2

Motion in One Dimension

[redacted]

* An asterisk indicates a question or problem new to this edition.

[redacted]

***TP2.1** **Conceptualize** The photo in Figure TP2.1 helps you to visualize the situation. In the first demonstrations, the object, which appears as a small silver-colored slider at the left end of the red lever in the photograph, rises up and just touches the bell at the top of the column. We can approximate this situation by saying that the slider just reaches zero velocity at the position of the bell.

In the final demonstration, the hitting of the hammer on the apparatus has caused the bell to shift to the side, so that the slider can fly vertically off the track when it is projected with a higher initial velocity. It looks like we have only two pieces of information: the height *h* of the object in the first situation and the fact that the initial velocity of the object in the second situation is twice that in the first. But we have two more pieces of information that can be gleaned from the description: in the first situation, the velocity of the object goes to zero just as its vertical position reaches height *h*; in the second situation, the initial and final vertical positions are the same.

Categorize The slider is an object in free fall, so we use the *particle under constant acceleration* model for its motion.

Analyze For the first situation, write an equation from the model relating velocity and position, using subscripts 1 to describe this situation. Evaluate the final values in the equation for the instant at which the object just touches the bell and momentarily comes to rest:

$$v_{f1}^2 = v_{i1}^2 - 2g\left(y_{f1} - y_{i1}\right) \quad \to \quad 0 = v_{i1}^2 - 2g\left(h - 0\right) \quad \to \quad v_{i1} = \sqrt{2gh} \quad (1)$$

For the second situation, write an equation from the model relating velocity and time, using subscripts 2. Solve for the time:

$$v_{f2} = v_{i2} - gt \quad \to \quad t = \frac{v_{i2} - v_{f2}}{g} \qquad (2)$$

Now evaluate the time at which the object reaches its highest position at the top of its motion:

$$t_{\text{top}} = \frac{v_{i2} - 0}{g} = \frac{v_{i2}}{g} \qquad (3)$$

Recognize that the initial velocity in the second situation is twice that of the first, and combine Equations (1) and (2):

$$t_{\text{top}} = \frac{2v_{i1}}{g} = \frac{2\left(\sqrt{2gh}\right)}{g} = 2\sqrt{\frac{2h}{g}}$$

Finally, recognize by symmetry that the time at which the object lands back on the ground is twice that at which it is at the top, and substitute numerical values:

$$t_{\text{ground}} = 2t_{\text{top}} = 4\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2(4.50\text{ m})}{9.80\text{ m/ s}^2}} = 3.83\text{ s}$$

Finalize Does this time interval for the flight of the object seem reasonable? Discuss with your partners the following. Is this time just twice the time for the object to fall back to the lever in the first situation? In the second situation, does the object rise to a maximum height of just twice that in the first situation? Does the fact that the object begins its upward motion a few centimeters above the ground make any significant difference in our calculation of the final time in the second situation?

Answer: 3.83 s

***TP2.2** **Conceptualize** Be sure you understand the setup of the problem. One person drops a rock from rest. The second person waits for a time interval and then throws the second rock with just the right speed so that it catches up to the first rock just as the two rocks enter the water.

Categorize The rocks are in free fall, so we use the *particle under constant acceleration* model for their motion.

Analyze Let us first look at the fall of the first rock through the vertical distance h. Using Equation 2.16, rewritten for the vertical direction, we find

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad -h = 0 + 0 - \tfrac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2h}{g}} \quad (1)$$

This is the time at which the dropped rock strikes the water. Now consider the fall of the second rock. The thrower waits for a time interval Δt after the first rock is dropped before throwing his rock downward. Let us denote time t' as the time at which the second rock reaches the water, with $t' = 0$ at the instant the second rock is thrown downward. Using Equation 2.16, rewritten for the vertical direction, we find

$$y_f = y_i + v_{yi}t' + \tfrac{1}{2}a_y(t')^2 \quad \rightarrow \quad -h = 0 - v_{yi}t' - \tfrac{1}{2}g(t')^2$$

$$\rightarrow \quad v_{yi} = \frac{h - \tfrac{1}{2}g(t')^2}{t'} \qquad (2)$$

In order for there to be a single splash, the time t at which the first rock strikes the water must be the same as the sum of the waiting time Δt and the time t' for the second rock to fall:

$$t = \Delta t + t' \quad \rightarrow \quad t' = t - \Delta t \qquad (3)$$

Substitute Equation (3) into Equation (2) and then substitute Equation (1) into Equation (2):

$$v_{yi} = \frac{h - \tfrac{1}{2}g(t - \Delta t)^2}{t - \Delta t} = \frac{h - \tfrac{1}{2}g\left(\sqrt{\dfrac{2h}{g}} - \Delta t\right)^2}{\sqrt{\dfrac{2h}{g}} - \Delta t} = \frac{\Delta t\sqrt{2gh} - \tfrac{1}{2}g(\Delta t)^2}{\sqrt{\dfrac{2h}{g}} - \Delta t} \qquad (4)$$

(a) Substitute numerical values:

$$v_{yi} = \frac{(1.00\text{ s})\sqrt{2(9.80\text{ m/ s}^2)(75.0\text{ m})} - \tfrac{1}{2}(9.80\text{ m/ s}^2)(1.00\text{ s})^2}{\sqrt{\dfrac{2(75.0\text{ m})}{(9.80\text{ m/ s}^2)}} - 1.00\text{ s}} = 11.5\text{ m/ s}$$

(b) One way to solve this problem is to solve Equation (4) for Δt. This leads to a quadratic equation whose appropriate solution is

$$\Delta t = \frac{v_{yi} + \sqrt{2gh} - \sqrt{v_{yi}^2 + 2gh}}{g} \qquad (5)$$

Substitute numerical values:

$$\Delta t = \frac{40.0\text{ m/ s} + \sqrt{2\left(9.80\text{ m/ s}^2\right)\left(75.0\text{ m}\right)} - \sqrt{\left(40.0\text{ m/ s}\right)^2 + 2\left(9.80\text{ m/ s}^2\right)\left(75.0\text{ m}\right)}}{9.80\text{ m/ s}^2}$$

$= 2.34\text{ s}$

Another way to solve the problem is to set up a spreadsheet using Equation (4) and generate a list of velocities v_{yi} for increasing values of Δt. You can then look in the list for the speed for 40.0 m/s and read the approximate value of Δt. Then the spreadsheet can be modified by looking at just the short range of Δt that gives a speed of about 40.0 m/s and using a smaller increment for Δt. By narrowing in using this technique, you find again that the time interval is 2.34 s.

(c) The second rock must be thrown before the first rock hits the water. Therefore, the longest time interval is given by Equation (1):

$$\Delta t_{\text{max}} = t = \sqrt{\frac{2h}{g}}$$

Substitute numerical values:

$$\Delta t_{\text{max}} = \sqrt{\frac{2\left(75.0\text{ m}\right)}{9.80\text{ m/ s}^2}} = 3.91\text{ s}$$

Finalize If the second person waited to throw the second rock for the time interval in part (c), it would have to be thrown with infinite speed. (We will find in Chapter 38 that this is not possible.) If you have set up the spreadsheet for part (b), you will see the values for the speed v_{yi} become very large as you approach 3.91 s and then become negative for larger values of the time interval.

Answers: (a) 11.5 m/s (b) 2.34 s (c) 3.91 s

***TP2.3** **Conceptualize** Think about the falling ruler. When it begins to fall, your eyes see the motion and send a signal to your brain. Your brain has to process the information and send a signal to your hand to grab the ruler. The muscles in your hand then have to respond to this signal and close your fingers onto the ruler. The time intervals for all of these things to happen are added to equal the reaction time.

Categorize The ruler falls with an acceleration due to gravity, so we use the *particle under constant acceleration* model for its motion.

Analyze There are no numbers given in the problem, so let's make one up. Suppose your fingers close on the ruler at the 8.0-inch mark. Then, using Equation 2.16 in the vertical direction,

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad -h = 0 + 0 - \tfrac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Substitute the value of h:

$$t = \sqrt{\frac{2(8.0\text{ in})}{9.80\text{ m/s}^2}\left(\frac{0.025\,4\text{ m}}{1\text{ in}}\right)} = 0.20\text{ s}$$

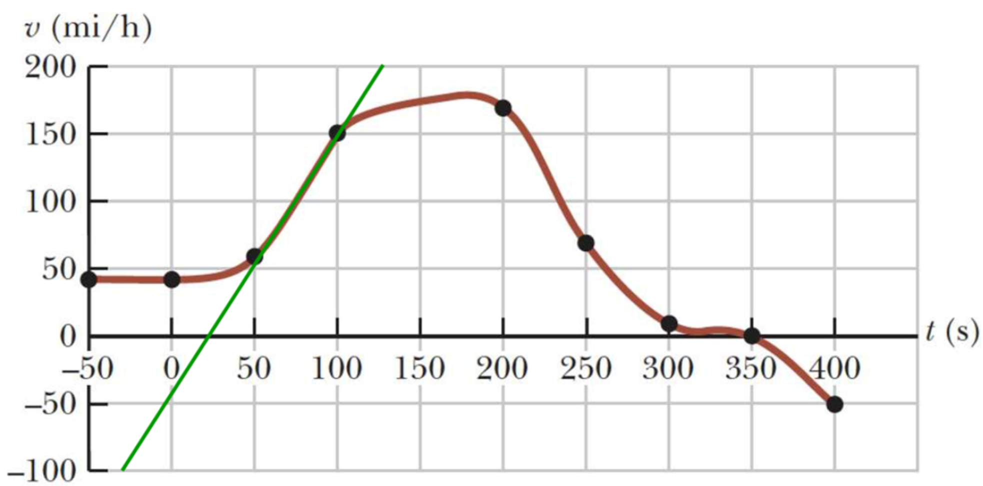
Finalize This value is a typical human reaction time.

Answer: Answers will vary depending on your individual reaction time. A typical human reaction time is about 0.2 s.

***TP2.4** **Conceptualize** From the graphical representation, generate a mental representation by running the motion in your mind. (a) From –50 s to 0, the Acela is cruising at a constant positive velocity in the $+x$ direction. Between 0 and 50 s, the constant velocity is maintained for a while and then the train begins to speed up. Between 50s and 100s, the train speeds up; notice that the graph is linear in this segment. Between 100 s and 200 s, the train reaches its maximum speed of about 175 mi/h. Between 200 s and 300 s, the engineer applies the brakes: the train is slowing down. The train is moving very slowly between 300 s and 350 s and then stops at 350 s. After 350 s, the train reverses direction: the velocity is negative. The train is backing up faster and faster until the data ends at 400 s.

Categorize Much of the motion of the train does not fit into a simple analysis model. However, between 50 s and 100 s, the train is well modeled as a *particle under constant acceleration*, since the *v*–*t* graph is linear.

Analyze (b) Draw a best-fit line between 50 s and 100 s on the graph, taking advantage of the full vertical extent of the graph:



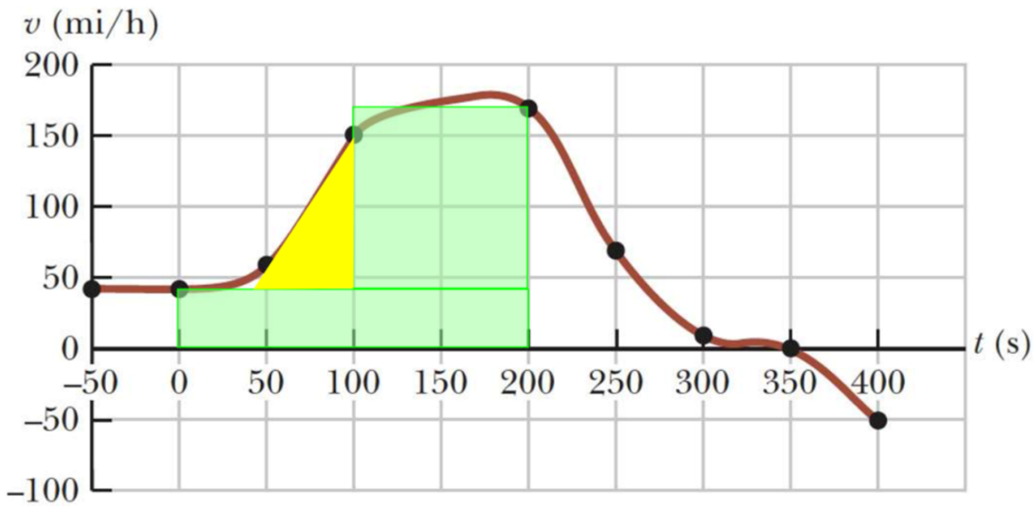
Now, find the slope of the green line, using its topmost and bottommost points, reading the coordinates from the graph:

$$a = slope = \frac{\Delta v}{\Delta t} = \frac{200 \text{ mi/ h} - (-100 \text{ mi/ h})}{125 \text{ s} - (-30 \text{ s})} = 1.9 \text{ mi/ h} \cdot \text{s}$$

This result can be converted to metric units:

$$a = 1.9 \text{ mi/ h} \cdot \text{s}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = 0.87 \text{ m/ s}^2$$

(c) To find the displacement, we need to use the notion in Section 2.9 that the displacement of a particle is the area under the *v*–*t* curve. Because we do not have a functional description of the velocity of the train, we will need to approximate the displacement. Let's approximate the area under the curve between 0 and 200 s with the following triangle (yellow) and two rectangles (green):



Find the area of each shape:

Lower rectangle:

$$\Delta x_1 = (40 \text{ mi/ h})(200 \text{ s}) = 8\,000 \text{ mi}\cdot\text{s/ h}$$

Upper rectangle:

$$\Delta x_2 = (170 \text{ mi/ h} - 40 \text{ mi/ h})(100 \text{ s}) = 13\,000 \text{ mi}\cdot\text{s/ h}$$

Triangle:

$$\Delta x_3 = \tfrac{1}{2}(110 \text{ mi/ h})(55 \text{ s}) = 3\,025 \text{ mi}\cdot\text{s/ h}$$

Therefore, the total displacement is

$$\begin{aligned}\Delta x_{\text{total}} &= \Delta x_1 + \Delta x_2 + \Delta x_3 \\ &= 8\,000 \text{ mi}\cdot\text{s/ h} + 13\,000 \text{ mi}\cdot\text{s/ h} + 3\,025 \text{ mi}\cdot\text{s/ h} \\ &= 2.4\times 10^4 \text{ mi}\cdot\text{s/ h}\end{aligned}$$

This is not a very useful unit! So let's reconcile the hours and seconds:

$$\Delta x_{\text{total}} = 2.4\times10^{4}\ \text{mi}\cdot\text{s/ h}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = 6.7\ \text{mi}$$

Finalize There are a variety of ways that the area under the curve in part (c) could be approximated, so the results may vary a bit depending on how this approximation is made.

Answers: (a) From –50 s to 0, the Acela is cruising at a constant positive velocity in the +*x* direction. Between 0 and 50 s, the constant velocity is maintained for a while and then the train begins to speed up. Between 50s and 100s, the train speeds up; notice that the graph is linear in this segment. Between 100 s and 200 s, the train reaches its maximum speed of about 175 mi/h. Between 200 s and 300 s, the engineer applies the brakes: the train is slowing down. The train is moving very slowly between 300 s and 350 s and then stops at 350 s. After 350 s, the train reverses direction: the velocity is negative. The train is backing up faster and faster until the data ends at 400 s. (b) 0.87 m/s^2 (c) 6.7 mi (answers may vary, depending on estimation from the graph.)

Section 2.1 Position, Velocity, and Speed

P2.1 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2\ \text{m}}{100\ \text{m/s}} = 2\times10^{-2}\ \text{s} = \boxed{0.02\ \text{s}}$$

P2.2 We substitute for t in $x = 10t^2$, then use the definition of average velocity:

t (s)	2.00	2.10	3.00
x (m)	40.0	44.1	90.0

(a) $$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{90.0 \text{ m} - 40.0 \text{ m}}{1.00 \text{ s}} = \frac{50.0 \text{ m}}{1.00 \text{ s}} = \boxed{50.0 \text{ m/s}}$$

(b) $$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{44.1 \text{ m} - 40.0 \text{ m}}{0.100 \text{ s}} = \frac{4.10 \text{ m}}{0.100 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

P2.3 We read the data from the table provided, assume three significant figures of precision for all the numbers, and use Equation 2.2 for the definition of average velocity.

(a) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2.30 \text{ m} - 0 \text{ m}}{1.00 \text{ s}} = \boxed{2.30 \text{ m/s}}$$

(b) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$$

Section 2.2 Instantaneous Velocity and Speed

P2.4 We use the definition of average velocity.

(a) $$v_{1,x,\text{ave}} = \frac{(\Delta x)_1}{(\Delta t)_1} = \frac{L - 0}{t_1} = \boxed{+L / t_1}$$

(b) $$v_{2,x,\text{ave}} = \frac{(\Delta x)_2}{(\Delta t)_2} = \frac{0 - L}{t_2} = \boxed{-L / t_2}$$

(c) To find the average velocity for the round trip, we add the displacement and time for each of the two halves of the swim:

$$v_{x,\text{ave,total}} = \frac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \frac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \frac{+L - L}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \boxed{0}$$

(d) The average speed of the round trip is the total distance the athlete travels divided by the total time for the trip:

$$v_{\text{ave,trip}} = \frac{\text{total distance traveled}}{(\Delta t)_{\text{total}}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1 + t_2}$$

$$= \frac{|+L| + |-L|}{t_1 + t_2} = \boxed{\frac{2L}{t_1 + t_2}}$$

P2.5 For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.

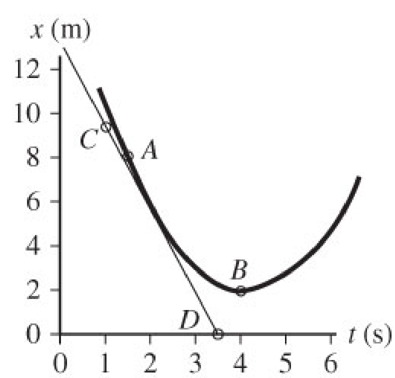
We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\text{ m}}{(4.0 - 1.5)\text{ s}}$$

$$= -\frac{6.0\text{ m}}{2.5\text{ s}} = \boxed{-2.4\text{ m/s}}$$



ANS. FIG. P2.5

(b) The slope of the tangent line can be found from points *C* and *D*. (t_C = 1.0 s, x_C = 9.5 m) and (t_D = 3.5 s, x_D = 0),

$$v \approx \boxed{-3.8 \text{ m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative *x* direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where *x* has its minimum value. This is at $t \approx \boxed{4.0 \text{ s}}$.

Section 2.3 Analysis Model: Particle Under Constant Velocity

P2.6 The trip has two parts: first the car travels at constant speed v_1 for distance *d*, then it travels at constant speed v_2 for distance *d*. The first part takes the time interval $\Delta t_1 = d/v_1$, and the second part takes the time interval $\Delta t_2 = d/v_2$.

(a) By definition, the average velocity for the entire trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = 2d$, and $\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$. Putting these together, we have

$$v_{\text{avg}} = \left(\frac{\Delta d}{\Delta t}\right) = \left(\frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}\right) = \left(\frac{2d}{d/v_1 + d/v_2}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right)$$

We know v_{avg} = 30 mi/h and v_1 = 60 mi/h.

Solving for v_2 gives

$$(v_1 + v_2)v_{avg} = 2v_1 v_2 \rightarrow v_2 = \left(\frac{v_1 v_{avg}}{2v_1 - v_{avg}}\right).$$

$$v_2 = \left[\frac{(30 \text{ mi/h})(60 \text{ mi/h})}{2(60 \text{ mi/h}) - (30 \text{ mi/h})}\right] = \boxed{20 \text{ mi/h}}$$

(b) The average velocity for this trip is $v_{avg} = \Delta x / \Delta t$, where

$\Delta x = \Delta x_1 + \Delta x_2 = d + (-d) = 0$; so, $v_{avg} = \boxed{0}$.

(c) The average speed for this trip is $v_{avg} = d / \Delta t$, where $d = d_1 + d_2 =$

$d + d = 2d$ and $\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$; so, the average speed is the

same as in part (a): $v_{avg} = \boxed{30 \text{ mi/h.}}$

P2.7 (a) The total time for the trip is $t_{total} = t_1 + 22.0$ min $= t_1 + 0.367$ h, where t_1 is the time spent traveling at $v_1 = 89.5$ km/h. Thus, the distance traveled is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, which gives

$$\begin{aligned}(89.5 \text{ km/h})t_1 &= (77.8 \text{ km/h})(t_1 + 0.367 \text{ h}) \\ &= (77.8 \text{ km/h})t_1 + 28.5 \text{ km}\end{aligned}$$

or $\quad (89.5 \text{ km/h} - 77.8 \text{ km/h})t_1 = 28.5 \text{ km}$

from which, $t_1 = 2.44$ h, for a total time of

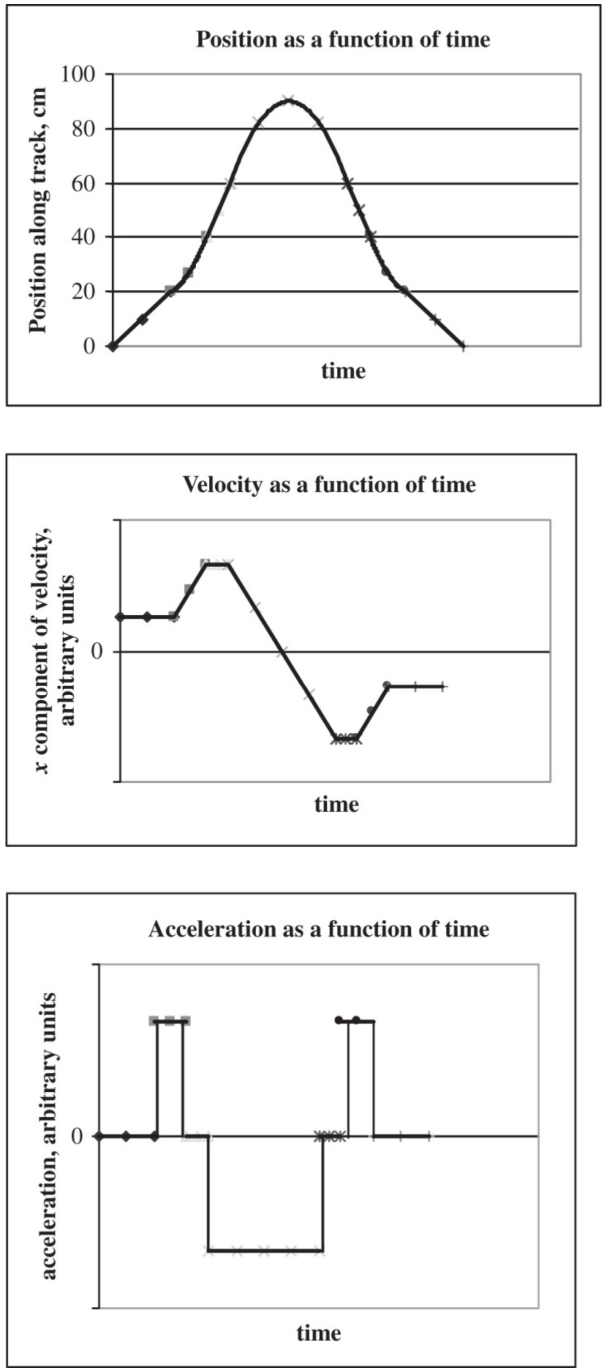
$$t_{total} = t_1 + 0.367 \text{ h} = \boxed{2.81 \text{ h}}$$

(b) The distance traveled during the trip is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, giving

$$\Delta x = v_{avg} t_{total} = (77.8 \text{ km/h})(2.81 \text{ h}) = \boxed{219 \text{ km}}$$

Section 2.5 Acceleration

P2.8 The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.



ANS. FIG. P2.8

P2.9 (a) In the interval $t_i = 0$ s and $t_f = 6.00$ s, the motorcyclist's velocity changes from $v_i = 0$ to $v_f = 8.00$ m/s. Then,

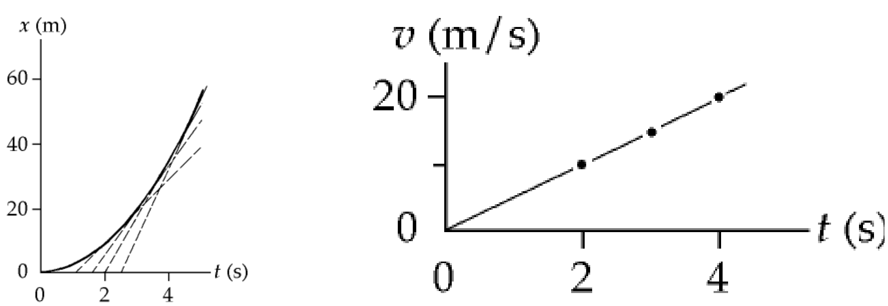
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{8.0\text{ m/s} - 0}{6.0\text{ s} - 0} = \boxed{1.3\text{ m/s}^2}$$

(b) Maximum positive acceleration occurs when the slope of the velocity-time curve is greatest, at t = 3 s, and is equal to the slope of the graph, approximately (6 m/s – 2 m/s)/(4 s – 2 s) = $\boxed{2\ \text{m/s}^2}$.

(c) The acceleration $a = 0$ when the slope of the velocity-time graph is zero, which occurs at $\boxed{t = 6\ \text{s}}$, and also for $\boxed{t > 10\ \text{s}}$.

(d) Maximum negative acceleration occurs when the velocity-time graph has its maximum negative slope, at t = 8 s, and is equal to the slope of the graph, approximately $\boxed{-1.5\ \text{m/s}^2}$.

P2.10(a) The graph is shown in ANS. FIG. P2.10 below.



ANS. FIG. P2.10

(b) At t = 5.0 s, the slope is $v \approx \dfrac{58\ \text{m}}{2.5\ \text{s}} \approx \boxed{23\ \text{m/s}}$.

At t = 4.0 s, the slope is $v \approx \dfrac{54\ \text{m}}{3\ \text{s}} \approx \boxed{18\ \text{m/s}}$.

At t = 3.0 s, the slope is $v \approx \dfrac{49\ \text{m}}{3.4\ \text{s}} \approx \boxed{14\ \text{m/s}}$.

At t = 2.0 s, the slope is $v \approx \dfrac{36\ \text{m}}{4.0\ \text{s}} \approx \boxed{9.0\ \text{m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \approx \dfrac{23\ \text{m/s}}{5.0\ \text{s}} \approx \boxed{4.6\ \text{m/s}^2}$

(d) The initial velocity of the car was $\boxed{\text{zero}}$.

P2.11 (a) The area under a graph of a vs. t is equal to the change in velocity, Δv. We can use Figure P2.11 to find the change in velocity during specific time intervals.

The area under the curve for the time interval 0 to 10 s has the shape of a rectangle. Its area is

$$\Delta v = (2\text{ m/s}^2)(10\text{ s}) = 20\text{ m/s}$$

The particle starts from rest, $v_0 = 0$, so its velocity at the end of the 10-s time interval is

$$v = v_0 + \Delta v = 0 + 20\text{ m/s} = \boxed{20\text{ m/s}}$$

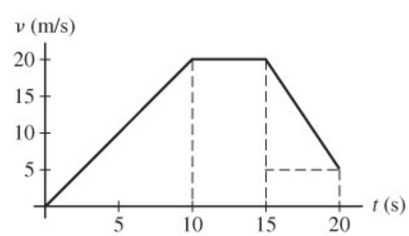
Between $t = 10$ s and $t = 15$ s, the area is zero: $\Delta v = 0$ m/s.

Between $t = 15$ s and $t = 20$ s, the area is a rectangle: $\Delta v =$ $(-3\text{ m/s}^2)(5\text{ s}) = -15\text{ m/s}$.

So, between $t = 0$ s and $t = 20$ s, the total area is $\Delta v = (20\text{ m/s}) + (0\text{ m/s}) + (-15\text{ m/s}) = 5\text{ m/s}$, and the velocity at $t = 20$ s is $\boxed{5\text{ m/s.}}$

(b) We can use the information we derived in part (a) to construct a graph of x vs. t; the area under such a graph is equal to the displacement, Δx, of the particle.

From (a), we have these points (t, v) = (0 s, 0 m/s), (10 s, 20 m/s), (15 s, 20 m/s), and (20 s, 5 m/s). The graph appears below.



The displacements are:

0 to 10 s (area of triangle): $\Delta x = (1/2)(20\text{ m/s})(10\text{ s}) = 100\text{ m}$

10 to 15 s (area of rectangle): $\Delta x = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$

15 to 20 s (area of triangle and rectangle):

$$\Delta x = (1/2)[(20 - 5)\text{ m/s}](5\text{ s}) + (5\text{ m/s})(5\text{ s})$$

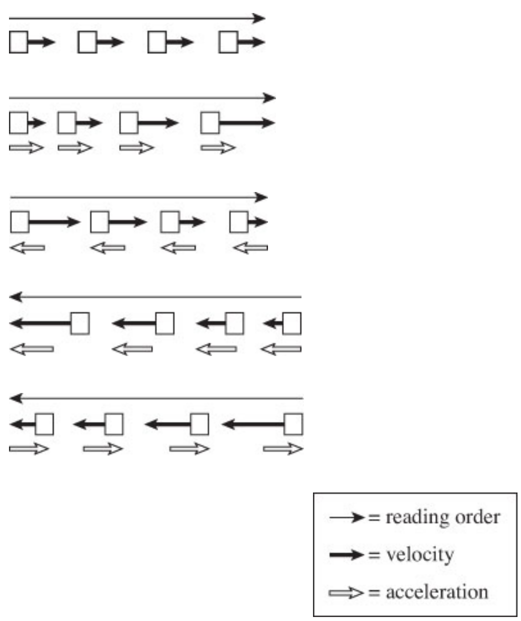
$$= 37.5\text{ m} + 25\text{ m} = 62.5\text{ m}$$

Total displacement over the first 20.0 s:

$$\Delta x = 100\text{ m} + 100\text{ m} + 62.5\text{ m} = 262.5\text{ m} = \boxed{263\text{ m}}$$

Section 2.6 Motion Diagrams

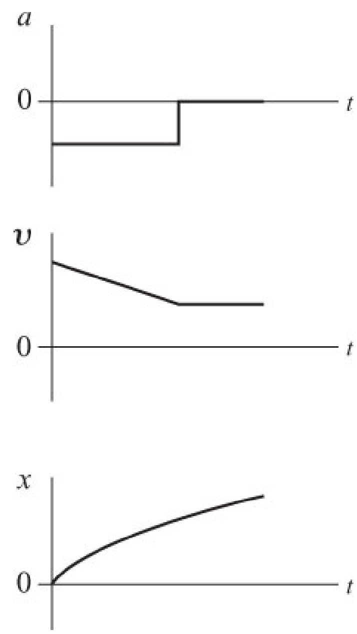
P2.12 (a) (b) (c) (d) (e)



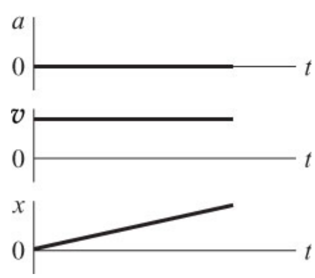
(f) One way of phrasing the answer: The spacing of the successive positions would change with **less regularity.**

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

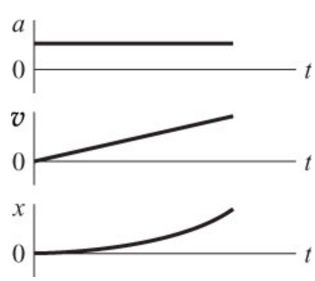
P2.13 (a) The motion is fast at first but slowing until the speed is constant. We assume the acceleration is constant as the object slows.



(b) The motion is constant in speed.



(c) The motion is speeding up, and we suppose the acceleration is constant.



Section 2.7 Analysis Model: Particle Under Constant Acceleration

P2.14 We have $v_i = 2.00 \times 10^4 \text{ m/s}$, $v_f = 6.00 \times 10^6 \text{ m/s}$, and $x_f - x_i = 1.50 \times 10^{-2} \text{ m}$.

(a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$:

$$t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}}$$
$$= \boxed{4.98 \times 10^{-9} \text{ s}}$$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})}$$
$$= \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

P2.15 In parts (a) – (c), we use Equation 2.13 to determine the velocity at the times indicated.

(a) The time given is 1.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0 \text{ m/s} + (-4.00 \text{ m/s}^2)(1.00 \text{ s}) = \boxed{9.00 \text{ m/s}}$$

(b) The time given is 4.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0 \text{ m/s} + (-4.00 \text{ m/s}^2)(4.00 \text{ s}) = \boxed{-3.00 \text{ m/s}}$$

(c) The time given is 1.00 s before 10:05:00 a.m., so

$$v_f = v_i + at = 13.0 \text{ m/s} + (-4.00 \text{ m/s}^2)(-1.00 \text{ s}) = \boxed{17.0 \text{ m/s}}$$

(d) The graph of velocity versus time is a slanting straight line, having the value 13.0 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4.00 m/s for every second thereafter.

(e) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants

P2.16 We think of the plane moving with maximum-size backward acceleration throughout the landing, so the acceleration is constant, the stopping time a minimum, and the stopping distance as short as it can be. The negative acceleration of the plane as it lands can be called deceleration, but it is simpler to use the single general term *acceleration* for all rates of velocity change.

(a) The plane can be modeled as a particle under constant acceleration, with $a_x = -5.00 \text{ m/s}^2$. Given $v_{xi} = 100 \text{ m/s}$ and $v_{xf} = 0$, we use the equation $v_{xf} = v_{xi} + a_x t$ and solve for t:

$$t = \frac{v_{xf} - v_{xi}}{a_x} = \frac{0 - 100 \text{ m/s}}{-5.00 \text{ m/s}^2} = \boxed{20.0 \text{ s}}$$

(b) Find the required stopping distance and compare this to the length of the runway. Taking x_i to be zero, we get

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

or $$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - (100 \text{ m/s})^2}{2(-5.00 \text{ m/s}^2)} = \boxed{1\,000 \text{ m}}$$

(c) The stopping distance is greater than the length of the runway; $\boxed{\text{the plane cannot land}}$.

P2.17 The velocity is always changing; there is always nonzero acceleration and the problem says it is constant. So we can use one of the set of equations describing constant-acceleration motion. Take the initial point to be the moment when x_i= 3.00 cmand v_{xi} = 12.0 cm/s. Also, at t = 2.00 s, x_f = –5.00 cm.

Once you have classified the object as a particle moving with constant acceleration and have the standard set of four equations in front of you, how do you choose which equation to use? Make a list of all of the six symbols in the equations: x_i, x_f, v_{xi}, v_{xf}, a_x, and t. On the list fill in values as above, showing that x_i, x_f, v_{xi}, and t are known. Identify a_x as the unknown. Choose an equation involving only one unknown and the knowns. That is, choose an equation *not* involving v_{xf}. Thus we choose the kinematic equation

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

and solve for a_x:

$$a_x = \frac{2\left[x_f - x_i - v_{xi}t\right]}{t^2}$$

We substitute:

$$a_x = \frac{2[-5.00\ \text{cm} - 3.00\ \text{cm} - (12.0\ \text{cm/s})(2.00\ \text{s})]}{(2.00\ \text{s})^2}$$

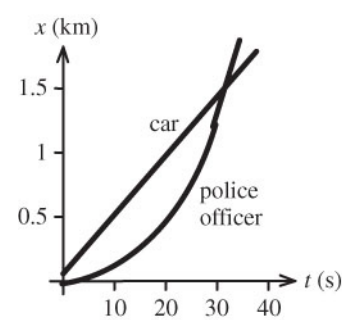
$$= \boxed{-16.0\ \text{cm/s}^2}$$

P2.18 As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and $$x_{\text{trooper}} = 1.5t^2$$

They intersect at $t = \boxed{31\ \text{s}}$.



ANS. FIG. P2.18

P2.19 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by

$v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.20 We ask whether the constant acceleration of the rhinoceros from rest over a period of 10.0 s can result in a final velocity of 8.00 m/s and a displacement of 50.0 m? To check, we solve for the acceleration in two ways.

1) $t_i = 0$, $v_i = 0$; $t = 10.0$ s, $v_f = 8.00$ m/s:

$$v_f = v_i + at \rightarrow a = \frac{v_f}{t}$$

$$a = \frac{8.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$$

2) $t_i = 0$, $x_i = 0$, $v_i = 0$; $t = 10.0$ s, $x_f = 50.0$ m:

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \rightarrow x_f = \frac{1}{2}at^2$$

$$a = \frac{2x_f}{t^2} = \frac{2(50.0 \text{ m})}{(10.0 \text{ s})^2} = 1.00 \text{ m/s}^2$$

$\boxed{\text{The accelerations do not match, therefore the situation is impossible.}}$

P2.21 (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and

$t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7 \text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between 0.628 + 1.39 = 2.02 s and 0.628 + 1.39 + 0.431 = 2.45 s is

12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from

$$\left[(28.8 - 19.7) \text{ cm/s}\right] / \left[(2.23 - 0.314) \text{ s}\right] = \boxed{4.70 \text{ cm/s}^2}$$

(c) The distance between A and B is not used, but the length of the glider is used to find the average velocity during a known time interval.

P2.22 Take any two of the standard four equations, such as

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

Solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_{xf} - a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t$$

Thus

$$\boxed{x_f - x_i = v_{xf} t - \frac{1}{2} a_x t^2}$$

We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$, this equation represents that displacement is a quadratic function of time.

P2.23 (a) For the first car, the speed as a function of time is

$$v_1 = v_{1i} + a_1 t = -3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)t$$

For the second car, the speed is

$$v_2 = v_{2i} + a_2 t = +5.5 \text{ cm/s} + 0$$

Setting the two expressions equal gives

$$-3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)t = 5.5 \text{ cm/s}$$

Solving for t gives

$$t = \frac{9.00 \text{ cm/s}}{2.40 \text{ cm/s}^2} = \boxed{3.75 \text{ s}}$$

(b) The first car then has speed

$$v_1 = v_{1i} + a_1 t = -3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)(3.75 \text{ s}) = \boxed{5.50 \text{ cm/s}}$$

and this is also the constant speed of the second car.

(c) For the first car, the position as a function of time is

$$\begin{aligned} x_1 &= x_{1i} + v_{1i}t + \frac{1}{2}a_1 t^2 \\ &= 15.0 \text{ cm} - (3.50 \text{ cm/s})t + \frac{1}{2}\left(2.40 \text{ cm/s}^2\right)t^2 \end{aligned}$$

For the second car, the position is

$$x_2 = 10.0 \text{ cm} + (5.50 \text{ cm/s})t$$

At the point where the cars pass one another, their positions are equal:

$$15.0 \text{ cm} - (3.50 \text{ cm/s})t + \frac{1}{2}\left(2.40 \text{ cm/s}^2\right)t^2 = 10.0 \text{ cm} + (5.50 \text{ cm/s})t$$

rearranging gives

$$\left(1.20 \text{ cm/s}^2\right)t^2 - (9.00 \text{ cm/s})t + 5.00 \text{ cm} = 0$$

We solve this with the quadratic formula. Suppressing units,

$$t = \frac{9 \pm \sqrt{(9)^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 \pm \sqrt{57}}{2.4} = 6.90 \text{ s, or } \boxed{0.604 \text{ s}}$$

(d) At $t = 0.604$ s, the second and also the first car's position is

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(0.604 \text{ s}) = \boxed{13.3 \text{ cm}}$$

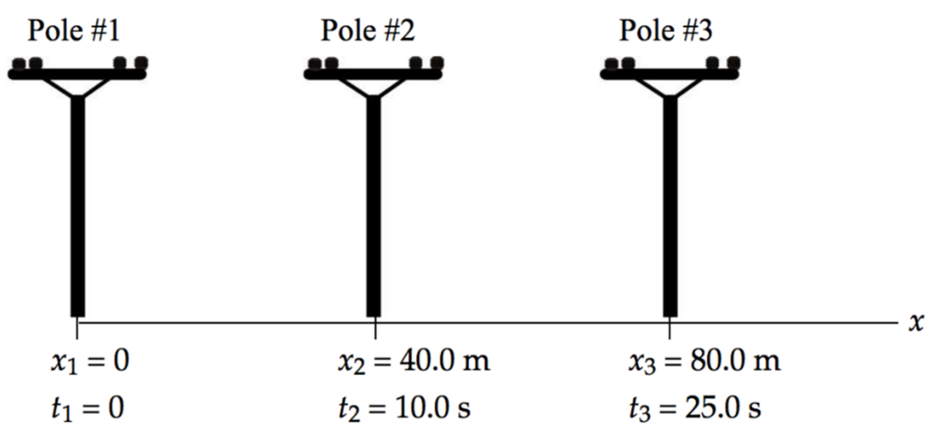
At $t = 6.90$ s, both are at position

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(6.90 \text{ s}) = \boxed{47.9 \text{ cm}}$$

(e) The cars are initially moving toward each other, so they soon arrive at the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car, but at this time the accelerating car is far behind the steadily moving car; thus, the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, but passing it at higher speed, and giving another answer to (c) that is not an answer to (a).

***P2.24** **Conceptualize** A diagram of the information provided will be helpful. The diagram below shows three positions of the car and the times at which the car is at those positions.

Categorize The phrase "slowing down the car uniformly" tells us to use the particle under constant acceleration analysis model for this problem.



Analyze We have no information about the velocity of the car, so that suggests we focus on the equation in the particle under constant acceleration model involving position: $x_f = x_i + v_i t + \frac{1}{2}at^2$. (a) Write the position equation twice, once for the car at pole #2 and once at pole #3, each referencing pole #1 as the initial situation:

$$x_2 = x_1 + v_1 t_2 + \tfrac{1}{2}at_2^2 = 0 + v_1 t_2 + \tfrac{1}{2}at_2^2 = v_1 t_2 + \tfrac{1}{2}at_2^2 \qquad (1)$$

$$x_3 = x_1 + v_1 t_3 + \tfrac{1}{2}at_3^2 = 0 + v_1 t_3 + \tfrac{1}{2}at_3^2 = v_1 t_3 + \tfrac{1}{2}at_3^2 \qquad (2)$$

Eliminate v_1 between Equations (1) and (2) and solve for the acceleration a:

$$a = \frac{2\left(x_3 t_2 - x_2 t_3\right)}{t_3^2 t_2 - t_2^2 t_3} \qquad (3)$$

Substitute numerical values:

$$a = \frac{2\left[(80.0\text{ m})(10.0\text{ s}) - (40.0\text{ m})(25.0\text{ s})\right]}{(25.0\text{ s})^2(10.0\text{ s}) - (10.0\text{ s})^2(25.0\text{ s})} = -0.107\text{ m/ s}^2$$

(b) Solve Equation (1) for the initial speed of the car:

$$x_2 = v_1 t_2 + \tfrac{1}{2}at_2^2 \quad \rightarrow \quad v_1 = \frac{x_2 - \frac{1}{2}at_2^2}{t_2} \qquad (4)$$

Substitute numerical values:

$$v_1 = \frac{40.0\text{ m} - \frac{1}{2}\left(-0.107\text{ m/ s}^2\right)(10.0\text{ s})^2}{(10.0\text{ s})} = 4.53\text{ m/ s}$$

(c) Using Equation 2.17 in the particle under constant acceleration model relating velocity and position, solve for the position at which the velocity goes to zero:

$$v_f^2 = v_1^2 + 2ax_f \quad \rightarrow \quad x_f = \frac{v_f^2 - v_1^2}{2a} = \frac{0 - (4.53\text{ m/ s})^2}{2\left(-0.107\text{ m/ s}^2\right)} = 96.3\text{ m}$$

Because the fourth pole would be at x = 120 m, the last pole passed is pole #3.

Finalize Part (c) can also be solved less directly by finding the time at which the velocity goes to zero and then substituting into the position equation to find the position at that time. In doing so, you find the time at which the car stops to be 42.5 s. It might be surprising that the car travels for an additional 42.5 – 25.0 = 17.5 s beyond pole #3 (longer than between poles #1 and #2, and between poles #2 and #3), but only moves 96.3 – 80.0 m = 16.3 m beyond that pole.

Answers: (a)–0.107 m/s^2(b) 4.53 m/s (c) pole #3 (stops at x = 96.3 m at t = 42.4 s

Section 2.8 Freely Falling Objects

P2.25 The bill starts from rest, $v_i = 0$, and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). For an average human reaction time of about 0.20 s, we can find the distance the bill will fall:

$$y_f = y_i + v_i t + \frac{1}{2}at^2 \rightarrow \Delta y = v_i t - \frac{1}{2}gt^2$$
$$\Delta y = 0 - \frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.20 \text{ s})^2 = -0.20 \text{ m}$$

The bill falls about 20 cm—this distance is about twice the distance between the center of the bill and its top edge, about 8 cm. Thus

David could not respond fast enough to catch the bill.

P2.26 We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$= (7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(3.65 \text{ m} - 1.55 \text{ m})$$
$$= 13.6 \text{ m}^2/\text{s}^2$$

which gives $v_f = 3.69 \text{ m/s}$.

$\boxed{\text{So the rock does reach the top of the wall with } v_f = 3.69 \text{ m/s}}$.

(c) The rock travels from $y_i = 3.65$ m to $y_f = 1.55$ m. We find the final speed of the rock thrown down:

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$= (-7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(1.55 \text{ m} - 3.65 \text{ m})$$
$$= 95.9 \text{ m}^2/\text{s}^2$$

which gives $v_f = -9.79 \text{ m/s}$.

The change in speed of the rock thrown down is

$$|9.79 \text{ m/s} - 7.40 \text{ m/s}| = \boxed{2.39 \text{ m/s}}$$

(d) The magnitude of the speed change of the rock thrown up is

$|7.40 \text{ m/s} - 3.69 \text{ m/s}| = 3.71 \text{ m/s}$. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

(e) $\boxed{\text{The upward-moving rock spends more time in flight because its average speed is smaller than the downward-moving rock, so the rock has more time to change its speed.}}$

P2.27 We are given the height of the helicopter: $y = h = 3.00t^3$.

At $t = 2.00$ s, $y = 3.00(2.00 \text{ s})^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36.0 m/s upward. The equation of motion of the mailbag is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$
$$y_f = (24.0 \text{ m}) + (36.0 \text{ m/s})t - \left(4.90 \text{ m/s}^2\right)t^2$$

Setting $y_f = 0$, dropping units, and rearranging the equation, we have

$$4.90t^2 - 36.0t - 24.0 = 0$$

We solve for t using the quadratic formula:

$$t = \frac{36.0 \pm \sqrt{(-36.0)^2 - 4(4.90)(-24.0)}}{2(4.90)}$$

Since only positive values of t count, we find $t = \boxed{7.96 \text{ s}}$.

P2.28 The falling ball moves a distance of (15 m – h) before they meet, where h is the height above the ground where they meet. We apply

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

to the falling ball to obtain

$$-(15.0 \text{ m} - h) = -\frac{1}{2}gt^2$$

or $$h = 15.0 \text{ m} - \frac{1}{2}gt^2 \qquad \textbf{[1]}$$

Applying $y_f = y_i + v_i t - \frac{1}{2}gt^2$ to the rising ball gives

$$h = (25 \text{ m/s})t - \frac{1}{2}gt^2 \qquad \textbf{[2]}$$

Combining equations [1] and [2] gives

$$(25\text{ m/s})t - \frac{1}{2}gt^2 = 15.0\text{ m} - \frac{1}{2}gt^2$$

or $$t = \frac{15\text{ m}}{25\text{ m/s}} = \boxed{0.60\text{ s}}$$

P2.29 We model the keys as a particle under the constant free-fall acceleration. Take the first student's position to be $y_i = 0$ and the second student's position to be $y_f = 4.00\text{ m}$. We are given that the time of flight of the keys is $t = 1.50$ s, and $a_y = -9.80\text{ m/s}^2$.

(a) We choose the equation $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ to connect the data and the unknown.

We solve:

$$v_{yi} = \frac{y_f - y_i - \frac{1}{2}a_y t^2}{t}$$

and substitute:

$$v_{yi} = \frac{4.00\text{ m} - \frac{1}{2}(-9.80\text{ m/s}^2)(1.50\text{ s})^2}{1.50\text{ s}} = \boxed{10.0\text{ m/s}}$$

(b) The velocity at any time $t > 0$ is given by $v_{yf} = v_{yi} + a_y t$.

Therefore, at $t = 1.50$ s,

$$v_{yf} = 10.0\text{ m/s} - (9.80\text{ m/s}^2)(1.50\text{ s}) = \boxed{-4.68\text{ m/s}}$$

The negative sign means that the keys are moving **downward** just before they are caught.

P2.30 (a) The keys, moving freely under the influence of gravity ($a = -g$), undergo a vertical displacement of $\Delta y = +h$ in time t. We use $\Delta y = v_i t + \frac{1}{2}at^2$ to find the initial velocity as

$$\Delta y = v_i t + \frac{1}{2}at^2 = h$$
$$\rightarrow h = v_i t - \frac{1}{2}gt^2$$
$$v_i = \frac{h + \frac{1}{2}gt^2}{t} = \boxed{\frac{h}{t} + \frac{gt}{2}}$$

(b) We find the velocity of the keys just before they were caught (at time t) using $v = v_i + at$:

$$v = v_i + at$$
$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) - gt$$
$$v = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

***P2.31** **Conceptualize** This is a simple situation in which an object is thrown straight upward and is in free fall after being thrown. We wish to relate the speed of the throw and the height of the projectile.

Categorize We recognize a simple one-dimensional free-fall problem, in which we use the *particle under constant acceleration* model for the motion of the box.

Analyze (a) From the particle under constant acceleration model, chose the equation that relates position and equation (Equation 2.17), noting that the motion is in the y direction:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right) \qquad (1)$$

Consider the point at which the box reaches its highest position. Note that the velocity of the box at that position is $v_{yf} = 0$, and define the point at which the box is thrown as the origin, so that $y_i = 0$. Also assign the acceleration to be that due to gravity, so that Equation (1) becomes

$$0 = v_{yi}^2 - 2g\left(y_f - 0\right) \quad \rightarrow \quad y_f = \frac{v_{yi}^2}{2g} \qquad (2)$$

Find the highest position for the box if thrown upward at the speed of 20.0 m/s found in the demonstration:

$$y_f = \frac{(20.0\text{ m/s})^2}{2\left(9.80\text{ m/s}^2\right)} = 20.4\text{ m}$$

Because this position is higher than the bottom of the window in which the alleged accomplice caught the box, the action of throwing the box to the accomplice is possible.

(b) The defense attorney could argue as follows: The demonstration involved throwing a baseball horizontally. It is more difficult to throw something upward, so the throwing speed could be in error. You might counter-argue that the throwing speed could actually be higher because the defendant was attempting to minimize his throwing speed during the demonstration in order to show that the feat could not be done.

Answers: (a) The box could reach the window according to the data provided.(b) Answers will vary.

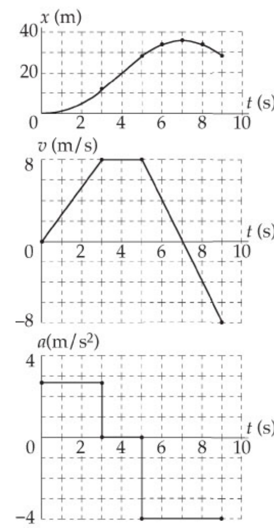
Section 2.9 Kinematic Equations Derived from Calculus

P2.32 (a) See the x vs. t graph on the top panel of ANS. FIG. P2.32.Choose $x = 0$ at $t = 0$.

$$\text{At } t = 3 \text{ s}, x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12 \text{ m}.$$

$$\text{At } t = 5 \text{ s}, x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28 \text{ m}.$$

$$\text{At } t = 7 \text{ s}, x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s}) = 36 \text{ m}$$



ANS. FIG. P2.32

(b) See the a vs. t graph at the bottom right.

$$\text{For } 0 < t < 3 \text{ s}, a = \frac{8 \text{ m/s}}{3 \text{ s}} = 2.67 \text{ m/s}^2.$$

For $3 < t < 5$ s, $a = 0$.

At the points of inflection, $t = 3$ and 5 s, the slope of the velocity curve changes abruptly, so the acceleration is not defined.

(c) For 5 s $< t <$ 9 s,

$$a = -\frac{16 \text{ m/s}}{4 \text{ s}} = \boxed{-4 \text{ m/s}^2}$$

(d) The average velocity between $t = 5$ and 7 s is

$v_{avg} = (8 \text{ m/s} + 0)/2 = 4 \text{ m/s}$

$$\text{At } t = 6 \text{ s}, x = 28 \text{ m} + (4 \text{ m/s})(1 \text{ s}) = \boxed{32 \text{ m}}$$

(e) The average velocity between $t = 5$ and 9 s is

$$v_{avg} = [(8 \text{ m/s}) + (-8 \text{ m/s})]/2 = 0 \text{ m/s}$$

At $t = 9$ s, $x = 28 \text{ m} + (0 \text{ m/s})(1 \text{ s}) = \boxed{28 \text{ m}}$

P2.33 This is a derivation problem. We start from basic definitions. We are given $J = da_x/dt$= constant, so we know that $da_x = Jdt$.

(a) Integrating from the 'initial' moment when we know the acceleration to any later moment,

$$\int_{a_{ix}}^{a_x} da = \int_0^t J\, dt \quad \rightarrow \quad a_x - a_{ix} = J(t-0)$$

Therefore, $\boxed{a_x = Jt + a_{xi}}$.

From $a_x = dv_x/dt$, $dv_x = a_x dt$.

Integration between the same two points tells us the velocity as a function of time:

$$\int_{v_{xi}}^{v_x} dv_x = \int_0^t a_x\, dt = \int_0^t (a_{xi} + Jt)\, dt$$

$$v_x - v_{xi} = a_{xi}t + \frac{1}{2}Jt^2 \quad \text{or} \quad \boxed{v_x = v_{xi} + a_{xi}t + \frac{1}{2}Jt^2}$$

From $v_x = dx/dt$, $dx = v_x dt$. Integrating a third time gives us $x(t)$:

$$\int_{x_i}^{x} dx = \int_0^t v_x\, dt = \int_0^t (v_{xi} + a_{xi}t + \frac{1}{2}Jt^2)\ dt$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_{xi}t^2 + \frac{1}{6}Jt^3$$

and $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_{xi}t^2 + v_{xi}t + x_i}$.

(b) Squaring the acceleration,

$$a_x^2 = (Jt + a_{xi})^2 = J^2t^2 + a_{xi}^2 + 2Ja_{xi}t$$

Rearranging,

$$a_x^2 = a_{xi}^2 + 2J\left(\frac{1}{2}Jt^2 + a_{xi}t\right)$$

The expression for v_x was

$$v_x = \frac{1}{2}Jt^2 + a_{xi}t + v_{xi}$$

So $\quad (v_x - v_{xi}) = \frac{1}{2}Jt^2 + a_{xi}t$

and by substitution

$$\boxed{a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})}$$

Additional Problems

P2.34 (a) Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore, $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$,

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}$$

(b) The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

P2.35 (a) From $v^2 = v_i^2 + 2a\Delta y$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a(\Delta y)}$$
$$= \sqrt{0 + 2(4\ 000\ \text{m/s}^2)(2.00\times 10^{-3}\ \text{m})} = \boxed{4.00\ \text{m/s}}$$

(b) The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.00\ \text{m/s} - 0}{4\ 000\ \text{m/s}^2} = 1.00\times 10^{-3}\ \text{s} = \boxed{1.00\ \text{ms}}$$

(c) The upward displacement of the insect between when its feet leave the ground and its speed is momentarily zero is

$$\Delta y = \frac{v_f^2 - v_i^2}{2a}$$
$$\Delta y = \frac{0 - (4.00\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{0.816\ \text{m}}$$

P2.36 Take downward as the positive y direction.

(a) While the woman was in free fall, $\Delta y = 144$ ft, $v_i = 0$, and we take $a = g = 32.0\ \text{ft/s}^2$. Thus,

$$\Delta y = v_i t + \frac{1}{2}at^2 \rightarrow 144\ \text{ft} = 0 + (16.0\ \text{ft/s}^2)t^2$$

giving $t_{\text{fall}} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0\ \text{ft/s}^2)(3.00\ \text{s}) = \boxed{96.0\ \text{ft/s}}$$

(b) While crushing the box, $v_i = 96.0\ \text{ft/s}$, $v_f = 0$, and $\Delta y = 18.0\ \text{in.} = 1.50\ \text{ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0\ \text{ft/s})^2}{2(1.50\ \text{ft})} = -3.07\times 10^3\ \text{ft/s}^2$$

or $\boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}} = 96.0g.$

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\overline{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}$

P2.37 (a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, d.

(b) During a time interval t the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where d is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

giving

$$v_1 t - v_2 t = d \quad \text{or} \quad t = \boxed{\frac{d}{(v_1 - v_2)}}$$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance D between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time t calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2 t = v_2 \left[\frac{d}{(v_1 - v_2)}\right] \quad \text{or} \quad \boxed{d_2 = \frac{v_2 d}{v_1 - v_2}}$$

P2.38 For the collision not to occur, the front of the passenger train must not have a position that is equal to or greater than the position of the back of the freight train at any time. We can write expressions of position to see whether the front of the passenger car (P) meets the back of the freight car (F) at some time.

Assume at $t = 0$, the coordinate of the front of the passenger car is $x_{Pi} = 0$; and the coordinate of the back of the freight car is $x_{Fi} = 58.5$ m. At later time t, the coordinate of the front of the passenger car is

$$x_P = x_{Pi} + v_{Pi}t + \frac{1}{2}a_P t^2$$
$$x_P = (40.0 \text{ m/s})t + \frac{1}{2}(-3.00 \text{ m/s}^2)t^2$$

and the coordinate of the back of the freight car is

$$x_F = x_{Fi} + v_{Fi}t + \frac{1}{2}a_F t^2$$
$$x_F = 58.5 \text{ m} + (16.0 \text{ m/s})t$$

Setting these expression equal to each other gives

$$x_P = x_F$$
$$(40.0 \text{ m/s})t + \frac{1}{2}(-3.00 \text{ m/s}^2)t^2 = 58.5 \text{ m} + (16.0 \text{ m/s})t$$

or $$(1.50)t^2 + (-24.0)t + 58.5 = 0$$

after simplifying and suppressing units.

We do not have to solve this equation, we just want to check if a solution exists; if a solution does exist, then the trains collide. A solution does exist:

$$t = \frac{-(-24.0) \pm \sqrt{(-24.0)^2 - 4(1.50)(58.5)}}{2(1.50)}$$

$$t = \frac{24.0 \pm \sqrt{576 - 351}}{3.00} \to t = \frac{24.0 \pm \sqrt{225}}{3.00} = \frac{24.0 \pm 15}{3.00}$$

The situation is impossible since there is a finite time for which the front of the passenger train and the back of the freight train are at the same location.

P2.39 We have constant-acceleration equations to apply to the two cars separately.

(a) Let the times of travel for Hannah and Stan be $t_K$and t_S, where

$t_S = t_K + 1.00$ s

Both start from rest ($v_{xi,K} = v_{xi,S} = 0$), so the expressions for the distances traveled are

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90 \text{ m/s}^2)t_K^2$$

and $x_S = \frac{1}{2}a_{x,S}t_S^2 = \frac{1}{2}(3.50 \text{ m/s}^2)(t_K + 1.00 \text{ s})^2$

When Hannah overtakes Stan, the two distances will be equal. Setting $x_K = x_S$gives

$$\frac{1}{2}(4.90 \text{ m/s}^2)t_K^2 = \frac{1}{2}(3.50 \text{ m/s}^2)(t_K + 1.00 \text{ s})^2$$

This we simplify and write in the standard form of a quadratic as

$$t_K^2 - (5.00\ t_K)\text{s} - 2.50 \text{ s}^2 = 0$$

We solve using the quadratic formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, suppressing units, to find

$$t_K = \frac{5 \pm \sqrt{5^2 - 4(1)(-2.5)}}{2(1)} = \frac{5 + \sqrt{35}}{2} = \boxed{5.46 \text{ s}}$$

Only the positive root makes sense physically, because the overtake point must be after the starting point in time.

(b) Use the equation from part (a) for distance of travel,

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90 \text{ m/s}^2)(5.46 \text{ s})^2 = \boxed{73.0 \text{ m}}$$

(c) Remembering that $v_{xi,K} = v_{xi,S} = 0$, the final velocities will be:

$$v_{xf,K} = a_{x,K}t_K = (4.90 \text{ m/s}^2)(5.46 \text{ s}) = \boxed{26.7 \text{ m/s}}$$

$$v_{xf,S} = a_{x,S}t_S = (3.50 \text{ m/s}^2)(6.46 \text{ s}) = \boxed{22.6 \text{ m/s}}$$

P2.40 We translate from a pictorial representation through a geometric model to a mathematical representation by observing that the distances x and y are always related by $x^2 + y^2 = L^2$.

(a) Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now the unknown velocity of B is $\frac{dy}{dt} = v_B$ and $\frac{dx}{dt} = -v$,

so the differentiated equation becomes

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\left(\frac{x}{y}\right)(-v) = v_B$$

But $\dfrac{y}{x} = \tan\theta$, so $\qquad v_B = \boxed{\left(\dfrac{1}{\tan\theta}\right)v}$

(b) We assume that θ starts from zero. At this instant $1/\tan\theta$ is infinite, and the velocity of B is infinitely larger than that of A. As θ increases, the velocity of object B decreases, becoming equal to v when $\theta = 45°$. After that instant, B continues to slow down with non-constant acceleration, coming to rest as θ goes to 90°.

P2.41 The average speed of every point on the train as the first car passes Lisa is given by:

$$\frac{\Delta x}{\Delta t} = \frac{8.60\text{ m}}{1.50\text{ s}} = 5.73\text{ m/s}$$

The train has this as its instantaneous speed halfway through the 1.50-s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\dfrac{8.60\text{ m}}{1.10\text{ s}} = 7.82\text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\text{ s}) + \frac{1}{2}(1.10\text{ s}) = 1.30\text{ s}$$

so the acceleration is: $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{7.82\text{ m/s} - 5.73\text{ m/s}}{1.30\text{ s}} = \boxed{1.60\text{ m/s}^2}$

Challenge Problems

P2.42 (a) The factors to consider are as follows. The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. We do note, however, that points Ⓐ, Ⓑ, and Ⓒ are the vertices of a right triangle with ⒶⒸ as the hypotenuse.

(b) The red bead is a particle under constant acceleration. Taking downward as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $D = \frac{1}{2}g t_R^2$

which gives $\boxed{t_R = \sqrt{\frac{2D}{g}}}$.

(c) The blue bead is a particle under constant acceleration, with $a = g\sin\theta$. Taking the direction along L as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $L = \frac{1}{2}(g\sin\theta)t_B^2$

which gives $\boxed{t_B = \sqrt{\frac{2L}{g\sin\theta}}}$.

(d) For the two beads to reach point Ⓒ simultaneously, $t_R = t_B$. Then,

$$\sqrt{\frac{2D}{g}} = \sqrt{\frac{2L}{g\sin\theta}}$$

Squaring both sides and cross-multiplying gives

$$2gD\sin\theta = 2gL$$

or $\sin\theta = \frac{L}{D}$.

We note that the angle between chords ⒶⒸ and ⒷⒸ is $90° - \theta$, so that the angle between chords ⒶⒸ and ⒶⒷ is θ. Then, $\sin\theta = \frac{L}{D}$, and the beads arrive at point Ⓒ simultaneously.

(e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

P2.43 Consider the runners in general. Each completes the race in a total time interval T. Each runs at constant acceleration a for a time interval Δt, so each covers a distance (displacement) $\Delta x_a = \frac{1}{2}a\Delta t^2$ where they eventually reach a final speed (velocity) $v = a\Delta t$, after which they run at this constant speed for the remaining time $(T-\Delta t)$ until the end of the race, covering distance $\Delta x_v = v(T-\Delta t) = a\Delta t(T-\Delta t)$. The total distance (displacement) each covers is the same:

$$\begin{aligned}\Delta x &= \Delta x_a + \Delta x_v \\ &= \frac{1}{2}a\Delta t^2 + a\Delta t(T-\Delta t) \\ &= a\left[\frac{1}{2}\Delta t^2 + \Delta t(T-\Delta t)\right]\end{aligned}$$

so $$a = \frac{\Delta x}{\frac{1}{2}\Delta t^2 + \Delta t(T-\Delta t)}$$

where $\Delta x = 100\text{ m}$ and $T = 10.4$ s.

(a) For Laura (runner 1), $\Delta t_1 = 2.00$ s:

$$a_1 = (100\text{ m})/(18.8\text{ s}^2) = \boxed{5.32\text{ m/s}^2}$$

For Healan (runner 2), $\Delta t_2 = 3.00$ s:

$$a_2 = (100\text{ m})/(26.7\text{ s}^2) = \boxed{3.75\text{ m/s}^2}$$

(b) Laura (runner 1): $v_1 = a_1\Delta t_1 = \boxed{10.6\text{ m/s}}$

Healan (runner 2): $v_2 = a_2\Delta t_2 = \boxed{11.2\text{ m/s}}$

(c) The 6.00-s mark occurs after either time interval Δt. From the reasoning above, each has covered the distance

$$\Delta x = a\left[\frac{1}{2}\Delta t^2 + \Delta t(t-\Delta t)\right]$$

where $t = 6.00$ s.

Laura (runner 1): $\Delta x_1 = 53.19$ m

Healan (runner 2): $\Delta x_2 = 50.56$ m

$\boxed{\text{So, Laura is ahead by (53.19 m – 50.56 m) = 2.63 m.}}$

(d) Laura accelerates at the greater rate, so she will be ahead of Healen at, and immediately after, the 2.00-s mark. After the 3.00-s mark, Healan is travelling faster than Laura, so the distance between them will shrink. In the time interval from the 2.00-s mark to the 3.00-s mark, the distance between them will be the greatest.

During that time interval, the distance between them (the position of Laura relative to Healan) is

$$D = \Delta x_1 - \Delta x_2 = a_1\left[\frac{1}{2}\Delta t_1^2 + \Delta t_1\left(t - \Delta t_1\right)\right] - \frac{1}{2}a_2 t^2$$

because Laura has ceased to accelerate but Healan is still accelerating. Differentiating with respect to time, (and doing some simplification), we can solve for the time t when D is an maximum:

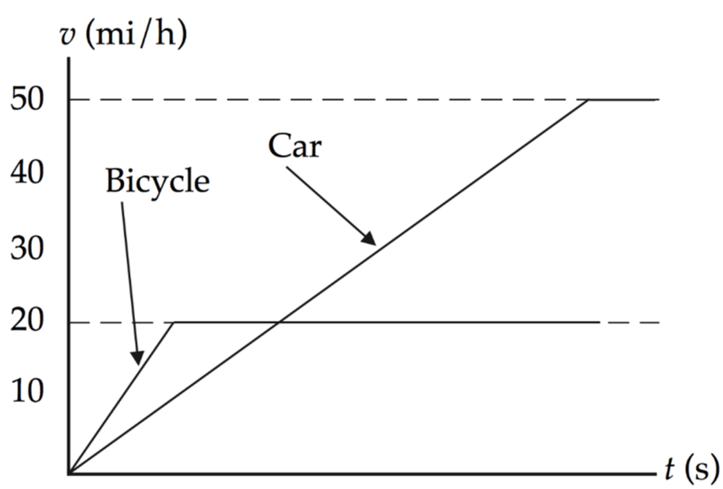
$$\frac{dD}{dt} = a_1\Delta t_1 - a_2 t = 0$$

which gives

$$t = \Delta t_1\left(\frac{a_1}{a_2}\right) = (2.00\ \text{s})\left(\frac{5.32\ \text{m/s}^2}{3.75\ \text{m/s}^2}\right) = 2.84\ \text{s}$$

Substituting this time back into the expression for D, we find that D = 4.47 m, that is, Laura ahead of Healan by 4.47 m.

***P2.44** **Conceptualize** You may have had this experience when driving. A bicycle can exhibit a high acceleration, so it can pull ahead of the car when leaving a traffic light. But the maximum speed of the bicycle is less than that of the car, so the car will eventually catch up and pass the bicycle. A graphical representation may be of some help here:



Because of the higher acceleration of the bicycle, the line representing its velocity is steeper in slope than that of the car. But it soon reaches its maximum speed of 20.0 mi/h, and the line for its velocity changes to horizontal. The slope of the line representing the acceleration of your car is less steep, but the acceleration lasts for a longer time, so that eventually your car is traveling faster than the bicycle.

Categorize The phrase "constant acceleration" tells us to use the *particle under constant acceleration* analysis model for this problem. Once the vehicles reach the indicated speeds, they travel at constant velocity, so we will also need to employ the *particle under constant velocity* analysis model.

Analyze Assuming that your car catches up with the bicycle before reaching its maximum speed, the motion of your car is described completely during the time interval of interest by the particle under constant acceleration model. Therefore, its position is given by

$$x_{\text{car}} = x_{\text{car},i} + v_{\text{car},i}t + \tfrac{1}{2}a_{\text{car}}t^2 = \tfrac{1}{2}a_{\text{car}}t^2 \quad (1)$$

where we have incorporated the fact that the car begins from rest ($v_{\text{car},i} = 0$), and have defined the position of the traffic light as the origin ($x_{\text{car},i} = 0$). The position of the bicycle is described by the particle under constant acceleration model until it reaches its maximum speed and by the particle under constant velocity model thereafter. Therefore, knowing that the bicycle also begins from the origin at rest,

$$x_{\text{bicycle}} = \begin{cases} \tfrac{1}{2}a_{\text{bicycle}}t^2 & (t < t_1) \quad (2) \\ \tfrac{1}{2}a_{\text{bicycle}}t_1^{\ 2} + v_{\text{bicycle,max}}(t - t_1) & (t > t_1) \quad (3) \end{cases}$$

where t_1 is the time at which the bicycle reaches its maximum speed and $v_{\text{bicyle,max}}$ is that maximum speed. As noted, the bicycle maintains that speed thereafter. The time at which the bicycle has a given speed is provided by the velocity equation in the particle under constant acceleration model:

$$v_{\text{bicycle}} = v_{\text{bicycle},i} + a_{\text{bicycle}}t = 0 + a_{\text{bicycle}}t \quad \rightarrow \quad t = \frac{v_{\text{bicycle}}}{a_{\text{bicycle}}} \qquad (4)$$

Evaluate this time when the bicycle first reaches its maximum speed:

$$t_1 = \frac{v_{\text{bicycle,max}}}{a_{\text{bicycle}}} \qquad (5)$$

(a) To find the time interval during which the bicycle is ahead of the car, we find the time at which the car and the bicycle are at the same position by using Equations (1) and (3):

$$x_{\text{bicycle}} = x_{\text{car}} \quad \rightarrow \quad \tfrac{1}{2}a_{\text{bicycle}}t_1^2 + v_{\text{bicycle,max}}\left(t - t_1\right) = \tfrac{1}{2}a_{\text{car}}t^2$$

Rearranging this equation gives the standard form of a quadratic equation:

$$\tfrac{1}{2}a_{\text{car}}t^2 - v_{\text{bicycle,max}}t - \left(\tfrac{1}{2}a_{\text{bicycle}}t_1^2 - v_{\text{bicycle,max}}t_1\right) = 0 \qquad (6)$$

Applying the quadratic formula, we can solve for *t* as follows:

$$t = \frac{v_{\text{bicycle,max}} \pm \sqrt{\left(-v_{\text{bicycle,max}}\right)^2 - 4\left(\tfrac{1}{2}a_{\text{car}}\right)\left[-\left(\tfrac{1}{2}a_{\text{bicycle}}t_1^2 - v_{\text{bicycle,max}}t_1\right)\right]}}{a_{\text{car}}}$$

$$= \frac{v_{\text{bicycle,max}} \pm \sqrt{\left(-v_{\text{bicycle,max}}\right)^2 + 2a_{\text{car}}\left(\tfrac{1}{2}a_{\text{bicycle}}t_1^2 - v_{\text{bicycle,max}}t_1\right)}}{a_{\text{car}}} \qquad (7)$$

Finally, substitute for t_1:

$$t = \frac{v_{\text{bicycle,max}} \pm \sqrt{\left(-v_{\text{bicycle,max}}\right)^2 + 2a_{\text{car}}\left[\frac{1}{2}a_{\text{bicycle}}\left(\frac{v_{\text{bicycle,max}}}{a_{\text{bicycle}}}\right)^2 - v_{\text{bicycle,max}}\left(\frac{v_{\text{bicycle,max}}}{a_{\text{bicycle}}}\right)\right]}}{a_{\text{car}}}$$

$$= \frac{v_{\text{bicycle,max}} \pm \sqrt{v_{\text{bicycle,max}}^2\left(1 - \frac{a_{\text{car}}}{a_{\text{bicycle}}}\right)}}{a_{\text{car}}}$$

Substitute numerical values:

$$t = \frac{20.0\text{ mi/ h} \pm \sqrt{(20.0\text{ mi/ h})^2\left[1 - \left(\frac{9.00\text{ mi/ h/ s}}{13.0\text{ mi/ h/ s}}\right)\right]}}{9.00\text{ mi/ h/ s}}$$

$$= \boxed{3.45\text{ s}}$$

(The other root is $t = 0.990$ s, but this turns out to be *less* than t_1, so it is not a realistic solution.)

(b) The speed of the your car at any time is found from the particle under constant acceleration model:

$$v_{\text{car}} = v_{\text{car},i} + a_{\text{car}}t \qquad (8)$$

The maximum separation occurs when the speeds of both vehicles are the same, because after that instant the car will be moving faster than the bicycle, so the separation distance will decrease. Therefore, set the speed of the car equal to the speed of the bicycle in Equation (8) and find the time t_2 at which the two objects have the same speed:

$$v_{\text{car}} = v_{\text{bicycle,max}} = v_{\text{car},i} + a_{\text{car}}t_2 \quad \rightarrow \quad t_2 = \frac{v_{\text{bicycle,max}}}{a_{\text{car}}} \qquad (9)$$

At any time after the bicycle reaches its maximum speed, the separation distance between the vehicles is found by subtracting Equation (1) from Equation (3):

$$x_{\text{bicycle}} - x_{\text{car}} = \tfrac{1}{2}a_{\text{bicycle}}t_1^2 + v_{\text{bicycle,max}}\left(t - t_1\right) - \tfrac{1}{2}a_{\text{car}}t^2 \qquad (10)$$

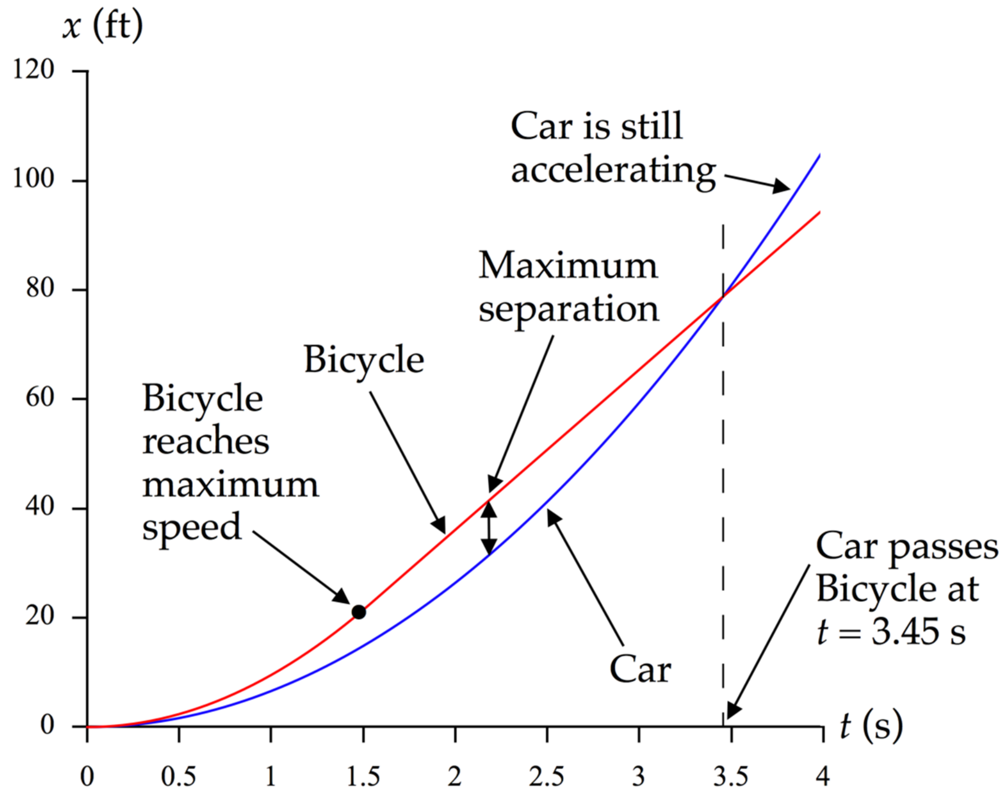
Evaluate the separation distance at the time found in Equation (9):

$$\begin{aligned} x_{\text{bicycle}} - x_{\text{car}} &= \tfrac{1}{2}a_{\text{bicycle}}t_1^2 + v_{\text{bicycle,max}}\left(t_2 - t_1\right) - \tfrac{1}{2}a_{\text{car}}t_1^2 \\ &= \tfrac{1}{2}a_{\text{bicycle}}\left(\frac{v_{\text{bicycle,max}}}{a_{\text{bicycle}}}\right)^2 + v_{\text{bicycle,max}}\left(\frac{v_{\text{bicycle,max}}}{a_{\text{car}}} - \frac{v_{\text{bicycle,max}}}{a_{\text{bicycle}}}\right) \\ &\qquad - \tfrac{1}{2}a_{\text{car}}\left(\frac{v_{\text{bicycle,max}}}{a_{\text{car}}}\right)^2 \\ &= \left(v_{\text{bicycle,max}}\right)^2\left[\frac{1}{2a_{\text{bicycle}}} + \left(\frac{1}{a_{\text{car}}} - \frac{1}{a_{\text{bicycle}}}\right) - \frac{1}{2a_{\text{car}}}\right] \end{aligned}$$

Substitute numerical values:

$$\begin{aligned} x_{\text{bicycle}} - x_{\text{car}} &= \left(20.0\text{ mi/ h}\right)^2\left[\begin{array}{l} \dfrac{1}{2\left(13.0\text{ mi/ h/ s}\right)} \\ \qquad + \left(\dfrac{1}{9.00\text{ mi/ h/ s}} - \dfrac{1}{13.0\text{ mi/ h/ s}}\right) \\ \qquad\qquad - \dfrac{1}{2\left(9.00\text{ mi/ h/ s}\right)} \end{array}\right] \\ &= 6.84\text{ mi}\cdot\text{s/ h}\left(\frac{5\,280\text{ ft}}{\text{mi}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = \boxed{10.0\text{ ft}} \end{aligned}$$

Finalize It would be helpful to graph the positions of the two vehicles against time as shown below:



This graph shows both the time at which the car passes the bicycle and the time at which the maximum separation occurs between the vehicles. Notice that the bicycle reaches its maximum speed at about 1.5 s, but the car is still accelerating at 4 seconds (the blue line is still curved). The black dot represents the instant that the bicycle stops accelerating. The corresponding black dot for the car would be to the right of the graph as drawn, at about 5.6 s.]

Answers: (a) 3.45 s (b) 10.0 ft

[redacted]

1. (b)

2. (c)

3. (b)

4. False.

5. (b)

6. (c)

7. (a)–(e), (b)–(d), (c)–(f)

8. (i) (e) (ii) (d)

[redacted]

P2.2 (a) 50.0 m/s; (b) 41.0 m/s

P2.4 (a) $+L/t_1$; (b) $-L/t_2$; (c) 0; (d) $2L/t_1 + t_2$

P2.6 (a) 20 mi/h; (b) 0; (c) 30 mi/h

P2.8 See graphs in P2.8.

P2.10 (a) See ANS. FIG. P2.10; (b) 23 m/s, 18 m/s, 14 m/s, and 9.0 m/s; (c) 4.6 m/s^2; (d) zero

P2.12 (a–e) See graphs in P2.12; (f) with less regularity

P2.14 (a) 4.98×10^{-19}s (b) $1.20 \times 10^{15}$$m/s^2$

P2.16 (a) 20.0s (b) 1000m (c) the plane cannot land

P2.18 31s

P2.20 The accelerations do not match, therefore the situation is impossible.

P2.22 The equation represents that displacement is a quadratic function of time.

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}$$

P2.24 (a)–0.107 m/s²(b) 4.53 m/s (c) pole #3 (stops at x = 96.3 m at t = 42.4 s

P2.26 (a and b) The rock does not reach the top of the wall with v_f = 3.69 m/s; (c) 2.39m/s; (d) does not agree; (e) The average speed of the upward-moving rock is smaller than the downward moving rock.

P2.28 0.60 s

P2.30 (a) $\frac{h}{t} + \frac{gt}{2}$; (b) $\frac{h}{t} - \frac{gt}{2}$

P2.32 (a) See graphs in P2.32; (b) See graph in P2.32; (c) −4 m/s^2; (d) 32 m; (e) 28 m

P2.34 (a) $A = v_{xi}t + \frac{1}{2}a_x t^2$; (b) The displacement is the same result for the total area.

P2.36 (a) 96.0 ft/s; (b) $3.07 \times 10^3\ \text{ft/s}^2$ upward; (c) 3.13×10^{-2} s

P2.38 The trains do collide.

P2.40 (a) v_B= (1/tan u)v (b) The velocity v_B starts off larger than v for small angles theta and then decreases, approaching zero as theta approaches 90°.

P2.42 (a) The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g \sin\theta$. A first guess would be that the blue bead "wins," but not by much. (b) $\sqrt{\frac{2D}{g}}$; (c) $\sqrt{\frac{2L}{g\sin\theta}}$; (d) the beads arrive at point © simultaneously; (e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

P2.44 (a) 3.45 s; (b) 10.0 ft.