CHAPTER 2

2-1. (a) Applying the voltage divider equation (2-10)

$$\frac{1.0}{10} = \frac{R_1}{R_1 + R_2 + R_3}$$

$$\frac{4.0}{10} = \frac{R_2}{R_1 + R_2 + R_3}$$

$$V_3 = 10.0\text{V} - 1.0\text{ V} - 4.0\text{ V} = 5.0\text{ V}$$

$$\frac{5.0}{10} = \frac{R_3}{R_1 + R_2 + R_3}$$

Dividing the first equation by the second, gives

$$R_1/R_2 = 1.0/4.0$$

Similarly, $R_2/R_3 = 4.0/5.0$

Letting $R_1 = 250\ \Omega$, $R_2 = 250 \times 4.0 = 1.0\ \text{k}\Omega$,

and $R_3 = 1.0\ \text{k}\Omega \times 5.0/4.0 = 1.25\ \text{k}\Omega$. Use a 1.0 kΩ resistor and a 250 kΩ resistor in series. The 500 kΩ resistor is not used.

(b) $V_3 = IR_3 = 10.0\text{ V} - 1.0\text{ V} - 4.0\text{ V} = 5.0\text{ V}$

(c) $I = V/(R_1 + R_2 + R_3) = 10.0\text{ V}/(250\ \Omega + 1000\ \Omega + 1250\ \Omega) = 0.004\text{ A } (4.0\text{ mA})$

(d) $P = IV = 0.004\text{ A} \times 10.0\text{ V} = 0.04\text{ W}$ (Equation 2-2)

2-2. (a) From Equation 2-10, $V_2 = 15 \times 400/(200 + 400 + 2000) = 2.31\text{ V}$

(b) $P = V_2^2/R_2 = (2.31)^2/400 = 0.013\text{ W}$

(c) Total $P = V^2/R_s = (15)^2/2600 = 0.087\text{ W}$

Percentage loss in R_2 = (0.013/0.087) × 100 = 15%

2-3. $V_{2,4} = 24.0 \times [(2.0 + 4.0) \times 10^3]/[(1.0 + 2.0 + 4.0) \times 10^3] = 20.6$ V

With the meter in parallel across contacts 2 and 4,

$$\frac{1}{R_{2,4}} = \frac{1}{(2.0+4.0)\text{ k}\Omega} + \frac{1}{R_M} = \frac{R_M + 6.0\text{ k}\Omega}{R_M \times 6.0\text{ k}\Omega}$$

$R_{2,4} = (R_M \times 6.0\text{ k}\Omega)/(R_M + 6.0\text{ k}\Omega)$

(a) $R_{2,4} = (4.0\text{ k}\Omega \times 6.0\text{ k}\Omega)/(4.0\text{ k}\Omega + 6.0\text{ k}\Omega) = 2.40\text{ k}\Omega$

$V_M = (24.0\text{ V} \times 2.40\text{ k}\Omega)/(1.00\text{ k}\Omega + 2.40\text{ k}\Omega) = 16.9\text{ V}$

$$\text{rel error} = \frac{16.9\text{ V} - 20.6\text{ V}}{20.6\text{ V}} \times 100\% = -18\%$$

Proceeding in the same way, we obtain (b) –1.2% and (c) –0.2%

2-4. Applying Equation 2-19, we can write

(a) $$-1.0\% = -\frac{1000\ \Omega}{(R_M - 1000\ \Omega)} \times 100\%$$

$R_M = (1000 \times 100 - 1000)\ \Omega = 99000\ \Omega$ or $99\text{ k}\Omega$

(b) $$-0.1\% = -\frac{1000\ \Omega}{(R_M - 1000\ \Omega)} \times 100\%$$

$R_M = 999\text{ k}\Omega$

2-5. Resistors R_2 and R_3 are in parallel, the parallel combination R_p is given by Equation 2-17

$R_p = (500 \times 250)/(500 + 250) = 166.67\ \Omega$

(a) This 166.67 Ω R_p is in series with R_1 and R_4. Thus, the voltage across R_1 is

$V_1 = (15.0 \times 250)/(250 + 166.67 + 1000) = 2.65$ V

$V_2 = V_3 = 15.0\text{ V} \times 166.67/1416.67 = 1.76\text{ V}$

$V_4 = 15.0\text{ V} \times 1000/1416.67 = 10.59\text{ V}$

(b) $I_1R_1 = V_1 = 2.647\text{ V}$ $I_1 = 2.647/250 = 0.01059\text{A}$ $(1.06 \times 10^{-2}\text{ A})$

$I_2 = 1.76\text{ V}/500\ \Omega = 3.5 \times 10^{-3}\text{ A}$

$I_3 = 1.76\text{ V}/250\ \Omega = 7.0 \times 10^{-3}\text{ A}$

$I_4 = 10.59\text{ V}/1000\ \Omega = 0.01059\text{ A}$ $(1.06 \times 10^{-2}\text{ A})$

(c) $P = IV = 1.76\text{ V} \times 7.0 \times 10^{-3}\text{ A} = 1.2 \times 10^{-2}\text{ W}$

(d) Since point 3 is at the same potential as point 2, the voltage between points 3 and 4 (V') is the sum of the drops across the 166.67 Ω and the 1000 Ω resistors. Or,

$V' = 1.76\text{ V} + 10.59\text{ V} = 12.35\text{ V}$. It is also the source voltage minus the V_1

$V' = 15.0 - 2.65 = 12.35\text{ V}$

2-6. The resistance between points 1 and 2 is the parallel combination or R_B and R_C

$R_{1,2} = 3.0\text{ k}\Omega \times 4.0\text{ k}\Omega/(3.0\text{ k}\Omega + 4.0\text{ k}\Omega) = 1.71\text{ k}\Omega$

Similarly the resistance between points 2 and 3 is

$R_{2,3} = 2.0\text{ k}\Omega \times 1.0\text{ k}\Omega/(2.0\text{ k}\Omega + 1.0\text{ k}\Omega) = 0.667\text{ k}\Omega$

These two resistors are in series with R_A for a total series resistance R_T of

$R_T = 1.71\text{ k}\Omega + 0.667\text{ k}\Omega + 2.0\text{ k}\Omega = 4.38\text{ k}\Omega$

$I = 24/(4380\ \Omega) = 5.5 \times 10^{-3}\text{ A}$

(a) $P_{1,2} = I^2R_{1,2} = (5.5 \times 10^{-3})^2 \times 1.71 \times 10^3 = 0.052\text{ W}$

(b) As above $I = 5.5 \times 10^{-3}\text{ A}$ (5.5 mA)

(c) $V_A = IR_A = 5.5 \times 10^{-3}\text{ A} \times 2.0 \times 10^3\ \Omega = 11.0\text{ V}$

(d) $V_D = 24 \times R_{2,3}/R_T = 24 \times 0.667/4.38 = 3.65$ V

(e) $V_{5,4} = 24 - V_A = 24 - 11.0 = 13$ V

2-7. With the standard cell in the circuit,

$V_{\text{std}} = V_b \times AC/AB$ where V_b is the battery voltage

$1.018 = V_b \times 84.3/AB$

With the unknown voltage V_x in the circuit,

$V_x = V_b \times 44.2/AB$

Dividing the third equation by the second gives,

$$\frac{1.018 \text{ V}}{V_x} = \frac{84.3 \text{ cm}}{44.3 \text{ cm}}$$

$V_x = 1.018 \times 44.3 \text{ cm}/84.3 \text{ cm} = 0.535$ V

2-8. $$E_r = -\frac{R_S}{R_M + R_S} \times 100\%$$

For $R_S = 20\ \Omega$ and $R_M = 10\ \Omega$, $E_r = -\frac{20}{10+20} \times 100\% = -67\%$

Similarly, for $R_M = 50\ \Omega$, $E_r = -\frac{20}{50+20} \times 100\% = -29\%$

The other values are shown in a similar manner.

2-9. Equation 2-20 is $E_r = -\frac{R_{\text{std}}}{R_L + R_{\text{std}}} \times 100\%$

For $R_{\text{std}} = 1\ \Omega$ and $R_L = 1\ \Omega$, $E_r = -\frac{1\ \Omega}{1\ \Omega + 1\ \Omega} \times 100\% = -50\%$

Similarly for $R_L = 10\ \Omega$, $E_r = -\dfrac{1\ \Omega}{10\ \Omega + 1\ \Omega} \times 100\% = -9.1\%$

The other values are shown in a similar manner.

2-10. (a) $R_s = V/I = 1.00\ \text{V}/20 \times 10^{-6}\ \text{A} = 50000\ \Omega$ or $50\ \text{k}\Omega$

(b) Using Equation 2-19

$$-1\% = -\frac{50\ \text{k}\Omega}{R_M + 50\ \text{k}\Omega} \times 100\%$$

$R_M = 50\ \text{k}\Omega \times 100 - 50\ \text{k}\Omega = 4950\ \text{k}\Omega$ or $\approx 5\ \text{M}\Omega$

2–11. $I_1 = 90/(25 + 5000) = 1.791 \times 10^{-2}$ A

$I_2 = 90/(45 + 5000) = 1.784 \times 10^{-2}$ A

% change = $[(1.784 \times 10^{-2}\ \text{A} - 1.791 \times 10^{-2}\ \text{A})/\ 1.791 \times 10^{-2}\ \text{A}] \times 100\% = -0.4\%$

2-12. $I_1 = 12.5/420 = 2.976 \times 10^{-2}$ A

$I_2 = 12.5/440 = 2.841 \times 10^{-2}$ A

% change = $[(2.841 \times 10^{-2} - 2.976 \times 10^{-2})/\ 2.976 \times 10^{-2}] \times 100\% = -4.5\%$

2-13. $i = I_{\text{init}}\, e^{-t/RC}$ (Equation 2-35)

$RC = 25 \times 10^6\ \Omega \times 0.2 \times 10^{-6}\ \text{F} = 5.00\ \text{s}$ $I_{\text{init}} = 24\text{V}/(25 \times 10^6\ \Omega) = 9.6 \times 10^{-7}$ A

$i = 9.6 \times 10^{-7}\, e^{-t/5.00}$ A or $0.96 \times e^{-t/5.0}$ µA

***t*, s**	***i*, µA**
0.00	0.96
0.010	0.958
0.10	0.941

***t*, s**	***i*, µA**
1.0	0.786
10	0.130

2-14. $v_C = V_C e^{-t/RC}$ (Equation 2-40)

$v_C/V_C = 1.00/100$ for discharge to 1%

$0.0100 = e^{-t/RC} = e^{-t/(R \times 0.025 \times 10^{-6})}$

$\ln 0.0100 = -4.61 = -t/(2.5 \times 10^{-8}R)$

$t = 4.61 \times 2.5 \times 10^{-8}R = 1.15 \times 10^{-7}R$

(a) When $R = 10$ MΩ or 10×10^6 Ω, $t = 1.15$ s

(b) Similarly, when $R = 1$ MΩ, $t = 0.115$ s

(c) When $R = 1$ kΩ, $t = 1.15 \times 10^{-4}$ s

2-15. (a) When $R = 10$ MΩ, $RC = 10 \times 10^6\ \Omega \times 0.025 \times 10^{-6}$ F = 0.25 s

(b) $RC = 1 \times 10^6 \times 0.025 \times 10^{-6} = 0.025$ s

(c) $RC = 1 \times 10^3 \times 0.025 \times 10^{-6} = 2.5 \times 10^{-5}$ s

2-16. Parts (a) and (b) are given in the spreadsheet below. For part (c), we calculate the quantities from

$i = I_{\text{init}}\, e^{-t/RC}$, $v_R = iR$, and $v_C = 25 - v_R$

For part (d) we calculate the quantities from

$$i = \frac{-v_C}{R} e^{-t/RC},\ v_R = iR, \text{ and } v_C = -v_R$$

The results are given in the spreadsheet.

	A	B	C	D	E	F	G	H	I
1	**Problem 2-16**								
2	*R*	5.00E+04	ohms						
3	*C*	3.50E-08	farads						
4	*V*	25	Volts						
6	(a)								
7	*RC*	1.75E-03	s						
8	(b)								
9	I_{init}	5.00E-04	A						
10			*t*, s	*i*, μA	v_R, V	v_C, V			
11			0	500	25	0.0			
12			1	282	14	11			
13			2	159	8.0	17			
14			3	90	4.5	20			
15			4	51	2.5	22			
16			5	29	1.4	24			
17			10	2	0.08	24.9			
18									
19	(c)		*t*, s	*i*, μA	v_R, V	v_C, V			
20			0	-498	-24.9	24.9			
21			1	-281	-14.1	14.1			
22			2	-159	-7.9	7.9			
23			3	-90	-4.5	4.5			
24			4	-51	-2.5	2.5			
25			5	-29	-1.4	1.4			
26			10	-1.6	-0.08	0.08			
27									
28	**Spreadsheet Documentation**								
29	Cell B7=B2*B3				Cell D20=-($F$17/$B$2)*1000000*EXP(-C20/$B$7*0.001)				
30	Cell B9=B4/B2				Cell E20=D20*0.000001*$B$2				
31	Cell D11=$B$9*1000000*EXP(-C11/$B$7*0.001)				Cell F20=-E20				
32	Cell E11=$B$2*0.000001*D11								
33	Cell F11=$B$4-E11								

2-17. Proceeding as in Problem 2-16, the results are in the spreadsheet

	A	B	C	D	E	F	G	H	I
1	**Problem 2-17**								
2	*R*	2.00E+07	Ω						
3	*C*	5.00E-08	F						
4	*V*	15	V						
6	(a)								
7	*RC*	1.00	s						
8	(b)								
9	I_{init}	7.50E-07							
10			*t*, s	*i*, μA	v_R, V	v_C, V			
11			0	0.75	15.0	0.0			
12			1	0.28	5.5	9.5			
13			2	0.10	2.0	13.0			
14			3	3.7E-02	0.75	14.3			
15			4	1.4E-02	0.27	14.7			
16			5	5.1E-03	0.10	14.9			
17			10	3.4E-05	0.00	15.0			
18									
19	(c)		*t*, s	*i*, μA	v_R, V	v_C, V			
20			0	-0.75	-15.0	15.0			
21			1	-0.28	-5.5	5.5			
22			2	-0.10	-2.0	2.0			
23			3	-3.7E-02	-0.75	0.75			
24			4	-1.4E-02	-0.27	0.27			
25			5	-5.1E-03	-0.10	0.10			
26			10	-3.4E-05	0.00	0.00			
27									
28	**Spreadsheet Documentation**								
29	Cell B7=B2*B3				Cell D20=-($F$17/$B$2)*1000000*EXP(-C20/$B$7)				
30	Cell B9=B4/B2				Cell E20=D20*0.000001*$B$2				
31	Cell D11=$B$9*1000000*EXP(-C11/$B$7)				Cell F20=-E20				
32	Cell E11=$B$2*0.000001*D11								
33	Cell F11=$B$4-E11								

2-18. In the spreadsheet we calculate X_C, Z, and ϕ from

$$X_C = 2/2\pi f C,\ Z = \sqrt{R^2 + X_C^2},\ \text{and}\ \phi = \text{arc tan}(X_C/R)$$

	A	B	C	D	E	F	G
1	**Problem 2-18**						
2		*f*, Hz	*R*, Ω	*C*, F	X_C, Ω	*Z*, Ω	ϕ, degrees
3	(a)	1	30000	3.30E-08	4.82E+06	4.8E+06	-90
4	(b)	1.00E+03	30000	3.30E-08	4.82E+03	3.0E+04	-9.1
5	(c)	1.00E+06	30000	3.30E-09	48.2	3.0E+04	-0.1
6	(d)	1	300	3.30E-09	4.82E+07	4.8E+07	-90.0
7	(e)	1.00E+03	300	3.30E-09	4.82E+04	4.8E+04	-89.6
8	(f)	1.00E+06	300	3.30E-09	48.2	3.0E+02	-9.1
9	(g)	1	3000	3.30E-07	4.82E+05	4.8E+05	-89.6
10	(h)	1.00E+03	3000	3.30E-07	4.82E+02	3.0E+03	-9.1
11	(i)	1.00E+06	3000	3.30E-07	0.48	3.0E+03	0.0
12							
13	**Spreadsheet Documentation**						
14	Cell E3=1/(2*PI()*B3*D3)						
15	Cell F3=SQRT(C3^2+E3^2)						
16	Cell G3=DEGREES(-ATAN(E3/C3))						

2-19. Let us rewrite Equation 2-54 in the form

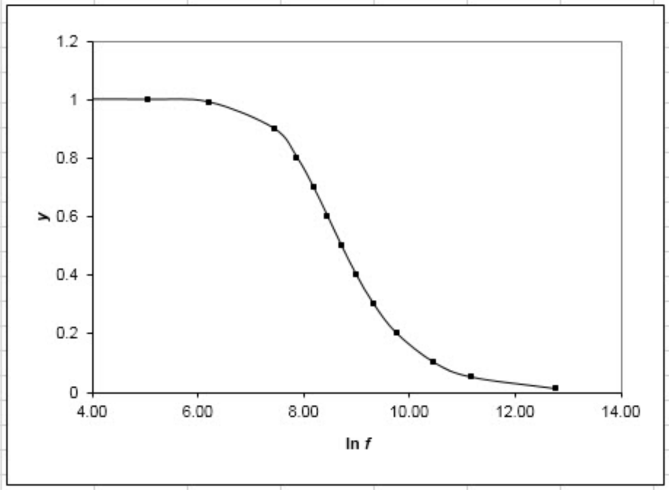
$$y = \frac{(V_p)_o}{(V_p)_i} = \frac{1}{\sqrt{(2\pi f RC)^2 + 1}}$$

$$y^2(2\pi f RC)^2 + y^2 = 1$$

$$f = \frac{1}{2\pi \text{RC}}\sqrt{\frac{1}{y^2} - 1} = \frac{1}{2\pi \text{RC}}\sqrt{\frac{1 - y^2}{y^2}}$$

The spreadsheet follows

	A	B	C	D	E
1	**Problem 2-19**				
2	*R*	3.00E+03			
3	*C*	1.50E-08			
4			$y = (V_p)_o/(V_p)_i$	*f*	ln*f*
5			0.01	3.54E+05	12.78
6			0.05	7.06E+04	11.17
7			0.1	3.52E+04	10.47
8			0.2	1.73E+04	9.76
9			0.3	1.12E+04	9.33
10			0.4	8104	9.00
11			0.5	6126	8.72
12			0.6	4716	8.46
13			0.7	3608	8.19
14			0.8	2653	7.88
15			0.9	1713	7.45
16			0.99	504	6.22
17			0.999	158	5.06
18			0.9999	50	3.91
19					
20					
21	**Spreadsheet Documentation**				
22	Cell D5=1/(2*PI()*$B$2*$B$3)*SQRT(-1+(1/C5^2))				
23	Cell E5=LN(D5)				



2-20. By dividing the numerator and denominator of the right side of Equation 2-53 by *R*, we obtain

$$y = \frac{(V_p)_o}{(V_p)_i} = \frac{1}{\sqrt{1 + (1/2\pi fRC)^2}}$$

Squaring this equation yields

$$y^2 + y^2/(2\pi fRC)^2 = 1$$

$$2\pi fRC = \sqrt{\frac{y^2}{1-y^2}}$$

$$f = \frac{1}{2\pi RC}\sqrt{\frac{y^2}{1-y^2}}$$

The results are shown in the spreadsheet that follows.

	A	B	C	D	E
1	**Problem 2-20**				
2	*R*	3.00E+05			
3	*C*	1.00E-10			
4			$y = (V_p)_o/(V_p)_i$	*f*	ln f
5			0.001	5.31E+00	1.668681
6			0.01	5.31E+01	3.971316
7			0.05	2.66E+02	5.581955
8			0.1	5.33E+02	6.278876
9			0.2	1.08E+03	6.987409
10			0.3	1.67E+03	7.419619
11			0.4	2.32E+03	7.747322
12			0.5	3.06E+03	8.02713
13			0.6	3.98E+03	8.288754
14			0.7	5.20E+03	8.556433
15			0.8	7.07E+03	8.864118
16			0.9	1.10E+04	9.301441
17			0.99	3.72E+04	10.5249
18			0.999	1.19E+05	11.68299
19			0.9999	3.75E+05	12.83496
20					
21	**Spreadsheet Documentation**				
22	Cell D5=SQRT(C5^2/(1-C5^2))/(2*PI()*$B$2*$B$3)				
23	Cell E5=LN(D5)