# 2 FUNCTIONS

**2.1** *What is a Function?*

**1.** Express the rule in function notation.

Multiply by $4$, then subtract $2$.

(a) $f(x) = 2x + 4$
(b) $f(x) = -18x$
(c) $f(x) = 4x - 2$
(d) $f(x) = 4(x + 2)$
(e) $f(x) = 4(x - 2)$

Answer: (c)

**2.** Express the rule in function notation.

Square, subtract $5$, then take the square root.

(a) $f(x) = \sqrt{x^2 - 5}$
(b) $f(x) = x - \sqrt{5}$
(c) $f(x) = x^2 - 5$
(d) $f(x) = \sqrt{5x^2}$
(e) $f(x) = \sqrt{-5x^2}$

Answer: (a)

**3.** Complete the table for the function $f(x) = \left| 3 - 2x^2 \right|$.

| $x$ | $f(x)$ |
|---|---|
| -2 | |
| 0 | |
| 1 | |
| 3 | |
| 5 | |

Answer:

| $x$ | $f(x)$ |
|---|---|
| -2 | 5 |
| 0 | 3 |
| 1 | 1 |
| 3 | 15 |
| 5 | 47 |

**4.** If $f(x) = 3x^2 + x - 2$, find $f(0)$, $f(1)$, $f(-1)$.

Answer: $f(0) = 3 \cdot 0^2 + 0 - 2 = -2$ , $f(1) = 3 \cdot 1^2 + 1 - 2 = 2$ , $f(-1) = 3(-1)^2 + (-1) - 2 = 0$ .

**5.** If $h(t)=t-2/t$, find $h(-2)$, $h(1)$, $h(2)$.

Answer: $h(-2)=-2-2/(-2)=-2+1=-1$, $h(1)=1-2/1=-1$, $h(2)=2-2/2=1$

**6.** If $f(x)=-4x+3$, find $f(-3)$, $f(-1)$, $f(2)$.

Answer: $f(-3)=-4(-3)+3=15$, $f(-1)=-4(-1)+3=7$, $f(2)=-4\cdot 2+3=-5$

**7.** If $f(x)=2x-1$, find $f(0)$, $f(-1)$, $f(2)$, $f(a)$, $f(2a-1)$, and $f\left(\frac{1}{a+1}\right)$

Answer: $f(0)=-1$, $f(-1)=-3$, $f(2)=3$, $f(a)=2a-1$, $f(2a-1)=4a-3$,
$f\left(\frac{1}{a+1}\right)=\frac{2}{a+1}-1=\frac{1-a}{a+1}$

**8.** If $f(x)=(x-2)^2+5$, find $f(1)$, $f(2)$, $f\left(\frac{5}{2}\right)$, $f(a)$, $f(1/a)$, and $f(a^2)$.

Answer: $f(1)=6$, $f(2)=(2-2)^2+5=5$, $f\left(\frac{5}{2}\right)=\left(\frac{5}{2}-2\right)^2+5=\frac{21}{4}$, $f(a)=(a-2)^2+5=9-4a+a^2$,
$f\left(\frac{1}{a}\right)=\left(\frac{1}{a}-2\right)^2+5=\frac{1-4a+9a^2}{a^2}$, $f(a^2)=(a^2-2)^2+5=a^4-4a^2+9$

**9.** If $g(x)=\frac{1}{x}+x^2$, find $g(-3)$, $g(-1)$, $g(1)$, $g(3)$, $g(a+1)$, and $g(a^2)$.

Answer: $g(-3)=\frac{1}{-3}+(-3)^2=\frac{26}{3}$, $g(-1)=\frac{1}{-1}+(-1)^2=0$, $g(1)=\frac{1}{1}+1^2=2$, $g(3)=\frac{1}{3}+3^2=\frac{28}{3}$,
$g(a+1)=\frac{1}{a+1}+(a+1)^2=\frac{a^3+3a^2+3a+2}{a+1}$, $g(a^2)=\frac{1}{a^2}+(a^2)^2=\frac{1+a^6}{a^2}$

**10.** If $h(t)=\frac{|t-1|}{t-1}$, find $h(-2)$, $h(-1)$, $h(0)$, $h(2)$, and $h(a+1)$.

Answer: $h(-2)=\frac{|-2-1|}{-2-1}=-1$, $h(-1)=\frac{|-1-1|}{-1-1}=-1$, $h(0)=\frac{|0-1|}{0-1}=-1$, $h(2)=\frac{|2-1|}{2-1}=1$,
$h(a+1)=\frac{|a+1-1|}{a+1-1}=\frac{|a|}{a}=\begin{cases}-1 & \text{if } a<0\\ 1 & \text{if } a>0\end{cases}$ ( $a\neq 0$ ).

**11.** Evaluate $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(5)$ for the piecewise-defined function.

$$f(x)=\begin{cases}3x^2 & \text{if } x<0\\ 2x+1 & \text{if } x\geq 0\end{cases}$$

Answer: $f(-2)=12$, $f(-1)=3$, $f(0)=1$, $f(1)=3$, $f(5)=11$

**12.** For the function $f(x)=2x^2-1$, find $f(x+1)$ and $f(x)+1$.

Answer: $f(x+1)=2(x+1)^2-1=2x^2+4x+1$, $f(x)+1=2x^2$

**13.** For the function $f(x)=5x-15$, find $f\left(\frac{x}{5}\right)$, and $\frac{f(x)}{5}$.

Answer: $f\left(\frac{x}{5}\right)=x-15$, $\frac{f(x)}{5}=x-3$

**14.** For the function $f(x)=\frac{2}{x-1}$, find $f(a)$, $f(a+h)$, and $\frac{f(a+h)-f(a)}{h}$, $h\neq 0$.

Answer: $\frac{f(a+h)-f(a)}{h}=\frac{\frac{2}{a+h-1}-\frac{2}{a-1}}{h}=\frac{\frac{2a-2-2a-2h+2}{(a+h-1)(a-1)}}{h}=\frac{-2}{(a+h-1)(a-1)}$

**15.** Caitlin kicks a soccer ball from the ground at an angle of $60°$ to the horizontal with an initial velocity of $35$ ft/s. The height $H$ of the soccer ball in feet is given by the function $H(x)=\sqrt{3}x-\frac{32x^2}{1225}$, where $x$ is the horizontal distance that the ball has traveled.

(a) Find $H(10)$ and $H(66.3)$.
(b) What do the answers in part (a) represent?
(c) Make a table of values of $H$ for $x=0, 5, 10, 20.5$, and $60$. Round all answers to one decimal place.

Answer: (a) $H(10)\approx 14.7$ feet, $H(66.3)\approx 0.0087\approx 0$ feet
(b) The height of the ball after traveling $10$ feet horizontally is about $14.7$ ft. The ball hits the ground after traveling about $66.3$ ft horizontally.
(c)

| $x$ | $H(x)$ |
|---|---|
| 0 | 0 |
| 5 | 8.0 |
| 10 | 14.7 |
| 20 | 24.2 |
| 60 | 9.9 |

**16.** The cost $C$ in dollars to manufacture $x$ integrated circuits is given by the function $C(x)=11834+0.3x+4.56x^3$, where $x$ is measured in hundreds.

(a) Find $C(0.05)$ and $C(5)$.
(b) What do the answers in part (a) represent?

Answer:(a) $C(0.05)=11,834$, $C(5)=12,406$ (b) It costs $\$11,834$ to produce $5$ integrated circuits, and $\$12,406$ to produce $500$ integrated circuits.

**17.** Find the domain of the function $f(x) = 5x^2 - 5,\ \ 0 \le x \le 5$.

Answer: $[0,5]$

**18.** Find the domain of the function $f(x) = \dfrac{x+1}{x^2+1}$.

Answer: There is no value of $x$ that makes the denominator $0$, thus the domain is all real numbers $(-\infty, \infty)$.

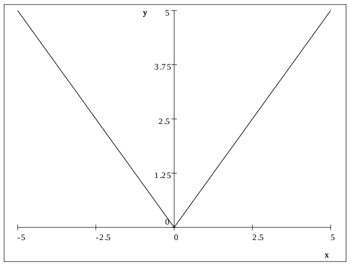
**19.** Find the domain of the function $h(x) = \sqrt{2x^2 + 3x + 1}$.

Answer: We need the radicand to be non-negative, so $2x^2 + 3x + 1 \ge 0 \ \Leftrightarrow\ (x+1)(2x+1) \ge 0$.

Case (1): $x+1 \ge 0$ and $(2x+1) \ge 0 \ \Leftrightarrow\ x \ge -1$ and $x \ge -\frac{1}{2} \ \Rightarrow\ x \ge -\frac{1}{2}$

Case (2): $x+1 \le 0$ and $(2x+1) \le 0 \ \Leftrightarrow\ x \le -1$ and $x \le -\frac{1}{2} \ \Rightarrow\ x \le -1$. The domain of the function is $(-\infty, -1] \cup [-\frac{1}{2}, \infty)$.

**1.** Determine whether the given curve is the graph of a function of $x$. If it is, state the domain and range of the function.




(a) Function, domain: $[0,\infty)$, range: $(-\infty,\infty)$

(b) Function, domain: $(-\infty,\infty)$, range: $[0,\infty)$

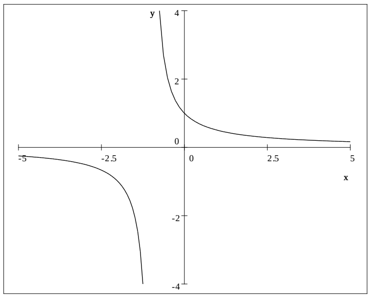
(c) Function, domain: $(0,\infty)$, range: $(-\infty,\infty)$

(d) Function, domain: $(-\infty,\infty)$ , range: $(-\infty,\infty)$

(e) Not a function

Answer: (b) Function, domain: $(-\infty,\infty)$, range: $[0,\infty)$

**2.** Determine whether the given curve is the graph of a function of $x$. If it is, state the domain and range of the function.




(a) Function, domain: $(-\infty,-1)\cup(-1,\infty)$, range: $(-\infty,0]$

(b) Function, domain: $(-\infty,\infty)$, range: $(-\infty,-1)\cup(-1,\infty)$

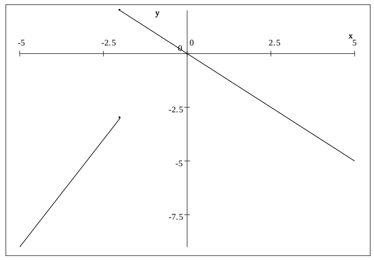
(c) Function, domain: $(-\infty,\infty)$, range: $(-\infty,\infty)$

(d) Function, domain: $(-\infty,-1)\cup(-1,\infty)$, range: $(-\infty,0)\cup(0,\infty)$

(e) Not a function

Answer: (d) Function, domain: $(-\infty,-1)\cup(-1,\infty)$, range: $(-\infty,0)\cup(0,\infty)$

**3.** Determine whether the given curve is the graph of a function of $x$. If it is, state the domain and range of the function.




(a) Function, domain: $(-\infty,-2)\cup(-2,\infty)$, range: $[2,\infty)$

(b) Function, domain: $(-\infty,-2)\cup(2,\infty)$, range: $[-2,\infty)$

(c) Function, domain: $(-\infty,0)$, range: $[2,\infty)$

(d) Function, domain: $(-\infty,-2)\cup(0,\infty)$, range: $[-2,\infty)$

(e) Not a function

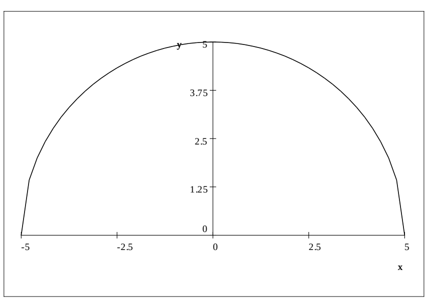
Answer: (e) The graph is not that of a function because it fails the vertical line test.

**4.** Let $f(x)=\sqrt{25-x^2}$

(a) Sketch the graph of $f$.

(b) Find the domain and range of $f$.

Answer:(a)

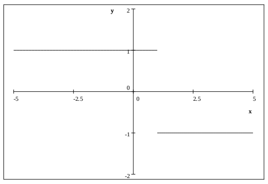



(b) Domain: $[-5,5]$, Range: $[0,5]$

**5.** Let $f(x) = \frac{|x-1|}{1-x}$.

(a) Sketch the graph of $f$.
(b) Find the domain of $f$.

Answer:(a)

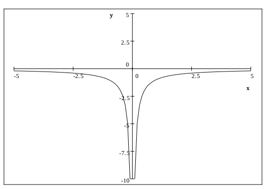


(b) Domain: $(-\infty,1)\cup(1,\infty)$

**6.** Let $f(x) = -\frac{|x|}{x^2}$.

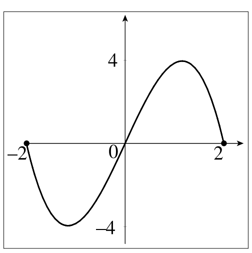
(a) Sketch the graph of $f$.
(b) Find the domain and the range of $f$.

Answer:(a)



(b) Domain $(-\infty,0)\cup(0,\infty)$, range: $(-\infty,0)$

**7.** State whether the curve is the graph of a function. If it is, state the domain and range of the function.



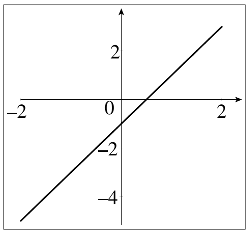

(a) Function, domain $[-2,2)$, range $(-4,4)$
(b) Function, domain $[-2,2]$, range $[-4,4]$
(c) Function, domain $(-2,2)$, range $(-4,4)$
(d) Function, domain $(-4,4)$, range $(-2,2)$
(e) Not a function

Answer: (b)

**8.** Let $f(x) = 2x - 1$.

(a) Sketch the graph of $f$.
(b) Find the domain of $f$.
(c) State the intervals on which $f$ is increasing and on which $f$ is decreasing.

Answer:(a)




(b) Domain $(-\infty,\infty)$ (c) $f$ is increasing on $(-\infty,\infty)$.

**9.** Determine if the equation $2x^2 + 3y = 4x + 5$ defines $y$ as a function of $x$. Explain your answer.

Answer: $2x^2 + 3y = 4x + 5 \Leftrightarrow 3y = 5 + 4x - 2x^2 \Leftrightarrow y = \frac{5 + 4x - 2x^2}{3}$. The last equation is a rule that gives one value of $y$ for each value of $x$.

**10.** Determine if the equation $x^2 + y^2 - 25 = 0$ defines $y$ as a function of $x$. Explain your answer.

Answer: $x^2 + y^2 - 25 = 0 \Leftrightarrow y^2 = 25 - x^2 \Leftrightarrow y = \pm\sqrt{25 - x^2}$. No. This equation gives two values of $y$ for a given value of $x$.

**11.** Determine if the equation $2x^2y - y - 5 = 0$ defines $y$ as a function of $x$. Explain your answer.

Answer: $2x^2y - y - 5 = 0 \Leftrightarrow y\left(2x^2 - 1\right) = 5 \Leftrightarrow y = \frac{5}{2x^2 - 1}$. Yes. The last equation is a rule that gives one value of $y$ for each value of $x$.

**12.** Determine if the equation $\sqrt{y} + x = 1$ defines $y$ as a function of $x$. Explain your answer.

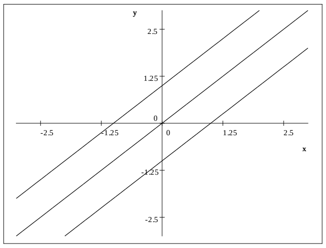
Answer: $\sqrt{y} + x = 1 \Leftrightarrow \sqrt{y} = 1 - x \Leftrightarrow y = (1 - x)^2$. Yes. The last equation is a rule that gives one value of $y$ for each value of $x$.

**13.** Determine if the equation $3x - |y| = 0$ defines $y$ as a function of $x$. Explain your answer.

Answer: $3x - |y| = 0 \Leftrightarrow |y| = 3x \Leftrightarrow y = 3x$ or $y = -3x$. No. Since this equation gives two values of $y$ for a given value of $x$, the given equation does not define a function of $x$.

**14.** Let $f(x) = x + c$. Graph the family of functions with $c = -1, 0$, and $-1$ in the viewing rectangle $[-3,3]$ by $[-3,3]$. How does the value of $c$ affect the graph?

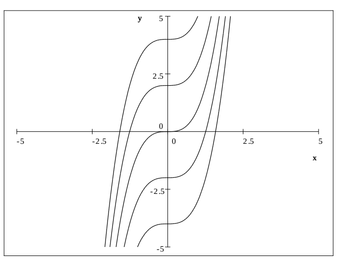
Answer:




For values of $c < 0$ the graph of the function $x$ is moved $c$ units down. For values of $c > 0$, the graph is moved $c$ units up.

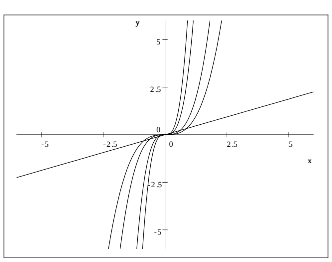
**15.** Let $f(x) = x^3 + c$. Graph the family of functions with $c = -4$, $-2$, $0$, $2$, and $4$ in the viewing rectangle $[-5,5]$ by $[-5,5]$. How does the value of $c$ affect the graph?

Answer: For values of $c < 0$ the graph of the function $x^3$ is moved $c$ units down. For values of $c > 0$, the graph is moved $c$ units up.



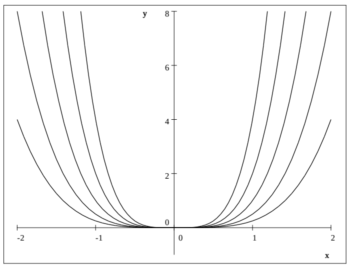

**16.** Let $f(x) = cx^3$. Graph the family of functions with $c = \frac{1}{8}$, $\frac{1}{2}$, $1$, $2$, and $8$ in the viewing rectangle $[-6,6]$ by $[-6,6]$. How does the value of $c$ affect the graph?

Answer: For values $0 < c < 1$, the graph of $cx^3$ is not as steep as for values of $c > 1$.



**17.** Let $f(x)=cx^4$. Graph the family of functions with $c=\frac{1}{4},\frac{1}{2},1,2$, and $4$ in the viewing rectangle $[-2,2]$ by $[-1,8]$. How does the value of $c$ affect the graph?

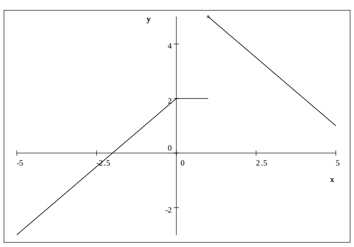
Answer: For $0<c<1$, the graph opens out more than if $c>1$.



**18.** Sketch the graph of the piecewise-defined function.

$$f(x)=\begin{cases} x+2 & \text{if } x<0 \\ 2 & \text{if } 0\le x\le 1 \\ -x+6 & \text{if } 1<x \end{cases}$$

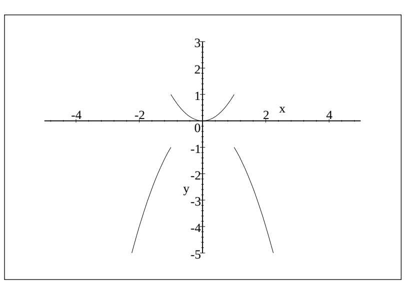
Answer:



**19.** Sketch the graph of the piecewise-defined function.

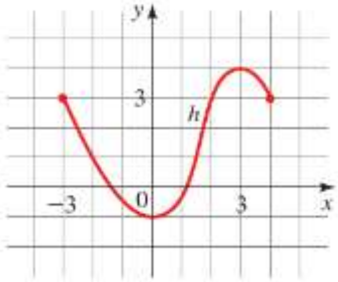
$$f(x)=\begin{cases} x^2 & \text{if } |x|\le 1 \\ -x^2 & \text{if } |x|>1 \end{cases}$$

Answer:



**1.** The graph of a function $h$ is given.

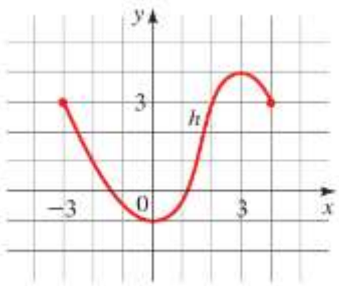
**(a)** Find $h(-3), h(-2), h(2)$, and $h(4)$
**(b)** Find the domain and range of $h$.
**(c)** Find the values of $x$ for which $h(x) = 3$
**(d)** Find the values of $x$ for which $h(x) \le 3$.




Answer: (a) $h(-3) = 3; h(-2) = 1; h(2) = 3; h(4) = 3$; (b) Domain $[-3, 4]$, Range $[-1, 4]$; (c) $-3, 2, 4$; (d) $-3 \le x \le 2$

**2.** The graph of a function $h$ is given.

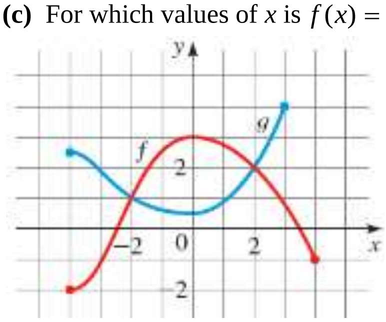
**(a)** Find $h(-3), h(-2), h(0)$, and $h(3)$
**(b)** Find the domain and range of $h$.
**(c)** Find the values of $x$ for which $h(x) = 3$
**(d)** Find the values of $x$ for which $h(x) \le 3$.




Answer: (a) $h(-3) = 3; h(-2) = 1; h(0) = -1; h(3) = 4$; (b) Domain $[-3, 4]$, Range $[-1, 4]$; (c) $-3, 2, 4$; (d) $-3 \le x \le 2$

**3.** Graphs of the functions f and g are given.

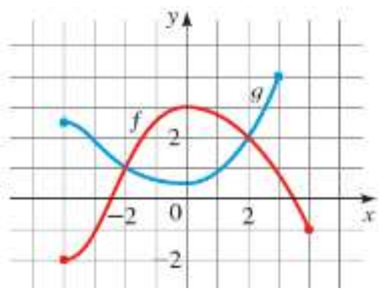
**(a)** Which is larger, $f(0)$ or $g(0)$?
**(b)** Which is larger, $f(-1)$ or $g(-1)$?
**(c)** For which values of $x$ is $f(x) = g(x)$?




Answer: (a) $f(0)$ (b) $f(-1)$ (c) $-2, 2$

**4.** Graphs of the functions f and g are given.

**(a)** Which is larger, $f(-3)$ or $g(3)$?
**(b)** Which is larger, $f(-1)$ or $g(-1)$?
**(c)** For which values of $x$ is $f(x) = g(x)$?




Answer: (a) $g(3)$ (b) $f(-1)$ (c) $-2, 2$

**5.** A function is given. Use a graphing calculator to draw the graph of $f$. Find the domain and range of $f$ from the graph.

$f(x) = 5, \quad 1 \le x \le 3$

Answer: Domain: $[1,3]$, Range $\{5\}$

**6.** A function is given. Use a graphing calculator to draw the graph of $f$. Find the domain and range of $f$ from the graph.

$f(x) = x^2, \quad -3 \le x \le 5$

Answer: Domain: $[-3,5]$, Range $[0,25]$

**7.** A function is given. Use a graphing calculator to draw the graph of $f$. Find the domain and range of $f$ from the graph.

$f(x) = \sqrt{9 - x^2}$

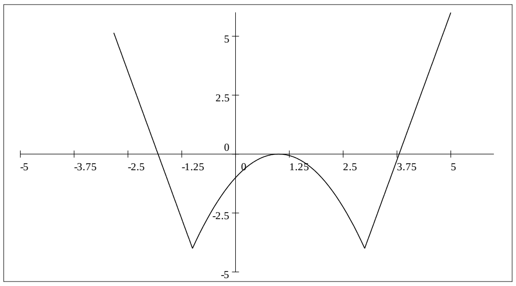
Answer: Domain: $[-3,3]$, Range $[0,3]$

**8.** A function is given. Use a graphing calculator to draw the graph of $f$. Find the domain and range of $f$ from the graph.

$f(x) = -\sqrt{16 - x^2}$

Answer: Domain: $[-4,4]$, Range $[-4,0]$

**9.** The graph of a function is given. Find the intervals on which the function is increasing and decreasing.

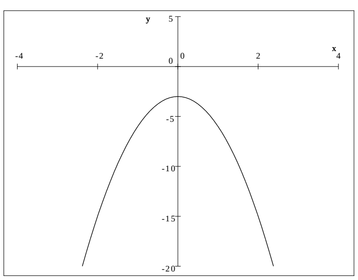



(a) Increasing on $[-1,1]$; $[3,\infty)$; decreasing on $(-\infty,-1]$, $[1,3]$
(b) Increasing on $[-1,1]$; decreasing on $(-\infty,-1]$
(c) Increasing on $[-4,1]$; decreasing on $[0,-4]$
(d) Increasing on $[-4,1]$; decreasing on $[-4,0]$
(e) Increasing on $[-1,1]$, $[3,\infty)$; decreasing on $[6,-4]$, $[-4,0]$

Answer (a) Increasing on $[-1,1]$, $[3,\infty)$; decreasing on $(-\infty,-1]$, $[1,3]$

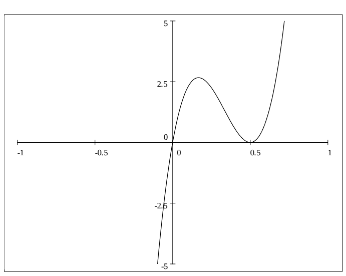
**10.** Use a graphing device to draw the graph of the function $f(x) = -3 - 3x^2$. State approximately the intervals on which the function is increasing and on which the function is decreasing.

Answer: $f$ is increasing on $(-\infty,0)$ and decreasing on $(0,\infty)$.



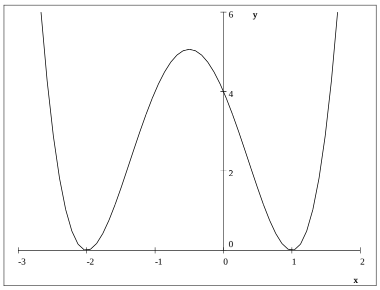

**11.** Use a graphing device to draw the graph of the function $f(x) = 144x^3 - 144x^2 + 36x$. State approximately the intervals on which the function is increasing and on which the function is decreasing.

Answer: $f$ is increasing on $\left(-\infty,\frac{1}{6}\right)$ and $\left(\frac{1}{2},\infty\right)$, and decreasing on $\left(\frac{1}{6},\frac{1}{2}\right)$.



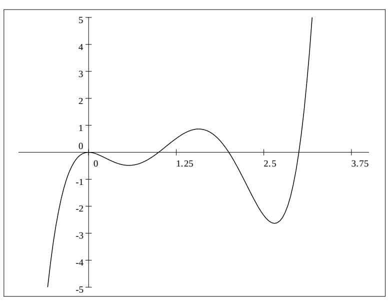
**12.** Use a graphing device to the graph of the function $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. State approximately the intervals on which the function is increasing and on which the function is decreasing.

Answer: $f$ is increasing on $\left(-2, -\frac{1}{2}\right)$ and $(1, \infty)$, and decreasing on $(-\infty, -2)$ and $\left(-\frac{1}{2}, -1\right)$.




**13.** Use a graphing device to draw the graph of the function $f(x) = x^5 - 6x^4 + 11x^3 - 6x^2$. State approximately the intervals on which the function is increasing and on which the function is decreasing.

Answer: $f$ is increasing on $(-\infty, 0)$, $(0.5, 1.5)$, and $(2.5, \infty)$, and decreasing on $(0, 0.5)$, and $(1.5, 2.5)$.




**14.** A function is given. **(a)** Find all the local maximum and minimum values of the function and the value of $x$ at which each occurs. **(b)** Find the intervals on which the function is increasing and on which the function is decreasing. State all answers correct to two decimal places.

$U(x) = 2\left(x^3 - x\right)$

Answer: **(a)** local maximum $\approx 0.77$ when $x \approx -0.58$; local minimum $\approx -0.77$ when $x \approx 0.58$
**(b)** increasing on $(-\infty, -0.58] \cup [0.58, \infty)$; decreasing on $[-0.58, 0.58]$

**15.** A function is given. **(a)** Find all the local maximum and minimum values of the function and the value of $x$ at which each occurs. **(b)** Find the intervals on which the function is increasing and on which the function is decreasing. State all answers correct to two decimal places.

$U(x) = 4\left(x^3 - x\right)$

Answer: **(a)** local maximum $\approx 1.54$ when $x \approx -0.58$; local minimum $\approx -1.54$ when $x \approx 0.58$ **(b)** increasing on $(-\infty, -0.58] \cup [0.58, \infty)$; decreasing on $[-0.58, 0.58]$

**16.** A function is given. **(a)** Find all the local maximum and minimum values of the function and the value of *x* at which each occurs. **(b)** Find the intervals on which the function is increasing and on which the function is decreasing. State all answers correct to two decimal places.

$$G(x) = \frac{2}{x^2 + x + 1}$$

Answer: **(a)** local maximum $\approx 2.67$ when $x \approx -0.50$; no local minimum **(b)** increasing on $(-\infty, -0.50]$ decreasing on $[-0.50, \infty)$

**17.** A function is given. **(a)** Find all the local maximum and minimum values of the function and the value of *x* at which each occurs. **(b)** Find the intervals on which the function is increasing and on which the function is decreasing. State all answers correct to two decimal places.

$$G(x) = \frac{3}{x^2 + x + 1}$$

Answer: **(a)** local maximum $\approx 4.00$ when $x \approx -0.50$; no local minimum **(b)** increasing on $(-\infty, -0.50]$ decreasing on $[-0.50, \infty)$

**1.** For the function $f(x)=2x-2$ determine the average rate of change between the values $x=5$ and $x=6$.

Answer: Average rate of change $=\dfrac{f(6)-f(5)}{6-5}=\dfrac{10-8}{6-5}=2$

**2.** For the function $f(x)=4+\frac{1}{2}x$ determine the average rate of change between the values $x=0$ and $x=6$ .

Answer: Average rate of change $=\dfrac{f(6)-f(0)}{6-0}=\dfrac{7-4}{6}=\dfrac{1}{2}$

**3.** For the function $f(x)=2x^2-x$ determine the average rate of change between the values $x=-1$ and $x=0$.

Answer: Average rate of change $=\dfrac{f(0)-f(-1)}{0-(-1)}=-3$

**4.** For the function $f(x)=4x^2-x^3+5$ determine the average rate of change between the values $x=0$ and $x=10$.

Answer: Average rate of change $=\dfrac{f(10)-f(0)}{10-0}=\dfrac{4(10)^2-(10)^3+5-5}{10}=-60$

**5.** For the function $f(x)=\frac{2}{3}x^4-\frac{1}{3}x^5-x+3$ determine the average rate of change between the values $x=-1$ and $x=1$.

Answer: $\dfrac{f(1)-f(-1)}{1-(-1)}=\dfrac{\frac{2}{3}(1)^4-\frac{1}{3}(1)^5-(1)+3-\left[\frac{2}{3}(-1)^4-\frac{1}{3}(-1)^5-(-1)+3\right]}{2}=-\dfrac{4}{3}$

**6.** For the function $f(t)=2t^2-t$ determine the average rate of change between the values $t=2$ and $t=2+h$ ( $h\neq 0$ ).

Answer: Average rate of change $=\dfrac{f(2+h)-f(2)}{2+h-2}=\dfrac{2(2+h)^2-8-h}{h}=7+2h$

**7.** For the function $g(t)=\dfrac{1}{3t-2}$ determine the average rate of change between the values $t=0$ and $t=a+1$.

Answer: Average rate of change $=\dfrac{g(a+1)-g(0)}{a+1-0}=\dfrac{\dfrac{1}{3a+1}+\dfrac{1}{2}}{a+1}=\dfrac{\dfrac{3}{2}\dfrac{a+1}{3a+1}}{a+1}=\dfrac{3}{2(3a+1)}$

**8.** For the function $f(t)=\frac{2}{t}-2$ determine the average rate of change between the values $t=a$ and $t=a+h$.

Answer: Average rate of change $=\frac{f(a+h)-f(a)}{a+h-a}=\frac{\frac{2}{a+h}-2-\left(\frac{2}{a}-2\right)}{h}=\frac{\frac{2}{a+h}-\frac{2}{a}}{h}=-\frac{2}{(a+h)a}$

**9.** For the function $f(x)=\sqrt{4x}$ determine the average rate of change between the values $x=a$ and $x=a+h$.

Answer: Average rate of change $=\frac{f(a+h)-f(a)}{a+h-a}=\frac{\sqrt{4(a+h)}-\sqrt{4a}}{h}=\frac{2\sqrt{(a+h)}-2\sqrt{a}}{h}$

**10.** For the linear function $f(x)=-\frac{1}{2}x+1$, find the average rate of change between $x=a$ and $x=a+h$. Show that the average rate of change is the same as the slope of the line.

Answer: The slope of the line $y=-\frac{1}{2}x+1$ is $m=-\frac{1}{2}$. The average rate of change for the function is

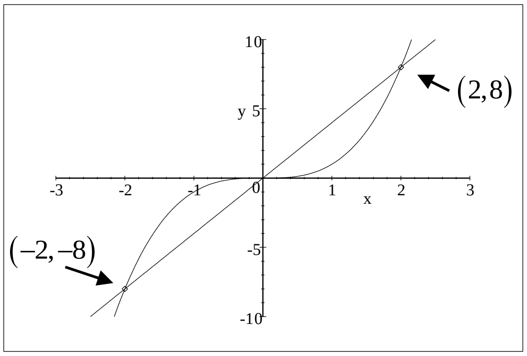
$$\frac{\text{Change in } y}{\text{Change in } x}=\frac{f(a+h)-f(a)}{a+h-a}=\frac{-\frac{1}{2}(a+h)+1-\left[-\frac{1}{2}a+1\right]}{h}=\frac{-\frac{1}{2}a-\frac{1}{2}h+1+\left[\frac{1}{2}a-1\right]}{h}=\frac{-\frac{1}{2}h}{h}=-\frac{1}{2}$$

**11.** For the linear function $f(x)=5-5x$, find the average rate of change between $x=a$ and $x=a+h$. Show that the average rate of change is the same as the slope of the line.

Answer: The slope of the line $y=5-5x$ is $m=-5$. The average rate of change for the function is

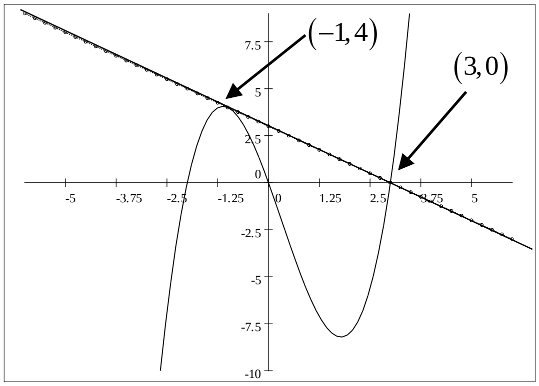
$$\frac{\text{Change in } y}{\text{Change in } x}=\frac{f(a+h)-f(a)}{a+h-a}=\frac{5-5(a+h)-[5-5(a)]}{h}=\frac{5-5a-5h-5+5a}{h}=\frac{-5h}{h}=-5$$

**12.** The graph of a function is given. Determine the average rate of change of the function between the indicated points on the graph.



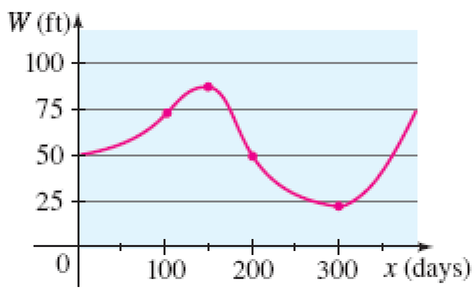

Answer: The average rate of change for the function between the points $(-2,-8)$ and $(2,8)$ is 4 .

**13.** The graph of a function is given. Determine the average rate of change of the function between the indicated points on the graph.



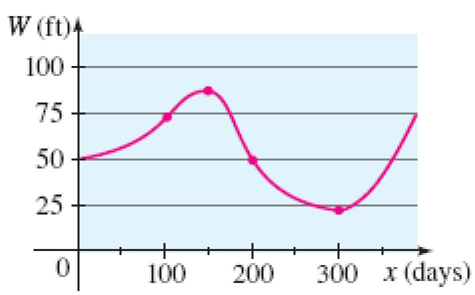

Answer: The average rate of change for the function between the points $(-1,4)$ and $(3,0)$ is $\dfrac{0-4}{3-(-1)}=-1$

**14.** The graph shows the depth of water $W$ in a reservoir over a one-year period as a function of the number of days $x$ since the beginning of the year. What was the average rate of change of $W$ between $x$ = 0 and $x$= 100?



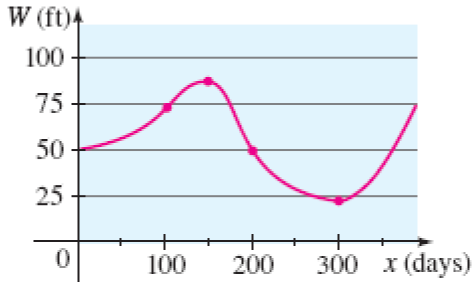

Answer: $\dfrac{75-50}{100-0}=\dfrac{25}{100}=\dfrac{1}{4}$ ft/day

**15.** The graph shows the depth of water $W$ in a reservoir over a one-year period as a function of the number of days $x$ since the beginning of the year. What was the average rate of change of $W$ between $x$ = 100 and $x$= 200?




Answer: $\dfrac{50-75}{200-100}=\dfrac{-25}{100}=-\dfrac{1}{4}$ ft/day

**16.** The graph shows the depth of water $W$ in a reservoir over a one-year period as a function of the number of days $x$ since the beginning of the year. Estimate the average rate of change of $W$ between $x = 200$ and $x = 300$.




Answer: $\frac{25-50}{300-200} = \frac{-25}{100} = -\frac{1}{4}$ ft/day

**17.** A man is running around a circular track that is 200 m in circumference. An observer uses a stopwatch to record the runner's time at the end of each lap, obtaining the data in the following table.

What was the man's average speed (rate) between 68 s and 203 s? Round the answer to two decimal places.

| Time (s) | Distance (m) |
|---|---|
| 32 | 200 |
| 68 | 400 |
| 108 | 600 |
| 152 | 800 |
| 203 | 1000 |
| 263 | 1200 |
| 335 | 1400 |
| 412 | 1600 |

Answer: 4.44 m/s

**18.** A man is running around a circular track that is 200 m in circumference. An observer uses a stopwatch to record the runner's time at the end of each lap, obtaining the data in the following table.

What was the man's average speed (rate) between 108 s and 203 s? Round the answer to two decimal places.

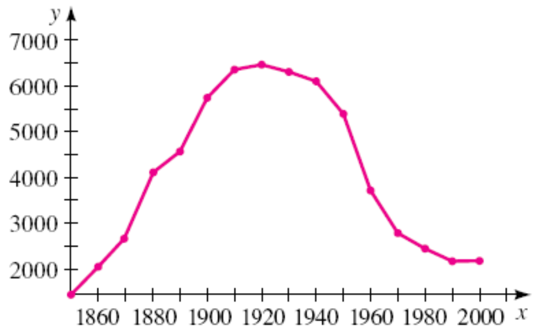
| Time (s) | Distance (m) |
|---|---|
| 32 | 200 |
| 68 | 400 |
| 108 | 600 |
| 152 | 800 |
| 203 | 1000 |
| 263 | 1200 |
| 335 | 1400 |
| 412 | 1600 |

Answer: 4.21 m/s

**19.** The graph gives the number of farms in the United States from 1850 to 2000.

Estimate the average rate of change in the number of farms between the following years.

(i) 1860 and 1890
(ii) 1920 and 1980




Answer: (i) $\approx 83$ farms/yr (ii) $\approx -67$ farms/yr

**20.** If an object is dropped from a high cliff or a tall building, then the distance it has fallen after $t$ seconds is given by the function $f(t) = 16t^2$. Find its average speed (average rate of change) over the following intervals:

(i) Between 1 s and 6 s
(ii) Between $t = c$ and $t = c + h$

Answer: (i) $\frac{f(6) - f(1)}{6 - 1} = 112\text{ ft/s}$, (ii) $\frac{f(c+h) - f(c)}{c + h - c} = \frac{16(c+h)^2 - 16c^2}{h} = 32c + 16h$

**21.** If an object is dropped from a high cliff or a tall building, then the distance it has fallen after $t$ seconds is given by the function $f(t) = 16t^2$. Find its average speed (average rate of change) over the following intervals:

(iii) Between 2 s and 7 s
(iv) Between $t = c$ and $t = c + h$

Answer: (i) $\frac{f(7) - f(2)}{7 - 2} = 144\text{ ft/s}$, (ii) $\frac{f(c+h) - f(c)}{c + h - c} = \frac{16(c+h)^2 - 16c^2}{h} = 32c + 16h$

**1.** Given the graph of $f$, describe how the graph of $y=\frac{4}{3}f(x-3)+\frac{4}{3}$ can be obtained from the graph of $f$.

(a) Shift $\frac{4}{3}$ units to the right, then stretching vertically by a factor of $3$, then shift upward $\frac{4}{3}$ units.
(b) Shift $3$ units to the right, then shrinking horizontally by a factor of $\frac{4}{3}$, then shift downward $\frac{4}{3}$ units.
(c) Shift $3$ units to the left, then shrinking vertically by a factor of $\frac{4}{3}$, then shift upward $3$ units.
(d) Shift $3$ units to the right, then stretching vertically by a factor of $\frac{4}{3}$, then shift upward $\frac{4}{3}$ units.
(e) Shift $3$ units to the left, then shrinking vertically by a factor of $3$, then shift upward $\frac{4}{3}$ units.

Answer: (d)

**2.** Given the graph of $f$, describe how the graph of $y=-f(2x)+2$ can be obtained from the graph of $f$.

(a) Stretch horizontally by a factor of $2$, then reflecting about the $y$ - axis, then shifting $2$ units up.
(b) Shrink vertically by a factor of $2$, then reflecting about the $x$ - axis, then shifting $2$ units up.
(c) Stretch horizontally by a factor of $2$, then reflecting about the $y$ - axis, then shifting $2$ units up.
(d) Shrink vertically by a factor of $\frac{1}{2}$, then reflecting about the $x$ - axis, then shifting downward $2$ units.
(e) Shrink horizontally by a factor of $\frac{1}{2}$, then reflecting about the $x$ - axis, then shifting $2$ units up.

Answer: (e)

**3.** Shift the graph of $f(x)=x^2$ by $6$ units to the left and then $4$ units downward, and choose the equation for the final transformed graph.

(a) $f(x)=(x^2+6)-4$
(b) $f(x)=(x+6)^2-4$
(c) $f(x)=(x+4)^2+6$
(d) $f(x)=(x-6)^2-4$
(e) $f(x)=(x-4)^2+6$

Answer: (b)

**4.** Shrink the graph of $f(x)=|x|$ vertically by a factor of $\frac{1}{6}$, then reflect it about the $x$ - axis, then shift it downward $\frac{5}{6}$ units and choose the equation for the final transformed graph.

(a) $f(x)=-\frac{1}{6}|x|+\frac{5}{6}$
(b) $f(x)=\frac{1}{6}\left|x-\frac{5}{6}\right|$
(c) $f(x)=-\frac{1}{6}|x|-\frac{5}{6}$
(d) $f(x)=\left|x-\frac{1}{6}\right|-\frac{5}{6}$
(e) $f(x)=|x|+\frac{2}{3}$

Answer: (c)

**5.** Explain how the graph of $g(x) = -\sqrt[3]{x+1}$ is obtained from the graph of $f = \sqrt[3]{x}$ .

(a) Shift the graph of $f$ one unit to the left, then reflect about the $y$ - axis.
(b) Shift the graph of $f$ one unit to the left, then reflect about the $y$ - axis.
(c) Shift the graph of $f$ one unit up, reflect about the $x$ - axis and stretch vertically by a factor of $\frac{1}{3}$ .
(d) Shift the graph of $f$ one unit to the left, then reflect about the $x$ - axis.
(e) Shift the graph of $f$ one unit down, reflect about the $x$ - axis and stretch vertically by a factor of $\frac{1}{3}$.

Answer: (d)

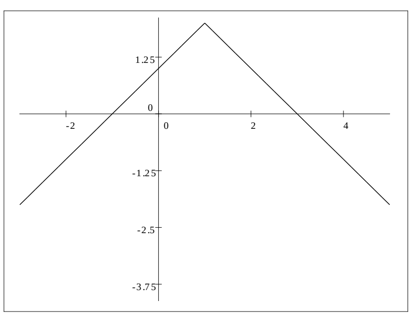
**6.** Explain how the graph of $g(x) = \dfrac{|x+1|-2}{3}$ is obtained from the graph of $f = |x|$ .

(a) Shift the graph of $f$ one unit to the right, then shift downward $\frac{2}{3}$ units.
(b) Shift the graph of $f$ one unit to the right, then shift downward $2$ units.
(c) Shift the graph of $f$ one unit to the left, then shrink vertically by a factor of $\frac{2}{3}$ .
(d) Shift the graph of $f$ one unit to the left, then shift upward $\frac{2}{3}$ units.
(e) Shift the graph of $f$ one unit to the left, shrink vertically by a factor of $\frac{1}{3}$ , shift downward $\frac{2}{3}$ units.

Answer: (e)

**7.** Sketch the graph of the function $f(x) = 2 - |x-1|$ , not by plotting points, but by starting with the graph of a standard function and applying transformations.

Answer: Shift $y = |x|$ one unit to the right, reflect about the $x$ - axis, then shift upward $2$ units.



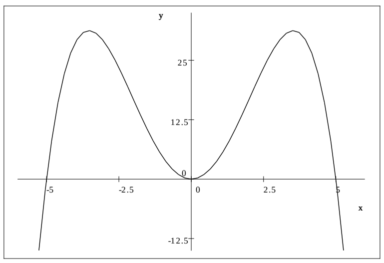
**8.** Graph the given functions on the same screen using the viewing rectangle $[-5,7]$ by $[-10,10]$. How is each part related to the graph in part (a)?

(a) $y=x^2$ (b) $y=\frac{1}{2}x^2$ (c) $y=\frac{1}{2}(x-3)^2$ (d) $y=-\frac{1}{2}(x-3)^2-3$

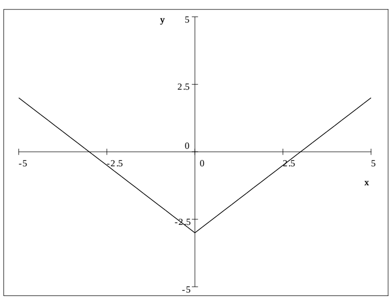
Answer: (b) Shrunk vertically by a factor of $2$. (c) Shifted $3$ units to the right, shrunk vertically by a factor of $2$. (d) Shifted $3$ units to the right, shrunk vertically by a factor of $2$ and reflected about the $x$ - axis, then shifted $3$ units down.

**9.** Determine whether $f(x)=5x^2-\dfrac{x^4}{5}$ is even or odd. If $f$ is even or odd, use symmetry to sketch its graph.

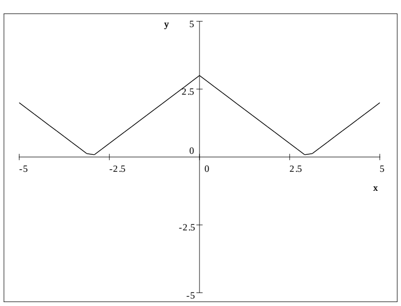
Answer: $f(-x)=5(-x)^2-\dfrac{(-x)^4}{5}=5x^2-\frac{x^4}{5}=f(x)$. So $f(x)=f(-x)$, and $f$ is even but not odd.




**10.** The graph of $f(x)=|x|-3$ is shown. Use this graph to sketch the graph of $g(x)=\left||x|-3\right|$.




Answer: The values of $g(x)$ between $[-3,3]$ must be positive. Hence the graph of $g(x)$ is shown.



**1.** Let $f(x)=2x-1$ and $g(x)=x^2+3x-5$. Find $f+g$, $f/g$ and their domains.

Answer: $(f+g)(x)=x^2+5x-6$, domain is □ ; $\dfrac{f}{g}(x)=\dfrac{2x-1}{x^2+x-6}$, domain is all reals except $-3$ and $2$.

**2.** Let $f(x)=3x+1$ and $g(x)=3x^2-2x+1$. Find $f-g$, $fg$, and their domains.

Answer: $(f-g)(x)=-3x^2+5x$, domain is □ ; $(fg)(x)=9x^3-3x^2+x+1$, domain is □ .

**3.** If $f(x)=2x^2+x$ and $g(x)=3x-1$, find $f+g$, $f-g$, and their domains.

Answer: $(f+g)(x)=2x^2+4x-1$, domain is □ ; $(f-g)(x)=2x^2-2x+1$, domain is □ .

**4.** If $f(x)=2x^2+1$ and $g(x)=x-1$, find $f+g$, $fg$, and their domains.

Answer: $(f+g)(x)=2x^2+1+x-1=2x^2+x$, domain is □ ; $(fg)(x)=2x^3-2x^2+x-1$, domain is □ .

**5.** If $f(x)=\sqrt{2-2x}$ and $g(x)=\sqrt{x^2-1}$, find $f+g$, $fg$, and their domains.

Answer: $(f+g)(x)=\sqrt{2-2x}+\sqrt{x^2-1}$, domain is $(-\infty,-1]$;
$(fg)=\sqrt{2-2x}\sqrt{x^2-1}=\sqrt{2}\sqrt{(1-x)(-1-x)(1-x)}=(1-x)\sqrt{-2(1+x)}$, domain is $(-\infty,-1]$.

**6.** For the functions $f(x)=2x^2-2$ and $g(x)=2x$, find $f+g$, $f-g$, $fg$, $\dfrac{f}{g}$ and their domains.

Answer:

$f+g=2x^2-2+2x=2x^2+2x-2$ domain: $(-\infty,\infty)$

$f-g=2x^2-2-2x=2x^2-2x-2$ domain: $(-\infty,\infty)$

$fg=(2x^2-2)2x=4x^3-4x$ domain: $(-\infty,\infty)$

$\dfrac{f}{g}=\dfrac{2x^2-2}{2x}=\dfrac{x^2-1}{x}$ domain: $(-\infty,0)\cup(0,\infty)$

**7.** For the functions $f(x)=3x^2-x^3$ and $g(x)=x^2-1$, find $f+g$, $f-g$, $fg$, $\dfrac{f}{g}$ and their domains.

Answer:

$f+g=4x^2-x^3-1$ domain: $(-\infty,\infty)$

$f-g=2x^2-x^3+1$ domain: $(-\infty,\infty)$

$fg=(3x^2-x^3)(x^2-1)=3x^4-3x^2-x^5+x^3$ domain: $(-\infty,\infty)$

$f/g=\dfrac{3x^2-x^3}{x^2-1}$ domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

**8.** For the functions $f(x)=\sqrt{x^2+1}$ and $g(x)=\sqrt{25-x^2}$, find $f+g$, $f-g$, $fg$, $\frac{f}{g}$ and their domains.

Answer: $f+g=\sqrt{x^2+1}+\sqrt{25-x^2}$ domain: $[-5,5]$

$f-g=\sqrt{x^2+1}-\sqrt{25-x^2}$ domain: $[-5,5]$

$fg=\sqrt{x^2+1}\sqrt{25-x^2}$ domain: $[-5,5]$

$f/g=\dfrac{\sqrt{x^2+1}}{\sqrt{25-x^2}}$ domain: $(-5,5)$

**9.** For the functions $f(x)=\dfrac{1}{x-2}$ and $g(x)=\dfrac{x}{x-1}$, find $f+g$, $f-g$, $fg$, $\frac{f}{g}$ and their domains.

Answer: $f+g=\dfrac{1}{x-2}+\dfrac{x}{x-1}=\dfrac{x^2-x-1}{(x-2)(x-1)}$ domain: $(-\infty,1)\cup(1,2)\cup(2,\infty)$

$f-g=\dfrac{1}{x-2}-\dfrac{x}{x-1}=-\dfrac{x^2-3x+1}{(x-2)(x-1)}$ domain: $(-\infty,1)\cup(1,2)\cup(2,\infty)$

$fg=\left(\dfrac{1}{x-2}\right)\left(\dfrac{x}{x-1}\right)=\dfrac{x}{(x-1)(x-2)}$ domain: $(-\infty,1)\cup(1,2)\cup(2,\infty)$

$\dfrac{f}{g}=\dfrac{(x-1)}{(x-2)x}$ domain: $(-\infty,0)\cup(0,1)\cup(1,2)\cup(2,\infty)$

**10.** Given $f(x)=2x-2$ and $g(x)=2x^2$, find $(g\circ f)(-1)$.

Answer: $(g\circ f)(-1)=g(f(-1))=g(-4)=32$

**11.** Use $f(x)=x-2$ and $g(x)=4-2x^2$ to evaluate the expression $g(f(0))$.

(a) –3 (b) –2 (c) –4 (d) 0 (e) 1

Answer: (c)

**12.** Use $f(x)=x-4$ and $g(x)=2-x^2$ to evaluate the expression $g(g(1))$.

(a) 1 (b) 3 (c) –1 (d) –3 (e) 0

Answer: (a)

**13.** Use $f(x)=x-3$ and $g(x)=3+x^2$ to evaluate the expression $(g\circ f)(-1)$.

(a) 14 (b) 19 (c) 3 (d) 12 (e) –12

Answer: (b)

**14.** Use $f(x)=3x-2$ and $g(x)=3+2x^2$ to evaluate the expression $(f\circ g)(2)$.

(a) 31 (b) 14 (c) 39 (d) 11 (e) 114

Answer: (a)

**15.** Use $f(x)=x^2-1$ and $g(x)=-\sqrt{x+1}$ to evaluate the expression $(f\circ g)(x)$.

(a) $-1$ (b) $-x^2$ (c) $x^2$ (d) $x$ (e) 0

Answer: (d)

**16.** Given $f(x)=\dfrac{1}{x+2}$ and $g(x)=\dfrac{1}{x-2}$, find $f\circ g$, $g\circ f$, $f\circ f$, and $g\circ g$, and their domains.

Answer: $(f\circ g)(x)=f\left(\dfrac{1}{x-2}\right)=\dfrac{1}{\frac{1}{x-2}+2}=\dfrac{x-2}{2x-3}$, with domain $\{x \mid x\neq \frac{3}{2},2\}$

$(g\circ f)(x)=g\left(\dfrac{1}{x+2}\right)=\dfrac{1}{\frac{1}{x+2}-2}=\dfrac{x+2}{-2x-3}$, with domain $\{x \mid x\neq -2,-\frac{3}{2}\}$

$(f\circ f)(x)=f\left(\dfrac{1}{x+2}\right)=\dfrac{1}{\frac{1}{x+2}+2}=\dfrac{x+2}{2x+5}$, with domain $\{x \mid x\neq -\frac{5}{2},-2\}$

$(g\circ g)(x)=g\left(\dfrac{1}{x-2}\right)=\dfrac{1}{\frac{1}{x-2}-2}=\dfrac{x-2}{-2x+5}$, with domain $\{x \mid x\neq 2,\frac{5}{2}\}$

**17.** Given $f(x)=\dfrac{1}{x}$, $g(x)=\dfrac{x}{x+1}$, and $h(x)=\dfrac{x+1}{x}$, find $f\circ g\circ h$.

Answer: $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\dfrac{x+1}{x}\right)\right)=f\left(\dfrac{x+1}{2x+1}\right)=\dfrac{2x+1}{x+1}$

**18.** Express the function $F(x)=3\sqrt{x}+2$ in the form $f\circ g$.

Answer: If $f(x)=3x+2$ and $g(x)=\sqrt{x}$ then $F(x)=(f\circ g)(x)$.

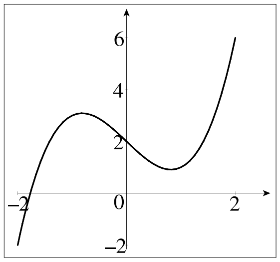
**19.** Express the function $G(x)=\dfrac{1}{\sqrt{x+2}}$ in the form $f\circ g$.

Answer: If $f(x)=\dfrac{1}{\sqrt{x}}$ and $g(x)=x+2$ then $G(x)=(f\circ g)(x)$.

**20.** Express the function $H(x)=\left(2+x^2\right)^3$ in the form $f\circ g$.

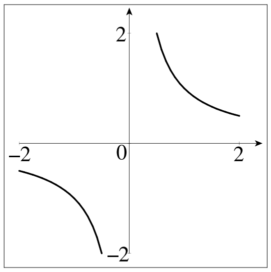
Answer: If $f(x)=x^3$ and $g(x)=2+x^2$ then $H(x)=(f\circ g)(x)$.

**1.** Determine whether or not the function given by the graph is one-to-one.




Answer: No, it fails the horizontal line test.

**2.** Determine whether or not the function given by the graph is one-to-one.




Answer: Yes, it passes the horizontal line test.

**3.** Determine whether or not the function $f(x) = x^2 - 3x + 2$ is one-to-one.

Answer: $f(x) = x^2 - 3x + 2 = (x-2)(x-1)$ , so $f(2) = 0 = f(1)$ , so $f$ is not one-to-one.

**4.** Determine whether or not the function $g(x) = |2x - 4|$ is one-to-one.

Answer: $g(0) = |-4| = 4 = |8 - 4| = g(2)$, so $g$ is not one-to-one.

**5.** Determine whether or not the function $h(x) = 3x^6 + 2$, $0 \le x \le \infty$ is one-to-one.

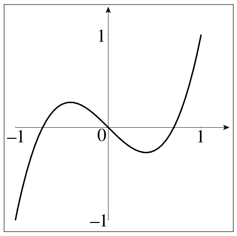
Answer: If $x_1 \ne x_2$ then $x_1^6 \ne x_2^6$ because different positive numbers differ in sixth powers. Thus, $3x_1^6 + 2 \ne 3x_2^6 + 2$ and $h$ is one-to-one.

**6.** Determine whether or not the function $h(x) = x^3 - x$ is one-to-one.

Answer: $h(x) = x^3 - x = x(x-1)(x+1)$. Since $h(-1) = h(0) = h(1) = 0$, $h$ is not one-to-one, or sketch the graph to see that the graph of the function fails the Horizontal Line Test.

**7.** Use a graphing calculator or computer to determine whether or not the function $f(x)=2x^3-x$ is one-to-one.

Answer: Using a graphing calculator and the horizontal line test we see that $f(x)=2x^3-x$ is not one-to-one.




**8.** Determine whether or not the function $g(x)=1-x^2, x\ge 0$ is one-to-one.

Answer: $g(x)=1-x^2$, $x\ge 0$. If $x_1\ne x_2$, then $x_1^2\ne x_2^2$. Two different *positive* numbers cannot have the same square. Thus $1-(x_1)^2\ne 1-(x_2)^2$. Therefore $g$ is one-to-one. (You can also use the Horizontal Line Test to show that no horizontal line intersects the graph of $g(x)=1-x^2$, $x\ge 0$ more than once.)

**9.** Use the Property of Inverse Functions to show that $f(x)=\frac{1}{2}x+1$ and $g(x)=2x-2$ are inverses of each other.

Answer: $f(g(x))=f(2x-2)=\frac{1}{2}(2x-2)+1=x$ and $g(f(x))=g\left(\frac{1}{2}x+1\right)=2\left(\frac{1}{2}x+1\right)-2=x.$

**10.** Use the Property of Inverse Functions to show that $f(x)=\dfrac{3x-5}{5}$ and $g(x)=\frac{5}{3}x+\frac{5}{3}$ are inverses of each other.

Answer: $f(g(x))=f\left(\frac{5}{3}x+\frac{5}{3}\right)=\dfrac{3\left(\frac{5}{3}x+\frac{5}{3}\right)-5}{5}=x$ and $g(f(x))=g\left(\dfrac{3x-5}{5}\right)=\dfrac{5}{3}\left(\dfrac{3x-5}{5}\right)+\dfrac{5}{3}=x$

**11.** Use the Property of Inverse Functions to show that $f(x)=\sqrt{25-x^2}$ , $0\le x\le 5$ and $g(x)=\sqrt{25-x^2}$ , $0\le x\le 5$ are inverses of each other.

Answer: $f(g(x))=\sqrt{25-\left(\sqrt{25-x^2}\right)^2}=x$ and $g(f(x))=\sqrt{25-\left(\sqrt{25-x^2}\right)^2}=x$

**12.** Find the inverse function of $f(x)=2-\dfrac{1}{x}$ , $x\ne 0$.

Answer: $f(x)=2-\dfrac{1}{x} \quad\Rightarrow\quad y=2-\frac{1}{x} \quad\Leftrightarrow\quad x=-\dfrac{1}{y-2}$ so $f^{-1}(x)=-\dfrac{1}{x-2}$

**13.** Find the inverse function of $f(x)=3(x+1)^2,\ x<-1$.

Answer: $y=3(x+1)^2 \Rightarrow \frac{y}{3}=(x+1)^2 \Rightarrow \pm\sqrt{\frac{y}{3}}=x+1$, but $x+1<0$, so $x+1=-\sqrt{\frac{y}{3}} \Rightarrow$
$x=-\sqrt{\frac{y}{3}}-1 \Rightarrow f^{-1}(x)=-\sqrt{\frac{x}{3}}-1,\ x\geq 0$.

**14.** Find the inverse function of $f(x)=x^2-4x+9,\ x\geq 2$.

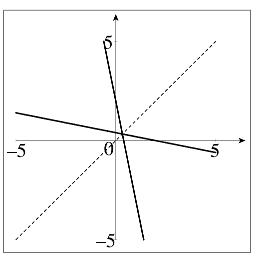
Answer: $f(x)=x^2-4x+9 \Rightarrow y=x^2-4x+9$.

Compete the square and write in standard form. $\Rightarrow y=(x-2)^2+5$.

Solve for $x \Rightarrow x-2=\pm\sqrt{(y-5)}$, but $x-2\geq 0$, so $x=2+\sqrt{(y-5)} \Rightarrow f^{-1}(x)=2+\sqrt{x-5}$.

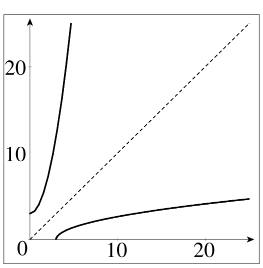
**15.** Let $f(x)=2-5x$. (a) Sketch the graph of $f$ then use it to sketch the graph of $f^{-1}$. (b) Find $f^{-1}$.

Answer: $y=2-5x \Leftrightarrow x=\frac{1}{5}(2-y)$. So $f^{-1}(x)=\frac{1}{5}(2-x)$



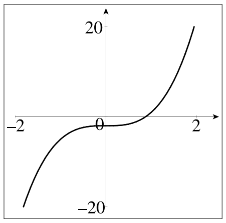

**16.** Let $f(x)=3+x^2,\ 0\leq x\leq 5$ (a) Sketch the graph of $f$ then use it to sketch the graph of $f^{-1}$.
(b) Find $f^{-1}$.

Answer: $y=3+x^2 \Leftrightarrow x=\sqrt{y-3}$. So the inverse function is $f^{-1}(x)=\sqrt{x-3}$.



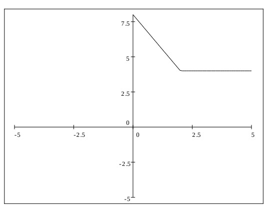
**17.** Use a graphing calculator or computer to draw a graph of $f(x)=3x^3-2$ and determine whether the function is one-to-one.

Answer: From the graph, we see that $f(x)=3x^3-2$ is one-to-one.




**18.** Determine whether or not the function $f(x)=\left||x-2|+|x-6|\right|$ is one-to-one.

Answer: Using a graphing calculator and the Horizontal Line Test, we see that $f(x)=\left||x-2|+|x-6|\right|$ is not one-to-one.



**19.** If $\$10,000$ is invested for $x$ years at $5\%$ simple interest, the total amount $A$ of the investment is given by the function $A(x)=500x+10,000$.(a) Find $A^{-1}$. (b) Find $A^{-1}(16,500)$. What does your answer represent?

Answer: (a) Let $y=A(x) \Rightarrow y=500x+10,000 \Rightarrow x=\dfrac{y-10,000}{500} \Rightarrow A^{-1}=\dfrac{x-10,000}{500}$.

(b) $A^{-1}(16,500)=\dfrac{16,500}{500}-20=13$ years, the number of years the investment took to achieve $\$16,500$ from an initial investment of $\$10,000$.

**20.** Suppose that the total cost $C$ (in thousands of dollars) to produce $x$ units (in millions) of a particular product is given by the function $C(x)=\dfrac{x^2}{10}+100$. (a) Find $C^{-1}$. (b) Find $C^{-1}(350)$. What does your answer represent?

Answer: (a) Let $C(x)=y=\dfrac{x^2}{10}+100 \Rightarrow \dfrac{x^2}{10}=100-y \Rightarrow x^2=1000-10y \Rightarrow x=\pm\sqrt{10y-1000}$, but $x\ge 0$, so $x=\sqrt{10y-1000} \Rightarrow C^{-1}(x)=\sqrt{10x-1000}$, the number of units produced as a function of total cost. (b) $C^{-1}(350)=\sqrt{10(350)-1000}=50$. Therefore 50 million units are produced when the total cost is $\$350,000$.