CONTENTS

PROLOGUE: Principles of Problem Solving

1. Let r be the rate of the descent. We use the formula time $= \dfrac{\text{distance}}{\text{rate}}$; the ascent takes $\dfrac{1}{15}$ h, the descent takes $\dfrac{1}{r}$ h, and the total trip should take $\dfrac{2}{30} = \dfrac{1}{15}$ h. Thus we have $\dfrac{1}{15} + \dfrac{1}{r} = \dfrac{1}{15} \quad \Leftrightarrow \quad \dfrac{1}{r} = 0$, which is impossible. So the car cannot go fast enough to average 30 mi/h for the 2-mile trip.

2. Let us start with a given price P. After a discount of 40%, the price decreases to $0.6P$. After a discount of 20%, the price decreases to $0.8P$, and after another 20% discount, it becomes $0.8\,(0.8P) = 0.64P$. Since $0.6P < 0.64P$, a 40% discount is better.

3. We continue the pattern. Three parallel cuts produce 10 pieces. Thus, each new cut produces an additional 3 pieces. Since the first cut produces 4 pieces, we get the formula $f(n) = 4 + 3(n - 1)$, $n \geq 1$. Since $f(142) = 4 + 3(141) = 427$, we see that 142 parallel cuts produce 427 pieces.

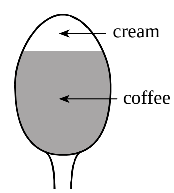
4. By placing two amoebas into the vessel, we skip the first simple division which took 3 minutes. Thus when we place two amoebas into the vessel, it will take $60 - 3 = 57$ minutes for the vessel to be full of amoebas.

5. The statement is false. Here is one particular counterexample:

	Player A	Player B
First half	1 hit in 99 at-bats: average $= \frac{1}{99}$	0 hit in 1 at-bat: average $= \frac{0}{1}$
Second half	1 hit in 1 at-bat: average $= \frac{1}{1}$	98 hits in 99 at-bats: average $= \frac{98}{99}$
Entire season	2 hits in 100 at-bats: average $= \frac{2}{100}$	99 hits in 100 at-bats: average $= \frac{99}{100}$

6. *Method 1:* After the exchanges, the volume of liquid in the pitcher and in the cup is the same as it was to begin with. Thus, any coffee in the pitcher of cream must be replacing an equal amount of cream that has ended up in the coffee cup.

Method 2: Alternatively, look at the drawing of the spoonful of coffee and cream mixture being returned to the pitcher of cream. Suppose it is possible to separate the cream and the coffee, as shown. Then you can see that the coffee going into the cream occupies the same volume as the cream that was left in the coffee.



Method 3 (an algebraic approach): Suppose the cup of coffee has y spoonfuls of coffee. When one spoonful of cream is added to the coffee cup, the resulting mixture has the following ratios: $\dfrac{\text{cream}}{\text{mixture}} = \dfrac{1}{y+1}$ and $\dfrac{\text{coffee}}{\text{mixture}} = \dfrac{y}{y+1}$.

So, when we remove a spoonful of the mixture and put it into the pitcher of cream, we are really removing $\dfrac{1}{y+1}$ of a spoonful of cream and $\dfrac{y}{y+1}$ spoonful of coffee. Thus the amount of cream left in the mixture (cream in the coffee) is $1 - \dfrac{1}{y+1} = \dfrac{y}{y+1}$ of a spoonful. This is the same as the amount of coffee we added to the cream.

7. Let r be the radius of the earth in feet. Then the circumference (length of the ribbon) is $2\pi r$. When we increase the radius by 1 foot, the new radius is $r + 1$, so the new circumference is $2\pi(r + 1)$. Thus you need $2\pi(r + 1) - 2\pi r = 2\pi$ extra feet of ribbon.

8. The north pole is such a point. And there are others: Consider a point a_1 near the south pole such that the parallel passing through a_1 forms a circle C_1 with circumference exactly one mile. Any point P_1 exactly one mile north of the circle C_1 along a meridian is a point satisfying the conditions in the problem: starting at P_1 she walks one mile south to the point a_1 on the circle C_1, then one mile east along C_1 returning to the point a_1, then north for one mile to P_1. That's not all. If a point a_2 (or a_3, a_4, a_5, ...) is chosen near the south pole so that the parallel passing through it forms a circle C_2 (C_3, C_4, C_5, ...) with a circumference of exactly $\frac{1}{2}$ mile ($\frac{1}{3}$ mi, $\frac{1}{4}$ mi, $\frac{1}{5}$ mi, ...), then the point P_2 (P_3, P_4, P_5, ...) one mile north of a_2 (a_3, a_4, a_5, ...) along a meridian satisfies the conditions of the problem: she walks one mile south from P_2 (P_3, P_4, P_5, ...) arriving at a_2 (a_3, a_4, a_5, ...) along the circle C_2 (C_3, C_4, C_5, ...), walks east along the circle for one mile thus traversing the circle twice (three times, four times, five times, ...) returning to a_2 (a_3, a_4, a_5, ...), and then walks north one mile to P_2 (P_3, P_4, P_5, ...).

1 FUNDAMENTALS

1.1 REAL NUMBERS

1. **(a)** The natural numbers are $\{1, 2, 3, \ldots\}$.
   (b) The numbers $\{\ldots, -3, -2, -1, 0\}$ are integers but not natural numbers.
   (c) Any irreducible fraction $\dfrac{p}{q}$ with $q \neq 1$ is rational but is not an integer. Examples: $\frac{3}{2}$, $-\frac{5}{12}$, $\frac{1729}{23}$.
   (d) Any number which cannot be expressed as a ratio $\dfrac{p}{q}$ of two integers is irrational. Examples are $\sqrt{2}$, $\sqrt{3}$, π, and e.
2. **(a)** $ab = ba$; Commutative Property of Multiplication
   (b) $a + (b + c) = (a + b) + c$; Associative Property of Addition
   (c) $a(b + c) = ab + ac$; Distributive Property
3. The set of numbers between but not including 2 and 7 can be written as **(a)** $\{x \mid 2 < x < 7\}$ in interval notation, or **(b)** $(2, 7)$ in interval notation.
4. The symbol $|x|$ stands for the *absolute value* of the number x. If x is not 0, then the sign of $|x|$ is always *positive*.
5. The distance between a and b on the real line is $d(a, b) = |b - a|$. So the distance between -5 and 2 is $|2 - (-5)| = 7$.
6. **(a)** Yes, the sum of two rational numbers is rational: $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$.
   (b) No, the sum of two irrational numbers can be irrational ($\pi + \pi = 2\pi$) or rational ($-\pi + \pi = 0$).
7. **(a)** No: $a - b = -(b - a) \neq b - a$ in general.
   (b) No; by the Distributive Property, $-2(a - 5) = -2a + -2(-5) = -2a + 10 \neq -2a - 10$.
8. **(a)** Yes, absolute values (such as the distance between two different numbers) are always positive.
   (b) Yes, $|b - a| = |a - b|$.

9. **(a)** Natural number: 100
   (b) Integers: 0, 100, -8
   (c) Rational numbers: -1.5, 0, $\frac{5}{2}$, 2.71, $3.1\overline{4}$, 100, -8
   (d) Irrational numbers: $\sqrt{7}$, $-\pi$

10. **(a)** Natural number: $\sqrt{16}\,(= 4)$
    (b) Integers: -500, $\sqrt{16}$, $-\frac{20}{5}\,(= -4)$
    (c) Rational numbers: 1.3, $1.3333\ldots$, 5.34, -500, $1\frac{2}{3}$, $\sqrt{16}$, $\frac{246}{579}$, $-\frac{20}{5}$
    (d) Irrational number: $\sqrt{5}$

11. Commutative Property of addition
12. Commutative Property of multiplication
13. Associative Property of addition
14. Distributive Property
15. Distributive Property
16. Distributive Property
17. Commutative Property of multiplication
18. Distributive Property
19. $x + 3 = 3 + x$
20. $7(3x) = (7 \cdot 3)x$
21. $4(A + B) = 4A + 4B$
22. $5x + 5y = 5(x + y)$
23. $3(x + y) = 3x + 3y$
24. $(a - b)8 = 8a - 8b$
25. $4(2m) = (4 \cdot 2)m = 8m$
26. $\frac{4}{3}(-6y) = \left[\frac{4}{3}(-6)\right]y = -8y$

27. $-\frac{5}{2}(2x-4y)=-\frac{5}{2}(2x)+\frac{5}{2}(4y)=-5x+10y$

28. $(3a)(b+c-2d)=3ab+3ac-6ad$

29. **(a)** $\frac{3}{10}+\frac{4}{15}=\frac{9}{30}+\frac{8}{30}=\frac{17}{30}$

(b) $\frac{1}{4}+\frac{1}{5}=\frac{5}{20}+\frac{4}{20}=\frac{9}{20}$

30. **(a)** $\frac{2}{3}-\frac{3}{5}=\frac{10}{15}-\frac{9}{15}=\frac{1}{15}$

(b) $1+\frac{5}{8}-\frac{1}{6}=\frac{24}{24}+\frac{15}{24}-\frac{4}{24}=\frac{35}{24}$

31. **(a)** $\frac{2}{3}\left(6-\frac{3}{2}\right)=\frac{2}{3}\cdot 6-\frac{2}{3}\cdot\frac{3}{2}=4-1=3$

(b) $\left(3+\frac{1}{4}\right)\left(1-\frac{4}{5}\right)=\left(\frac{12}{4}+\frac{1}{4}\right)\left(\frac{5}{5}-\frac{4}{5}\right)=\frac{13}{4}\cdot\frac{1}{5}=\frac{13}{20}$

32. **(a)** $\dfrac{2}{\frac{2}{3}}-\dfrac{\frac{2}{3}}{2}=2\cdot\frac{3}{2}-\frac{2}{3}\cdot\frac{1}{2}=3-\frac{1}{3}=\frac{9}{3}-\frac{1}{3}=\frac{8}{3}$

(b) $\dfrac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}}=\dfrac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{1}{5}}=\dfrac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{1}{5}}\cdot\frac{10}{10}=\frac{4+5}{1+2}=\frac{9}{3}=3$

33. **(a)** $2\cdot 3=6$ and $2\cdot\frac{7}{2}=7$, so $3<\frac{7}{2}$

(b) $-6>-7$

(c) $3.5=\frac{7}{2}$

34. **(a)** $3\cdot\frac{2}{3}=2$ and $3\cdot 0.67=2.01$, so $\frac{2}{3}<0.67$

(b) $\frac{2}{3}>-0.67$

(c) $|0.67|=|-0.67|$

35. **(a)** False

(b) True

36. **(a)** False: $\sqrt{3}\approx 1.73205<1.7325$.

(b) False

37. **(a)** True **(b)** False

38. **(a)** True **(b)** True

39. **(a)** $x>0$ **(b)** $t<4$

(c) $a\geq\pi$ **(d)** $-5<x<\frac{1}{3}$

(e) $|3-p|\leq 5$

40. **(a)** $y<0$ **(b)** $z>1$

(c) $b\leq 8$ **(d)** $0<w\leq 17$

(e) $|y-\pi|\geq 2$

41. **(a)** $A\cup B=\{1,2,3,4,5,6,7,8\}$

(b) $A\cap B=\{2,4,6\}$

42. **(a)** $B\cup C=\{2,4,6,7,8,9,10\}$

(b) $B\cap C=\{8\}$

43. **(a)** $A\cup C=\{1,2,3,4,5,6,7,8,9,10\}$

(b) $A\cap C=\{7\}$

44. **(a)** $A\cup B\cup C=\{1,2,3,4,5,6,7,8,9,10\}$

(b) $A\cap B\cap C=\varnothing$

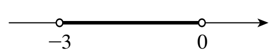
45. **(a)** $B\cup C=\{x\mid x\leq 5\}$

(b) $B\cap C=\{x\mid -1<x<4\}$

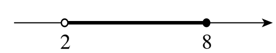
46. **(a)** $A\cap C=\{x\mid -1<x\leq 5\}$

(b) $A\cap B=\{x\mid -2\leq x<4\}$

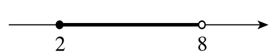
47. $(-3,0)=\{x\mid -3<x<0\}$



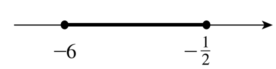
48. $(2,8]=\{x\mid 2<x\leq 8\}$



49. $[2,8)=\{x\mid 2\leq x<8\}$



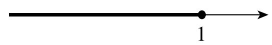
50. $\left[-6,-\frac{1}{2}\right]=\left\{x\mid -6\leq x\leq -\frac{1}{2}\right\}$



51. $[2,\infty)=\{x\mid x\geq 2\}$

52. $(-\infty,1)=\{x\mid x<1\}$

53. $x \le 1 \Leftrightarrow x \in (-\infty, 1]$

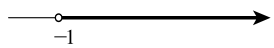


54. $1 \le x \le 2 \Leftrightarrow x \in [1, 2]$

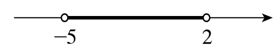
55. $-2 < x \le 1 \Leftrightarrow x \in (-2, 1]$

56. $x \ge -5 \Leftrightarrow x \in [-5, \infty)$

57. $x > -1 \Leftrightarrow x \in (-1, \infty)$



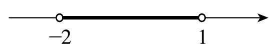
58. $-5 < x < 2 \Leftrightarrow x \in (-5, 2)$



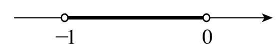
59. **(a)** $[-3, 5]$ **(b)** $(-3, 5]$

60. **(a)** $[0, 2)$ **(b)** $(-2, 0]$

61. $(-2, 0) \cup (-1, 1) = (-2, 1)$



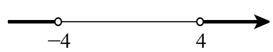
62. $(-2, 0) \cap (-1,) = (-1, 0)$



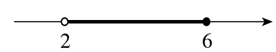
63. $[-4, 6] \cap [0, 8) = [0, 6]$

64. $[-4, 6] \cup [0, 8) = [-4, 8)$

65. $(-\infty, -4) \cup (4, \infty)$



66. $(-\infty, 6] \cap (2, 10) = (2, 6]$



67. **(a)** $|100| = 100$

(b) $|-73| = 73$

68. **(a)** $\left|\sqrt{5} - 5\right| = -\left(\sqrt{5} - 5\right) = 5 - \sqrt{5}$, since $5 > \sqrt{5}$.

(b) $|10 - \pi| = 10 - \pi$, since $10 > \pi$.

69. **(a)** $||-6| - |-4|| = |6 - 4| = |2| = 2$

(b) $\frac{-1}{|-1|} = \frac{-1}{1} = -1$

70. **(a)** $|2 - |-12|| = |2 - 12| = |-10| = 10$

(b) $-1 - |1 - |-1|| = -1 - |1 - 1| = -1 - |0| = -1$

71. **(a)** $|(-2) \cdot 6| = |-12| = 12$

(b) $\left|\left(-\frac{1}{3}\right)(-15)\right| = |5| = 5$

72. **(a)** $\left|\frac{-6}{24}\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

(b) $\left|\frac{7-12}{12-7}\right| = \left|-\frac{5}{5}\right| = |-1| = 1$

73. $|(-2) - 3| = |-5| = 5$

74. $|-2.5 - 1.5| = |-4| = 4$

75. **(a)** $|17 - 2| = 15$

(b) $|21 - (-3)| = |21 + 3| = |24| = 24$

(c) $\left|-\frac{3}{10} - \frac{11}{8}\right| = \left|-\frac{12}{40} - \frac{55}{40}\right| = \left|-\frac{67}{40}\right| = \frac{67}{40}$

76. **(a)** $\left|\frac{7}{15} - \left(-\frac{1}{21}\right)\right| = \left|\frac{49}{105} + \frac{5}{105}\right| = \left|\frac{54}{105}\right| = \left|\frac{18}{35}\right| = \frac{18}{35}$

(b) $|-38 - (-57)| = |-38 + 57| = |19| = 19$.

(c) $|-2.6 - (-1.8)| = |-2.6 + 1.8| = |-0.8| = 0.8$.

77. **(a)** Let $x = 0.777\ldots$. So $10x = 7.7777\ldots \Leftrightarrow x = 0.7777\ldots \Leftrightarrow 9x = 7$. Thus, $x = \frac{7}{9}$.

(b) Let $x = 0.2888\ldots$. So $100x = 28.8888\ldots \Leftrightarrow 10x = 2.8888\ldots \Leftrightarrow 90x = 26$. Thus, $x = \frac{26}{90} = \frac{13}{45}$.

(c) Let $x = 0.575757\ldots$. So $100x = 57.5757\ldots \Leftrightarrow x = 0.5757\ldots \Leftrightarrow 99x = 57$. Thus, $x = \frac{57}{99} = \frac{19}{33}$.

78. (a) Let $x = 5.2323\ldots$. So $100x = 523.2323\ldots \Leftrightarrow 1x = 5.2323\ldots \Leftrightarrow 99x = 518$. Thus, $x = \frac{518}{99}$.

(b) Let $x = 1.3777\ldots$. So $100x = 137.7777\ldots \Leftrightarrow 10x = 13.7777\ldots \Leftrightarrow 90x = 124$. Thus, $x = \frac{124}{90} = \frac{62}{45}$.

(c) Let $x = 2.13535\ldots$. So $1000x = 2135.3535\ldots \Leftrightarrow 10x = 21.3535\ldots \Leftrightarrow 990x = 2114$. Thus, $x = \frac{2114}{990} = \frac{1057}{495}$.

79. $\pi > 3$, so $|\pi - 3| = \pi - 3$.

80. $\sqrt{2} > 1$, so $\left|1 - \sqrt{2}\right| = \sqrt{2} - 1$.

81. $a < b$, so $|a - b| = -(a - b) = b - a$.

82. $a + b + |a - b| = a + b + b - a = 2b$

83. (a) $-a$ is negative because a is positive.

(b) bc is positive because the product of two negative numbers is positive.

(c) $a - ba + (-b)$ is positive because it is the sum of two positive numbers.

(d) $ab + ac$ is negative: each summand is the product of a positive number and a negative number, and the sum of two negative numbers is negative.

84. (a) $-b$ is positive because b is negative.

(b) $a + bc$ is positive because it is the sum of two positive numbers.

(c) $c - a = c + (-a)$ is negative because c and $-a$ are both negative.

(d) ab^2 is positive because both a and b^2 are positive.

85. Distributive Property

86.

Day	T_O	T_G	$T_O - T_G$	$\lvert T_O - T_G \rvert$
Sunday	68	77	−9	9
Monday	72	75	−3	3
Tuesday	74	74	0	0
Wednesday	80	75	5	5
Thursday	77	69	8	8
Friday	71	70	1	1
Saturday	70	71	−1	1

$T_O - T_G$ gives more information because it tells us which city had the higher temperature.

87. (a) When $L = 60$, $x = 8$, and $y = 6$, we have $L + 2(x + y) = 60 + 2(8 + 6) = 60 + 28 = 88$. Because $88 \le 108$ the post office will accept this package.

When $L = 48$, $x = 24$, and $y = 24$, we have $L + 2(x + y) = 48 + 2(24 + 24) = 48 + 96 = 144$, and since $144 \not\le 108$, the post office will *not* accept this package.

(b) If $x = y = 9$, then $L + 2(9 + 9) \le 108 \Leftrightarrow L + 36 \le 108 \Leftrightarrow L \le 72$. So the length can be as long as 72 in. = 6 ft.

88. Let $x = \dfrac{m_1}{n_1}$ and $y = \dfrac{m_2}{n_2}$ be rational numbers. Then $x + y = \dfrac{m_1}{n_1} + \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 + m_2 n_1}{n_1 n_2}$, $x - y = \dfrac{m_1}{n_1} - \dfrac{m_2}{n_2} = \dfrac{m_1 n_2 - m_2 n_1}{n_1 n_2}$, and $x \cdot y = \dfrac{m_1}{n_1} \cdot \dfrac{m_2}{n_2} = \dfrac{m_1 m_2}{n_1 n_2}$. This shows that the sum, difference, and product of two rational numbers are again rational numbers. However the product of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. Also, the sum of two irrational numbers is not necessarily irrational; for example, $\sqrt{2} + \left(-\sqrt{2}\right) = 0$ which is rational.

89. $\frac{1}{2}+\sqrt{2}$ is irrational. If it were rational, then by Exercise 6(a), the sum $\left(\frac{1}{2}+\sqrt{2}\right)+\left(-\frac{1}{2}\right)=\sqrt{2}$ would be rational, but this is not the case.

Similarly, $\frac{1}{2}\cdot\sqrt{2}$ is irrational.

(a) Following the hint, suppose that $r+t=q$, a rational number. Then by Exercise 6(a), the sum of the two rational numbers $r+t$ and $-r$ is rational. But $(r+t)+(-r)=t$, which we know to be irrational. This is a contradiction, and hence our original premise—that $r+t$ is rational—was false.

(b) r is rational, so $r=\dfrac{a}{b}$ for some integers a and b. Let us assume that $rt=q$, a rational number. Then by definition, $q=\dfrac{c}{d}$ for some integers c and d. But then $rt=q\Leftrightarrow\dfrac{a}{b}t=\dfrac{c}{d}$, whence $t=\dfrac{bc}{ad}$, implying that t is rational. Once again we have arrived at a contradiction, and we conclude that the product of a rational number and an irrational number is irrational.

90.

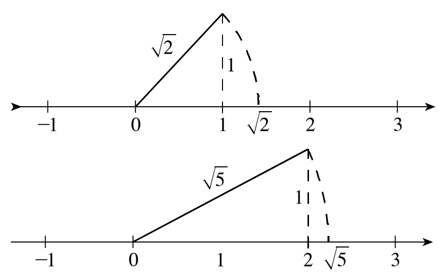
x	1	2	10	100	1000
$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

As x gets large, the fraction $1/x$ gets small. Mathematically, we say that $1/x$ goes to zero.

x	1	0.5	0.1	0.01	0.001
$\frac{1}{x}$	1	$\frac{1}{0.5}=2$	$\frac{1}{0.1}=10$	$\frac{1}{0.01}=100$	$\frac{1}{0.001}=1000$

As x gets small, the fraction $1/x$ gets large. Mathematically, we say that $1/x$ goes to infinity.

91. We can construct the number $\sqrt{2}$ on the number line by transferring the length of the hypotenuse of a right triangle with legs of length 1 and 1.



Similarly, to locate $\sqrt{5}$, we construct a right triangle with legs of length 1 and 2. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2+2^2}=\sqrt{5}$. Then transfer the length of the hypotenuse to the number line.

The square root of any rational number can be located on a number line in this fashion.

The circle in the second figure in the text has circumference π, so if we roll it along a number line one full rotation, we have found π on the number line. Similarly, any rational multiple of π can be found this way.

92. **(a)** Suppose that $a>b$, so $\max(a,b)=a$ and $|a-b|=a-b$. Then $\dfrac{a+b+|a-b|}{2}=\dfrac{a+b+a-b}{2}=a$.

On the other hand, if $b>a$, then $\max(a,b)=b$ and $|a-b|=-(a-b)=b-a$. In this case, $\dfrac{a+b+|a-b|}{2}=\dfrac{a+b+b-a}{2}=b$.

If $a=b$, then $|a-b|=0$ and the result is trivial.

(b) If $a<b$, then $\min(a,b)=a$ and $|a-b|=b-a$. In this case $\dfrac{a+b-|a-b|}{2}=\dfrac{a+b-(b-a)}{2}=a$.

Similarly, if $b<a$, then $\dfrac{a+b-|a-b|}{2}=b$; and if $a=b$, the result is trivial.

93. Answers will vary.

94. **(a)** Subtraction is not commutative. For example, $5-1\neq1-5$.

(b) Division is not commutative. For example, $5\div1\neq1\div5$.

(c) Putting on your socks and putting on your shoes are not commutative. If you put on your socks first, then your shoes, the result is not the same as if you proceed the other way around.

(d) Putting on your hat and putting on your coat are commutative. They can be done in either order, with the same result.

(e) Washing laundry and drying it are not commutative.

95. **(a)** If $x = 2$ and $y = 3$, then $|x + y| = |2 + 3| = |5| = 5$ and $|x| + |y| = |2| + |3| = 5$.
If $x = -2$ and $y = -3$, then $|x + y| = |-5| = 5$ and $|x| + |y| = 5$.
If $x = -2$ and $y = 3$, then $|x + y| = |-2 + 3| = 1$ and $|x| + |y| = 5$.
In each case, $|x + y| \le |x| + |y|$ and the Triangle Inequality is satisfied.

(b) *Case 0:* If either x or y is 0, the result is equality, trivially.

Case 1: If x and y have the same sign, then $|x + y| = \left\{ \begin{array}{ll} x + y & \text{if } x \text{ and } y \text{ are positive} \\ -(x + y) & \text{if } x \text{ and } y \text{ are negative} \end{array} \right\} = |x| + |y|$.

Case 2: If x and y have opposite signs, then suppose without loss of generality that $x < 0$ and $y > 0$. Then $|x + y| < |-x + y| = |x| + |y|$.

1.2 EXPONENTS AND RADICALS

1. **(a)** Using exponential notation we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as 5^6.

(b) In the expression 3^4, the number 3 is called the *base* and the number 4 is called the *exponent*.

2. **(a)** When we multiply two powers with the same base, we *add* the exponents. So $3^4 \cdot 3^5 = 3^9$.

(b) When we divide two powers with the same base, we *subtract* the exponents. So $\frac{3^5}{3^2} = 3^3$.

3. **(a)** Using exponential notation we can write $\sqrt[3]{5}$ as $5^{1/3}$.

(b) Using radicals we can write $5^{1/2}$ as $\sqrt{5}$.

(c) No. $\sqrt{5^2} = \left(5^2\right)^{1/2} = 5^{2(1/2)} = 5$ and $\left(\sqrt{5}\right)^2 = \left(5^{1/2}\right)^2 = 5^{(1/2)2} = 5$.

4. $\left(4^{1/2}\right)^3 = 2^3 = 8$; $\left(4^3\right)^{1/2} = 64^{1/2} = 8$

5. Because the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

6. $5^{1/3} \cdot 5^{2/3} = 5^1 = 5$

7. **(a)** No, $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

(b) Yes, $(-5)^4 = 625$, while $-5^4 = -\left(5^4\right) = -625$.

8. **(a)** No, $\left(x^2\right)^3 = x^{2 \cdot 3} = x^6$.

(b) No, $\left(2x^4\right)^3 = 2^3 x^{4 \cdot 3} = 8x^{12}$.

(c) No; if a is negative, then $\sqrt{4a^2} = -2a$.

(d) No, because $(a + 2)^2 \ne a^2 + 4$.

9. $\frac{1}{\sqrt{3}} = 3^{-1/2}$

10. $\sqrt[3]{7^2} = 7^{2/3}$

11. $4^{2/3} = \sqrt[3]{4^2} = \sqrt[3]{16}$

12. $10^{-3/2} = \left(10^{3/2}\right)^{-1} = \left(\sqrt{10^3}\right)^{-1} = \frac{1}{\sqrt{10^3}}$

13. $\sqrt[5]{5^3} = 5^{3/5}$

14. $2^{-1.5} = 2^{-3/2} = \frac{1}{\sqrt{2^3}} = \frac{1}{\sqrt{8}}$

15. $a^{2/5} = \sqrt[5]{a^2}$

16. $\frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}} = x^{-5/2}$

17. **(a)** $-2^6 = -\left(2^6\right) = -64$

(b) $(-2)^6 = 64$

(c) $\left(\frac{1}{5}\right)^2 \cdot (-3)^3 = \frac{(-3)^3}{5^2} = -\frac{27}{25}$

18. **(a)** $(-5)^3 = -125$

(b) $-5^3 = -\left(5^3\right) = -125$

(c) $(-5)^2 \cdot \left(\frac{2}{5}\right)^2 = 4$

19. **(a)** $\left(\frac{5}{3}\right)^0 \cdot 2^{-1} = \frac{1}{2}$

(b) $\frac{2^{-3}}{3^0} = \frac{1}{2^3} = \frac{1}{8}$

(c) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

20. **(a)** $-2^3 \cdot (-2)^0 = -\left(2^3\right) \cdot 1 = -8$

(b) $-2^{-3} \cdot (-2)^0 = -\frac{1}{2^3} \cdot 1 = -\frac{1}{8}$

(c) $\left(\frac{-3}{5}\right)^{-3} = \frac{5^3}{(-3)^3} = -\frac{125}{27}$

21. **(a)** $5^3 \cdot 5 = 5^{3+1} = 625$

(b) $5^4 \cdot 5^{-2} = 5^{4-2} = 25$

(c) $\left(2^2\right)^3 = 2^{2 \cdot 3} = 64$

22. **(a)** $3^8 \cdot 3^5 = 3^{8+5} = 3^{13}$

(b) $\frac{10^7}{10^4} = 10^{7-4} = 1000$

(c) $\left(3^5\right)^4 = 3^{5 \cdot 4} = 3^{20}$

23. **(a)** $3\sqrt[3]{16} = 3\sqrt[3]{8 \cdot 2} = 3\sqrt[3]{8}\sqrt[3]{2} = 6\sqrt[3]{2}$

(b) $\frac{\sqrt{18}}{\sqrt{81}} = \sqrt{\frac{18}{81}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

(c) $\sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2} = \frac{\sqrt{9 \cdot 3}}{\sqrt{4}}$

24. **(a)** $2\sqrt[3]{81} = 2\sqrt[3]{27 \cdot 3} = 2\sqrt[3]{27}\sqrt[3]{3} = 6\sqrt[3]{3}$

(b) $\frac{\sqrt{18}}{\sqrt{25}} = \frac{\sqrt{9 \cdot 2}}{5} = \frac{3\sqrt{2}}{5}$

(c) $\sqrt{\frac{12}{49}} = \frac{\sqrt{4 \cdot 3}}{\sqrt{49}} = \frac{2\sqrt{3}}{7}$

25. **(a)** $\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

(b) $\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$

(c) $\sqrt[3]{24}\sqrt[3]{18} = \sqrt[3]{24 \cdot 18} = \sqrt[3]{6^3 \cdot 2} = 6\sqrt[3]{2}$

26. **(a)** $\sqrt{10}\sqrt{32} = \sqrt{320} = \sqrt{64 \cdot 5} = 8\sqrt{5}$

(b) $\frac{\sqrt{54}}{\sqrt{6}} = \sqrt{\frac{54}{6}} = 3$

(c) $\sqrt[3]{15}\sqrt[3]{75} = \sqrt[3]{15 \cdot 75} = \sqrt[3]{5^3 \cdot 9} = 5\sqrt[3]{9}$

27. **(a)** $\frac{\sqrt{132}}{\sqrt{3}} = \sqrt{\frac{132}{3}} = \sqrt{44} = \sqrt{4 \cdot 11} = 2\sqrt{11}$

(b) $\sqrt[3]{2}\sqrt[3]{32} = \sqrt[3]{64} = 4$

(c) $\sqrt[4]{\frac{1}{4}}\sqrt[4]{\frac{1}{64}} = \sqrt[4]{\frac{1}{4 \cdot 4^3}} = \frac{1}{4}$

28. **(a)** $\sqrt[5]{\frac{1}{8}}\sqrt[5]{\frac{1}{4}} = \sqrt[5]{\frac{1}{2^3 \cdot 2^2}} = \frac{1}{2}$

(b) $\sqrt[6]{\frac{1}{2}}\sqrt[6]{128} = \sqrt[6]{\frac{2^7}{2}} = 2$

(c) $\frac{\sqrt[3]{4}}{\sqrt[3]{108}} = \sqrt[3]{\frac{4}{108}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

29. **(a)** $x^3 \cdot x^4 = x^{3+4} = x^7$

(b) $\left(2y^2\right)^3 = 2^3 y^{2 \cdot 3} = 8y^6$

(c) $y^{-2}y^7 = y^{-2+7} = y^5$

30. **(a)** $y^5 \cdot y^2 = y^{5+2} = y^7$

(b) $(8x)^2 = 8^2 x^2 = 64x^2$

(c) $x^4 x^{-3} = x^{4-3} = x^1 = x$

31. **(a)** $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$

(b) $w^{-2}w^{-4}w^5 = w^{-2-4+5} = w^{-1} = \frac{1}{w}$

(c) $\frac{x^{16}}{x^{10}} = x^{16-10} = x^6$

32. **(a)** $y^2 \cdot y^{-5} = y^{2-5} = y^{-3} = \frac{1}{y^3}$

(b) $z^5 z^{-3} z^{-4} = z^{5-3-4} = z^{-2} = \frac{1}{z^2}$

(c) $\frac{y^7 y^0}{y^{10}} = y^{7+0-10} = y^{-3} = \frac{1}{y^3}$

33. (a) $\dfrac{a^9a^{-2}}{a} = a^{9-2-1} = a^6$

(b) $\left(a^2a^4\right)^3 = \left(a^{2+4}\right)^3 = \left(a^6\right)^3 = a^{6\cdot3} = a^{18}$

(c) $\left(\dfrac{x}{2}\right)^3\left(5x^6\right) = \dfrac{x^3\cdot5x^6}{2^3} = \dfrac{5x^9}{8}$

34. (a) $\dfrac{z^2z^4}{z^3z^{-1}} = z^{2+4-3-(-1)} = z^4$

(b) $\left(2a^3a^2\right)^4 = 2^4\left(a^{3+2}\right)^4 = 16\left(a^5\right)^4 = 16a^{20}$

(c) $\left(-3z^2\right)^3\left(2z^3\right) = (-3)^3\cdot z^{2\cdot3}\cdot2z^3 = -54z^9$

35. (a) $\left(3x^3y^2\right)\left(2y^3\right) = 3\cdot2x^3y^2y^3 = 6x^3y^5$

(b) $\left(5w^2z^{-2}\right)^2\left(z^3\right) = \left(\dfrac{5w^2}{z^2}\right)^2\left(z^3\right) = \dfrac{5^2\left(w^2\right)^2z^3}{\left(z^2\right)^2} = \dfrac{25w^4}{z}$

36. (a) $\left(8m^{-2}n^4\right)\left(\frac{1}{2}n^{-2}\right) = 8\cdot\frac{1}{2}m^{-2}n^{4-2} = \dfrac{4n^2}{m^2}$

(b) $\left(3a^4b^{-2}\right)^3\left(a^2b^{-1}\right) = \left(3^3a^{4\cdot3}b^{-2\cdot3}\right)\left(a^2b^{-1}\right) = 3^3a^{12+2}b^{-6-1} = \dfrac{27a^{14}}{b^7}$

37. (a) $\dfrac{x^2y^{-1}}{x^{-5}} = x^{2-(-5)}y^{-1} = \dfrac{x^7}{y}$

(b) $\left(\dfrac{a^3}{2b^2}\right)^3 = \dfrac{a^{3\cdot3}}{2^3\left(b^2\right)^3} = \dfrac{a^9}{8b^6}$

38. (a) $\dfrac{y^{-2}z^{-3}}{y^{-1}} = \dfrac{y}{y^2z^3} = \dfrac{1}{yz^3}$

(b) $\left(\dfrac{x^3y^{-2}}{x^{-3}y^2}\right)^{-2} = \left[x^{3-(-3)}y^{-2-2}\right]^{-2} = \left(x^6y^{-4}\right)^{-2} = \dfrac{y^8}{x^{12}}$

39. (a) $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{a^3b^2}{c^3}\right)^3 = a^{2\cdot5+3\cdot3}b^{-5+2\cdot3}c^{-3\cdot3} = \dfrac{a^{19}b}{c^9}$

(b) $\dfrac{\left(u^{-1}v^2\right)^2}{\left(u^3v^{-2}\right)^3} = u^{-1\cdot2-3\cdot3}v^{2\cdot2-(-2)3} = \dfrac{v^{10}}{u^{11}}$

40. (a) $\left(\dfrac{x^4z^2}{4y^5}\right)\left(\dfrac{2x^3y^2}{z^3}\right)^2 = \dfrac{2^2}{4}x^{4+3\cdot2}y^{-5+2\cdot2}z^{2+(-3)2} = \dfrac{x^{10}}{yz^4}$

(b) $\dfrac{\left(rs^2\right)^3}{\left(r^{-3}s^2\right)^2} = r^{3-(-3)2}s^{2\cdot3-2\cdot2} = r^9s^2$

41. (a) $\dfrac{8a^3b^{-4}}{2a^{-5}b^5} = 4a^{3-(-5)}b^{-4-5} = \dfrac{4a^8}{b^9}$

(b) $\left(\dfrac{y}{5x^{-2}}\right)^{-3} = 5^{-1(-3)}x^{-(-2)(-3)}y^{-3} = \dfrac{125}{x^6y^3}$

42. (a) $\dfrac{5xy^{-2}}{x^{-1}y^{-3}} = 5x^{1-(-1)}y^{-2-(-3)} = 5x^2y$

(b) $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3} = 2^{-3}a^{-1(-3)-2(-3)}b^{-3-(-3)(-3)} = \dfrac{a^9}{8b^{12}}$

43. (a) $\left(\dfrac{3a}{b^3}\right)^{-1} = 3^{-1}a^{-1}b^{-3(-1)} = \dfrac{b^3}{3a}$

(b) $\left(\dfrac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1} = \dfrac{r^{-5}sq^{-8}}{q^{-1}r^{-1}s^{-2}} = q^{-8-(-1)}r^{-5-(-1)}s^{1-(-2)} = \dfrac{s^3}{q^7r^4}$

44. (a) $\left(\dfrac{s^2t^{-4}}{5s^{-1}t}\right)^{-2} = s^{2(-2)-(-1)(-2)}t^{-4(-2)-1(-2)}5^{-(-2)} = \dfrac{25t^{10}}{s^6}$

(b) $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3} = x^{-3-2(-3)}y^{-2(-3)-3(-3)}z^{-3(-3)-(-4)(-3)} = \dfrac{x^3y^{15}}{z^3}$

45. (a) $\sqrt[4]{x^4} = |x|$

(b) $\sqrt[4]{16x^8} = \sqrt[4]{2^4x^8} = 2x^2$

46. (a) $\sqrt[5]{x^{10}} = \left(x^{10}\right)^{1/5} = x^2$

(b) $\sqrt[3]{x^3y^6} = \left(x^3y^6\right)^{1/3} = xy^2$

47. (a) $\sqrt[6]{64a^6b^7} = \sqrt[6]{2^6 \cdot a^6 \cdot b^6 \cdot b} = 2ab\sqrt[6]{b}$

(b) $\sqrt[3]{a^2b}\sqrt[3]{64a^4b} = \sqrt[3]{a^2b \cdot 64a^4b} = \sqrt[3]{64a^6b^2} = \sqrt[3]{4^3\left(a^2\right)^3 b^2} = 4a^2\sqrt[3]{b^2}$

48. (a) $\sqrt[4]{x^4y^2z^2} = \sqrt[4]{x^4}\sqrt[4]{y^2z^2} = |x|\sqrt[4]{y^2z^2} = |x|\sqrt{|yz|}$

(b) $\sqrt[3]{\sqrt{64x^6}} = \left(8\left|x^3\right|\right)^{1/3} = 2|x|$

49. (a) $\sqrt{32} + \sqrt{18} = \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2} = \sqrt{16}\sqrt{2} + \sqrt{9}\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

(b) $\sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = \sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

50. (a) $\sqrt{125} + \sqrt{45} = \sqrt{25 \cdot 5} + \sqrt{9 \cdot 5} = 5\sqrt{5} + 3\sqrt{3} = 8\sqrt{5}$

(b) $\sqrt[3]{54} - \sqrt[3]{16} = \sqrt[3]{27 \cdot 2} - \sqrt[3]{8 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} - \sqrt[3]{8}\sqrt[3]{2} = \sqrt[3]{2}$

51. (a) $\sqrt{9a^3} + \sqrt{a} = \sqrt{9}\sqrt{a^2 \cdot a} + \sqrt{a} = 3a\sqrt{a} + \sqrt{a} = (3a+1)\sqrt{a}$

(b) $\sqrt{16x} + \sqrt{x^5} = \sqrt{16}\sqrt{x} + \sqrt{x^4}\sqrt{x} = \left(x^2+4\right)\sqrt{x}$

52. (a) $\sqrt[3]{x^4} + \sqrt[3]{8x} = \sqrt[3]{x^3 \cdot x} + \sqrt[3]{2^3x} = (x+2)\sqrt[3]{x}$

(b) $4\sqrt{18rt^3} + 5\sqrt{32r^3t^5} = 4\sqrt{\left(3^2 \cdot 2\right)r\left(t^2 \cdot t\right)} + 5\sqrt{\left(4^2 \cdot 2\right)\left(r^2 \cdot r\right)\left(t^4 \cdot t\right)} = 4(3)(t)\sqrt{2rt} + 5(4)(r)\left(t^2\right)\sqrt{2rt}$

$= \left(20rt^2 + 12t\right)\sqrt{2rt}$

53. (a) $\sqrt{81x^2+81} = \sqrt{9^2\left(x^2+1\right)} = 9\sqrt{x^2+1}$

(b) $\sqrt{36x^2+36y^2} = \sqrt{6^2\left(x^2+y^2\right)} = 6\sqrt{x^2+y^2}$

54. (a) $\sqrt{27a^2+63a} = \sqrt{3^2a(3a+7)} = 3\sqrt{a(3a+7)}$

(b) $\sqrt{75t+100t^2} = \sqrt{5^2t(3+4t)} = 5\sqrt{t(4t+3)}$

55. (a) $16^{1/4} = \sqrt[4]{16} = 2$

(b) $-8^{1/3} = \sqrt[3]{-8} = -2$

(c) $9^{-1/2} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$

56. (a) $27^{1/3} = \sqrt[3]{27} = 3$

(b) $(-8)^{1/3} = \sqrt[3]{-8} = -2$

(c) $-\left(\dfrac{1}{8}\right)^{1/3} = -\dfrac{1}{\sqrt[3]{8}} = -\dfrac{1}{2}$

57. (a) $32^{2/5} = \left(\sqrt[5]{32}\right)^2 = 2^2 = 4$

(b) $\left(\dfrac{4}{9}\right)^{-1/2} = \dfrac{1}{\sqrt{4/9}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

(c) $\left(\dfrac{16}{81}\right)^{3/4} = \left(\dfrac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3 = \dfrac{8}{27}$

58. (a) $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

(b) $\left(\dfrac{25}{64}\right)^{3/2} = \left(\dfrac{\sqrt{25}}{\sqrt{64}}\right)^3 = \left(\dfrac{5}{8}\right)^3 = \dfrac{125}{512}$

(c) $27^{-4/3} = \dfrac{1}{\left(\sqrt[3]{27}\right)^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

59. (a) $5^{2/3} \cdot 5^{1/3} = 5^{(2/3)+(1/3)} = 5^1 = 5$

(b) $\dfrac{3^{3/5}}{3^{2/5}} = 3^{(3/5)-(2/5)} = 3^{1/5} = \sqrt[5]{3}$

(c) $\left(\sqrt[3]{4}\right)^3 = \left(4^{1/3}\right)^3 = 4^{(1/3)\cdot 3} = 4^1 = 4$

60. (a) $3^{2/7} \cdot 3^{12/7} = 3^{2/7+12/7} = 3^2 = 9$

(b) $\dfrac{7^{2/3}}{7^{5/3}} = 7^{(2/3)-(5/3)} = 7^{-1} = \dfrac{1}{7}$

(c) $\left(\sqrt[5]{6}\right)^{-10} = \left(6^{1/5}\right)^{-10} = 6^{(1/5)(-10)} = 6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

61. (a) $x^{3/4}x^{5/4} = x^{3/4+5/4} = x^2$

(b) $y^{2/3}y^{4/3} = y^{2/3+4/3} = y^2$

62. (a) $(4b)^{1/2}\left(8b^{1/4}\right) = 4^{1/2} \cdot 8b^{1/2+1/4} = 16b^{3/4}$

(b) $\left(3a^{3/4}\right)^2\left(5a^{1/2}\right) = 3^2 \cdot 5a^{(3/4)\cdot 2+(1/2)} = 45a^2$

63. (a) $\dfrac{w^{4/3}w^{2/3}}{w^{1/3}} = w^{4/3+2/3-1/3} = w^{5/3}$

(b) $\dfrac{a^{5/4}\left(2a^{3/4}\right)^3}{a^{1/4}} = 2^3a^{(5/4)+(3/4)3-1/4} = 8a^{13/4}$

64. (a) $\left(8y^3\right)^{-2/3} = 8^{-2/3}y^{3(-2/3)} = \dfrac{1}{4y^2}$

(b) $\left(u^4v^6\right)^{-1/3} = u^{4(-1/3)}v^{6(-1/3)} = \dfrac{1}{u^{4/3}v^2}$

65. (a) $\left(8a^6b^{3/2}\right)^{2/3} = 8^{2/3}a^{6(2/3)}b^{(3/2)(2/3)} = 4a^4b$

(b) $\left(4a^6b^8\right)^{3/2} = 4^{3/2}a^{6(3/2)}b^{8(3/2)} = 8a^9b^{12}$

66. (a) $\left(x^{-5}y^{1/3}\right)^{-3/5} = x^{-5(-3/5)}y^{(1/3)(-3/5)} = \dfrac{x^3}{y^{1/5}}$

(b) $\left(4r^8s^{-1/2}\right)^{1/2}\left(32s^{-5/4}\right)^{-1/5} = 4^{1/2}r^{8(1/2)}s^{(-1/2)(1/2)}32^{-1/5}s^{(-5/4)(-1/5)} = 2r^4s^{-1/4} \cdot \frac{1}{2}s^{1/4} = r^4$

67. (a) $\dfrac{\left(8s^3t^3\right)^{2/3}}{\left(s^4t^{-8}\right)^{1/4}} = 8^{2/3}s^{3(2/3)-4(1/4)}t^{3(2/3)-(-8)(1/4)} = 4st^4$

(b) $\dfrac{\left(32x^5y^{-3/2}\right)^{2/5}}{\left(x^{5/3}y^{2/3}\right)^{3/5}} = \dfrac{32^{2/5}x^{5(2/5)}y^{(-3/2)(2/5)}}{x^{(5/3)(3/5)}y^{(2/3)(3/5)}} = \dfrac{4x^2y^{-3/5}}{xy^{2/5}} = 4x^{2-1}y^{(-3/5)-(2/5)} = \dfrac{4x}{y}$

68. (a) $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4} = 16^{-(-1/4)}x^{8(-1/4)}y^{-4(-1/4)-4/3(-1/4)} = \dfrac{2y^{4/3}}{x^2}$

(b) $\left(\dfrac{4s^3t^4}{s^2t^{9/2}}\right)^{-1/2} = \left[4s^{3-2}t^{4-(9/2)}\right]^{-1/2} = \left(4st^{-1/2}\right)^{-1/2} = \dfrac{\sqrt[4]{t}}{2\sqrt{s}}$

69. (a) $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4\left(\dfrac{x^{-2}}{y^3}\right) = x^{(3/2)4}y^{-(-1/2)4}x^{-2}y^{-3} = x^{6-2}y^{2-3} = \dfrac{x^4}{y}$

(b) $\left(\dfrac{4y^3z^{2/3}}{x^{1/2}}\right)^2\left(\dfrac{x^{-3}y^6}{8z^4}\right)^{1/3} = 4^28^{-1/3}x^{-1/2(2)-3(1/3)}y^{3(2)+6(1/3)}z^{2/3(2)-4(1/3)} = \dfrac{8y^8}{x^2}$

70. (a) $\left(\dfrac{a^{1/6}b^{-3}}{x^{-1}y}\right)^3\left(\dfrac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right) = a^{1/6(3)-3/2}b^{-3(3)-1}x^{-(-1)(3)-2}y^{-1(3)-1/3} = \dfrac{x}{ab^{10}y^{10/3}}$

(b) $\dfrac{(9st)^{3/2}}{\left(27s^3t^{-4}\right)^{2/3}}\left(\dfrac{3s^{-2}}{4t^{1/3}}\right)^{-1} = 9^{3/2}27^{-2/3}3^{-1}4^{-(-1)}s^{3/2-3(2/3)-2(-1)}t^{3/2-(-4)(2/3)-1/3(-1)} = 4s^{3/2}t^{9/2}$

71. **(a)** $\sqrt{x^3} = \left(x^3\right)^{1/2} = x^{3(1/2)} = x^{3/2}$

(b) $\sqrt[5]{x^6} = \left(x^6\right)^{1/5} = x^{6(1/5)} = x^{5/6}$

72. **(a)** $\sqrt{x^5} = \left(x^5\right)^{1/2} = x^{5(1/2)} = x^{5/2}$

(b) $\sqrt[4]{x^6} = \left(x^6\right)^{1/4} = x^{6(1/4)} = x^{3/2}$

73. **(a)** $\sqrt[6]{y^5}\sqrt[3]{y^2} = y^{5/6} \cdot y^{2/3} = y^{5/6+2/3} = y^{3/2}$

(b) $\left(5\sqrt[3]{x}\right)\left(2\sqrt[4]{x}\right) = 5 \cdot 2x^{1/3+1/4} = 10x^{7/12}$

74. **(a)** $\sqrt[4]{b^3}\sqrt{b} = b^{3/4+1/2} = b^{5/4}$

(b) $\left(2\sqrt{a}\right)\left(\sqrt[3]{a^2}\right) = 2a^{1/2+2/3} = 2a^{7/6}$

75. **(a)** $\sqrt{4st^3}\sqrt[6]{s^3t^2} = 4^{1/2}s^{1/2+3/6}t^{3/2+2/6} = 2st^{11/6}$

(b) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}} = x^{7/4-3/4} = x$

76. **(a)** $\sqrt[5]{x^3y^2}\sqrt[10]{x^4y^{16}} = x^{3/5+4/10}y^{2/5+16/10} = xy^2$

(b) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}} = 8^{1/3}x^{2/3-1/2} = 2x^{1/6}$

77. **(a)** $\sqrt[3]{y\sqrt{y}} = \left(y^{1+1/2}\right)^{1/3} = y^{(3/2)(1/3)} = y^{1/2}$

(b) $\sqrt{\dfrac{16u^3v}{uv^5}} = \sqrt{\dfrac{16u^2}{v^4}} = \dfrac{4u}{v^2}$

78. **(a)** $\sqrt{s\sqrt{s^3}} = \left(s^{1+3/2}\right)^{1/2} = s^{5/4}$

(b) $\sqrt[3]{\dfrac{54x^2y^4}{2x^5y}} = \sqrt[3]{\dfrac{27y^3}{x^3}} = \dfrac{3y}{x}$

79. **(a)** $\dfrac{1}{\sqrt{6}} = \dfrac{1}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{6}}{6}$

(b) $\sqrt{\dfrac{3}{2}} = \sqrt{\dfrac{3}{2}}\sqrt{\dfrac{2}{2}} = \dfrac{\sqrt{6}}{2}$

(c) $\dfrac{9}{\sqrt[4]{2}} = \dfrac{9}{2^{1/4}} \cdot \dfrac{2^{3/4}}{2^{3/4}} = \dfrac{9\sqrt[4]{8}}{2}$

80. **(a)** $\dfrac{12}{\sqrt{3}} = \dfrac{12}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{12\sqrt{3}}{3} = 4\sqrt{3}$

(b) $\sqrt{\dfrac{12}{5}} = \sqrt{\dfrac{4 \cdot 3}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{2\sqrt{15}}{5}$

(c) $\dfrac{8}{\sqrt[3]{5^2}} = \dfrac{8}{5^{2/3}} \cdot \dfrac{5^{1/3}}{5^{1/3}} = \dfrac{8\sqrt[3]{5}}{5}$

81. **(a)** $\dfrac{1}{\sqrt{5x}} = \dfrac{1}{\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{\sqrt{5x}}{5x}$

(b) $\sqrt{\dfrac{x}{5}} = \sqrt{\dfrac{x}{5}}\sqrt{\dfrac{5}{5}} = \dfrac{\sqrt{5x}}{5}$

(c) $\sqrt[5]{\dfrac{1}{x^3}} = \dfrac{1}{x^{3/5}} \cdot \dfrac{x^{2/5}}{x^{2/5}} = \dfrac{\sqrt[5]{x^2}}{x}$

82. **(a)** $\sqrt{\dfrac{s}{3t}} = \sqrt{\dfrac{s}{3t}}\sqrt{\dfrac{3t}{3t}} = \dfrac{\sqrt{3st}}{3t}$

(b) $\dfrac{a}{\sqrt[6]{b^2}} = \dfrac{a}{\sqrt[3]{b}}\dfrac{\sqrt[3]{b^2}}{\sqrt[3]{b^2}} = \dfrac{a\sqrt[3]{b^2}}{b}$

(c) $\dfrac{1}{c^{3/5}} = \dfrac{1}{c^{3/5}} \cdot \dfrac{c^{2/5}}{c^{2/5}} = \dfrac{c^{2/5}}{c}$

83. **(a)** $69{,}300{,}000 = 6.93 \times 10^7$

(b) $7{,}200{,}000{,}000{,}000 = 7.2 \times 10^{12}$

(c) $0.000028536 = 2.8536 \times 10^{-5}$

(d) $0.0001213 = 1.213 \times 10^{-4}$

84. **(a)** $129{,}540{,}000 = 1.2954 \times 10^8$

(b) $7{,}259{,}000{,}000 = 7.259 \times 10^9$

(c) $0.0000000014 = 1.4 \times 10^{-9}$

(d) $0.0007029 = 7.029 \times 10^{-4}$

85. **(a)** $3.19 \times 10^5 = 319{,}000$

(b) $2.721 \times 10^8 = 272{,}100{,}000$

(c) $2.670 \times 10^{-8} = 0.00000002670$

(d) $9.999 \times 10^{-9} = 0.000000009999$

86. **(a)** $7.1 \times 10^{14} = 710{,}000{,}000{,}000{,}000$

(b) $6 \times 10^{12} = 6{,}000{,}000{,}000{,}000$

(c) $8.55 \times 10^{-3} = 0.00855$

(d) $6.257 \times 10^{-10} = 0.0000000006257$

87. **(a)** 5,900,000,000,000 mi$= 5.9 \times 10^{12}$ mi

(b) 0.0000000000004 cm $= 4 \times 10^{-13}$ cm

(c) 33 billion billion molecules $= 33 \times 10^9 \times 10^9 = 3.3 \times 10^{19}$ molecules

88. **(a)** 93,000,000 mi $= 9.3 \times 10^7$ mi

(b) 0.000000000000000000000053 g $= 5.3 \times 10^{-23}$ g

(c) 5,970,000,000,000,000,000,000,000 kg $= 5.97 \times 10^{24}$ kg

89. $\left(7.2 \times 10^{-9}\right)\left(1.806 \times 10^{-12}\right) = 7.2 \times 1.806 \times 10^{-9} \times 10^{-12} \approx 13.0 \times 10^{-21} = 1.3 \times 10^{-20}$

90. $\left(1.062 \times 10^{24}\right)\left(8.61 \times 10^{19}\right) = 1.062 \times 8.61 \times 10^{24} \times 10^{19} \approx 9.14 \times 10^{43}$

91. $\dfrac{1.295643 \times 10^9}{\left(3.610 \times 10^{-17}\right)\left(2.511 \times 10^6\right)} = \dfrac{1.295643}{3.610 \times 2.511} \times 10^{9+17-6} \approx 0.1429 \times 10^{19} = 1.429 \times 10^{19}$

92. $\dfrac{(73.1)\left(1.6341 \times 10^{28}\right)}{0.0000000019} = \dfrac{(7.31 \times 10)\left(1.6341 \times 10^{28}\right)}{1.9 \times 10^{-9}} = \dfrac{7.31 \times 1.6341}{1.9} \times 10^{1+28-(-9)} \approx 6.3 \times 10^{38}$

93. $\dfrac{(0.0000162)(0.01582)}{(594621000)(0.0058)} = \dfrac{\left(1.62 \times 10^{-5}\right)\left(1.582 \times 10^{-2}\right)}{\left(5.94621 \times 10^8\right)\left(5.8 \times 10^{-3}\right)} = \dfrac{1.62 \times 1.582}{5.94621 \times 5.8} \times 10^{-5-2-8+3} = 0.074 \times 10^{-12}$
$= 7.4 \times 10^{-14}$

94. $\dfrac{\left(3.542 \times 10^{-6}\right)^9}{\left(5.05 \times 10^4\right)^{12}} = \dfrac{(3.542)^9 \times 10^{-54}}{(5.05)^{12} \times 10^{48}} = \dfrac{87747.96}{275103767.10} \times 10^{-54-48} \approx 3.19 \times 10^{-4} \times 10^{-102} \approx 3.19 \times 10^{-106}$

95. **(a)** b^5 is negative since a negative number raised to an odd power is negative.

(b) b^{10} is positive since a negative number raised to an even power is positive.

(c) ab^2c^3 we have (positive) (negative)^2 (negative)^3 = (positive) (positive) (negative) which is negative.

(d) Since $b - a$ is negative, $(b - a)^3 = (\text{negative})^3$ which is negative.

(e) Since $b - a$ is negative, $(b - a)^4 = (\text{negative})^4$ which is positive.

(f) $\dfrac{a^3c^3}{b^6c^6} = \dfrac{(\text{positive})^3\,(\text{negative})^3}{(\text{negative})^6\,(\text{negative})^6} = \dfrac{(\text{positive})\,(\text{negative})}{(\text{positive})\,(\text{positive})} = \dfrac{\text{negative}}{\text{positive}}$ which is negative.

96. **(a)** Since $\frac{1}{2} > \frac{1}{3}$, $2^{1/2} > 2^{1/3}$.

(b) $\left(\frac{1}{2}\right)^{1/2} = 2^{-1/2}$ and $\left(\frac{1}{2}\right)^{1/3} = 2^{-1/3}$. Since $-\frac{1}{2} < -\frac{1}{3}$, we have $\left(\frac{1}{2}\right)^{1/2} < \left(\frac{1}{2}\right)^{1/3}$.

(c) We find a common root: $7^{1/4} = 7^{3/12} = \left(7^3\right)^{1/12} = 343^{1/12}$; $4^{1/3} = 4^{4/12} = \left(4^4\right)^{1/12} = 256^{1/12}$. So $7^{1/4} > 4^{1/3}$.

(d) We find a common root: $\sqrt[3]{5} = 5^{1/3} = 5^{2/6} = \left(5^2\right)^{1/6} = 25^{1/6}$; $\sqrt{3} = 3^{1/2} = 3^{3/6} = \left(3^3\right)^{1/6} = 27^{1/6}$. So $\sqrt[3]{5} < \sqrt{3}$.

97. Since one light year is 5.9×10^{12} miles, Centauri is about $4.3 \times 5.9 \times 10^{12} \approx 2.54 \times 10^{13}$ miles away or 25,400,000,000,000 miles away.

98. $9.3 \times 10^7 \text{ mi} = 186,000 \dfrac{\text{mi}}{\text{s}} \times t \text{ s} \Leftrightarrow t = \dfrac{9.3 \times 10^7}{186,000} \text{ s} = 500 \text{ s} = 8\frac{1}{3} \text{ min}.$

99. Volume = (average depth) (area) $= \left(3.7 \times 10^3 \text{ m}\right)\left(3.6 \times 10^{14} \text{ m}^2\right)\left(\dfrac{10^3 \text{ liters}}{\text{m}^3}\right) \approx 1.33 \times 10^{21}$ liters

100. Each person's share is equal to $\dfrac{\text{national debt}}{\text{population}} = \dfrac{1.674 \times 10^{13}}{3.164 \times 10^8} \approx 5.2908 \times 10^4 \approx \$52{,}900$.

101. The number of molecules is equal to

$$(\text{volume}) \cdot \left(\frac{\text{liters}}{\text{m}^3}\right) \cdot \left(\frac{\text{molecules}}{22.4 \text{ liters}}\right) = (5 \cdot 10 \cdot 3) \cdot \left(10^3\right) \cdot \left(\frac{6.02 \times 10^{23}}{22.4}\right) \approx 4.03 \times 10^{27}$$

102. First convert 1135 feet to miles. This gives $1135 \text{ ft} = 1135 \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}} = 0.215$ mi. Thus the distance you can see is given by $D = \sqrt{2rh + h^2} = \sqrt{2\,(3960)\,(0.215) + (0.215)^2} \approx \sqrt{1702.8} \approx 41.3$ miles.

103. **(a)** Using $f = 0.4$ and substituting $d = 65$, we obtain $s = \sqrt{30fd} = \sqrt{30 \times 0.4 \times 65} \approx 28$ mi/h.

(b) Using $f = 0.5$ and substituting $s = 50$, we find d. This gives $s = \sqrt{30fd} \Leftrightarrow 50 = \sqrt{30 \cdot (0.5)\, d} \Leftrightarrow 50 = \sqrt{15d} \Leftrightarrow 2500 = 15d \Leftrightarrow d = \frac{500}{3} \approx 167$ feet.

104. Since 1 day = 86,400 s, 365.25 days = 31,557,600 s. Substituting, we obtain $d = \left(\dfrac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{4\pi^2}\right)^{1/3} \cdot \left(3.15576 \times 10^7\right)^{2/3} \approx 1.5 \times 10^{11}$ m $= 1.5 \times 10^8$ km.

105. Since $10^6 = 10^3 \cdot 10^3$ it would take 1000 days ≈ 2.74 years to spend the million dollars.
Since $10^9 = 10^3 \cdot 10^6$ it would take $10^6 = 1{,}000{,}000$ days ≈ 2739.72 years to spend the billion dollars.

106. **(a)** $\dfrac{18^5}{9^5} = \left(\dfrac{18}{9}\right)^5 = 2^5 = 32$

(b) $20^6 \cdot (0.5)^6 = (20 \cdot 0.5)^6 = 10^6 = 1{,}000{,}000$

107. **(a)**

n	1	2	5	10	100
$2^{1/n}$	$2^{1/1} = 2$	$2^{1/2} = 1.414$	$2^{1/5} = 1.149$	$2^{1/10} = 1.072$	$2^{1/100} = 1.007$

So when n gets large, $2^{1/n}$ decreases to 1.

(b)

n	1	2	5	10	100
$\left(\frac{1}{2}\right)^{1/n}$	$\left(\frac{1}{2}\right)^{1/1} = 0.5$	$\left(\frac{1}{2}\right)^{1/2} = 0.707$	$\left(\frac{1}{2}\right)^{1/5} = 0.871$	$\left(\frac{1}{2}\right)^{1/10} = 0.933$	$\left(\frac{1}{2}\right)^{1/100} = 0.993$

So when n gets large, $\left(\frac{1}{2}\right)^{1/n}$ increases to 1.

108. **(a)** $\dfrac{a^m}{a^n} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}}$. Because $m > n$, we can cancel n factors of a from numerator and denominator and are left with $m - n$ factors of a in the numerator. Thus, $\dfrac{a^m}{a^n} = a^{m-n}$.

(b) $\left(\dfrac{a}{b}\right)^n = \overbrace{\dfrac{a}{b} \cdot \dfrac{a}{b} \cdot \cdots \cdot \dfrac{a}{b}}^{n \text{ factors}} = \dfrac{\overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ factors}}}{\underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}} = \dfrac{a^n}{b^n}$

109. **(a)** $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{1}{\left(\frac{a}{b}\right)^n} = \dfrac{1}{\frac{a^n}{b^n}} = \dfrac{b^n}{a^n}$

(b) $\dfrac{a^{-n}}{b^{-m}} = \dfrac{\frac{1}{a^n}}{\frac{1}{b^m}} = \dfrac{1}{a^n} \cdot b^m = \dfrac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

1. **(a)** The polynomial $2x^5 + 6x^4 + 4x^3$ has three terms: $2x^5$, $6x^4$, and $4x^3$.

(b) The factor $2x^3$ is common to each term, so $2x^5 + 6x^4 + 4x^3 = 2x^3\left(x^2 + 3x + 2\right)$. [In fact, the polynomial can be factored further as $2x^3(x + 2)(x + 1)$.]

2. To factor the trinomial $x^2 + 7x + 10$ we look for two integers whose product is 10 and whose sum is 7. These integers are 5 and 2, so the trinomial factors as $(x + 5)(x + 2)$.

3. The greatest common factor in the expression $3x^3 + x^2$ is x^2, and the expression factors as $x^2(3x + 1)$.

4. The Special Product Formula for the "square of a sum" is $(A+B)^2 = A^2 + 2AB + B^2$. So $(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.

5. The Special Product Formula for the "product of the sum and difference of terms" is $(A+B)(A-B) = A^2 - B^2$. So $(5+x)(5-x) = 5^2 - x^2 = 25 - x^2$.

6. The Special Factoring Formula for the "difference of squares" is $A^2 - B^2 = (A-B)(A+B)$. So $4x^2 - 25 = (2x-5)(2x+5)$.

7. The Special Factoring Formula for a "perfect square" is $A^2 + 2AB + B^2 = (A+B)^2$. So $x^2 + 10x + 25 = (x+5)^2$.

8. **(a)** No; $(x+5)^2 = x^2 + 2(5x) + 25 \neq x^2 + 25$.

(b) Yes; $(x+a)^2 = x^2 + 2xa + a^2$.

(c) Yes; by a Special Product Formula, $(x+5)(x-5) = x^2 - 25$.

(d) Yes, $(x+a)(x-a) = x^2 - a^2$, by a Special Product Formula.

9. Type: binomial. Terms: $5x^3$ and 6. Degree: 3.

10. Type: trinomial. Terms: $-2x^2$, $5x$, and -3. Degree: 2.

11. Type: monomial. Term: -8. Degree: 0.

12. Type: monomial. Terms: $\frac{1}{2}x^7$. Degree: 7.

13. Type: four-term polynomial. Terms: x, $-x^2$, x^3, and $-x^4$. Degree: 4.

14. Type: binomial. Terms: $\sqrt{2}x$ and $-\sqrt{3}$. Degree: 1.

15. $(12x-7) - (5x-12) = 12x - 7 - 5x + 12 = 7x + 5$

16. $(5-3x) + (2x-8) = -x - 3$

17. $\left(-2x^2 - 3x + 1\right) + \left(3x^2 + 5x - 4\right) = -2x^2 - 3x + 1 + 3x^2 + 5x - 4 = x^2 + 2x - 3$

18. $\left(3x^2 + x + 1\right) - \left(2x^2 - 3x - 5\right) = 3x^2 + x + 1 - 2x^2 + 3x + 5 = x^2 + 4x + 6$

19. $\left(5x^3 + 4x^2 - 3x\right) - \left(x^2 + 7x + 2\right) = 5x^3 + 4x^2 - 3x - x^2 - 7x - 2 = 5x^3 + 3x^2 - 10x - 2$

20. $3(x-1) + 4(x+2) = 3x - 3 + 4x + 8 = 7x + 5$

21. $8(2x+5) - 7(x-9) = 16x + 40 - 7x + 63 = 9x + 103$

22. $4\left(x^2 - 3x + 5\right) - 3\left(x^2 - 2x + 1\right) = 4x^2 - 12x + 20 - 3x^2 + 6x - 3 = x^2 - 6x + 17$

23. $2(2-5t) + t^2(t-1) - \left(t^4 - 1\right) = 4 - 10t + t^3 - t^2 - t^4 + 1 = -t^4 + t^3 - t^2 - 10t + 5$

24. $5(3t-4) - \left(t^2 + 2\right) - 2t(t-3) = 15t - 20 - t^2 - 2 - 2t^2 + 6t = -3t^2 + 21t - 22$

25. $(3t-2)(7t-4) = 21t^2 - 12t - 14t + 8 = 21t^2 - 26t + 8$

26. $(4s-1)(2s+5) = 8s^2 + 18s - 5$

27. $(3x+5)(2x-1) = 6x^2 + 10x - 3x - 5 = 6x^2 + 7x - 5$

28. $(7y-3)(2y-1) = 14y^2 - 13y + 3$

29. $(x+3y)(2x-y) = 2x^2 + 5xy - 3y^2$

30. $(4x-5y)(3x-y) = 12x^2 - 19xy + 5y^2$

31. $(5x+1)^2 = 25x^2 + 10x + 1$

32. $(2-7y)^2 = 49y^2 - 28y + 4$

33. $(2u+v)^2 = 4u^2 + 4uv + v^2$

34. $(x-3y)^2 = x^2 - 6xy + 9y^2$

35. $(2x+3y)^2 = 4x^2 + 12xy + 9y^2$

36. $(r-2s)^2 = r^2 - 4rs + 4s^2$

37. $(x+6)(x-6) = x^2 - 36$

38. $(5-y)(5+y) = 25 - y^2$

39. $(3x-4)(3x+4) = (3x)^2 - 4^2 = 9x^2 - 16$

40. $(2y+5)(2y-5) = 4y^2 - 25$

41. $\left(\sqrt{x} + 2\right)\left(\sqrt{x} - 2\right) = x - 4$

42. $\left(\sqrt{y} + \sqrt{2}\right)\left(\sqrt{y} - \sqrt{2}\right) = y - 2$

43. $(y+2)^3 = y^3 + 3y^2(2) + 3y\left(2^2\right) + 2^3 = y^3 + 6y^2 + 12y + 8$

44. $(x-3)^3 = x^3 - 9x^2 + 27x - 27$

45. $(1-2r)^3 = 1^3 - 3\left(1^2\right)(2r) + 3(1)(2r)^2 - (2r)^3 = -8r^3 + 12r^2 - 6r + 1$

46. $(3+2y)^3 = 8y^3 + 36y^2 + 54y + 27$

47. $(x+2)\left(x^2+2x+3\right) = x^3 + 2x^2 + 3x + 2x^2 + 4x + 6 = x^3 + 4x^2 + 7x + 6$

48. $(x+1)\left(2x^2-x+1\right) = 2x^3 + x^2 + 1$

49. $(2x-5)\left(x^2-x+1\right) = 2x^3 - 2x^2 + 2x - 5x^2 + 5x - 5 = 2x^3 - 7x^2 + 7x - 5$

50. $(1+2x)\left(x^2-3x+1\right) = 2x^3 - 5x^2 - x + 1$

51. $\sqrt{x}\left(x-\sqrt{x}\right) = x\sqrt{x} - \left(\sqrt{x}\right)^2 = x\sqrt{x} - x$

52. $x^{3/2}\left(\sqrt{x} - 1/\sqrt{x}\right) = x^2 - x$

53. $y^{1/3}\left(y^{2/3} + y^{5/3}\right) = y^{1/3+2/3} + y^{1/3+5/3} = y^2 + y$

54. $x^{1/4}\left(2x^{3/4} - x^{1/4}\right) = 2x - \sqrt{x}$

55. $\left(x^2-a^2\right)\left(x^2+a^2\right) = x^4 - a^4$

56. $\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right) = x - y$

57. $\left(\sqrt{a}-b\right)\left(\sqrt{a}+b\right) = a - b^2$

58. $\left(\sqrt{h^2+1}+1\right)\left(\sqrt{h^2+1}-1\right) = h^2$

59. $\left((x-1)+x^2\right)\left((x-1)-x^2\right) = (x-1)^2 - \left(x^2\right)^2 = x^2 - 2x + 1 - x^4 = -x^4 + x^2 - 2x + 1$

60. $\left(x+\left(2+x^2\right)\right)\left(x-\left(2+x^2\right)\right) = -x^4 - 3x^2 - 4$

61. $(2x+y-3)(2x+y+3) = (2x+y)^2 - 3^2 = 4x^2 + 4xy + y^2 - 9$

62. $(x+y+z)(x-y-z) = x^2 - y^2 - z^2 - 2yz$

63. $-2x^3 + x = x\left(1-2x^2\right)$

64. $3x^4 - 6x^3 - x^2 = x^2\left(3x^2 - 6x - 1\right)$

65. $y(y-6) + 9(y-6) = (y-6)(y+9)$

66. $(z+2)^2 - 5(z+2) = (z+2)[(z+2)-5] = (z+2)(z-3)$

67. $2x^2y - 6xy^2 + 3xy = xy(2x - 6y + 3)$

68. $-7x^4y^2 + 14xy^3 + 21xy^4 = 7xy^2\left(-x^3 + 2y + 3y^2\right)$

69. $x^2 + 8x + 7 = (x+7)(x+1)$

70. $x^2 + 4x - 5 = (x+5)(x-1)$

71. $8x^2 - 14x - 15 = (2x-5)(4x+3)$

72. $6y^2 + 11y - 21 = (y+3)(6y-7)$

73. $3x^2 - 16x + 5 = (3x-1)(x-5)$

74. $5x^2 - 7x - 6 = (5x+3)(x-2)$

75. $(3x+2)^2 + 8(3x+2) + 12 = [(3x+2)+2][(3x+2)+6] = (3x+4)(3x+8)$

76. $2(a+b)^2 + 5(a+b) - 3 = [(a+b)+3][2(a+b)-1] = (a+b+3)(2a+2b-1)$

77. $9a^2 - 16 = (3a)^2 - 4^2 = (3a-4)(3a+4)$

78. $(x+3)^2 - 4 = (x+3)^2 - 2^2 = [(x+3)-2][(x+3)+2] = (x+1)(x+5)$

79. $27x^3 + y^3 = (3x)^3 + y^3 = (3x+y)\left[(3x)^2 + 3xy + y^2\right] = (3x+y)\left(9x^2 - 3xy + y^2\right)$

80. $a^3 - b^6 = a^3 - \left(b^2\right)^3 = \left(a-b^2\right)\left[a^2 + ab^2 + \left(b^2\right)^2\right] = \left(a-b^2\right)\left(a^2 + ab^2 + b^4\right)$

81. $8s^3 - 125t^3 = (2s)^3 - (5t)^3 = (2s-5t)\left[(2s)^2 + (2s)(5t) + (5t)^2\right] = (2s-5t)\left(4s^2 + 10st + 25t^2\right)$

82. $1 + 1000y^3 = 1 + (10y)^3 = (1+10y)\left[1 - 10y + (10y)^2\right] = (1+10y)\left(1 - 10y + 100y^2\right)$

83. $x^2 + 12x + 36 = x^2 + 2(6x) + 6^2 = (x + 6)^2$

84. $16z^2 - 24z + 9 = (4z)^2 - 2(4z)(3) + 3^2 = (4z - 3)^2$

85. $x^3 + 4x^2 + x + 4 = x^2(x + 4) + 1(x + 4) = (x + 4)\left(x^2 + 1\right)$

86. $3x^3 - x^2 + 6x - 2 = x^2(3x - 1) + 2(3x - 1) = (3x - 1)\left(x^2 + 2\right)$

87. $5x^3 + x^2 + 5x + 1 = x^2(5x + 1) + (5x + 1) = \left(x^2 + 1\right)(5x + 1)$

88. $18x^3 + 9x^2 + 2x + 1 = 9x^2(2x + 1) + (2x + 1) = \left(9x^2 + 1\right)(2x + 1)$

89. $x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1) = (x + 1)\left(x^2 + 1\right)$

90. $x^5 + x^4 + x + 1 = x^4(x + 1) + 1(x + 1) = (x + 1)\left(x^4 + 1\right)$

91. $x^{5/2} - x^{1/2} = x^{1/2}\left(x^2 - 1\right) = \sqrt{x}(x - 1)(x + 1)$

92. $3x^{-1/2} + 4x^{1/2} + x^{3/2} = x^{-1/2}\left(3 + 4x + x^2\right) = \left(\frac{1}{\sqrt{x}}\right)(3 + x)(1 + x)$

93. Start by factoring out the power of x with the smallest exponent, that is, $x^{-3/2}$. So

$$x^{-3/2} + 2x^{-1/2} + x^{1/2} = x^{-3/2}\left(1 + 2x + x^2\right) = \frac{(1 + x)^2}{x^{3/2}}.$$

94. $(x - 1)^{7/2} - (x - 1)^{3/2} = (x - 1)^{3/2}\left[(x - 1)^2 - 1\right] = (x - 1)^{3/2}[(x - 1) - 1][(x - 1) + 1]$

$$= (x - 1)^{3/2}(x - 2)(x)$$

95. Start by factoring out the power of $\left(x^2 + 1\right)$ with the smallest exponent, that is, $\left(x^2 + 1\right)^{-1/2}$.

Thus, $\left(x^2 + 1\right)^{1/2} + 2\left(x^2 + 1\right)^{-1/2} = \left(x^2 + 1\right)^{-1/2}\left[\left(x^2 + 1\right) + 2\right] = \dfrac{x^2 + 3}{\sqrt{x^2 + 1}}$.

96. $x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2} = x^{-1/2}(x + 1)^{-1/2}[(x + 1) + x] = \dfrac{2x + 1}{\sqrt{x}\sqrt{x + 1}}$

97. $12x^3 + 18x = 6x\left(2x^2 + 3\right)$

98. $30x^3 + 15x^4 = 15x^3(2 + x)$

99. $x^2 - 2x - 8 = (x - 4)(x + 2)$

100. $x^2 - 14x + 48 = (x - 8)(x - 6)$

101. $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

102. $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

103. $9x^2 - 36x - 45 = 9\left(x^2 - 4x - 5\right) = 9(x - 5)(x + 1)$

104. $8x^2 + 10x + 3 = (4x + 3)(2x + 1)$

105. $49 - 4y^2 = (7 - 2y)(7 + 2y)$

106. $4t^2 - 9s^2 = (2t - 3s)(2t + 3s)$

107. $t^2 - 6t + 9 = (t - 3)^2$

108. $x^2 + 10x + 25 = (x + 5)^2$

109. $4x^2 + 4xy + y^2 = (2x + y)^2$

110. $r^2 - 6rs + 9s^2 = (r - 3s)^2$

111. $(a + b)^2 - (a - b)^2 = [(a + b) - (a - b)][(a + b) + (a - b)] = (2b)(2a) = 4ab$

112. $\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2 = \left[\left(1 + \frac{1}{x}\right) - \left(1 - \frac{1}{x}\right)\right]\left[\left(1 + \frac{1}{x}\right) + \left(1 - \frac{1}{x}\right)\right]$

$$= \left(1 + \frac{1}{x} - 1 + \frac{1}{x}\right)\left(1 + \frac{1}{x} + 1 - \frac{1}{x}\right) = \left(\frac{2}{x}\right)(2) = \frac{4}{x}$$

113. $x^2\left(x^2 - 1\right) - 9\left(x^2 - 1\right) = \left(x^2 - 1\right)\left(x^2 - 9\right) = (x - 1)(x + 1)(x - 3)(x + 3)$

114. $\left(a^2 - 1\right)b^2 - 4\left(a^2 - 1\right) = \left(a^2 - 1\right)\left(b^2 - 4\right) = (a - 1)(a + 1)(b - 2)(b + 2)$

115. $8x^3 - 125 = (2x)^3 - 5^3 = (2x - 5)\left[(2x)^2 + (2x)(5) + 5^2\right] = (2x - 5)\left(4x^2 + 10x + 25\right)$

116. $x^6 + 64 = x^6 + 2^6 = \left(x^2\right)^3 + (4)^3 = \left(x^2 + 4\right)\left[\left(x^2\right)^2 - 4\left(x^2\right) + (4)^2\right] = \left(x^2 + 4\right)\left(x^4 - 4x^2 + 16\right)$

117. $x^3 + 2x^2 + x = x\left(x^2 + 2x + 1\right) = x(x + 1)^2$

118. $3x^3 - 27x = 3x\left(x^2 - 9\right) = 3x(x - 3)(x + 3)$

119. $x^4y^3 - x^2y^5 = x^2y^3\left(x^2 - y^2\right) = x^2y^3(x + y)(x - y)$

120. $18y^3x^2 - 2xy^4 = 2xy^3(9x - y)$

121. $3x^3 - x^2 - 12x + 4 = 3x\left(x^2 - 4\right) - \left(x^2 - 4\right) = (3x - 1)\left(x^2 - 4\right) = (3x - 1)(x + 2)(x - 2)$

122. $9x^3 + 18x^2 - x - 2 = 9x^2(x + 2) - (x + 2) = \left(9x^2 - 1\right)(x + 2) = (3x + 1)(3x - 1)(x + 2)$

123. $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2) = (x - 1)(x + 2)[(x + 2) - (x - 1)] = 3(x - 1)(x + 2)$

124. $y^4(y + 2)^3 + y^5(y + 2)^4 = y^4(y + 2)^3[(1) + y(y + 2)] = y^4(y + 2)^3\left(y^2 + 2y + 1\right) = y^4(y + 2)^3(y + 1)^2$

125. Start by factoring $y^2 - 7y + 10$, and then substitute $a^2 + 1$ for y. This gives

$$\left(a^2 + 1\right)^2 - 7\left(a^2 + 1\right) + 10 = \left[\left(a^2 + 1\right) - 2\right]\left[\left(a^2 + 1\right) - 5\right] = \left(a^2 - 1\right)\left(a^2 - 4\right) = (a - 1)(a + 1)(a - 2)(a + 2)$$

126. $\left(a^2 + 2a\right)^2 - 2\left(a^2 + 2a\right) - 3 = \left[\left(a^2 + 2a\right) - 3\right]\left[\left(a^2 + 2a\right) + 1\right] = \left(a^2 + 2a - 3\right)\left(a^2 + 2a + 1\right)$
$= (a - 1)(a + 3)(a + 1)^2$

127. $5\left(x^2 + 4\right)^4(2x)(x - 2)^4 + \left(x^2 + 4\right)^5(4)(x - 2)^3 = 2\left(x^2 + 4\right)^4(x - 2)^3\left[(5)(x)(x - 2) + \left(x^2 + 4\right)(2)\right]$
$= 2\left(x^2 + 4\right)^4(x - 2)^3\left(5x^2 - 10x + 2x^2 + 8\right) = 2\left(x^2 + 4\right)^4(x - 2)^3\left(7x^2 - 10x + 8\right)$

128. $3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3\left(\frac{1}{2}\right)(x + 3)^{-1/2} = (2x - 1)^2(x + 3)^{-1/2}\left[6(x + 3) + (2x - 1)\left(\frac{1}{2}\right)\right]$
$= (2x - 1)^2(x + 3)^{-1/2}\left(6x + 18 + x - \frac{1}{2}\right) = (2x - 1)^2(x + 3)^{-1/2}\left(7x + \frac{35}{2}\right)$

129. $\left(x^2 + 3\right)^{-1/3} - \frac{2}{3}x^2\left(x^2 + 3\right)^{-4/3} = \left(x^2 + 3\right)^{-4/3}\left[\left(x^2 + 3\right) - \frac{2}{3}x^2\right] = \left(x^2 + 3\right)^{-4/3}\left(\frac{1}{3}x^2 + 3\right) = \dfrac{\frac{1}{3}x^2 + 3}{\left(x^2 + 3\right)^{4/3}}$

130. $\frac{1}{2}x^{-1/2}(3x + 4)^{1/2} - \frac{3}{2}x^{1/2}(3x + 4)^{-1/2} = \frac{1}{2}x^{-1/2}(3x + 4)^{-1/2}[(3x + 4) - 3x] = \frac{1}{2}x^{-1/2}(3x + 4)^{-1/2}(4)$
$= 2x^{-1/2}(3x + 4)^{-1/2}$

131. **(a)** $\frac{1}{2}\left[(a + b)^2 - \left(a^2 + b^2\right)\right] = \frac{1}{2}\left[a^2 + 2ab + b^2 - a^2 - b^2\right] = \frac{1}{2}(2ab) = ab.$

(b) $\left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2 = \left[\left(a^2 + b^2\right) - \left(a^2 - b^2\right)\right]\left[\left(a^2 + b^2\right) + \left(a^2 - b^2\right)\right]$
$= \left(a^2 + b^2 - a^2 + b^2\right)\left(a^2 + b^2 + a^2 - b^2\right) = \left(2b^2\right)\left(2a^2\right) = 4a^2b^2$

132. LHS $= \left(a^2 + b^2\right)\left(c^2 + d^2\right) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

RHS $= (ac + bd)^2 + (ad - bc)^2 = a^2c^2 + 2abcd + b^2d^2 + a^2d^2 - 2abcd + b^2c^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2.$

So LHS = RHS, that is, $\left(a^2 + b^2\right)\left(c^2 + d^2\right) = (ac + bd)^2 + (ad - bc)^2.$

133. $4a^2c^2 - \left(c^2 - b^2 + a^2\right)^2 = (2ac)^2 - \left(c^2 - b^2 + a^2\right)$

$= \left[(2ac) - \left(c^2 - b^2 + a^2\right)\right]\left[(2ac) + \left(c^2 - b^2 + a^2\right)\right]$ (difference of squares)

$= \left(2ac - c^2 + b^2 - a^2\right)\left(2ac + c^2 - b^2 + a^2\right)$

$= \left[b^2 - \left(c^2 - 2ac + a^2\right)\right]\left[\left(c^2 + 2ac + a^2\right) - b^2\right]$ (regrouping)

$= \left[b^2 - (c-a)^2\right]\left[(c+a)^2 - b^2\right]$ (perfect squares)

$= [b - (c-a)][b + (c-a)][(c+a) - b][(c+a) + b]$ (each factor is a difference of squares)

$= (b - c + a)(b + c - a)(c + a - b)(c + a + b)$

$= (a + b - c)(-a + b + c)(a - b + c)(a + b + c)$

134. **(a)** $x^4 + x^2 - 2 = \left(x^2 - 1\right)\left(x^2 + 2\right) = (x-1)(x+1)\left(x^2 + 2\right)$

(b) $x^4 + 2x^2 + 9 = \left(x^4 + 6x^2 + 9\right) - 4x^2 = \left(x^2 + 3\right)^2 - (2x)^2 = \left[\left(x^2 + 3\right) - 2x\right]\left[\left(x^2 + 3\right) + 2x\right]$

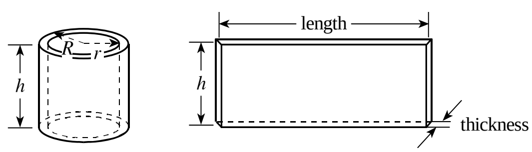
$= \left(x^2 - 2x + 3\right)\left(x^2 + 2x + 3\right)$

(c) $x^4 + 4x^2 + 16 = \left(x^4 + 8x^2 + 16\right) - 4x^2 = \left(x^2 + 4\right)^2 - (2x)^2$

$= \left[\left(x^2 + 4\right) - 2x\right]\left[\left(x^2 + 4\right) + 2x\right] = \left(x^2 - 2x + 4\right)\left(x^2 + 2x + 4\right)$

(d) $x^4 + 2x^2 + 1 = \left(x^2 + 1\right)^2$

135. The volume of the shell is the difference between the volumes of the outside cylinder (with radius R) and the inside cylinder (with radius r). Thus $V = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h = \pi(R-r)(R+r)h = 2\pi \cdot \dfrac{R+r}{2} \cdot h \cdot (R-r)$. The average radius is $\dfrac{R+r}{2}$ and $2\pi \cdot \dfrac{R+r}{2}$ is the average circumference (length of the rectangular box), h is the height, and $R - r$ is the thickness of the rectangular box. Thus $V = \pi R^2 h - \pi r^2 h = 2\pi \cdot \dfrac{R+r}{2} \cdot h \cdot (R-r) = 2\pi \cdot$ (average radius) $\cdot$ (height) $\cdot$ (thickness)



136. **(a)** Moved portion = field − habitat

(b) Using the difference of squares, we get $b^2 - (b-2x)^2 = [b - (b-2x)][b + (b-x)] = 2x(2b - 2x) = 4x(b-x)$.

137. **(a)** The degree of the product is the sum of the degrees.

(b) The degree of a sum is at most the largest of the degrees — it could be smaller than either. For example, the degree of $\left(x^3\right) + \left(-x^3 + x\right) = x$ is 1.

138. **(a)** $528^2 - 527^2 = (528 - 527)(528 + 527) = 1(1055) = 1055$

(b) $122^2 - 120^2 = (122 - 120)(122 + 120) = 2(242) = 484$

(c) $1020^2 - 1010^2 = (1020 - 1010)(1020 + 1010) = 10(2030) = 20{,}300$

139. **(a)** $501 \cdot 499 = (500 + 1)(500 - 1) = 500^2 - 1 = 250{,}000 - 1 = 249{,}999$

(b) $79 \cdot 61 = (70 + 9)(70 - 9) = 70^2 - 9^2 = 4900 - 81 = 4819$

(c) $2007 \cdot 1993 = (2000 + 7)(2000 - 7) = 2000^2 - 7^2 = 4{,}000{,}000 - 49 = 3{,}999{,}951$

140. **(a)** $A^4 - B^4 = \left(A^2 - B^2\right)\left(A^2 + B^2\right) = (A - B)(A + B)\left(A^2 + B^2\right)$

$A^6 - B^6 = \left(A^3 - B^3\right)\left(A^3 + B^3\right)$ (difference of squares)

$= (A - B)\left(A^2 + AB + B^2\right)(A + B)\left(A^2 - AB + B^2\right)$ (difference and sum of cubes)

(b) $12^4 - 7^4 = 20{,}736 - 2{,}401 = 18{,}335$; $12^6 - 7^6 = 2{,}985{,}984 - 117{,}649 = 2{,}868{,}335$

(c) $18{,}335 = 12^4 - 7^4 = (12 - 7)(12 + 7)\left(12^2 + 7^2\right) = 5(19)(144 + 49) = 5(19)(193)$

$2{,}868{,}335 = 12^6 - 7^6 = (12 - 7)(12 + 7)\left[12^2 + 12(7) + 7^2\right]\left[12^2 - 12(7) + 7^2\right]$

$= 5(19)(144 + 84 + 49)(144 - 84 + 49) = 5(19)(277)(109)$

141. **(a)** $(A - 1)(A + 1) = A^2 + A - A - 1 = A^2 - 1$

$(A - 1)\left(A^2 + A + 1\right) = A^3 + A^2 + A - A^2 - A - 1 = A^3 - 1$

$(A - 1)\left(A^3 + A^2 + A + 1\right) = A^4 + A^3 + A^2 + A - A^3 - A^2 - A - 1$

(b) We conjecture that $A^5 - 1 = (A - 1)\left(A^4 + A^3 + A^2 + A + 1\right)$. Expanding the right-hand side, we have $(A - 1)\left(A^4 + A^3 + A^2 + A + 1\right) = A^5 + A^4 + A^3 + A^2 + A - A^4 - A^3 - A^2 - A - 1 = A^5 - 1$, verifying our conjecture. Generally, $A^n - 1 = (A - 1)\left(A^{n-1} + A^{n-2} + \cdots + A + 1\right)$ for any positive integer n.

142. **(a)**

$$\begin{array}{r} A + 1 \\ \times \quad A - 1 \\ \hline -A - 1 \\ A^2 + A \\ \hline A^2 - 1 \end{array} \qquad \begin{array}{r} A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^2 - A - 1 \\ A^3 + A^2 + A \\ \hline A^3 - 1 \end{array} \qquad \begin{array}{r} A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^3 - A^2 - A - 1 \\ A^4 + A^3 + A^2 + A \\ \hline A^4 - 1 \end{array}$$

(b) Based on the pattern in part (a), we suspect that $A^5 - 1 = (A - 1)\left(A^4 + A^3 + A^2 + A + 1\right)$. Check:

$$\begin{array}{r} A^4 + A^3 + A^2 + A + 1 \\ \times \quad A - 1 \\ \hline -A^4 - A^3 - A^2 - A - 1 \\ A^5 + A^4 + A^3 + A^2 + A \\ \hline A^5 - 1 \end{array}$$

The general pattern is $A^n - 1 = (A - 1)\left(A^{n-1} + A^{n-2} + \cdots + A^2 + A + 1\right)$, where n is a positive integer.

1.4 RATIONAL EXPRESSIONS

1. **(a)** $\dfrac{3x}{x^2 - 1}$ is a rational expression.

(b) $\dfrac{\sqrt{x + 1}}{2x + 3}$ is not a rational expression. A rational expression must be a polynomial divided by a polynomial, and the numerator of the expression is $\sqrt{x + 1}$, which is not a polynomial.

(c) $\dfrac{x(x^2 - 1)}{x + 3} = \dfrac{x^3 - x}{x + 3}$ is a rational expression.

2. To simplify a rational expression we cancel factors that are common to the *numerator* and *denominator*. So, the expression $\dfrac{(x+1)(x+2)}{(x+3)(x+2)}$ simplifies to $\dfrac{x+1}{x+3}$.

3. To multiply two rational expressions we multiply their *numerators* together and multiply their *denominators* together. So $\dfrac{2}{x+1}\cdot\dfrac{x}{x+3}$ is the same as $\dfrac{2\cdot x}{(x+1)\cdot(x+3)}=\dfrac{2x}{x^2+4x+3}$.

4. **(a)** $\dfrac{1}{x}-\dfrac{2}{(x+1)}-\dfrac{x}{(x+1)^2}$ has three terms.

(b) The least common denominator of all the terms is $x(x+1)^2$.

(c) $$\frac{1}{x}-\frac{2}{(x+1)}-\frac{x}{(x+1)^2}=\frac{(x+1)^2}{x(x+1)^2}-\frac{2x(x+1)}{(x+1)}-\frac{x(x)}{(x+1)^2}=\frac{(x+1)^2-2x(x+1)-x^2}{x(x+1)^2}$$
$$=\frac{x^2+2x+1-2x^2-2x-x^2}{x(x+1)^2}=\frac{-2x^2+1}{x(x+1)^2}$$

5. **(a)** Yes. Cancelling $x+1$, we have $\dfrac{x(x+1)}{(x+1)^2}=\dfrac{x}{x+1}$.

(b) No; $(x+5)^2=x^2+10x+25\neq x^2+25$, so $x+5=\sqrt{x^2+10x+25}\neq\sqrt{x^2+25}$.

6. **(a)** Yes, $\dfrac{3+a}{3}=\dfrac{3}{3}+\dfrac{a}{3}=1+\dfrac{a}{3}$.

(b) No. We cannot "separate" the denominator in this way; only the numerator, as in part (a). (See also Exercise 101.)

7. The domain of $4x^2-10x+3$ is all real numbers.

8. The domain of $-x^4+x^3+9x$ is all real numbers.

9. Since $x-3\neq 0$ we have $x\neq 3$. Domain: $\{x\mid x\neq 3\}$

10. Since $3t+6\neq 0$ we have $t\neq -2$. Domain: $\{t\mid t\neq -2\}$

11. Since $x+3\geq 0$, $x\geq -3$. Domain; $\{x\mid x\geq -3\}$

12. Since $x-1>0$, $x>1$. Domain; $\{x\mid x>1\}$

13. $x^2-x-2=(x+1)(x-2)\neq 0\Leftrightarrow x\neq -1$ or 2, so the domain is $\{x\mid x\neq -1,2\}$.

14. $2x\geq 0$ and $x+1\neq 0\Leftrightarrow x\geq 0$ and $x\neq -1$, so the domain is $\{x\mid x\geq 0\}$.

15. $\dfrac{5(x-3)(2x+1)}{10(x-3)^2}=\dfrac{5(x-3)(2x+1)}{5(x-3)\cdot 2(x-3)}=\dfrac{2x+1}{2(x-3)}$

16. $\dfrac{4\left(x^2-1\right)}{12(x+2)(x-1)}=\dfrac{4(x+1)(x-1)}{12(x+2)(x-1)}=\dfrac{x+1}{3(x+2)}$

17. $\dfrac{x-2}{x^2-4}=\dfrac{x-2}{(x-2)(x+2)}=\dfrac{1}{x+2}$

18. $\dfrac{x^2-x-2}{x^2-1}=\dfrac{(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x-2}{x-1}$

19. $\dfrac{x^2+5x+6}{x^2+8x+15}=\dfrac{(x+2)(x+3)}{(x+5)(x+3)}=\dfrac{x+2}{x+5}$

20. $\dfrac{x^2-x-12}{x^2+5x+6}=\dfrac{(x-4)(x+3)}{(x+2)(x+3)}=\dfrac{x-4}{x+2}$

21. $\dfrac{y^2+y}{y^2-1}=\dfrac{y(y+1)}{(y-1)(y+1)}=\dfrac{y}{y-1}$

22. $\dfrac{y^2-3y-18}{2y^2+7y+3}=\dfrac{(y-6)(y+3)}{(2y+1)(y+3)}=\dfrac{y-6}{2y+1}$

23. $\dfrac{2x^3-x^2-6x}{2x^2-7x+6}=\dfrac{x\left(2x^2-x-6\right)}{(2x-3)(x-2)}=\dfrac{x(2x+3)(x-2)}{(2x-3)(x-2)}=\dfrac{x(2x+3)}{2x-3}$

24. $\dfrac{1-x^2}{x^3-1}=\dfrac{(1-x)(1+x)}{(x-1)\left(x^2+x+1\right)}=\dfrac{-(x-1)(1+x)}{(x-1)\left(x^2+x+1\right)}=\dfrac{-(x+1)}{x^2+x+1}$

25. $\dfrac{4x}{x^2-4}\cdot\dfrac{x+2}{16x}=\dfrac{4x}{(x-2)(x+2)}\cdot\dfrac{x+2}{16x}=\dfrac{1}{4(x-2)}$

26. $\dfrac{x^2-25}{x^2-16}\cdot\dfrac{x+4}{x+5}=\dfrac{(x-5)(x+5)}{(x-4)(x+4)}\cdot\dfrac{x+4}{x+5}=\dfrac{x-5}{x-4}$

27. $\dfrac{x^2+2x-15}{x^2-25}\cdot\dfrac{x-5}{x+2}=\dfrac{(x+5)(x-3)(x-5)}{(x+5)(x-5)(x+2)}=\dfrac{x-3}{x+2}$

28. $\dfrac{x^2+2x-3}{x^2-2x-3}\cdot\dfrac{3-x}{3+x}=\dfrac{(x+3)(x-1)}{(x-3)(x+1)}\cdot\dfrac{-(x-3)}{x+3}=\dfrac{-(x-1)}{x+1}=\dfrac{-x+1}{x+1}=\dfrac{1-x}{1+x}$

29. $\dfrac{t-3}{t^2+9}\cdot\dfrac{t+3}{t^2-9}=\dfrac{(t-3)(t+3)}{\left(t^2+9\right)(t-3)(t+3)}=\dfrac{1}{t^2+9}$

30. $\dfrac{x^2-x-6}{x^2+2x}\cdot\dfrac{x^3+x^2}{x^2-2x-3}=\dfrac{(x-3)(x+2)}{x(x+2)}\cdot\dfrac{x^2(x+1)}{(x-3)(x+1)}=x$

31. $\dfrac{x^2+7x+12}{x^2+3x+2}\cdot\dfrac{x^2+5x+6}{x^2+6x+9}=\dfrac{(x+3)(x+4)}{(x+1)(x+2)}\cdot\dfrac{(x+2)(x+3)}{(x+3)(x+3)}=\dfrac{x+4}{x+1}$

32. $\dfrac{x^2+2xy+y^2}{x^2-y^2}\cdot\dfrac{2x^2-xy-y^2}{x^2-xy-2y^2}=\dfrac{(x+y)(x+y)}{(x-y)(x+y)}\cdot\dfrac{(x-y)(2x+y)}{(x-2y)(x+y)}=\dfrac{2x+y}{x-2y}$

33. $\dfrac{x+3}{4x^2-9}\div\dfrac{x^2+7x+12}{2x^2+7x-15}=\dfrac{x+3}{4x^2-9}\cdot\dfrac{2x^2+7x-15}{x^2+7x+12}=\dfrac{x+3}{(2x-3)(2x+3)}\cdot\dfrac{(x+5)(2x-3)}{(x+3)(x+4)}=\dfrac{x+5}{(2x+3)(x+4)}$

34. $\dfrac{2x+1}{2x^2+x-15}\div\dfrac{6x^2-x-2}{x+3}=\dfrac{2x+1}{(x+3)(2x-5)}\cdot\dfrac{x+3}{(2x+1)(3x-2)}=\dfrac{1}{(2x-5)(3x-2)}$

35. $\dfrac{\dfrac{x^3}{x+1}}{\dfrac{x}{x^2+2x+1}}=\dfrac{x^3}{x+1}\cdot\dfrac{x^2+2x+1}{x}=\dfrac{x^3(x+1)(x+1)}{(x+1)x}=x^2(x+1)$

36. $\dfrac{\dfrac{2x^2-3x-2}{x^2-1}}{\dfrac{2x^2+5x+2}{x^2+x-2}}=\dfrac{2x^2-3x-2}{x^2-1}\cdot\dfrac{x^2+x-2}{2x^2+5x+2}=\dfrac{(x-2)(2x+1)}{(x-1)(x+1)}\cdot\dfrac{(x-1)(x+2)}{(x+2)(2x+1)}=\dfrac{x-2}{x+1}$

37. $\dfrac{x/y}{z}=\dfrac{x}{y}\cdot\dfrac{1}{z}=\dfrac{x}{yz}$

38. $\dfrac{x}{y/z}=x\div\dfrac{y}{z}=\dfrac{x}{1}\cdot\dfrac{z}{y}=\dfrac{xz}{y}$

39. $1+\dfrac{1}{x+3}=\dfrac{x+3}{x+3}+\dfrac{1}{x+3}=\dfrac{x+4}{x+3}$

40. $\dfrac{3x-2}{x+1}-2=\dfrac{3x-2}{x+1}-\dfrac{2(x+1)}{x+1}=\dfrac{3x-2-2x-2}{x+1}=\dfrac{x-4}{x+1}$

41. $\dfrac{1}{x+5}+\dfrac{2}{x-3}=\dfrac{x-3}{(x+5)(x-3)}+\dfrac{2(x+5)}{(x+5)(x-3)}=\dfrac{x-3+2x+10}{(x+5)(x-3)}=\dfrac{3x+7}{(x+5)(x-3)}$

42. $\dfrac{1}{x+1}+\dfrac{1}{x-1}=\dfrac{x-1}{(x+1)(x-1)}+\dfrac{x+1}{(x+1)(x-1)}=\dfrac{x-1+x+1}{(x+1)(x-1)}=\dfrac{2x}{(x+1)(x-1)}$

43. $\dfrac{3}{x+1}-\dfrac{1}{x+2}=\dfrac{3(x+2)}{(x+1)(x+2)}-\dfrac{1(x+1)}{(x+1)(x+2)}=\dfrac{3x+6-x-1}{(x+1)(x+2)}=\dfrac{2x+5}{(x+1)(x+2)}$

44. $\dfrac{x}{x-4}-\dfrac{3}{x+6}=\dfrac{x(x+6)}{(x-4)(x+6)}+\dfrac{-3(x-4)}{(x-4)(x+6)}=\dfrac{x^2+6x-3x+12}{(x-4)(x+6)}=\dfrac{x^2+3x+12}{(x-4)(x+6)}$

45. $\dfrac{5}{2x-3}-\dfrac{3}{(2x-3)^2}=\dfrac{5(2x-3)}{(2x-3)^2}-\dfrac{3}{(2x-3)^2}=\dfrac{10x-15-3}{(2x-3)^2}=\dfrac{10x-18}{(2x-3)^2}=\dfrac{2(5x-9)}{(2x-3)^2}$

46. $\dfrac{x}{(x+1)^2}+\dfrac{2}{x+1}=\dfrac{x}{(x+1)^2}+\dfrac{2(x+1)}{(x+1)(x+1)}=\dfrac{x+2x+2}{(x+1)^2}=\dfrac{3x+2}{(x+1)^2}$

47. $u+1+\dfrac{u}{u+1}=\dfrac{(u+1)(u+1)}{u+1}+\dfrac{u}{u+1}=\dfrac{u^2+2u+1+u}{u+1}=\dfrac{u^2+3u+1}{u+1}$

48. $\dfrac{2}{a^2}-\dfrac{3}{ab}+\dfrac{4}{b^2}=\dfrac{2b^2}{a^2b^2}-\dfrac{3ab}{a^2b^2}+\dfrac{4a^2}{a^2b^2}=\dfrac{2b^2-3ab+4a^2}{a^2b^2}$

49. $\dfrac{1}{x^2}+\dfrac{1}{x^2+x}=\dfrac{1}{x^2}+\dfrac{1}{x(x+1)}=\dfrac{x+1}{x^2(x+1)}+\dfrac{x}{x^2(x+1)}=\dfrac{2x+1}{x^2(x+1)}$

50. $\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}=\dfrac{x^2}{x^3}+\dfrac{x}{x^3}+\dfrac{1}{x^3}=\dfrac{x^2+x+1}{x^3}$

51. $\dfrac{2}{x+3}-\dfrac{1}{x^2+7x+12}=\dfrac{2}{x+3}-\dfrac{1}{(x+3)(x+4)}=\dfrac{2(x+4)}{(x+3)(x+4)}+\dfrac{-1}{(x+3)(x+4)}$
$=\dfrac{2x+8-1}{(x+3)(x+4)}=\dfrac{2x+7}{(x+3)(x+4)}$

52. $\dfrac{x}{x^2-4}+\dfrac{1}{x-2}=\dfrac{x}{(x-2)(x+2)}+\dfrac{1}{x-2}=\dfrac{x}{(x-2)(x+2)}+\dfrac{x+2}{(x-2)(x+2)}$
$=\dfrac{2x+2}{(x-2)(x+2)}=\dfrac{2(x+1)}{(x-2)(x+2)}$

53. $\dfrac{1}{x+3}+\dfrac{1}{x^2-9}=\dfrac{1}{x+3}+\dfrac{1}{(x-3)(x+3)}=\dfrac{x-3}{(x-3)(x+3)}+\dfrac{1}{(x-3)(x+3)}=\dfrac{x-2}{(x-3)(x+3)}$

54. $\dfrac{x}{x^2+x-2}-\dfrac{2}{x^2-5x+4}=\dfrac{x}{(x-1)(x+2)}+\dfrac{-2}{(x-1)(x-4)}$
$=\dfrac{x(x-4)}{(x-1)(x+2)(x-4)}+\dfrac{-2(x+2)}{(x-1)(x+2)(x-4)}=\dfrac{x^2-4x-2x-4}{(x-1)(x+2)(x-4)}=\dfrac{x^2-6x-4}{(x-1)(x+2)(x-4)}$

55. $\dfrac{2}{x}+\dfrac{3}{x-1}-\dfrac{4}{x^2-x}=\dfrac{2}{x}+\dfrac{3}{x-1}-\dfrac{4}{x(x-1)}=\dfrac{2(x-1)}{x(x-1)}+\dfrac{3x}{x(x-1)}+\dfrac{-4}{x(x-1)}=\dfrac{2x-2+3x-4}{x(x-1)}=\dfrac{5x-6}{x(x-1)}$

56. $\dfrac{x}{x^2-x-6}-\dfrac{1}{x+2}-\dfrac{2}{x-3}=\dfrac{x}{(x-3)(x+2)}+\dfrac{-1}{x+2}+\dfrac{-2}{x-3}$
$=\dfrac{x}{(x-3)(x+2)}+\dfrac{-1(x-3)}{(x-3)(x+2)}+\dfrac{-2(x+2)}{(x-3)(x+2)}=\dfrac{x-x+3-2x-4}{(x-3)(x+2)}=\dfrac{-2x-1}{(x-3)(x+2)}$

57. $\dfrac{1}{x^2+3x+2}-\dfrac{1}{x^2-2x-3}=\dfrac{1}{(x+2)(x+1)}-\dfrac{1}{(x-3)(x+1)}$
$=\dfrac{x-3}{(x-3)(x+2)(x+1)}+\dfrac{-(x+2)}{(x-3)(x+2)(x+1)}=\dfrac{x-3-x-2}{(x-3)(x+2)(x+1)}=\dfrac{-5}{(x-3)(x+2)(x+1)}$

58. $\dfrac{1}{x+1}-\dfrac{2}{(x+1)^2}+\dfrac{3}{x^2-1}=\dfrac{1}{x+1}+\dfrac{-2}{(x+1)^2}+\dfrac{3}{(x-1)(x+1)}$
$=\dfrac{(x+1)(x-1)}{(x-1)(x+1)^2}+\dfrac{-2(x-1)}{(x-1)(x+1)^2}+\dfrac{3(x+1)}{(x-1)(x+1)^2}$
$=\dfrac{x^2-1}{(x-1)(x+1)^2}+\dfrac{-2x+2}{(x-1)(x+1)^2}+\dfrac{3x+3}{(x-1)(x+1)^2}=\dfrac{x^2-1-2x+2+3x+3}{(x-1)(x+1)^2}=\dfrac{x^2+x+4}{(x-1)(x+1)^2}$

59. $\dfrac{1+\frac{1}{x}}{\frac{1}{x}-2}=\dfrac{x\left(1+\frac{1}{x}\right)}{x\left(\frac{1}{x}-2\right)}=\dfrac{x+1}{1-2x}$

60. $\dfrac{1-\frac{2}{y}}{\frac{3}{y}-1}=\dfrac{y\left(1-\frac{2}{y}\right)}{y\left(\frac{3}{y}-1\right)}=\dfrac{y-2}{3-y}$

61. $\dfrac{1+\dfrac{1}{x+2}}{1-\dfrac{1}{x+2}}=\dfrac{(x+2)\left(1+\dfrac{1}{x+2}\right)}{(x+2)\left(1-\dfrac{1}{x+2}\right)}=\dfrac{(x+2)+1}{(x+2)-1}=\dfrac{x+3}{x+1}$

62. $\dfrac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}}=\dfrac{c-1+1}{c-1-1}=\dfrac{c}{c-2}$

63. $$\frac{\dfrac{1}{x-1}+\dfrac{1}{x+3}}{x+1}=\frac{(x-1)(x+3)\left(\dfrac{1}{x-1}+\dfrac{1}{x+3}\right)}{(x-1)(x+3)(x+1)}=\frac{(x+3)+(x-1)}{(x-1)(x+1)(x+3)}=\frac{2(x+1)}{(x-1)(x+1)(x+3)}$$
$$=\frac{2}{(x-1)(x+3)}$$

64. $$\frac{\dfrac{x-3}{x-4}-\dfrac{x+2}{x+1}}{x+3}=\frac{(x-3)(x+1)-(x+2)(x-4)}{(x-4)(x+3)(x+1)}=\frac{x^2-2x-3-\left(x^2-2x-8\right)}{(x-4)(x+3)(x+1)}=\frac{5}{(x-4)(x+3)(x+1)}$$

65. $$\frac{x-\dfrac{x}{y}}{y-\dfrac{y}{x}}=\frac{xy\left(x-\dfrac{x}{y}\right)}{xy\left(y-\dfrac{y}{x}\right)}=\frac{x^2y-x^2}{xy^2-y^2}=\frac{x^2(y-1)}{y^2(x-1)}$$

66. $$\frac{x+\dfrac{y}{x}}{y+\dfrac{x}{y}}=\frac{xy\left(x+\dfrac{y}{x}\right)}{xy\left(y+\dfrac{x}{y}\right)}=\frac{x^2y+y^2}{xy^2+x^2}=\frac{y\left(y+x^2\right)}{x\left(x+y^2\right)}$$

67. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}=\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{y^2-x^2}{x^2y^2}}=\dfrac{x^2-y^2}{xy}\cdot\dfrac{x^2y^2}{y^2-x^2}=\dfrac{xy}{-1}=-xy$. An alternative method is to multiply the numerator and denominator by the common denominator of both the numerator and denominator, in this case x^2y^2:

$$\frac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}=\frac{\left(\dfrac{x}{y}-\dfrac{y}{x}\right)}{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}\cdot\frac{x^2y^2}{x^2y^2}=\frac{x^3y-xy^3}{y^2-x^2}=\frac{xy\left(x^2-y^2\right)}{y^2-x^2}=-xy.$$

68. $$x-\frac{y}{\dfrac{x}{y}+\dfrac{y}{x}}=x-\frac{y}{\dfrac{x}{y}+\dfrac{y}{x}}\cdot\frac{xy}{xy}=x-\frac{xy^2}{x^2+y^2}=\frac{x\left(x^2+y^2\right)}{x^2+y^2}-\frac{xy^2}{x^2+y^2}=\frac{x^3+xy^2-xy^2}{x^2+y^2}=\frac{x^3}{x^2+y^2}$$

69. $$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}=\frac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x}+\dfrac{1}{y}}=\frac{\dfrac{y^2}{x^2y^2}-\dfrac{x^2}{x^2y^2}}{\dfrac{y}{xy}+\dfrac{x}{xy}}=\frac{y^2-x^2}{x^2y^2}\cdot\frac{xy}{y+x}=\frac{(y-x)(y+x)xy}{x^2y^2(y+x)}=\frac{y-x}{xy}$$

Alternatively, $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}=\dfrac{\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)}{\left(\dfrac{1}{x}+\dfrac{1}{y}\right)}\cdot\dfrac{x^2y^2}{x^2y^2}=\dfrac{y^2-x^2}{xy^2+x^2y}=\dfrac{(y-x)(y+x)}{xy(y+x)}=\dfrac{y-x}{xy}$.

70. $$\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}=\frac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x+y}}=\frac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x+y}}\cdot\frac{xy(x+y)}{xy(x+y)}=\frac{y(x+y)+x(x+y)}{xy}$$
$$=\frac{xy+y^2+x^2+xy}{xy}=\frac{x^2+2xy+y^2}{xy}=\frac{(x+y)^2}{xy}$$

71. $$1-\frac{1}{1-\dfrac{1}{x}}=1-\frac{x}{x-1}=\frac{x-1-x}{x-1}=\frac{1}{1-x}$$

72. $$1+\frac{1}{1+\dfrac{1}{1+x}}=1+\frac{1+x}{(1+x)+1}=1+\frac{x+1}{x+2}=\frac{x+2+x+1}{x+2}=\frac{2x+3}{x+2}$$

73. $\dfrac{\dfrac{1}{1+x+h}-\dfrac{1}{1+x}}{h}=\dfrac{(1+x)-(1+x+h)}{h\,(1+x)\,(1+x+h)}=-\dfrac{1}{(1+x)\,(1+x+h)}$

74. In calculus it is necessary to eliminate the h in the denominator, and we do this by rationalizing the numerator:

$$\frac{\dfrac{1}{\sqrt{x+h}}-\dfrac{1}{\sqrt{x}}}{h}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=-\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}.$$

75. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\dfrac{x^2-(x+h)^2}{hx^2\,(x+h)^2}=\dfrac{x^2-\left(x^2+2xh+h^2\right)}{hx^2\,(x+h)^2}=-\dfrac{2x+h}{x^2\,(x+h)^2}$

76. $\dfrac{(x+h)^3-7\,(x+h)-\left(x^3-7x\right)}{h}=\dfrac{x^3+3x^2h+3xh^2+h^3-7x-7h-x^3+7x}{h}=\dfrac{3x^2h+3xh^2+h^3-7h}{h}$

$$=\frac{h\left(3x^2+3xh+h^2-7\right)}{h}=3x^2+3xh+h^2-7$$

77. $\sqrt{1+\left(\dfrac{x}{\sqrt{1-x^2}}\right)^2}=\sqrt{1+\dfrac{x^2}{1-x^2}}=\sqrt{\dfrac{1-x^2}{1-x^2}+\dfrac{x^2}{1-x^2}}=\sqrt{\dfrac{1}{1-x^2}}=\dfrac{1}{\sqrt{1-x^2}}$

78. $\sqrt{1+\left(x^3-\dfrac{1}{4x^3}\right)^2}=\sqrt{1+x^6-\dfrac{2x^3}{4x^3}+\dfrac{1}{16x^6}}=\sqrt{1+x^6-\frac{1}{2}+\dfrac{1}{16x^6}}=\sqrt{x^6+\frac{1}{2}+\dfrac{1}{16x^6}}$

$$=\sqrt{\left(x^3+\frac{1}{4x^3}\right)^2}=\left|x^3+\frac{1}{4x^3}\right|$$

79. $\dfrac{3\,(x+2)^2\,(x-3)^2-(x+2)^3\,(2)\,(x-3)}{(x-3)^4}=\dfrac{(x+2)^2\,(x-3)\,[3\,(x-3)-(x+2)\,(2)]}{(x-3)^4}$

$$=\frac{(x+2)^2\,(3x-9-2x-4)}{(x-3)^3}=\frac{(x+2)^2\,(x-13)}{(x-3)^3}$$

80. $\dfrac{2x\,(x+6)^4-x^2\,(4)\,(x+6)^3}{(x+6)^8}=\dfrac{(x+6)^3\left[2x\,(x+6)-4x^2\right]}{(x+6)^8}=\dfrac{2x^2+12x-4x^2}{(x+6)^5}=\dfrac{12x-2x^2}{(x+6)^5}=\dfrac{2x\,(6-x)}{(x+6)^5}$

81. $\dfrac{2\,(1+x)^{1/2}-x\,(1+x)^{-1/2}}{1+x}=\dfrac{(1+x)^{-1/2}\,[2\,(1+x)-x]}{1+x}=\dfrac{x+2}{(1+x)^{3/2}}$

82. $\dfrac{\left(1-x^2\right)^{1/2}+x^2\left(1-x^2\right)^{-1/2}}{1-x^2}=\dfrac{\left(1-x^2\right)^{-1/2}\left(1-x^2+x^2\right)}{1-x^2}=\dfrac{1}{\left(1-x^2\right)^{3/2}}$

83. $\dfrac{3\,(1+x)^{1/3}-x\,(1+x)^{-2/3}}{(1+x)^{2/3}}=\dfrac{(1+x)^{-2/3}\,[3\,(1+x)-x]}{(1+x)^{2/3}}=\dfrac{2x+3}{(1+x)^{4/3}}$

84. $\dfrac{(7-3x)^{1/2}+\frac{3}{2}x\,(7-3x)^{-1/2}}{7-3x}=\dfrac{(7-3x)^{-1/2}\left(7-3x+\frac{3}{2}x\right)}{7-3x}=\dfrac{7-\frac{3}{2}x}{(7-3x)^{3/2}}$

85. $\dfrac{1}{5-\sqrt{3}}=\dfrac{1}{5-\sqrt{3}}\cdot\dfrac{5+\sqrt{3}}{5+\sqrt{3}}=\dfrac{5+\sqrt{3}}{25-3}=\dfrac{5+\sqrt{3}}{22}$

86. $\dfrac{3}{2-\sqrt{5}}=\dfrac{\left(2+\sqrt{5}\right)3}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}=\dfrac{6+3\sqrt{5}}{4-5}=-6-3\sqrt{5}$

87. $\dfrac{2}{\sqrt{2}+\sqrt{7}}=\dfrac{2}{\sqrt{2}+\sqrt{7}}\cdot\dfrac{\sqrt{2}-\sqrt{7}}{\sqrt{2}-\sqrt{7}}=\dfrac{2\left(\sqrt{2}-\sqrt{7}\right)}{2-7}=\dfrac{2\left(\sqrt{2}-\sqrt{7}\right)}{-5}=\dfrac{2\left(\sqrt{7}-\sqrt{2}\right)}{5}$

88. $\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{\sqrt{x}+1}\cdot\dfrac{\sqrt{x}-1}{\sqrt{x}-1}=\dfrac{\sqrt{x}-1}{x-1}$

89. $\dfrac{y}{\sqrt{3}+\sqrt{y}}=\dfrac{y}{\sqrt{3}+\sqrt{y}}\cdot\dfrac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}}=\dfrac{y\left(\sqrt{3}-\sqrt{y}\right)}{3-y}=\dfrac{y\sqrt{3}-y\sqrt{y}}{3-y}$

90. $\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}=\dfrac{2(x-y)}{\sqrt{x}-\sqrt{y}}\cdot\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\dfrac{2(x-y)\left(\sqrt{x}+\sqrt{y}\right)}{x-y}=2\left(\sqrt{x}+\sqrt{y}\right)=2\sqrt{x}+2\sqrt{y}$

91. $\dfrac{1-\sqrt{5}}{3}=\dfrac{1-\sqrt{5}}{3}\cdot\dfrac{1+\sqrt{5}}{1+\sqrt{5}}=\dfrac{1-5}{3\left(1+\sqrt{5}\right)}=\dfrac{-4}{3\left(1+\sqrt{5}\right)}$

92. $\dfrac{\sqrt{3}+\sqrt{5}}{2}=\dfrac{\sqrt{3}+\sqrt{5}}{2}\cdot\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\dfrac{3-5}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-2}{2\left(\sqrt{3}-\sqrt{5}\right)}=\dfrac{-1}{\sqrt{3}-\sqrt{5}}$

93. $\dfrac{\sqrt{r}+\sqrt{2}}{5}=\dfrac{\sqrt{r}+\sqrt{2}}{5}\cdot\dfrac{\sqrt{r}-\sqrt{2}}{\sqrt{r}-\sqrt{2}}=\dfrac{r-2}{5\left(\sqrt{r}-\sqrt{2}\right)}$

94. $\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}=\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

$=\dfrac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\dfrac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

95. $\sqrt{x^2+1}-x=\dfrac{\sqrt{x^2+1}-x}{1}\cdot\dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}=\dfrac{x^2+1-x^2}{\sqrt{x^2+1}+x}=\dfrac{1}{\sqrt{x^2+1}+x}$

96. $\sqrt{x+1}-\sqrt{x}=\dfrac{\sqrt{x+1}-\sqrt{x}}{1}\cdot\dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

97. (a) $R=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}\cdot\dfrac{R_1R_2}{R_1R_2}=\dfrac{R_1R_2}{R_2+R_1}$

(b) Substituting $R_1=10$ ohms and $R_2=20$ ohms gives $R=\dfrac{(10)(20)}{(20)+(10)}=\dfrac{200}{30}\approx 6.7$ ohms.

98. (a) The average cost $A=\dfrac{\text{Cost}}{\text{number of shirts}}=\dfrac{500+6x+0.01x^2}{x}$.

(b)

x	10	20	50	100	200	500	1000
Average cost	\$56.10	\$31.20	\$16.50	\$12.00	\$10.50	\$12.00	\$16.50

99.

x	2.80	2.90	2.95	2.99	2.999	3	3.001	3.01	3.05	3.10	3.20
$\dfrac{x^2-9}{x-3}$	5.80	5.90	5.95	5.99	5.999	?	6.001	6.01	6.05	6.10	6.20

From the table, we see that the expression $\dfrac{x^2-9}{x-3}$ approaches 6 as x approaches 3. We simplify the expression: $\dfrac{x^2-9}{x-3}=\dfrac{(x-3)(x+3)}{x-3}=x+3$, $x\neq 3$. Clearly as x approaches 3, $x+3$ approaches 6. This explains the result in the table.

100. No, squaring $\dfrac{2}{\sqrt{x}}$ changes its value by a factor of $\dfrac{2}{\sqrt{x}}$.