Chapter 3:
Introduction to Solid-State Switching Circuits

Problem 3.1:

A half-wave, single-phase ac-to-dc converter is loaded by an impedance of l0 mH inductance in series with l0 Ω resistance. The ac voltage is 110 v (rms). For α equal to 30º and 90º, calculate the following:

a. Conduction period
b. Average current of the load
c. Average voltage of the load
d. Power

Solution:

$$L = 10 \text{ mH}$$
$$R = 10\ \Omega$$
$$Vs(\omega t) = 100\sqrt{2}\sin(\omega t)V$$
$$\omega = 2\pi 60 \text{ radians}$$
$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 20.66^0,\ \ \tau = \frac{L}{R} = 1 \text{ ms}$$

a. Conduction Period:

From the voltage and the current waveforms for the load, the voltage and the current waveforms for the load can be found:

The conduction period is $\gamma = \alpha - \beta$

To find β, we use Eq. 3.37 where current I is equated to zero at $\omega t = \beta$

For $\alpha = \frac{\pi}{6}$, we have,

$$\sin(\beta - \frac{20.66\pi}{180}) + \sin\left(\frac{20.66\pi}{180} - \frac{\pi}{6}\right)e^{-\left(\frac{(\beta - \frac{\pi}{6}}{0.37699}\right)} = 0$$

Using any non-linear solution algorithm, we get,

$\beta = 3.502048 \text{ radians} = 200.6525°$
for the condition that $\beta > \pi$ radians

Therefore, conduction period, $\gamma = 170.6525°$

For $\alpha = \frac{\pi}{2}$, we have,

$$\beta = 3.49645 \text{ radians} = 200.3317°$$
for the condition that $\beta > \pi$ radians

Therefore, conduction period, $\gamma = 110.3317°$

b. and c. <u>Average current and average voltage of the load:</u>

$$V_{ave} = \frac{V_{Rave}}{R}$$

$$V_{Rave} = V_{Ave}$$

For V_{ave}, use Eq. 3.38.

For $\alpha = \frac{\pi}{6}$

$V_{ave} = 44.61$ V
$I_{ave} = 4.461$ A

For $\alpha = \frac{\pi}{2}$, we have,

$V_{ave} = 23.22$ V
$I_{ave} = 2.322$ A

d. Power delivered by the source = Power consumed by the resistor

$$= \frac{1}{2\pi}\int_{\alpha}^{\beta}\left(i(\omega t)\right)^2 R d\omega t$$

Use Eq. 3.41 for $i(wt)$ and evaluate the above integral.

For $\alpha = \frac{\pi}{6}$

P = 511.5874 W

For $\alpha = \frac{\pi}{2}$, we have,

P = 213.8638 W

Problem 3.2:

Repeat Problem 3.1 for $\alpha = 10°$, assuming that a freewheeling diode is used.

Solution:

$\alpha = 10^0$

a. With a freewheeling diode, conduction angle of the

SCR = $\gamma = \beta - \alpha(\beta = \pi) = 170^{\circ}$

b. Average Voltage = 49.1412 V (Eq. 3.38)

c. Average Current = 49.1412 V/R = 4.91412 A

d. Power = Power delivered by the source

$$= \frac{1}{2\pi}\int_{\alpha}^{\beta} V(\omega t) i(\omega t) d\omega t$$

Here, we use the power delivered by the source, because the power delivered to the load (dissipated in R) during freewheeling actually comes from the energy stored in the inductor when the SCR is conducting. We can also use the square of the current as we did in Problem 3.1, but the power calculated in that manner will always be an approximation since the angle β when i(wt) dies down to zero will have to be approximated by the angle at which i(wt) is very small.

$$P = \frac{1}{2\pi}\int_{\alpha}^{\beta} \frac{V_{\max}^2}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t)\sin(\omega t - \varphi) + \sin(\varphi - \alpha) e^{-\left(\frac{\omega t - \alpha}{\omega \tau}\right)} d\omega t = 543.028 \text{ W}$$

Problem 3.3:

Assume that an additional inductance can be inserted in series with the load. Also assume that the converter has no freewheeling diodes. Calculate the added inductance that leads to a conduction period of 180° when α = 30°.

Solution:

For $\alpha = 30°$ and $\gamma = 180°$

$$\beta = 210°$$

Using Eq. 3.42, substituting

$$\beta = \frac{7\pi}{6} \text{ rad}$$

$$\varphi = \tan^{-1}\left(\frac{\omega L_t}{R}\right) \text{ rad}$$

$$\omega = 2\pi 60 = 376.99 \text{ rad/sec}$$

$$L_t = L_{added} + L = L_a + 10 \text{ mH}$$

$$\tau = \frac{L_t}{R}$$

Solving the equation gives,

L_t = 15.3152 mH

Therefore, La = 5.3152 mH

Problem 3.4:

Calculate the average current, average voltage, and the power of the load for the case described in Problem 3.3

Solution:

$\alpha = 30°$, $\beta = 210°$

$$V_{ave} = \frac{V_{\max}}{2\pi}(\cos\alpha - \cos\beta) = 42.883 \text{ V}$$

Please note the reduction in V_{ave}, due to the higher conduction angle for the same α and V_{max} as Problem 3.1.

$$I_{ave} = \frac{V_{ave}}{R} = 4.288 \text{ A}$$

Here,

$$\phi = \tan^{-1}\left(\frac{\omega L_t}{R}\right) = 30^O$$

$$\omega\tau = \frac{\omega L_t}{R} = 0.57735 rad$$

$$\therefore Power = \frac{1}{2\pi}\int_{\pi/6}^{7\pi/6} i(\omega t)^2 R.d\omega t$$

Using Eq. 3.42 for $i(\omega t)$ and evaluation of the integral gives consumed power P = 453.744 W.

Problem 3.5:

A single-phase, half-wave SCR circuit is used to control the power consumption of an inductive load. The resistive component of the load is 5 Ω. The source voltage is 120 V (rms). When the triggering angle is adjusted to 60°, the average current of the load is 6A. Calculate the following:

a. Average voltage across the load
b. Conduction period in degrees

Solution:

$$R = 5\ \Omega$$

$$Vs = 120\sqrt{2}\sin(\omega t)$$

$$\alpha = 60°$$

$$I_{ave} = 6 \text{ A}$$

a. $\therefore V_{ave} = I_{ave}R = 30 \text{ V}$

b.

$$V_{ave} = \frac{V_{\max}}{2\pi}[\cos\alpha - \cos\beta]$$

Therefore,

$$\cos\beta = -0.61072$$
$$\Rightarrow \beta = 127.642° \text{ or } 232.36°$$
$$\because \beta > 180°,$$
$$\beta = 232.36°$$
$$\therefore \gamma = \beta - \alpha = 172.36°$$

Problem 3.6:

A dc/dc converter consists of a 100 V dc source in series with a 10 Ω load resistance and a bipolar transistor. Assume that the transistor is an ideal switch. In each cycle, the transistor is turned on for 100 μs and turned off for 300 μs. Calculate the following:

a. Switching frequency of the converter
b. Average voltage across the load
c. Average load current
d. rms voltage across the load
e. rms current
f. rms power consumed by the load

Solution:

a.

$$f_{sw} = \frac{1}{400\mu s} = 2.5\text{kHz}$$

b.

$$V_{ave} = KV_s = \left(\frac{t_{on}}{\tau}\right)V_s = \left(\frac{100\mu s}{400\mu s}\right)100\text{V} = 25\text{V}$$

c.

$$I_{ave} = \frac{25V}{10\Omega} = 2.5\text{A}$$

d.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}(V_s^2 t_{on})} = V_s\sqrt{K} = 100\text{V}\sqrt{(0.25)} = 50\text{V}$$

e.

$$I_{rms} = \frac{50V}{10\Omega} = 5\text{A}$$

f.

$$P = (5\text{A}^2)10\Omega = 250\text{W}$$

Problem 3.7:

A 120 V (rms), 60 Hz source is connected to a full-wave bridge as shown in Figure 3.58. The load is an arc welding machine that can be represented by a resistance of 1 Ω in series with an inductive reactance of 3 Ω. At a triggering angle of 60°, the current of the load is continuous. Calculate the following:

a. Average voltage across the load
b. Average voltage across the resistive element of the load
c. Average current of the load

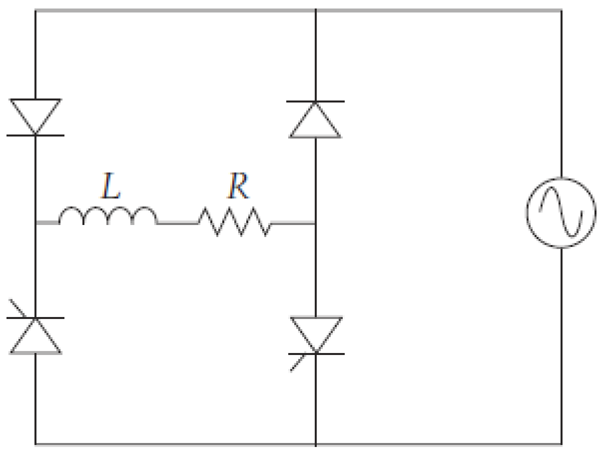

Figure 3.58

Solution:

For continuous current, the conduction angle for every SCR is $(\pi - \alpha)^C$

a.

$$\therefore \alpha = 60°, \beta = \pi^C$$

$$\therefore V_{Avg} = \frac{V_{Max}}{\pi}(1 - \cos\alpha) = 81.03 \text{ V}$$

b. $V_{RAvg} = V_{Avg} = 81.03 \text{ V}$

c. $I_{Avg} = \dfrac{V_{RAvg}}{R} = 81.03 \text{ A}$

Problem 3.8:

An inductive load consists of a resistance and an inductive reactance connected in series. The circuit is excited by a full-wave ac/dc SCR converter. The ac voltage (input to the converter) is 120 V (rms), and the circuit resistance is 5 Ω. At a triggering angle of 30°, the load current is continuous. Calculate the following:

a. Average voltage across the load

b. Average load current
c. rms voltage across the load

Solution:

$R = 5\ \Omega$

$V_{rms} = 120$

Full wave rectifier feeding an R-L load.

$$\alpha = 30° = \left(\frac{\pi}{6}\right)^c$$

Since the current through the load is continuous, SCR1 will turn off <u>only when</u> SCR2 is turned on at $\pi + \alpha$. Hence, the conduction period for each SCR is from α to $\pi + \alpha$.

$$\therefore \beta = \pi + \alpha = 210°$$

a. To find the average load voltage:

$$V_{LoadAve} = \frac{V_{\max}}{\pi}[\cos\alpha - \cos\beta] = \frac{2V_{\max}}{\pi}\cos\alpha$$

$$\therefore V_{LoadAve} = 93.56\ \text{V}$$

b. Average load current:

$$\because V_{\text{InductorAve}} = 0$$

$$\Rightarrow V_{\text{ResistanceAve}} = V_{\text{LoadAve}}$$

$$\therefore I_{\text{ResistanceAve}} = \frac{V_{\text{ResistanceAve}}}{R} = 18.71\ \text{A}$$

c. Since the current is continuous,

$$\Rightarrow V_{LoadRMS} = V_{SourceRMS} = 120\ \text{V}$$

Mathematically,

$$V_{LoadRMS} = \frac{V_{Max}}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\gamma - \frac{\sin 2\beta - \sin 2\alpha}{2}\right)}$$

Multiply Eq. 3.42 by $\sqrt{2}$ for full wave.
But $\beta = \gamma + \alpha$ and $\gamma = \pi$.
Therefore, $V_{LoadRMS}$ = 120 V.

Problem 3.9:

A purely inductive load of 10 Ω is connected to an ac source of 120 V (rms) through a half-wave SCR circuit.

a. If the SCR is triggered at 90^o, calculate the angle at the maximum instantaneous current.

b. If the triggering angle is changed to 120^o, calculate the angle at the maximum instantaneous current.

c. Calculate the conduction period for the case in (b).

Solution:

a. As stated on page 48, the peak of the current occurs at $\omega t = 90^\circ + \phi$ where $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ hence for a pure inductive load $\phi = 90°$ and so the peak current occurs at $\omega t = 180°$

b. As given above, the angle is 180°.

c. From $V_L = L\frac{di}{dt}$, $\omega t = \theta$ and taking the integral between α and θ we can write

$$i_L = \frac{V_L}{L}(\cos(\theta) - \cos(\alpha)) = \frac{V_L}{L}\left(\cos(\theta) - \cos\left(\frac{2\pi}{3}\right)\right)$$

Hence, the current extinction occurs when $\theta = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3} rads = 240^\circ$

The conduction period is: $\gamma = \theta - \alpha = \frac{2\pi}{3}\ rads = 120^\circ$

Problem 3.10:

A resistive load of 5 Ω is connected to an ac source of 120 V (rms) through an SCR circuit.

a. If the SCR circuit consists of a single SCR, and if the triggering angle is adjusted to 30^o, calculate the power consumption of the load.

b. If the SCR circuit consists of two back-to-back SCRs, calculate the power consumption of the load assuming that the triggering angle is kept at 30^o.

Solution:

a. $P = \frac{V_{max}^2}{8\pi R}\left(2(\pi - \alpha) + \sin 2\alpha\right) = 1398.48\ \text{W}$

b. $P_{fw} = 2\,P_{hw} = 2 * 1398.48 = 2796.96\ \text{W}$

Problem 3.11:

An inductive load that has a resistive component of 4 Ω is connected to an ac source of 120 V (rms) through a half-wave SCR circuit. When the triggering angle of the SCR is 50^o, the conduction period is 160^o. Calculate the following:

a. Average voltage across the load
b. Average voltage across the resistive element of the load
c. rms voltage across the load
d. Average current of the load

e. If a freewheeling diode is connected across the load, calculate the load rms voltage. Assume that the current of the diode flows for a complete half cycle.

Solution:

a. $V_{avg} = \dfrac{1}{2\pi}\int_{\alpha}^{\beta} v\, d\theta = \dfrac{V_{\max}{}^{2}}{2\pi}\left(\cos\alpha - \cos\beta\right)$ = 40.75 V

b. Voltage across the resistance is = 40.75 V (since the average voltage across the inductance = 0)

c. $V_{rms} = \sqrt{\dfrac{1}{2\pi}\int_{\alpha}^{\beta} v^2\, d\theta} = \sqrt{\dfrac{V_{\max}{}^{2}}{4\pi}\left(\beta - \alpha + \dfrac{\sin 2\alpha - \sin 2\beta}{2}\right)}$ = 80.846 V

d. $I_{avg} = \dfrac{V_{avg}}{R}$ = 40.75 / 4 = 10.19 A

e. $V_{rms} = \sqrt{\dfrac{1}{2\pi}\int_{\alpha}^{\pi} v^2 d\theta} = \sqrt{\dfrac{V_{\max}{}^{2}}{4\pi}\left(\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)}$ = 79.54 V

Problem 3.12:

The full-wave, ac/dc converter shown in Figure 3.59 is operating under continuous current (conduction period = 180º). The source voltage is 120 V (rms), and the load resistance is 2 Ω. For an average load current of 40 A, calculate the triggering angle of the SCRs.

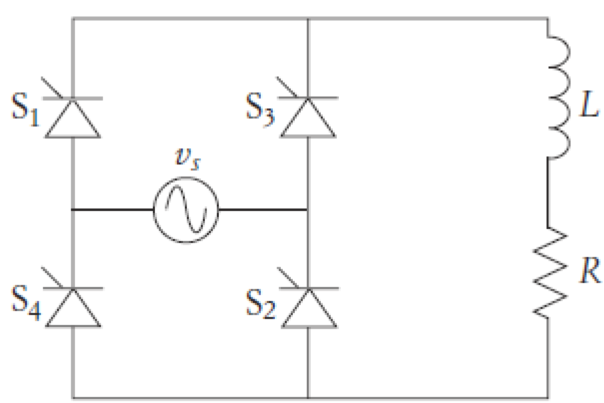

Figure 3.59

Solution:

$$V_{ave} = I_{ave}\ R = 40*2 = 80\,\mathrm{V}$$

$$V_{ave} = \frac{1}{\pi}\int_{\alpha}^{\alpha+180} V_{\max}\sin\theta\ d\theta$$

$$V_{ave} = \frac{2V_{\max}}{\pi}\cos\alpha$$

$$80 = \frac{2\sqrt{2}120}{\pi}\cos\alpha$$

$$\alpha = 42.27$$

Problem 3.13:

Draw the waveforms of the load voltage for the circuit in Figure 3.21, assuming that the triggering angle is -30^{o}.

Solution:

See Figure 3.22 for the solution.

Problem 3.14:

A three-phase, ac /dc converter is excited by a three-phase source of 480 V (rms and line-to-line). Compute the following:

a. The rms voltage across the load when the triggering angle is 30^{o}.

b. The average voltage across the load when the triggering angle is 140^{o}. Keep in mind that the conduction is incomplete when the triggering angle is greater than 90^{o}.

Solution:

a. Full wave converter:

$$V_{rms}=\sqrt{\frac{6}{2\pi}\int_{\alpha}^{\alpha+60} v_{ab}^2\ d\omega t}=\sqrt{\frac{3}{\pi}\int_{\alpha}^{\alpha+60} V_{\max}^2\left[\sin\omega t-\sin\left(\omega t-120\right)\right]^2\ d\omega t}$$

$$V_{rms}=\sqrt{\frac{3V_{\max}^2}{\pi}\int_{\alpha}^{\alpha+60}\left[\sqrt{3}\sin\left(\omega t+30\right)\right]^2\ d\omega t}=\sqrt{\frac{9V_{\max}^2}{\pi}\int_{\alpha}^{\alpha+60}\left[\sin\left(\omega t+30\right)\right]^2\ d\omega t}$$

$$V_{rms}=\sqrt{\frac{9V_{\max}^2}{\pi}\left[\frac{\sqrt{3}}{8}\sin\omega t-\cos\omega t\right]\sin\omega t\Bigg|_{30}^{90}}=\sqrt{\frac{99\sqrt{3}\,V_{\max}^2}{32\pi}}=361.76\,\text{V}$$

b. Eq. 3.54 is valid for $\alpha\le 90°$. For $\alpha>90°$, we can develop the following equation

$$V_{ave}=\frac{6}{2\pi}\int_{\alpha}^{150} v_{ab}\,d\omega t=\frac{3}{\pi}\int_{\alpha}^{150} V_{\max}\left[\sin(\omega t)-\sin(\omega t-120)\right]d\omega t$$

$$V_{ave}=\frac{3\sqrt{3}\ V_{\max}}{\pi}\int_{\alpha}^{150}\sin(\omega t+30)\ d\omega t$$

$$V_{ave}=\frac{3\sqrt{3}\ V_{\max}}{\pi}\left[1+\cos(\alpha+30)\right]=\frac{3*\sqrt{2}*480}{\pi}\left[1+\cos(140+30)\right]=9.84\,\text{V}$$

Problem 3.15:

The three-phase circuit shown in Figure 3.43 is used to charge a battery bank of 250 V. The line-to-line ac voltage is 208 V. The system resistance between the battery bank and the source during conduction is 3 Ω. Compute the triggering angle of the IGBTs that limits the average charging current to 5 A.

Solution:

$$V_r=V_{dc}\frac{3\gamma}{2\pi}+\frac{3\sqrt{3}.V_{\max}}{2\pi}\left(\cos\beta-\cos\alpha\right)$$

Taking $\gamma=\frac{2\pi}{3}$

$$V_r=V_{dc}+\frac{9V_{\max}}{2\pi}\left(\cos(\alpha+150)\right)$$

$$\because i_R=\frac{V_r}{R}=5\ \text{A}$$

$$\Rightarrow\alpha=-32.91°$$