for steady one-dimensional conduction is to be expressed for the following cases:



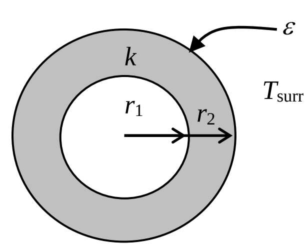
(*a*) Specified temperature of 50°C: $T(r_1) = 50°\text{C}$

(*b*) Specified heat flux of 30 W/m^2 towards the center: $k\dfrac{dT(r_1)}{dr} = 30\text{ W/m}^2$

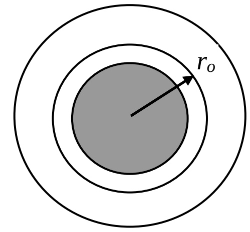
(*c*) Convection to a medium at T_∞ with a heat transfer coefficient of h: $k\dfrac{dT(r_1)}{dr} = h[T(r_1) - T_\infty]$

2-50 A spherical shell of inner radius r_1, outer radius r_2, and thermal conductivity k is considered. The outer surface of the shell is subjected to radiation to surrounding surfaces at T_{surr}. Assuming no convection and steady one-dimensional conduction in the radial direction, the radiation boundary condition on the outer surface of the shell can be expressed as

$$-k\frac{dT(r_2)}{dr} = \varepsilon\sigma\left[T(r_2)^4 - T_{surr}^4\right]$$



2-51 A spherical container consists of two spherical layers *A* and *B* that are at perfect contact. The radius of the interface is r_o. Assuming transient one-dimensional conduction in the radial direction, the boundary conditions at the interface can be expressed as



$$T_A(r_o,t) = T_B(r_o,t)$$

and
$$-k_A \frac{\partial T_A(r_o,t)}{\partial r} = -k_B \frac{\partial T_B(r_o,t)}{\partial r}$$

2-52 A spherical metal ball that is heated in an oven to a temperature of T_i throughout is dropped into a large body of water at T_∞ where it is cooled by convection. Assuming constant thermal conductivity and transient one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be transient and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_0$ is subjected to convection.

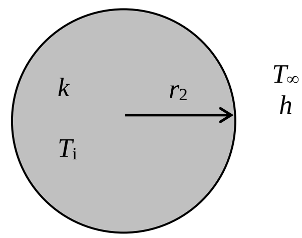
Analysis Noting that there is thermal symmetry about the midpoint and convection at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial T(0,t)}{\partial r} = 0$$

$$-k\frac{\partial T(r_o,t)}{\partial r} = h[T(r_o) - T_\infty]$$

$$T(r,0) = T_i$$



2-53 A spherical metal ball that is heated in an oven to a temperature of T_i throughout is allowed to cool in ambient air at T_∞ by convection and radiation. Assuming constant thermal conductivity and transient one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be transient and one-dimensional. **2** Thermal conductivity is given to be variable. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_o$ is subjected to convection and radiation.

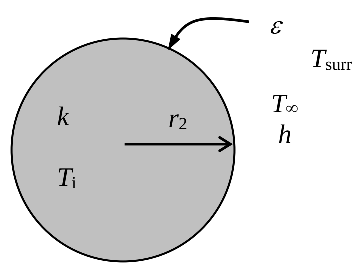
Analysis Noting that there is thermal symmetry about the midpoint and convection and radiation at the outer surface and expressing all temperatures in Rankine, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r}\right) = \rho c\frac{\partial T}{\partial t}$$

$$\frac{\partial T(0,t)}{\partial r} = 0$$

$$-k\frac{\partial T(r_o,t)}{\partial r} = h[T(r_o) - T_\infty] + \varepsilon\sigma[T(r_o)^4 - T_{\text{surr}}^4]$$

$$T(r,0) = T_i$$



Solution of Steady One-Dimensional Heat Conduction Problems

2-54C Yes, the temperature in a plane wall with constant thermal conductivity and no heat generation will vary linearly during steady one-dimensional heat conduction even when the wall loses heat by radiation from its surfaces. This is because the steady heat conduction equation in a plane wall is $d^2T/dx^2 = 0$ whose solution is $T(x) = C_1x + C_2$ regardless of the boundary conditions. The solution function represents a straight line whose slope is C_1.

2-55C Yes, this claim is reasonable since in the absence of any heat generation the rate of heat transfer through a plain wall in steady operation must be constant. But the value of this constant must be zero since one side of the wall is perfectly insulated. Therefore, there can be no temperature difference between different parts of the wall; that is, the temperature in a plane wall must be uniform in steady operation.

2-56C Yes, this claim is reasonable since no heat is entering the cylinder and thus there can be no heat transfer from the cylinder in steady operation. This condition will be satisfied only when there are no temperature differences within the cylinder and the outer surface temperature of the cylinder is the equal to the temperature of the surrounding medium.

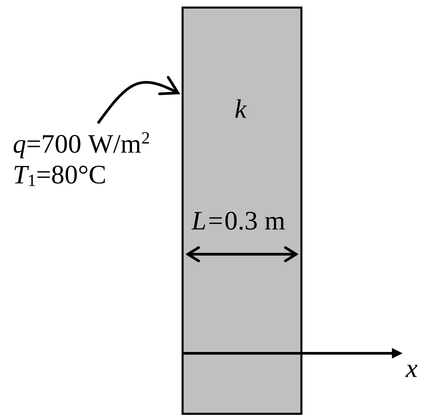
2-57C Yes, in the case of constant thermal conductivity and no heat generation, the temperature in a solid cylindrical rod whose ends are maintained at constant but different temperatures while the side surface is perfectly insulated will vary linearly during steady one-dimensional heat conduction. This is because the steady heat conduction equation in this case is $d^2T/dx^2 = 0$ whose solution is $T(x) = C_1x + C_2$ which represents a straight line whose slope is C_1.

2-58 A large plane wall is subjected to specified heat flux and temperature on the left surface and no conditions on the right surface. The mathematical formulation, the variation of temperature in the plate, and the right surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall.

Properties The thermal conductivity is given to be k =2.5 W/m·°C.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with *x* = 0 at the left surface, the mathematical formulation of this problem can be expressed as



$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 700 \text{ W/m}^2$$

$$T(0) = T_1 = 80°\text{C}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

Heat flux at *x* = 0: $-kC_1 = \dot{q}_0 \;\rightarrow\; C_1 = -\frac{\dot{q}_0}{k}$

Temperature at *x* = 0: $T(0) = C_1 \times 0 + C_2 = T_1 \;\rightarrow\; C_2 = T_1$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{700 \text{ W/m}^2}{2.5 \text{ W/m}\cdot°\text{C}}x + 80°\text{C} = -280x + 80$$

(*c*) The temperature at $x = L$ (the right surface of the wall) is

$$T(L) = -280 \times (0.3 \text{ m}) + 80 = \mathbf{-4°C}$$

Note that the right surface temperature is lower as expected.

2-59 The base plate of a household iron is subjected to specified heat flux on the left surface and to specified temperature on the right surface. The mathematical formulation, the variation of temperature in the plate, and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity is given to be $k = 20$ W/m·°C.

Analysis (*a*) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

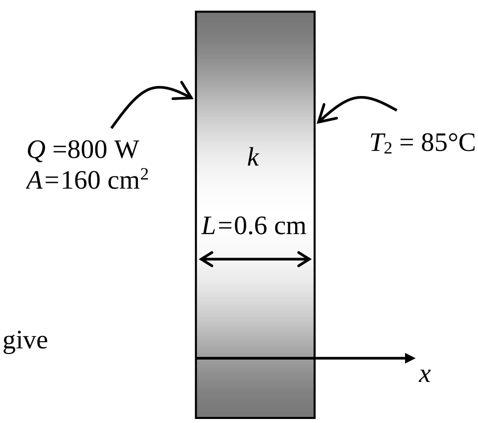
$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{800\text{ W}}{160 \times 10^{-4}\text{ m}^2} = 50{,}000\text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 50{,}000\text{ W/m}^2$$

$$T(L) = T_2 = 85°\text{C}$$



(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $$-kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$x = L$: $$T(L) = C_1L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L \rightarrow C_2 = T_2 + \frac{\dot{q}_0 L}{k}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_2 + \frac{\dot{q}_0 L}{k} = \frac{\dot{q}_0(L-x)}{k} + T_2$$

$$= \frac{(50{,}000\text{ W/m}^2)(0.006 - x)\text{m}}{20\text{ W/m}\cdot°\text{C}} + 85°\text{C}$$

$$= 2500\,(0.006 - x) + 85$$

(*c*) The temperature at $x = 0$ (the inner surface of the plate) is

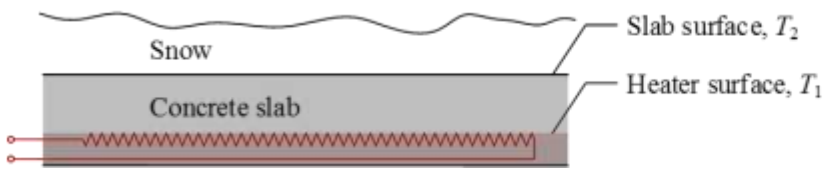
$$T(0) = 2500\,(0.006 - 0) + 85 = \mathbf{100°C}$$

Note that the inner surface temperature is higher than the exposed surface temperature, as expected.

2-60 A concrete slab with embedded heating cable to provide 1200 W/m^2 of heat to melt snow. Formulate the temperature profile in the concrete slab. Determine the slab thickness so that the temperature difference between the heater surface and the slab surface does not exceed 21°C, as recommended in ASHRAE Handbook to minimize thermal stress.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the concrete slab. **3** The bottom surface at $x = 0$ is subjected to uniform heat flux from the heating cable. **4** The upper surface at $x = L$ is at a constant temperature of 0°C from the snow melt. **5** There is no heat generation in the concrete slab. **6** Thermal properties are constant.

Properties The thermal conductivity of concrete is given as 1.4 W/m·K.



Analysis Taking the direction normal to the surface of the concrete slab to be the x direction with $x = 0$ at the bottom surface (the surface that is in contact with the heater surface), the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_2 = C_1L + C_2 \qquad \rightarrow \qquad C_2 = T_L - C_1L = T_2 + \frac{\dot{q}_0}{k}L$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the concrete slab is determined to be

$$\boldsymbol{T(x) = \frac{\dot{q}_0}{k}(L - x) + T_2}$$

Note that at $x = L$, $T(L) = T_2 = 0$°C, where the snow melt occurs.

The concrete slab thickness such that the temperature difference between the heater surface (T_1) and the slab surface (T_2) does not exceed 21°C is

$$L = \frac{k}{\dot{q}_0}[T_1 - T_2] = \frac{1.4\ \text{W/m·K}}{1200\ \text{W/m}^2}[21\ \text{K}] = 0.0245\ \text{m} = \mathbf{24.5\ mm}$$

Discussion As the concrete slab thickness increases, the temperature difference between the heater surface and the slab surface will increase. So, 24.5 mm is the maximum thickness for the concrete slab to comply with the recommendation by the 2015 ASHRAE Handbook—HVAC Applications, Chapter 51, for $T_1 - T_2 \leq 21$°C.

2-61 A large plane wall is subjected to specified temperature on the left surface and convection on the right surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be k = 2.3 W/m·°C.

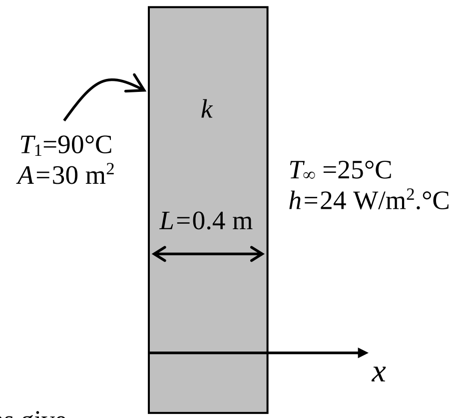
Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with *x* = 0 at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and

$$T(0) = T_1 = 90°\text{C}$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$



(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

x = 0: $T(0) = C_1 \times 0 + C_2 \;\rightarrow\; C_2 = T_1$

x = *L*: $-kC_1 = h[(C_1L + C_2) - T_\infty] \rightarrow C_1 = -\dfrac{h(C_2 - T_\infty)}{k + hL} \rightarrow C_1 = -\dfrac{h(T_1 - T_\infty)}{k + hL}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{h(T_1 - T_\infty)}{k + hL}x + T_1 \\ &= -\frac{(24\ \text{W/m}^2\cdot°\text{C})(90 - 25)°\text{C}}{(2.3\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})}x + 90°\text{C} \\ &= 90 - 131.1x \end{aligned}$$

(*c*) The rate of heat conduction through the wall is

$$\begin{aligned} \dot{Q}_{\text{wall}} &= -kA\frac{dT}{dx} = -kAC_1 = kA\frac{h(T_1 - T_\infty)}{k + hL} \\ &= (2.3\ \text{W/m}\cdot°\text{C})(30\ \text{m}^2)\frac{(24\ \text{W/m}^2\cdot°\text{C})(90 - 25)°\text{C}}{(2.3\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})} \\ &= \mathbf{9045\ W} \end{aligned}$$

Note that under steady conditions the rate of heat conduction through a plain wall is constant.

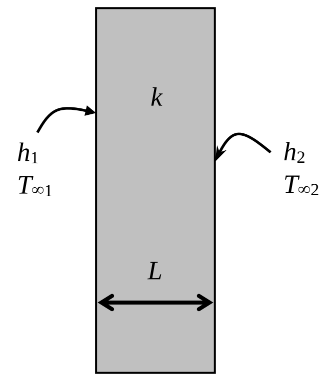
2-62 A large plane wall is subjected to convection on the inner and outer surfaces. The mathematical formulation, the variation of temperature, and the temperatures at the inner and outer surfaces to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k = 0.77$ W/m·K.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with $x = 0$ at the inner surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



The boundary conditions for this problem are:

$$h_1[T_{\infty 1} - T(0)] = -k\frac{dT(0)}{dx}$$

$$-k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $\quad h_1[T_{\infty 1} - (C_1 \times 0 + C_2)] = -kC_1$

$x = L$: $\quad -kC_1 = h_2[(C_1 L + C_2) - T_{\infty 2}]$

Substituting the given values, the above boundary condition equations can be written as

$$5(27 - C_2) = -0.77C_1$$

$$-0.77C_1 = (12)(0.2C_1 + C_2 - 8)$$

Solving these equations simultaneously give

$$C_1 = -45.45 \qquad C_2 = 20$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = 20 - 45.45x$$

(*c*) The temperatures at the inner and outer surfaces are

$$T(0) = 20 - 45.45 \times 0 = \mathbf{20°C}$$

$$T(L) = 20 - 45.45 \times 0.2 = \mathbf{10.9°C}$$

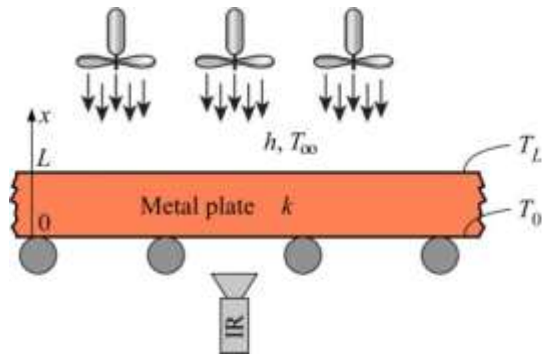
2-63 In this example, the concepts of Prevention through Design (PtD) are applied in conjunction with the solution of steady one-dimensional heat conduction problem. The top surface of the plate is cooled by convection, and temperature at the bottom surface is measured by an IR thermometer. The variation of temperature in the metal plate and the convection heat transfer coefficient necessary to keep the top surface below 47°C are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** The bottom surface at $x = 0$ is at constant temperature while the top surface at $x = L$ is subjected to convection.

Properties The thermal conductivity of the metal plate is given to be $k = 13.5$ W/m·K.

Analysis Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the lower surface, the mathematical formulation can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



with boundary conditions

$$T(0) = T_0$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the first boundary condition yields

$$T(0) = C_1 \times 0 + C_2 \rightarrow C_2 = T_0$$

The application of the second boundary condition gives

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \rightarrow -kC_1 = h(C_1L + C_2 - T_\infty)$$

Solving for C_2 yields

$$C_1 = \frac{h(T_\infty - C_2)}{k + hL} = \frac{T_\infty - T_0}{(k/h) + L}$$

Now substituting C_1 and C_2 into the general solution and the variation of temperature is

$$\boldsymbol{T(x) = \frac{T_\infty - T_0}{(k/h) + L}x + T_0}$$

The minimum convection heat transfer coefficient necessary to maintain the top surface below 47°C can be determined from the variation of temperature:

$$T(L) = T_L = \frac{T_\infty - T_0}{(k/h) + L}L + T_0$$

Solving for h gives

$$h = \frac{k}{L}\frac{T_L - T_0}{T_\infty - T_L} = \left(\frac{13.5\text{ W/m}\cdot\text{K}}{0.025\text{ m}}\right)\frac{(47-60)\,^\circ\text{C}}{(30-47)\,^\circ\text{C}} = \mathbf{413\,W/m^2 \cdot K}$$

Discussion To keep the top surface of the metal plate below 47°C, the convection heat transfer coefficient should be greater than 413 W/m^2·K. A convection heat transfer coefficient value of 413 W/m^2·K is very high for forced convection of gases. The typical values for forced convection of gases are 25–250 W/m^2·K (see Table 1-5 in Chapter 1). To protect workers from thermal burn, appropriate apparel should be worn when operating in an area where hot surfaces are present.

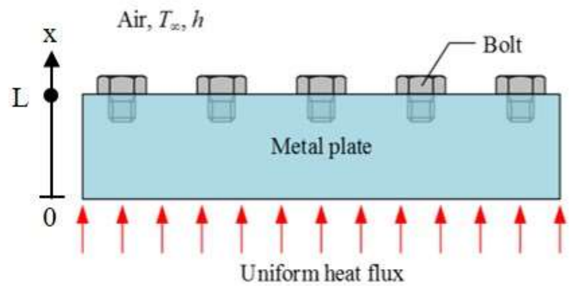
2-64 A series of ASME SA-193 carbon steel bolts of 1 cm thread length are bolted on the upper surface of a metal plate. The upper surface is exposed to convection with the ambient air. The bottom surface is subjected to a uniform heat flux. Formulate the temperature profile in the metal plate, and determine the location in the plate where the temperature begins to exceed 260°C. The compliance of the SA-193 bolts with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300) is to be determined.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the metal plate. **3** The bottom surface at $x = 0$ is subjected to uniform heat flux while the upper surface at $x = L$ is at uniform temperature. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The thermal conductivity of the metal plate is given as 15 W/m·K.

Analysis The uniform heat flux on the bottom plate surface ($x = 0$) is equal to the heat flux transferred by convection on the upper surface ($x = L$):

$$\dot{q}_0 = h[T(L) - T_\infty] \qquad \rightarrow \qquad T(L) = \frac{\dot{q}_0}{h} + T_\infty$$



Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = -C_1 L + T_L = \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the metal plate is determined to be

$$\boldsymbol{T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty = \frac{\dot{q}_0}{k}(L - x) + \frac{\dot{q}_0}{h} + T_\infty}$$

Using the temperature profile of the metal plate, the location where the temperature begins to exceed 260°C is

$$T(x) = 260°\text{C}: \qquad x = L + \frac{k}{h} + \frac{k}{\dot{q}_0}[T_\infty - T(x)] = 0.05\text{ m} + \frac{15\text{ W/m·K}}{10\text{ W/m}^2\text{·K}} + \frac{15\text{ W/m·K}}{2250\text{ W/m}^2}(30 - 260)\text{K}$$

$$= 0.0167\text{ m} = \mathbf{1.67\text{ cm}}$$

Discussion Since the thread length of the SA-193 bolts is 1 cm; the bolt tips would only reach the location $x = 0.04$ cm in the metal plate. This is where the bolts are subjected to the highest temperature in the metal plate. At this location, the temperature in the plate can be determined from the temperature profile:

$$T(0.04) = \frac{2250\text{ W/m}^2}{15\text{ W/m·K}}(0.05 - 0.04)\text{m} + \frac{2250\text{ W/m}^2}{10\text{ W/m}^2\text{·K}} + 30°\text{C} = 256.5°\text{C}$$

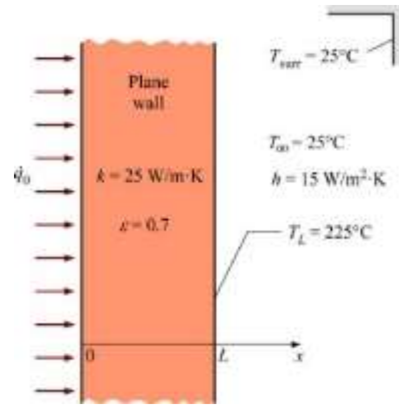
Therefore, the SA-193 bolts would comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300).

2-65 A plane wall is subjected to uniform heat flux on the left surface, while the right surface is subjected to convection and radiation heat transfer. The variation of temperature in the wall and the left surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Temperatures on both sides of the wall are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the wall. **5** The surrounding temperature $T_\infty = T_{surr}$ = 25°C.

Properties Emissivity and thermal conductivity are given to be 0.70 and 25 W/m·K, respectively.

Analysis(a) Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation can be expressed as



$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad T(L) = T_L = C_1 L + C_2 \rightarrow C_2 = -C_1 L + T_L = \frac{\dot{q}_0}{k}L + T_L$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + T_L \rightarrow T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

(b) The uniform heat flux subjected on the left surface is equal to the sum of heat fluxes transferred by convection and radiation on the right surface:

$$\dot{q}_0 = h(T_L - T_\infty) + \varepsilon\sigma(T_L^4 - T_{surr}^4)$$

$$\dot{q}_0 = (15\text{ W/m}^2\cdot\text{K})(225 - 25)\text{K} + (0.70)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(225+273)^4 - (25+273)^4]\text{K}^4$$

$$\dot{q}_0 = \mathbf{5130\ W/m^2}$$

(c) The temperature at x = 0 (the left surface of the wall) is

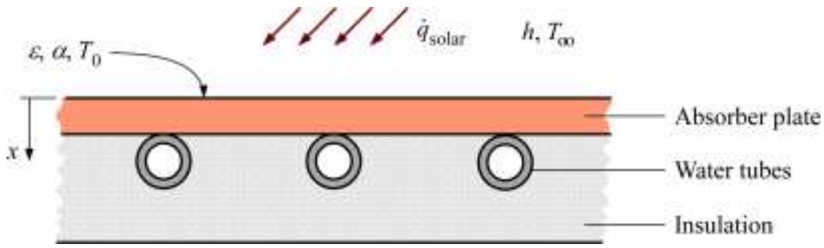
$$T(0) = \frac{\dot{q}_0}{k}(L - 0) + T_L = \frac{5130\text{ W/m}^2}{25\text{ W/m}\cdot\text{K}}(0.50\text{ m}) + 225\,°\text{C} = \mathbf{327.6\ °C}$$

Discussion As expected, the left surface temperature is higher than the right surface temperature. The absence of radiative boundary condition may lower the resistance to heat transfer at the right surface of the wall resulting in a temperature drop on the left wall surface by about 40°C.

2-66 A flat-plate solar collector is used to heat water. The top surface (x = 0) is subjected to convection, radiation, and incident solar radiation. The variation of temperature in the solar absorber and the net heat flux absorbed by the solar collector are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** The top surface at x = 0 is subjected to convection, radiation, and incident solar radiation.

Properties The absorber surface has an absorptivity of 0.9 and an emissivity of 0.9.



Analysis Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the top surface, the mathematical formulation can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = 0: \qquad T(0) = T_0 = C_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_0$$

At the top surface (x = 0), the net heat flux absorbed by the solar collector is

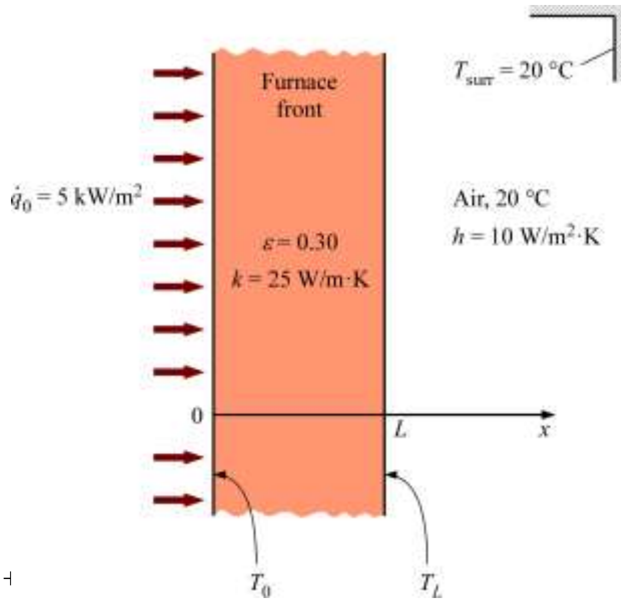
$$\dot{q}_0 = \alpha\dot{q}_{\text{solar}} - \varepsilon\sigma(T_0^4 - T_{\text{surr}}^4) - h(T_0 - T_\infty)$$

$$\dot{q}_0 = (0.9)(500\,\text{W/m}^2) - (0.9)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(35+273)^4 - (0+273)^4)]\text{K}^4 - (5\,\text{W/m}^2\cdot\text{K})(35-25)\,\text{K}$$

$$\dot{q}_0 = \mathbf{224\,W/m^2}$$

Discussion The absorber plate is generally very thin. Thus, the temperature difference between the top and bottom surface temperatures of the plate is miniscule. The net heat flux absorbed by the solar collector increases with the increase in the ambient and surrounding temperatures and thus the use of solar collectors is justified in hot climatic conditions.

2-67 A 20-mm thick draw batch furnace front is subjected to uniform heat flux on the inside surface, while the outside surface is subjected to convection and radiation heat transfer. The inside surface temperature of the furnace front is to be determined.



Assumptions **1** Heat conduction is steady. **2** One dimensional heat conduction across the furnace front thickness. **3** Thermal properties are constant. **4** Inside and outside surface temperatures are constant.

Properties Emissivity and thermal conductivity are given to be 0.30 and 25 W/m · K, respectively

Analysis The uniform heat flux subjected on the inside surface is equal to the sum of heat fluxes transferred by convection and radiation on the outside surface:

$$\dot{q}_0 = h(T_L - T_\infty) + \varepsilon\sigma(T_L^4 - T_{\text{surr}}^4)$$

$$5000\ \text{W/m}^2 = (10\ \text{W/m}^2\cdot\text{K})[T_L - (20+273)]\ \text{K} + (0.30)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_L^4 - (20+$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
5000=10*(T_L-(20+273))+0.30*5.67e-8*(T_L^4-(20+273)^4)
```

Solving by EES software, the outside surface temperature of the furnace front is

$$T_L = 594\ \text{K}$$

For steady heat conduction, the Fourier's law of heat conduction can be expressed as

$$\dot{q}_0 = -k\frac{dT}{dx}$$

Knowing that the heat flux and thermal conductivity are constant, integrating the differential equation once with respect to *x* yields

$$T(x) = -\frac{\dot{q}_0}{k}x + C_1$$

Applying the boundary condition gives

$$x = L: \quad T(L) = T_L = -\frac{\dot{q}_0}{k}L + C_1 \quad \rightarrow \quad C_1 = \frac{\dot{q}_0}{k}L + T_L$$

Substituting C_1 into the general solution, the variation of temperature in the furnace front is determined to be

$$T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

The inside surface temperature of the furnace front is

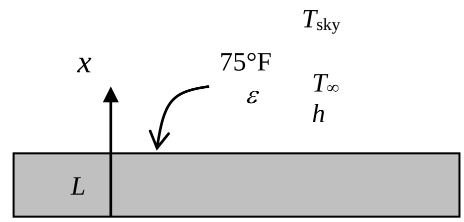
$$T(0) = T_0 = \frac{\dot{q}_0}{k}L + T_L = \frac{5000\ \text{W/m}^2}{25\ \text{W/m}\cdot\text{K}}(0.020\ \text{m}) + 594\ \text{K} = \mathbf{598\,K}$$

Discussion By insulating the furnace front, heat loss from the outer surface can be reduced.

2-68E A large plate is subjected to convection, radiation, and specified temperature on the top surface and no conditions on the bottom surface. The mathematical formulation, the variation of temperature in the plate, and the bottom surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate.

Properties The thermal conductivity and emissivity are given to be k =7.2 Btu/h·ft·°F and ε = 0.7.



Analysis (a) Taking the direction normal to the surface of the plate to be the x direction with x = 0 at the bottom surface, and the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{sky}}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]$$

$$T(L) = T_2 = 75°\text{F}$$

(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

Convection at $x = L$: $$-kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]$$

$$\rightarrow C_1 = -\{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]\}/k$$

Temperature at $x = L$: $T(L) = C_1 \times L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = C_1x + (T_2 - C_1L) = T_2 - (L - x)C_1 = T_2 + \frac{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]}{k}(L - x)$$

$$= 75°\text{F} + \frac{(12\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(75 - 90)°\text{F} + 0.7(0.1714 \times 10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(535\text{ R})^4 - (480\text{ R})^4]}{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}(4/12 - x)\text{ ft}$$

$$= 75 - 20.2(1/3 - x)$$

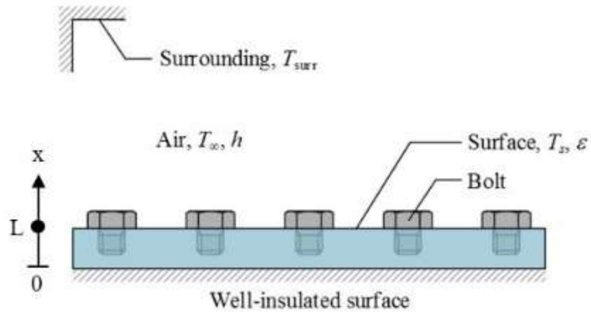
(c) The temperature at x = 0 (the bottom surface of the plate) is

$$T(0) = 75 - 20.2 \times (1/3 - 0) = \mathbf{68.3°F}$$

2-69 A series of ASTM B21 naval brass bolts are bolted on the upper surface of a plate. The upper surface is exposed to convection with air and radiation with the surrounding surface. Formulate the temperature profile in the plate, and determine if the bolts comply with the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the plate. **3** The bottom surface at $x = 0$ is well-insulated while the upper surface at $x = L$ is subjected to convection and radiation. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The emissivity of the plate and bolts is given as 0.3.



Analysis Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the well- insulated surface boundary condition at the bottom surface ($x = 0$) yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = -kC_1 = 0 \qquad \rightarrow \qquad C_1 = 0$$

Since $C_1 = 0$, the temperature profile in the plate is constant

$$T(x) = C_2$$

At the upper surface ($x = L$) we have

$$x = L: \qquad -k\frac{dT(L)}{dx} = -kC_1 = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = 0$$

$$x = L: \qquad T(x) = T(L) = C_2$$

Solving for C_2 (note that absolute temperatures are used for the calculation),

$$h[C_2 - T_\infty] + \varepsilon\sigma[C_2^4 - T_{\text{surr}}^4] = 0$$

$$(2)\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(C_2 - 323)(\text{K}) + (0.3)(5.67\times10^{-8})\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(C_2^4 - 473^4)(\text{K}^4) = 0$$

$$T(x) = C_2 = 437\text{ K} = \mathbf{164}\text{°C}$$

Discussion The temperature in the plate exceeds the maximum use temperature by 15°C. The use of the ASTM B21 bolts under these conditions does not comply with the ASME Code for Process Piping (ASME B31.3-2014). One way to reduce the plate temperature is by increasing the convection heat transfer coefficient with forced convection. If the convection heat transfer coefficient were to increase to higher than 3.15 W/m²·K, then the plate temperature would reduce to below 149°C:

$$h > \varepsilon\sigma\frac{T_{\text{surr}}^4 - T(x)^4}{T(x) - T_\infty}$$

$$> (0.3)(5.67\times10^{-8})\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)\frac{473^4 - 422^4}{422 - 323}(\text{K}^3)$$

$$> 3.15\text{ W/m}^2\cdot\text{K}$$

2-70 Chilled water flows in a pipe that is well insulated from outside. The mathematical formulation and the variation of temperature in the pipe are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

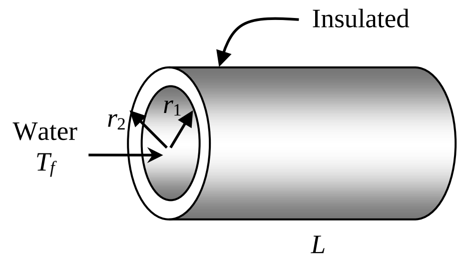
Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

and

$$-k\frac{dT(r_1)}{dr}=h[T_f-T(r_1)]$$

$$\frac{dT(r_2)}{dr}=0$$



(*b*) Integrating the differential equation once with respect to *r* gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$\frac{C_1}{r_2}=0\rightarrow C_1=0$$

$r = r_1$: $$-k\frac{C_1}{r_1}=h[T_f-(C_1\ln r_1+C_2)]$$

$$0=h(T_f-C_2)\rightarrow C_2=T_f$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=T_f$$

This result is not surprising since steady operating conditions exist.

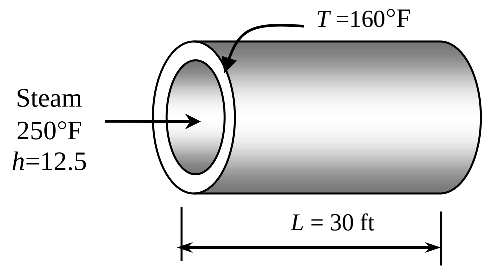
2-71E A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Properties The thermal conductivity is given to be k = 7.2 Btu/h·ft·°F.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$



and $$-k\frac{dT(r_1)}{dr}=h[T_\infty-T(r_1)]$$

$$T(r_2)=T_2=160°\text{F}$$

(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$-k\frac{C_1}{r_1}=h[T_\infty-(C_1\ln r_1+C_2)]$$

$r = r_2$: $$T(r_2)=C_1\ln r_2+C_2=T_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1=\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}\quad\text{and}\quad C_2=T_2-C_1\ln r_2=T_2-\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}\ln r_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=C_1\ln r+T_2-C_1\ln r_2=C_1(\ln r-\ln r_2)+T_2=\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}\ln\frac{r}{r_2}+T_2$$

$$=\frac{(160-250)°\text{F}}{\ln\frac{2.4}{2}+\frac{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}}\ln\frac{r}{2.4\text{ in}}+160°\text{F}=-24.74\ln\frac{r}{2.4\text{ in}}+160°\text{F}$$

(*c*) The rate of heat conduction through the pipe is

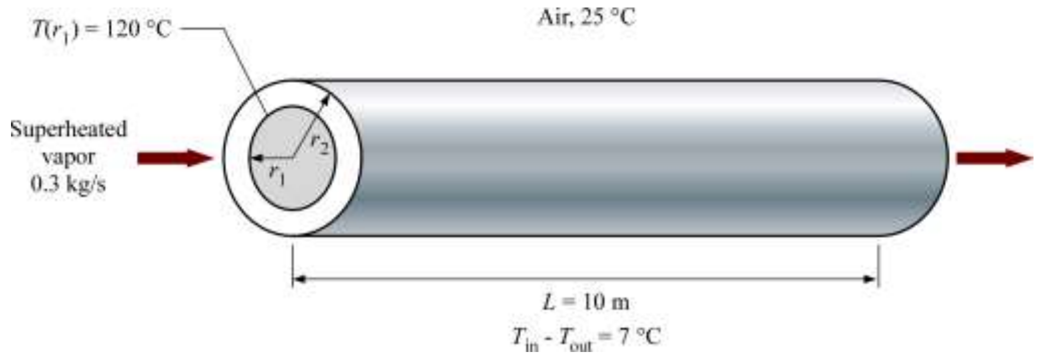
$$\dot{Q}=-kA\frac{dT}{dr}=-k(2\pi rL)\frac{C_1}{r}=-2\pi Lk\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}$$

$$=-2\pi(30\text{ ft})(7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(160-250)°\text{F}}{\ln\frac{2.4}{2}+\frac{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}}=\mathbf{33{,}600\ Btu/h}$$

2-72 The convection heat transfer coefficient between the surface of a pipe carrying superheated vapor and the surrounding air is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipe. **4** Heat transfer by radiation is negligible.

Properties The constant pressure specific heat of vapor is given to be 2190 J/kg · °C and the pipe thermal conductivity is 17 W/m · °C.



Analysis The inner and outer radii of the pipe are

$$r_1 = 0.05\text{ m}/2 = 0.025\text{ m}$$

$$r_2 = 0.025\text{ m} + 0.006\text{ m} = 0.031\text{ m}$$

The rate of heat loss from the vapor in the pipe can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (0.3\text{ kg/s})(2190\text{ J/kg}\cdot\text{°C})(7)\text{ °C} = 4599\text{ W}$$

For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $$-k\frac{dT(r_1)}{dr} = \frac{\dot{Q}_{\text{loss}}}{A} = \frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} \quad \text{(heat flux at the inner pipe surface)}$$

$$T(r_1) = 120\text{ °C} \quad \text{(inner pipe surface temperature)}$$

Integrating the differential equation once with respect to r gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to r again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \quad \frac{dT(r_1)}{dr} = -\frac{1}{k}\frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} = \frac{C_1}{r_1} \quad \rightarrow \quad C_1 = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}$$

$$r = r_1: \quad T(r_1) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + C_2 \quad \rightarrow \quad C_2 = \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r + \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$

$$= -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r/r_1) + T(r_1)$$

The outer pipe surface temperature is

$$T(r_2) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r_2/r_1) + T(r_1)$$
$$= -\frac{1}{2\pi}\frac{4599\ \text{W}}{(17\ \text{W/m}\cdot°\text{C})(10\ \text{m})}\ln\left(\frac{0.031}{0.025}\right) + 120\ °\text{C}$$
$$= 119.1°\text{C}$$

From Newton's law of cooling, the rate of heat loss at the outer pipe surface by convection is

$$\dot{Q}_{\text{loss}} = h(2\pi\, r_2 L)\left[T(r_2) - T_\infty\right]$$

Rearranging and the convection heat transfer coefficient is determined to be

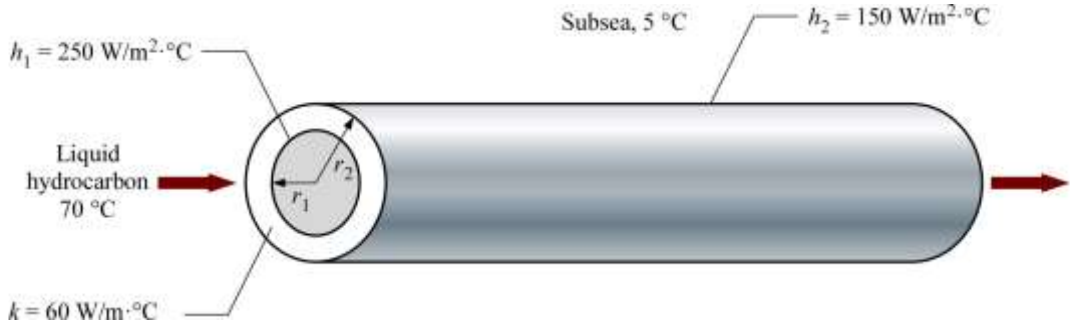
$$h = \frac{\dot{Q}_{\text{loss}}}{2\pi\, r_2 L[T(r_2) - T_\infty]} = \frac{4599\ \text{W}}{2\pi(0.031\ \text{m})(10\ \text{m})(119.1 - 25)\ °\text{C}} = \mathbf{25.1\ W/m^2\cdot °C}$$

Discussion If the pipe wall is thicker, the temperature difference between the inner and outer pipe surfaces will be greater. If the pipe has very high thermal conductivity or the pipe wall thickness is very small, then the temperature difference between the inner and outer pipe surfaces may be negligible.

2-73 A subsea pipeline is transporting liquid hydrocarbon. The temperature variation in the pipeline wall, the inner surface temperature of the pipeline, the mathematical expression for the rate of heat loss from the liquid hydrocarbon, and the heat flux through the outer pipeline surface are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipeline.

Properties The pipeline thermal conductivity is given to be 60 W/m · °C.



Analysis The inner and outer radii of the pipeline are

$$r_1 = 0.5\text{ m}/2 = 0.25\text{ m}$$

$$r_2 = 0.25\text{ m} + 0.008\text{ m} = 0.258\text{ m}$$

(*a*) For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $$-k\frac{dT(r_1)}{dr} = h_1[T_{\infty,1} - T(r_1)] \quad \text{(convection at the inner pipeline surface)}$$

$$-k\frac{dT(r_2)}{dr} = h_2[T(r_2) - T_{\infty,2}] \quad \text{(convection at the outer pipeline surface)}$$

Integrating the differential equation once with respect to *r* gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to *r* again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad -k\frac{dT(r_1)}{dr} = -k\frac{C_1}{r_1} = h_1(T_{\infty,1} - C_1 \ln r_1 - C_2)$$

$$r = r_2: \qquad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = h_2(C_1 \ln r_2 + C_2 - T_{\infty,2})$$

C_1 and C_2 can be expressed explicitly as

$$C_1 = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}$$

$$C_2 = T_{\infty,1} - \frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left(\frac{k}{r_1 h_1} - \ln r_1\right)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left[\frac{k}{r_1 h_1} + \ln(r/r_1)\right] + T_{\infty,1}$$

(*b*) The inner surface temperature of the pipeline is

$$T(r_1) = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left[\frac{k}{r_1 h_1} + \ln(r_1/r_1)\right] + T_{\infty,1}$$

$$= -\frac{(70-5)\ ^\circ\text{C}\left[\dfrac{60\ \text{W/m}\cdot{}^\circ\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot{}^\circ\text{C})}\right]}{\dfrac{60\ \text{W/m}\cdot{}^\circ\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot{}^\circ\text{C})} + \ln\left(\dfrac{0.258}{0.25}\right) + \dfrac{60\ \text{W/m}\cdot{}^\circ\text{C}}{(0.258\ \text{m})(150\ \text{W/m}^2\cdot{}^\circ\text{C})}} + 70\ ^\circ\text{C}$$

$$= \mathbf{45.5\ ^\circ C}$$

(*c*) The mathematical expression for the rate of heat loss through the pipeline can be determined from Fourier’s law to be

$$\dot{Q}_{\text{loss}} = -kA\frac{dT}{dr}$$

$$= -k(2\pi r_2 L)\frac{dT(r_2)}{dr} = -2\pi LkC_1$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{1}{2\pi r_1 L h_1} + \dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{2\pi r_2 L h_2}}$$

(*d*) Again from Fourier’s law, the heat flux through the outer pipeline surface is

$$\dot{q}_2 = -k\frac{dT}{dr} = -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2}$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\frac{k}{r_2}$$

$$= \frac{(70-5)\ ^\circ\text{C}}{\dfrac{60\ \text{W/m}\cdot{}^\circ\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot{}^\circ\text{C})} + \ln\left(\dfrac{0.258}{0.25}\right) + \dfrac{60\ \text{W/m}\cdot{}^\circ\text{C}}{(0.258\ \text{m})(150\ \text{W/m}^2\cdot{}^\circ\text{C})}}\left(\frac{60\ \text{W/m}\cdot{}^\circ\text{C}}{0.258\ \text{m}}\right)$$

$$= \mathbf{5947\ W/m^2}$$

Discussion Knowledge of the inner pipeline surface temperature can be used to control wax deposition blockages in the pipeline.

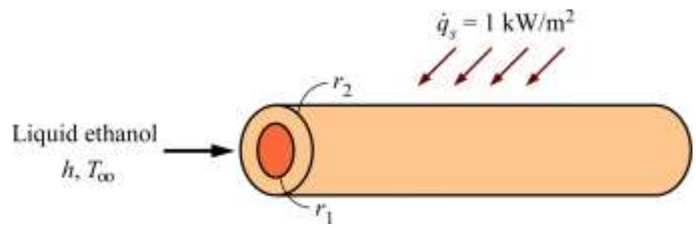
2-74 Liquid ethanol is being transported in a pipe where the outer surface is subjected to heat flux. Convection heat transfer occurs on the inner surface of the pipe. The variation of temperature in the pipe wall and the inner and outer surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to convection while the outer surface at $r = r_2$ is subjected to uniform heat flux.

Properties Thermal conductivity is given to be 15 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = -\dot{q}_s = -k\frac{C_1}{r_2} \quad \rightarrow \quad C_1 = \dot{q}_s\frac{r_2}{k}$$

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)] \quad \rightarrow \quad k\frac{C_1}{r_1} = h(C_1 \ln r_1 + C_2 - T_\infty)$$

Solving for C_2 gives

$$C_2 = C_1\left(\frac{k}{h}\frac{1}{r_1} - \ln r_1\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + C_2 = C_1 \ln r + C_1\left(\frac{k}{h}\frac{1}{r_1} - \ln r_1\right) + T_\infty \quad \rightarrow \quad T(r) = \dot{q}_s\frac{r_2}{k}\left(\frac{k}{h}\frac{1}{r_1} + \ln\frac{r}{r_1}\right) + T_\infty$$

The temperature at $r = r_1$ = 0.015 m (the inner surface of the pipe) is

$$T(r_1) = \frac{\dot{q}_s}{h}\frac{r_2}{r_1} + T_\infty = \frac{1000\ \text{W/m}^2}{50\ \text{W/m}^2\cdot\text{K}}\left(\frac{0.018\ \text{m}}{0.015\ \text{m}}\right) + 10°\text{C} = \mathbf{34.0°C}$$

$$T(r_1) = \mathbf{34.0°C}$$

The temperature at $r = r_2$ = 0.018 m (the outer surface of the pipe) is

$$T(r_2) = \dot{q}_s\frac{r_2}{k}\left(\frac{k}{h}\frac{1}{r_1} + \ln\frac{r_2}{r_1}\right) + T_\infty = (1000\ \text{W/m}^2)\frac{0.018\ \text{m}}{15\ \text{W/m}\cdot\text{K}}\left[\left(\frac{15\ \text{W/m}\cdot\text{K}}{50\ \text{W/m}^2\cdot\text{K}}\right)\frac{1}{0.015\ \text{m}} + \ln\frac{0.018}{0.015}\right] + 10°\text{C}$$

$$T(r_2) = \mathbf{34.2°C}$$

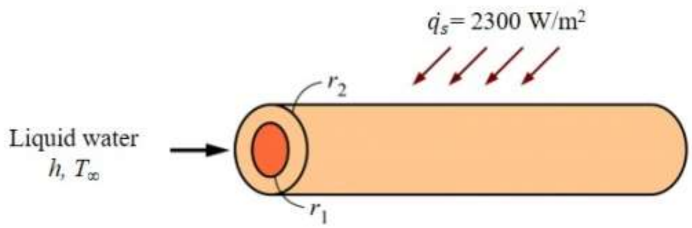
Both the inner and outer surfaces of the pipe are at higher temperatures than the flashpoint of ethanol (16.6°C).

Discussion The outer surface of the pipe should be wrapped with protective insulation to keep the heat input from heating the ethanol inside the pipe.

2-75 Liquid water flows in a tube with the inner surface lined with PVDC lining. The tube outer surface is subjected to a known uniform heat flux. The tube inner diameter, the tube wall thickness, the water temperature, and the convection heat transfer coefficient are known. Formulate the temperature profile in the tube wall, and determine if the PVDC lining is in compliance with the ASME Code for Process Piping.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the tube wall. **4** The inner surface at $r = r_1$ is subjected to convection while the outer surface at $r = r_2$ is subjected to uniform heat flux. **5** The PVDC lining is very thin and the temperature gradient in the lining is negligible.

Properties Thermal conductivity of the tube wall is given to be 15 W/m·K.



AnalysisFor one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \text{or} \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)] \quad \rightarrow \quad k\frac{C_1}{r_1} = h(C_1 \ln r_1 + C_2 - T_\infty)$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = -\dot{q}_s \quad \rightarrow \quad C_1 = \dot{q}_s\frac{r_2}{k}$$

Solving for C_2 yields

$$C_2 = C_1\left(\frac{1}{r_1}\frac{k}{h} - \ln r_1\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the tube wall is determined to be

$$T(r) = C_1 \ln r + C_2 = C_1 \ln r + C_1\left(\frac{1}{r_1}\frac{k}{h} - \ln r_1\right) + T_\infty \quad \rightarrow \quad \boldsymbol{T(r) = \dot{q}_s\frac{r_2}{k}\left(\ln\frac{r}{r_1} + \frac{1}{r_1}\frac{k}{h}\right) + T_\infty}$$

At the tube inner surface ($r = r_1$ = D$_1$/2 = 0.012 m), which is lined with PVDC lining, the temperature is

$$T(r_1) = \dot{q}_s\frac{r_2}{k}\left(\ln\frac{r_1}{r_1} + \frac{1}{r_1}\frac{k}{h}\right) + T_\infty = \left(2300\ \frac{\text{W}}{\text{m}^2}\right)\frac{(0.017\text{ m})}{\left(15\frac{\text{W}}{\text{m}\cdot\text{K}}\right)}\left(\frac{1}{0.012\text{ m}} \times \frac{\left(15\frac{\text{W}}{\text{m}\cdot\text{K}}\right)}{50\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}}\right) + 20°\text{C}$$

$$= \mathbf{85.2}°\text{C} > 79°\text{C}$$

where and

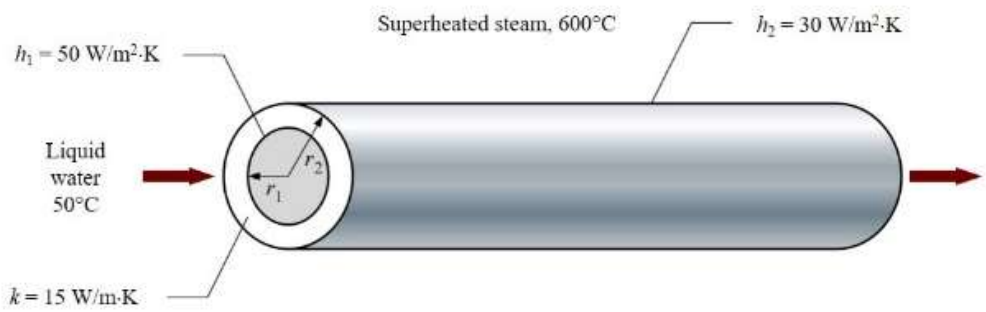
$r_2 = r_1$ + wall thickness = 0.012 m + 0.005 m = 0.017 m.

Discussion The tube inner surface temperature at $r = r_1$ =0.012 m is more than 6°C higher than the recommended maximum temperature of 79°C by the ASME Code for Process Piping (ASME B31.3-2014, A323) for PVDC. Therefore, having the tube inner surface lined with PVDC lining is not in compliance with the code.

2-76 Liquid water flows in a tube with the inner surface lined with PTFE lining. The tube inner surface is subjected to convection with water, the tube outer surface is subjected to convection with superheated steam. Formulate the temperature profile in the tube wall, and determine if the PTFE lining is in compliance with the ASME Code for Process Piping.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the tube wall. **4** The inner surface at $r = r_1$ is subjected to convection with water. **5** The outer surface at $r = r_2$ is subjected to convection with superheated steam. **6** The PTFE lining is very thin and the temperature gradient in the lining is negligible.

Properties Thermal conductivity of the tube wall is given to be 15 W/m·K.



AnalysisFor one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \text{or} \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$r = r_1: \qquad -k\frac{dT(r_1)}{dr} = -k\frac{C_1}{r_1} = h_1[T_{\infty,1} - T(r_1)]$$

$$r = r_2: \qquad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = h_2[T(r_2) - T_{\infty,2}]$$

C_1 and C_2 can be expressed as

$$C_1 = -\frac{T_{\infty,1} - T_{\infty,2}}{\frac{k}{r_1 h_1} + \ln\frac{r_2}{r_1} + \frac{k}{r_2 h_2}}$$

$$C_2 = T_{\infty,1} - \frac{T_{\infty,1} - T_{\infty,2}}{\frac{k}{r_1 h_1} + \ln\frac{r_2}{r_1} + \frac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} - \ln r_1\right)$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the tube wall is determined to be

$$\boldsymbol{T(r) = -\frac{T_{\infty,1} - T_{\infty,2}}{\frac{k}{r_1 h_1} + \ln\frac{r_2}{r_1} + \frac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} + \ln\frac{r}{r_1}\right) + T_{\infty,1}}$$

At the tube inner surface ($r = r_1 = 0.012$ m), which is lined with PTFE lining, the temperature is

$$T(r_1) = -\frac{T_{\infty,1} - T_{\infty,2}}{\frac{k}{r_1 h_1} + \ln\frac{r_2}{r_1} + \frac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} + \ln\frac{r_1}{r_1}\right) + T_{\infty,1}$$

$$T(r_1) = -\frac{(50-600)°\text{C}}{\frac{15\,\frac{\text{W}}{\text{m·K}}}{(0.012\text{ m})\left(50\,\frac{\text{W}}{\text{m}^2\text{·K}}\right)} + \ln\frac{0.017\text{ m}}{0.012\text{ m}} + \frac{15\,\frac{\text{W}}{\text{m·K}}}{(0.017\text{ m})\left(30\,\frac{\text{W}}{\text{m}^2\text{·K}}\right)}}\left[\frac{15\,\frac{\text{W}}{\text{m·K}}}{(0.012\text{ m})\left(50\,\frac{\text{W}}{\text{m}^2\text{·K}}\right)}\right] + 50°\text{C}$$

$$= \mathbf{301}℃ > 260℃$$

whereand $r_2 = r_1$ + wall thickness = 0.012 m + 0.005 m = 0.017 m.

Discussion The tube inner surface temperature at $r = r_1$ =0.012 m is 41°C higher than the recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323) for PTFE. Therefore, having the tube inner surface lined with PTFE lining is not in compliance with the code.

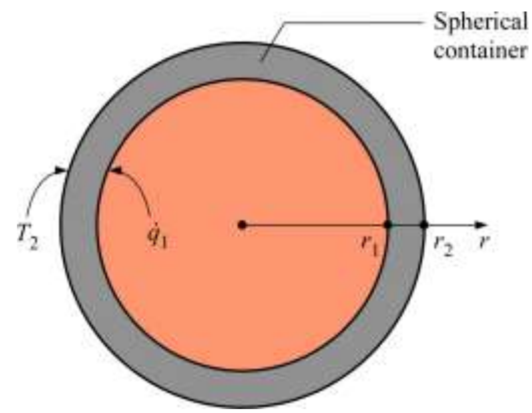
2-77 A spherical container is subjected to uniform heat flux on the inner surface, while the outer surface maintains a constant temperature. The variation of temperature in the container wall and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Temperatures on both surfaces are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the wall. **5** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is at constant temperature T_2.

Properties Thermal conductivity is given to be k = 1.5 W/m·K.

Analysis For one-dimensional heat transfer in the radial direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r^2 \frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1 = -k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1 = -\dot{q}_1 \frac{r_1^2}{k}$$

$$r = r_2: \quad T(r_2) = T_2 = -\frac{C_1}{r_2} + C_2 \quad \rightarrow \quad C_2 = T_2 + \frac{C_1}{r_2} = T_2 - \frac{\dot{q}_1}{k}\frac{r_1^2}{r_2}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = \frac{\dot{q}_1}{k}\frac{r_1^2}{r} + T_2 - \frac{\dot{q}_1}{k}\frac{r_1^2}{r_2} \quad \rightarrow \quad T(r) = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{1}{r} - \frac{1}{r_2}\right) + T_2$$

The temperature at $r = r_1 = 1$ m (the inner surface of the container) is

$$T(r_1) = T_1 = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{1}{r} - \frac{1}{r_2}\right) + T_2$$

$$T_1 = (7000\ \text{W/m}^2)\frac{(1\ \text{m})^2}{(1.5\ \text{W/m}\cdot\text{K})}\left(\frac{1}{1\ \text{m}} - \frac{1}{1.05\ \text{m}}\right) + 25°\text{C} = \mathbf{247°C}$$

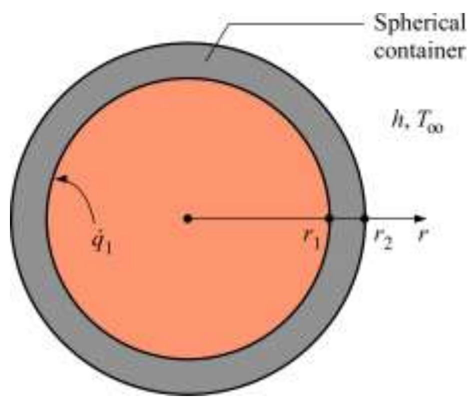
Discussion As expected the inner surface temperature is higher than the outer surface temperature.

2-78 A spherical shell is subjected to uniform heat flux on the inner surface, while the outer surface is subjected to convection heat transfer. The variation of temperature in the shell wall and the outer surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r^2 \frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1 = -k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1 = -\dot{q}_1 \frac{r_1^2}{k}$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty] \quad \rightarrow \quad -k\frac{C_1}{r_2^2} = h\left(-\frac{C_1}{r_2} + C_2 - T_\infty\right)$$

Solving for C_2 gives

$$C_2 = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = \dot{q}_1 \frac{r_1^2}{k}\frac{1}{r} + \dot{q}_1 \frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty \quad \rightarrow \quad T(r) = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{1}{r} + \frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty$$

The temperature at $r = r_2$ (the outer surface of the shell) can be expressed as

$$T(r_2) = \frac{\dot{q}_1}{h}\left(\frac{r_1}{r_2}\right)^2 + T_\infty$$

Discussion Increasing the convection heat transfer coefficient h would decrease the outer surface temperature $T(r_2)$.

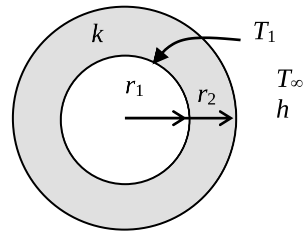
2-79 A spherical container is subjected to specified temperature on the inner surface and convection on the outer surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the midpoint. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k = 30$ W/m·°C.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$



and $T(r_1)=T_1=0°\text{C}$

$$-k\frac{dT(r_2)}{dr}=h[T(r_2)-T_\infty]$$

(*b*) Integrating the differential equation once with respect to *r* gives

$$r^2\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r=r_1$: $\quad T(r_1)=-\frac{C_1}{r_1}+C_2=T_1$

$r=r_2$: $\quad -k\frac{C_1}{r_2^2}=h\left(-\frac{C_1}{r_2}+C_2-T_\infty\right)$

Solving for C_1 and C_2 simultaneously gives

$$C_1=\frac{r_2(T_1-T_\infty)}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}}\quad\text{and}\quad C_2=T_1+\frac{C_1}{r_1}=T_1+\frac{T_1-T_\infty}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}}\frac{r_2}{r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=-\frac{C_1}{r}+T_1+\frac{C_1}{r_1}=C_1\left(\frac{1}{r_1}-\frac{1}{r}\right)+T_1=\frac{T_1-T_\infty}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}}\left(\frac{r_2}{r_1}-\frac{r_2}{r}\right)+T_1$$

$$=\frac{(0-25)°\text{C}}{1-\frac{2.1}{2}-\frac{30\text{ W/m}\cdot°\text{C}}{(18\text{ W/m}^2\cdot°\text{C})(2.1\text{ m})}}\left(\frac{2.1}{2}-\frac{2.1}{r}\right)+0°\text{C}=29.63(1.05-2.1/r)$$

(*c*) The rate of heat conduction through the wall is

$$\dot{Q}=-kA\frac{dT}{dr}=-k(4\pi r^2)\frac{C_1}{r^2}=-4\pi kC_1=-4\pi k\frac{r_2(T_1-T_\infty)}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}}$$

$$=-4\pi(30\text{ W/m}\cdot°\text{C})\frac{(2.1\text{ m})(0-25)°\text{C}}{1-\frac{2.1}{2}-\frac{30\text{ W/m}\cdot°\text{C}}{(18\text{ W/m}^2\cdot°\text{C})(2.1\text{ m})}}=\mathbf{23{,}460\text{ W}}$$

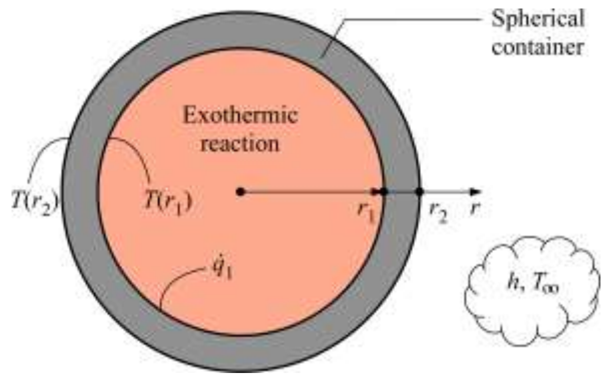
2-80 A spherical container is used for storing chemicals undergoing exothermic reaction that provides a uniform heat flux to its inner surface. The outer surface is subjected to convection heat transfer. The variation of temperature in the container wall and the inner and outer surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection.

Properties Thermal conductivity is given to be 15 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r^2 \frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1 = -k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1 = -\dot{q}_1\frac{r_1^2}{k}$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty] \quad \rightarrow \quad -k\frac{C_1}{r_2^2} = h\left(-\frac{C_1}{r_2} + C_2 - T_\infty\right)$$

Solving for C_2 gives

$$C_2 = \dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = \dot{q}_1\frac{r_1^2}{k}\frac{1}{r} + \dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty \quad \rightarrow \quad T(r) = \dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} + \frac{1}{r} - \frac{1}{r_2}\right) + T_\infty$$

The temperature at $r = r_1 = 0.5$ m (the inner surface of the container) is

$$T(r_1) = \dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} + \frac{1}{r_1} - \frac{1}{r_2}\right) + T_\infty$$

$$T(r_1) = (60000\ \text{W/m}^2)\frac{(0.5\ \text{m})^2}{(15\ \text{W/m}\cdot\text{K})}\left[\left(\frac{15\ \text{W/m}\cdot\text{K}}{1000\ \text{W/m}^2\cdot\text{K}}\right)\frac{1}{(0.55\ \text{m})^2} + \frac{1}{(0.5\ \text{m})} - \frac{1}{(0.55\ \text{m})}\right] + 23°\text{C}$$

$$T(r_1) = \mathbf{254.4°C}$$

The temperature at $r = r_2 = 0.55$ m (the outer surface of the container) is

$$T(r_2) = \frac{\dot{q}_1}{h}\left(\frac{r_1}{r_2}\right)^2 + T_\infty = \frac{60000\ \text{W/m}^2}{1000\ \text{W/m}^2\cdot\text{K}}\left(\frac{0.5\ \text{m}}{0.55\ \text{m}}\right)^2 + 23°\text{C} = \mathbf{72.6°C}$$

The outer surface temperature of the container is above the safe temperature of 50°C.

Discussion To prevent thermal burn, the container's outer surface should be covered with insulation.

2-81 A spherical container is subjected to uniform heat flux on the outer surface and specified temperature on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the outer surface temperature, and the maximum rate of hot water supply are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the mid point. **2** Thermal conductivity is constant. **3** There is no heat generation in the container.

Properties The thermal conductivity is given to be k = 1.5 W/m·°C. The specific heat of water at the average temperature of (100+20)/2 = 60°C is 4.185 kJ/kg·°C (Table A-9).

Analysis (*a*) Noting that the 90% of the 500 W generated by the strip heater is transferred to the container, the heat flux through the outer surface is determined to be

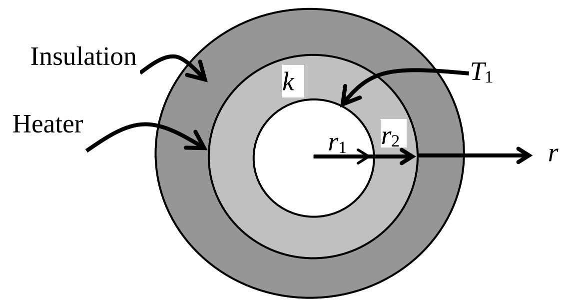
$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{4\pi r_2^2} = \frac{0.90 \times 500 \text{ W}}{4\pi (0.41 \text{ m})^2} = 213.0 \text{ W/m}^2$$

Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

and $$T(r_1) = T_1 = 100°\text{C}$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$



(*b*) Integrating the differential equation once with respect to r gives

$$r^2 \frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r^2 and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$k\frac{C_1}{r_2^2} = \dot{q}_s \rightarrow C_1 = \frac{\dot{q}_s r_2^2}{k}$$

$r = r_1$: $$T(r_1) = T_1 = -\frac{C_1}{r_1} + C_2 \rightarrow C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{\dot{q}_s r_2^2}{k r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)C_1 = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)\frac{\dot{q}_s r_2^2}{k}$$

$$= 100°\text{C} + \left(\frac{1}{0.40 \text{ m}} - \frac{1}{r}\right)\frac{(213 \text{ W/m}^2)(0.41 \text{ m})^2}{1.5 \text{ W/m}\cdot°\text{C}} = 100 + 23.87\left(2.5 - \frac{1}{r}\right)$$

(*c*) The outer surface temperature is determined by direct substitution to be

Outer surface ($r = r_2$): $$T(r_2) = 100 + 23.87\left(2.5 - \frac{1}{r_2}\right) = 100 + 23.87\left(2.5 - \frac{1}{0.41}\right) = \mathbf{101.5°C}$$

Noting that the maximum rate of heat supply to the water is 0.9×500 W = 450 W, water can be heated from 20 to 100°C at a rate of

$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow \dot{m} = \frac{\dot{Q}}{c_p\Delta T} = \frac{0.450 \text{ kJ/s}}{(4.185 \text{ kJ/kg}\cdot°\text{C})(100 - 20)°\text{C}} = 0.00134 \text{ kg/s} = \mathbf{4.84\,kg/h}$$

Heat Generation in a Solid

2-82C Heat generation in a solid is simply conversion of some form of energy into sensible heat energy. Some examples of heat generations are resistance heating in wires, exothermic chemical reactions in a solid, and nuclear reactions in nuclear fuel rods.

2-83C No. Heat generation in a solid is simply the conversion of some form of energy into sensible heat energy. For example resistance heating in wires is conversion of electrical energy to heat.

2-84C The cylinder will have a higher center temperature since the cylinder has less surface area to lose heat from per unit volume than the sphere.

2-85C The rate of heat generation inside an iron becomes equal to the rate of heat loss from the iron when steady operating conditions are reached and the temperature of the iron stabilizes.

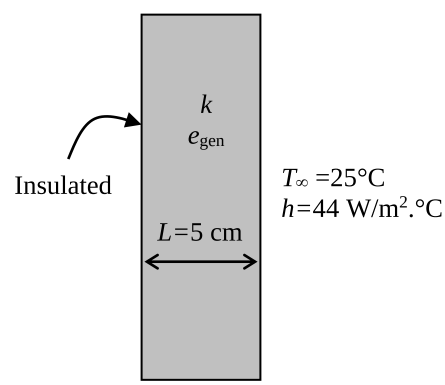
2-86C No, it is not possible since the highest temperature in the plate will occur at its center, and heat cannot flow "uphill."

2-87 Heat is generated uniformly in a large brass plate. One side of the plate is insulated while the other side is subjected to convection. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =111 W/m·°C.

Analysis This insulated plate whose thickness is L is equivalent to one-half of an uninsulated plate whose thickness is 2L since the midplane of the uninsulated plate can be treated as insulated surface. The highest temperature will occur at the insulated surface while the lowest temperature will occur at the surface which is exposed to the environment. Note that L in the following relations is the full thickness of the given plate since the insulated side represents the center surface of a plate whose thickness is doubled. The desired values are determined directly from



$$T_s = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h} = 25°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05 \text{ m})}{44 \text{ W/m}^2\cdot°\text{C}} = \mathbf{252.3°C}$$

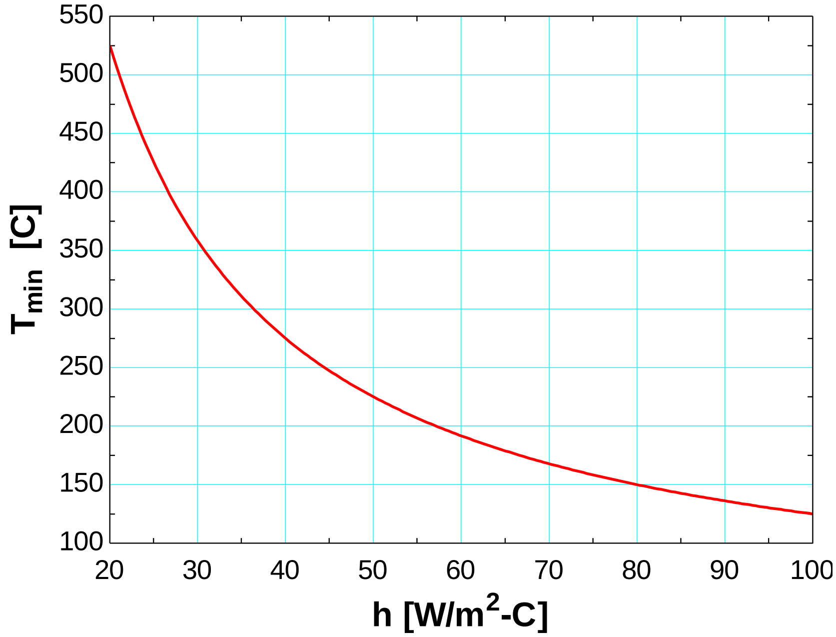
$$T_o = T_s + \frac{\dot{e}_{\text{gen}} L^2}{2k} = 252.3°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05 \text{ m})^2}{2(111 \text{ W/m}\cdot°\text{C})} = \mathbf{254.6°C}$$

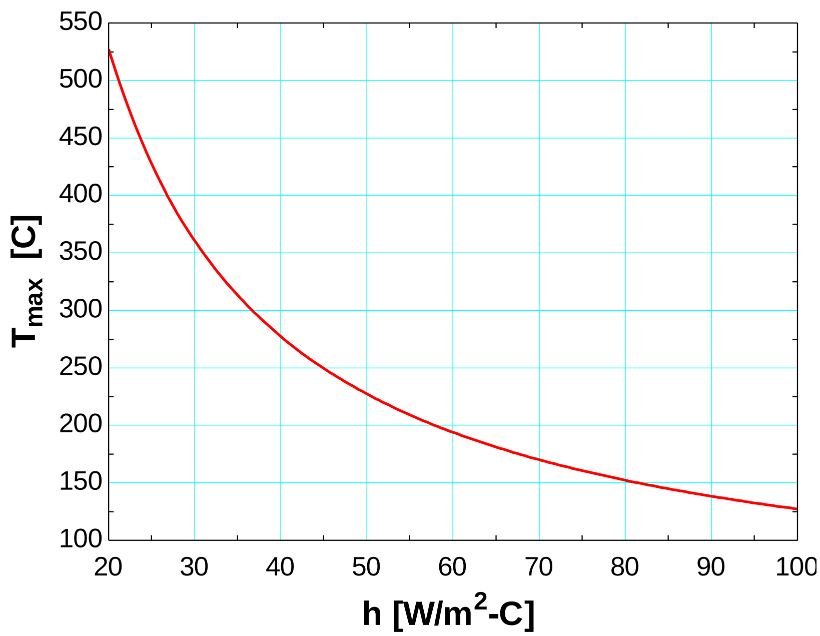
2-88 Prob. 2-87 is reconsidered. The effect of the heat transfer coefficient on the highest and lowest temperatures in the plate is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.05 [m]
k=111 [W/m-C]
g_dot=2E5 [W/m^3]
T_infinity=25 [C]
h=44 [W/m^2-C]
"ANALYSIS"
T_min=T_infinity+(g_dot*L)/h
T_max=T_min+(g_dot*L^2)/(2*k)
```

h [W/m^2.C]	T_{min} [C]	T_{max} [C]
20	525	527.3
25	425	427.3
30	358.3	360.6
35	310.7	313
40	275	277.3
45	247.2	249.5
50	225	227.3
55	206.8	209.1
60	191.7	193.9
65	178.8	181.1
70	167.9	170.1
75	158.3	160.6
80	150	152.3
85	142.6	144.9
90	136.1	138.4
95	130.3	132.5
100	125	127.3

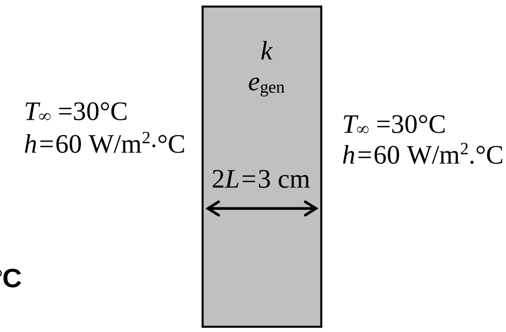




2-89 Both sides of a large stainless steel plate in which heat is generated uniformly are exposed to convection with the environment. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =15.1 W/m·°C.



Analysis The lowest temperature will occur at surfaces of plate while the highest temperature will occur at the midplane. Their values are determined directly from

$$T_s = T_\infty + \frac{\dot{e}_{\rm gen} L}{h} = 30^\circ\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\ \text{m})}{60\ \text{W/m}^2\cdot{}^\circ\text{C}} = \mathbf{155^\circ C}$$

$$T_o = T_s + \frac{\dot{e}_{\rm gen} L^2}{2k} = 155^\circ\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\ \text{m})^2}{2(15.1\ \text{W/m}\cdot{}^\circ\text{C})} = \mathbf{158.7^\circ C}$$

2-90 Heat is generated in a large plane wall whose one side is insulated while the other side is subjected to convection. The mathematical formulation, the variation of temperature in the wall, the relation for the surface temperature, and the relation for the maximum temperature rise in the plate are to be determined for steady one-dimensional heat transfer.

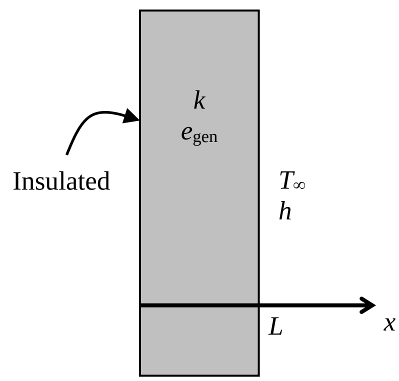
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in *x* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $\quad \dfrac{dT(0)}{dx}=0 \quad$ (insulated surface at $x = 0$)

$$-k\frac{dT(L)}{dx}=h[T(L)-T_\infty]$$



(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{\dot{e}_{\text{gen}}}{k} \rightarrow \frac{dT}{dx}=-\frac{\dot{e}_{\text{gen}}}{k}x+C_1$$

Integrating one more time,

$$T(x)=\frac{-\dot{e}_{\text{gen}}x^2}{2k}+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at $x = 0$: $\quad \dfrac{dT(0)}{dx}=0\rightarrow \dfrac{-\dot{e}_{\text{gen}}}{k}(0)+C_1=0 \rightarrow C_1=0$

B. C. at $x = L$:

$$-k\left(\frac{-\dot{e}_{\text{gen}}}{k}L\right)=h\left(\frac{-\dot{e}_{\text{gen}}L^2}{2k}+C_2-T_\infty\right)$$

$$\dot{e}_{\text{gen}}L=\frac{-h\dot{e}_{\text{gen}}L^2}{2k}-hT_\infty+C_2 \rightarrow C_2=\dot{e}_{\text{gen}}L+\frac{h\dot{e}_{\text{gen}}L^2}{2k}+hT_\infty$$

Dividing by h: $\quad C_2=\dfrac{\dot{e}_{\text{gen}}L}{h}+\dfrac{\dot{e}_{\text{gen}}L^2}{2k}+T_\infty$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging give

$$T(x)=\frac{-\dot{e}_{\text{gen}}x^2}{2k}+\frac{\dot{e}_{\text{gen}}L}{h}+\frac{\dot{e}_{\text{gen}}L^2}{2k}+T_\infty=\frac{\dot{e}_{\text{gen}}}{2k}(L^2-x^2)+\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

which is the desired solution for the temperature distribution in the wall as a function of *x*.

(*c*) The temperatures at two surfaces and the temperature difference between these surfaces are

$$T(0)=\frac{\dot{e}_{\text{gen}}L^2}{2k}+\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

$$T(L)=\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

$$\Delta T_{\max}=T(0)-T(L)=\frac{\dot{e}_{\text{gen}}L^2}{2k}$$

Discussion These relations are obtained without using differential equations in the text (see Eqs. 2-67 and 2-73).

2-91E Heat is generated in a large plane wall whose one side is insulated while the other side is maintained at a specified temperature. The mathematical formulation, the variation of temperature in the wall, and the highest temperature in the wall are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional, and there is thermal symmetry about the center plane. **3** Thermal conductivity is constant. **4** Heat generation varies with location in the x direction.

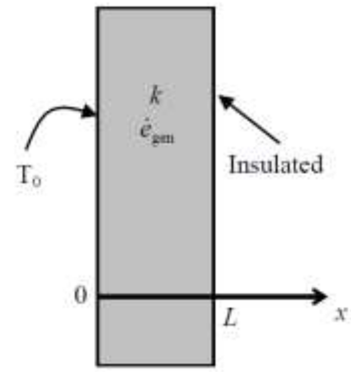
Properties The thermal conductivity is given to be k = 5 Btu/h·ft·°F.

Analysis (a) Noting that heat transfer is steady and one-dimensional in x direction, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}(x)}{k}=0$$

where $\dot{e}_{gen}=ax^2$

$$\frac{d^2T}{dx^2}=-\frac{\dot{e}_{\text{gen}}(x)}{k}=-\frac{a}{k}x^2$$



The boundary conditions for this problem are:

$T(0)=T_0$ (specified surface temperature at x = 0)

$\frac{dT(L)}{dx}=0$ (insulated surface at x = L)

(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{a}{k}x^2 \rightarrow \frac{dT}{dx}=-\frac{1}{3}\frac{a}{k}x^3+C_1$$

Integrating one more time,

$$T(x)=-\frac{1}{12}\frac{a}{k}x^4+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at x = 0: $T(0)=T_0=C_2$

B.C. at x = L: $\frac{dT(L)}{dx}=-\frac{1}{3}\frac{a}{k}L^3+C_1=0 \rightarrow C_1=\frac{aL^3}{3k}$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging gives

$$T(x)=-\frac{1}{12}\frac{a}{k}x^4+\frac{aL^3}{3k}x+T_0 \qquad (2)$$

(c) The highest (maximum) temperature occurs at the insulate surface (x = L) and is determined by substituting the given quantities into Eq. (2), the result is

$$T(L)=T_{\text{max}}=-\frac{1}{12}\frac{a}{k}L^4+\frac{aL^3}{3k}L+T_0=\frac{aL^4}{4k}+T_0$$

$$=\frac{(1200\ \text{Btu/h}\cdot\text{ft}^5)(1\,\text{ft}^4)}{4(5\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})}+700\ {}^\circ\text{F}$$

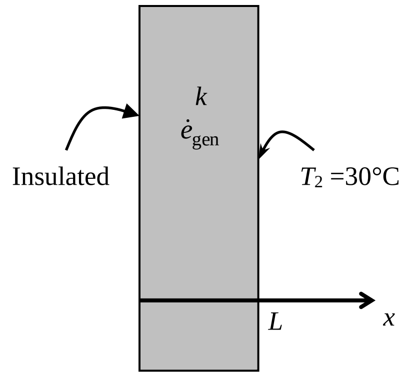
$$=\mathbf{760\ {}^\circ F}$$

2-92 Heat is generated in a large plane wall whose one side is insulated while the other side is maintained at a specified temperature. The mathematical formulation, the variation of temperature in the wall, and the temperature of the insulated surface are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and there is thermal symmetry about the center plane. **3** Thermal conductivity is constant. **4** Heat generation varies with location in the *x* direction.

Properties The thermal conductivity is given to be $k = 30$ W/m·°C.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in *x* direction, the mathematical formulation of this problem can be expressed as



$$\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}(x)}{k} = 0$$

where $\dot{e}_{gen} = \dot{e}_0 e^{-0.5x/L}$ and $\dot{e}_0 = 8\times10^6$ W/m^3

and $\frac{dT(0)}{dx} = 0$ (insulated surface at $x = 0$)

$T(L) = T_2 = 30$°C (specified surface temperature)

(*b*) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2} = -\frac{\dot{e}_0}{k} e^{-0.5x/L} \rightarrow \frac{dT}{dx} = -\frac{\dot{e}_0}{k}\frac{e^{-0.5x/L}}{-0.5/L} + C_1 \rightarrow \frac{dT}{dx} = \frac{2\dot{e}_0 L}{k} e^{-0.5x/L} + C_1$$

Integrating one more time,

$$T(x) = \frac{2\dot{e}_0 L}{k}\frac{e^{-0.5x/L}}{-0.5/L} + C_1 x + C_2 \rightarrow T(x) = -\frac{4\dot{e}_0 L^2}{k} e^{-0.5x/L} + C_1 x + C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at $x = 0$: $\frac{dT(0)}{dx} = \frac{2\dot{e}_0 L}{k} e^{-0.5\times 0/L} + C_1 \rightarrow 0 = \frac{2\dot{e}_0 L}{k} + C_1 \rightarrow C_1 = -\frac{2\dot{e}_0 L}{k}$

B. C. at $x = L$: $T(L) = T_2 = -\frac{4\dot{e}_0 L^2}{k} e^{-0.5L/L} + C_1 L + C_2 \rightarrow C_2 = T_2 + \frac{4\dot{e}_0 L^2}{k} e^{-0.5} + \frac{2\dot{e}_0 L^2}{k}$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging give

$$T(x) = T_2 + \frac{\dot{e}_0 L^2}{k}[4(e^{-0.5} - e^{-0.5x/L}) + 2(1 - x/L)]$$

which is the desired solution for the temperature distribution in the wall as a function of *x*.

(*c*) The temperature at the insulate surface ($x = 0$) is determined by substituting the known quantities to be

$$\begin{aligned} T(0) &= T_2 + \frac{\dot{e}_0 L^2}{k}[4(e^{-0.5} - e^0) + (2 - 0/L)] \\ &= 30°\text{C} + \frac{(8\times10^6 \text{ W/m}^3)(0.05 \text{ m})^2}{(30 \text{ W/m}\cdot°\text{C})}[4(e^{-0.5} - 1) + (2 - 0)] \\ &= \mathbf{314°C} \end{aligned}$$

Therefore, there is a temperature difference of almost 300°C between the two sides of the plate.

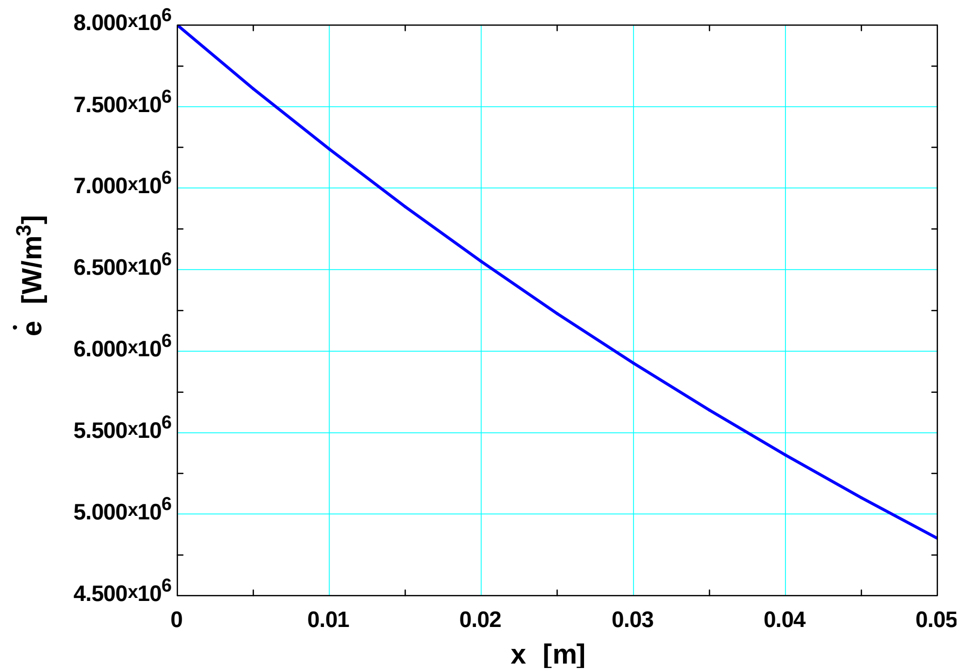
2-93 Prob. 2-92 is reconsidered. The heat generation as a function of the distance is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.05 [m]
T_s=30 [C]
k=30 [W/m-C]
e_dot_0=8E6 [W/m^3]

"ANALYSIS"
e_dot=e_dot_0*exp((-0.5*x)/L) "Heat generation as a function of x"
```

x [m]	e [W/m^3]
0	8.000E+06
0.005	7.610E+06
0.01	7.239E+06
0.015	6.886E+06
0.02	6.550E+06
0.025	6.230E+06
0.03	5.927E+06
0.035	5.638E+06
0.04	5.363E+06
0.045	5.101E+06
0.05	4.852E+06



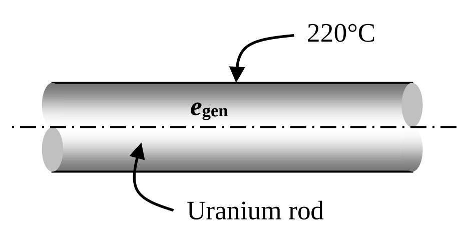
2-94 A nuclear fuel rod with a specified surface temperature is used as the fuel in a nuclear reactor. The center temperature of the rod is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the rod is uniform.

Properties The thermal conductivity is given to be k = 29.5 W/m·°C.

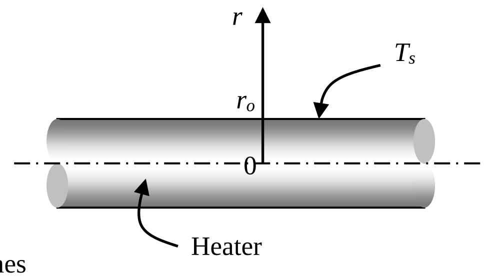
Analysis The center temperature of the rod is determined from

$$T_o = T_s + \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = 220\,°\text{C} + \frac{(4\times10^7\ \text{W/m}^3)(0.005\ \text{m})^2}{4(29.5\ \text{W/m.°C})} = \mathbf{228\,°C}$$



2-95E Heat is generated uniformly in a resistance heater wire. The temperature difference between the center and the surface of the wire is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.



Properties The thermal conductivity is given to be k = 5.8 Btu/h·ft·°F.

Analysis The resistance heater converts electric energy into heat at a rate of 3 kW. The rate of heat generation per unit length of the wire is

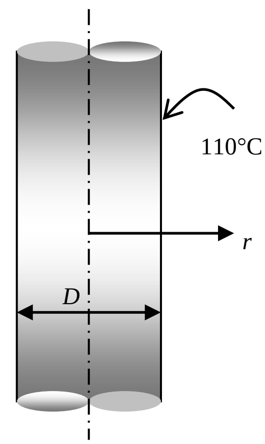
$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{gen}}{\pi r_o^2 L} = \frac{(3\times 3412.14 \text{ Btu/h})}{\pi(0.04/12 \text{ ft})^2 (1 \text{ ft})} = 2.933\times 10^8 \text{ Btu/h}\cdot\text{ft}^3$$

Then the temperature difference between the centerline and the surface becomes

$$\Delta T_{\max} = \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = \frac{(2.933\times 10^8 \text{ Btu/h}\cdot\text{ft}^3)(0.04/12 \text{ ft})^2}{4(5.8 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})} = \mathbf{140.5°F}$$

2-96 A 2-kW resistance heater wire with a specified surface temperature is used to boil water. The center temperature of the wire is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.



Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The resistance heater converts electric energy into heat at a rate of 2 kW. The rate of heat generation per unit volume of the wire is

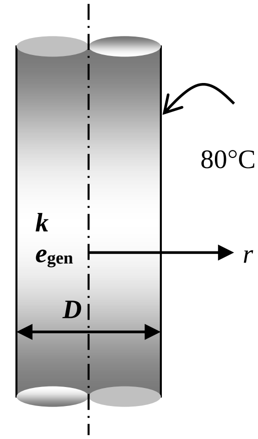
$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi r_o^2 L} = \frac{2000 \text{ W}}{\pi(0.002 \text{ m})^2 (0.9 \text{ m})} = 1.768\times 10^8 \text{ W/m}^3$$

The center temperature of the wire is then determined from Eq. 2-71 to be

$$T_o = T_s + \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = 110°\text{C} + \frac{(1.768\times 10^8 \text{ W/m}^3)(0.002 \text{ m})^2}{4(20 \text{ W/m.°C})} = \mathbf{118.8°C}$$

2-97 Heat is generated in a long solid cylinder with a specified surface temperature. The variation of temperature in the cylinder is given by

$$T(r) = \frac{\dot{e}_{\text{gen}} r_o^2}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right]+T_s$$



(*a*) Heat conduction is steady since there is no time *t* variable involved.

(*b*) Heat conduction is a one-dimensional.

(*c*) Using Eq. (1), the heat flux on the surface of the cylinder at $r = r_o$ is determined from its definition to be

$$\dot{q}_s = -k\frac{dT(r_o)}{dr} = -k\left[\frac{\dot{e}_{\text{gen}} r_o^2}{k}\left(-\frac{2r}{r_o^2}\right)\right]_{r=r_0}$$

$$= -k\left[\frac{\dot{e}_{\text{gen}} r_o^2}{k}\left(-\frac{2r_o}{r_o^2}\right)\right] = 2\dot{e}_{\text{gen}} r_o = 2(35\ \text{W/cm}^3)(4\ \text{cm}) = \mathbf{280\ W/cm^2}$$

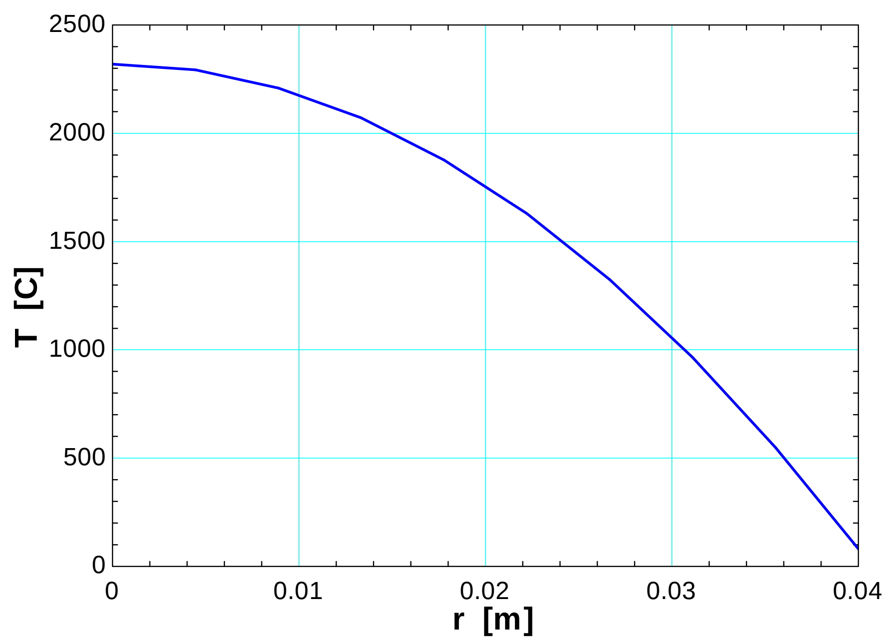
2-98  Prob. 2-97 is reconsidered. The temperature as a function of the radius is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_0=0.04 [m]
k=25 [W/m-C]
e_dot_gen=35E+6 [W/m^3]
T_s=80 [C]

"ANALYSIS"
T=(e_dot_gen*r_0^2)/k*(1-(r/r_0)^2)+T_s "Variation of temperature"
```

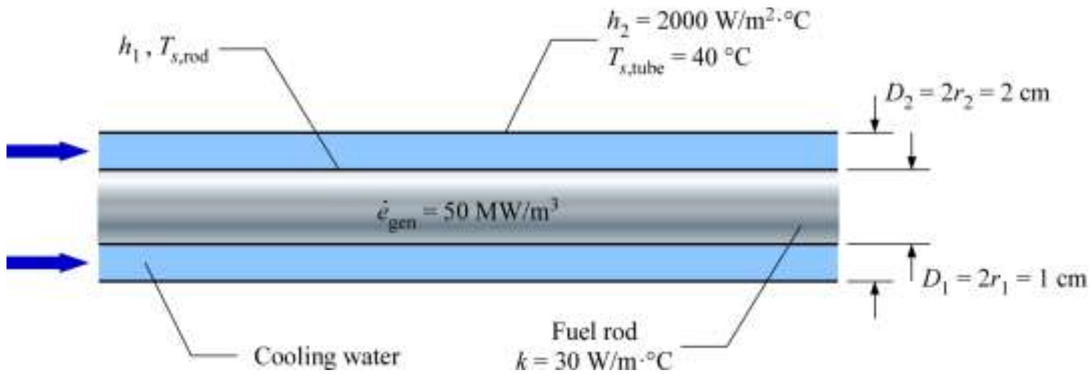
r [m]	T [C]
0	2320
0.004444	2292
0.008889	2209
0.01333	2071
0.01778	1878
0.02222	1629
0.02667	1324
0.03111	964.9
0.03556	550.1
0.04	80



2-99 A cylindrical nuclear fuel rod is cooled by water flowing through its encased concentric tube. The average temperature of the cooling water is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat generation in the fuel rod is uniform.

Properties The thermal conductivity is given to be 30 W/m · °C.



Analysis The rate of heat transfer by convection at the fuel rod surface is equal to that of the concentric tube surface:

$$h_1 A_{s,1}(T_{s,\text{rod}} - T_\infty) = h_2 A_{s,2}(T_\infty - T_{s,\text{tube}})$$

$$h_1(2\pi\, r_1 L)(T_{s,\text{rod}} - T_\infty) = h_2(2\pi\, r_2 L)(T_\infty - T_{s,\text{tube}})$$

$$T_{s,\text{rod}} = \frac{h_2 r_2}{h_1 r_1}(T_\infty - T_{s,\text{tube}}) + T_\infty \qquad (a)$$

The average temperature of the cooling water can be determined by applying Eq. 2-68:

$$T_{s,\text{rod}} = T_\infty + \frac{\dot{e}_{\text{gen}} r_1}{2h_1} \qquad (b)$$

Substituting Eq. (*a*) into Eq. (*b*) and solving for the average temperature of the cooling water gives

$$\frac{h_2 r_2}{h_1 r_1}(T_\infty - T_{s,\text{tube}}) + T_\infty = T_\infty + \frac{\dot{e}_{\text{gen}} r_1}{2h_1}$$

$$T_\infty = \frac{r_1}{r_2}\frac{\dot{e}_{\text{gen}} r_1}{2h_2} + T_{s,\text{tube}}$$

$$= \frac{0.005\text{ m}}{0.010\text{ m}}\left[\frac{(50\times 10^6\text{ W/m}^3)(0.005\text{ m})}{2(2000\text{ W/m}^2\cdot{}^\circ\text{C})}\right] + 40\ {}^\circ\text{C}$$

$$= \mathbf{71.3\ {}^\circ C}$$

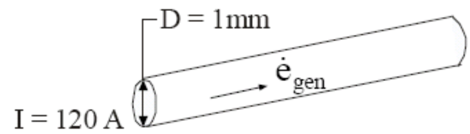
Discussion The given information is not sufficient for one to determine the fuel rod surface temperature. The convection heat transfer coefficient for the fuel rod surface (h_1) or the centerline temperature of the fuel rod (T_0) is needed to determine the fuel rod surface temperature.

2-100 The heat generation and the maximum temperature rise in a solid stainless steel wire.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be k = 14 W/m·K.

Analysis (*a*) The heat generation per unit volume of the wire is



$$\dot{e}_{gen} = \frac{\dot{E}_{gen,electric}}{V_{wire}} = \frac{I^2 R_e}{\pi r_o^2 L}$$

With electrical resistance defined as

$$R_e = \frac{\rho L}{A} \quad (\Omega)$$

where ρ = electrical resistivity (Ω·m), L = wire length (m), A = wire cross-sectional area $\pi D^2/4$ (m^2)

Combining equations for $\dot{e}_{gen}$ and R_e, we have

$$\dot{e}_{gen} = \frac{I^2 \rho}{A^2} = \frac{I^2 \rho}{(\pi D^2/4)^2} = \frac{16 I^2 \rho}{\pi^2 D^4}$$

$$\dot{e}_{gen} = \frac{16\,(120\text{A})^2\,(45\times 10^{-8}\ \Omega\cdot\text{m})}{\pi^2\,(0.001\ \text{m})^4} = \mathbf{1.05 \times 10^{10}\ W/m^3}$$

(*b*) The maximum temperature rise in the solid stainless steel wire is obtained from

$$T_o - T_s = \Delta T_{max,cylinder} = \frac{\dot{e}_{gen}\, r_o^2}{4k} \quad (\text{W/m}^3)$$

$$\Delta T_{max} = \frac{(1.05\times 10^{10}\ \text{W/m}^3)(0.0005\,m)^2}{4\,(14\ \text{W/m}\cdot\text{K})} = \mathbf{47^oC}$$

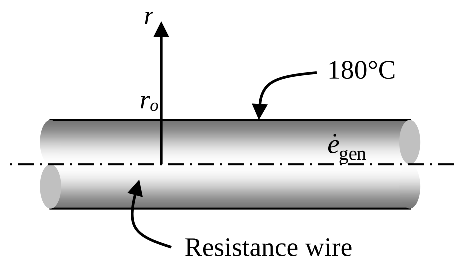
Discussion The maximum temperature rise in the wire can be reduced by increasing the convective heat transfer coefficient and thus reducing the surface temperature.

2-101 A long homogeneous resistance heater wire with specified surface temperature is used to heat the air. The temperature of the wire 3.5 mm from the center is to be determined in steady operation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 8 W/m·°C.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $T(r_o)=T_s=180°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the boundary condition at the center since it is related to the first derivative of the temperature. It yields

B.C. at r = 0: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}\times 0+C_1 \rightarrow C_1=0$$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r$$

and $$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2 \qquad (b)$$

Applying the other boundary condition at $r=r_o$,

B. C. at $r=r_o$: $$T_s=-\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+C_2 \rightarrow C_2=T_s+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2$$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_s+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r. The temperature 3.5 mm from the center line (r = 0.0035 m) is determined by substituting the known quantities to be

$$T(0.0035\text{ m})=T_s+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)=180\,°\text{C}+\frac{5\times10^7\text{ W/m}^3}{4\times(8\text{ W/m}\cdot°\text{C})}[(0.005\text{ m})^2-(0.0035\text{ m})^2]=\mathbf{200°C}$$

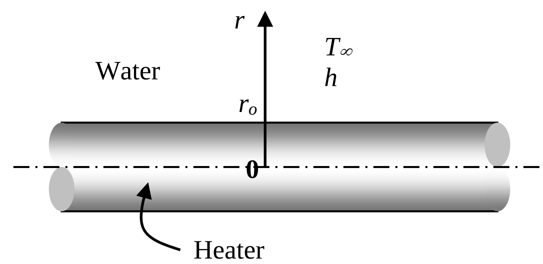
Thus the temperature at that location will be about 20°C above the temperature of the outer surface of the wire.

2-102 A long homogeneous resistance heater wire with specified convection conditions at the surface is used to boil water. The mathematical formulation, the variation of temperature in the wire, and the temperature at the centerline of the wire are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.2 W/m·K.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $$-k\frac{dT(r_o)}{dr}=h[T(r_o)-T_\infty] \quad \text{(convection at the outer surface)}$$

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the second boundary condition since it is related to the first derivative of the temperature by replacing all occurrences of r and dT/dr in the equation above by zero. It yields

B.C. at $r = 0$: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}\times 0+C_1 \quad \rightarrow \quad C_1=0$$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r$$

and $$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2 \qquad (b)$$

Applying the second boundary condition at $r=r_o$,

B. C. at $r=r_o$: $$k\frac{\dot{e}_{\text{gen}}r_o}{2k}=h\left(-\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+C_2-T_\infty\right) \quad \rightarrow \quad C_2=T_\infty+\frac{\dot{e}_{\text{gen}}r_o}{2h}+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2$$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_\infty+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)+\frac{\dot{e}_{\text{gen}}r_o}{2h}$$

which is the desired solution for the temperature distribution in the wire as a function of r. Then the temperature at the center line (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_\infty+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+\frac{\dot{e}_{\text{gen}}r_o}{2h}$$

$$=100^\circ\text{C}+\frac{(16.4\times10^6\ \text{W/m}^3)(0.006\ \text{m})^2}{4\times(15.2\ \text{W/m}\cdot\text{K})}+\frac{(16.4\times10^6\ \text{W/m}^3)(0.006\ \text{m})}{2\times(3200\ \text{W/m}^2\cdot\text{K})}=\mathbf{125^\circ C}$$

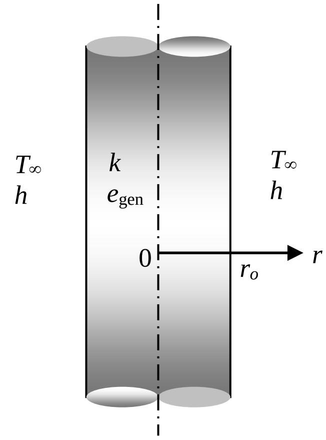
Thus the centerline temperature will be 25°C above the temperature of the surface of the wire.

2-103 A long resistance heater wire is subjected to convection at its outer surface. The surface temperature of the wire is to be determined using the applicable relations directly and by solving the applicable differential equation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.1 W/m·°C.

Analysis (a) The heat generation per unit volume of the wire is



$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi r_o^2 L} = \frac{2000\text{ W}}{\pi(0.001\text{ m})^2(6\text{ m})} = 1.061\times10^8\text{ W/m}^3$$

The surface temperature of the wire is then (Eq. 2-68)

$$T_s = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{2h} = 20°\text{C} + \frac{(1.061\times10^8\text{ W/m}^3)(0.001\text{ m})}{2(175\text{ W/m}^2\cdot°\text{C})} = \mathbf{323°C}$$

(b) The mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

and $\quad -k\dfrac{dT(r_o)}{dr} = h[T(r_o) - T_\infty]$ (convection at the outer surface)

$$\frac{dT(0)}{dr} = 0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and integrating gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{e}_{\text{gen}}}{k}r \;\rightarrow\; r\frac{dT}{dr} = -\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2} + C_1 \qquad (a)$$

Applying the boundary condition at the center line,

B.C. at r = 0: $\quad 0\times\dfrac{dT(0)}{dr} = -\dfrac{\dot{e}_{\text{gen}}}{2k}\times 0 + C_1 \;\rightarrow\; C_1 = 0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr} = -\frac{\dot{e}_{\text{gen}}}{2k}r \qquad \rightarrow \qquad T(r) = -\frac{\dot{e}_{\text{gen}}}{4k}r^2 + C_2 \qquad (b)$$

Applying the boundary condition at $r = r_o$,

B. C. at $r = r_o$: $\quad -k\dfrac{\dot{e}_{\text{gen}} r_o}{2k} = h\left(-\dfrac{\dot{e}_{\text{gen}}}{4k}r_o^2 + C_2 - T_\infty\right) \;\rightarrow\; C_2 = T_\infty + \dfrac{\dot{e}_{\text{gen}} r_o}{2h} + \dfrac{\dot{e}_{\text{gen}}}{4k}r_o^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r) = T_\infty + \frac{\dot{e}_{\text{gen}}}{4k}(r_o^2 - r^2) + \frac{\dot{e}_{\text{gen}} r_o}{2h}$$

which is the temperature distribution in the wire as a function of r. Then the temperature of the wire at the surface ($r = r_o$) is determined by substituting the known quantities to be

$$T(r_0) = T_\infty + \frac{\dot{e}_{\text{gen}}}{4k}(r_o^2 - r_o^2) + \frac{\dot{e}_{\text{gen}} r_0}{2h} = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{2h} = 20°\text{C} + \frac{(1.061\times10^8\text{ W/m}^3)(0.001\text{ m})}{2(175\text{ W/m}^2\cdot°\text{C})} = \mathbf{323°C}$$

Note that both approaches give the same result.

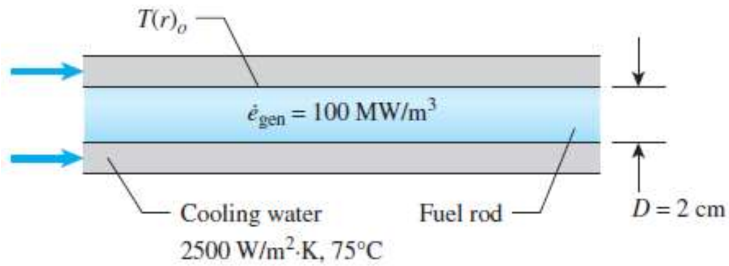
2-104 A cylindrical fuel rod is cooled by water flowing through its encased concentric tube while generating a uniform heat. The variation of temperature in the fuel rod and the center and surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady and one-dimensional with thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** The rod surface at $r = r_o$ is subjected convection. **4** Heat generation in the rod is uniform.

Properties The thermal conductivity is given to be 30 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate with heat generation can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0 \quad \text{or} \quad \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r^2+C_1 \quad \text{or} \quad \frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r+\frac{C_1}{r}$$

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0: \quad \frac{dT(0)}{dr}=0 \quad \rightarrow \quad C_1=0$$

$$r=r_o: \quad -k\frac{dT(r_o)}{dr}=h[T(r_o)-T_\infty] \quad \rightarrow \quad k\frac{\dot{e}_{\text{gen}}}{2k}r_o=h\left(-\frac{\dot{e}_{\text{gen}}}{4k}r_0^2+C_2-T_\infty\right)$$

Solving for C_2 gives

$$C_2=\frac{\dot{e}_{\text{gen}}}{2h}r_o+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+\frac{\dot{e}_{\text{gen}}}{2h}r_o+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+T_\infty \quad \rightarrow \quad T(r)=\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)+\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty$$

The temperature at $r = 0$ (the centerline of the rod) is

$$T(0)=\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty=\frac{100\times10^6\ \text{W/m}^3}{4(30\ \text{W/m}\cdot\text{K})}(0.01\ \text{m})^2+\frac{100\times10^6\ \text{W/m}^3}{2(2500\ \text{W/m}^2\cdot\text{K})}(0.01\ \text{m})+75°\text{C}$$

$$T(0)=\mathbf{358°C}$$

The temperature at $r = r_o = 0.01$ m (the surface of the rod) is

$$T(r_o)=\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty=\frac{100\times10^6\ \text{W/m}^3}{2(2500\ \text{W/m}^2\cdot\text{K})}(0.01\ \text{m})+75°\text{C}=\mathbf{275°C}$$

Fuel rod surface not cooled adequately.

Discussion The temperature of the fuel rod surface is 75°C higher than the temperature necessary to prevent the cooling water from reaching the CHF. To keep the temperature of the fuel rod surface below 200°C, the convection heat transfer coefficient of the cooling water should be kept above 4000 W/m^2·K. This can be done either by increasing the mass flow rate of the cooling water or by decreasing the inlet temperature of the cooling water. The topic of critical heat flux is covered in Chapter 10 (Boiling and Condensation).

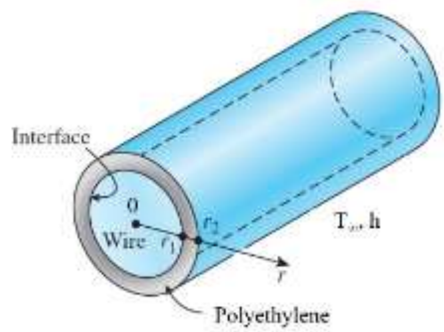
2-105 A long electrical resistance wire that is generating heat uniformly is covered with polyethylene insulation. Formulate the temperature profiles for the wire and the polyethylene insulation. Determine the temperature at the interface of the wire and the insulation, and the temperature at the center of the wire. Conclude whether the polyethylene insulation for the wire meets the ASTM D1351 standard.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Heat generation in the wire is uniform. **4** There is no contact resistance at the interface of the wire and the insulation, $r = r_1$. **5** At the center of the wire, $r = 0$, is a symmetry boundary. **6** The outer surface of the insulation, $r = r_2$, is subjected to convection and radiation.

PropertiesThe thermal conductivities of the wire and the polyethylene insulation are given to be k_{wire} = 15 W/m·K and k_{ins} = 0.4 W/m·K, respectively.

AnalysisFor one-dimensional heat transfer in the radial r direction with uniform heat generation, the differential equation for heat conduction in cylindrical coordinate for the wire can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}=0 \qquad \text{or}\ \frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{wire}}}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k_{\text{wire}}}r^2+C_1$$

$$T_{\text{wire}}(r)=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r^2+C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0:\qquad \frac{dT_{\text{wire}}(0)}{dr}=0 \qquad\rightarrow\qquad C_1=0$$

$$r=r_1:\qquad T_{\text{wire}}(r_1)=T_I=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2+C_2 \qquad\rightarrow\qquad C_2=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the wire is determined to be

$$\boldsymbol{T_{\text{wire}}(r)=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}(r_1^2-r^2)}\ \text{for}\qquad 0\le r\le r_1$$

The insulation layer does not involve any heat generation, the heat conduction equation in the insulation layer is

$$\frac{d}{dr}\left(r\frac{dT_{\text{ins}}}{dr}\right)=0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{ins}}}{dr}=C_3\ \text{or}\ \frac{dT_{\text{ins}}}{dr}=\frac{C_3}{r}$$

$$T_{\text{ins}}(r)=C_3\ln r+C_4$$

where C_3 and C_4 are arbitrary constants. Applying the boundary conditions yields

$$r=r_1:\qquad -k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr}=-k_{\text{ins}}\frac{dT_{\text{ins}}(r_1)}{dr} \qquad\rightarrow\qquad \frac{\dot{e}_{\text{gen}}}{2}r_1=-k_{\text{ins}}\frac{C_3}{r_1}$$

$$r=r_2:\qquad -k_{\text{ins}}\frac{dT_{\text{ins}}(r_2)}{dr}=-k_{\text{ins}}\frac{C_3}{r_2}=h_{\text{combined}}[T_{\text{ins}}(r_2)-T_{\text{surr}}]$$

Note that $h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}}$. The arbitrary constants C_3 and C_4 can be expressed as

$$C_3 = -\frac{\dot{e}_{\text{gen}}}{2}\frac{r_1^2}{k_{\text{ins}}}$$

$$C_4 = T_{\text{surr}} + \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln r_2\right)$$

Substituting C_3 and C_4 into the general solution, the temperature profile in the insulation layer is determined to be

$$\boldsymbol{T_{\text{ins}}(r) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln\frac{r_2}{r}\right) + T_{\text{surr}}} \text{ for } r_1 \le r \le r_2$$

At the interface of the wire and the insulation, $r = r_1$, we have

$$T_I = T_{\text{ins}}(r_1) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln\frac{r_2}{r_1}\right) + T_{\text{surr}}$$

$$T_I = \frac{\left(1.2\times10^6\,\frac{\text{W}}{\text{m}^3}\right)(0.002\text{ m})^2}{2(0.4\text{ W/m·K})}\left(\frac{0.4\text{ W/m·K}}{7\,\frac{\text{W}}{\text{m}^2\cdot\text{K}}}\times\frac{1}{0.007\text{ m}} + \ln\frac{0.007\text{ m}}{0.002\text{ m}}\right) + 20°\text{C} = \mathbf{76.5}°\text{C} > 75°\text{C}$$

whereand $r_2 = r_1$ + wall thickness = 0.002 m + 0.005 m = 0.007 m.
The temperature at the center of the wire, $r = 0$, is

$$T_{\text{wire}}(0) = T_I + \frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}} r_1^2 = 76.5°\text{C} + \frac{\left(1.2\times10^6\,\frac{\text{W}}{\text{m}^3}\right)(0.002\text{ m})^2}{4(15\text{ W/m·K})} = \mathbf{76.6}°\text{C}$$

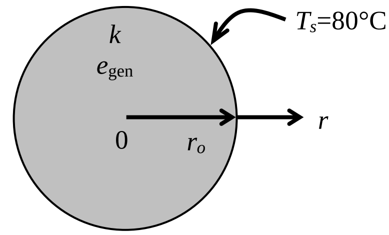
Discussion With the temperature at the interface of the wire and the insulation being 1.5°C higher than the specification of the ASTM D1351 standard for polyethylene insulation, the ASTM standard is not met. We can consider using a different insulation material with a higher temperature rating. From the ASTM database, the crosslinked polyethylene insulation (ASTM D2655) is rated up to 90°C for normal operation.

2-106 Heat is generated uniformly in a spherical radioactive material with specified surface temperature. The mathematical formulation, the variation of temperature in the sphere, and the center temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any changes with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the mid point. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k = 15 W/m·°C.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0 \quad \text{with} \quad \dot{e}_{\text{gen}}=\text{constant}$$

and $T(r_o)=T_s=80°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the mid point)}$$

(*b*) Multiplying both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the mid point,

B.C. at r = 0: $0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}\times 0+C_1 \quad \rightarrow \quad C_1=0$

Dividing both sides of Eq. (*a*) by r^2 to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}r$$

and $T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+C_2 \qquad (b)$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_o$: $T_s=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+C_2 \quad \rightarrow \quad C_2=T_s+\frac{\dot{e}_{\text{gen}}}{6k}r_o^2$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r)=T_s+\frac{\dot{e}_{\text{gen}}}{6k}(r_o^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r.

(*c*) The temperature at the center of the sphere (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_s+\frac{\dot{e}_{\text{gen}}}{6k}(r_o^2-0^2)=T_s+\frac{\dot{e}_{\text{gen}}r_o^2}{6k}=80°\text{C}+\frac{(4\times10^7\ \text{W/m}^3)(0.04\ \text{m})^2}{6\times(15\ \text{W/ m}\cdot°\text{C})}=\mathbf{791°C}$$

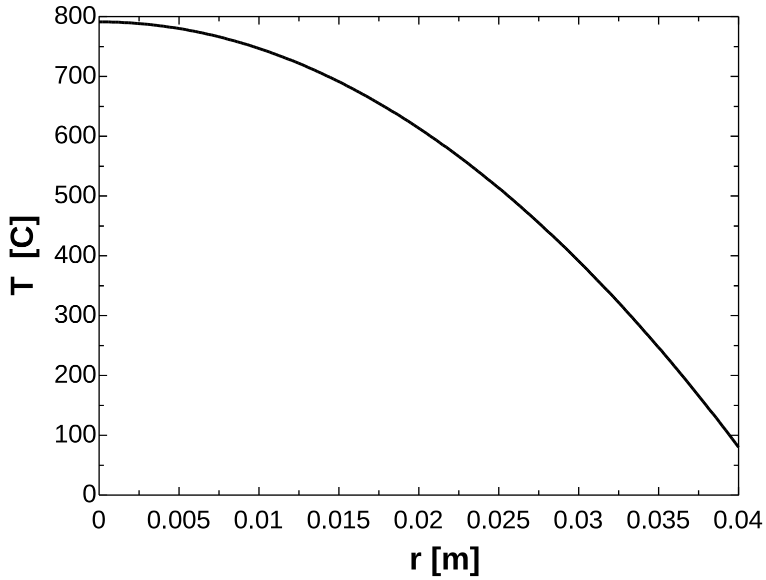
Thus the temperature at center will be about 711°C above the temperature of the outer surface of the sphere.

2-107 Prob. 2-106 is reconsidered. The temperature as a function of the radius is to be plotted. Also, the center temperature of the sphere as a function of the thermal conductivity is to be plotted.

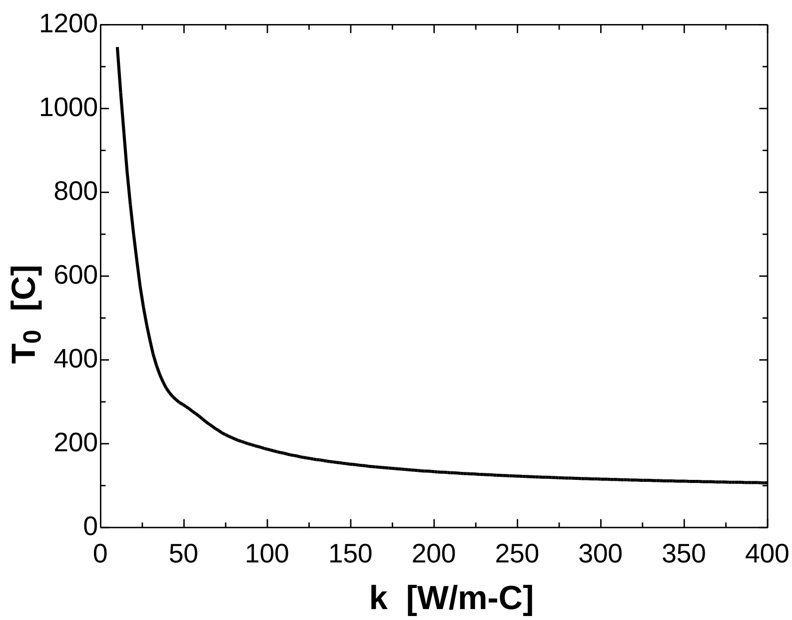
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_0=0.04 [m]
g_dot=4E7 [W/m^3]
T_s=80 [C]
k=15 [W/m-C]
"ANALYSIS"
T=T_s+g_dot/(6*k)*(r_0^2-r^2) "Temperature distribution as a function of r"
T_0=T_s+g_dot/(6*k)*r_0^2 "Temperature at the center (r=0)"
```

r [m]	T [C]
0	791.1
0.002105	789.1
0.004211	783.2
0.006316	773.4
0.008421	759.6
0.01053	741.9
0.01263	720.2
0.01474	694.6
0.01684	665
0.01895	631.6
0.02105	594.1
0.02316	552.8
0.02526	507.5
0.02737	458.2
0.02947	405
0.03158	347.9
0.03368	286.8
0.03579	221.8
0.03789	152.9
0.04	80



k [W/m.C]	T_0 [C]
10	1147
30.53	429.4
51.05	288.9
71.58	229
92.11	195.8
112.6	174.7
133.2	160.1
153.7	149.4
174.2	141.2
194.7	134.8
215.3	129.6
235.8	125.2
256.3	121.6
276.8	118.5
297.4	115.9
317.9	113.6
338.4	111.5
358.9	109.7
379.5	108.1
400	106.7



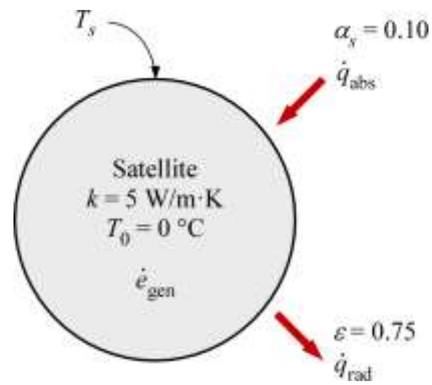
2-108 A spherical communication satellite orbiting in space absorbs solar radiation while losing heat to deep space by thermal radiation. The heat generation rate and the surface temperature of the satellite are to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** Heat generation is uniform. **3** Thermal properties are constant.

Properties The properties of the satellite are given to be $\varepsilon = 0.75$, $\alpha = 0.10$, and $k = 5$ W/m · K.

Analysis For steady one-dimensional heat conduction in sphere, the differential equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$



and $\quad T(0) = T_0 = 273$ K $\quad$ (midpoint temperature of the satellite)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the midpoint)}$$

Multiply both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the midpoint (thermal symmetry about the midpoint),

$$r=0: \qquad 0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\times 0+C_1 \quad \rightarrow \quad C_1=0$$

Dividing both sides of Eq. (a) by r^2 and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}r$$

and

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+C_2 \qquad (b)$$

Applying the boundary condition at the midpoint (midpoint temperature of the satellite),

$$r=0: \qquad T_0=-\frac{\dot{e}_{\text{gen}}}{6k}\times 0+C_2 \quad \rightarrow \quad C_2=T_0$$

Substituting C_2 into Eq. (b), the variation of temperature is determined to be

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+T_0$$

At the satellite surface ($r = r_o$), the temperature is

$$T_s=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+T_0 \qquad (c)$$

Also, the rate of heat transfer at the surface of the satellite can be expressed as

$$\dot{e}_{\text{gen}}\left(\frac{4}{3}\pi r_o^3\right)=A_s\varepsilon\sigma(T_s^4-T_{\text{space}}^4)-A_s\alpha_s\dot{q}_{\text{solar}} \quad \text{where} \quad T_{\text{space}}=0$$

The surface temperature of the satellite can be explicitly expressed as

$$T_s=\left[\frac{1}{A_s\varepsilon\sigma}\left(\frac{4}{3}\pi r_o^3\dot{e}_{\text{gen}}+A_s\alpha_s\dot{q}_{\text{solar}}\right)\right]^{1/4}=\left(\frac{\dot{e}_{\text{gen}}r_o/3+\alpha_s\dot{q}_{\text{solar}}}{\varepsilon\sigma}\right)^{1/4} \qquad (d)$$

Substituting Eq. (c) into Eq. (d)

$$\left(\frac{\dot{e}_{\text{gen}}r_o/3+\alpha_s\dot{q}_{\text{solar}}}{\varepsilon\sigma}\right)^{1/4}=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+T_0$$

$$\left[\frac{\dot{e}_{\text{gen}}(1.25\text{ m})/3+(0.10)(1000\text{ W/m}^2)}{(0.75)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4}=-\frac{\dot{e}_{\text{gen}}(1.25\text{ m})^2}{6(5\text{ W/m}\cdot\text{K})}+273\text{ K}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
((e_gen*1.25/3+0.10*1000)/(0.75*5.67e-8))^(1/4)=-e_gen*1.25^2/(6*5)+273
```

Solving by EES software, the heat generation rate is

$$\dot{e}_{\text{gen}} = \mathbf{233\,W/m^3}$$

Using Eq. (*c*), the surface temperature of the satellite is determined to be

$$T_s = -\frac{(233\text{ W/m}^3)}{6(5\text{ W/m}\cdot\text{K})}(1.25\text{ m})^2 + 273\text{ K} = \mathbf{261\,K}$$

Discussion The surface temperature of the satellite in space is well below freezing point of water.

Variable Thermal Conductivity, *k*(*T*)

2-109C The thermal conductivity of a medium, in general, varies with temperature.

2-110C Yes, when the thermal conductivity of a medium varies linearly with temperature, the average thermal conductivity is always equivalent to the conductivity value at the average temperature.

2-111C No, the temperature variation in a plain wall will not be linear when the thermal conductivity varies with temperature.

2-112C During steady one-dimensional heat conduction in a plane wall in which the thermal conductivity varies linearly, the error involved in heat transfer calculation by assuming constant thermal conductivity at the average temperature is (*a*) *none*.

2-113C During steady one-dimensional heat conduction in a plane wall, long cylinder, and sphere with constant thermal conductivity and no heat generation, the temperature in only the *plane wall* will vary linearly.

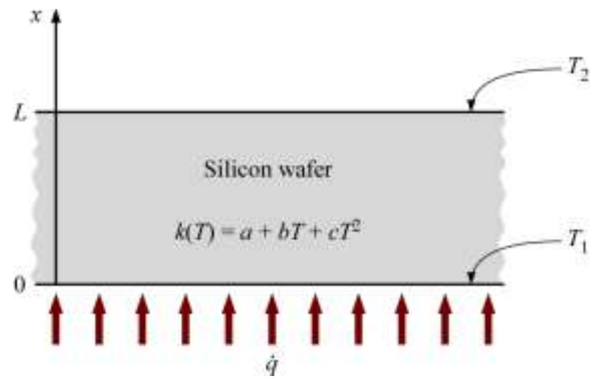
2-114 A silicon wafer with variable thermal conductivity is subjected to uniform heat flux at the lower surface. The maximum allowable heat flux such that the temperature difference across the wafer thickness does not exceed 2 °C is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = (a + bT + cT^2)$ W/m · K.

Analysis For steady heat transfer, the Fourier's law of heat conduction can be expressed as

$$\dot{q} = -k(T)\frac{dT}{dx} = -(a + bT + cT^2)\frac{dT}{dx}$$



Separating variable and integrating from $x = 0$ where $T(0) = T_1$ to $x = L$ where $T(L) = T_2$, we obtain

$$\int_0^L \dot{q}dx = -\int_{T_1}^{T_2} (a + bT + cT^2)dT$$

Performing the integration gives

$$\dot{q}L = -\left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) + \frac{c}{3}(T_2^3 - T_1^3)\right]$$

The maximum allowable heat flux such that the temperature difference across the wafer thickness does not exceeding 2°C (where $T_1 = 602$ K and $T_2 = 600$ K) is

$$\dot{q} = -\frac{\left[437(600 - 602) - \frac{1.29}{2}(600^2 - 602^2) + \frac{0.00111}{3}(600^3 - 602^3)\right]\text{W/m}}{(925 \times 10^{-6}\ \text{m})}$$

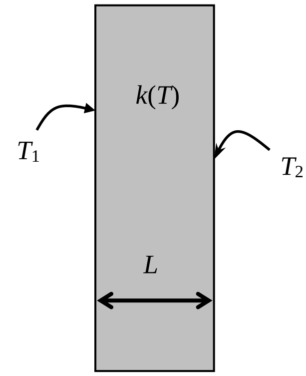
$$= \mathbf{1.35 \times 10^5\ W/m^2}$$

Discussion For heat flux less than 135 kW/m^2, the temperature difference across the silicon wafer thickness will be maintained below 2°C.

2-115 A plate with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the plate is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.



Analysis The average thermal conductivity of the medium in this case is simply the conductivity value at the average temperature since the thermal conductivity varies linearly with temperature, and is determined to be

$$k_{\text{ave}} = k(T_{\text{avg}}) = k_0\left(1+\beta\frac{T_2+T_1}{2}\right)$$

$$= (25\text{ W/m}\cdot\text{K})\left(1+(8.7\times10^{-4}\text{ K}^{-1})\frac{(500+350)\text{ K}}{2}\right)$$

$$= 34.24\text{ W/m}\cdot\text{K}$$

Then the rate of heat conduction through the plate becomes

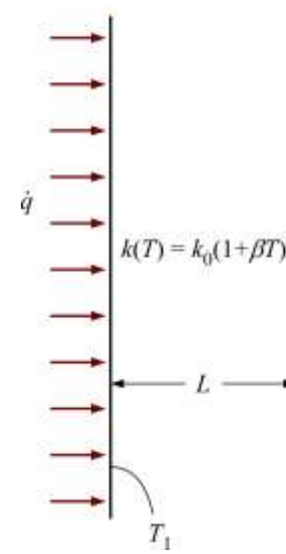
$$\dot{Q} = k_{\text{avg}}A\frac{T_1-T_2}{L} = (34.24\text{ W/m}\cdot\text{K})(1.5\text{ m}\times0.6\text{ m})\frac{(500-350)\text{K}}{0.15\text{ m}} = 30{,}820\text{ W} = \mathbf{30.8\,kW}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq, 2-76, and performed the indicated integration.

2-116 On one side, a steel plate is subjected to a uniform heat flux and maintained at a constant temperature. On the other side, the temperature is maintained at a lower temperature. The plate thickness is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.



Analysis For steady heat transfer, the Fourier's law of heat conduction can be expressed as

$$\dot{q} = k_{\text{avg}}\frac{T_1-T_2}{L}$$

Solving for the plate thickness from the above equation

$$L = k_{\text{avg}}\frac{T_1-T_2}{\dot{q}}$$

The average thermal conductivity of the steel plate is

$$k_{\text{avg}} = k_0\left(1+\beta\frac{T_2+T_1}{2}\right) = (9.14\text{ W/m}\cdot\text{K})\left[1+(0.0023\text{ K}^{-1})\frac{(600+800)\text{K}}{2}\right] = 23.86\text{ W/m}\cdot\text{K}$$

Substituting into the equation for the plate thickness,

$$L = (23.86\text{ W/m}\cdot\text{K})\frac{(800-600)\text{K}}{50000\text{ W/m}^2} = \mathbf{0.095\,m}$$

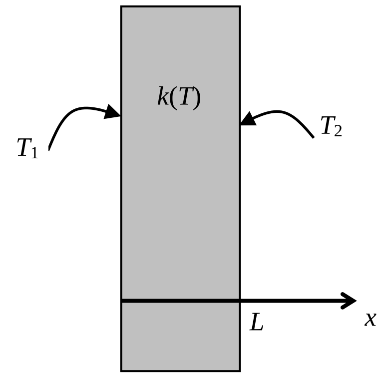
Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-76, and performed the indicated integration.

2-117 A plate with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the plate is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies quadratically. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T^2)$.

Analysis When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from



$$k_{avg} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} k_0(1+\beta T^2)dT}{T_2 - T_1} = \frac{k_0\left(T + \frac{\beta}{3}T^3\right)\Big|_{T_1}^{T_2}}{T_2 - T_1}$$

$$= \frac{k_0\left[(T_2 - T_1) + \frac{\beta}{3}\left(T_2^3 - T_1^3\right)\right]}{T_2 - T_1}$$

$$= k_0\left[1 + \frac{\beta}{3}\left(T_2^2 + T_1T_2 + T_1^2\right)\right]$$

This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{avg} equals the rate of heat transfer through the same medium with variable conductivity $k(T)$. Then the rate of heat conduction through the plate can be determined to be

$$\dot{Q} = k_{avg}A\frac{T_1 - T_2}{L} = k_0\left[1 + \frac{\beta}{3}\left(T_2^2 + T_1T_2 + T_1^2\right)\right]A\frac{T_1 - T_2}{L}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-76, and performed the indicated integration.

2-118 The thermal conductivity of stainless steel has been characterized experimentally to vary with temperature. The average thermal conductivity over a given temperature range and the $k(T) = k_0(1 + \beta T)$ expression are to be determined.

***Assumptions* 1** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = 9.14 + 0.021T$ for $273 < T < 1500$ K.

Analysis The average thermal conductivity can be determined using

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)\,dT}{T_2 - T_1} = \frac{\int_{300}^{1200} (9.14 + 0.021T)\,dT}{1200 - 300} = \frac{(9.14T + 0.0105T^2)\Big|_{300}^{1200}}{1200 - 300} = \mathbf{24.9\ W/m \cdot K}$$

To express $k(T) = 9.14 + 0.021T$ as $k(T) = k_0(1 + \beta T)$, we have

$$k(T) = k_0 + k_0 \beta T$$

and comparing with $k(T) = 9.14 + 0.021T$, we have

$$k_0 = 9.14\ \text{W/m} \cdot \text{K} \quad \text{and} \quad k_0 \beta = 0.021\ \text{W/m} \cdot \text{K}^2$$

which gives

$$\beta = \frac{0.021\ \text{W/m} \cdot \text{K}^2}{k_0} = \frac{0.021\ \text{W/m} \cdot \text{K}^2}{9.14\ \text{W/m} \cdot \text{K}} = 0.0023\ \text{K}^{-1}$$

Thus,

$$k(T) = k_0(1 + \beta T) \quad \text{where} \quad k_0 = \mathbf{9.14\ W/m \cdot K} \quad \text{and} \quad \beta = \mathbf{0.0023 K^{-1}}$$

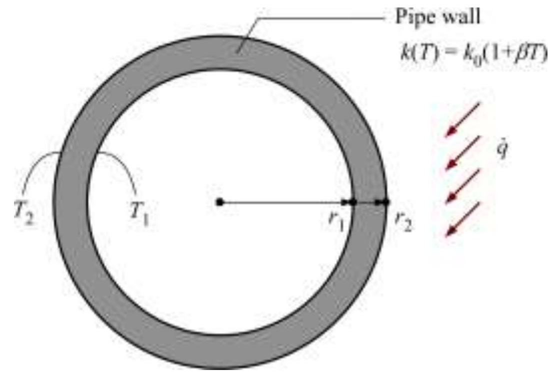
Discussion The average thermal conductivity can also be determined using the average temperature:

$$k_{\text{avg}} = k(T_{\text{avg}}) = 9.14 + 0.021\left(\frac{1200 + 300}{2}\right) = 24.9\ \text{W/m} \cdot \text{K}$$

2-119 A pipe outer surface is subjected to a uniform heat flux and has a known temperature. The metal pipe has a variable thermal conductivity. The inner surface temperature of the pipe is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.



Analysis For steady heat transfer, the heat conduction through a cylindrical layer can be expressed as

$$\dot{q} = \frac{\dot{Q}}{2\pi r_2 L} = \frac{2\pi L k_{avg}}{2\pi r_2 L}\frac{T_2 - T_1}{\ln(r_2/r_1)} = \frac{k_{avg}}{r_2}\frac{T_2 - T_1}{\ln(r_2/r_1)}$$

The inner and outer radii of the pipe are

$$r_1 = 0.1/2\,\text{m} = 0.05\,\text{m} \quad \text{and} \quad r_2 = (0.05 + 0.01)\,\text{m} = 0.06\,\text{m}$$

The average thermal conductivity is

$$k_{avg} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (7.5\,\text{W/m}\cdot\text{K})\left[1 + (0.0012\ \text{K}^{-1})\frac{(773)\,\text{K} + T_1}{2}\right]$$
$$= [7.5 + 0.0045\,(773 + T_1)]\ \text{W/m}\cdot\text{K}$$

Thus,

$$5000\ \text{W/m}^2 = \frac{[7.5 + 0.0045\,(773 + T_1)]\,\text{W/m}\cdot\text{K}}{0.06\,\text{m}}\left[\frac{773 - T_1}{\ln(0.06/0.05)}\ \text{K}\right]$$

Solving for the inner pipe temperature T_1,

$$T_1 = 769.21\,\text{K} = \mathbf{496.2°C}$$

Discussion There is about 4°C drop in temperature across the pipe wall.

2-120 A pipe is used for transporting boiling water with a known inner surface temperature in surroundings of cooler ambient temperature and known convection heat transfer coefficient. The pipe wall has a variable thermal conductivity. The outer surface temperature of the pipe is to be determined to ensure that it is below 50°C.

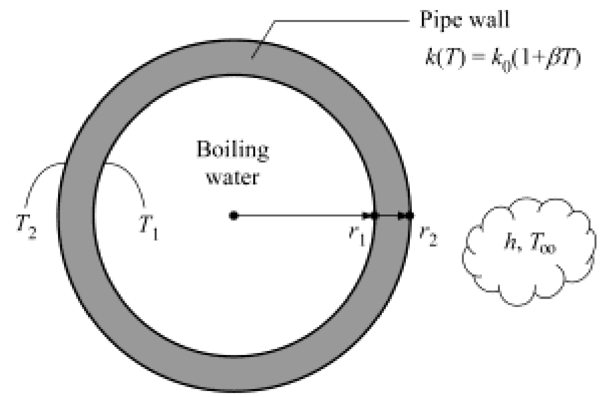
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature. **4** Inner pipe surface temperature is constant at 100°C.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the pipe are

$$r_1 = 0.030/2\ \text{m} = 0.015\ \text{m}$$

$$r_2 = (0.015 + 0.003)\ \text{m} = 0.018\ \text{m}$$



The rate of heat transfer at the pipe's outer surface can be expressed as

$$\dot{Q}_{\text{cylinder}} = \dot{Q}_{\text{conv}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty)$$

$$\frac{k_{\text{avg}}}{r_2}\frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) \qquad (1)$$

where

h = 70 W/m^2 K, T_1 = 373 K, and T_∞ = 283 K

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.23\ \text{W/m}\cdot\text{K})\left[1 + (0.002\ \text{K}^{-1})\frac{T_2 + (373\ \text{K})}{2}\right]$$

$$k_{\text{avg}} = [1.23 + 0.00123(\,T_2 + 373)]\ \text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{364.3\ K = 91.3°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=70 [W/(m^2*K)]   "convection heat transfer coefficient"
r_1=0.030/2 [m]   "inner radius"
r_2=r_1+0.003 [m]   "outer radius"
T_1=100+273 [K]   "inner surface temperature"
T_inf=10+273 [K]   "ambient temperature"
k_0=1.23 [W/(m*K)]
beta=0.002 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
Q_dot_cylinder=2*pi*k_avg*(T_1-T_2)/ln(r_2/r_1)   "heat rate through the cylindrical layer"
Q_dot_conv=h*2*pi*r_2*(T_2-T_inf)   "heat rate by convection"
Q_dot_cylinder=Q_dot_conv
```

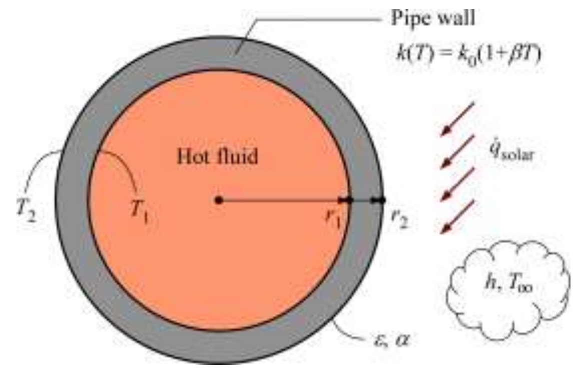
The outer surface temperature of the pipe is more than 40°C above the safe temperature of 50°C to prevent thermal burn on skin tissues.

Discussion It is necessary to wrap the pipe with insulation to prevent thermal burn.

2-121 A pipe is used for transporting hot fluid with a known inner surface temperature. The pipe wall has a variable thermal conductivity. The pipe's outer surface is subjected to radiation and convection heat transfer. The outer surface temperature of the pipe is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$, $\alpha = \varepsilon = 0.9$ at the outer pipe surface.



Analysis The inner and outer radii of the pipe are

$$r_1 = 0.15/2\,\text{m} = 0.075\text{ m}$$

$$r_2 = (0.075 + 0.005)\,\text{m} = 0.08\text{ m}$$

The rate of heat transfer at the pipe's outer surface can be expressed as

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} - \dot{Q}_{\text{abs}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty) + \varepsilon\sigma(2\pi r_2 L)(T_2^4 - T_{\text{surr}}^4) - \alpha(2\pi r_2 L)\dot{q}_{\text{solar}}$$

$$\frac{k_{\text{avg}}}{r_2}\frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{\text{surr}}^4) - \alpha\dot{q}_{\text{solar}} \qquad (1)$$

where $h = 60\text{ W/m}^2\text{ K}$, $\dot{q}_{\text{solar}} = 100\text{ W/m}^2$, $T_1 = 423\text{ K}$, and $T_\infty = T_{\text{surr}} = 273\text{ K}$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (8.5\,\text{W/m}\cdot\text{K})\left[1 + (0.001\text{ K}^{-1})\frac{\text{T}_2 + (423\text{ K})}{2}\right]$$

$$k_{\text{avg}} = [8.5 + 0.00425(\,T_2 + 423)]\text{ W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{418.8\,K = 145.8°C}$$

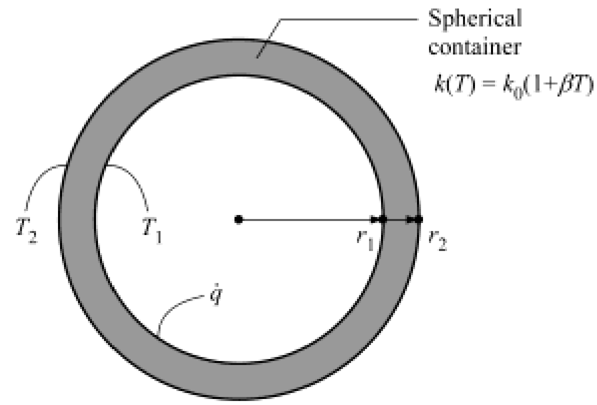
Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=60 [W/(m^2*K)]   "outer surface h"
r_1=0.15/2 [m]   "inner radius"
r_2=r_1+0.005 [m]   "outer radius"
T_1=423 [K]   "inner surface T"
T_inf=273 [K]   "ambient T"
T_surr=273 [K]  "surrounding surface T"
alpha=0.9   "outer surface absorptivity"
epsilon=0.9   "outer surface emissivity"
q_dot_solar=100 [W/m^2]   "incident solar radiation"
k_0=8.5 [W/(m*K)]
beta=0.001 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_cyl=k_avg/r_2*(T_1-T_2)/ln(r_2/r_1)   "heat flux through the cylindrical layer"
q_dot_conv=h*(T_2-T_inf)   "heat flux by convection"
q_dot_rad=epsilon*sigma#*(T_2^4-T_surr^4)   "heat flux by radiation emission"
q_dot_abs=alpha*q_dot_solar   "heat flux by radiation absorption"
q_dot_cyl-q_dot_conv-q_dot_rad+q_dot_abs=0
```

Discussion Increasing h or decreasing k_{avg} would decrease the pipe's outer surface temperature.

2-122 A spherical container has its inner surface subjected to a uniform heat flux and its outer surface is at a known temperature. The container wall has a variable thermal conductivity. The temperature drop across the container wall thickness is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.



Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis For steady heat transfer, the heat conduction through a spherical layer can be expressed as

$$\dot{q} = \frac{\dot{Q}}{4\pi r_1^2} = \frac{4\pi k_{avg} r_1 r_2}{4\pi r_1^2}\frac{T_1 - T_2}{r_2 - r_1} = k_{avg}\frac{r_2}{r_1}\frac{T_1 - T_2}{r_2 - r_1}$$

The inner and outer radii of the container are

$$r_1 = 1\,\text{m}$$

$$r_2 = 1\,\text{m} + 0.005\text{ m} = 1.005\text{ m}$$

The average thermal conductivity is

$$k_{avg} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.33\,\text{W/m}\cdot\text{K})\left[1 + (0.0023\text{ K}^{-1})\frac{(293)\text{K} + T_1}{2}\right]$$
$$= [1.33 + 0.00153(293 + T_1)]\,\text{W/m}\cdot\text{K}$$

Thus,

$$7000\text{ W/m}^2 = [1.33 + 0.00153(293 + T_1)]\,\text{W/m}\cdot\text{K}\left(\frac{1.005\text{ m}}{1\text{m}}\right)\left(\frac{T_1 - 293}{0.005}\frac{\text{K}}{\text{m}}\right)$$

Solving for the inner pipe temperature T_1,

$$T_1 = 308.5\,\text{K}$$

The temperature drop across the container wall is,

$$T_1 - T_2 = 308.5\,\text{K} - 293\text{ K} = \mathbf{15.5^\circ C}$$

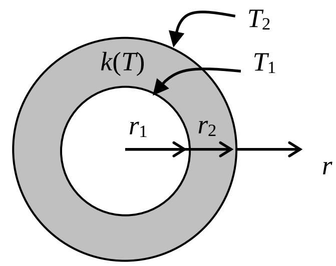
Discussion The temperature drop across the container wall would decrease if a material with a higher k_{avg} value is used.

2-123 A spherical shell with variable conductivity is subjected to specified temperatures on both sides. The variation of temperature and the rate of heat transfer through the shell are to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.

Analysis (*a*) The rate of heat transfer through the shell is expressed as



$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1}$$

where r_1 is the inner radius, r_2 is the outer radius, and

$$k_{\text{avg}} = k(T_{\text{avg}}) = k_0\left(1 + \beta \frac{T_2 + T_1}{2}\right)$$

is the average thermal conductivity.

(*b*) To determine the temperature distribution in the shell, we begin with the Fourier's law of heat conduction expressed as

$$\dot{Q} = -k(T)A\frac{dT}{dr}$$

where the rate of conduction heat transfer $\dot{Q}$ is constant and the heat conduction area $A = 4\pi r^2$ is variable. Separating the variables in the above equation and integrating from $r = r_1$ where $T(r_1) = T_1$ to any r where $T(r) = T$, we get

$$\dot{Q}\int_{r_1}^{r} \frac{dr}{r^2} = -4\pi\int_{T_1}^{T} k(T)dT$$

Substituting $k(T) = k_0(1+\beta T)$ and performing the integrations gives

$$\dot{Q}\left(\frac{1}{r_1} - \frac{1}{r}\right) = -4\pi k_0[(T - T_1) + \beta(T^2 - T_1^2)/2]$$

Substituting the $\dot{Q}$ expression from part (*a*) and rearranging give

$$T^2 + \frac{2}{\beta}T + \frac{2k_{\text{avg}}}{\beta k_0}\frac{r_2(r - r_1)}{r(r_2 - r_1)}(T_1 - T_2) - T_1^2 - \frac{2}{\beta}T_1 = 0$$

which is a *quadratic* equation in the unknown temperature *T*. Using the quadratic formula, the temperature distribution $T(r)$ in the cylindrical shell is determined to be

$$T(r) = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} - \frac{2k_{\text{avg}}}{\beta k_0}\frac{r_2(r - r_1)}{r(r_2 - r_1)}(T_1 - T_2) + T_1^2 + \frac{2}{\beta}T_1}$$

Discussion The proper sign of the square root term (+ or -) is determined from the requirement that the temperature at any point within the medium must remain between T_1 and T_2.

2-124 A spherical vessel, filled with chemicals undergoing an exothermic reaction, has a known inner surface temperature. The wall of the vessel has a variable thermal conductivity. Convection heat transfer occurs on the outer surface of the vessel. The minimum wall thickness of the vessel is to be determined so that the outer surface temperature is 50°C or lower.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

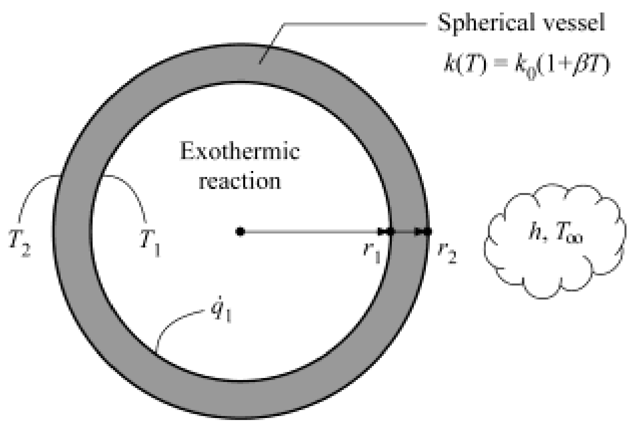
Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the vessel are

$$r_1 = 5/2\,\text{m} = 2.5\,\text{m}$$

and $$r_2 = (r_1 + t)$$

where t = wall thickness



The rate of heat transfer at the vessel's outer surface can be expressed as

$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}}$$

$$4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_2 - T_\infty)$$

$$k_{\text{avg}} \frac{r_1}{r_2} \frac{T_1 - T_2}{r_2 - r_1} = h(T_2 - T_\infty) \qquad (1)$$

where

$h = 80$ W/m^2 K, $T_1 = 393$ K, $T_2 = 323$ K, and $T_\infty = 288$ K

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta \frac{T_2 + T_1}{2}\right) = (1.01\,\text{W/m}\cdot\text{K})\left[1 + (0.0018\ \text{K}^{-1})\frac{(323\ \text{K}) + (393\ \text{K})}{2}\right] = 1.6611\ \text{W/m}\cdot\text{K}$$

Solving Eq. (1) for r_2 yields

$$r_2 = 2.541\ \text{m}$$

Thus, the minimum wall thickness of the vessel should be

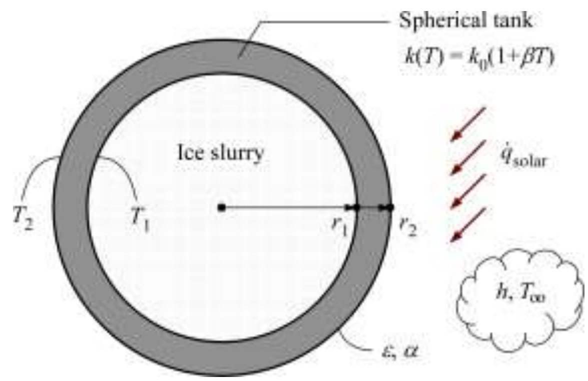
$$t = r_2 - r_1 = 0.041\ \text{m} = \mathbf{41\ mm}$$

Discussion To prevent the outer surface temperature of the vessel from causing thermal burn, the wall thickness should be at least 41 mm. As the wall thickness increases, it would decrease the outer surface temperature.

2-125 A spherical tank, filled with ice slurry, has a known inner surface temperature. The tank wall has a variable thermal conductivity. The tank's outer surface is subjected to radiation and convection heat transfer. The outer surface temperature of the tank is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$, $\alpha = \varepsilon = 0.75$ at the outer tank surface.



Analysis The inner and outer radii of the tank are

$$r_1 = 9/2\,\text{m} = 4.5\,\text{m}$$

and $$r_2 = (4.5 + 0.02)\,\text{m} = 4.52\,\text{m}$$

The rate of heat transfer at the tank's outer surface can be expressed as

$$\dot{Q}_{sph} = \dot{Q}_{conv} + \dot{Q}_{rad} + \dot{Q}_{abs}$$

$$4\pi k_{avg} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_\infty - T_2) + \varepsilon\sigma(4\pi r_2^2)(T_{surr}^4 - T_2^4) + \alpha(4\pi r_2^2)\dot{q}_{solar}$$

$$k_{avg}\frac{r_1}{r_2}\frac{T_1 - T_2}{r_2 - r_1} = h(T_\infty - T_2) + \varepsilon\sigma(T_{surr}^4 - T_2^4) + \alpha\dot{q}_{solar} \qquad (1)$$

where

$$h = 70\ \text{W/m}^2\,\text{K},\quad \dot{q}_{solar} = 150\ \text{W/m}^2,\quad T_1 = 273\ \text{K},\quad \text{and}\quad T_\infty = T_{surr} = 308\ \text{K}$$

The average thermal conductivity is

$$k_{avg} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (0.33\ \text{W/m}\cdot\text{K})\left[1 + (0.0025\ \text{K}^{-1})\frac{T_2 + (273.15\ \text{K})}{2}\right]$$

$$k_{avg} = [0.33 + 0.0004125(\ T_2 + 273)]\ \text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{299.5\,K} = \mathbf{26.5°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=70 [W/(m^2*K)]   "outer surface h"
r_1=9/2 [m]   "inner radius"
r_2=r_1+0.020 [m]   "outer radius"
T_1=273 [K]   "inner surface T"
T_inf=308 [K]   "ambient T"
T_surr=308 [K]  "surrounding surface T"
alpha=0.75   "outer surface absorptivity"
epsilon=0.75   "outer surface emissivity"
q_dot_solar=150 [W/m^2]   "incident solar radiation"
k_0=0.33 [W/(m*K)]
beta=0.0025 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_sph=k_avg*r_1/r_2*(T_1-T_2)/(r_2-r_1)   "heat flux through the spherical layer"
q_dot_conv=h*(T_inf-T_2)   "heat flux by convection"
q_dot_rad=epsilon*sigma#*(T_surr^4-T_2^4)   "heat flux by radiation emission"
q_dot_abs=alpha*q_dot_solar   "heat flux by radiation absorption"
q_dot_sph+q_dot_conv+q_dot_rad+q_dot_abs=0
```

Discussion Increasing the tank wall thickness would increase the tanks' outer surface temperature.

Special Topic: Review of Differential equations

2-126C We utilize appropriate simplifying assumptions when deriving differential equations to obtain an equation that we can deal with and solve.

2-127C A **variable** is a quantity which may assume various values during a study. A variable whose value can be changed arbitrarily is called an **independent variable** (or argument). A variable whose value depends on the value of other variables and thus cannot be varied independently is called a **dependent variable** (or a function).

2-128C A differential equation may involve more than one dependent or independent variable. For example, the equation $\frac{\partial^2 T(x,t)}{\partial x^2}+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}$ has one dependent (T) and 2 independent variables (x and t). the equation $\frac{\partial^2 T(x,t)}{\partial x^2}+\frac{\partial W(x,t)}{\partial x}=\frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}+\frac{1}{\alpha}\frac{\partial W(x,t)}{\partial t}$ has 2 dependent (T and W) and 2 independent variables (x and t).

2-129C Geometrically, the **derivative** of a function $y(x)$ at a point represents the *slope* of the tangent line to the graph of the function at that point. The derivative of a function that depends on two or more independent variables with respect to one variable while holding the other variables constant is called the partial derivative. Ordinary and partial derivatives are equivalent for functions that depend on a single independent variable.

2-130C The order of a derivative represents the number of times a function is differentiated, whereas the degree of a derivative represents how many times a derivative is multiplied by itself. For example, y''' is the third order derivative of y, whereas $(y')^3$ is the third degree of the first derivative of y.

2-131C For a function $f(x,y)$, the partial derivative $\partial f/\partial x$ will be equal to the ordinary derivative df/dx when f does not depend on y or this dependence is negligible.

2-132C For a function $f(x)$, the derivative df/dx does not have to be a function of x. The derivative will be a constant when the f is a linear function of x.

2-133C Integration is the inverse of derivation. Derivation increases the order of a derivative by one, integration reduces it by one.

2-134C A differential equation involves derivatives, an algebraic equation does not.

2-135C A differential equation that involves only ordinary derivatives is called an ordinary differential equation, and a differential equation that involves partial derivatives is called a partial differential equation.

2-136C The order of a differential equation is the order of the highest order derivative in the equation.

2-137C A differential equation is said to be **linear** if the dependent variable and all of its derivatives are of the first degree, and their coefficients depend on the independent variable only. In other words, a differential equation is linear if it can be written in a form which does not involve (1) any powers of the dependent variable or its derivatives such as y^3 or $(y')^2$, (2) any products of the dependent variable or its derivatives such as yy' or $y'y'''$, and (3) any other nonlinear functions of the dependent variable such as $\sin y$ or e^y. Otherwise, it is **nonlinear**.

2-138C A linear homogeneous differential equation of order *n* is expressed in the most general form as

$$y^{(n)} + f_1(x)y^{(n-1)} + \cdots + f_{n-1}(x)y' + f_n(x)y = 0$$

Each term in a linear homogeneous equation contains the dependent variable or one of its derivatives after the equation is cleared of any common factors. The equation $y'' - 4x^2y = 0$ is linear and homogeneous since each term is linear in y, and contains the dependent variable or one of its derivatives.

2-139C A differential equation is said to have **constant coefficients** if the coefficients of all the terms which involve the dependent variable or its derivatives are constants. If, after cleared of any common factors, any of the terms with the dependent variable or its derivatives involve the independent variable as a coefficient, that equation is said to have **variable coefficients** The equation $y'' - 4x^2y = 0$ has variable coefficients whereas the equation $y'' - 4y = 0$ has constant coefficients.

2-140C A linear differential equation that involves a single term with the derivatives can be solved by direct integration.

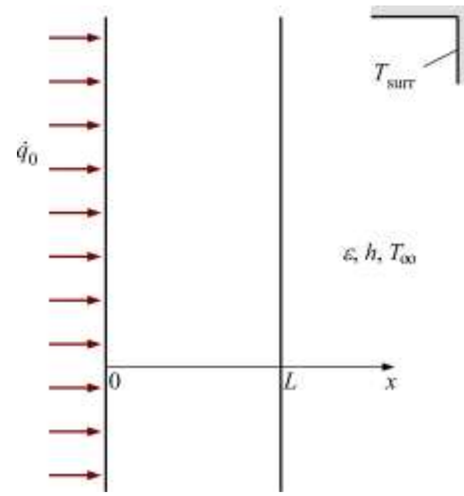
2-141C The general solution of a 3rd order linear and homogeneous differential equation will involve 3 arbitrary constants.

Review Problems

2-142 A plane wall is subjected to uniform heat flux on the left surface, while the right surface is subjected to convection and radiation heat transfer. The boundary conditions and the differential equation of this heat conduction problem are to be obtained.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The left surface at $x = 0$ is subjected to uniform heat flux while the right surface at $x = L$ is subjected to convection and radiation. **5** The surrounding temperature is $T_\infty = T_{surr}$.

Analysis Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the differential equation for heat conduction can be expressed as



$$\frac{d^2T}{dx^2} = 0$$

The boundary conditions for the left and right surfaces are

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0$$

$$x = L: \qquad -k\frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{surr}^4]$$

where

$$T_\infty = T_{surr}$$

Discussion Due to the radiation heat transfer equation, all temperatures are expressed in absolute temperatures, i.e. K or °R.

2-143 A long rectangular bar is initially at a uniform temperature of T_i. The surfaces of the bar at $x = 0$ and $y = 0$ are insulated while heat is lost from the other two surfaces by convection. The mathematical formulation of this heat conduction problem is to be expressed for transient two-dimensional heat transfer with no heat generation.

Assumptions **1** Heat transfer is transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Analysis The differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

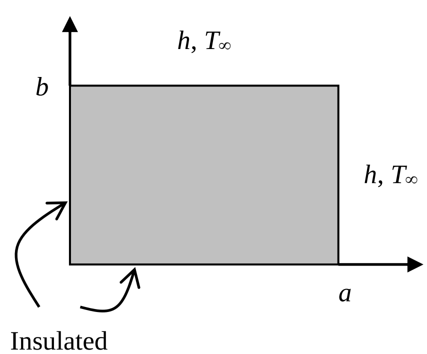
$$\frac{\partial T(x,0,t)}{\partial x} = 0$$

$$\frac{\partial T(0,y,t)}{\partial y} = 0$$

$$-k\frac{\partial T(a,y,t)}{\partial y} = h[T(a,y,t) - T_\infty]$$

$$-k\frac{\partial T(x,b,t)}{\partial x} = h[T(x,b,t) - T_\infty]$$

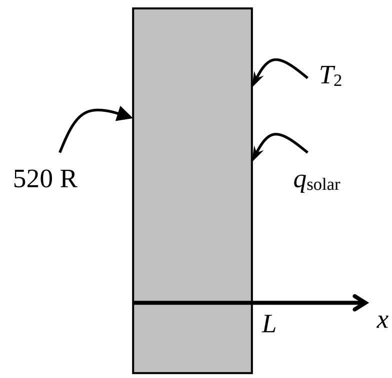
$$T(x,y,0) = T_i$$



2-144E A large plane wall is subjected to a specified temperature on the left (inner) surface and solar radiation and heat loss by radiation to space on the right (outer) surface. The temperature of the right surface of the wall and the rate of heat transfer are to be determined when steady operating conditions are reached.

Assumptions **1** Steady operating conditions are reached. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **3** Thermal properties are constant. **4** There is no heat generation in the wall.

Properties The properties of the plate are given to be k = 1.2 Btu/h·ft·°F and ε = 0.80, and $\alpha_s = 0.60$.



Analysis In steady operation, heat conduction through the wall must be equal to net heat transfer from the outer surface. Therefore, taking the outer surface temperature of the plate to be T_2 (absolute, in R),

$$kA_s \frac{T_1 - T_2}{L} = \varepsilon \sigma A_s T_2^4 - \alpha_s A_s \dot{q}_{\text{solar}}$$

Canceling the area A and substituting the known quantities,

$$(1.2 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})\frac{(520\text{ R}) - T_2}{0.8\text{ ft}} = 0.8(0.1714\times10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)T_2^4 - 0.60(300\text{ Btu/h}\cdot\text{ft}^2)$$

Solving for T_2 gives the outer surface temperature to be

T_2 = **553.9 R**

Then the rate of heat transfer through the wall becomes

$$\dot{q} = k\frac{T_1 - T_2}{L} = (1.2\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})\frac{(520-553.9)\text{ R}}{0.8\text{ ft}} = \mathbf{-50.9\text{ Btu/h}\cdot\text{ft}^2} \quad \text{(per unit area)}$$

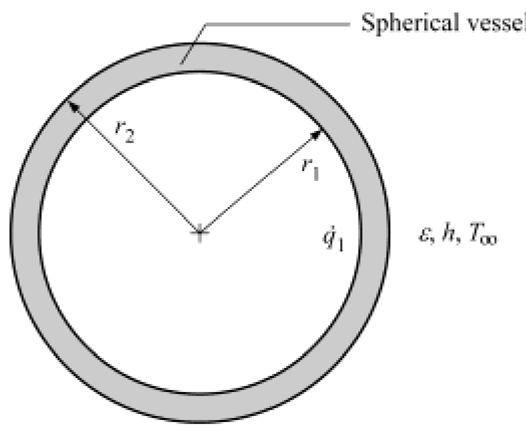
Discussion The negative sign indicates that the direction of heat transfer is from the outside to the inside. Therefore, the structure is gaining heat.

2-145 A spherical vessel is subjected to uniform heat flux on the inner surface, while the outer surface is subjected to convection and radiation heat transfer. The boundary conditions and the differential equation of this heat conduction problem are to be obtained.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection and radiation. **5** The surrounding temperature is $T_\infty = T_{\text{surr}}$.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$



The boundary conditions for the inner and outer surfaces are

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty] + \varepsilon\sigma[T(r_2)^4 - T_{\text{surr}}^4]$$

where $\quad T_\infty = T_{\text{surr}}$

Discussion Due to the radiation heat transfer equation, all temperatures are expressed in absolute temperatures, i.e. K or °R.

2-146 Heat is generated at a constant rate in a short cylinder. Heat is lost from the cylindrical surface at $r = r_o$ by convection to the surrounding medium at temperature T_∞ with a heat transfer coefficient of h. The bottom surface of the cylinder at $r = 0$ is insulated, the top surface at $z = H$ is subjected to uniform heat flux $\dot{q}_h$, and the cylindrical surface at $r = r_o$ is subjected to convection. The mathematical formulation of this problem is to be expressed for steady two-dimensional heat transfer.

Assumptions **1** Heat transfer is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** Heat is generated uniformly.

Analysis The differential equation and the boundary conditions for this heat conduction problem can be expressed as

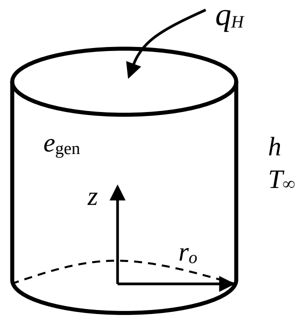
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{e}_{\text{gen}}}{k}=0$$

$$\frac{\partial T(r,0)}{\partial z}=0$$

$$k\frac{\partial T(r,H)}{\partial z}=\dot{q}_H$$

$$\frac{\partial T(0,z)}{\partial r}=0$$

$$-k\frac{\partial T(r_o,z)}{\partial r}=h[T(r_o,z)-T_\infty]$$

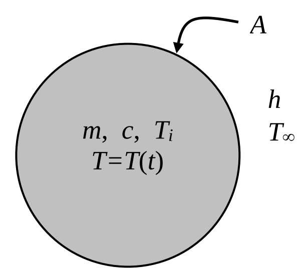


2-147 A small hot metal object is allowed to cool in an environment by convection. The differential equation that describes the variation of temperature of the ball with time is to be derived.

Assumptions **1** The temperature of the metal object changes uniformly with time during cooling so that $T = T(t)$. **2** The density, specific heat, and thermal conductivity of the body are constant. **3** There is no heat generation.

Analysis Consider a body of arbitrary shape of mass m, volume V, surface area A, density ρ, and specific heat c_p initially at a uniform temperature T_i. At time $t = 0$, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment with a heat transfer coefficient h.

During a differential time interval dt, the temperature of the body rises by a differential amount dT. Noting that the temperature changes with time only, an energy balance of the solid for the time interval dt can be expressed as



$$\left(\begin{array}{c}\text{Heat transfer from the body}\\ \text{during } dt\end{array}\right)=\left(\begin{array}{c}\text{The decrease in the energy}\\ \text{of the body during } dt\end{array}\right)$$

or $\quad hA_s(T-T_\infty)dt = mc_p(-dT)$

Noting that $m=\rho V$ and $dT = d(T-T_\infty)$ since $T_\infty =$ constant, the equation above can be rearranged as

$$\frac{d(T-T_\infty)}{T-T_\infty}=-\frac{hA_s}{\rho V c_p}dt$$

which is the desired differential equation.

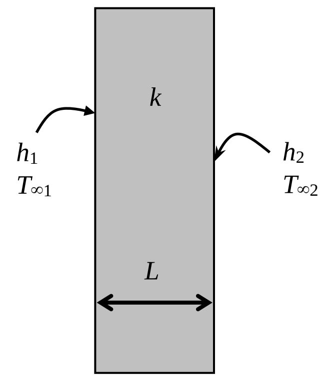
2-148 A large plane wall is subjected to convection on the inner and outer surfaces. The mathematical formulation, the variation of temperature, and the temperatures at the inner and outer surfaces to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be k = 0.77 W/m·°C.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with *x* = 0 at the inner surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



and

$$h_1[T_{\infty 1} - T(0)] = -k\frac{dT(0)}{dx}$$

$$-k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

x = 0: $\quad h_1[T_{\infty 1} - (C_1 \times 0 + C_2)] = -kC_1$

x = *L*: $\quad -kC_1 = h_2[(C_1 L + C_2) - T_{\infty 2}]$

Substituting the given values, these equations can be written as

$$8(22 - C_2) = -0.77C_1$$

$$-0.77C_1 = (12)(0.2C_1 + C_2 - 8)$$

Solving these equations simultaneously give

$$C_1 = -38.84 \qquad C_2 = 18.26$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = 18.26 - 38.84x$$

(*c*) The temperatures at the inner and outer surfaces are

$$T(0) = 18.26 - 38.84 \times 0 = \mathbf{18.3°C}$$

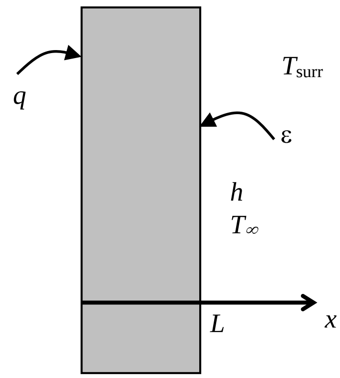
$$T(L) = 18.26 - 38.84 \times 0.2 = \mathbf{10.5°C}$$

2-149 The base plate of an iron is subjected to specified heat flux on the left surface and convection and radiation on the right surface. The mathematical formulation, and an expression for the outer surface temperature and its value are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation. **4** Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity and emissivity are given to be k = 18 W/m·°C and ε = 0.7.

Analysis (*a*) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be



$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{base}} = \frac{1000\ \text{W}}{150\times10^{-4}\ \text{m}^2} = 66{,}667\ \text{W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 66{,}667\ \text{W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L)-T_\infty] + \varepsilon\sigma[T(L)^4 - T_{surr}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2+273)^4 - T_{surr}^4]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $$-kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$x = L$: $$-kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2+273)^4 - T_{surr}^4]$$

Eliminating the constant C_1 from the two relations above gives the following expression for the outer surface temperature T_2,

$$h(T_2 - T_\infty) + \varepsilon\sigma[(T_2+273)^4 - T_{surr}^4] = \dot{q}_0$$

(*c*) Substituting the known quantities into the implicit relation above gives

$$(30\ \text{W/m}^2\cdot°\text{C})(T_2 - 26) + 0.7(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(T_2+273)^4 - 295^4] = 66{,}667\ \text{W/m}^2$$

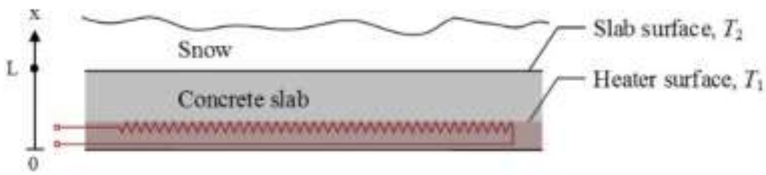
Using an equation solver (or a trial and error approach), the outer surface temperature is determined from the relation above to be

T_2 = **759°C**

2-150 A 30 m^2 concrete slab with embedded heating cable melts snow at a rate of 0.1 kg/s. Formulate the temperature profile in the concrete slab in terms of the snow melt rate. The power density for the embedded heater is to be determined whether it is in compliance with the NFPA 70 code.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the concrete slab. **3** The bottom surface at $x = 0$ is subjected to uniform heat flux from the heating cable. **4** The upper surface at $x = L$ is at a constant temperature of 0°C from the snow melt. **5** There is no heat generation in the concrete slab. **6** Thermal properties are constant.

Properties The latent heat of fusion for water is 333.7 kJ/kg (Table A-2).



Analysis Taking the direction normal to the surface of the concrete slab to be the x direction with $x = 0$ at the bottom surface (the surface that is in contact with the heater surface), the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = T_L - C_1 L = T_L + \frac{\dot{q}_0}{k}L$$

Substituting C_1 and C_2 into the general solution yields

$$T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

The heat rate required for melting snow can be determined from the latent heat of fusion for water,

$$\dot{Q} = \dot{m}_{\text{ice}} h_{if}$$

For a surface area of 30 m^2, the heat flux is determined using

$$\dot{q}_0 = \frac{\dot{Q}}{A_s} = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s}$$

Therefore, the temperature profile in the concrete slab in terms of the snow melt rate is

$$\boldsymbol{T(x) = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s k}(L - x) + T_L}$$

The power density (heat flux) for the heater to melt snow at 0.1 kg/s is

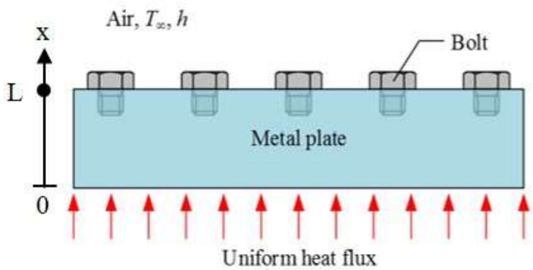
$$\dot{q}_0 = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s} = \frac{(0.1\text{ kg/s})(333700\text{ J/kg})}{30\text{ m}^2} = \mathbf{1112\ W/m^2} < 1300\text{ W/m}^2$$

Discussion The power density for the embedded heating cable in the concrete slab is below the limit set by the National Electrical Code® (NFPA 70) of 1300 W/m^2. So, melting the snow at 0.1 kg/s is in compliance with the code.

2-151 A series of ASME SA-193 carbon steel bolts are bolted on the upper surface of a metal plate. The upper surface is exposed to convection with the ambient air. The bottom surface is subjected to a uniform heat flux. Formulate the variation of temperature in the metal plate, and determine the temperatures at x = 0, 1.5, and 3.0 cm. The compliance of the SA-193 bolts with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300) is to be determined.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the metal plate. **3** The bottom surface at x = 0 is subjected to uniform heat flux while the upper surface at $x = L$ is at uniform temperature. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The thermal conductivity of the metal plate is given as 15 W/m·K.



Analysis Taking the direction normal to the surface of the plate to be the x direction with x = 0 at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = -C_1 L + T_L = \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty$$

Note that the uniform heat flux on the bottom plate surface (x = 0) is equal to the heat flux transferred by convection on the upper surface ($x = L$):

$$\dot{q}_0 = h[T(L) - T_\infty] \qquad \rightarrow \qquad T(L) = \frac{\dot{q}_0}{h} + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\boldsymbol{T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty = \frac{\dot{q}_0}{k}(L - x) + \frac{\dot{q}_0}{h} + T_\infty}$$

The temperatures in the metal plate at x = 0, 1.5, and 3.0 cm are

$$\text{At } x = 0: \qquad T(0) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + 30°\text{C} = \mathbf{540}°\text{C}$$

$$\text{At } x = 0.015\ \text{m}: \qquad T(0.015) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0.015\ \text{m}) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + 30°\text{C} = \mathbf{535}°\text{C}$$

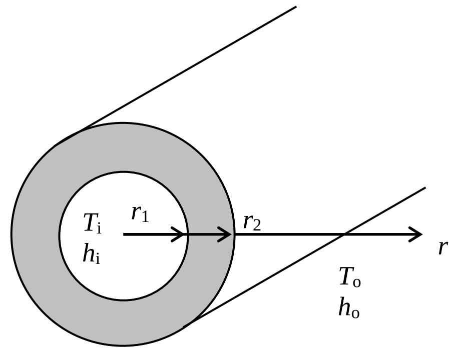
$$\text{At } x = 0.030\ \text{m}: \qquad T(0.030) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0.030\ \text{m}) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + 30°\text{C} = \mathbf{530}°\text{C}$$

Discussion The entire metal plate is above 260°C. The minimum temperature in the metal plate is at the upper surface that is exposed to convection with the ambient air, 530°C at x = 3 cm. Therefore, the SA-193 bolts would not comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300).

2-152 A steam pipe is subjected to convection on both the inner and outer surfaces. The mathematical formulation of the problem and expressions for the variation of temperature in the pipe and on the outer surface temperature are to be obtained for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as



$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

and $$-k\frac{dT(r_1)}{dr}=h_i[T_i-T(r_1)]$$

$$-k\frac{dT(r_2)}{dr}=h_o[T(r_2)-T_o]$$

(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$-k\frac{C_1}{r_1}=h_i[T_i-(C_1\ln r_1+C_2)]$$

$r = r_2$: $$-k\frac{C_1}{r_2}=h_o[(C_1\ln r_2+C_2)-T_o]$$

Solving for C_1 and C_2 simultaneously gives

$$C_1=\frac{T_0-T_i}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{h_ir_1}+\dfrac{k}{h_or_2}} \quad\text{and}\quad C_2=T_i-C_1\left(\ln r_1-\frac{k}{h_ir_1}\right)=T_i-\frac{T_0-T_i}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{h_ir_1}+\dfrac{k}{h_or_2}}\left(\ln r_1-\frac{k}{h_ir_1}\right)$$

Substituting C_1 and C_2 into the general solution and simplifying, we get the variation of temperature to be

$$T(r)=C_1\ln r+T_i-C_1(\ln r_1-\frac{k}{h_ir_1})=T_i+\frac{(T_0-T_i)\ln\dfrac{r}{r_1}+\dfrac{k}{h_ir_1}}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{h_ir_1}+\dfrac{k}{h_or_2}}$$

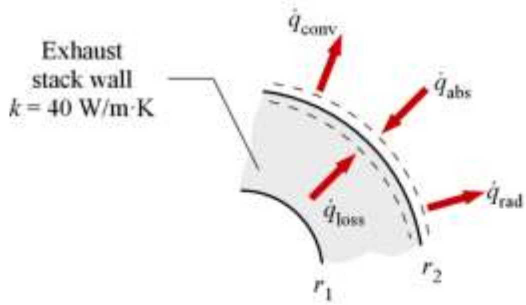
(*c*) The outer surface temperature is determined by simply replacing r in the relation above by r_2. We get

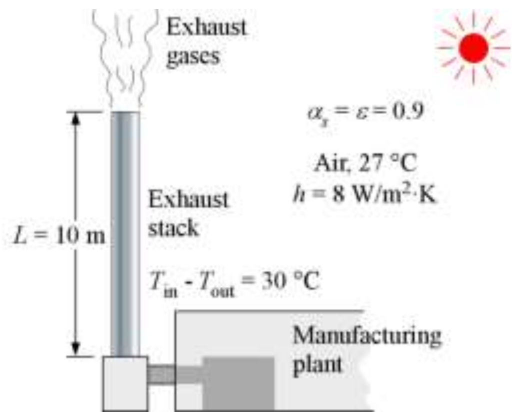
$$T(r_2)=T_i+\frac{(T_0-T_i)\ln\dfrac{r_2}{r_1}+\dfrac{k}{h_ir_1}}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{h_ir_1}+\dfrac{k}{h_or_2}}$$

2-153 A 10-m tall exhaust stack discharging exhaust gases at a rate of 1.2 kg/s is subjected to solar radiation and convection at the outer surface. The variation of temperature in the exhaust stack and the inner surface temperature of the exhaust stack are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipe.

Properties The constant pressure specific heat of exhaust gases is given to be 1600 J/kg · °C and the pipe thermal conductivity is 40 W/m · K. Both the emissivity and solar absorptivity of the exhaust stack outer surface are 0.9.





Analysis The outer and inner radii of the pipe are

$$r_2 = 1\,\text{m}/2 = 0.5\,\text{m}$$

$$r_1 = 0.5\,\text{m} - 0.1\,\text{m} = 0.4\,\text{m}$$

The outer surface area of the exhaust stack is

$$A_{s,2} = 2\pi\, r_2 L = 2\pi(0.5\,\text{m})(10\,\text{m}) = 31.42\,\text{m}^2$$

The rate of heat loss from the exhaust gases in the exhaust stack can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (1.2\,\text{kg/s})(1600\,\text{J/kg}\cdot{}^\circ\text{C})(30)\,{}^\circ\text{C} = 57600\,\text{W}$$

The heat loss on the outer surface of the exhaust stack by radiation and convection can be expressed as

$$\frac{\dot{Q}_{\text{loss}}}{A_{s,2}} = h[T(r_2) - T_\infty] + \varepsilon\sigma\,[T(r_2)^4 - T_{\text{surr}}^4] - \alpha_s \dot{q}_{\text{solar}}$$

$$\frac{57600\,\text{W}}{31.42\,\text{m}^2} = (8\,\text{W/m}^2\cdot\text{K})[T(r_2) - (27+273)]\,\text{K}$$

$$+(0.9)(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)[T(r_2)^4 - (27+273)^4]\,\text{K}^4 - (0.9)(150\,\text{W/m}^2)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
57600/31.42=8*(T_r2-(27+273))+0.9*5.67e-8*(T_r2^4-(27+273)^4)-0.9*150
```

Solving by EES software, the outside surface temperature of the furnace front is

$$T(r_2) = 412.7\,\text{K}$$

(*a*) For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $\quad -k\dfrac{dT(r_1)}{dr} = \dfrac{\dot{Q}_{\text{loss}}}{A_{s,1}} = \dfrac{\dot{Q}_{\text{loss}}}{2\pi\, r_1 L}$ (heat flux at the inner exhaust stack surface)

$$T(r_2) = 412.7\,\text{K} \quad \text{(outer exhaust stack surface temperature)}$$

Integrating the differential equation once with respect to *r* gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to *r* again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad \frac{dT(r_1)}{dr} = -\frac{1}{k}\frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} = \frac{C_1}{r_1} \quad \rightarrow \quad C_1 = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}$$

$$r = r_2: \qquad T(r_2) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + C_2 \quad \rightarrow \quad C_2 = \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + T(r_2)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r + \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + T(r_2)$$

$$= -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r/r_2) + T(r_2)$$

(*b*) The inner surface temperature of the exhaust stack is

$$T(r_1) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r_1/r_2) + T(r_2)$$

$$= -\frac{1}{2\pi}\frac{57600\ \text{W}}{(40\ \text{W/m}\cdot\text{K})(10\ \text{m})}\ln\left(\frac{0.4}{0.5}\right) + 412.7\ \text{K}$$

$$= 417.7\ \text{K} = \mathbf{418\ K}$$

Discussion There is a temperature drop of 5 °C from the inner to the outer surface of the exhaust stack.

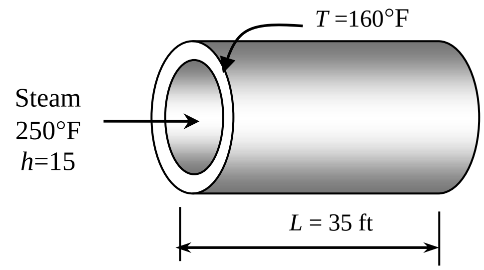
2-154E A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Properties The thermal conductivity is given to be $k = 8$ Btu/h·ft·°F.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



The boundary conditions for this problem are:

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$T(r_2) = T_2 = 160\,°\text{F}$$

(*b*) Integrating the differential equation once with respect to *r* gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$-k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)]$$

$r = r_2$: $$T(r_2) = C_1 \ln r_2 + C_2 = T_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{T_2 - T_\infty}{ln\frac{r_2}{r_1} + \frac{k}{hr_1}} \quad \text{and} \quad C_2 = T_2 - C_1 ln\, r_2 = T_2 - \frac{T_2 - T_\infty}{ln\frac{r_2}{r_1} + \frac{k}{hr_1}} ln\, r_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_2 - C_1 \ln r_2 = C_1(\ln r - \ln r_2) + T_2 = \frac{T_2 - T_\infty}{\ln\frac{r_2}{r_1} + \frac{k}{hr_1}} \ln\frac{r}{r_2} + T_2$$

$$= \frac{(160 - 250)\,°\text{F}}{\ln\frac{2.4}{2} + \frac{8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(15\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}} \ln\frac{r}{2.4\text{ in}} + 160\,°\text{F} = -26.61 \ln\frac{r}{2.4\text{ in}} + 160\,°\text{F}$$

(*c*) The rate of heat conduction through the pipe is

$$\dot{Q} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi Lk\frac{T_2 - T_\infty}{\ln\frac{r_2}{r_1} + \frac{k}{hr_1}}$$

$$= -2\pi(35\text{ ft})(8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(160 - 250)\,°\text{F}}{\ln\frac{2.4}{2} + \frac{8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(15\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}} = \mathbf{46{,}813\text{ Btu/h}}$$

2-155 A compressed air pipe is subjected to uniform heat flux on the outer surface and convection on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

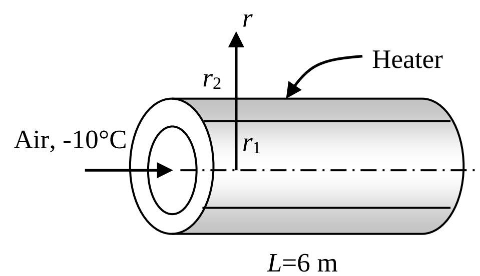
Properties The thermal conductivity is given to be $k = 14$ W/m·K.

Analysis (*a*) Noting that the 85% of the 300 W generated by the strip heater is transferred to the pipe, the heat flux through the outer surface is determined to be

$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{0.85\times 300\text{ W}}{2\pi(0.04\text{ m})(6\text{ m})} = 169.1\text{ W/m}^2$$

Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



The boundary conditions for this problem are:

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$

(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $\quad k\frac{C_1}{r_2} = \dot{q}_s \ \rightarrow\ C_1 = \frac{\dot{q}_s r_2}{k}$

$r = r_1$: $\quad -k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)] \ \rightarrow\ C_2 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln r - \ln r_1 + \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln\frac{r}{r_1} + \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$= -10°\text{C} + \left(\ln\frac{\text{r}}{\text{r}_1} + \frac{14\text{ W/m}\cdot\text{K}}{(30\text{ W/m}^2\cdot\text{K})(0.037\text{ m})}\right)\frac{(169.1\text{ W/m}^2)(0.04\text{ m})}{14\text{ W/m}\cdot\text{K}} = -10 + 0.483\left(\ln\frac{r}{r_1} + 12.61\right)$$

(*c*) The inner and outer surface temperatures are determined by direct substitution to be

Inner surface ($r = r_1$): $\quad T(r_1) = -10 + 0.483\left(\ln\frac{r_1}{r_1} + 12.61\right) = -10 + 0.483(0 + 12.61) = \mathbf{-3.91°C}$

Outer surface ($r = r_2$): $\quad T(r_2) = -10 + 0.483\left(\ln\frac{r_2}{r_1} + 12.61\right) = -10 + 0.483\left(\ln\frac{0.04}{0.037} + 12.61\right) = \mathbf{-3.87°C}$

Discussion Note that the pipe is essentially isothermal at a temperature of about -3.9°C.

2-156 A hollow pipe is subjected to specified temperatures at the inner and outer surfaces. There is also heat generation in the pipe. The variation of temperature in the pipe and the center surface temperature of the pipe are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the centerline. **2** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 14$ W/m·°C.

Analysis The rate of heat generation is determined from

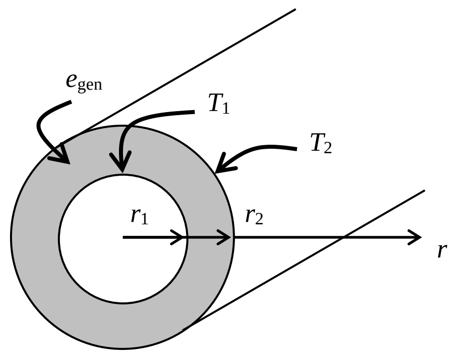
$$\dot{e}_{\text{gen}} = \frac{\dot{W}}{V} = \frac{\dot{W}}{\pi(D_2^2 - D_1^2)L/4} = \frac{25{,}000 \text{ W}}{\pi\left[(0.4\text{ m})^2 - (0.3\text{ m})^2\right](17\text{ m})/4} = 26{,}750 \text{ W/m}^3$$

Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

and $T(r_1) = T_1 = 60°\text{C}$

$T(r_2) = T_2 = 80°\text{C}$



Rearranging the differential equation

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{e}_{\text{gen}}r}{k} = 0$$

and then integrating once with respect to r,

$$r\frac{dT}{dr} = \frac{-\dot{e}_{\text{gen}}r^2}{2k} + C_1$$

Rearranging the differential equation again

$$\frac{dT}{dr} = \frac{-\dot{e}_{\text{gen}}r}{2k} + \frac{C_1}{r}$$

and finally integrating again with respect to r, we obtain

$$T(r) = \frac{-\dot{e}_{\text{gen}}r^2}{4k} + C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$T(r_1) = \frac{-\dot{e}_{\text{gen}}r_1^2}{4k} + C_1 \ln r_1 + C_2$$

$r = r_2$: $$T(r_2) = \frac{-\dot{e}_{\text{gen}}r_2^2}{4k} + C_1 \ln r_2 + C_2$$

Substituting the given values, these equations can be written as

$$60 = \frac{-(26{,}750)(0.15)^2}{4(14)} + C_1 \ln(0.15) + C_2$$

$$80 = \frac{-(26{,}750)(0.20)^2}{4(14)} + C_1 \ln(0.20) + C_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = 98.58 \qquad C_2 = 257.8$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = \frac{-26{,}750r^2}{4(14)} + 98.58 \ln r + 257.8 = 257.8 - 477.7r^2 + 98.58 \ln r$$

The temperature at the center surface of the pipe is determined by setting radius r to be 17.5 cm, which is the average of the inner radius and outer radius.

$$T(r) = 257.8 - 477.7(0.175)^2 + 98.58\ln(0.175) = \mathbf{71.3°C}$$

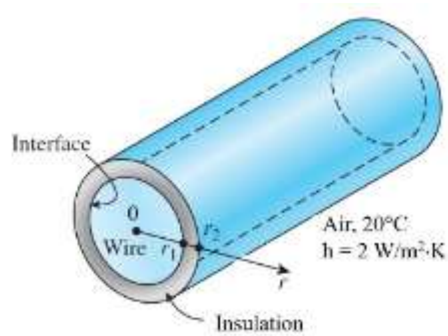
2-157 A long electrical resistance wire that is generating heat uniformly is covered with polyethylene insulation. Formulate the temperature profiles for the wire and the polyethylene insulation. Determine the temperature at the interface of the wire and the insulation, and the temperature at the center of the wire. Conclude whether the polyethylene insulation for the wire meets the ASTM D1351 standard.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Heat generation in the wire is uniform. **4** There is no contact resistance at the interface of the wire and the insulation, $r = r_1$. **5** At the center of the wire, $r = 0$, is a symmetry boundary. **6** The outer surface of the insulation, $r = r_2$, is subjected to convection.

PropertiesThe thermal conductivities of the wire and the polyethylene insulation are given to be k_{wire} = 15 W/m·K and k_{ins} = 0.4 W/m·K, respectively.

AnalysisFor one-dimensional heat transfer in the radial r direction with uniform heat generation, the differential equation for heat conduction in cylindrical coordinate for the wire can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}=0 \qquad \text{or}\ \frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{wire}}}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k_{\text{wire}}}r^2+C_1$$

$$T_{\text{wire}}(r)=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r^2+C_1\ln r+C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0:\qquad \frac{dT_{\text{wire}}(0)}{dr}=0 \qquad \rightarrow \qquad C_1=0$$

$$r=r_1:\qquad T_{\text{wire}}(r_1)=T_I=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2+C_2 \qquad \rightarrow \qquad C_2=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the wire is determined to be

$$\boldsymbol{T_{\text{wire}}(r)=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}\left(r_1^2-r^2\right)}\text{for} \qquad 0\le r\le r_1$$

The insulation layer does not involve any heat generation, the heat conduction equation in the insulation layer is

$$\frac{d}{dr}\left(r\frac{dT_{\text{ins}}}{dr}\right)=0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{ins}}}{dr}=C_3\text{or}\frac{dT_{\text{ins}}}{dr}=\frac{C_3}{r}$$

$$T_{\text{ins}}(r)=C_3\ln r+C_4$$

whereC_3 and C_4 are arbitrary constants. Applying the boundary conditions yields

$$r=r_1:\qquad -k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr}=-k_{\text{ins}}\frac{dT_{\text{ins}}(r_1)}{dr} \qquad \rightarrow \qquad \frac{\dot{e}_{\text{gen}}}{2}r_1=-k_{\text{ins}}\frac{C_3}{r_1}$$

$$r=r_2:\qquad -k_{\text{ins}}\frac{dT_{\text{ins}}(r_2)}{dr}=-k_{\text{ins}}\frac{C_3}{r_2}=h[T_{\text{ins}}(r_2)-T_\infty]$$

C_3 and C_4 can be expressed as

$$C_3 = -\frac{\dot{e}_{\text{gen}}}{2}\frac{r_1^2}{k_{\text{ins}}}$$

$$C_4 = T_\infty + \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln r_2\right)$$

Substituting C_3 and C_4 into the general solution, the temperature profile in the insulation layer is determined to be

$$\boldsymbol{T_{\text{ins}}(r) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln\frac{r_2}{r}\right) + T_\infty} \text{ for } r_1 \le r \le r_2$$

At the interface of the wire and the insulation, $r = r_1$, we have

$$T_I = T_{\text{ins}}(r_1) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln\frac{r_2}{r_1}\right) + T_\infty$$

$$T_I = \frac{\left(5\times 10^5\,\frac{\text{W}}{\text{m}^3}\right)(0.0025\text{ m})^2}{2(0.4\text{ W/m}\cdot\text{K})}\left(\frac{0.4\text{ W/m}\cdot\text{K}}{2}\times\frac{1}{0.005\text{ m}} + \ln\frac{0.005\text{ m}}{0.0025\text{ m}}\right) + 20°\text{C} = \mathbf{178.96}°\text{C} > 75°\text{C}$$

whereand $r_2 = r_1$ + wall thickness = 0.0025 m + 0.0025 m = 0.005 m.

The temperature at the center of the wire, $r = 0$, is

$$T_{\text{wire}}(0) = T_I + \frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2 = 178.96 + \frac{\left(5\times 10^5\,\frac{\text{W}}{\text{m}^3}\right)(0.0025\text{ m})^2}{4(15\text{ W/m}\cdot\text{K})} = \mathbf{179.01}°\text{C}$$

Discussion With the temperature at the interface of the wire and the insulation being about 104°C higher than the specification of the ASTM D1351 standard for polyethylene insulation, the ASTM standard is not met. If the convection heat transfer coefficient at the outer surface of the insulation is increase to 6 W/m^2·K or higher, the temperature at the interface of the wire and the insulation would be lower than 75°C.

2-158 In a quenching process, steel ball bearings at a given instant have a rate of temperature decrease of 50 K/s. The rate of heat loss is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** There is no heat generation. **3** Thermal properties are constant.

Properties The properties of the steel ball bearings are given to be c = 500 J/kg · K, k = 60 W/m · K, and ρ = 7900 kg/m^3.

Analysis The thermal diffusivity on the steel ball bearing is

$$\alpha = \frac{k}{\rho c} = \frac{60 \text{ W/m}\cdot\text{K}}{(7900 \text{ kg/m}^3)(500 \text{ J/kg}\cdot\text{K})} = 15.19\times10^{-6} \text{ m}^2\text{/s}$$

The given rate of temperature decrease can be expressed as

$$\frac{dT(r)}{dt} = -50 \text{ K/s}$$

For one-dimensional transient heat conduction in a sphere with no heat generation, the differential equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Substituting the thermal diffusivity and the rate of temperature decrease, the differential equation can be written as

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}$$

Multiply both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr} = \frac{-50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}\left(\frac{r^3}{3}\right) + C_1 \qquad (a)$$

Applying the boundary condition at the midpoint (thermal symmetry about the midpoint),

$$r = 0: \qquad 0\times\frac{dT(0)}{dr} = \frac{-50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}\left(\frac{0}{3}\right) + C_1 \quad\rightarrow\quad C_1 = 0$$

Dividing both sides of Eq. (a) by r^2 gives

$$\frac{dT}{dr} = \frac{-50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}\left(\frac{r}{3}\right)$$

The rate of heat loss through the steel ball bearing surface can be determined from Fourier's law to be

$$\begin{aligned}\dot{Q}_{\text{loss}} &= -kA\frac{dT}{dr}\\ &= -k(4\pi r_o^2)\frac{dT(r_o)}{dr} = k(4\pi r_o^2)\frac{50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}\left(\frac{r_o}{3}\right)\\ &= (60 \text{ W/m}\cdot\text{K})(4\pi)(0.125 \text{ m})^2\,\frac{50 \text{ K/s}}{15.19\times10^{-6} \text{ m}^2\text{/s}}\left(\frac{0.125 \text{ m}}{3}\right)\\ &= \mathbf{1.62\ kW}\end{aligned}$$

Discussion The rate of heat loss through the steel ball bearing surface determined here is for the given instant when the rate of temperature decrease is 50 K/s.

2-159 A spherical reactor of 5-cm diameter operating at steady condition has its heat generation suddenly set to 9 MW/m^3. The time rate of temperature change in the reactor is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Heat generation is uniform. **3** Thermal properties are constant.

Properties The properties of the reactor are given to be c = 200 J/kg·°C, k = 40 W/m·°C, and ρ = 9000 kg/m^3.

Analysis The thermal diffusivity of the reactor is

$$\alpha = \frac{k}{\rho c} = \frac{40 \text{ W/m}\cdot°\text{C}}{(9000 \text{ kg/m}^3)(200 \text{ J/kg}\cdot°\text{C})} = 22.22\times10^{-6} \text{ m}^2/\text{s}$$

For one-dimensional transient heat conduction in a sphere with heat generation, the differential equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{or} \qquad \frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{\text{gen}}}{k}\right]$$

At the instant when the heat generation of reactor is suddenly set to 90 MW/m^3 (t = 0), the temperature variation can be expressed by the given $T(r) = a - br^2$, hence

$$\frac{\partial T}{\partial t} = \alpha\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(a - br^2)\right] + \frac{\dot{e}_{\text{gen}}}{k}\right\} = \alpha\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(-2br)\right] + \frac{\dot{e}_{\text{gen}}}{k}\right\}$$

$$= \alpha\left[\frac{1}{r^2}(-6br^2) + \frac{\dot{e}_{\text{gen}}}{k}\right] = \alpha\left(-6b + \frac{\dot{e}_{\text{gen}}}{k}\right)$$

The time rate of temperature change in the reactor when the heat generation suddenly set to 9 MW/m^3 is determined to be

$$\frac{\partial T}{\partial t} = \alpha\left(-6b + \frac{\dot{e}_{\text{gen}}}{k}\right) = (22.22\times10^{-6} \text{ m}^2/\text{s})\left[-6(5\times10^5 \text{ °C/m}^2) + \frac{9\times10^6 \text{ W/m}^3}{40 \text{ W/m}\cdot°\text{C}}\right]$$

$$= \mathbf{-61.7\ °C/s}$$

Discussion Since the time rate of temperature change is a negative value, this indicates that the heat generation of reactor is suddenly decreased to 9 MW/m^3.

2-160 A cylindrical shell with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the shell is to be determined.

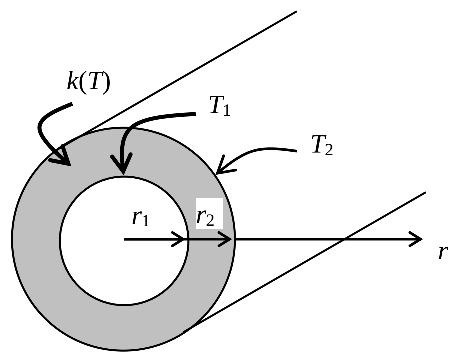
Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies quadratically. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T^2)$.

Analysis When the variation of thermal conductivity with temperature *k*(*T*) is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 is determined from

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} k_0(1+\beta T^2)dT}{T_2 - T_1} = \frac{k_0\left(T + \frac{\beta}{3}T^3\right)_{T_1}^{T_2}}{T_2 - T_1} = \frac{k_0\left[(T_2 - T_1) + \frac{\beta}{3}(T_2^3 - T_1^3)\right]}{T_2 - T_1}$$

$$= k_0\left[1 + \frac{\beta}{3}\left(T_2^2 + T_1T_2 + T_1^2\right)\right]$$



This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{avg} equals the rate of heat transfer through the same medium with variable conductivity *k*(*T*). Then the rate of heat conduction through the cylindrical shell can be determined from Eq. 2-77 to be

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = 2\pi k_0\left[1 + \frac{\beta}{3}\left(T_2^2 + T_1T_2 + T_1^2\right)\right] L \frac{T_1 - T_2}{\ln(r_2/r_1)}$$

Discussion We would obtain the same result if we substituted the given *k*(*T*) relation into the second part of Eq. 2-77, and performed the indicated integration.

2-161 A pipe is used for transporting boiling water with a known inner surface temperature in a surrounding of cooler ambient temperature and known convection heat transfer coefficient. The pipe wall has a variable thermal conductivity. The outer surface temperature of the pipe is to be determined.

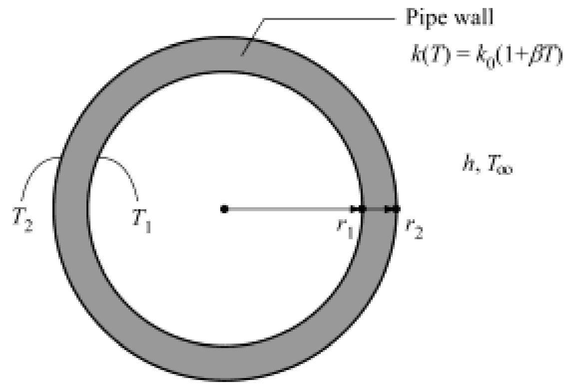
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature. **4** Inner pipe surface temperature is constant at 100°C.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the pipe are

$$r_1 = 0.025/2 \text{ m} = 0.0125 \text{ m} \quad \text{and} \quad r_2 = (0.0125 + 0.003)\text{ m} = 0.0155 \text{ m}$$

The rate of heat transfer at the pipe's outer surface can be expressed as



$$\dot{Q}_{\text{cylinder}} = \dot{Q}_{\text{conv}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty)$$

$$\frac{k_{\text{avg}}}{r_2}\frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) \qquad (1)$$

where

$$h = 50 \text{ W/m}^2\text{ K}, \quad T_1 = 373 \text{ K}, \quad \text{and} \quad T_\infty = 293 \text{ K}$$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.5 \text{W/m}\cdot\text{K})\left[1 + (0.003 \text{ K}^{-1})\frac{T_2 + (373 \text{ K})}{2}\right]$$

$$k_{\text{avg}} = [1.5 + 0.00225(T_2 + 373)] \text{ W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{369\,K} = \mathbf{96°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=50 [W/(m^2*K)]   "convection heat transfer coefficient"
r_1=0.025/2 [m]   "inner radius"
r_2=r_1+0.003 [m]   "outer radius"
T_1=373 [K]   "inner surface temperature"
T_inf=293 [K]   "ambient temperature"
k_0=1.5 [W/(m*K)]
beta=0.003 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
Q_dot_cylinder=2*pi*k_avg*(T_1-T_2)/ln(r_2/r_1)   "heat rate through the cylindrical layer"
Q_dot_conv=h*2*pi*r_2*(T_2-T_inf)   "heat rate by convection"
Q_dot_cylinder=Q_dot_conv
```

Discussion Increasing h or decreasing k_{avg} would decrease the pipe's outer surface temperature.

2-162 A metal spherical tank, filled with chemicals undergoing an exothermic reaction, has a known inner surface temperature. The tank wall has a variable thermal conductivity. Convection heat transfer occurs on the outer tank surface. The heat flux on the inner surface of the tank is to be determined.

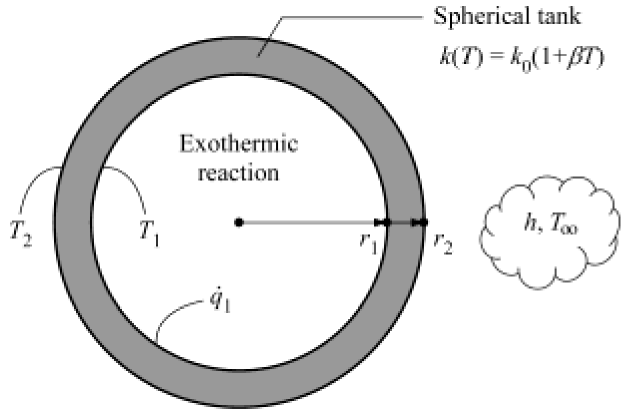
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the tank are

$$r_1 = 5/2\,\text{m} = 2.5\,\text{m}$$

and $$r_2 = (2.5 + 0.01)\,\text{m} = 2.51\,\text{m}$$



The rate of heat transfer at the tank's outer surface can be expressed as

$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}}$$

$$4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_2 - T_\infty)$$

$$k_{\text{avg}} \frac{r_1}{r_2} \frac{T_1 - T_2}{r_2 - r_1} = h(T_2 - T_\infty) \qquad (1)$$

where

$h = 80$ W/m^2 K, $T_1 = 393$ K, and $T_\infty = 288$ K

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (9.1\,\text{W/m}\cdot\text{K})\left[1 + (0.0018\ \text{K}^{-1})\frac{T_2 + (393\ \text{K})}{2}\right]$$

$$k_{\text{avg}} = [9.1 + 0.00819(\ T_2 + 393)]\ \text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for T_2 and k_{avg} yields

$$T_2 = 387.8\,\text{K} \quad \text{and} \quad k_{\text{avg}} = 15.5\,\text{W/m}\cdot\text{K}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=80 [W/(m^2*K)]   "outer surface h"
r_1=5/2 [m]   "inner radius"
r_2=r_1+0.010 [m]   "outer radius"
T_1=120+273 [K]   "inner surface T"
T_inf=15+273 [K]   "ambient T"
k_0=9.1 [W/(m*K)]
beta=0.0018 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE AND k_avg"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_sph=k_avg*r_1/r_2*(T_1-T_2)/(r_2-r_1)   "heat flux through the spherical layer"
q_dot_conv=h*(T_inf-T_2)   "heat flux by convection"
q_dot_sph+q_dot_conv=0
```

Thus, the heat flux on the inner surface of the tank is

$$\dot{q}_1 = \frac{\dot{Q}_{\text{sph}}}{4\pi r_1^2} = \frac{4\pi k_{\text{avg}} r_1 r_2}{4\pi r_1^2}\frac{T_1 - T_2}{r_2 - r_1} = k_{\text{avg}}\frac{r_2}{r_1}\frac{T_1 - T_2}{r_2 - r_1} = (15.5\ \text{W/m}\cdot\text{K})\left(\frac{2.51}{2.5}\right)\frac{(393 - 387.8)\ \text{K}}{0.01\ \text{m}}$$

$$\dot{q}_1 = \mathbf{8092.2\,W/m^2}$$

Discussion The inner-to-outer surface heat flux ratio can be related to r_1 and r_2: $\dot{q}_1/\dot{q}_2 = (r_2/r_1)^2$.

Fundamentals of Engineering (FE) Exam Problems

2-163 The heat conduction equation in a medium is given in its simplest form as $\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right)+\dot{e}_{\text{gen}}=0$ Select the wrong statement below.

(a) the medium is of cylindrical shape.
(b) the thermal conductivity of the medium is constant.
(c) heat transfer through the medium is steady.
(d) there is heat generation within the medium.
(e) heat conduction through the medium is one-dimensional.

Answer (b) thermal conductivity of the medium is constant

2-164 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?
(*e*) Is the medium a plane wall, a cylinder, or a sphere?
(*f*) Is this differential equation for heat conduction linear or nonlinear?

Answers: (*a*) transient, (*b*) one-dimensional, (*c*) no, (*d*) constant, (*e*) sphere, (*f*) linear

2-165 Consider a large plane wall of thickness *L*, thermal conductivity *k*, and surface area *A*. The left surface of the wall is exposed to the ambient air at T_∞ with a heat transfer coefficient of *h* while the right surface is insulated. The variation of temperature in the wall for steady one-dimensional heat conduction with no heat generation is

(a) $T(x)=\frac{h(L-x)}{k}T_\infty$ (b) $T(x)=\frac{k}{h(x+0.5L)}T_\infty$ (c) $T(x)=\left(1-\frac{xh}{k}\right)T_\infty$ (d) $T(x)=(L-x)T_\infty$

(e) $T(x)=T_\infty$

Answer (e) $T(x)=T_\infty$

2-166 A solar heat flux $\dot{q}_s$ is incident on a sidewalk whose thermal conductivity is k, solar absorptivity is α_s and convective heat transfer coefficient is h. Taking the positive x direction to be towards the sky and disregarding radiation exchange with the surroundings surfaces, the correct boundary condition for this sidewalk surface is

(a) $-k\dfrac{dT}{dx} = \alpha_s \dot{q}_s$ (b) $-k\dfrac{dT}{dx} = h(T - T_\infty)$ (c) $-k\dfrac{dT}{dx} = h(T - T_\infty) - \alpha_s \dot{q}_s$

(d) $h(T - T_\infty) = \alpha_s \dot{q}_s$ (e) None of them

Answer (c) $-k\dfrac{dT}{dx} = h(T - T_\infty) - \alpha_s \dot{q}_s$

2-167 A plane wall of thickness L is subjected to convection at both surfaces with ambient temperature $T_{\infty 1}$ and heat transfer coefficient h_1 at inner surface, and corresponding $T_{\infty 2}$ and h_2 values at the outer surface. Taking the positive direction of x to be from the inner surface to the outer surface, the correct expression for the convection boundary condition is

(a) $k\dfrac{dT(0)}{dx} = h_1[T(0) - T_{\infty 1})]$ (b) $k\dfrac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2})]$

(c) $-k\dfrac{dT(0)}{dx} = h_1[T_{\infty 1} - T_{\infty 2})]$ (d) $-k\dfrac{dT(L)}{dx} = h_2[T_{\infty 1} - T_{\infty 2})]$ (e) None of them

Answer (a) $k\dfrac{dT(0)}{dx} = h_1[T(0) - T_{\infty 1})]$

2-168 Consider steady one-dimensional heat conduction through a plane wall, a cylindrical shell, and a spherical shell of uniform thickness with constant thermophysical properties and no thermal energy generation. The geometry in which the variation of temperature in the direction of heat transfer be linear is

(a) plane wall (b) cylindrical shell (c) spherical shell (d) all of them (e) none of them

Answer (a) plane wall

2-169 The conduction equation boundary condition for an adiabatic surface with direction n being normal to the surface is

(a) $T = 0$ (b) $dT/dn = 0$ (c) $d^2T/dn^2 = 0$ (d) $d^3T/dn^3 = 0$ (e) $-kdT/dn = 1$

Answer (b) $dT/dn = 0$

2-170 The variation of temperature in a plane wall is determined to be $T(x)=65x+25$ where x is in m and T is in °C. If the temperature at one surface is 38ºC, the thickness of the wall is

(a) 2 m (b) 0.4 m (c) 0.2 m (d) 0.1 m (e) 0.05 m

Answer (c) 0.2 m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
38=65*L+25
```

2-171 The variation of temperature in a plane wall is determined to be $T(x)=110-48x$ where x is in m and T is in °C. If the thickness of the wall is 0.75 m, the temperature difference between the inner and outer surfaces of the wall is

(a) 110ºC (b) 74ºC (c) 55ºC (d) 36ºC (e) 18ºC

Answer (d) 36ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=110 [C]
L=0.75
T2=110-48*L
DELTAT=T1-T2
```

2-172 The temperatures at the inner and outer surfaces of a 15-cm-thick plane wall are measured to be 40ºC and 28ºC, respectively. The expression for steady, one-dimensional variation of temperature in the wall is

(a) $T(x)=28x+40$ (b) $T(x)=-40x+28$ (c) $T(x)=40x+28$

(d) $T(x)=-80x+40$ (e) $T(x)=40x-80$

Answer (d) $T(x)=-80x+40$

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=40 [C]
T2=28 [C]
L=0.15 [m]
"T(x)=C1x+C2"
C2=T1
T2=C1*L+T1
```

2-173 The thermal conductivity of a solid depends upon the solid's temperature as $k = aT + b$ where a and b are constants. The temperature in a planar layer of this solid as it conducts heat is given by

(a) $aT + b = x + C_2$ (b) $aT + b = C_1x^2 + C_2$ (c) $aT^2 + bT = C_1x + C_2$
(d) $aT^2 + bT = C_1x^2 + C_2$ (e) None of them

Answer (c) $aT^2 + bT = C_1x + C_2$

2-174 Hot water flows through a PVC (k = 0.092 W/m·K) pipe whose inner diameter is 2 cm and outer diameter is 2.5 cm. The temperature of the interior surface of this pipe is 35°C and the temperature of the exterior surface is 20°C. The rate of heat transfer per unit of pipe length is

(a) 22.8 W/m (b) 38.9 W/m (c) 48.7 W/m (d) 63.6 W/m (e) 72.6 W/m

Answer (b) 38.9 W/m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
do=2.5 [cm]
di=2.0 [cm]
k=0.092 [W/m-C]
T2=35 [C]
T1=20 [C]
Q=2*pi*k*(T2-T1)/LN(do/di)
```

2-175 Heat is generated in a long 0.3-cm-diameter cylindrical electric heater at a rate of 150 W/cm^3. The heat flux at the surface of the heater in steady operation is

(a) 42.7 W/cm^2 (b) 159 W/cm^2 (c) 150 W/cm^2 (d) 10.6 W/cm^2 (e) 11.3 W/cm^2

Answer (e) 11.3 W/cm^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"Consider a 1-cm long heater:"
L=1 [cm]
e=150 [W/cm^3]
D=0.3 [cm]
V=pi*(D^2/4)*L
A=pi*D*L "[cm^2]”
Egen=e*V "[W]"
Qflux=Egen/A "[W/cm^2]"

“Some Wrong Solutions with Common Mistakes:”
W1=Egen "Ignoring area effect and using the total"
W2=e/A "Threating g as total generation rate"
W3=e “ignoring volume and area effects”
```

2-176 Heat is generated uniformly in a 4-cm-diameter, 16-cm-long solid bar (*k* = 2.4 W/m·ºC). The temperatures at the center and at the surface of the bar are measured to be 210ºC and 45ºC, respectively. The rate of heat generation within the bar is

(a) 240 W (b) 796 W b) 1013 W (c) 79,620 W (d) 3.96×10^6 W

Answer (b) 796 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
L=0.16 [m]
k=2.4 [W/m-C]
T0=210 [C]
T_s=45 [C]
T0-T_s=(e*(D/2)^2)/(4*k)
V=pi*D^2/4*L
E_dot_gen=e*V

"Some Wrong Solutions with Common Mistakes"
W1_V=pi*D*L "Using surface area equation for volume"
W1_E_dot_gen=e*W14_1
T0=(W2_e*(D/2)^2)/(4*k) "Using center temperature instead of temperature difference"
W2_Q_dot_gen=W2_e*V
W3_Q_dot_gen=e "Using heat generation per unit volume instead of total heat generation as the result"
```

2-177 Heat is generated in a 8-cm-diameter spherical radioactive material whose thermal conductivity is 25 W/m.°C uniformly at a rate of 15 W/cm^3. If the surface temperature of the material is measured to be 120°C, the center temperature of the material during steady operation is

(a) 160°C (b) 280°C (c) 212°C (d) 360°C (e) 600°C

Answer (b) 280°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.08
Ts=120
k=25
e_gen=15E+6
T=Ts+g*(D/2)^2/(6*k)

“Some Wrong Solutions with Common Mistakes:”
W1_T= e_gen*(D/2)^2/(6*k) "Not using Ts"
W2_T= Ts+e_gen*(D/2)^2/(4*k) "Using the relation for cylinder"
W3_T= Ts+e_gen*(D/2)^2/(2*k) "Using the relation for slab"
```

2-178 Heat is generated in a 3-cm-diameter spherical radioactive material uniformly at a rate of 15 W/cm^3. Heat is dissipated to the surrounding medium at 25°C with a heat transfer coefficient of 120 W/m^2·°C. The surface temperature of the material in steady operation is

(a) 56°C (b) 84°C (c) 494°C (d) 650°C (e) 108°C

Answer (d) 650°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=120 [W/m^2-C]
e=15 [W/cm^3]
Tinf=25 [C]
D=3 [cm]
V=pi*D^3/6 "[cm^3]"
A=pi*D^2/10000 "[m^2]"
Egen=e*V "[W]"
Qgen=h*A*(Ts-Tinf)
```

2-179 .... 2-181 Design and Essay Problems