1. Differentiate.

$$y = \frac{\sin x}{6 + \cos x}$$

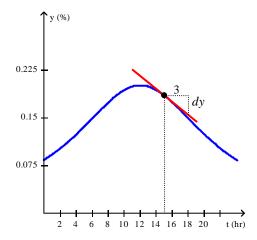
2. Find the limit.

$$\lim_{\theta \to 0} 4 \frac{\sin(\sin 4\theta)}{\sec 4\theta}$$

3. Differentiate.

$$y = \frac{\sin x}{3 + \cos x}$$

4. The graph shows the percentage of households in a certain city watching television during a 24-hr period on a weekday (t = 0 corresponds to 6 a.m.). By computing the slope of the respective tangent line, estimate the rate of change of the percentage of households watching television at a-12 p.m. Note that dy = 0.03



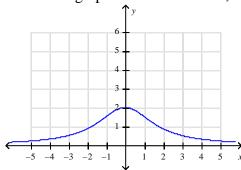
- 5. Suppose the total cost in maunufacturing x units of a certain product is C(x) dollars.
 - **a.** What does C'(x) measure? Give units.
 - **b.** What can you say about the sign of C'?
 - **c.** Given that C'(3000) = 11, estimate the additional cost in producing the 3001st unit of the product.

6. The level of nitrogen dioxide present on a certain June day in downtown Megapolis is approximated by

$$A(t) = 0.03t^3(t-7)^4 + 64.8$$
 $0 \le t \le 7$

where A(t) is measured in pollutant standard index and t is measured in hours with t = 0 corresponding to 7 a.m. What is the average level of nitrogen dioxide in the atmosphere from 1 a.m. to 2 p.m. on that day? Round to three decimal places.

7. Sketch the graph of the derivative f' of the function f whose graph is given.



- 8. Let $f(x) = x |x^3|$.
 - **a.** Sketch the graph of f.
 - **b.** For what values of x is f differentiable?
 - **c.** Find a formula for f'(x).
- 9. Suppose that f and g are functions that are differentiable at x = 1 and that f(1) = 1, f'(1) = -3, g(1) = 2, and g'(1) = 5. Find h'(1).

$$h(x) = \frac{xf(x)}{x + g(x)}$$

10. Find the derivative of the function.

$$f(x) = -x^2 + x + 2$$

11. Identify the "inside function" u = f(x) and the "outside function" y = g(u). Then find dy/dx using the Chain Rule.

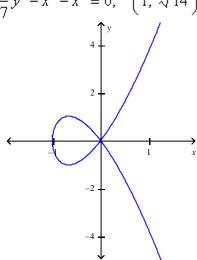
$$y = \sqrt{x^2 - 2}$$

12. Find the derivative of the function.

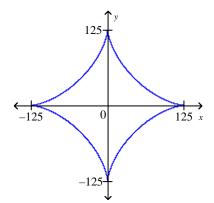
$$f(x) = x \sin^8 x$$

13. Find an equation of the tangent line to the given curve at the indicated point.

$$\frac{1}{7}y^2 - x^3 - x^2 = 0; \quad \left(1, \sqrt{14}\right)$$

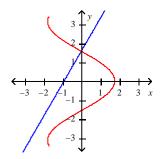


14. The curve with the equation $x^{2/3} + y^{2/3} = 25$ is called an asteroid. Find an equation of the tangent to the curve at the point $(48\sqrt{6}, 1)$.



15. Two curves are said to be **orthogonal** if their tangent lines are perpendicular at each point of intersection of the curves. Show that the curves of the given equations are orthogonal.

$$y - \frac{7}{4}x = \frac{\pi}{2}, \quad x = \frac{7}{4} \cos y$$

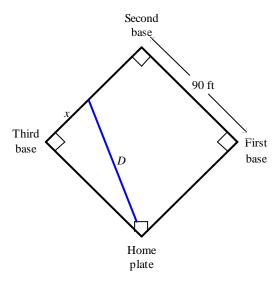


16. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{4t}{t^2 + 1}; \qquad t = 3$$

- 17. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.
- 18. The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose that the radius and height of the cone are changing with respect to time t.
 - **a.** Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.
 - **b.** At a certain instant of time, the radius and height of the cone are 12 in. and 13 in. and are increasing at the rate of 0.2 in./sec and 0.5 in./sec, respectively. How fast is the volume of the cone increasing?
- 19. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest

20. The sides of a square baseball diamond are 90 ft long. When a player who is between the second and third base is 30 ft from second base and heading toward third base at a speed of 24 ft/sec, how fast is the distance between the player and home plate changing? Round to two decimal places.

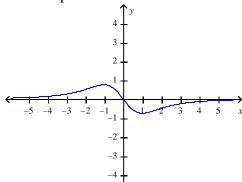


Answer Key

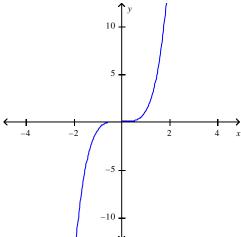
$$1. \quad \frac{dy}{dx} = \frac{6\cos x + 1}{\left(6 + \cos x\right)^2}$$

3.
$$\frac{dy}{dx} = \frac{3\cos x + 1}{(3 + \cos x)^2}$$

- 4. Falling at 1%/hr
- 5. **a.** C'(x), measured in dollars per unit, gives the instantaneous rate of changes of the total manufacturing cost C when x units of a certain product are produced.
 - **b.** Positive
 - **c.** \$11
- 6. 153.037 pollutant standard index



- 7.
- 8. **a.**



b.
$$x \in (-\infty, \infty)$$

b.
$$x \in (-\infty, \infty)$$
c. $f'(x) = \begin{cases} -4x^3 & \text{if } x < 0 \end{cases}$

9.
$$-\frac{4}{3}$$

10.
$$-2x + 1$$

11.
$$u = x^{2} - 2$$

$$y = \sqrt{u}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^{2} - 2}}$$

12.
$$\sin^8 x + 8x \cos x \sin^7 x$$

13.
$$y = \frac{5\sqrt{14}}{4}x - \frac{\sqrt{14}}{4}$$

14.
$$y = -\frac{\sqrt{6}}{12}x + 25$$

15. The curves intersect at
$$\left(0, \frac{\pi}{2}\right)$$
.

For
$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $m = \frac{7}{4}$.

For
$$x = \frac{7}{4} \cos y$$
, $m = -\frac{4}{7} \csc y$; at $\left(0, \frac{\pi}{2}\right)$, $m = -\frac{4}{7}$.

16.
$$\frac{6}{5}$$
 ft, $-\frac{8}{25}$ ft/sec, $\frac{8}{25}$ ft/sec

17.
$$20.9 \text{ ft}^2/\text{sec}$$

18. **a.**
$$\frac{dV}{dt} = \frac{\pi}{3} \left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt} \right)$$

b.
$$44.8\pi \, \text{in.}^{3}/\text{sec}$$

1. The position function of a particle is given by

$$s = t^3 - 10.5t^2 - 2t, t \le 0$$

When does the particle reach a velocity of 22 m/s?

- 2. Find an equation of the tangent line to the graph of $f(x) = 2x^2 7$ at the point (3, 11).
- 3. Find $\frac{dy}{dx}$ by implicit differentiation.

$$8\sqrt{x} + \sqrt{y} = 8$$

4. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{4t}{t^2 + 1};$$
 $t = 3$

- 5. The circumference of a sphere was measured to be 86 cm with a possible error of 0.8 cm. Use differentials to estimate the maximum error in the calculated volume.
- 6. If a cylindrical tank holds 10000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after *t* minutes as

$$V(t) = 10000 \left(1 - \frac{1}{60} t \right)^2, 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

- 7. Suppose the total cost in maunufacturing x units of a certain product is C(x) dollars.
 - **a.** What does C'(x) measure? Give units.
 - **b.** What can you say about the sign of C'?
 - **c.** Given that C'(3000) = 11, estimate the additional cost in producing the 3001st unit of the product.
- 8. Find the derivative of the function.

$$f(x) = -x^2 + x + 2$$

9. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = \frac{3t}{t^2 + 1}; \qquad t = 2$$

10. s(t) is the position of a body moving along a coordinate line, where $t \ge 0$, and s(t) is measured in feet and t in seconds.

$$s(t) = -3 + 2t - t^2$$

- **a.** Determine the time(s) and the position(s) when the body is stationary.
- **b.** When is the body moving in the positive direction? In the negative direction?
- **c.** Sketch a schematic showing the position of the body at any time t.
- 11. Find the equation of the tangent to the curve at the given point.

$$y = \sqrt{16 + 4\sin x}$$
, (0,4)

12. Find the rate of change of y with respect to x at the given values of x and y.

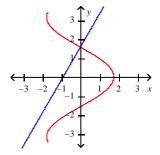
$$2xy^2 - 5x^2y + 192 = 0;$$
 $x = 4, y = 4$

13. Find an equation of the tangent line to the curve

$$xe^{y} + x + 2y = 2$$
 at $(1, 0)$.

14. Two curves are said to be **orthogonal** if their tangent lines are perpendicular at each point of intersection of the curves. Show that the curves of the given equations are orthogonal.

$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $x = \frac{7}{4} \cos y$



15. A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the radius r when r = 1 ft.

16. s(t) is the position of a body moving along a coordinate line; s(t) is measured in feet and t in seconds, where $t \ge 0$. Find the position, velocity, and speed of the body at the indicated time.

$$s(t) = t^{10} e^{-t};$$
 $t = 1$

17. Find the differential of the function at the indicated number.

$$f(x) = e^{7x} + \ln(x+8); x = 0$$

18. Two chemicals react to form another chemical. Suppose that the amount of chemical formed in time *t* (in hours) is given by

$$x(t) = \frac{11\left[1 - \left(\frac{2}{3}\right)^{3t}\right]}{1 - \frac{1}{4}\left(\frac{2}{3}\right)^{3t}}$$

where x(t) is measured in pounds.

- a. Find the rate at which the chemical is formed when t = 4. Round to two decimal places.
- b. How many pounds of the chemical are formed eventually?

19. The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose that the radius and height of the cone are changing with respect to time t.

a. Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.

b. At a certain instant of time, the radius and height of the cone are 12 in. and 13 in. and are increasing at the rate of 0.2 in./sec and 0.5 in./sec, respectively. How fast is the volume of the cone increasing?

20. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the polluted area is circular, determine how fast the area is increasing when the radius of the circle is 20 ft and is increasing at the rate of $\frac{1}{6}$ ft/sec. Round to the nearest tenth if necessary.

Answer Key

2.
$$y = 12x - 25$$

$$3. -\frac{8\sqrt{y}}{\sqrt{x}}$$

4.
$$\frac{6}{5}$$
 ft, $-\frac{8}{25}$ ft/sec, $\frac{8}{25}$ ft/sec

6.
$$V'(t) = \frac{-1000}{3} + \frac{50t}{9}$$

- 7. **a.** C'(x), measured in dollars per unit, gives the instantaneous rate of changes of the total manufacturing cost C when x units of a certain product are produced.
 - **b.** Positive

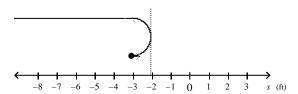
8.
$$-2x + 1$$

9.
$$\frac{6}{5}$$
 ft, $-\frac{9}{25}$ ft/sec, $\frac{9}{25}$ ft/sec

10. **a.**
$$s(1) = -2$$

b. Positive when
$$0 < t < 1$$
, negative when $t > 1$

c.



11.
$$y = \frac{1}{2}x + 4$$

13.
$$y = -\frac{2}{3}x + \frac{2}{3}$$

14. The curves intersect at
$$\left(0, \frac{\pi}{2}\right)$$
.

For
$$y - \frac{7}{4}x = \frac{\pi}{2}$$
, $m = \frac{7}{4}$.

For
$$x = \frac{7}{4} \cos y$$
, $m = -\frac{4}{7} \csc y$; at $\left[0, \frac{\pi}{2}\right]$, $m = -\frac{4}{7}$.

15.
$$8\pi$$

16.
$$\frac{1}{e}$$
 ft, $\frac{9}{e}$ ft/sec, $\frac{9}{e}$ ft/sec

17.
$$\frac{57}{8}dx$$

18. a. 0.08 lb/hr, b. 11 lbs

19. **a.**
$$\frac{dV}{dt} = \frac{\pi}{3} \left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt} \right)$$

b.
$$44.8\pi \, \text{in.}^3/\text{sec}$$
 20. $20.9 \, \text{ft}^2/\text{sec}$

Select the correct answer for each question.

- 1. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.0017 cm thick to a hemispherical dome with diameter 70 m.
 - a. 4.165π
 - b. 2.52π
 - c. 4.11π
 - d. 3.82π
 - e. 2.28π
- 2. Determine the values of x for which the given linear approximation is accurate to within 0.07 at a = 0.

 $tan x \approx x$

a.
$$-0.19 < x < 0.28$$

b.
$$-0.57 < x < 0.57$$

c.
$$0.06 < x < 0.68$$

d.
$$-1.04 < x < 1.55$$

e.
$$-0.71 < x < 0.48$$

3. Find the differential of the function at the indicated number.

$$f(x) = 13\sin x + 4\cos x; \quad x = \frac{\pi}{4}$$

a.
$$\frac{9\sqrt{2}}{2} dx$$

b.
$$-\frac{17\sqrt{2}}{2} dx$$

c.
$$\frac{17\sqrt{2}}{2} dx$$

d.
$$-\frac{9\sqrt{2}}{2} dx$$

4. Find the linearization *L* (*x*) of the function at *a*.

$$f(x) = x^{2/3}$$
; $a = 64$

a.
$$\frac{8}{3}x + \frac{464}{3}$$

b.
$$\frac{8}{3}x + \frac{512}{3}$$

c.
$$\frac{8}{3}x - \frac{512}{3}$$

d.
$$\frac{8}{3}x - \frac{464}{3}$$

5. The slope of the tangent line to the graph of the exponential function
$$y = 6^x$$
 at the point (0, 1) is

$$\lim_{x \to 0} \frac{6^x - 1}{x}$$
. Estimate the slope to three decimal places.

6. If
$$g(x) = \sqrt{2-3x}$$
, use the definition of derivative to find $g'(x)$.

a.
$$g'(x) = -\frac{1}{2}(2-3x)^{-1/2}$$

b.
$$g'(x) = -(2-3x)^{-1/2}$$

c.
$$g'(x) = -\frac{3}{2}(2-3x)^{1/2}$$

d.
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

7. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 4, f'(14) = 15, and f'(2) = 13.

Find F'(14).

- a. 140
- b. 20
- c. 24
- d. 52
- e. 17

8. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

- a. f'(x) = 4g(x)
- b. $f'(x) = 4[g(x)]^3 g'(x)$
- c. $f'(x) = 4 \left[g'(x) \right]^3$
- d. f'(x) = 4[gx][xg'+g]
- e. f'(x) = 4g'(x)

9. Find the point(s) on the graph of f where the tangent line is horizontal.

$$f(x) = x^2 e^{-x}$$

- a. $(0,0), \left[2, \frac{2^2}{e^2}\right]$
- b. $\left(1, \frac{1}{e}\right)$
- c. (0,0)d. $\left[2, \frac{2^2}{e^2}\right]$

10. Find the derivative of the function.

$$f(x) = (4x + 9)^9$$

- a. $36(4x+9)^8$
- b. $9(4x+9)^8$
- c. $9x(4x+9)^8$ d. $36x(4x+9)^8$

11. Find an equation of the tangent line to the curve $120(x^2 + y^2)^2 = 2312(x^2 - y^2)$ at the point (4,1).

- a. y = -1.11x + 17
- b. y = -1.11x + 3.43
- c. y = 1.11x + 5.43
- d. y = -1.11x + 5.43
- e. None of these

12. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is

$$S = 4x^2$$
.

Find the linear density when x is 3 m.

- a. 20
- b. 24
- c. 18
- d. 12
- e. 4

 13.	In an adiabatic process (one in which no heat transfer takes place), the pressure P and volume V of
	an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
,

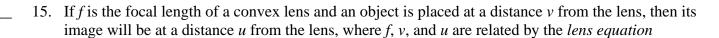
where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

c.
$$C - \frac{1}{14}$$
 L/sec

d.
$$\frac{1}{14}$$
 L/sec

$$O(t) = t^3 - 3t^2 + 4t + 3$$
.

Find the current when t = 1s.



$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

Find the rate of change of v with respect to u.

a.
$$\frac{dv}{du} = -\frac{f}{\left(u - f\right)^2}$$

b.
$$\frac{dv}{du} = -\frac{f^2}{u - f}$$

c.
$$\frac{dv}{du} = \frac{2f^2}{\left(u - f\right)^2}$$

$$\frac{\mathrm{d} v}{\mathrm{d} u} = \frac{f^2}{\left(u - f\right)^2}$$

e.
$$\frac{dv}{du} = -\frac{f^2}{\left(u - f\right)^2}$$

16. Find the instantaneous rate of change of the function $f(x) = \sqrt{3x}$ when x = 3.

a.
$$\frac{1}{3}$$

d.
$$\frac{1}{2}$$

17. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 1.5 m from the wall, it slides away from the wall at a rate of 0.3 m/s. How long is the ladder?

_____ 18. Find equations of the tangent lines to the curve $y = \frac{x - 10}{x + 10}$ that are parallel to the line x - y = 10.

a.
$$x - y = -4.5$$

b.
$$x - y = -2$$

c.
$$x - y = -12.5$$

d.
$$x - y = -19.75$$

e.
$$x - y = -15$$

____ 19. If $f(x) = 6\cos x + \sin^2 x$, find f'(x) and f''(x).

a.
$$f''(x) = -6\cos(2x) + 2\cos(x)$$

b.
$$f'(x) = -6\sin(x) + \sin(2x)$$

c.
$$f'(x) = -6\sin(2x) + \sin(x)$$

d.
$$f''(x) = -6\cos(x) + 2\cos(2x)$$

e.
$$f''(x) = -2\cos(2x) + 6\cos(x)$$

____ 20. Find equations of the tangent lines to the curve $y = \frac{x-8}{x+8}$ that are parallel to the line x-y=8.

a.
$$x - y = -18.5$$

b.
$$x - y = -4.5$$

c.
$$x - y = -1.5$$

d.
$$x - y = -12.5$$

e.
$$x - y = -15$$

Answer Key

- 1. A
- 2. B
- 3. A
- 4. D
- 5. D
- 6. D
- 7. D
- 8. B
- 9. A
- 10. A
- 11. D
- 12. B
- 13. D
- 14. E
- 15. E
- 16. D
- 17. D
- 18. B, D
- 19. B, D
- 20. A, C

Select the correct answer for each question.

1. Find the differential of the function at the indicated number.

$$f(x) = 13\sin x + 4\cos x; \quad x = \frac{\pi}{4}$$

a.
$$9\sqrt{2}$$
 dx

b.
$$-\frac{17\sqrt{2}}{2} dx$$

c.
$$\frac{17\sqrt{2}}{2} dx$$

d.
$$-\frac{9\sqrt{2}}{2} dx$$

2. The cost (in dollars) of producing x units of a certain commodity is

$$C(x) = 4,280 + 13x + 0.03x^2$$
.

Find the average rate of change with respect to x when the production level is changed from x = 102 to x = 122.

3. If $g(x) = \sqrt{2-3x}$, use the definition of derivative to find g'(x).

a.
$$g'(x) = -\frac{1}{2}(2-3x)^{-1/2}$$

b.
$$g'(x) = -(2-3x)^{-1/2}$$

c.
$$g'(x) = -\frac{3}{2}(2-3x)^{1/2}$$

d.
$$g'(x) = -\frac{3}{2}(2-3x)^{-1/2}$$

- e. None of these
- 4. Differentiate.

$$K(x) = \left(3x^5 + 1\right)\left(x^6 - 4x\right)$$

a.
$$15x^4(x^6-4x)+(3x^5+1)(6x^5-4)$$

b.
$$(x^6 - 4x) + (3x^5 + 1)$$

c.
$$15x^4(6x^5-4)+(3x^5+1)(x^6-4x)$$

d.
$$15x^4(6x^5)+(3x^5)(x^6-4x)$$

e.
$$(3x^5+1)(x^6-4x)+15x^4(6x^5-4)+1$$

5. Find f' in terms of g'.

$$f(x) = x^7 g(x)$$

a.
$$f'(x) = 7x^6 f'(x) + x^7 g'(x)$$

b.
$$f'(x) = 7x^6 g'(x)$$

c.
$$f'(x) = 7x^6 g(x) + 7x^7 g'(x)$$

d.
$$f'(x) = 7x^6g(x) + x^7g'(x)$$

e.
$$f'(x) = 7x^6 + g'(x)$$

____ 6. Find the derivative of the function.

$$f(x) = 0.2x^{-17}$$

a.
$$-\frac{0.34}{x^{0.7}}$$

b.
$$-\frac{0.34}{x^{2.7}}$$

c.
$$-0.34x^{27}$$

d.
$$-0.34x^{07}$$

7. Find the derivative of the function.

$$f(x) = \frac{2\sqrt{x}}{x^2 + 9}$$

a.
$$\frac{-3x^2+9}{\sqrt{x}(x^2+9)^2}$$

b.
$$\frac{1}{2x\sqrt{x}\left(x^2+9\right)}$$

c.
$$\frac{1}{2x\sqrt{x}}$$

$$d. \quad \frac{-3x^2 + 9}{\sqrt{x}\left(x^2 + 9\right)}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

a.
$$\frac{63x^3 + 135x^2 + 49x + 9}{7(7x + 1)^2}$$

b.
$$\frac{63x^3 + 135x^2 + 49x + 9}{7(7x+1)}$$

c.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)^2}$$

d.
$$\frac{126x^3 + 20x^2 - 2x + 16}{(7x+1)}$$

9. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.

a.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$$

b.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) + x^2 f'(x)$$

c.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 2x^2 f(x) - x^3 f'(x)$$

d.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^2 f(x) - x^3 f'(x)$$

e.
$$\frac{d}{dx}\left(x^3f(x)\right) = 3x^3f(x) - x^2f'(x)$$

10. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 19$ where the tangent is horizontal.

a.
$$(-3, 100), (2, -25)$$

11. If $f(t) = \sqrt{9t+1}$, find f''(5).

12. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y\sin 3x = x\cos 3y, \left(\frac{\pi}{3}, \frac{\pi}{6}\right)$$

a.
$$y = \frac{x}{2}$$

b.
$$y = 2x - \frac{3\pi}{3}$$

c.
$$y = -\frac{x}{2} + \frac{\pi}{2}$$

d.
$$y = \frac{x}{6}$$

e.
$$y = \frac{x}{3} + \frac{\pi}{6}$$

13. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

- a. $y' = \frac{1 y^3 3x^2y}{3xy^2 + x^3 3}$
- b. $y' = \frac{1 y^3 2x^3}{2x^2 + x^2 x^2}$
- c. $y' = \frac{-y^4 3xy}{4xy^3 + x^2}$
- d. $y' = \frac{xy^2 + 2x 3}{x^2y^2(3x 1)}$
- e. none of these
- 14. Find the derivative of the function.

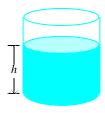
$$y = 3\cos^{-1}\left(\sin^{-1}t\right)$$

- a. $y' = -\frac{3}{\sqrt{\left(1 t^2\right)\left(1 \left(\sin^{-1}(t)\right)^2\right)}}$ b. $y' = -\frac{3}{\sqrt{\left(1 t^2\right)\left(1 \sin^{-1}(t)\right)}}$
- c. $y' = -\frac{3}{\sqrt{(1-t^2)}}$
- d. $y' = -\frac{3}{\sqrt{1 (\sin^{-1}(t))^2}}$
- e. $y' = -\frac{3}{\sqrt{\left(1+t^2\right)\left(1+\left(\sin^{-1}(t)\right)^2\right)}}$

15. Water flows from a tank of constant cross-sectional area 50 ft² through an orifice of constant cross-sectional area $\frac{1}{4}$ ft² located at the bottom of the tank. Initially, the height of the water in the tank was 20 ft, and t sec later it was given by the equation

$$2\sqrt{h} + \frac{1}{25}t - 2\sqrt{20} = 0 \qquad 0 \le t \le 50\sqrt{20}$$

How fast was the height of the water decreasing when its height was 2 ft?



a.
$$100\sqrt{5} - 50\sqrt{2}$$
 ft/sec
b. $100\sqrt{5} - 50\sqrt{2}$ ft/sec

b.
$$100\sqrt{5} - 50\sqrt{2}$$
 ft/sec

c.
$$\frac{2}{25}$$
 ft/sec

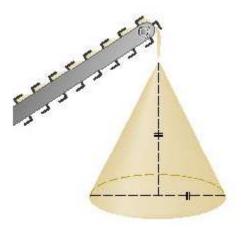
d.
$$\frac{\sqrt{2}}{25}$$
ft/sec

16. The mass of part of a wire is $x(1+\sqrt{x})$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when x = 36m.

- a. 6 kg/m
- b. 4 kg/m
- c. 9 kg/m
- d. $1.5 \, \text{kg/m}$
- e. None of these

- _____ 17. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.
 - a. ≈ 476 mi/h
 - b. ≈670 mi/h
 - c. ≈ 455 mi/h
 - d. $\approx 570 \, \text{mi/h}$
 - e. ≈ 495 mi/h
 - 18. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.
 - a. $1.145 \,\mathrm{m}^2/\mathrm{s}$
 - b. $-0.955 \,\mathrm{m}^2/\mathrm{s}$
 - c. $0.090 \,\mathrm{m}^2/\mathrm{s}$
 - d. $5.045 \,\mathrm{m}^2/\mathrm{s}$
 - e. $-1.955 \,\mathrm{m}^2/\mathrm{s}$

19. Gravel is being dumped from a conveyor belt at a rate of 34 ft/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 13 ft high? Round the result to the nearest hundredth.



- a. 0.6 ft/min
- b. 0.26 ft/min
- c. 0.14 ft/min
- d. 0.27 ft/min
- e. 1.24 ft/min

20. A point moves along the curve $3y + y^2 - 8x = 2$. When the point is at $\left(-\frac{1}{2}, -1\right)$, its x-coordinate is increasing at the rate of 3 units per second. How fast is its y-coordinate changing at that instant of time?

- a. 24 units/sec
- b. 26 units/sec
- c. -24 units/sec
- d. -22 units/sec

Answer Key

- 1. A
- 2. E
- 3. D
- 4. A
- 5. D
- 6. B
- 7. A
- 8. C
- 9. A
- 10. A
- 11. A
- 12. A
- 13. A
- 14. A
- 15. D
- 16. C
- 17. A
- 18. C
- 19. B
- 20. A

- 1. Use the linear approximation of the function $f(x) = \sqrt{9-x}$ at a = 0 to approximate the number $\sqrt{9.08}$.
- 2. Compute $\triangle y$ and dy for the given values of x and $dx = \triangle x$.

$$v = x^2$$
, $x = 1$, $\triangle x = 0.5$

3. If the tangent line to y = f(x) at (8, 4) passes through the point (4, -32), find f'(8). Select the correct answer.

a.
$$f'(8) = 29$$

b.
$$f'(8) = 19$$

c.
$$f'(8) = 9$$

d.
$$f'(8) = 34$$

e.
$$f'(8) = -9$$

- 4. If $g(x) = \sqrt{8-7x}$, find the domain of g'(x).
- 5. Differentiate.

$$K(x) = \left(3x^5 + 1\right)\left(x^6 - 4x\right)$$

6. Use the Product Rule to find the derivative of the function. Select the correct answer.

$$f(x) = (4x+5)\left(x^2 - 8\right)$$

b.
$$2x + 4$$

c.
$$12x^2 + 10x - 32$$

d. $8x^2 - 40$

d.
$$8x^2 - 40$$

7. Use the Quotient Rule to find the derivative of the function.

$$P(t) = \frac{1-t}{7-8t}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

9. Find f''(x).

$$f(x) = (2x)^5 - (7x)^2 + 5$$

10. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

11. Find f' in terms of g'.

$$f(x) = x^5 g(x)$$

Select the correct answer.

a.
$$f'(x) = 5x^4 + g'(x)$$

b.
$$f'(x) = x^5 g(x) + 5x^5 g'(x)$$

c.
$$f'(x) = 5x^4 g(x) + x^5 g'(x)$$

d.
$$f'(x) = 5x^4 g'(x)$$

e.
$$f'(x) = 5x f'(x) + 5xg'(x)$$

12. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 5, f'(14) = 15, and f'(2) = 16.

Find
$$F'(14)$$
.

13. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

14. Find the derivative of the function.

$$y = 3\cos^{-1}\left(\sin^{-1}t\right)$$

15. Find an equation of the tangent line to the curve $120(x^2 + y^2)^2 = 2312(x^2 - y^2)$ at the point (4,1).

Select the correct answer.

a.
$$y = -1.11x + 17$$

b.
$$y = -1.11x + 3.43$$

c.
$$y = 1.11x + 5.43$$

d.
$$y = -1.11x + 5.43$$

- e. None of these
- 16. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is

$$S = 4x^2$$

Find the linear density when x is 3 m.

17. In an adiabatic process (one in which no heat transfer takes place), the pressure *P* and volume *V* of an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
.

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

- 18. Find the instantaneous rate of change of the function $f(x) = \sqrt{3x}$ when x = 3.
- 19. Let *C* (*t*) be the total value of US currency (coins and banknotes) in circulation at time. The table gives values of this function from 1980 to 2000, as of September 30, in billions of dollars. Estimate the value of *C* (1990).

t	1980	1985	1990	1995	2000
C(t)	129.9	176.3	275.9	405.3	568.6

Answers are in billions of dollars per year. Round your answer to two decimal places.

20. A car leaves an intersection traveling west. Its position 4 sec later is 26 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position 4 sec later is 26 ft from the intersection. If the speeds of the cars at that instant of time are 12 ft/sec and 10 ft/sec, respectively, find the rate at which the distance between the two cars is changing. Round to the nearest tenth if necessary.

Select the correct answer.

- a. 15.6 ft/sec
- b. 3.7 ft/sec
- c. 3.1 ft/sec
- d. 36.8 ft/sec

Answer Key

- 1. 3.0133
- 2. $\triangle y = 1.25$, dy = 1
- 3. C
- 4. $\left(-\infty, \frac{8}{7}\right)$
- 5. $15x^4(x^6-4x)+(3x^5+1)(6x^5-4)$
- 6. C
- 7. $\frac{1}{(7-8t)^2}$
- 8. $\frac{126x^3 + 20x^2 2x + 16}{(7x+1)^2}$
- 9. $640x^3 98$
- 10. $f'(x) = 4[g(x)]^3 g'(x)$
- 11. C
- 12. 80
- 13. $y' = \frac{1 y^3 3x^2y}{3xy^2 + x^3 3}$
- 14. $y' = -\frac{3}{\sqrt{\left(1 t^2\right)\left(1 \left(\sin^{-1}(t)\right)^2\right)}}$
- 15. D
- 16. 24
- 17. $\frac{1}{14}$ L/sec
- 18. $\frac{1}{2}$
- 19. 22.90
- 20. A

1. Find the differential of the function at the indicated number.

Select the correct answer.

$$f(x) = \sqrt{x^2 + 7}; \quad x = 3$$

- a. $\frac{3}{8} dx$ b. $\frac{3}{4} dx$ c. $\frac{3}{2} dx$
- d. $\frac{1}{8} dx$
- 2. The slope of the tangent line to the graph of the exponential function $y = 6^x$ at the point (0, 1) is $\lim_{x \to 0} \frac{6^x - 1}{x}$. Estimate the slope to three decimal places.
- 3. A turkey is removed from the oven when its temperature reaches 175 °F and is placed on a table in a room where the temperature is 70 °F. After 10 minutes the temperature of the turkey is 161 °F and after 20 minutes it is 149 °F. Use a linear approximation to predict the temperature of the turkey after 30 minutes.
- 4. If $g(x) = \sqrt{8-7x}$, find the domain of g'(x)
- 5. Suppose that F(x) = f(g(x)) and g(14) = 2, g'(14) = 4, f'(14) = 15, and f'(2) = 13. Find F'(14).
- 6. Plot the graph of the function f in an appropriate viewing window.

$$f(x) = \frac{x^4}{x^4 + 1}$$

7. Find the derivative of the function.

Select the correct answer.

$$f(x) = \frac{2\sqrt{x}}{x^2 + 9}$$

a.
$$\frac{-3x^2 + 9}{\sqrt{x}(x^2 + 9)^2}$$

b.
$$\frac{1}{2x\sqrt{x}\left(x^2+9\right)}$$

c.
$$\frac{1}{2x\sqrt{x}}$$

$$\frac{d.}{\sqrt{x}\left(x^2+9\right)}$$

8. Find the derivative of the function.

$$f(x) = \left(x^2 + 1\right) \left(\frac{9x - 1}{7x + 1}\right)$$

- 9. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.
- 10. Find the derivative of the function.

$$g(v) = \sin v - 8v \csc v$$

11. Find f' in terms of g'.

$$f(x) = \left[g(x) \right]^4$$

____ 12. Find the second derivative of the function.

Select the correct answer.

$$f(x) = x \left(3x^2 - 1\right)^4$$

a.
$$4x(3x^2-1)^3$$

b.
$$72x(3x^2-1)^2(9x^2-1)$$

c.
$$12x(3x^2-1)^2$$

d.
$$(27x^2-1)(3x^2-1)^3$$

13. Use implicit differentiation to find an equation of the tangent line to the curve at the indicated point.

$$y = \sin xy^6; \quad \left(\frac{\pi}{2}, 1\right)$$

14. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$$x^7 - y^7 = 1$$

15. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

16. If $f(t) = \sqrt{9t+1}$, find f''(4).

17.	In an adiabatic process (one in which no heat transfer takes place), the pressure P and volume V of
	an ideal gas such as oxygen satisfy the equation

$$P^5V^7=C$$
.

where C is a constant. Suppose that at a certain instant of time, the volume of the gas is 2L, the pressure is 100 kPa, and the pressure is decreasing at the rate of 5 kPa/sec. Find the rate at which the volume is changing.

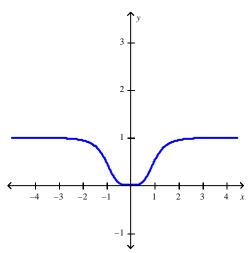
18. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

Select the correct answer.

- a. ≈ 476 mi/h
- b. ≈670 mi/h
- c. ≈ 455 mi/h
- d. ≈ 570 mi/h
- e. ≈ 495 mi/h
- 19. Two sides of a triangle are 2 m and 3 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.
- 20. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 1.5 m from the wall, it slides away from the wall at a rate of 0.3 m/s. How long is the ladder?

Answer Key

- 1. B
- 2. 1.792
- 3. 136°F
- 4. $\left(-\infty, \frac{8}{7}\right)$
- 5. 52
- 6.



- 7. A
- 8. $\frac{126x^3 + 20x^2 2x + 16}{(7x+1)^2}$
- 9. $\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$
- 10. $\cos v 8 \csc v + 8v \csc v \cot v$
- 11. $f'(x) = 4[g(x)]^3 g'(x)$
- 12. B
- 13. y = 1
- $14. \quad \frac{6x^5}{y^6} \frac{6x^{12}}{y^{13}}$
- 15. $y' = \frac{1 y^3 3x^2y}{3xy^2 + x^3 3}$
- 16. -0.090
- 17. $\frac{1}{14}$ L/sec

- 18. A
- 19. $0.090 \,\mathrm{m}^2/\mathrm{s}$
- 20. 3.4 m

1. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.0017 cm thick to a hemispherical dome with diameter 70 m.

Select the correct answer.

- a. 4.165π
- b. 2.52π
- c. 4.11π
- d. 3.82π
- e. 2.28π
- 2. A turkey is removed from the oven when its temperature reaches $175^{\circ}F$ and is placed on a table in a room where the temperature is $70^{\circ}F$. After 10 minutes the temperature of the turkey is $160^{\circ}F$ and after 20 minutes it is $150^{\circ}F$. Use a linear approximation to predict the temperature of the turkey after 40 minutes.

Select the correct answer.

- a. 160
- b. 36
- c. 134
- d. 135
- e. 130
- 3. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.
- 4. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

$$\frac{d^{89}}{dx^{89}}(\sin x)$$

Select the correct answer.

- a. $-\sin x$
- b. $\sin x$
- $c. \cos x$
- d. cosx
- e. None of these

5. If
$$f(0) = 4$$
, $f'(0) = 3$, $g(0) = 1$ and $g'(0) = -6$, find $(f+g)'(0)$

6. Find
$$\frac{d^2y}{dx^2}$$
 in terms of x and y.

$$x^7 - y^7 = 1$$

7. Calculate y'.

$$xy^3 + x^3y = x + 3y$$

- 8. Find the average rate of change of the area of a circle with respect to its radius *r* as *r* changes from 3 to 8.
- 9. Two cars start moving from the same point. One travels south at 70 mi/h and the other travels west at 20 mi/h. At what rate is the distance between the cars increasing 2 hours later? Round the result to the nearest hundredth.
- 10. A plane flying horizontally at an altitude of 1 mi and a speed of 550 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.
 - a. ≈ 476 mi/h
 - b. ≈670 mi/h
 - c. ≈ 455 mi/h
 - d. ≈ 570 mi/h
 - e. ≈ 495 mi/h
- 11. If a cylindrical tank holds 10000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after *t* minutes as

$$V(t) = 10000 \left(1 - \frac{1}{60} t \right)^2, 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

12. Differentiate the function.

$$f(t) = \frac{1}{3}t^3 - 2t^7 + t$$

- 13. Find an equation of the tangent line to the curve $y = 7 \tan x$ at the point $\left(\frac{\pi}{4}, 7\right)$.
- 14. Find the limit.

$$\lim_{\theta \to 0} 4 \frac{\sin(\sin 4\theta)}{\sec 4\theta}$$

15. Differentiate.

$$y = \frac{\sin x}{3 + \cos x}$$

16. Find the equation of the tangent to the curve at the given point.

$$y = \sqrt{16 + 4\sin x}$$
, (0,4)

17. Find y' by implicit differentiation.

$$10\cos x \sin y = 16$$

- 18. A spherical balloon is being inflated. Find the rate of increase of the surface area $S = 4\pi r^2$ with respect to the radius r when r = 1 ft.
- 19. If a snowball melts so that its surface area decreases at a rate of 4 cm²/min, find the rate at which the diameter decreases when the diameter is 37 cm.
- 20. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm²/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm².

Answer Key

- 1. A
- 2. E
- 3. $\frac{d}{dx}\left(x^3f(x)\right) = 3x^2f(x) + x^3f'(x)$
- 4. D
- 5. -3
- 6. $\frac{6x^5}{y^6} \frac{6x^{12}}{y^{13}}$
- 7. $y' = \frac{1 y^3 3x^2y}{3xy^2 + x^3 3}$
- 8. 11π
- 9. 72.80 mi/h
- 10. A
- 11. $V'(t) = \frac{-1000}{3} + \frac{50t}{9}$
- 12. $f'(t) = t^2 14t^6 + 1$
- 13. $y = 14x + 7\left(1 \frac{\pi}{2}\right)$
- 14. 0
- 15. $\frac{dy}{dx} = \frac{3\cos x + 1}{(3 + \cos x)^2}$
- 16. $y = \frac{1}{2}x + 4$
- 17. tan(x)tan(y)
- 18. 8π
- 19. $\frac{2}{37\pi}$
- 20. -1.6

- 1. Find an equation of the tangent line to the curve $y = x^3 6x$ at the point (6, 8).
- 2. A turkey is removed from the oven when its temperature reaches 175 °F and is placed on a table in a room where the temperature is 70 °F. After 10 minutes the temperature of the turkey is 161 °F and after 20 minutes it is 149 °F. Use a linear approximation to predict the temperature of the turkey after 30 minutes.
- 3. The equation of motion is given for a particle, where *s* is in meters and *t* is in seconds. Find the acceleration after 5 seconds.

$$s = t^3 - 3t$$

4. If f is a differentiable function, find an expression for the derivative of $y = x^3 f(x)$.

Select the correct answer.

a.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^2 f(x) + x^3 f'(x)$$

b.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) + x^2 f'(x)$$

c.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 2x^2 f(x) - x^3 f'(x)$$

d.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^2 f(x) - x^3 f'(x)$$

e.
$$\frac{d}{dx} \left(x^3 f(x) \right) = 3x^3 f(x) - x^2 f'(x)$$

5. Differentiate the function.

$$B(y) = cy^{-4}$$

6. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 19$ where the tangent is horizontal.