2.1 A Preview of Calculus

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

Find the distance traveled in 16 seconds by an object traveling at a constant velocity of 20 feet per second.

a. calculus, 320 ft
b. calculus, 340 ft
c. precalculus, 320 ft
d. calculus, 640 ft
e. precalculus, 640 ft

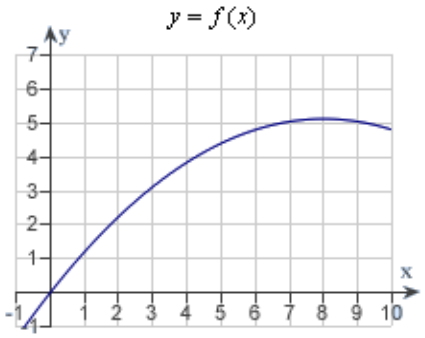
____ 2. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

Find the distance traveled in 20 seconds by an object moving with a velocity of $v(t) = 8 + 6\cos t$ feet per second.

a. calculus, 162.4485 ft
b. precalculus, 163.7985 ft
c. calculus, 165.4777 ft
d. precalculus, 165.4777 ft
e. precalculus, 162.4485 ft

____ 3. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

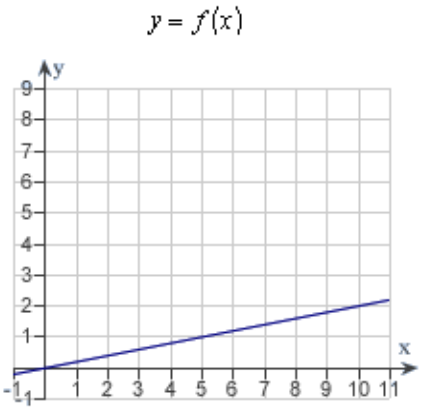
A cyclist is riding on a path whose elevation is modeled by the function $f(x) = 0.08\left(16x - x^2\right)$ where x and $f(x)$ are measured in miles. Find the rate of change of elevation when $x = 4$.



a. precalculus, 0.08
b. calculus, 0.2
c. calculus, 0.64
d. calculus, 0.08
e. precalculus, 0.2

____ 4. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

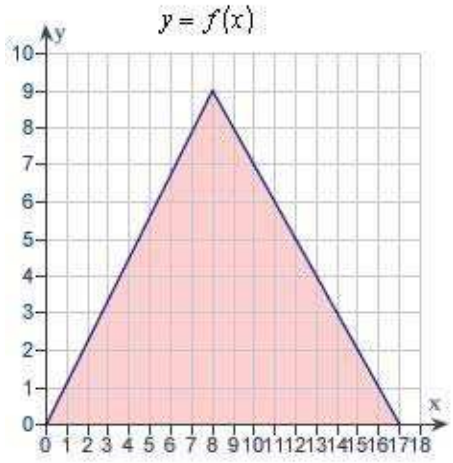
A cyclist is riding on a path whose elevation is modeled by the function $f(x) = 0.2x$ where x and $f(x)$ are measured in miles. Find the rate of change of elevation when $x = 5$.



a. calculus, 2
b. precalculus, 0.2
c. calculus, 0.2
d. precalculus, 2
e. precalculus, 0.45

____ 5. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

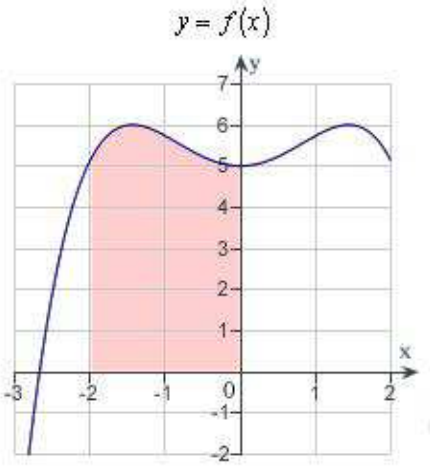
Find the area of the shaded region bounded by the triangle with vertices (0,0), (8,9), (17,0).



a. precalculus , 153
b. calculus , 229.5
c. precalculus , 76.5
d. precalculus , 229.5
e. calculus , 153

____ 6. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

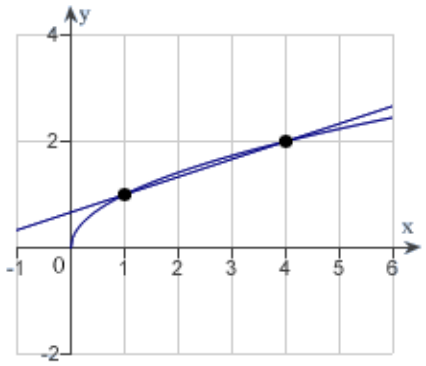
Find the area of the shaded region.



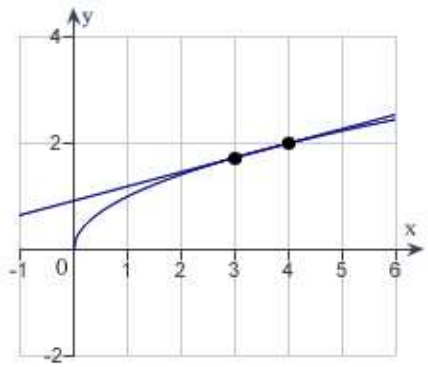
a. calculus , 11
b. precalculus , 11
c. precalculus , 13
d. calculus , 16
e. precalculus , 16

____ 7. Consider the function $f(x) = \sqrt{x}$ and the point $P(4, 2)$ on the graph of *f*. Graph *f* and the secant line passing through $P(4, 2)$ and $Q\left(x, f(x)\right)$ for $x = 3$.

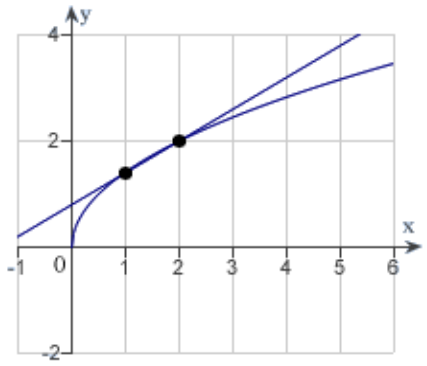
a.



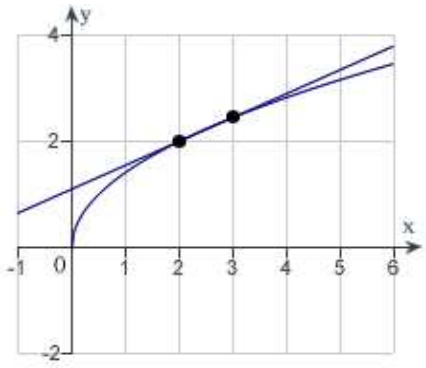
d.



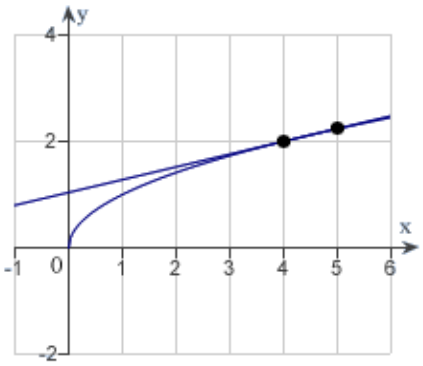
b.



e.



c.



____ 8. Consider the function $f(x) = \sqrt{x}$ and the point $P(81, 9)$ on the graph of *f*. Find the slope of the secant line passing through $P(81, 9)$ and $Q(x, f(x))$ for $x = 1$. Round your answer to four decimal places.

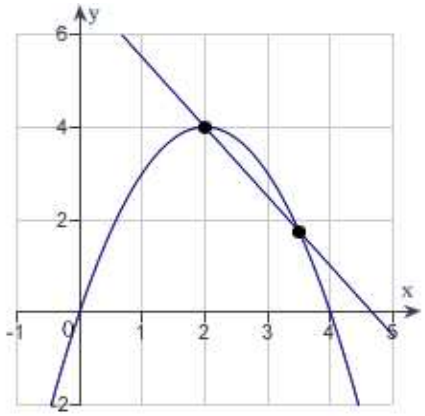
a. m=0.1000
b. m=0.0122
c. m=0.0122
d. m=0.3133
e. m=0.1000

____ 9. Consider the function $f(x) = \sqrt{x}$ and the point $P(9, 3)$ on the graph of *f*. Estimate the slope *m* of the tangent line of *f* at $P(9, 3)$. Round your answer to four decimal places.

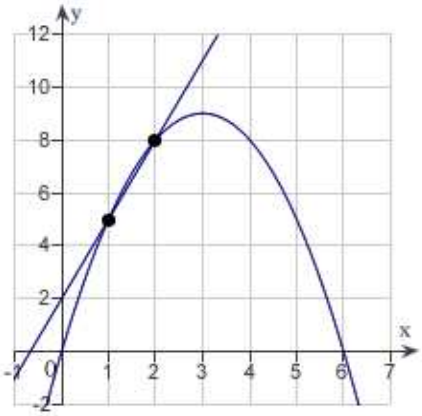
a. m=0.1667
b. m=0.0832
c. m=0.3800
d. m=0.0556
e. m=0.0833

____ 10. Consider the function $f(x) = 6x - x^2$ and the point $P(2, 8)$ on the graph of *f*. Graph *f* and the secant line passing through $P(2, 8)$ and $Q(x, f(x))$ for $x = 3$.

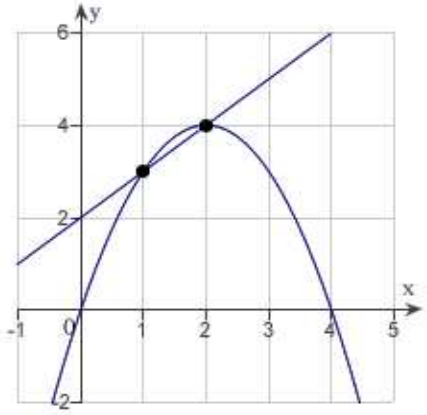
a.



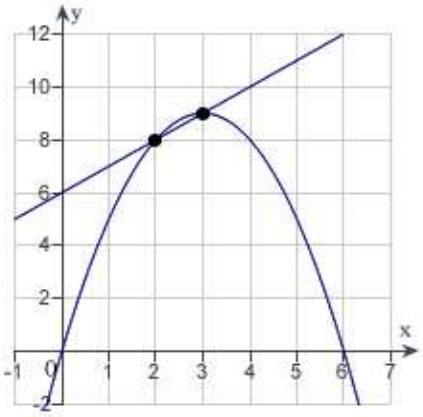
b.



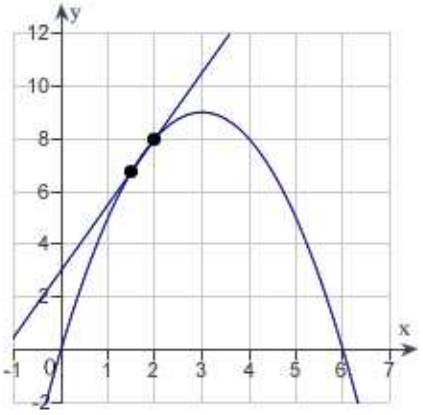
c.



d.



e.



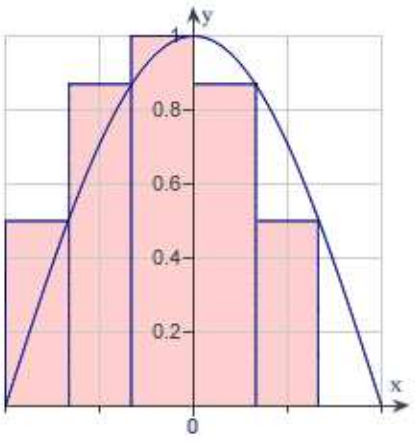
____ 11. Consider the function $f(x) = 11x - x^2$ and the point $P(4, 28)$ on the graph of *f*. Find the slope of the secant line passing through $P(4, 28)$ and $Q\left(x, f(x)\right)$ for $x = 5$. Round your answer to one decimal place.

a. 3.5
b. 2.0
c. 3.0
d. 4.5
e. 9.0

____ 12. Consider the function $f(x) = 8x - x^2$ and the point $P(3, 15)$ on the graph of *f*. Estimate the slope of the tangent line of *f* at $P(3, 15)$.

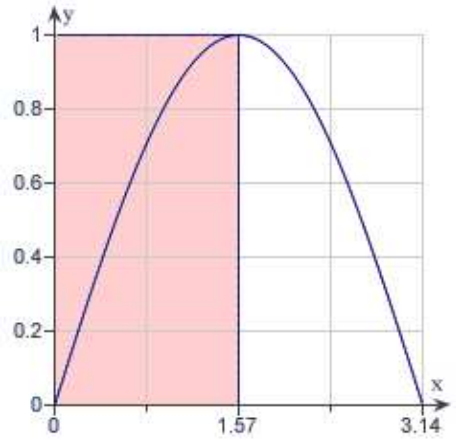
a. 10
b. 3
c. 8
d. 2
e. 9

____ 13. Use the rectangles in the following graph to approximate the area of the region bounded by $y = \cos x$, $y = 0$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{2}$.



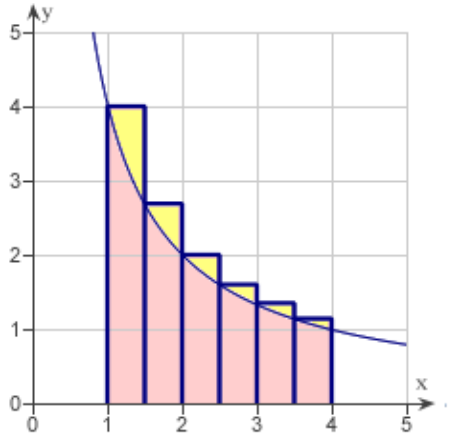
a. 3.9082
b. 2.6055
c. 1.9541
d. 1.4656
e. 0.9770

____ 14. Use the rectangles in the following graph to approximate the area of the region bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$.



a. 0.7850
b. 1.5700
c. 3.1400
d. 1.1775
e. 1.0519

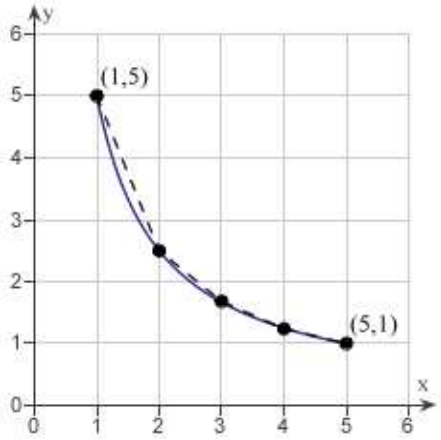
____ 15. Use the rectangles in the graph given below to approximate the area of the region bounded by $y = 4 / x$, $y = 0$, $x = 1$, and $x = 4$ Round your answer to three decimal places.



a. 2.481 units^2
b. 6.371 units^2
c. 3.585 units^2
d. 6.872 units^2
e. 6.903 units^2

____ 16. Consider the length of the graph of $f(x) = 5/x$ from $(1, 5)$ to $(5, 1)$

Approximate the length of the curve by finding the sum of the lengths of four line segments, as shown in following figure. Round your answer to two decimal places.



a. 6.11
b. 8.12
c. 5.66
d. 8.49
e. 7.11

2.1 A Preview of Calculus
Answer Section

1. ANS: C PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Recognize problems requiring precalculus and find the solution MSC: Skill
2. ANS: C PTS: 1 DIF: Med REF: Section 2.1
OBJ: Recognize problems requiring calculus and estimate solutions MSC: Skill
3. ANS: C PTS: 1 DIF: Med REF: Section 2.1
OBJ: Recognize problems requiring calculus and estimate solutions MSC: Skill
4. ANS: B PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Recognize problems requiring precalculus and find the solution MSC: Skill
5. ANS: C PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Recognize problems requiring precalculus and find the solution MSC: Skill
6. ANS: A PTS: 1 DIF: Med REF: Section 2.1
OBJ: Recognize problems requiring calculus and estimate solution MSC: Skill
7. ANS: D PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Graph a function and the secant line passing through given points MSC: Skill
8. ANS: A PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Calculate the slope of a secant line passing through given points MSC: Skill
9. ANS: A PTS: 1 DIF: Med REF: Section 2.1
OBJ: Estimate the slope of a tangent line MSC: Skill
10. ANS: D PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Graph a function and the secant line passing through given points MSC: Skill
11. ANS: B PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Calculate the slope of a secant line passing through given points MSC: Skill
12. ANS: D PTS: 1 DIF: Med REF: Section 2.1
OBJ: Calculate the slope of secant line passing through the given points MSC: Skill
13. ANS: C PTS: 1 DIF: Med REF: Section 2.1
OBJ: Estimate the area of a region using rectangles MSC: Skill
14. ANS: B PTS: 1 DIF: Med REF: Section 2.1
OBJ: Estimate the area of a region using rectangles MSC: Skill
15. ANS: B PTS: 1 DIF: Med REF: Section 2.1
OBJ: Estimate the area of a region using rectangles MSC: Skill
16. ANS: A PTS: 1 DIF: Med REF: Section 2.1
OBJ: Estimate the length of the curve using a piecewise linear function MSC: Skill

2.2 Finding Limits Graphically and Numerically

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Complete the table and use the result to estimate the limit.

$$\lim_{x \to 3} \frac{x-3}{x^2 - 16x + 39}$$

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

a. 0.525000
b. 0.275000
c. –0.100000
d. 0.400000
e. –0.475000

____ 2. Complete the table and use the result to estimate the limit.

$$\lim_{x \to 7} \frac{\frac{1}{x-3} - \frac{1}{4}}{x-7}$$

x	6.9	6.99	6.999	7.001	7.01	7.1
$f(x)$						

a. –0.062500
b. 0.067500
c. –0.192500
d. 0.047500
e. –0.172500

____ 3. Complete the table and use the result to estimate the limit.

$$\lim_{x \to -10} \frac{\sqrt{-6x - 54} - \sqrt{6}}{x + 10}$$

x	–10.1	–10.01	–10.001	–9.999	–9.99	–9.9
$f(x)$						

a. 0.974745
b. –1.099745
c. –1.224745
d. 1.058078
e. 1.224745

____ 4. Complete the table and use the result to estimate the limit.

$$\lim_{x \to 0} \frac{\sin^3 x}{x^3}$$

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$						

a. -0.5
b. 0
c. 1
d. 0.5
e. -1

____ 5. Complete the table and use the result to estimate the limit.

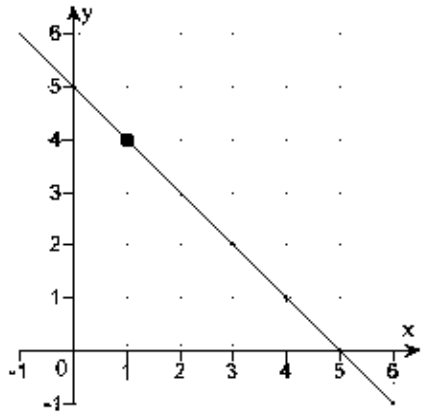
$$\lim_{x \to 0} \frac{\cos(3x) - 1}{3x}$$

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$						

a. -1
b. -0.5
c. 0
d. 0.5
e. 1

____ 6. Determine the following limit. (Hint: Use the graph to calculate the limit.)

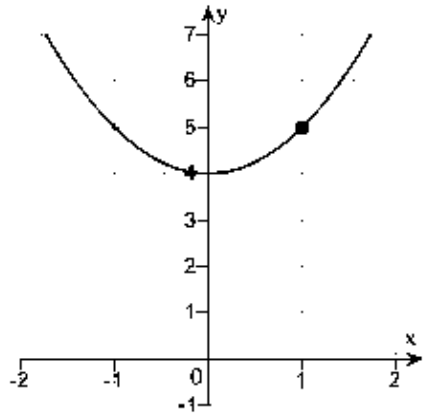
$\lim_{x \to 1} (5 - x)$



a. 6
b. 1
c. 5
d. 4
e. does not exist

____ 7. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$\lim_{x \to 1} \left(x^2 + 4\right)$

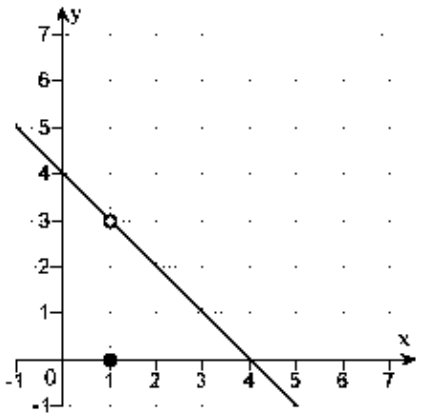


a. 5
b. 1
c. 0
d. 4
e. does not exist

____ 8. Let $f(x) = \begin{cases} 4 - x, & x \neq 1 \\ 0, & x = 1 \end{cases}$.

Determine the following limit. (Hint: Use the graph to calculate the limit.)

$\lim_{x \to 1} f(x)$

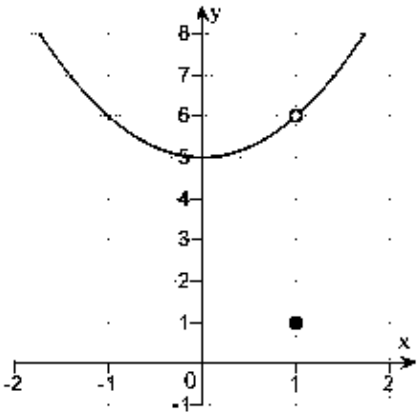


a. 5
b. 4
c. 3
d. 0
e. does not exist

____ 9. Let $f(x) = \begin{cases} x^2 + 5, & x \neq 1 \\ 1, & x = 1 \end{cases}$.

Determine the following limit. (Hint: Use the graph to calculate the limit.)

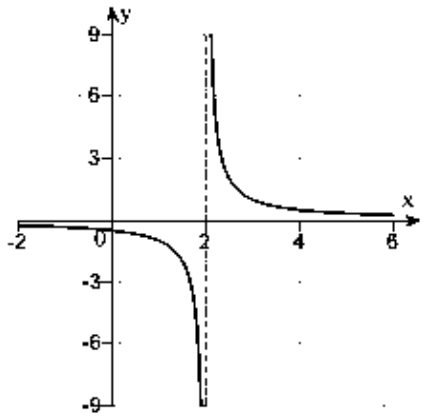
$\lim_{x \to 1} f(x)$



a. 6
b. 25
c. 1
d. 5
e. does not exist.

____ 10. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$\lim_{x \to 2} \frac{1}{x - 2}$



a. –2
b. 0
c. –4
d. 2
e. does not exist

____ 11. A ring has a inner circumference of 10 centimeters. What is the radius of the ring? Round your answer to four decimal places.

a. 0.7958 centimeter
b. 3.1831 centimeters
c. 1.5915 centimeters
d. 1.7841 centimeters
e. 10.1321 centimeters

____ 12. A ring has a inner circumference of 9 centimeters. If the ring's inner circumference can vary between 8 centimeters and 10 centimeters how can the radius vary? Round your answer to five decimal places.

a. Radius can vary between 6.48456 centimeters and 10.13212 centimeters.
b. Radius can vary between 1.59577 centimeters and 1.78412 centimeters.
c. Radius can vary between 1.27324 centimeters and 1.59155 centimeters.
d. Radius can vary between 2.54648 centimeters and 3.18310 centimeters.
e. Radius can vary between 0.43239 centimeter and 2.43239 centimeters.

____ 13. A sphere has a volume of 4.76 cubic inches. What is the radius of the sphere? Round your answer to four decimal places.

a. 1.0435 inches
b. 1.6565 inches
c. 1.0660 inches
d. 2.1320 inches
e. 1.9335 inches

____ 14. A sphere has a volume of 5.2 cubic inches. If the sphere's volume can vary between 4.4 cubic inches and 6.1 cubic inches , how can the radius vary? Round your answer to five decimal places.

a. Radius can vary between 1.01653 inches and 1.13348 inches.
b. Radius can vary between 1.61365 inches and 1.79929 inches.
c. Radius can vary between 0.27474 inch and 1.97474 inches.
d. Radius can vary between 1.85897 inches and 2.18882 inches.
e. Radius can vary between 1.02490 inches and 1.20676 inches.

2.2 Finding Limits Graphically and Numerically
Answer Section

1.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate a limit from a table of values						MSC:	Skill
2.	ANS:	A	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate a limit from a table of values						MSC:	Skill
3.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate a limit from a table of values						MSC:	Skill
4.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate a limit from a table of values						MSC:	Skill
5.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate a limit from a table of values						MSC:	Skill
6.	ANS:	D	PTS:	1	DIF:	Easy	REF:	Section 2.1
OBJ:	Estimate the limit of a function from its graph						MSC:	Skill
7.	ANS:	A	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate the limit of a function from its graph						MSC:	Skill
8.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate the limit of a function from its graph						MSC:	Skill
9.	ANS:	A	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate the limit of a function from its graph						MSC:	Skill
10.	ANS:	E	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Estimate the limit of a function from its graph						MSC:	Skill
11.	ANS:	C	PTS:	1	DIF:	Easy	REF:	Section 2.1
OBJ:	Solve a linear equation in applications						MSC:	Application
12.	ANS:	C	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Solve a linear equation in applications						MSC:	Application
13.	ANS:	A	PTS:	1	DIF:	Easy	REF:	Section 2.1
OBJ:	Solve a cubic equation in applications						MSC:	Application
14.	ANS:	A	PTS:	1	DIF:	Med	REF:	Section 2.1
OBJ:	Solve a linear equation in applications						MSC:	Application

2.3 Evaluating Limits Analytically

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Find the limit.

$$\lim_{x \to -4} 9x^2 + 36x$$

a. 108
b. −108
c. 288
d. −288
e. 0

____ 2. Find the limit.

$$\lim_{x \to 6} \frac{x}{x^2 + 8}$$

a. $\frac{1}{14}$
b. $\frac{1}{10}$
c. $\frac{3}{22}$
d. $\frac{3}{7}$
e. $\frac{3}{10}$

____ 3. Find the limit.

$$\lim_{x \to 4} \frac{\sqrt{x+5}}{x-1}$$

a. 3
b. −1
c. –3
d. 1
e. 9

____ 4. Find the limit.

$$\lim_{x \to \frac{3\pi}{4}} \sin x$$

a. $\frac{\sqrt{3}}{2}$
b. $-\frac{\sqrt{2}}{2}$
c. $-\frac{1}{2}$
d. $\frac{\sqrt{2}}{2}$
e. does not exist

____ 5. Find the limit.

$$\lim_{x \to 2} \cos \frac{\pi x}{3}$$

a. $\frac{1}{2}$
b. $-\frac{1}{2}$
c. $-\frac{\sqrt{3}}{2}$
d. $\frac{\sqrt{3}}{2}$
e. 0

____ 6. Find the limit.

$$\lim_{x \to 5} \cos\left(\frac{\pi x}{6}\right)$$

a. $-\frac{1}{2}$
b. 0
c. $\frac{1}{2}$
d. $-\frac{\sqrt{3}}{2}$
e. $\frac{\sqrt{3}}{2}$

____ 7. Find the lmit.

$$\lim_{x \to \pi} \tan\left(\frac{x}{3}\right)$$

a. $\frac{-1}{\sqrt{3}}$
b. $\sqrt{3}$
c. $-\sqrt{3}$
d. $\frac{1}{\sqrt{3}}$
e. does not exixt

____ 8. Let $f(x) = -x^2 - 5$ and $g(x) = 2x$. Find the limit.

$$\lim_{x \to -2} g\big(f(x)\big)$$

a. –18
b. 25
c. 21
d. 8
e. 9

____ 9. Let $f(x) = 4x - 2$ and $g(x) = x^3$. Find the limit.

$$\lim_{x \to 1} g\big(f(x)\big)$$

a. 2
b. 1
c. 8
d. –8
e. –4

____ 10. Let $f(x) = 3 + 2x^2$ and $g(x) = \sqrt{x+3}$. Find the limit.

$$\lim_{x \to 2} g\big(f(x)\big)$$

a. $\sqrt{6}$
b. $\sqrt{14}$
c. $\sqrt{11}$
d. $\sqrt{10}$
e. $\sqrt{2}$

____ 11. Let $f(x) = x^2 - x - 5$ and $g(x) = \sqrt[3]{x + 14}$. Find the limits.

$$\lim_{x \to 3} g\big(f(x)\big)$$

a. $-\sqrt[3]{1}$
b. $\sqrt[3]{29}$
c. $-\sqrt[3]{15}$
d. $\sqrt[3]{15}$
e. $\sqrt[3]{1}$

____ 12. Suppose that $\lim_{x \to c} f(x) = -13$ and $\lim_{x \to c} g(x) = -10$. Find the following limit.

$$\lim_{x \to c} \big[f(x) + g(x)\big]$$

a. 0
b. –10
c. –3
d. –23
e. 130

____ 13. Suppose that $\lim_{x \to c} f(x) = -15$ and $\lim_{x \to c} g(x) = -10$. Find the following limit.

$$\lim_{x \to c} \big[f(x)g(x)\big]$$

a. 10
b. –5
c. –25
d. –15
e. 150

____ 14. Suppose that $\lim_{x \to c} f(x) = 7$ and $\lim_{x \to c} g(x) = 3$. Find the following limit.

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

a. 21
b. $\frac{3}{7}$
c. –21
d. $\frac{7}{3}$
e. does not exist

____ 15. Suppose that $\lim_{x \to c} f(x) = -11$ and $\lim_{x \to c} g(x) = -3$. Find the following limit.

$$\lim_{x \to c} \left[f(x) - g(x) \right]$$

a. –11
b. –8
c. 33
d. –14
e. 0

____ 16. Suppose that $\lim_{x \to c} f(x) = 5$. Find the following limit.

$$\lim_{x \to c} \left[f(x)^3 \right]$$

a. 2
b. 125
c. 8
d. 0
e. 15

____ 17. Suppose that $\lim_{x \to c} f(x) = -5$. Find the following limit.

$$\lim_{x \to c} 3f(x)$$

a. –5
b. 15
c. –15
d. $3c$
e. 3

____ 18. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \to -4} \frac{8x^2 + 40x + 32}{x + 4}$$

a. 40
b. –24
c. 24
d. –40
e. does not exist

____ 19. Find the limit (if it exists).

$$\lim_{x \to -8} \frac{x+8}{x^2-64}$$

a. $-\frac{1}{16}$
b. $-\frac{1}{32}$
c. –32
d. –8
e. $\frac{1}{16}$

____ 20. Find the limit (if it exists).

$$\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$$

a. 6
b. 1
c. 0
d. $\frac{1}{6}$
e. Limit does not exist.

____ 21. Find the limit (if it exists).

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 9(x+\Delta x) + 2 - \left(x^2 - 9x + 2\right)}{\Delta x}$$

a. $\frac{1}{3}x^3 - \frac{9}{2}x^2 + 2x$
b. $2x - 9$
c. $x^3 - 9x^2 + 2x$
d. $x^2 - 9x + 2$
e. does not exist

____ 22. Determine the limit (if it exists).

$$\lim_{x \to 0} \frac{12(1 - \cos x)}{x^2}$$

a. 6
b. 48
c. 10
d. 24
e. does not exist

____ 23. Determine the limit (if it exists).

$$\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{2x^8}$$

a. 8
b. 1
c. 0
d. 2
e. does not exist

____ 24. Determine the limit (if it exists).

$$\lim_{x \to 0} \frac{\sin^4 x}{x^3}$$

a. 1
b. 0
c. 2
d. ∞
e. does not exist

____ 25. Find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ where $f(x) = 4x - 3$.

a. 1
b. 4
c. –3
d. 0
e. Limit does not exist.

2.3 Evaluating Limits Analytically
Answer Section

1. ANS: E PTS: 1 DIF: Easy REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
2. ANS: C PTS: 1 DIF: Easy REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
3. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
4. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
5. ANS: B PTS: 1 DIF: Easy REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
6. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
7. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of the function MSC: Skill
8. ANS: A PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of composite functions MSC: Skill
9. ANS: C PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of composite functions MSC: Skill
10. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of composite functions MSC: Skill
11. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of composite functions MSC: Skill
12. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
13. ANS: E PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
14. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
15. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
16. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
17. ANS: C PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
18. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of the function and simplify it to an identical function except at the discontinuity point MSC: Skill
19. ANS: A PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill
20. ANS: D PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill
21. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill
22. ANS: A PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill

23. ANS: E PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill
24. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a function analytically MSC: Skill
25. ANS: B PTS: 1 DIF: Med REF: Section 2.3
OBJ: Evaluate the limit of a difference quotient MSC: Skill

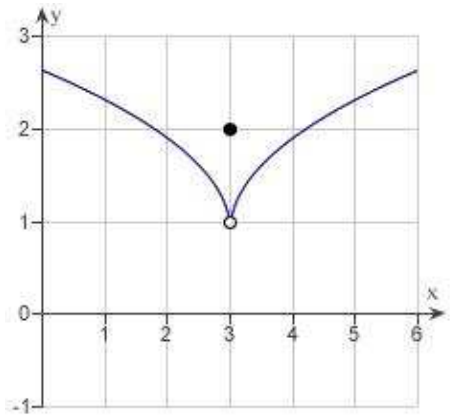
2.4 Continuity and One-Sided Limits

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = 3$.

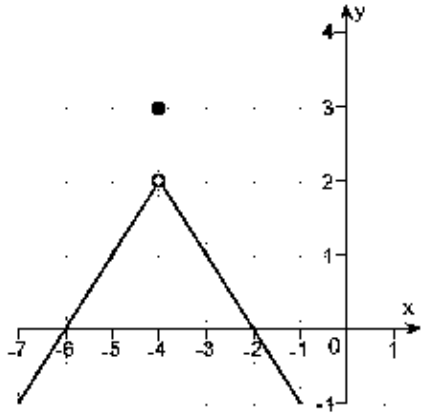
(i) $\lim_{x \to 3^+} f(x)$ (ii) $\lim_{x \to 3^-} f(x)$ (iii) $\lim_{x \to 3} f(x)$



a. 1 , 1 , 1 , not continuous
b. 2 , 2 , 2 , continuous
c. 4 , 4 , 4 , not continuous
d. 2 , 2 , 2 , not continuous
e. 1 , 1 , 1 , continuous

____ 2. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = -4$.

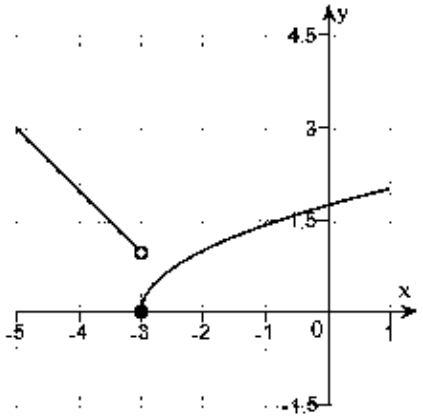
(i) $\lim_{x \to -4^+} f(x)$ (ii) $\lim_{x \to -4^-} f(x)$ (iii) $\lim_{x \to -4} f(x)$



a. $3, 3, 3$, continuous
b. $2, 2, 2$, not continuous
c. $3, 3, 3$, not continuous
d. $-4, -4, -4$, continuous
e. $2, 2, 2$, continuous

____ 3. Use the graph to determine the following limits, and discuss the continuity of the function at $x = -3$.

(i) $\lim_{x \to -3^+} f(x)$ (ii) $\lim_{x \to -3^-} f(x)$ (iii) $\lim_{x \to -3} f(x)$



a. 1, − 1, does not exist, not continuous
b. 1, 0, does not exist, not continuous
c. 0, 1, does not exist, not continuous
d. −3, 0, does not exist, not continuous
e. 0, 1, 0, continuous

____ 4. Find the limit (if it exists).

$$\lim_{x \to 11^+} \frac{11 - x}{x^2 - 121}$$

a. $\frac{1}{22}$
b. 0
c. Limit does not exist.
d. $-\frac{1}{22}$
e. $\frac{1}{242}$

____ 5. Find the limit (if it exists).

$$\lim_{x \to 36^-} \frac{\sqrt{x} - 6}{x - 36}$$

a. 0
b. $-\frac{1}{12}$
c. $\frac{1}{72}$
d. $\frac{1}{12}$
e. Limit does not exist.

____ 6. Find the limit (if it exists).

$$\lim_{x \to 1^-} f(x), \text{ where } f(x) = \begin{cases} x^3 + 10, & x < 1 \\ x + 10, & x \geq 1 \end{cases}$$

a. Limit does not exist.
b. 0
c. 10
d. 11
e. 30

____ 7. Find the limit (if it exists). Note that $f(x) = \left[|x| \right]$ represents the greatest integer function.

$$\lim_{x \to -6^+} \left(-3\left[|x| \right] - 8 \right)$$

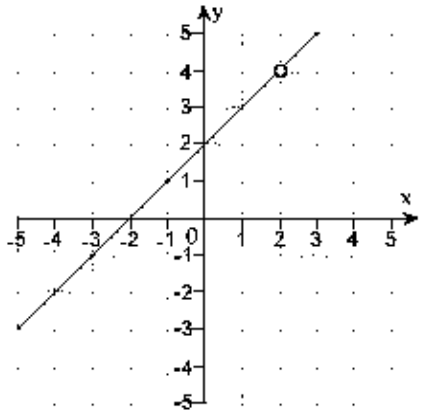
a. 13
b. -10
c. 10
d. -13
e. does not exist

____ 8. Find the limit (if it exists). Note that $f(x) = \left[|x| \right]$ represents the greatest integer function.

$$\lim_{x \to 5^+} \left(2x - \left[|x| \right] \right)$$

a. 6
b. Limit does not exist.
c. 5
d. 0
e. 4

____ 9. Discuss the continuity of the function $f(x) = \dfrac{x^2 - 4}{x - 2}$.



a. $f(x)$ is discontinuous at $x = -2$.
b. $f(x)$ is discontinuous at $x = -2, 2$.
c. $f(x)$ is discontinuous at $x = 2$.
d. $f(x)$ is continuous for all real x.
e. $f(x)$ is continuous at $x = 4$.

____ 10. Find the *x*-values (if any) at which the function $f(x) = 13x^2 - 15x - 15$ is not continuous. Which of the discontinuities are removable?

a. $x = 4$, removable
b. $x = 0$, removable
c. $x = \dfrac{15}{26}$, not removable.
d. continuous everywhere
e. $x = \dfrac{15}{26}$, removable.

____ 11. Find the x-values (if any) at which $f(x) = \dfrac{x}{x^2 - 2x}$ is not continuous.

a. $f(x)$ is not continuous at $x = 0$ and $f(x)$ has a removable discontinuity at $x = 0$.
b. $f(x)$ is not continuous at $x = 0, 2$ and both the discontinuities are nonremovable.
c. $f(x)$ is not continuous at $x = 2$ and $f(x)$ has a removable discontinuity at $x = 2$.
d. $f(x)$ is not continuous at $x = 0, 2$ and $f(x)$ has a removable discontinuity at $x = 0$.
e. $f(x)$ is continuous for all real x.

____ 12. Find the x-values (if any) at which the function $f(x) = \dfrac{x}{x^2 - 100}$ is not continuous.

Which of the discontinuities are removable?

a. 10 and -10, removable
b. discontinuous everywhere
c. continuous everywhere
d. 10 and -10, not removable
e. 0, removable

____ 13. Find the x-values (if any) at which the function $f(x) = \dfrac{x+2}{x^2 + 6x + 8}$ is not continuous.

Which of the discontinuities are removable?

a. no points of discontinuity
b. $x = -2$ (not removable), $x = -4$ (removable)
c. $x = -2$ (removable), $x = -4$ (not removable)
d. no points of continuity
e. $x = -2$ (not removable), $x = -4$ (not removable)

____ 14. Find the x-values (if any) at which $f(x) = \dfrac{|x-3|}{x-3}$ is not continuous.

a. $f(x)$ is not continuous at $x = 3$ and the discontinuity is nonremovable.
b. $f(x)$ is not continuous at $x = 0$ and the discontinuity is removable.
c. $f(x)$ is continuous for all real x.
d. $f(x)$ is not continuous at $x = 3$ and the discontinuity is removable.
e. $f(x)$ is not continuous at $x = 0, -3$ and $x = 0$ is a removable discontinuity.

____ 15. Find the constant *a* such that the function

$$f(x) = \begin{cases} -4 \cdot \dfrac{\sin x}{x}, & x < 0 \\ a + 7x, & x \geq 0 \end{cases}$$

is continuous on the entire real line.

a. 1
b. -7
c. 7
d. 4
e. -4

____ 16. Find the constant *a* such that the function

$$f(x) = \begin{cases} 6, & x \leq -5 \\ ax + b, & -5 < x < 1 \\ -6, & x \geq 1 \end{cases}$$

is continuous on the entire real line.

a. $a = 2, b = 0$
b. $a = 2, b = -4$
c. $a = -2, b = -4$
d. $a = -2, b = 4$
e. $a = 2, b = 4$

____ 17. Find the value of *c* guaranteed by the Intermediate Value Theorem.

$f(x) = x^2 - 2x + 8, [2, 6], f(c) = 11$

a. 0
b. 3
c. 5
d. 1
e. 4

____ 18. Find the value of c guaranteed by the Intermediate Value Theorem.

$$f(x) = \frac{x^2 - 5x}{x-3}, \left[\frac{9}{2}, 18\right], f(c) = 6$$

a. 11
b. 2
c. 1
d. 9
e. 10

____ 19. A long distance phone service charges $\$0.35$ for the first 10 minutes and $\$0.1$ for each additional minute or fraction thereof. Use the greatest integer function to write the cost C of a call in terms of time t (in minutes).

a. $$C = \begin{cases} 0.35 & 0 < t \le 10 \\ 0.35 + 0.1\left[\left|t - 10\right|\right] & t > 10, t \text{ is not an integer} \\ 0.35 + 0.1(t-9) & t > 10, t \text{ is an integer} \end{cases}$$

b. $$C = \begin{cases} 0.35 & 0 < t \le 10 \\ 0.35 + 0.1(t-10) & t > 10 \end{cases}$$

c. $$C = \begin{cases} 0.35 & 0 < t \le 10 \\ 0.35 + 0.1\left[\left|t - 9\right|\right] & t > 10 \end{cases}$$

d. $$C = \begin{cases} 0.35 & 0 < t \le 10 \\ 0.35 + 0.1\left[\left|t - 10\right|\right] & t > 10 \end{cases}$$

e. $$C = \begin{cases} 0.35 & 0 < t \le 10 \\ 0.35 + 0.1\left[\left|t - 9\right|\right] & t > 10, t \text{ is not an integer} \\ 0.35 + 0.1(t-10) & t > 10, t \text{ is an integer} \end{cases}$$

____ 20. Find all values of *c* such that f is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 4 - x^2, & x \le c \\ x, & x > c \end{cases}$$

a. $c = 3$
b. $c = 0$
c. $\dfrac{-1 + \sqrt{17}}{2}$
d. $\dfrac{1 + \sqrt{17}}{2}, \dfrac{1 - \sqrt{17}}{2}$
e. $\dfrac{-1 + \sqrt{17}}{2}, \dfrac{-1 - \sqrt{17}}{2}$

2.4 Continuity and One-Sided Limits
Answer Section

1. ANS: A PTS: 1 DIF: Med REF: Section 2.4
OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill
2. ANS: B PTS: 1 DIF: Med REF: Section 2.4
OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill
3. ANS: C PTS: 1 DIF: Med REF: Section 2.4
OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill
4. ANS: D PTS: 1 DIF: Easy REF: Section 2.4
OBJ: Evaluate one-sided limits MSC: Skill
5. ANS: D PTS: 1 DIF: Med REF: Section 2.4
OBJ: Evaluate one-sided limits MSC: Skill
6. ANS: D PTS: 1 DIF: Med REF: Section 2.4
OBJ: Evaluate one-sided limits MSC: Skill
7. ANS: A PTS: 1 DIF: Med REF: Section 2.4
OBJ: Evaluate one-sided limits MSC: Skill
8. ANS: C PTS: 1 DIF: Med REF: Section 2.4
OBJ: Evaluate one-sided limits MSC: Skill
9. ANS: C PTS: 1 DIF: Easy REF: Section 2.4
OBJ: Identify the discontinuities of a function if any exist MSC: Skill
10. ANS: D PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
11. ANS: D PTS: 1 DIF: Easy REF: Section 2.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
12. ANS: D PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
13. ANS: C PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
14. ANS: A PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
15. ANS: E PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the value of a parameter to ensure a function is continuous MSC: Skill
16. ANS: C PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the value of a parameter to ensure a function is continuous MSC: Skill
17. ANS: B PTS: 1 DIF: Easy REF: Section 2.4
OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem MSC: Skill
18. ANS: D PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem MSC: Skill
19. ANS: E PTS: 1 DIF: Med REF: Section 2.4
OBJ: Create functions in applications MSC: Application
20. ANS: E PTS: 1 DIF: Med REF: Section 2.4
OBJ: Identify the value of a parameter to ensure a function is continuous MSC: Skill

2.5 Infinite Limits

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Determine whether $f(x) = \dfrac{x^{10}}{x^2 - 9}$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the tables below.

x	-3.5	–3.1	–3.01	–3.001
$f(x)$				

x	–2.999	–2.99	–2.9	–2.5
$f(x)$				

a. $\lim_{x \to -3^-} f(x) = -\infty, \quad \lim_{x \to -3^+} f(x) = \infty$

b. $\lim_{x \to -3^-} f(x) = \infty, \quad \lim_{x \to -3^+} f(x) = -\infty$

c. $\lim_{x \to -3^-} f(x) = \infty, \quad \lim_{x \to -3^+} f(x) = \infty$

d. $\lim_{x \to -3^-} f(x) = -\infty, \quad \lim_{x \to -3^+} f(x) = -\infty$

____ 2. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \dfrac{5}{(x-3)^2}$.

a. $x = -3$
b. $x = 5$
c. $x = 3, -3$
d. $x = 3$
e. no vertical asymptotes

____ 3. Find the vertical asymptotes (if any) of the function $f(x) = \dfrac{x^2 - 4}{x^2 + 3x + 2}$.

a. $x = 2$
b. $x = -1$
c. $x = 1$
d. $x = -2$
e. $x = -2$

____ 4. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \dfrac{1+x}{x^2(1-x)}$.

a. $x = -1$
b. $x = 1$
c. $x = 0$
d. $x = 1,\ x = 0$
e. no vertical asymptotes

____ 5. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \dfrac{x^3+8}{x+2}$.

a. $x = -2$
b. $x = 8$
c. $x = 2$
d. $x = 2, -2$
e. no vertical asymptotes

____ 6. Find all vertical asymptotes (if any) of the function $f(x) = \dfrac{x^2+4x+3}{x^3-4x^2-x+4}$.

a. $x = 4, 1$
b. $x = 4,\ 1,\ -1$
c. $x = -4, -1$
d. $x = 1$
e. $x = -1$

____ 7. Find the vertical asymptotes (if any) of the function $f(x) = \tan(15x)$.

a. $x = \dfrac{k}{15}\pi \quad \left(k = 0, \pm 1,\ \pm 2, \ldots\right)$
b. $x = \dfrac{2k+1}{30}\pi \quad \left(k = 0, \pm 1,\ \pm 2, \ldots\right)$
c. $x = \dfrac{2k}{15}\pi \quad \left(k = 0, \pm 1,\ \pm 2, \ldots\right)$
d. $x = \dfrac{2k+1}{15}\pi \quad \left(k = 0, \pm 1, \pm 2, \ldots\right)$
e. no vertical asymptotes

____ 8. Find the limit.

$$\lim_{x \to 14^+} \frac{x-3}{x-14}$$

a. 1
b. $-\infty$
c. 0
d. ∞
e. -1

____ 9. Find the limit.

$$\lim_{x \to -10} \frac{x^2 + 10x}{\left(x^2 + 100\right)(x+10)}$$

a. $\frac{1}{20}$
b. $-\frac{1}{20}$
c. 20
d. -10
e. -20

____ 10. Find the limit.

$$\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right)$$

a. 1
b. 0
c. -1
d. $-\infty$
e. ∞

____ 11. Find the following limit if it exists: $\lim_{x \to 3^+} \ln(x-3)$. Use $\pm\infty$ when appropriate.

a. ∞
b. 3
c. 1
d. $-\infty$
e. does not exist

____ 12. Find the limit (if it exists).

$$\lim_{x \to \frac{1}{2}} x \tan \pi x$$

a. $-\infty$
b. $\frac{1}{2}$
c. 0
d. ∞
e. Limit does not exist

____ 13. Use a graphing utility to graph the function $f(x) = \frac{x^2 - 2x + 4}{x^3 + 8}$ and determine the one-sided limit $\lim_{x \to -2^+} f(x)$.

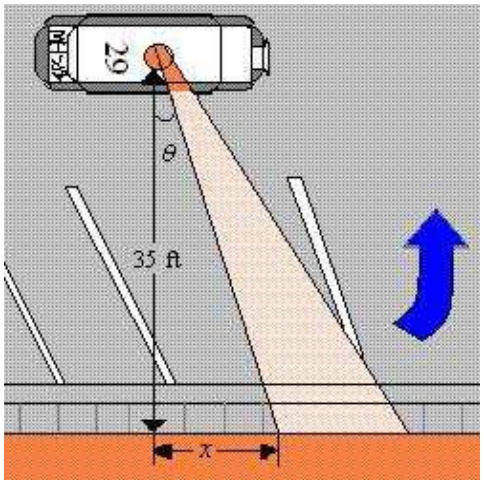
a. $-\infty$
b. ∞
c. 0
d. 12
e. 8

____ 14. Use a graphing utility to graph the function $f(x) = \csc \frac{\pi x}{2}$ and determine the following one-sided limit.

$$\lim_{x \to 2^-} f(x)$$

a. $-\infty$
b. 2
c. -2
d. ∞
e. 0

____ 15. A petrol car is parked 35 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is $r = 35\pi \sec^2 \theta$ ft/sec. Find the rate r when θ is $\frac{\pi}{6}$.



a. $r = \frac{140}{3}$ ft / sec

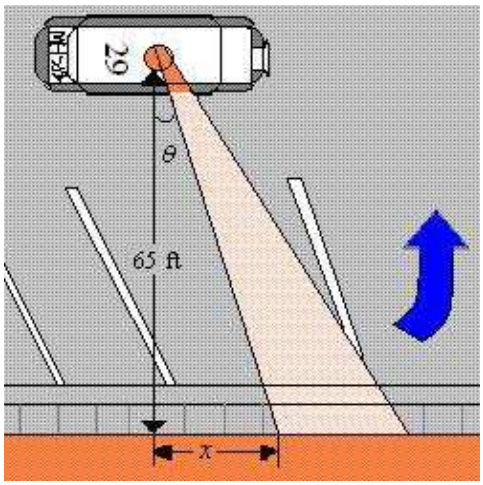
b. $r = \frac{70\sqrt{3}\,\pi}{3}$ ft / sec

c. $r = \frac{70\sqrt{3}}{3}$ ft / sec

d. $r = \frac{140\pi}{3}$ ft / sec

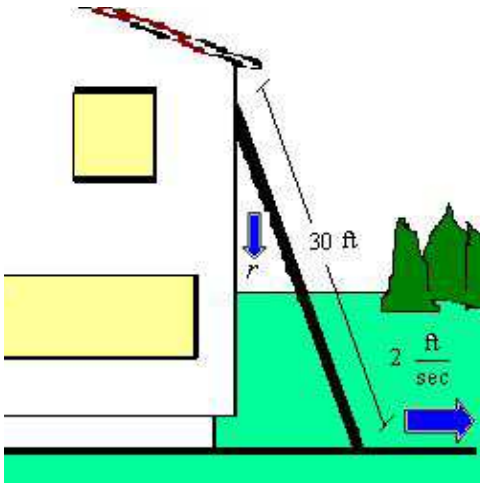
e. $r = \frac{70\pi}{3}$ ft / sec

____ 16. A petrol car is parked 65 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is $r = 65\pi \sec^2 \theta \text{ ft/sec}$. Find the limit of r as $\theta \to (\pi/2)^-$.



a. ∞
b. 65π
c. 0
d. 65
e. $-\infty$

____ 17. A 30-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of $r = \frac{2x}{\sqrt{900 - x^2}} \text{ ft/sec}$, where x is the distance between the base of the ladder and the house. Find the rate r when x is 18 feet.



a. $r = \frac{3}{2}$ ft/sec

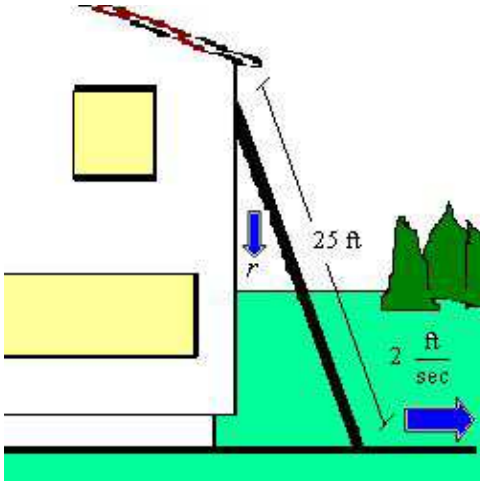
b. $r = \frac{4}{3}$ ft/sec

c. $r = \frac{48}{5}$ ft/sec

d. $r = \frac{2}{3}$ ft/sec

e. $r = \frac{3}{4}$ ft/sec

____ 18. A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of $r = \frac{2x}{\sqrt{625 - x^2}}$ ft / sec where x is the distance between the base of the ladder and the house. Find the limit of r as $x \to 25^-$.



a. $-\infty$
b. 50
c. 0
d. ∞
e. 25

2.5 Infinite Limits
Answer Section

1. ANS: B PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate an infinite limit from a table of values MSC: Skill
2. ANS: D PTS: 1 DIF: Easy REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
3. ANS: B PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
4. ANS: D PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
5. ANS: E PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
6. ANS: A PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
7. ANS: B PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function MSC: Skill
8. ANS: D PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate one-sided limits MSC: Skill
9. ANS: B PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate the limit of a function MSC: Skill
10. ANS: E PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate one-sided limits MSC: Skill
11. ANS: D PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate limits involving logarithmic functions MSC: Skill
12. ANS: E PTS: 1 DIF: Med REF: Section 2.5
OBJ: Identify a limit that does not exist MSC: Skill
13. ANS: B PTS: 1 DIF: Med REF: Section
2.OBJ: Estimate one-sided limits from a graph MSC: Skill
14. ANS: D PTS: 1 DIF: Med REF: Section 2.5
OBJ: Estimate one-sided limits from a graph MSC: Skill
15. ANS: D PTS: 1 DIF: Easy REF: Section 2.5
OBJ: Evaluate functions in applications MSC: Application
16. ANS: A PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate limits in applications MSC: Application
17. ANS: A PTS: 1 DIF: Easy REF: Section 2.5
OBJ: Evaluate functions in applications MSC: Application
18. ANS: D PTS: 1 DIF: Med REF: Section 2.5
OBJ: Evaluate limits in applications MSC: Application