Complete Solutions Manual

Elementary Geometry for College Students

SEVENTH EDITION

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Suggestions for Course Design

The authors believe that this textbook would be appropriate for a 3-hour, 4-hour, or 5-hour course. Some instructors may choose to include all or part of Appendix A (Algebra Review) due to their students' background in algebra. There may also be a desire to include The Introduction to Logic, found at our website, as a portion of the course requirement. Inclusion of some laboratory work with a geometry package such as *Geometry Sketchpad* is an option for course work.

3-hour course

Include most of Chapters 1–8. Optional sections could include:

Section 2.2	*Indirect Proof*
Section 2.3	*Proving Lines Parallel*
Section 2.6	*Symmetry and Transformations*
Section 3.5	*Inequalities in a Triangle*
Section 6.4	*Some Constructions and Inequalities for the Circle*
Section 8.5	*More Area Relationships in the Circle*

4-hour course

Include most of Chapters 1–8 and include all/part of at least one of these chapters:

Chapter 9	*Surfaces and Solids (Solid Geometry)*
Chapter 10	*Analytic Geometry (Coordinate Geometry)*
Chapter 11	*Introduction to Trigonometry*

5-hour course

Include most of Chapters 1–11 as well as topics desired from Appendix A and/or The Introduction to Logic (see website).

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Chapter-by-Chapter Commentary for Instructors

Chapter P: Preliminary Concepts

Section P.1: Sets and Sets of Numbers
In this section, students review the notions and basic terms related to sets of objects. Given a set or description of a set, the student should be able to classify that set as empty, finite, or infinite. For the path provided by a set of points, the student should be able to characterize the path as continuous or discontinuous and also to describe that path as straight, curved, circular, or scattered. The student should recognize certain subsets of a straight line as a line segment or ray. Given two sets, the student should be able to form their union or their intersection. In turn, students should utilize Venn diagrams to display two sets that are disjoint or the union or intersection of the two sets.

Section P.2: Statements and Reasoning
The student should realize that statements of geometry appear in both words or symbols and can be classified as true or false. Of the compound statements (conjunction, disjunction, and implication), the instructor should warn the student of the significance of the implication in that it (the "If . . ., then . . ." statement) is most relevant in deductive reasoning. For the implication (also known as a conditional statement), the student should be able to determine its hypothesis and conclusion; this determination acts as an important prerequisite for preparing a proof. Also, the student should be able to recognize and distinguish the three type of reasoning (intuition, induction, and deduction). Further, the Law of Detachment plays a major role in the development/advancement of geometry. Emphasizing that valid arguments can be confused with invalid arguments will alert students to potential pitfalls.

Section P.3: Informal Geometry and Measurement
In this section, many terms of geometry are introduced *informally*; in Chapter 1, these vocabulary terms will be presented *formally*. For students who seem to be poorly prepared, this approach (both an informal and a formal introduction to geometric terminology) may prove quite helpful. Measuring the line segment's length with a ruler prepares the student intuitively for the Ruler Postulate and the Segment Addition Postulate of Chapter 1; similarly, measuring angles with a protractor also prepares the student with the insights needed to deal with topics found in Section 1.2. Students that have difficulty with measures of angles (likely due to the dual scales found on protractors) can correct this situation by considering an activity sheet which focuses upon measuring angles with a protractor.

Chapter One: Line and Angle Relationships

Section 1.1: Early Definitions and Postulates
So that the student can understand the concept "branch of mathematics," he or she should be introduced to the four parts of a mathematical system. The basic terminology and symbolism for lines (and their subsets) must be given due attention because these will be

utilized throughout the textbook. The instructor should alert students to undefined terms such as "building blocks." Also, characterize definitions and postulates as significant in that they lead to conclusions known as theorems, or statements that can be proven. For the instructor, pens and pencils can be used to visualize relationships among lines, line segments, and rays. Table tops and pieces of cardboard can be used to represent planes.

Section 1.2: Angles and Their Relationships
It is most important, once again, that students be able to not only recognize the terminology for angles, but also to be able to state definitions and principles in their own terms. Measuring angles with the protractor should enable the student to understand principles such as the Protractor Postulate and the Angle Addition Postulate. Constructions may also provide understanding of certain concepts (like congruence and angle bisector). Many examples will remind the student of algebra's role in the solution of problems of geometry. Students can be referred to the Algebra Review (Appendix A) as needed.

Section 1.3: Introduction to Geometric Proof
The purpose of this section is to introduce the student to geometric proof. Many of the little things (hypothesis = given information, order, statements and reasons, etc.) are of tremendous importance as you prepare the student for proof. In the Sixth Edition, many of the techniques are emphasized in the feature *Strategy for Proof*; be sure that your students are aware of this feature and utilize these techniques. The two-column proof is used at this time because it emphasizes all the written elements of proof.

Section 1.4: Relationships: Perpendicular Lines
The "perpendicular relationship" is most important to many later discoveries. For now, be sure that students know that this relation extends itself to combinations such as line-line, line-plane, and plane-plane. For the general concept of *relation*, we explore the reflexive, symmetric, and transitive properties—particularly those that relate geometric figures. Some discussion of *uniqueness* is productive in that it will provide background for the notion of auxiliary lines (introduced in a later section).

Section 1.5: The Formal Proof of a Theorem
Be sure that your students know *in order* the five written parts of the *written proof*: Statement of proof (the theorem), drawing (from hypothesis), given (from hypothesis), prove (from conclusion), and proof. The instructor must help the student understand that the unwritten Plan for Proof is far and away the most important step; for this part, suggest scratch paper, reviewing the textbook, and use of the *Strategy for Proof* feature. Several theorems that have already been stated or proven in part are left as exercises; many of these have a similar counterpart (an example) in the textbook.

Chapter Two: Parallel Lines

Section 2.1: The Parallel Postulate and Special Angles
From the outset of Chapter 2, the instructor should emphasize that parallel lines must be coplanar. It is suggested that the instructor illustrate parallel and perpendicular (even skew lines) relationships by using pens and pencils for lines and pieces of construction

paper or cardboard for planes. Even though it is nearly impossible for students to grasp the significance of this fact, tell students that the Parallel Postulate characterizes the branch of mathematics known as Euclidean Geometry (plane geometry). While this characterization suggests that "the Earth is flat," it is adequate for our study even though spherical geometry is required at the global level. Beginning with Postulate 11, students should be able to complete several statements of the form, "If two parallel lines are cut by a transversal, then . . . ."

Section 2.2: Indirect Proof

Note: If there is insufficient time allowed for the complete development of geometry from a theoretical perspective, this section can be treated as optional. This section provides the opportunity to review the negation of a statement as well as the implication and its related statements (converse, inverse, and contrapositive). Based upon the deductive form Law of Negative Inference, the primary goal of this section is the introduction of the indirect proof. It is important that students be aware that the indirect proof is often used in proving negations and uniqueness theorems. In the construction of an indirect proof, the student often makes the mistake of assuming that the negation of the hypothesis (rather than negation of conclusion) is true.

Section 2.3: Proving Lines Parallel

Due to the similarity among statements of this section and those in Section 2.1, caution students that parallel lines were a *given* in Section 2.1. However, theorems in Section 2.3 *prove* that lines meeting specified conditions are parallel; that is, statements in this section take the form, "If . . . , then these lines are parallel." For this section, have students draw up a list of conditions that *lead to* parallel lines.

Section 2.4: The Angles of a Triangle

Students will need to become familiar with much of the terminology of triangles (sides, angles, vertices, etc.). Also, students should classify triangles by using both side relationships (scalene, isosceles, etc.) and angle relationships (obtuse, right, etc.). Some persuasion may be needed to have students accept the use of an auxiliary line. For an auxiliary line, you must (1) explain its uniqueness, (2) verify its existence in a proof, and (3) explain *why* that particular line was chosen. The instructor cannot emphasize enough the role of the theorem, "The sum of the measures of the interior angles of a triangle is 180°." Because of the relation of remaining theorems to Theorem 2.4.1, note that each statement is called a *corollary* of that theorem.

Section 2.5: Convex Polygons

Again, terminology for the polygon must be given due attention. The student should be able to classify several polygons due to the number of sides (triangle, quadrilateral, pentagon, etc.). Terms such as equilateral, equiangular, and regular should be known. Rather than count the number of diagonals D for a polygon of n sides, the student should be able to use the formula $D = \frac{n(n-3)}{2}$. The student should be able to state and use formulas for the sum of the interior angles (or exterior angles) of a polygon; in turn, the student should know and be able to apply formulas that lead to the measures of an interior angle or exterior angle of a regular polygon. Polygrams can be treated as optional.

Section 2.6: Symmetry and Transformations
Appealing to the student's intuitive sense of symmetry, the student can be taught that line symmetry exists when one-half of the figure is the mirror image (*reflection*) of the other half, with the line of symmetry as the mirror. For point symmetry, ask the student "is there a point (not necessarily on the figure) that is the midpoint of a line segment determined by two corresponding points on the figure in question." While a figure may have more than one line of symmetry, emphasize that the figure can have only one point of symmetry. Transformations (slides, reflections, and rotations) always produce an image (figure) that is congruent to the original figure. In Chapter 3, many examples of pairs of congruent triangles can be interpreted as the result of a slide, reflection, or rotation of one triangle to produce another triangle (its image).

Chapter Three: Triangles

Section 3.1: Congruent Triangles
As you begin the study of congruent triangles, stress the need to pair corresponding vertices, corresponding sides, and corresponding angles. Also, students should realize that the methods for proving triangles congruent (SSS, SAS, ASA, and AAS) are useful throughout the remainder of their study of geometry. Due to the simplicity and brevity of some proof problems found in this section, encourage students to attempt proof without fear. Also, have students utilize suggestions found in the *Strategy for Proof* feature.
Section 3.2: Corresponding Parts of Congruent Triangles
Students should know the acronym CPCTC and know that it represents, "Corresponding Parts of Congruent Triangles are Congruent." Emphasize that CPCTC allows them to prove that a pair of line segments (or a pair of angles) are congruent; however, warn them that CPCTC cannot be cited as a reason unless a pair of congruent triangles have already been established. Let students know that CPCTC empowers them to take an additional step; for instance, proving that 2 line segments are congruent may enable the student to establish a midpoint relationship. Once terminology for the right triangle has been introduced, caution students that the HL method for proving triangles congruent is valid only for right triangles. In order to give it due attention, the Pythagorean Theorem is introduced here without proof. The connection of the Pythagorean Theorem to this section lies in the fact that it will later be used to prove the HL theorem.

Section 3.3: Isosceles Triangles
Students should become familiar with terms (base, legs, etc.) that characterize the isosceles triangle. Students should know meanings of (and be able to differentiate between) these figures related to a triangle: an angle-bisector, the perpendicular-bisector of a side, an altitude, and a median. Of course, every triangle will have three angle-bisectors, three altitudes, etc. With unsuspecting students, it may be best to show them that the three-perpendicular bisectors of sides (or three altitudes) can intersect at a point *outside* the triangle; perhaps a drawing session would help! The most important theorems of this section are converses: (1) If two sides of a triangle are congruent, then the angles

opposite these sides are congruent, and (2) If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

Section 3.4: Basic Constructions Justified
Note: If there is insufficient time or constructions are not to be emphasized, this section can be treated as optional.

The first goal of this section is to validate (prove) the construction methods introduced in earlier sections. For instance, we validate the method for bisecting an angle through the use of congruent triangles and CPCTC. The second goal of this section is that of constructing line segments of a particular length or of constructing angles of a particular measure (such as 45° or 60°).

Section 3.5: Inequalities in a Triangle
Note: If there is insufficient time or inequality relationships are not to be emphasized, this section can be treated as optional.

To enable the proofs of theorems in this section, we must begin with a concrete definition of the term *greater than.* Note that some theorems involving inequalities are referred to as *lemmas* because these theorems help us to prove other theorems. The inequality theorems involving the lengths of sides and measures of angles of a triangle are very important because they will be applied in Chapters 4 and 6. For some students, the Triangle Inequality will later be applied in the coursework of trigonometry and calculus.

Chapter Four: Quadrilaterals

Section 4.1: Properties of a Parallelogram
Alert students to the fact that principles of parallel lines, perpendicular lines, and congruent triangles are extremely helpful in developments of this chapter. Be sure to define the parallelogram, but caution students not to confuse this definition with any of several properties of parallelograms found in theorems of this section. These theorems have the form, "If a quadrilateral is a parallelogram, then . . . ." In Section 4.3, these properties will also characterize the rectangle, square, and rhombus, because each is actually a special type of parallelogram. The final topic (bearing of airplane or ship) can be treated as optional.

Section 4.2: The Parallelogram and Kite
In this section, parallelograms are *not* a given in the theorems of the form, "If a quadrilateral . . . , then the quadrilateral is a parallelogram." That is, we will be proving that certain quadrilaterals are parallelograms. Like the parallelogram, a kite has two pairs of congruent sides; by definition, the congruent pairs of sides in the kite are adjacent sides. A kite has its own properties (like perpendicular diagonals) as well.

Section 4.3: The Rectangle, Square, and Rhombus
Consider carefully the definition of each figure (rectangle, square, and rhombus); with each being a type of parallelogram, the properties of parallelograms are also those of the rectangle, square, and rhombus. Of course, each type of parallelogram found in this

section has its own properties. For example, the rectangle and square have four right angles while the diagonals of a rhombus are perpendicular; as a consequence of these properties, the Pythagorean Theorem can be applied toward solving many problems involving these special types of parallelograms.

Section 4.4: The Trapezoid
Because the trapezoid has only two sides that are parallel, it does not assume the properties of parallelograms. If the trapezoid is isosceles, then it will have special properties such as congruent diagonals and congruent base angles. Remaining theorems describe the length of a median of a trapezoid and characterize certain quadrilaterals as trapezoids or isosceles trapezoids.

Quadrilateral types can be compared by use of a Venn diagram or the following outline:

1. Quadrilaterals
    - A. Parallelograms
        - 1. Rectangle
            - a. Square
        - 2. Rhombus
    - B. Kites
    - C. Trapezoids
        - 1. Isosceles Trapezoids

Chapter Five: Similar Triangles

Section 5.1: Ratios, Rates, and Proportions
Note: For work in Chapter 5, the instructor may want to refer those students who need a review of the methods of solving quadratic equations to Appendix Sections A.4 and A.5. In this section, emphasize the difference between a ratio (quotient comparing *like* units) and a rate (quotient comparing *unlike units).* Students should understand that a proportion is an equation in which two ratios (or rates) are equal. Terminology for proportions (means, extremes, etc.) are important because the student better understands a property like the Means-Extremes Property.

Section 5.2: Similar Polygons
In this section, similar polygons are defined and their related terminology (corresponding sides, corresponding angles, etc.) are introduced. The definition of similar polygons allows students to (1) equate measures of corresponding angles, and (2) form proportions that compare lengths of corresponding sides. Thus, this section focuses on problem solving strategies, including an ancient technique known as *shadow reckoning.*

Section 5.3: Proving Triangles Similar
Whereas Section 5.2 focuses on problem solving, Section 5.3 emphasizes methods for proving that triangles are similar. Due to its simplicity, the instructor should emphasize that the AA method for proving triangles similar should be used whenever possible. The definition of similar triangles forces two relationships among parts of similar triangles:
(1.) CASTC means “Corresponding Angles of Similar Triangles are Congruent,” while
(2.) CSSTP means “Corresponding Sides of Similar Triangles are Proportional.”

Other methods for proving triangles similar are SAS and SSS; in application, these methods are difficult due to the necessity of showing lengths of sides to be proportional. Warn students not to use SAS and SSS (methods of proving triangles congruent) as reasons for claiming that triangles are similar.

Section 5.4: The Pythagorean Theorem
Theorem 5.3.1 leads to a proof of the Pythagorean Theorem and its converse. Students should be aware that many (more than 100) proofs exist for the Pythagorean Theorem. For emphasis, note that the Pythagorean Theorem allows one to find the length of a side of a right triangle; however, its converse enables one to conclude that a given triangle may be a right triangle. Because these are commonly applied, Pythagorean Triples such as (3,4,5) and (5,12,13) are best memorized by the student.
With *c* being the length of the longest side of a given triangle, this triangle is:
an *acute* triangle if $c^2 < a^2 + b^2$, a *right* triangle if $c^2 = a^2 + b^2$,
or an *obtuse* triangle if $c^2 > a^2 + b^2$.

Section 5.5: Special Right Triangles
In Section 5.4, some right triangles were special because of their integral lengths of sides (*a*,*b*,*c*). In Section 5.5, a right triangle with angle measures of 45°, 45°, and 90° always has congruent legs while the hypotenuse is $\sqrt{2}$ times as long as either leg. Also, a right triangle with angle measures of 30°, 60°, and 90° has a longer leg that is $\sqrt{3}$ times as long as the shorter leg, while the hypotenuse is two times as long as the shorter leg. These relationships, and their converses, also have applications in trigonometry and calculus.

Section 5.6: Segments Divided Proportionally
Note: In this section, Ceva's Theorem is optional in that it is not applied in later sections. The phrase *divided proportionally* can be compared to profit sharing among unequal partners in a business venture. This concept is, of course, the essence of numerous applications found in this section. The Angle-Bisector Theorem states that an angle-bisector of an angle in a triangle leads to equal ratios among the parts of the lengths of the two sides forming the bisected angle and the lengths of parts of the third side.

Chapter Six: Circles

Section 6.1: Circles and Related Segments and Angles
Terminology for the circle is reviewed and extended in this section. Students will have difficulty with the definition of *congruent arcs* in that they must have both equal measures and lie within the *same* circle or *congruent* circles. Many of the principles of this section are intuitive and therefore easily accepted. Contrast the sides and vertex locations of the central angle and the inscribed angle. Stress these angle-measurement relationships in that further angle-measurement relationships will be added in Section 6.2.

Section 6.2: More Angle Measures in the Circle
The terms *tangent* and *secant* are introduced and will be given further attention in later sections as well as in the coursework of trigonometry and calculus. Again emphasize the

new angle-measurement techniques with the circle. A summary of methods (Table 6.1) is provided for students.

Section 6.3: Line and Segment Relationships in the Circle
The early theorems in this section sound similar, yet make different assertions; for this reason, it may be best that students draw the hypothesis of each theorem to "see" that the conclusion must follow. Students will also need to distinguish the concepts of *common tangent for two circles* and *tangent circles.* Each of the relationships found in Theorems 6.3.5–6.3.7 is difficult to believe without proof; however, with the help of an auxiliary line, each proof of theorem is easily and quickly proved.

Section 6.4: Some Constructions and Inequalities for the Circle
Note: If there is insufficient time or constructions and inequality relationships are not to be emphasized, this section can be treated as optional.

Because the construction methods of this section are fairly involved, be sure to assign homework exercises that have students perform them. The inequality relationships involving circles are intuitive (easily believed); due to the difficulty found in constructing proofs of these theorems, the instructor may wish to treat proofs as optional.

Chapter Seven: Locus and Concurrence

Section 7.1: Locus of Points
So that the term *locus* is less confusing for students, the instructor should tie this word to its Latin meaning: "location." For the locus concept, quantity makes a difference; that is, students will need to see several examples. While construction of a locus is optional, a drawing of the locus is imperative. Theorems 7.1.1 and 7.1.2 are most important in that they lay the groundwork for later sections. The instructor should be sure to distinguish between the locus of points in a plane and the locus of points in space.

Section 7.2: Concurrence of Lines
The discussion of locus leads indirectly to the notion of concurrence. In particular, the concurrence of the three angle-bisectors of a triangle follows directly from the first locus theorem in Section 7.1; in turn, a triangle has an inscribed circle whose center is the incenter of the triangle. Likewise, the three perpendicular-bisectors of the sides of a triangle are concurrent at the circumcenter of the triangle, the point that is the center of the circumscribed circle of every triangle. In this section, not only have students memorize the terms *incenter*, *circumcenter*, *orthocenter*, and *centroid*, but also have them know which concurrency (angle-bisectors, etc.) leads to each result.

Section 7.3: More About Regular Polygons
Based upon our findings in Section 7.2, the student should know that a circle can be inscribed in *every* triangle and also be circumscribed about *every* triangle. Further, the center for both circles (inscribed and circumscribed) is the same point for the equilateral triangle and regular polygons in general. The new terminology for the regular polygon (center, radius, apothem, central angle, etc) should be memorized because it will also be applied in Section 8.3.

Chapter Eight: Areas of Polygons and Circles

Section 8.1: Area and Initial Postulates
Even though most students say *area of a triangle,* they should realize that the more accurate description would be *area of triangular region.* Stress the difference between linear units (used to measure length) and square units (used to measure area). With each area formula serving as a "stepping stone" to the next formula, the given order for the area formulas is natural. Perhaps the most significant formula in the list is that of the parallelogram ($A = bh$) in that this is derived from the area of rectangle formula while it leads to the remaining formulas.

Section 8.2: Perimeter and Area of Polygons
Given its practical applications, the notion of perimeter should be reviewed and extended. Heron's Formula is difficult to state and apply; however, it is common to find the area of a triangle whose lengths of sides are known. The proof of Heron's Formula is found at the website that accompanies this textbook. Emphasize Theorem 8.2.3 and that the area formulas for the rhombus and kite are just special cases of this theorem.

Section 8.3: Regular Polygons and Area
In this section, we first consider formulas for the area of the equilateral triangle and square. For regular polygons in general, be sure to introduce or review the terminology (center, radius, apothem, central angle, etc.) that was found in Section 7.3; if studied, the work in both Chapters 9 and 11 use this terminology as well. The ultimate goal of this section is to establish the formula for the area of a regular polygon, namely $A = \frac{1}{2}aP$.

Section 8.4: Circumference and Area of a Circle
Begin with the definition of π as a ratio and then provide some approximations of its value (such as $\frac{22}{7}$ and 3.1416). Using $\pi = \frac{C}{d}$, we can show that $C = \pi d$ and $C = 2\pi r$. Using a proportion, we find the length of an arc of circle (as part of the circumference). Developed as the limit of areas of inscribed regular polygons, we show that the area of a circle is given by $A = \pi r^2$. Note that the concept of *limit* needs a few examples. For students to distinguish between $2\pi r$ and πr^2 (for circumference and area), have them compare units, where $r = 3$ cm, $2\pi r = 2\pi \cdot 3$ cm $= 6\pi$ cm (a linear measure) while $\pi r^2 = \pi \cdot 3$ cm $\cdot 3$ cm or 9π cm^2 (a measure of area).

Section 8.5: More Area Relationships in the Circle
Note: If there is insufficient time for the study of this section, it can be treated as optional in that none of the content is used in later sections.

Formulas for the area of a sector and segment depend upon the formula for the area of a circle; however, an understanding of these area concepts is far more important than the memorization of formulas. The area of segment applications require that the related central angle have a convenient measure, such as 60°, 90°, or 120°; otherwise, trigonometry would be necessary to solve the problem. When a triangle has perimeter P and inscribed circle of radius r, the area of the triangle is given by $A = \frac{1}{2}rP$.

Chapter Nine: Surfaces and Solids

Section 9.1: Prisms, Area, and Volume
The student should consider three-dimensional objects in this section and chapter; for that purpose, the instructor should use a set of models displaying various prisms and other solids or space figures. Students need to become familiar with prisms and related terminology. To calculate the lateral area and the total area of a prism, a student must apply formulas from Chapter 8. For the volume formula for a prism ($V = Bh$), emphasize that B is the area of the base and that V is always measured in *cubic units*.

Section 9.2: Pyramids, Area, and Volume
Again, the instructor should use a set of models to display various pyramids. Students need to become familiar with pyramids and related terminology, including the *slant height* of a regular pyramid. Calculating the lateral area and the total area of a pyramid requires the application of formulas from Chapter 8. To find the length of the slant height of a regular pyramid requires the use of the Pythagorean Theorem. Compare the formula for the volume of a pyramid ($V = \frac{1}{3}Bh$) to that of the prism ($V = Bh$).

Section 9.3: Cylinders and Cones
Comparing the prism to cylinder and the pyramid to cone will help to motivate students in learning the area and volume formulas of this section. Three-dimensional models will motivate the formula for the lateral area of cylinder and to explain the slant height of the right circular cone. The length of the slant height of the right circular cone can be found by using the Pythagorean Theorem. Compare volume formulas for the prism (V = Bh) and right circular cylinder ($V = Bh$ or $V = \pi r^2 h$); likewise compare the volume formulas for the pyramid ($V = \frac{1}{3}Bh$) and right circular cylinder ($V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$). While the material involving solids of revolution is a preparatory topic for calculus, it can be treated as optional.

Section 9.4: Polyhedrons and Spheres
Students should recognize (or be told) that prisms and pyramids are merely examples of polyhedrons (or polyhedra). Students should verify Euler's Formula (V + F = E + 2) for polyhedra with a small number of vertices by using solid models from a kit. For the sphere, compare its terminology with that of the circle; however, note that a sphere also has tangent planes. To develop the volume of sphere formula, it is necessary to interpret the volume as the limit of the volumes of inscribed regular polyhedra with an increasing number of faces. Due to limitations, we only apply the surface area of sphere formula.

Chapter Ten: Analytic Geometry

Section 10.1: The Rectangular Coordinate System
The student should become familiar with the rectangular coordinate system and its related terminology. Warn students that the definitions for lengths of horizontal and vertical line segments are fairly important in the development of the chapter. For the formulas developed, P_1 is read *first* point and x_1 as the *value of x for the first point.* The Distance Formula and Midpoint Formulas are must be memorized in that they will be used throughout Chapter 10; of course, these formulas are also useful in later coursework as well.

Section 10.2: Graphs of Linear Equations and Slope
At first, a point-plot approach for graphing equations is used. However, graphing linear equations leads to graphs that are lines and, in turn, the notion of *slope* of a line. The student must memorize the Slope Formula. By sight, a student should be able to recognize that a given line has a positive, negative, zero, or undefined slope.
Many students have difficulty drawing a line based upon its provided slope; for this reason, it is important to treat slope as $m = \frac{rise}{run}$. To draw a line with slope m, move from one point to the second point by simultaneously using a vertical change (rise) that corresponds to the horizontal change (run). Using the slopes of two given lines, the student should be able to classify lines as parallel, perpendicular, or neither.

Section 10.3: Preparing to Do Analytic Proofs
This section is a "warm up" for completing analytic proofs that follow in Section 10.4. Specific goals that need to be achieved are:

1. The student should know the formulas found in the summary on the first page.
2. The student should follow the suggestions for placement of a drawing so that the proof of the theorem can be completed. See the *Strategy for Proof*.
3. The student should study the relationship between desired theorem conclusions and formulas needed to obtain such conclusions. See the *Strategy for Proof*.

Section 10.4: Analytic Proofs
This section utilizes all formulas and suggestions from previous sections of Chapter 10. In each classroom, the instructor must warn students of the amount of rigor required. For instance, suppose that we are trying to prove a theorem such as, "If a quadrilateral is a parallelogram, then its diagonals bisect each other." Does the student provide a figure with certain vertices that is known to be a parallelogram, *or* does that figure have to be proven a parallelogram before the proof can be continued? You may wish to prove each claim once and then accept it at a later time as given (not needing proof); if it was shown in an earlier section that the triangle with vertices at $A(-a,0)$, $B(a,0)$, and $C(0,b)$ is isosceles, then it will be given as such in a later proof.

Section 10.5: Equations of Lines
In this section, we use given information about a line (like slope and *y*-intercept) to find its equation. Students will need to memorize and apply both the Slope-Intercept and the Point-Slope forms of a line. Emphasize that solving systems of linear equations is the

algebraic equivalent of finding the point of intersection of two lines using geometry. Emphasize that the method (algebra or geometry) used to find this point of intersection always leads to the same result. Point out that the Slope-Intercept and Point-Slope forms of a line can be used to prove further theorems by the analytic approach.

Section 10.6: The Three-Dimensional Coordinate System
In this section, we plot points of the form (x,y,z) in three dimensions. Warn students that the forms of equations of a line will seem unfamiliar; however, the equation of a plane in Cartesian space is similar to the general form for the equation of line in the Cartesian plane. Students should easily adapt to the natural extensions of the Distance Formula and Midpoint Formula. Ironically, there is no Slope Formula. For the concept of direction vector, the student will need some convincing of its importance; however, the direction vectors for two lines will determine whether these lines have the same direction (parallel or coincident) or different directions (intersecting or skew). To consider the relationships between planes, the instructor will need to give considerable attention to algebraic techniques in that the methods will be more involved. This section concludes with the equation of a sphere in Cartesian space, again seen by students as a natural extension of the equation of a circle in the Cartesian plane.

Chapter Eleven: Introduction to Trigonometry

Section 11.1: The Sine Ratio and Applications
Related to the right triangle, ask students to memorize the sine ratio of an angle in the form $\frac{opposite}{hypotenuse}$; while this seems rather informal, the remaining definitions of trigonometric ratios will be given in a similar form. While students are encouraged to use the calculator to find sine ratios for angles, they should also know these results from memory: $\sin 0° = 0$, $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, and $\sin 90° = 1$. Students should realize that the sine ratios increase as the angle measure increases. Emphasize the terms *angle of elevation* and *angle of depression* and be able to perform applications that require the use of the sine ratio.

Section 11.2: The Cosine Ratio and Applications
Ask students to memorize the cosine ratio of an angle in the form $\frac{adjacent}{hypotenuse}$.
In addition to using the calculator to find cosine ratios, students should memorize results such as $\cos 0° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$, etc. Students should recognize that an increase in angle measures produces a decrease in cosine measures. Students need to be able to complete applications that require the cosine ratio. The instructor should include and perhaps require that the student be able to prove the theorem $\sin^2 \theta + \cos^2 \theta = 1$. Emphasize that many geometry problems (such as Example 7 of this section) cannot be solved without the use of trigonometry.

Section 11.3: The Tangent Ratio and Other Ratios

Students should memorize the tangent ratio as $\frac{opposite}{adjacent}$ and memorize exact values for tan 0°, tan 30°, tan 45°, etc. Some attention and discussion should be devoted to the claim that "tan 90° is undefined." Now that three ratios are available, some practice and discussion should be given to determination of the ratio needed to solve a particular problem. While the remaining ratios (cotangent, secant, and cosecant) are included for completeness, the students can solve all problems by using only the sine, cosine, and tangent ratios. The final ratios can be recalled as reciprocals of the first three; for instance, if $\sin\theta = \frac{a}{b}$, then $\csc\theta = \frac{b}{a}$.

Section 11.4: Applications with Acute Triangles

Only the most basic trigonometric identities are included in this section. Due to the Reciprocal Identities, remind students that only the sine, cosine, and tangent ratios are needed in application. The instructor should demonstrate the use of the calculator in finding a ratio such as sec 34° (as reciprocal of cos 34°). Because the Quotient Identities and Pythagorean Identities are easily proved, some time should be devoted to proving at least one identity of each type. The area formula, $A = \frac{1}{2}bc\sin\alpha$, is easily proved; however, students should focus on its application. Students should also know the general form of the the Law of Sines and the Law of Cosines; also, the student should be able to determine which form of each is used to solve a problem. In this textbook, we do not touch on ratios, identities, or formulas involving an obtuse angle.

Chapter 1 Line and Angle Relationships

SECTION 1.1: Early Definitions and Postulates

1. *AC*

2. Midpoint

3. 6.25 ft · 12 in./ft = 75 in.

4. 52 in. ÷ 12 in./ft = $4\frac{1}{3}$ ft or 4 ft 4 in.

5. $\frac{1}{2}\text{m} \cdot 3.28 \text{ ft/m} = 1.64 \text{ feet}$

6. 16.4 ft ÷ 3.28 ft/m = 5 m

7. 18 – 15 = 3 mi

8. $300 + 450 + 600 = 1350$ ft
   1350 ft ÷ 15 ft/s = 90 s or 1 min 30 s

9. **a.** *A-C-D*

   b. *A*, *B*, *C* or *B*, *C*, *D* or *A*, *B*, *D*

10. **a.** Infinite

    b. One

    c. None

    d. None

11. $\overleftrightarrow{CD}$ means line *CD*;
    $\overline{CD}$ means segment *CD*;
    CD means the measure or length of $\overline{CD}$;
    $\overrightarrow{CD}$ means ray *CD* with endpoint *C*.

12. **a.** No difference

    b. No difference

    c. No difference

    d. $\overrightarrow{CD}$ is the ray starting at *C* and going toward *D*.
    $\overrightarrow{DC}$ is the ray starting at *D* and going toward *C*.

13. **a.** *m* and *t*

    b. *m* and *p* or *p* and *t*

14. **a.** False

    b. False

    c. True

    d. True

    e. False

15. $$\begin{aligned} 2x+1 &= 3x-2 \\ -x &= -3 \\ x &= 3 \\ AM &= 7 \end{aligned}$$

16. $$\begin{aligned} 2(x+1) &= 3(x-2) \\ 2x+2 &= 3x-6 \\ -1x &= -8 \\ x &= 8 \\ AB &= AM + MB \\ AB &= 18+18 = 36 \end{aligned}$$

17. $$\begin{aligned} 2x+1+3x+2 &= 6x-4 \\ 5x+3 &= 6x-4 \\ -1x &= -7 \\ x &= 7 \\ AB &= 38 \end{aligned}$$

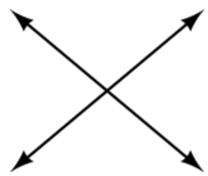
18. No; Yes; Yes; No

19. **a.** $\overrightarrow{OA}$ and $\overrightarrow{OD}$

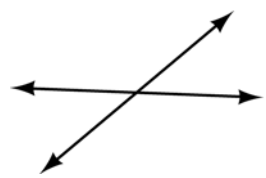
    b. $\overrightarrow{OA}$ and $\overrightarrow{OB}$
    (There are other possible answers.)

20. $\overleftrightarrow{CD}$ lies on plane *X*.

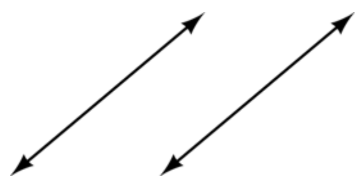
21. **a.**



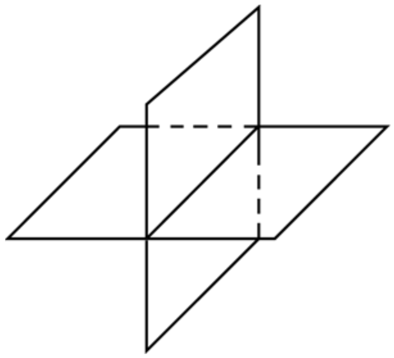
b.



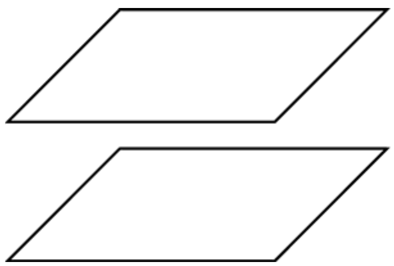
c.



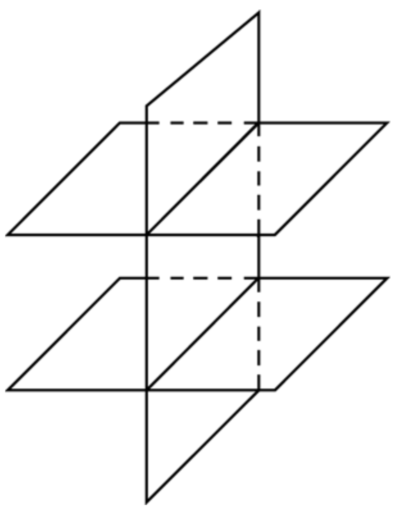
22. **a.**



b.



c.



23. Planes *M* and *N* intersect at $\overleftrightarrow{AB}$.

24. *B*

25. *A*

26. **a.** One

b. Infinite

c. One

d. None

27. **a.** *C*

b. *C*

c. *H*

28. **a.** Equal

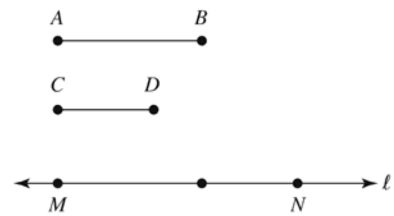
b. Equal

c. *AC* is twice *CD*.

29. Given: $\overline{AB}$ and $\overline{CD}$ as shown $(AB > CD)$

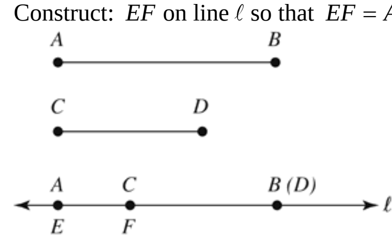
Construct $\overline{MN}$ on line ℓ so that

$MN = AB + CD$



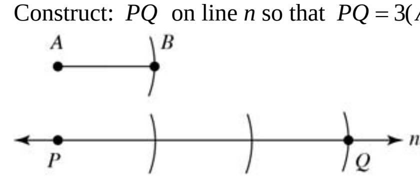
30. Given: $\overline{AB}$ and $\overline{CD}$ as shown $(AB > CD)$

Construct: $\overline{EF}$ on line ℓ so that $EF = AB - CD$.



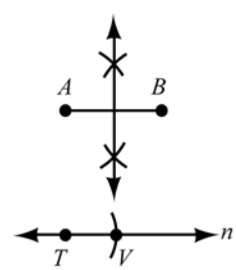
31. Given: $\overline{AB}$ as shown

Construct: $\overline{PQ}$ on line *n* so that $PQ = 3(AB)$



32. Given: $\overline{AB}$ as shown

Construct: $\overline{TV}$ on line *n* so that $TV = \frac{1}{2}(AB)$



33. **a.** No

b. Yes

c. No

d. Yes

34. A segment can be divided into 2^n congruent parts, where $n \geq 1$.

35. Six

36. Four

37. Nothing

38. **a.** One

b. One

c. None

d. One

e. One

f. One

g. None

39. **a.** Yes

b. Yes

c. No

40. **a.** Yes

b. No

c. Yes

41. $\frac{1}{3}a + \frac{1}{2}b$ or $\frac{2a+3b}{6}$

SECTION 1.2: Angles and Their Relationships

1. **a.** Acute

b. Right

c. Obtuse

2. **a.** Obtuse

b. Straight

c. Acute

3. **a.** Complementary

b. Supplementary

4. **a.** Congruent

b. None

5. Adjacent

6. Vertical

7. Complementary (also adjacent)

8. Supplementary

9. Yes; No

10. **a.** True

b. False

c. False

d. False

e. True

11. **a.** Obtuse

b. Straight

c. Acute

d. Obtuse

12. B is not in the interior of $\angle FAE$; the Angle-Addition Postulate does not apply.

13. $\text{m}\angle FAC + \text{m}\angle CAD = 180$
$\angle FAC$ and $\angle CAD$ are supplementary.

14. **a.** $x + y = 180$

b. $x = y$

15. **a.** $x + y = 90$

b. $x = y$

16. 62°

17. 42°

18.
$$\begin{aligned} 2x + 9 + 3x - 2 &= 67 \\ 5x + 7 &= 67 \\ 5x &= 60 \\ x &= 12 \end{aligned}$$

19.
$$\begin{aligned} 2x - 10 + x + 6 &= 4(x - 6) \\ 3x - 4 &= 4x - 24 \\ 20 &= x \\ x &= 20 \\ \text{m}\angle RSV &= 4(20 - 6) = 56° \end{aligned}$$

20.
$$\begin{aligned} 5(x+1) - 3 + 4(x-2) + 3 &= 4(2x+3) - 7 \\ 5x + 5 - 3 + 4x - 8 + 3 &= 8x + 12 - 7 \\ 9x - 3 &= 8x + 5 \\ x &= 8 \\ \text{m}\angle RSV = 4(2 \cdot 8 + 3) - 7 &= 69° \end{aligned}$$

21. $\frac{x}{2} + \frac{x}{4} = 45$

Multiply by LCD, 4

$$\begin{aligned} 2x + x &= 180 \\ 3x &= 180 \\ x &= 60;\ m\angle RST = 30° \end{aligned}$$

22. $\frac{2x}{3}+\frac{x}{2}=49$

Multiply by LCD, 6

$$\begin{aligned}4x+3x&=294\\7x&=294\\x&=42;\ m\angle TSV=\frac{x}{2}=21^\circ\end{aligned}$$

23.
$$\begin{aligned}x+y&=2x-2y\\x+y+2x-2y&=64\end{aligned}$$

$$\begin{aligned}-1x+3y&=0\\3x-1y&=64\end{aligned}$$

$$\begin{aligned}-3x+9y&=0\\3x-y&=64\\\hline 8y&=64\\y&=8;\ x=24\end{aligned}$$

24.
$$\begin{aligned}2x+3y&=3x-y+2\\2x+3y+3x-y+2&=80\end{aligned}$$

$$\begin{aligned}-1x+4y&=2\\5x+2y&=78\end{aligned}$$

$$\begin{aligned}-5x+20y&=10\\5x+2y&=78\\\hline 22y&=88\\y&=4;\ x=14\end{aligned}$$

25. $\angle CAB\cong\angle DAB$

26.
$$\begin{aligned}x+y&=90\\x&=12+y\end{aligned}$$

$$\begin{aligned}x+y&=90\\x-y&=12\\\hline 2x\quad&=102\\x&=51\end{aligned}$$

$$\begin{aligned}51+y&=90\\y&=39\end{aligned}$$

$\angle$s are 51° and 39°.

27.
$$\begin{aligned}x+y&=180\\x&=24+2y\end{aligned}$$

$$\begin{aligned}x+y&=180\\x-2y&=24\end{aligned}$$

$$\begin{aligned}2x+2y&=360\\x-2y&=24\\\hline 3x\quad&=384\\x&=128;\ y=52\end{aligned}$$

$\angle$s are 128° and 52°.

28. **a.** $(90-x)^\circ$

b. $(90-(3x-12))^\circ=(102-3x)^\circ$

c. $(90-(2x+5y))^\circ=(90-2x-5y)^\circ$

29. **a.** $(180-x)^\circ$

b. $(180-(3x-12))^\circ=(192-3x)^\circ$

c. $(180-(2x+5y))^\circ=(180-2x-5y)^\circ$

30.
$$\begin{aligned}x-92&=92-53\\x-92&=39\\x&=131\end{aligned}$$

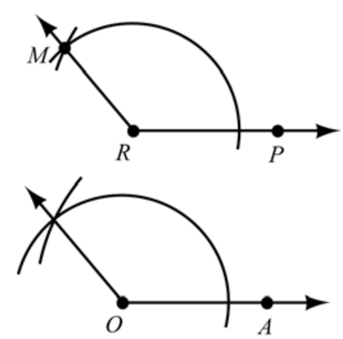
31.
$$\begin{aligned}x-92+(92-53)&=90\\x-92+39&=90\\x-53&=90\\x&=143\end{aligned}$$

32. **a.** True

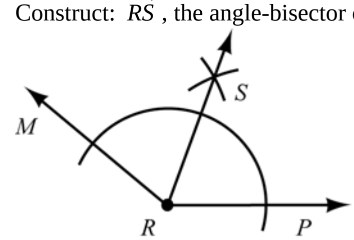
b. False

c. False

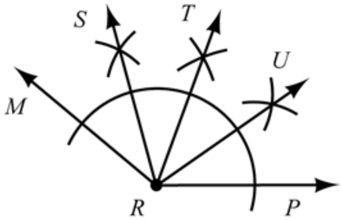
33. Given: Obtuse $\angle MRP$
Construct: With $\overrightarrow{OA}$ as one side, an angle $\cong\angle MRP$



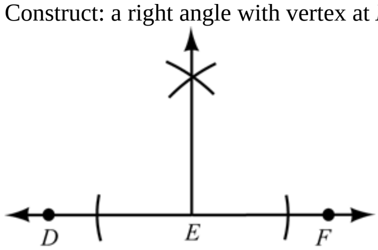
34. Given: Obtuse $\angle MRP$
Construct: $\overrightarrow{RS}$, the angle-bisector of $\angle MRP$



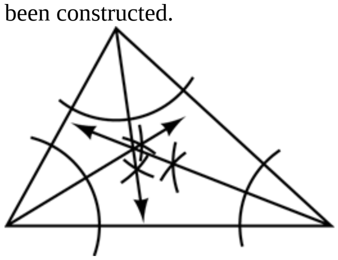
35. Given: Obtuse $\angle MRP$
Construct: Rays RS, RT, and RU so that $\angle MRP$ is divided into 4 $\cong$ angles



36. Given: Straight angle DEF
Construct: a right angle with vertex at E

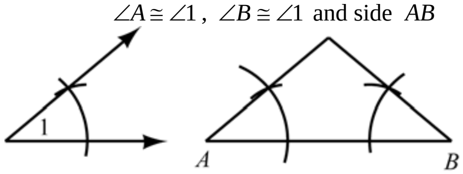


37. For the triangle shown, the angle bisectors have been constructed.



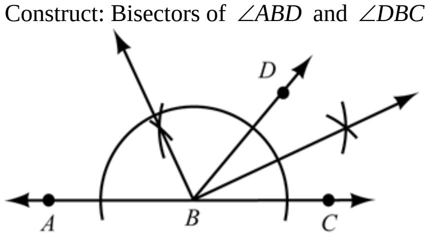
It appears that the angle bisectors meet at one point.

38. Given: Acute $\angle 1$ and $\overleftrightarrow{AB}$
Construct: Triangle ABC which has $\angle A \cong \angle 1$, $\angle B \cong \angle 1$ and side $\overline{AB}$



39. It appears that the two sides opposite $\angle$ s A and B are congruent.

40. Given: Straight $\angle ABC$ and $\overrightarrow{BD}$
Construct: Bisectors of $\angle ABD$ and $\angle DBC$



It appears that a right angle is formed.

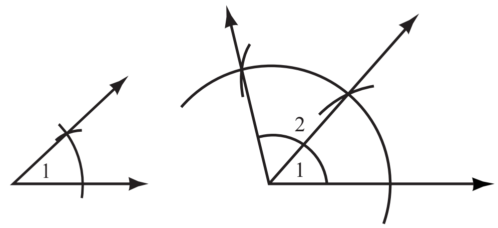
41. $m\angle 1 + m\angle 2 = 90°$

If $\angle$s 1 and 2 are bisected, then

$\frac{1}{2} \cdot m\angle 1 + \frac{1}{2} \cdot m\angle 2 = 45°$

42. Given: Acute $\angle 1$

Construct: $\angle 2$, an angle whose measure is twice that of $\angle 1$



43. **a.** 90°

b. 90°

c. Equal

44. Let $m\angle USV = x$, then $m\angle TSU = 38 - x$

$$\begin{aligned} 38 - x + 40 &= 61 \\ 78 - x &= 61 \\ 78 - 61 &= x \end{aligned}$$

$$x = 17;\ m\angle USV = 17°$$

45. $x + 2z + x - z + 2x - z = 60$

$$\begin{aligned} 4x &= 60 \\ x &= 15 \end{aligned}$$

If $x = 15$, then $m\angle USV = 15 - z$,
$m\angle VSW = 2(15) - z$, and
$m\angle USW = 3x - 6 = 3(15) - 6 = 39$

So $15 - z + 2(15) - z = 39$

$$\begin{aligned} 45 - 2z &= 39 \\ 6 &= 2z \\ z &= 3 \end{aligned}$$

46. **a.** 52°

b. 52°

c. Equal

47. $90 + x + x = 360$

$$\begin{aligned} 2x &= 270 \\ x &= 135° \end{aligned}$$

48. 90°

SECTION 1.3: Introduction to Geometric Proof

1. Division Property of Equality or Multiplication Property of Equality
2. Distributive Property $[x+x=(1+1)x=2x]$
3. Subtraction Property of Equality
4. Addition Property of Equality
5. Multiplication Property of Equality
6. Addition Property of Equality
7. If 2 angles are supplementary, then the sum of their measures is 180°.
8. If the sum of the measures of 2 angles is 180°, then the angles are supplementary.
9. Angle-Addition Property
10. Definition of angle-bisector
11. $AM + MB = AB$
12. $AM = MB$
13. $\overrightarrow{EG}$ bisects $\angle DEF$
14. $m\angle 1 = m\angle 2$ or $\angle 1 \cong \angle 2$
15. $m\angle 1 + m\angle 2 = 90^\circ$
16. $\angle 1$ and $\angle 2$ are complementary
17. $2x = 10$
18. $x = 7$
19. $7x + 2 = 30$
20. $\frac{1}{2} = 50\%$
21. $6x - 3 = 27$
22. $x = -20$
23. **1.** Given
    2. Distributive Property
    3. Addition Property of Equality
    4. Division Property of Equality
24. **1.** Given
    2. Subtraction Property of Equality
    3. Division Property of Equality
25. **1.** $2(x+3)-7=11$
    2. $2x+6-7=11$
    3. $2x-1=11$
    4. $2x=12$
    5. $x=6$
26. **1.** $\frac{x}{5}+3=9$
    2. $\frac{x}{5}=6$
    3. $x=30$
27. **1.** Given
    2. Segment-Addition Postulate
    3. Subtraction Property of Equality
28. **1.** Given
    2. The midpoint forms 2 segments of equal measure.
    3. Segment-Addition Postulate
    4. Substitution
    5. Distributive Property
    6. Multiplication (or Division) Property of Equality
29. **1.** Given
    2. If an angle is bisected, then the two angles formed are equal in measure.
    3. Angle-Addition Postulate
    4. Substitution
    5. Distribution Property
    6. Multiplication (or Division) Property of Equality
30. **1.** Given
    2. Angle-Addition Postulate
    3. Subtraction Property of Equality
31. **S1.** M-N-P-Q on $\overline{MQ}$
    R1. Given
    2. Segment-Addition Postulate
    3. Segment-Addition Postulate
    4. $MN + NP + PQ = MQ$
32. **S1.** $\angle TSW$ with $\overrightarrow{SU}$ and $\overrightarrow{SV}$
    R1. Given
    2. Angle-Addition Postulate
    3. Angle-Addition Postulate
    4. $m\angle TSW = m\angle TSU + m\angle USV + m\angle VSW$

33. $5 \cdot x + 5 \cdot y = 5(x + y)$

34. $5 \cdot x + 7 \cdot x = (5 + 7)x = 12x$

35. $(-7)(-2) > 5(-2)$ or $14 > -10$

36. $\frac{12}{-4} < \frac{-4}{-4}$ or $-3 < 1$

37. $ac > bc$

38. $x > -5$

39. **1**. Given

2. Addition Property of Equality

3. Given

4. Substitution

40.

1. $a = b$	**1**. Given
2. $a - c = b - c$	**2**. Subtraction Property of Equality
3. $c = d$	**3**. Given
4. $a - c = b - d$	**4**. Substitution

SECTION 1.4: Relationships: Perpendicular Lines

1. **1.** Given

2. If 2 $\angle$ s are $\cong$, then they are equal in measure.

3. Angle-Addition Postulate

4. Addition Property of Equality

5. Substitution

6. If 2 $\angle$ s are = in measure, then they are $\cong$.

2. **1.** Given

2. The measure of a straight angle is 180°.

3. Angle-Addition Postulate

4. Substitution

5. Given

6. The measure of a right $\angle = 90^\circ$.

7. Substitution

8. Subtraction Property of Equality

9. Angle-Addition Postulate

10. Substitution

11. If the sum of measures of 2 angles is 90°, then the angles are complementary.

3. **1.** $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$

2. $\angle 1 \cong \angle 3$

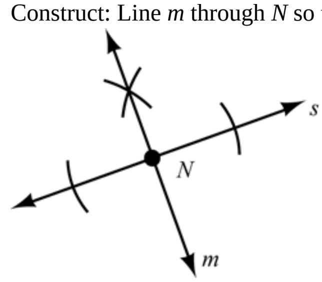
4. **1.** $m\angle AOB = m\angle 1$ and $m\angle BOC = m\angle 1$

2. $m\angle AOB = m\angle BOC$

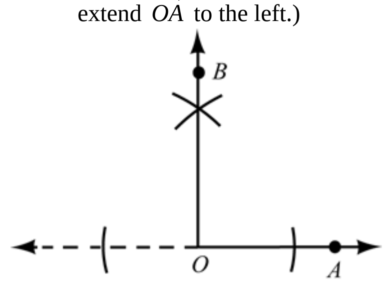
3. $\angle AOB \cong \angle BOC$

4. $\overrightarrow{OB}$ bisects $\angle AOC$

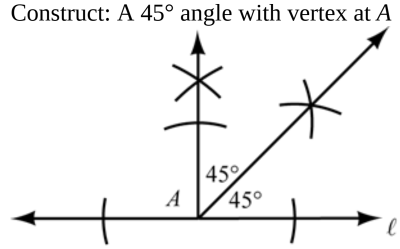
5. Given: Point *N* on line *s*.
Construct: Line *m* through *N* so that $m \perp s$



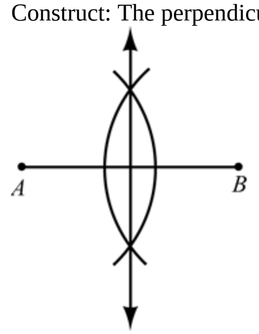
6. Given: $\overrightarrow{OA}$
Construct: Right angle *BOA*
(Hint: Use the straightedge to extend $\overrightarrow{OA}$ to the left.)



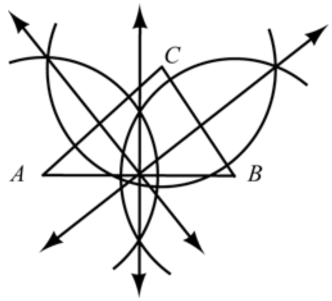
7. Given: Line ℓ containing point *A*
Construct: A 45° angle with vertex at *A*



8. Given: $\overline{AB}$
Construct: The perpendicular bisector of $\overline{AB}$



9. Given: Triangle ABC
Construct: The perpendicular bisectors of sides, $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$



10. It appears that the perpendicular bisectors meet at one point.

11. **R1.** Given

R3. Substitution

S4. $m\angle 1 = m\angle 2$

S5. $\angle 1 \cong \angle 2$

12. **R1.** Given

S2. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

R3. Given

S4. $m\angle 2 + m\angle 3 = 90$

R5. Substitution

S6. $\angle$s 1 and 4 are complementary.

13. No; Yes; No

14. No; No; Yes

15. No; Yes; No

16. No; No; Yes

17. No; Yes; Yes

18. No; No; No

19. **a.** perpendicular

b. angles

c. supplementary

d. right

e. measure of angle

20. **a**. postulate

b. union

c. empty set

d. less than

e. point

21. **a**. adjacent

b. complementary

c. ray AB

d. is congruent to

e. vertical

22. In space, there is an infinite number of lines perpendicular to a given line at a point on the line.

23.

STATEMENTS	REASONS
1. M-N-P-Q on $\overline{MQ}$	**1.** Given
2. $MN + NQ = MQ$	**2.** Segment-Addition Postulate
3. $NP + PQ = NQ$	**3.** Segment-Addition Postulate
4. $MN + NP + PQ = MQ$	**4.** Substitution

24. $AE = AB + BC + CD + DE$

25.

STATEMENTS	REASONS
1. $\angle TSW$ with $\overrightarrow{SU}$ and $\overrightarrow{SV}$	**1.** Given
2. $m\angle TSW = m\angle TSU + m\angle USW$	**2.** Angle-Addition Postulate
3. $m\angle USW = m\angle USV + m\angle VSW$	**3.** Angle-Addition Postulate
4. $m\angle TSW = m\angle TSU + m\angle USV + m\angle VSW$	**4.** Substitution

26. $m\angle GHK = m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4$

27. In space, there is an infinite number of lines that perpendicularly bisect a given line segment at its midpoint.

28. **1.** Given

2. If 2 $\angle$s are complementary, then the sum of their measures is 90°.

3. Given

4. The measure of an acute angle is between 0 and 90°.

5. Substitution

6. Subtraction Property of Equality

7. Subtraction Property of Inequality

8. Addition Property of Inequality

9. Transitive Property of Inequality

10. Substitution

11. If the measure of an angle is between 0 and 90°, then the angle is an acute $\angle$.

29. Angles 1, 2, 3, and 4 are adjacent and form the straight angle *AOB*, which measures 180. Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180$.

30. If $\angle 2$ and $\angle 3$ are complementary, then $m\angle 2 + m\angle 3 = 90$. From Exercise 29, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180$. Therefore, $m\angle 1 + m\angle 4 = 90$ and $\angle 1$ and $\angle 4$ are complementary.

SECTION 1.5: The Formal Proof of a Theorem

1. H: A line segment is bisected.

C: Each of the equal segments has half the length of the original segment.

2. H: Two sides of a triangle are congruent.

C: The triangle is isosceles.

3. First write the statement in the "If, then" form. If a figure is a square, then it is a quadrilateral.

H: A figure is a square.

C: It is a quadrilateral.

4. First write the statement in the "If, then" form. If a polygon is a regular polygon, then it has congruent interior angles.

H: A polygon is a regular polygon.

C: It has congruent interior angles.

5. First write the statement in the "If, then" form. If each is right angle, then two angles are congruent.

H: Each is a right angle.

C: Two angles are congruent.

6. First write the statement in the "If, then" form. If polygons are similar, then the lengths of corresponding sides are proportional.

H: Polygons are similar.

C: The lengths of corresponding sides are proportional.

7. Statement, Drawing, Given, Prove, Proof

8. **a.** Hypothesis

b. Hypothesis

c. Conclusion

9. **a**. Given **b**. Prove

10. *a*, *c*, *d*

11. After the theorem has been proved.

12. No

13. Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$
Prove: $\angle AEC$ is a right angle.

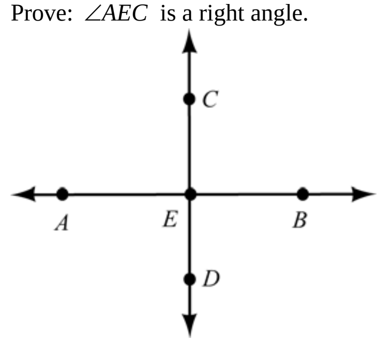
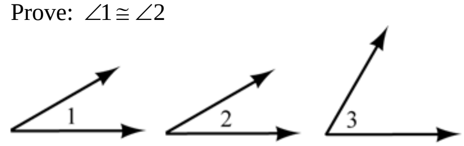


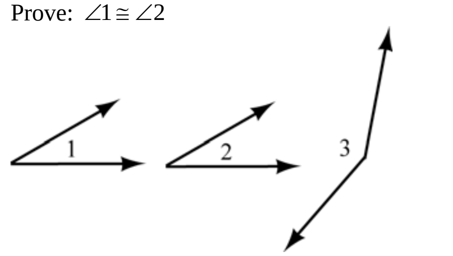
Figure for exercises 13 and 14.

14. Given: $\angle AEC$ is a right angle
Prove: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

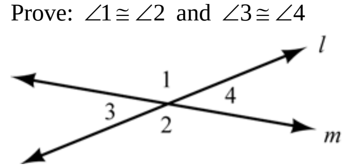
15. Given: $\angle 1$ is complementry to $\angle 3$
$\angle 2$ is complementry to $\angle 3$
Prove: $\angle 1 \cong \angle 2$



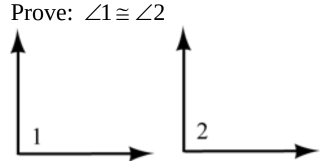
16. Given: $\angle 1$ is supplementary to $\angle 3$
$\angle 2$ is supplementary to $\angle 3$
Prove: $\angle 1 \cong \angle 2$



17. Given: Lines *l* and *m* intersect as shown
Prove: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



18. Given: $\angle 1$ and $\angle 2$ are right angles
Prove: $\angle 1 \cong \angle 2$



19. $m\angle 2 = 55^\circ$, $m\angle 3 = 125^\circ$, $m\angle 4 = 55^\circ$

20. $m\angle 1 = 133^\circ$, $m\angle 3 = 133^\circ$, $m\angle 4 = 47^\circ$

21.
$$m\angle 1 = m\angle 3$$
$$3x + 10 = 4x - 30$$
$$x = 40;\ m\angle 1 = 130^\circ$$

22. $m\angle 2 = m\angle 4$

$6x + 8 = 7x$

$x = 8;\ m\angle 2 = 56°$

23. $m\angle 1 + m\angle 2 = 180°$

$2x + x = 180$

$3x = 180$

$x = 60;\ m\angle 1 = 120$

24. $m\angle 2 + m\angle 3 = 180°$

$x + 15 + 2x = 180$

$3x = 165$

$x = 55;\ m\angle 2 = 70°$

25. $\frac{x}{2} - 10 + \frac{x}{3} + 40 = 180$

$\frac{x}{2} + \frac{x}{3} + 30 = 180$

$\frac{x}{2} + \frac{x}{3} = 150$

Multiply by 6

$3x + 2x = 900$

$5x = 900$

$x = 180;\ m\angle 2 = 80°$

26. $x + 20 + \frac{x}{3} = 180$

$x + \frac{x}{3} = 160$

Multiply by 3

$3x + x = 480$

$4x = 480$

$x = 120;\ m\angle 4 = 40°$

27. **1.** Given

2. If 2 $\angle$ s are complementary, the sum of their measures is 90.

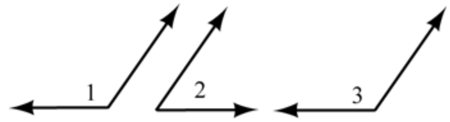
3. Substitution

4. Subtraction Property of Equality

5. If 2 $\angle$ s are = in measure, then they are $\cong$.

28. Given: $\angle 1$ is supplementary to $\angle 2$
$\angle 3$ is supplementary to $\angle 2$

Prove: $\angle 1 \cong \angle 3$

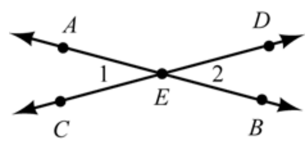


STATEMENTS	REASONS
1. $\angle 1$ is supplementary to $\angle 2$ $\angle 3$ is supplementary to $\angle 2$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 2 = 180$	**2.** If 2 $\angle$s are supplementary, then the sum of their measures is 180.
3. $m\angle 1 + m\angle 2$ $= m\angle 3 + m\angle 2$	**3.** Substitution
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** If 2 $\angle$s are = in measure, then they are $\cong$.

29. If 2 lines intersect, the vertical angles formed are congruent.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E

Prove: $\angle 1 \cong \angle 2$



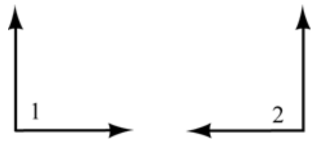
STATEMENTS	REASONS
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E	**1.** Given
2. $\angle 1$ is supplementary to $\angle AED$ $\angle 2$ is supplementary to $\angle AED$	**2.** If the exterior sides of two adjacent $\angle$s form a straight line, then these $\angle$s are supplementary
3. $\angle 1 \cong \angle 2$	**3.** If 2 $\angle$s are supplementary to the same $\angle$, then these $\angle$s are $\cong$.

30. Any two right angles are congruent.

Given: $\angle 1$ is a right $\angle$

$\angle 2$ is a right $\angle$

Prove: $\angle 1 \cong \angle 2$



STATEMENTS	REASONS
1. $\angle 1$ is a right $\angle$ $\angle 2$ is a right $\angle$	**1.** Given
2. $m\angle 1 = 90$ $m\angle 2 = 90$	**2.** Measure of a right $\angle = 90$.
3. $m\angle 1 = m\angle 2$	**3.** Substitution
4. $\angle 1 \cong \angle 2$	**4.** If 2 $\angle$s are = in measure, then they are $\cong$.

31. **R1.** Given

S2. $\angle ABC$ is a right $\angle$.

R3. The measure of a right $\angle = 90$.

R4. Angle-Addition Postulate

S6. $\angle 1$ is complementary to $\angle 2$.

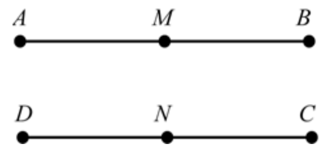
32. If 2 segments are congruent, then their midpoints separate these segments into four congruent segments.

Given: $\overline{AB} \cong \overline{DC}$

M is the midpoint of $\overline{AB}$

N is the midpoint of $\overline{DC}$

Prove: $\overline{AM} \cong \overline{MB} \cong \overline{DN} \cong \overline{NC}$



STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DC}$	**1.** Given
2. $AB = DC$	**2.** If 2 segments are $\cong$, then their lengths are $=$.
3. $AB = AM + MB$ $DC = DN + NC$	**3.** Segment-Addition Postulate
4. $AM + MB = DN + NC$	**4.** Substitution
5. M is the midpoint of $\overline{AB}$ N is the midpoint of $\overline{DC}$	**5.** Given
6. $AM = MB$ and $DN = NC$	**6.** If a point is the midpoint of a segment, it forms 2 segments equal in measure.
7. $AM + AM = DN + DN$ or $2 \cdot AM = 2 \cdot DN$	**7.** Substitution
8. $AM = DN$	**8.** Division Property of Equality
9. $AM = MB = DN = NC$	**9.** Substitution
10. $\overline{AM} \cong \overline{MB} \cong \overline{DN} \cong \overline{NC}$	**10.** If segments are = in length, then they are $\cong$.

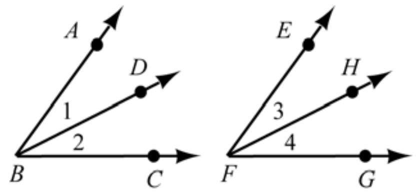
33. If 2 angles are congruent, then their bisectors separate these angles into four congruent angles.

Given: $\angle ABC \cong \angle EFG$

$\overrightarrow{BD}$ bisects $\angle ABC$

$\overrightarrow{FH}$ bisects $\angle EFG$

Prove: $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$



STATEMENTS	REASONS
1. $\angle ABC \cong \angle EFG$	**1.** Given
2. $m\angle ABC = m\angle EFG$	**2.** If 2 angles are $\cong$, their measures are $=$.
3. $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle EFG = m\angle 3 + m\angle 4$	**3.** Angle-Addition Postulate
4. $m\angle 1 + m\angle 2$ $= m\angle 3 + m\angle 4$	**4.** Substitution
5. $\overrightarrow{BD}$ bisects $\angle ABC$ $\overrightarrow{FH}$ bisects $\angle EFG$	**5.** Given
6. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$	**6.** If a ray bisects an $\angle$, then 2 $\angle$s of equal measure are formed.
7. $m\angle 1 + m\angle 1$ $= m\angle 3 + m\angle 3$ or $2 \cdot m\angle 1 = 2 \cdot m\angle 3$	**7.** Substitution
8. $m\angle 1 = m\angle 3$	**8.** Division Property of Equality
9. $m\angle 1 = m\angle 2$ $= m\angle 3 = m\angle 4$	**9.** Substitution
10. $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$	**10.** If $\angle$s are = in measure, then they are $\cong$.

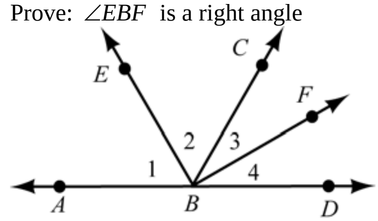
34. The bisectors of two adjacent supplementary angles form a right angle.

Given: $\angle ABC$ is supplementary to $\angle CBD$

$\overrightarrow{BE}$ bisects $\angle ABC$

$\overrightarrow{BF}$ bisects $\angle CBD$

Prove: $\angle EBF$ is a right angle



STATEMENTS	REASONS
1. $\angle ABC$ is supplementary to $\angle CBD$	**1.** Given
2. $m\angle ABC + m\angle CBD$ $= 180$	**2.** The sum of the measures of supplementary angles is 180.
3. $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle CBD = m\angle 3 + m\angle 4$	**3.** Angle-Addition Postulate
4. $m\angle 1 + m\angle 2 + m\angle 3$ $+ m\angle 4 = 180$	**4.** Substitution
5. $\overrightarrow{BE}$ bisects $\angle ABC$ $\overrightarrow{BF}$ bisects $\angle CBD$	**5.** Given
6. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$	**6.** If a ray bisects an $\angle$, then 2 $\angle$s of equal measure are formed.
7. $m\angle 2 + m\angle 2 + m\angle 3$ $+ m\angle 3 = 180$ or $2 \cdot m\angle 2 + 2 \cdot m\angle 3 = 180$	**7.** Substitution
8. $m\angle 2 + m\angle 3 = 90$	**8.** Division Property of Equality
9. $m\angle EBF = m\angle 2 + m\angle 3$	**9.** Angle-Addition Postulate
10. $m\angle EBF = 90$	**10.** Substitution
11. $\angle EBF$ is a right angle	**11.** If the measure of an $\angle$ is 90, then the $\angle$ is a right $\angle$.