Chapter 2: The Logic of Compound Statements

The ability to reason using the principles of logic is essential for solving problems in abstract mathematics and computer science and for understanding the reasoning used in mathematical proof and disproof. Because a significant number of students who come to college have had limited opportunity to develop this ability, a primary aim of Chapters 2 and 3 is to help students develop an inner voice that speaks with logical precision. Consequently, the various rules used in logical reasoning are developed both symbolically and in the context of their somewhat limited but very important use in everyday language. Exercise sets for Sections 2.1–2.3 and 3.1–3.4 contain sentences for students to negate, write the contrapositive for, and so forth. Virtually all students benefit from doing these exercises. Another aim of Chapters 2 and 3 is to teach students the rudiments of symbolic logic as a foundation for a variety of upper-division courses. Symbolic logic is used in, among others, the study of digital logic circuits, relational databases, artificial intelligence, and program verification.

Suggestions

1. In Section 2.1 a surprising number of students apply De Morgan's law to write the negation of, for example, "$1 < x \leq 3$" as "$1 \geq x > 3$." You may find that it takes some effort to teach them to avoid making this mistake.

2. In Sections 2.1 and 2.4, students have more difficulty than you might expect simplifying statement forms and circuits. Only through trial and error can you learn the extent to which this is the case at your institution. If it is, you might either assign only the easier exercises or build in extra time to teach students how to do the more complicated ones. Discussion of simplification techniques occurs again in Chapter 6 in the context of set theory. At this later point in the course most students are able to deal with it successfully.

3. In ordinary English, the phrase "only if" is often used as a synonym for "if and only if." But it is possible to find informal sentences for which the intuitive interpretation is the same as the logical definition. It is helpful to give examples of such statements when you introduce the logical definition. For instance, it is not hard to get students to agree that "The team will win the championship only if it wins the semifinal game" means the same as "If the team does not win the semifinal game then it will not win the championship." Once students see this, you can suggest that they remember this example when they encounter more abstract "only if" statements.

Through guided discussion, students also come to agree that the statement "Winning the semifinal game is a necessary condition for winning the championship" translates to "If the team does not win the semifinal game then it will not win the championship." They can be encouraged to use this (or a similar statement) as a reference to help develop intuition for general statements of the form "A is a necessary condition for B."

With students who have weaker backgrounds, you may find yourself tying up excessive amounts of class time discussing "only if" and "necessary and sufficient conditions." You might just assign the easier exercises, or you might assign exercises on these topics to be done for extra credit (putting corresponding extra credit problems on exams) and use the results to help distinguish A from B students. It is probably best not to omit these topics altogether, though, because the language of "only if" and "necessary and sufficient conditions" is a standard part of the technical vocabulary of textbooks used in upper-division courses, as well as occurring regularly in non-mathematical writing.

4. In Section 2.3, many students mistakenly conclude that an argument is valid if, when they compute the truth table, they find a single row in which both the premises and the conclusion are true. The source of students' difficulty appears to be their tendency to ignore quantification and to misinterpret if-then statements as "and" statements. Since the definition of validity includes both a universal quantifier and if-then, it is helpful to go back over the definition and the procedures for testing for validity and invalidity after discussing the general topic of universal conditional statements in Section 3.1. As a practical measure to help students assess validity and invalidity correctly, the first example in Section 2.3 is of an invalid argument whose truth table has eight rows, several of which have true premises and a true conclusion. To further focus students' attention on the situations where all the premises are true, the truth values for the conclusions of arguments are omitted when at least one premise is false.

5. In Section 2.3, you might suggest that students just familiarize themselves with, but not memorize, the various forms of valid arguments covered in Section 2.3. It is wise, however, to have them learn the terms modus ponens and modus tollens (because these are used in some upper-division computer science courses) and converse and inverse errors (because these errors are so common).

Section 2.1

1. Common form: If p then q.
   p
   Therefore, q

   $(a+2b)(a^2-b)$ can be written in prefix notation. All algebraic expressions can be written in prefix notation.

2. Common form: If p then q.
   $\sim q$
   Therefore, $\sim p$

   All prime numbers are odd. 2 is odd

3. Common form: $p \vee q$
   $\sim p$
   Therefore, q

   My mind is shot. Logic is confusing.

4. Common form: If p then q.
   If q then r.
   Therefore, If p then r.

   Has 4 vertices and 6 edges. Is complete; Any two of its vertices can be joined by a path

5. *a.* It is a statement because it is a true sentence. 1,024 is a perfect square because $1{,}024 = 32^2$, and the next smaller perfect square is $31^2 = 961$, which has fewer than four digits.

   b. The truth or falsity of this sentence depends on the reference for the pronoun "she." Considered on its own, the sentence cannot be said to be either true or false, and so it is not a statement.

   c. This sentence is false; hence it is a statement.

   d. This is not a statement because its truth or falsity depends on the value of x.

6. *a.* $s \wedge i$ *b.* $\sim s \wedge \sim i$

7. $m \wedge \sim c$

8. *a.* $(h \wedge w) \wedge \sim s$ *b.* $\sim w \wedge (h \wedge s)$ *c.* $\sim w \wedge \sim h \wedge \sim s$
   d. $(\sim w \wedge \sim s) \wedge h$ *e.* $w \wedge \sim (h \wedge s)$ ($w \wedge (\sim h \vee \sim s)$ is also acceptable)

9. *a.* $p \vee q$ **b.** $r \wedge p$ **c.** $r \wedge (p \vee q)$

10. *a.* $p \wedge q \wedge r$ *b.* $p \wedge \sim q$ **c.** $p \wedge (\sim q \vee \sim r)$ **d.** $(\sim p \wedge q) \wedge \sim r$ **e.** $\sim p \vee (q \wedge r)$

11. Inclusive or. For instance, a team could win the playoff by winning games 1, 3, and 4 and losing game 2. Such an outcome would satisfy both conditions.

12.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

13.

p	q	$p \wedge q$	$p \vee q$	$\sim (p \wedge q)$	$\sim (p \wedge q) \vee (p \vee q)$
T	T	T	T	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	F	T	T

14.

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

15.

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

16.

p	q	$p \wedge q$	$p \vee (p \wedge q)$	p
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F

same truth values

The truth table shows that $p \vee (p \wedge q)$ and p always have the same truth values. Thus they are logically equivalent. (This proves one of the absorption laws.)

17.

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim (p \wedge q)$	$\sim p \wedge \sim q$	
T	T	T	F	F	F	F	
T	F	F	F	T	T	F	$\leftarrow$
F	T	F	T	F	T	F	$\leftarrow$
F	F	F	T	T	T	T	

different truth values in rows 2 and 3

The truth table shows that $\sim (p \wedge q)$ and $\sim p \wedge \sim q$ do not always have the same truth values. Therefore they are not logically equivalent.

18.

p	$\mathbf{t}$	$p \vee \mathbf{t}$
T	T	T
F	T	T

same truth values

The truth table shows that $p \vee \mathbf{t}$ and $\mathbf{t}$ always have the same truth values. Thus they are logically equivalent. (This proves one of the universal bound laws.)

19.

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p
T	T	T	T
F	T	F	F

same truth values

The truth table shows that $p \wedge\ \mathbf{t}$ and p always have the same truth values. Thus they are logically equivalent. This proves the identity law for $\wedge$.

20.

p	$\mathbf{c}$	$p \wedge \mathbf{c}$	$p \vee \mathbf{c}$	
T	F	F	T	$\leftarrow$
F	F	F	F	

different truth values in row 1

The truth table shows that $p \wedge\ \mathbf{c}$ and $p \vee \mathbf{c}$ do not always have the same truth values. Thus they are not logically equivalent.

21.

p	q	$p \quad q \quad r$	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ always have the same truth values. Thus they are logically equivalent. (This proves the associative law for $\wedge$.)

22.

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

same truth values

The truth table shows that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ always have the same truth values. Therefore they are logically equivalent. This proves the distributive law for $\wedge$ over $\vee$.

23.

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \vee r$	$p \wedge (q \vee r)$	
T	T	T	T	T	T	T	
T	T	F	T	T	T	T	
T	F	T	F	T	T	T	
T	F	F	F	F	F	F	
F	T	T	F	T	T	F	$\leftarrow$
F	T	F	F	T	F	F	
F	F	T	F	T	T	F	$\leftarrow$
F	F	F	F	F	F	F	

different truth values in rows 5 and 7

The truth table shows that $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ have different truth values in rows 5 and 7. Thus they are not logically equivalent. (This proves that parentheses are needed with $\wedge$ and $\vee$.)

24.

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$	
T	T	T	T	T	T	T	
T	T	F	T	F	T	F	$\leftarrow$
T	F	T	T	T	T	T	$\leftarrow$
T	F	F	T	F	T	F	
F	T	T	T	F	T	T	
F	T	F	T	F	T	F	$\leftarrow$
F	F	T	F	F	F	F	
F	F	F	F	F	F	F	

different truth values in rows 2, 3, and 6

The truth table shows that $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ have different truth values in rows 2, 3, and 6. Hence they are not logically equivalent.

25. Hal is not a math major or Hal's sister is not a computer science major.

26. Sam is not an orange belt or Kate is not a red belt.

27. The connector is not loose and the machine is not unplugged.

28. The train is not late and my watch is not fast.

29. This computer program does not have a logical error in the first ten lines and it is not being run with an incomplete data set.

30. The dollar is not at an all-time high or the stock market is not at a record low.

31. **a.** 01, 02, 11, 12 **b.** 21, 22 **c.** 11, 10, 21, 20

32. $-2 \geq x$ or $x \geq 7$

33. $-10 \geq x$ or $x \geq 2$

34. $2 \leq x \leq 5$

35. $x > -1$ and $x \leq 1$

36. $1 \leq x$ or $x < -3$

37. $0 \leq x$ or $x < -7$

38. This statement's logical form is $(p \wedge q) \vee r$, so its negation has the form $\sim((p \wedge q) \vee r) \equiv \sim(p \wedge q) \wedge \sim r \equiv (\sim p \vee \sim q) \wedge \sim r$. Thus a negation for the statement is ($num\_orders \leq 100$ or $num\_instock > 500$) and $num\_instock \geq 200$.

39. The statement's logical form is $(p \wedge q) \vee ((r \wedge s) \wedge t)$, so its negation has the form

$$\begin{aligned} \sim((p \wedge q) \vee ((r \wedge s) \wedge t)) &\equiv \sim(p \wedge q) \wedge \sim((r \wedge s) \wedge t)) \\ &\equiv (\sim p \vee \sim q) \wedge (\sim (r \wedge s) \vee \sim t)) \\ &\equiv (\sim p \vee \sim q) \wedge ((\sim r \vee \sim s) \vee \sim t)). \end{aligned}$$

Thus a negation is ($num\_orders \geq 50$ or $num\_instock \leq 300$) and (($50 > num\_orders$ or $num\_orders \geq 75$) or $num\_instock \leq 500$).

40.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Since all the truth values of $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ are T, $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.

41.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q)(p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Since all the truth values of $(p \wedge \sim q) \wedge (\sim p \vee q)$ are F, $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

42.

p	q	r	$\sim p$	$\sim q$	$\sim p \wedge q$	$q \wedge r$	$((\sim p \wedge q) \wedge (q \wedge r))$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	F	F	F

all $F's$

Since all the truth values of $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ are F, $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ is a contradiction.

43.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

all $T's$

44. **a.** No real numbers satisfy this inequality

b. No real numbers satisfy this inequality.

45. Let b be "Bob is a double math and computer science major," m be "Ann is a math major," and a be "Ann is a double math and computer science major." Then the two statements can be symbolized as follows: a. $(b \wedge m) \wedge \sim a$ and **b.** $\sim (b \wedge a) \wedge (m \wedge b)$. *Note*: The entries in the truth table assume that a person who is a double math and computer science major is also a math major and a computer science major.

b	m	a	$\sim a$	$b \wedge m$	$m \wedge b$	$b \wedge a$	$\sim (b \wedge a)$	$(b \wedge m) \wedge \sim a$	$\sim (b \wedge a) \wedge (m \wedge b)$
T	T	T	F	T	T	T	F	F	F
T	T	F	T	F	T	F	T	T	T
T	F	T	F	T	F	T	F	F	F
T	F	F	T	F	F	F	T	F	F
F	T	T	F	F	F	F	T	F	F
F	T	F	T	F	F	F	T	F	F
F	F	T	F	F	F	F	T	F	F
F	F	F	T	F	F	F	T	F	F

same truth values

The truth table shows that $(b \wedge m) \wedge \sim a$ and $\sim (b \wedge a) \wedge (m \wedge b)$ always have the same truth values. Hence they are logically equivalent.

46. **a.** *Solution 1:* Construct a truth table for $p \oplus p$ using the truth values for *exclusive or.*

p	$p \oplus p$
T	F
F	F

because an *exclusive or* statement is false when both components are true and when both components are false, and the two components in $p \oplus p$ are both p.

Since all its truth values are false, $p \oplus p \equiv \mathbf{c}$, a contradiction.

Solution 2: Replace q by p in the logical equivalence $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$, and simplify the result.

$$\begin{aligned} p \oplus p &\equiv (p \vee p) \wedge \sim(p \wedge p) && \text{by definition of } \oplus \\ &\equiv p \wedge \sim p && \text{by the identity laws} \\ &\equiv \mathbf{c} && \text{by the negation law for } \wedge \end{aligned}$$

b. Yes.

p	q	r	$p \oplus q$	$q \oplus r$	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	F	T	T
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \oplus r$ and $p \oplus (q \oplus r)$ always have the same truth values. So they are logically equivalent.

c. Yes.

p	q	r	$p \oplus q$	$p \wedge r$	$q \wedge r$	$(p \oplus q) \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \wedge r$ and $(p \wedge r) \oplus (q \wedge r)$ always have the same truth values. So they are logically equivalent.

47. There is a famous story about a philosopher who once gave a talk in which he observed that whereas in English and many other languages a double negative is equivalent to a positive, there is no language in which a double positive is equivalent to a negative. To this, another philosopher, Sidney Morgenbesser, responded sarcastically, "Yeah, yeah."

[Strictly speaking, sarcasm functions like negation. When spoken sarcastically, the words "Yeah, yeah" are not a true double positive; they just mean "no."]

48. **a.** the distributive law **b.** the commutative law for $\vee$
c. the negation law for $\vee$ **d.** the identity law for $\wedge$

49. **a.** the commutative law for $\vee$ **b.** the distributive law
c. the negation law for $\wedge$ **d.** the identity law for $\vee$

50. $$\begin{aligned} (p \wedge \sim q) \vee p &\equiv p \vee (p \wedge \sim q) && \text{by the commutative law for } \vee \\ &\equiv p && \text{by the absorption law (with } \sim q \text{ in place of } q) \end{aligned}$$

51. *Solution 1*: $$\begin{aligned} p \wedge (\sim q \vee p) &\equiv p \wedge (p \vee \sim q) && \text{commutative law for } \vee \\ &\equiv p && \text{absorption law} \end{aligned}$$

Solution 2: $p \wedge (\sim q \vee p) \equiv (p \wedge \sim q) \vee (p \wedge p)$ distributive law
$\equiv (p \wedge \sim q) \vee p$ identity law for $\wedge$
$\equiv p$ by exercise 50.

52. $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv (\sim p \wedge \sim (\sim q)) \vee (\sim p \wedge \sim q)$ De Morgan's law
$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ double negative law
$\equiv \sim p \wedge (q \vee \sim q)$ distributive law
$\equiv \sim p \wedge \mathbf{t}$ negation law for $\vee$
$\equiv \sim p$ identity law for $\wedge$

53. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv \sim[\sim p \wedge (q \vee \sim q)] \vee (p \wedge q)$ by the distributive law
$\equiv \sim(\sim p \wedge \mathbf{t}) \vee (p \wedge q)$ by the negation law for $\vee$
$\equiv \sim(\sim p) \vee (p \wedge q)$ by the identity law for $\wedge$
$\equiv p \vee (p \wedge q)$ by the double negative law
$\equiv p$ by the absorption law

54. $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv (p \wedge (\sim (\sim p) \wedge \sim q)) \vee (p \wedge q)$ De Morgan's law
$\equiv (p \wedge (p \wedge \sim q)) \vee (p \wedge q)$ double negative law
$\equiv ((p \wedge p) \wedge \sim q)) \vee (p \wedge q)$ associative law for $\wedge$
$\equiv (p \wedge \sim q)) \vee (p \wedge q)$ idempotent law for $\wedge$
$\equiv p \wedge (\sim q \vee q)$ distributive law
$\equiv p \wedge (q \vee \sim q)$ commutative law for $\vee$
$\equiv p \wedge \mathbf{t}$ negation law for $\vee$
$\equiv p$ identity law for $\wedge$

Section 2.2

1. If this loop does not contain a **stop** or a **go to**, then it will repeat exactly N times.
2. If I catch the 8:05 bus, then I am on time for work.
3. If you do not freeze, then I'll shoot.
4. If you don't fix my ceiling, then I won't pay my rent.
5.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim p \vee q \to \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

6.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \vee (\sim p \wedge q)$	$(p \vee q) \vee (\sim p \wedge q) \to q$
T	T	F	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

7.

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \to r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

8.

p	q	r	$\sim p$	$\sim p \vee q$	$\sim p \vee q \to r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

9.

p	q	r	$\sim r$	$p \wedge \sim r$	$q \wedge r$	$p \wedge \sim r \leftrightarrow q \vee r$
T	T	T	F	F	T	F
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	T	F	T	F	T	F
F	F	T	F	F	T	F
F	F	F	T	F	F	T

10.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

11.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$p \wedge q \to r$	$(p \to (q \to r)) \leftrightarrow (p \wedge q \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

12. If $x > 2$ then $x^2 > 4$, and if $x < -2$ then $x^2 > 4$.

13. **a.**

p	q	$\sim p$	$p \to q$	$\sim p \wedge q$
T	T	F	T	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

same truth values

The truth table shows that $p \to q$ and $\sim p \vee q$ always have the same truth values. Hence they are logically equivalent.

b.

p	q	$\sim q$	$p \to q$	$\sim (p \to q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

same truth values

The truth table shows that $\sim (p \to q)$ and $p \wedge \sim q$ always have the same truth values. Hence they are logically equivalent.

14. **a.**

p	q	r	$\sim q$	$\sim r$	$q \vee r$	$p \wedge \sim q$	$p \wedge \sim r$	$p \to q \vee r$	$p \wedge \sim q \to r$	$p \wedge \sim r \to q$
T	T	T	F	F	T	F	F	T	T	T
T	T	F	F	T	T	F	T	T	T	T
T	F	T	T	F	T	T	F	T	T	T
T	F	F	T	T	F	T	T	F	F	F
F	T	T	F	F	T	F	F	T	T	T
F	T	F	F	T	T	F	F	T	T	T
F	F	T	T	F	T	F	F	T	T	T
F	F	F	T	T	F	F	F	T	T	T

same truth values

The truth table shows that the three statement forms $p \to q \vee r$, $p \wedge \sim q \to r$, and $p \wedge \sim r \to q$ always have the same truth values. Thus they are all logically equivalent.

b. If n is prime and n is not odd, then n is 2.
And: If n is prime and n is not 2, then n is odd.

15.

p	q	r	$q \to r$	$p \to q$	$p \to (q \to r)$	$(p \to q) \to r$	
T	T	T	T	T	T	T	
T	T	F	F	T	F	F	
T	F	T	T	F	T	T	
T	F	F	T	F	T	T	
F	T	T	T	T	T	T	
F	T	F	F	T	T	F	$\leftarrow$
F	F	T	T	T	T	F	$\leftarrow$
F	F	F	T	T	T	F	$\leftarrow$

different truth values

The truth table shows that $p \to (q \to r)$ and $(p \to q) \to r$ do not always have the same truth values. (They differ for the combinations of truth values for p, q, and r shown in rows 6, 7, and 8.) Therefore they are not logically equivalent.

16. Let p represent "You paid full price" and q represent "You didn't buy it at Crown Books." Thus, "If you paid full price, you didn't buy it at Crown Books" has the form $p \to q$. And "You didn't buy it at Crown Books or you paid full price" has the form $q \vee p$.

p	q	$p \to q$	$q \vee p$	
T	T	T	T	
T	F	F	T	$\leftarrow$
F	T	T	T	$\leftarrow$
F	F	T	F	

different truth values

These two statements are not logically equivalent because their forms have different truth values in rows 2 and 4.
(An alternative representation for the forms of the two statements is $p \to \sim q$ and $\sim q \vee p$. In this case, the truth values differ in rows 1 and 3.)

17. Let p represent "2 is a factor of n," q represent "3 is a factor of n," and r represent "6 is a factor of n." The statement "If 2 is a factor of n and 3 is a factor of n, then 6 is a factor of n" has the form $p \wedge q \to r$. And the statement "2 is not a factor of n or 3 is a not a factor of n or 6 is a factor of n" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge q \to r$	$\sim p \vee \sim q \vee r$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	F	F
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $\sim p \vee \sim q \vee r$ always have the same truth values. Therefore they are logically equivalent.

18. *Part 1*: Let p represent "It walks like a duck," q represent "It talks like a duck," and r represent "It is a duck." The statement "If it walks like a duck and it talks like a duck, then it is a duck" has the form $p \wedge q \to r$. And the statement "Either it does not walk like a duck or it does not talk like a duck or it is a duck" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$p \wedge q \to r$	$(\sim p \vee \sim q) \vee r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $(\sim p \vee \sim q) \vee r$ always have the same truth values. Thus the following statements are logically equivalent: "If it walks like a duck and it talks like a duck, then it is a duck" and "Either it does not walk like a duck or it does not talk like a duck or it is a duck."

Part 2: The statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck" has the form $\sim p \wedge \sim q \to \sim r$.

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge q$	$\sim p \wedge \sim q$	$p \wedge q \to r$	$(\sim p \wedge \sim q) \to \sim r$	
T	T	T	F	F	F	T	F	T	T	
T	T	F	F	F	T	T	F	F	T	$\leftarrow$
T	F	T	F	T	F	F	F	T	T	
T	F	F	F	T	T	F	F	T	T	
F	T	T	T	F	F	F	F	T	T	
F	T	F	T	F	T	F	F	T	T	
F	F	T	T	T	F	F	T	T	F	$\leftarrow$
F	F	F	T	T	T	F	T	T	T	

different truth values

The truth table shows that $p \wedge q \to r$ and $(\sim p \wedge \sim q) \to \sim r$ do not always have the same truth values. (They differ for the combinations of truth values of p, q, and r shown in rows 2 and 7.) Thus they are not logically equivalent, and so the statement "If it walks like a duck and it talks like a duck, then it is a duck" is not logically equivalent to the statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck." In addition, because of the logical equivalence shown in Part 1, we can also conclude that the following two statements are not logically equivalent: "Either it does not walk like a duck or it does not talk like a duck or it is a duck" and "If it does not walk like a duck and it does not talk like a duck then it is not a duck."

19. False. The negation of an if-then statement is not an if-then statement. It is an *and* statement.

20. **a.** *Negation*: P is a square and P is not a rectangle.

b. *Negation*: Today is New Year's Eve and tomorrow is not January.

c. *Negation*: The decimal expansion of r is terminating and r is not rational.

d. *Negation*: n is prime and both n is not odd and n is not 2. Or: n is prime and n is neither odd nor 2.

e. *Negation*: x is nonnegative and x is not positive and x is not 0.
Or: x is nonnegative but x is not positive and x is not 0.
Or: x is nonnegative and x is neither positive nor 0.

f. *Negation*: Tom is Ann's father and either Jim is not her uncle or Sue is not her aunt.

g. *Negation*: n is divisible by 6 and either n is not divisible by 2 or n is not divisible by 3.

21. By assumption, $p \to q$ is false. By definition of a conditional statement, the only way this can happen is for the hypothesis, p, to be true and the conclusion, q, to be false.

a. The only way $\sim p \to q$ can be false is for $\sim p$ to be true and q to be false. But since p is true, $\sim p$ is false. Hence $\sim p \to q$ is not false and so it is true.

b. Since p is true, then $p \vee q$ is true because if one component of an *and* statement is true, then the statement as a whole is true.

c The only way $q \to p$ can be false is for q to be true and p to be false. Thus, since q is false, $q \to p$ is not false and so it is true.

22. **a.** *Contrapositive*: If P is not a rectangle, then P is not a square.

b. *Contrapositive*: If tomorrow is not January, then today is not New Year's Eve.

c. *Contrapositive*: If r is not rational, then the decimal expansion of r is not terminating.

d. *Contrapositive*: If n is not odd and n is not 2, then n is not prime.

e. *Contrapositive*: If x is not positive and x is not 0, then x is not nonnegative.
Or: If x is neither positive nor 0, then x is negative.

f. *Contrapositive*: If either Jim is not Ann's uncle or Sue is not her aunt, then Tom is not her father.

g. *Contrapositive*: If n is not divisible by 2 or n is not divisible by 3, then n is not divisible by 6.

23. **a.** *Converse*: If P is a rectangle, then P is a square.
Inverse: If P is not a square, then P is not a rectangle.

b. *Converse*: If tomorrow is January, then today is New Year's Eve.
Inverse: If today is not New Year's Eve, then tomorrow is not January.

c. *Converse*: If r is rational then the decimal expansion of r is terminating.
Inverse: If the decimal expansion of r is not terminating, then r is not rational.

d. *Converse*: If n is odd or n is 2, then n is prime.
Inverse: If n is not prime, then n is not odd and n is not 2.

e. *Converse*: If x is positive or x is 0, then x is nonnegative.
Inverse: If x is not nonnegative, then both x is not positive and x is not 0.
Or: If x is negative, then x is neither positive nor 0.

f. *Converse*: If Jim is Ann's uncle and Sue is her aunt, then Tom is her father.
Inverse: If Tom is not Ann's father, then Jim is not her uncle or Sue is not her aunt

g. *Converse*: If n is divisible by 2 and n is divisible by 3, then n is divisible by 6
Inverse: If n is not divisible by 6, then n is not divisible by 2 or n is not divisible by 3.

24.

p	q	$p \to q$	$q \to p$	
T	T	T	T	
T	F	F	T	$\leftarrow$
F	T	T	F	$\leftarrow$
F	F	T	T	

different truth values

The truth table shows that $p \to q$ and $q \to p$ have different truth values in the second and third rows. Hence they are not logically equivalent.

25.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	
T	T	F	F	T	T	
T	F	F	T	F	T	$\leftarrow$
F	T	T	F	T	F	$\leftarrow$
F	F	T	T	T	T	

different truth values

The truth table shows that $p \to q$ *and* $\sim p \to \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent. Thus a conditional statement is not logically equivalent to its inverse.

26.

p	q	$\sim p$	$\sim q$	$\sim q \to \sim p$	$p \to q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

same truth values

The truth table shows that $\sim q \to \sim p$ and $p \to q$ always have the same truth values and thus are logically equivalent. It follows that a conditional statement and its contrapositive are logically equivalent to each other.

27.

p	q	$\sim p$	$\sim q$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

same truth values

The truth table shows that $q \to p$ and $\sim p \to \sim q$ always have the same truth values and thus are logically equivalent. It follows that the converse and inverse of a conditional statement are logically equivalent to each other.

28. The if-then form of "I say what I mean" is "If I mean something, then I say it."
The if-then form of "I mean what I say" is "If I say something, then I mean it."
Thus "I mean what I say" is the converse of "I say what I mean," and so the two statements are not logically equivalent.

29. The corresponding tautology is $(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$

p	q	r	$\sim q$	$q \vee r$	$p \wedge \sim q$	$p \to (q \vee r)$	$p \wedge \sim q \to r$	$(p \to (q \vee r)) \leftrightarrow$ $((p \wedge \sim q) \to r)$
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	F	F	T	T	T

The truth table shows that $(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$ is a tautology because all of its truth values are T.

30. The corresponding tautology is $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow$ $(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

all T's

The truth table shows that $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$ is always true. Hence it is a tautology.

31. The corresponding tautology is $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$.

p	q	r	$q \to r$	$p \wedge q$	$p \to (q \to r)$	$(p \wedge q) \to r)$	$p \to (q \to r) \leftrightarrow (p \wedge q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

all T's

The truth table shows that $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ is always true. Hence it is a tautology.

32. If this quadratic equation has two distinct real roots, then its discriminant is greater than zero, and if the discriminant of this quadratic equation is greater than zero, then the equation has two real roots.

33. If this integer is even, then it equals twice some integer, and if this integer equals twice some integer, then it is even.

34. If the Cubs do not win tomorrow's game, then they will not win the pennant.
If the Cubs win the pennant, then they will have won tomorrow's game.

35. If Sam is not an expert sailor, then he will not be allowed on Signe's racing boat.
If Sam is allowed on Signe's racing boat, then he is an expert sailor.

36. The Personnel Director did not lie. By using the phrase "only if," the Personnel Director set forth conditions that were necessary but not sufficient for being hired: if you did not satisfy those conditions then you would not be hired. The Personnel Director's statement said nothing about what would happen if you did satisfy those conditions.

37. If a new hearing is not granted, payment will be made on the fifth.

38. If it doesn't rain, then Ann will go.

39. If a security code is not entered, then the door will not open.

40. If I catch the 8:05 bus, then I am on time for work.

41. If this triangle has two 45° angles, then it is a right triangle.

42. If this number is not divisible by 3, then it is not divisible by 9.
If this number is divisible by 9, then it is divisible by 3.

43. If Jim does not do his homework regularly, then Jim will not pass the course.

If Jim passes the course, then he will have done his homework regularly.

44. If Jon's team wins the rest of its games, then it will win the championship.

45. If this computer program produces error messages during translation, then it is not correct.

If this computer program is correct, then it does not produce error messages during translation.

46. **a.** This statement is the converse of the given statement, and so it is not necessarily true. For instance, if the actual boiling point of compound X were 200°C, then the given statement would be true but this statement would be false.

b. This statement must be true. It is the contrapositive of the given statement.

c. must be true **d.** not necessarily true **e.** must be true **f.** not necessarily true

Note: To solve this problem, it may be helpful to imagine a compound whose boiling point is greater than 150° C. For concreteness, suppose it is 200° C. Then the given statement would be true for this compound, but statements a, d, and f would be false.

47. **a.** $p \wedge {\sim}q \to r \equiv {\sim}(p \wedge {\sim}q) \vee r$

b. $p \wedge {\sim}q \to r \equiv {\sim}(p \wedge {\sim}q) \vee r$ by the identity for $\to$ shown in the directions *[an acceptable answer]*

$\equiv {\sim}[{\sim}({\sim}(p \wedge {\sim}q)) \wedge {\sim}r]$ by De Morgan's law *[another acceptable answer]*

$\equiv {\sim}[(p \wedge {\sim}q) \wedge {\sim}r]$ by the double negative law *[another acceptable answer]*

Any of the expressions in part (b) would also be acceptable answers for part (a).

48. **a.** $p \vee {\sim} q \to r \vee q \equiv {\sim} (p \vee {\sim} q) \vee (r \vee q)$ by the identity for $\to$ shown in the directions *[an acceptable answer]*

$\equiv ({\sim} p \wedge {\sim} ({\sim} q)) \vee (r \vee q)$ by De Morgan's law *[another acceptable answer]*

$\equiv ({\sim} p \wedge q) \vee (r \vee q)$ by the double negative law *[another acceptable answer]*

b. $p \vee {\sim} q \to r \vee q \equiv ({\sim} p \wedge q) \vee (r \vee q)$ by part (a)

$\equiv {\sim} ({\sim} ({\sim} p \wedge q) \wedge {\sim} (r \vee q))$ by De Morgan's law

$\equiv {\sim} ({\sim} ({\sim} p \wedge q) \wedge ({\sim} r \wedge {\sim} q))$ by De Morgan's law

Any of the expressions in part (b) would also be acceptable answers for part (a).

49. **a.** $(p \to r) \leftrightarrow (q \to r) \equiv ({\sim}p \vee r) \leftrightarrow ({\sim}q \vee r)$

$\equiv [{\sim}({\sim}p \vee r) \vee ({\sim}q \vee r)] \wedge [{\sim}({\sim}q \vee r) \vee ({\sim}p \vee r)]$
by the identity for $\leftrightarrow$ shown in the directions *[an acceptable answer]*

$\equiv [(p \wedge {\sim}r) \vee ({\sim}q \vee r)] \wedge [(q \wedge {\sim}r) \vee ({\sim}p \vee r)]$
by De Morgan's law *[another acceptable answer]*

b. $({\sim}p \vee r) \leftrightarrow ({\sim}q \vee r) \equiv {\sim}[{\sim}(p \wedge {\sim}r) \wedge {\sim}({\sim}q \vee r)] \wedge {\sim}[{\sim}(q \wedge {\sim}r) \wedge {\sim}({\sim}p \vee r)]$
by De Morgan's law

$\equiv {\sim}[{\sim}(p \wedge {\sim}r) \wedge (q \wedge {\sim}r)] \wedge {\sim}[{\sim}(q \wedge {\sim}r) \wedge (p \wedge {\sim}r)]$
by De Morgan's law

Any of the expressions in part (b) would also be acceptable answers for part (a).

50. **a.**
$$
\begin{aligned}
(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) &\equiv [\sim p \vee (q \to r)] \leftrightarrow [\sim (p \wedge q) \vee r] \\
&\equiv [\sim p \vee (\sim q \vee r)] \leftrightarrow [\sim (p \wedge q) \vee r] \\
&\equiv \sim [\sim p \vee (\sim q \vee r)] \vee [\sim (p \wedge q) \vee r] \\
&\qquad \wedge \sim [\sim (p \wedge q) \vee r] \vee [\sim p \vee (\sim q \vee r)]
\end{aligned}
$$

b. By part (a), De Morgan's law, and the double negative law,

$$
\begin{aligned}
(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) &\equiv \sim [\sim p \vee (\sim q \vee r)] \vee [\sim (p \wedge q) \vee r] \\
&\qquad \wedge \sim [\sim (p \wedge q) \vee r] \vee [\sim p \vee (\sim q \vee r)] \\
&\equiv \sim [\sim p \vee (\sim q \vee r)] \wedge \sim [\sim (p \wedge q) \vee r] \\
&\qquad \wedge \sim \sim [(p \wedge q) \wedge \sim r] \wedge \sim [\sim p \vee (\sim q \vee r)] \\
&\equiv \sim \sim [p \wedge \sim (\sim q \vee r)] \wedge [(p \wedge q) \wedge \sim r] \\
&\qquad \wedge \sim \sim [(p \wedge q) \wedge \sim r] \wedge [p \wedge \sim (\sim q \vee r)] \\
&\equiv \sim \sim [p \wedge (q \wedge \sim r)] \wedge [(p \wedge q) \wedge \sim r] \\
&\qquad \wedge \sim \sim [(p \wedge q) \wedge \sim r] \wedge [p \wedge (q \wedge \sim r)].
\end{aligned}
$$

Any of the expressions in the right-hand column would also be acceptable answers for part (a).

51. Yes. As in exercises 47-50, the following logical equivalences can be used to rewrite any statement form in a logically equivalent way using only $\sim$ and $\wedge$:

$$
\begin{array}{ll}
p \to q \equiv \sim p \vee q & \qquad p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p) \\
p \vee q \equiv \sim (\sim p \wedge \sim q) & \qquad \sim (\sim p) \equiv p
\end{array}
$$

The logical equivalence $p \wedge q \equiv \sim (\sim p \vee \sim q)$ can then be used to rewrite any statement form in a logically equivalent way using only $\sim$ and $\vee$.

Section 2.3

1. $\sqrt{2}$ is not rational.

2. $1 - 0.99999...$ is less than every positive real number.

3. Logic is not easy.

4. This graph cannot be colored with two colors.

5. They did not telephone.

6.

		premises		*conclusion*	
p	q	$p \to q$	$p \to q$	$p \vee q$	
T	T	T	T	T	← *critical row*
T	F	F	T		
F	T	T	F		
F	T	T	T	F	← *critical row*

Rows 2 and 4 of the truth table are the critical rows in which all the premises are true, but row 4 shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

7.

				premises			*conclusion*	
p	q	r	$\sim q$	p	$p \to q$	$\sim q \vee r$	r	
T	T	T	F	T	T	T	T	*critical row*
T	T	F	F	T	T	F		
T	F	T	T	T	F	T		
T	F	F	T	T	F	T		
F	T	T	F	F	T	T		
F	T	F	F	F	T	F		
F	F	T	T	F	T	T		
F	F	F	T	F	T	T		

This row describes the only situation in which all the premises are true. Because the conclusion is also true here, the argument form is valid.

8.

					premises		*conclusion*	
p	q	r	$\sim q$	$p \vee q$	$p \to \sim q$	$p \to r$	r	
T	T	T	F	T	F	T		
T	T	F	F	T	F	F		
T	F	T	T	T	T	T	T	*critical row*
T	F	F	T	T	T	F		
F	T	T	F	T	T	T	T	*critical row*
F	T	F	F	T	T	T	F	*critical row*
F	F	T	T	F	T	T		
F	F	F	T	F	T	T		

This row shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

9.

							premises		*conclusion*	
p	q	r	$\sim q$	$\sim r$	$p \wedge q$	$p \wedge q \to \sim r$	$p \vee \sim q$	$\sim q \to p$	$\sim r$	
T	T	T	F	F	T	F	T	T		
T	T	F	F	T	T	T	T	T	T	*critical row*
T	F	T	T	F	F	T	T	T	F	*critical row*
T	F	F	T	T	F	T	T	T	T	*critical row*
F	T	T	F	F	F	T	F	T		
F	T	F	F	T	F	T	F	T		
F	F	T	T	F	F	T	T	F		
F	F	F	T	T	F	T	T	F		

Rows 2, 3, and 4 of the truth table are the critical rows in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

10.

									premise	*conclusion*	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p \wedge \sim q$	$p \vee q$	r	$p \vee q \to r$	$\sim r \to \sim p \wedge \sim q$	
T	T	T	F	F	F	F	T	T	T	T	*critical row*
T	T	F	F	F	T	F	T	F	F	F	
T	F	T	F	T	F	F	T	T	T	T	*critical row*
T	F	F	F	T	T	F	T	F	F	F	
F	T	T	T	F	F	F	T	T	T	T	*critical row*
F	T	F	T	F	T	F	T	F	F	F	
F	F	T	T	T	F	T	F	T	T	T	*critical row*
F	F	F	T	T	T	T	F	F	T	T	*critical row*

This form of argument has just one premise. Rows 1, 3, 5, 7, and 8 of the truth table represent all the situations in which the premise is true, and in each of these rows the conclusion is also true. Therefore, the argument form is valid.

11.

							premises		conclusion	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \vee r$	$p \to q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim r$	
T	T	T	F	F	F	T	T	F		
T	T	F	F	F	T	T	T	T	T	← *critical row*
T	F	T	F	T	F	T	T	T	F	← *critical row*
T	F	F	F	T	T	F	F	T		
F	T	T	T	F	F	T	T	F		
F	T	F	T	F	T	T	T	T	T	← *critical row*
F	F	T	T	T	F	T	T	T	T	← *critical row*
F	F	F	T	T	T	F	T	T	T	← *critical row*

Rows 2, 3, 6, 7, and 8 of the truth table represent the situations in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

12. **a.**

		premises		conclusion	
p	q	$p \to q$	q	p	
T	T	T	T	T	← *critical row*
T	F	F	F		
F	T	T	T	F	← *critical row*
F	T	T	F		

Rows 1 and 3 of the truth table represent the situations in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

b.

		premises		conclusion	
p	q	$p \to q$	$\sim p$	$\sim q$	
T	T	T	F		
T	F	F	T	T	← *critical row*
F	T	T	F		
F	T	T	T	F	← *critical row*

Rows 2 and 4 of the truth table represent the situations in which all the premises are true, but row 4 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

13.

		premises		conclusion	
p	q	$p \to q$	$\sim q$	$\sim p$	
T	T	T	F		
T	F	F	T		
F	T	T	F		
F	F	T	T	T	← *critical row*

Row 4 of the truth table represents the only situation in which all the premises are true, and in this row the conclusion is also true. Therefore, the argument form (modus tollens) is valid.

14.

		premise	conclusion	
p	q	p	$p \vee q$	
T	T	T	T	← *critical row*
T	F	F		
F	T	T	T	← *critical row*
F	F	F		

The truth table shows that in the two situations (represented by rows 1 and 3) in which the premise is true, the conclusion is also true. Therefore, Generalization, version (a), is valid.

15.

		premise	conclusion	
p	q	q	$p \vee q$	
T	T	T	T	← *critical row*
T	F	F		
F	T	T	T	← *critical row*
F	F	F		

The truth table shows that in the two situations (represented by rows 1 and 3) in which the premise is true, the conclusion is also true. Therefore, Generalization, version (b), is valid.

16.

		premise	conclusion	
p	q	$p \wedge q$	p	
T	T	T	T	← *critical row*
T	F	F		
F	T	F		
F	F	F		

The truth table shows that in the only situation (represented by row 1) in which both premises are true, the conclusion is also true. Therefore, Specialization, version (sa), is valid.

17.

		premise	conclusion	
p	q	$p \wedge q$	q	
T	T	T	T	← *critical row*
T	F	F		
F	T	F		
F	F	F		

The truth table shows that in the only situation (represented by row 1) in which both premises are true, the conclusion is also true. Therefore, Specialization, version (b), is valid.

18.

		premises		*conclusion*	
p	q	$p \vee q$	$\sim q$	p	
T	T	T	F		
T	F	T	T	T	← *critical row*
F	T	T	F		
F	T	F	T		

Row 2 represents the only situation in which both premises are true. Because the conclusion is also true here the argument form is valid.

19.

		premises		*conclusion*	
p	q	$p \vee q$	$\sim p$	q	
T	T	T	F		
T	F	T	F		
F	T	T	T	T	← *critical row*
F	F	F	T		

The truth table shows that in the only situation (represented by row 3) in which both premises are true, the conclusion is also true. Therefore, Elimination, version (b), is valid.

20.

			premises		*conclusion*	
p	q	r	$p \to q$	$q \to r$	$p \to r$	
T	T	T	T	T	T	← *critical row*
T	T	F	T	F		
T	F	T	F	T		
T	F	F	F	T		
F	T	T	T	T	T	← *critical row*
F	T	F	T	F		
F	F	T	T	T	T	← *critical row*
F	F	F	T	T	T	← *critical row*

The truth table shows that in the four situations (represented by rows 1, 5, 7, and 8) in which both premises are true, the conclusion is also true. Therefore, Transitivity is valid.

21.

				premises		*conclusion*	
p	q	r	$p \vee q$	$p \to r$	$q \to r$	r	
T	T	T	T	T	T	T	← *critical row*
T	T	F	T	F	F		
T	F	T	T	T	T	T	← *critical row*
T	F	F	T	F	T		
F	T	T	T	T	T	T	← *critical row*
F	T	F	T	T	F		
F	F	T	F	T	T		
F	F	F	F	T	T		

The truth table shows that in the three situations (represented by rows 1, 3, 5) in which all three premises are true, the conclusion is also true. Therefore, proof by division into cases is valid.

22. Let p represent "Tom is on team A" and q represent "Hua is on team B." Then the argument has the form

$$\begin{array}{ll} & \sim p \to q \\ & \sim q \to p \\ \therefore & \sim p \vee \sim q \end{array}$$

p	q	$\sim p$	$\sim q$	$\sim p \to q$ (premises)	$\sim q \to p$ (premises)	$\sim p \vee \sim q$ (conclusion)	
T	T	F	F	T	T	F	*critical row*
T	F	F	T	T	T	T	*critical row*
F	T	T	F	T	T	T	*critical row*
F	F	T	T	F	F		

Rows 1, 2, and 3 of the truth table are the critical rows in which all the premises are true, but row 1 shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

23. form:

$$\begin{array}{ll} & p \vee q \\ & p \to r \\ \therefore & q \vee \sim r \end{array}$$

p	q	r	$\sim r$	$p \vee q$ (premises)	$p \to r$ (premises)	$q \vee \sim r$ (conclusion)	
T	T	T	F	T	T	T	*critical row*
T	T	F	T	T	F		
T	F	T	F	T	T	F	*critical row*
T	F	F	T	T	F		
F	T	T	F	T	T	T	*critical row*
F	T	F	T	T	T	T	*critical row*
F	F	T	F	F	T		
F	F	F	T	F	T		

Rows 1, 3, 5, and 6 represent the situations in which both premises are true, but in row 3 the conclusion is false. Hence, it is possible for an argument of this form to have true premises and a false conclusion, and so the given argument is invalid.

24. form:

$$\begin{array}{ll} & p \to q \\ & q \\ \therefore & p \end{array}$$

invalid: converse error

25. form:

$$\begin{array}{ll} & p \vee q \\ & \sim p \\ \therefore & q \end{array}$$

valid: elimination

26. form:

$$\begin{array}{ll} & p \to q \\ & q \to r \\ \therefore & p \to r \end{array}$$

valid: transitivity

27. form:

$$\begin{array}{ll} & p \to q \\ & \sim p \\ \therefore & \sim q \end{array}$$

invalid: inverse error

28. form: $p \to q$ invalid, converse error
q
$\therefore\ p$

29. form: $p \to q$ invalid, inverse error
$\sim p$
$\therefore\ \sim q$

30. form: $p \to q$ invalid, converse error
q
$\therefore\ p$

31. form: $p \wedge q$ valid, generalization
$\therefore\ q$

32. form: $p \to r$ valid, proof by division into cases
$q \to r$
$\therefore\ p \vee q \to r$

33. A valid argument with a false conclusion must have at least one false premise. In the following example, the second premise is false. (The first premise is true because its hypothesis is false.)

    If the square of every real number is positive, then no real number is negative.

    The square of every real number is positive.

    Therefore, no real number is negative.

34. An invalid argument with a true conclusion can have premises that are either true or false. In the following example the first premise is true for either one of following two reasons: its hypothesis is false and its conclusion is true.

    If the square of every real number is positive, then some real numbers are positive.

    Some real numbers are positive.

    Therefore, the square of every real number is positive.

35. A correct answer should indicate that for a valid argument, any argument of the same form that has true premises has a true conclusion, whereas for an invalid argument, it is possible to find an argument of the same form that has true premises and a false conclusion. The validity of an argument does not depend on whether the conclusion is true or not. The validity of an argument only depends on the formal relationship between its premises and its conclusion.

36. The program contains an undeclared variable.

    One explanation:
    1. There is not a missing semicolon and there is not a misspelled variable name. *(by (c) and (d) and definition of* $\wedge$*)*
    2. It is not the case that there is a missing semicolon or a misspelled variable name. *(by (1) and De Morgan's laws)*
    3. There is not a syntax error in the first five lines. *(by (b) and (2) and modus tollens)*
    4. There is an undeclared variable. *(by (a) and (3) and elimination)*

37. The treasure is buried under the flagpole.

    One explanation:
    1. The treasure is not in the kitchen. *(by (c) and (a) and modus ponens)*
    2. The tree in the front yard is not an elm. *(by (b) and (1) and modus tollens)*
    3. The treasure is buried under the flagpole. *(by (d) and (2) and elimination)*

38. **a.** A is a knave and B is a knight.

One explanation:
1. Suppose A is a knight.
2. $\therefore$ What A says is true. *(by definition of knight)*
3. $\therefore$ B is a knight also. *(That's what A said.)*
4. $\therefore$ What B says is true. *(by definition of knight)*
5. $\therefore$ A is a knave. *(That's what B said.)*
6. $\therefore$ We have a contradiction: A is a knight and a knave. *(by (1) and (5))*
7. $\therefore$ The supposition that A is a knight is false. *(by the contradiction rule)*
8. $\therefore$ A is a knave. *(negation of supposition)*
9. $\therefore$ What B says is true. *(B said A was a knave, which we now know to be true.)*
10. $\therefore$ B is a knight. *(by definition of knight)*

b. C is a knave and D is a knight

One explanation:
1. Suppose C is a knight.
2. $\therefore$ C is a knave (because what C said was true).
3. $\therefore$ C is both a knight and a knave (by (1) and (2)), which is a contradiction.
4. $\therefore$ C is not a knight (because by the contradiction rule the supposition is false).
5. $\therefore$ What C says is false (because since C is not a knight he is a knave and knaves always speak falsely).
6. $\therefore$ At least one of C or D is a knight (by De Morgan's law).
7. $\therefore$ D is a knight (by (4) and (6) and elimination).
8. $\therefore$ C is a knave and D is a knight (by (4) and (7)).
To check that the problem situation is not inherently contradictory, note that if C is a knave and D is a knight, then each could have spoken as reported.

c. One is a knight and the other is a knave.
One explanation:
There is one knave. E and F cannot both be knights because then both would also be knaves (since each would have spoken the truth), which is a contradiction. Nor can E and F both be knaves because then both would be telling the truth which is impossible for knaves. Hence, the only possible answer is that one is a knight and the other is a knave. But in this case both E and F could have spoken as reported, without contradiction.

d. U, Z, X, and V are knaves and W and Y are knights.
One explanation:
1. The statement made by U must be false because if it were true then U would not be a knight (since none would be a knight), but since he spoke the truth he would be a knight and this would be a contradiction.
2. $\therefore$ there is at least one knight, and U is a knave (since his statement that there are no knights is false).
3. Suppose Z spoke the truth. Then so did W (since if there is exactly one knight then it is also true that there are at most three knights). But this implies that there are at least two knights, which contradicts $Z's$ statement. Hence Z cannot have spoken the truth.
4. $\therefore$ there are at least two knights, and Z is a knave (since his statement that there is exactly one knight is false). Also $X's$ statement is false because since both U and Z are knaves it is impossible for there to be exactly five knights. Hence X also is a knave.
5. $\therefore$ there are at least three knaves (U, Z, and X), and so there are at most three knights.
6. $\therefore$ $W's$ statement is true, and so W is a knight.

7. Suppose V spoke the truth. Then V, W, and Y are all knights (otherwise there would not be at least three knights because U, Z, and X are known to be knaves). It follows that Y spoke the truth. But Y said that exactly two were knights. This contradicts the result that V, W, and Y are all knights.
8. $\therefore$ V cannot have spoken the truth, and so V is a knave.
9. $\therefore$ U, Z, X, and V are all knaves, and so there are at most two knights.
10. Suppose that Y is a knave. Then the only knight is W, which means that Z spoke the truth. But we have already seen that this is impossible. Hence Y is a knight.
11. By 6, 9, and 10, the only possible solution is that U, Z, X, and V are knaves and W and Y are knights. Examination of the statements shows that this solution is consistent: in this case, the statements of U, Z, X, and V are false and those of W and Y are true.

39. The chauffeur killed Lord Hazelton.

One explanation:
1. Suppose the cook was in the kitchen at the time of the murder.
2. $\therefore$ The butler killed Lord Hazelton with strychnine. *(by (c) and (1) and modus ponens)*
3. $\therefore$ We have a contradiction: Lord Hazelton was killed by strychnine and a blow on the head. *(by (2) and (a))*
4. $\therefore$ The supposition that the cook was in the kitchen is false. *(by the contradiction rule)*
5. $\therefore$ The cook was not in the kitchen at the time of the murder. *(negation of supposition)*
6. $\therefore$ Sara was not in the dining room when the murder was committed. *(by (e) and (5) and modus ponens)*
7. $\therefore$ Lady Hazelton was in the dining room when the murder was committed. *(by (b) and (6) and elimination)*
8. $\therefore$ The chauffeur killed Lord Hazelton. *(by (d) and (7) and modus ponens)*

40. *One solution:* Suppose Socko is telling the truth. Then Fats is also telling the truth because if Lefty killed Sharky then Muscles didn't kill Sharky. Consequently, two of the men were telling the truth, which contradicts the fact that all were lying except one. Therefore, Socko is not telling the truth: Lefty did not kill Sharky. Hence Muscles is telling the truth and all the others are lying. It follows that Fats is lying, and so Muscles killed Sharky.

Another solution: The statements of Socko and Muscles contradict each other, which implies that one is lying and the other is telling the truth. If Socko is telling the truth, then Fats is also telling the truth, which contradicts the fact that only one person told the truth. So Muscles is the only one who told the truth. Hence Muscles is telling the truth and all the others are lying. It follows that Fats is lying, and so Muscles killed Sharky.

41. (1)
$p \to t$ by premise (d)
$\sim p$ by premise (c)
$\therefore\ \sim p$ by modus tollens

(2)
$\sim p$ by (1)
$\therefore\ \sim p \vee q$ by generalization

(3)
$\sim p \vee q \to r$ by premise (a)
$\sim p \vee q$ by (2)
$\therefore\ r$ by modus ponens

(4)
$\sim p$ by (1)
r by (3)
$\therefore\ \sim p \wedge r$ by conjunction

(5)
$\sim p \wedge r \to \sim s$ by premise (e)
$\sim p \wedge r$ by (4)
$\therefore\ \sim s$ by modus ponens

(6) $s \vee \sim q$ by premise (b)
$\sim s$ by (5)
$\therefore \sim q$ by elimination

42. (1) $q \to r$ by premise (b)
$\sim r$ by premise (d)
$\therefore \sim q$ by modus tollens

(2) $p \vee q$ by premise (a)
$\sim q$ by (1)
$\therefore p$ by elimination

(3) $\sim q \to u \wedge s$ by premise (e)
$\sim q$ by (1)
$\therefore u \wedge s$ by modus ponens

(4) $u \wedge s$ by (3)
$\therefore s$ by specialization

(5) p by (2)
s by (4)
$\therefore p \wedge s$ by conjunction

(6) $p \wedge s \to t$ by premise (c)
$p \wedge s$ by (5)
$\therefore t$ by modus ponens

43. (1) $\sim w$ by premise (d)
$u \vee w$ by premise (e)
$\therefore u$ by elimination

(2) $u \to \sim p$ by premise (c)
u by (1)
$\therefore \sim p$ by modus ponens

(3) $\sim p \to r \wedge \sim s$ by premise (a)
$\sim p$ by (2)
$\therefore r \wedge \sim s$ by modus ponens

(4) $r \wedge \sim s$ by (3)
$\therefore \sim s$ by specialization

(5) $\sim t \to s$ by premise (b)
$\sim s$ by (4)
$\therefore \sim t$ by modus tollens

44. (1) $\sim q \vee s$ by premise (d)
$\sim s$ by premise (e)
$\therefore \sim q$ by elimination

(2) $p \to q$ by premise (a)
$\sim q$ by (1)
$\therefore \sim p$ by modus tollens

(3) $r \vee s$ by premise (b)
$\sim s$ by premise (e)
$\therefore r$ by elimination

(4) $\sim p$ by (2)
r by (3)
$\therefore \sim p \wedge r$ by conjunction

(5)	$\sim p \wedge r \to u$	by premise (f)
	$\sim p \wedge r$	by (4)
$\therefore$	u	by modus ponens

(6)	$\sim s \to \sim t$	by premise (c)
	$\sim s$	by premise (e)
$\therefore$	$\sim t$	by modus ponens

(7)	$w \vee t$	by premise (g)
	$\sim t$	by (6)
$\therefore$	w	by elimination

(8)	u	by (5)
	w	by (7)
$\therefore$	$u \wedge w$	by conjunction

Section 2.4

1. $R = 1$
2. $R = 1$
3. $S = 1$
4. $S = 1$
5. The input/output table is as follows:

Input		**Output**
P	Q	R
1	1	1
1	0	1
0	1	0
0	0	1

6. The input/output table is as follows:

Input			**Output**
P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

7. The input/output table is as follows:

Input		**Output**
P	Q	R
1	1	0
1	0	1
0	1	0
0	0	0

8. The input/output table is as follows:

Input			**Output**
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

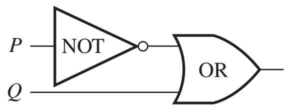
9. $P \vee {\sim}Q$

10. $(P \vee Q) \wedge \sim Q$

11. $(P \wedge {\sim}Q) \vee R$

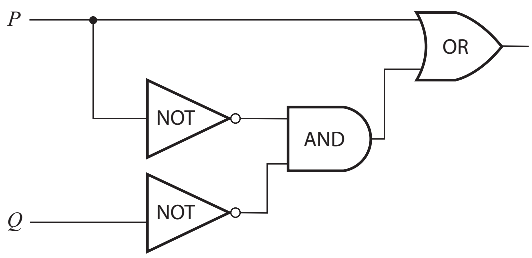
12. $(P \vee Q) \vee \sim (Q \wedge R)$

13.



14.

15.



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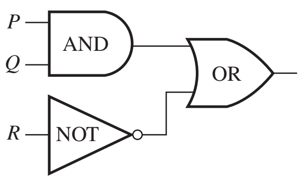
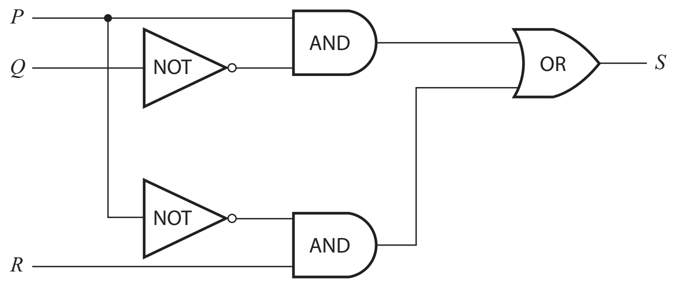
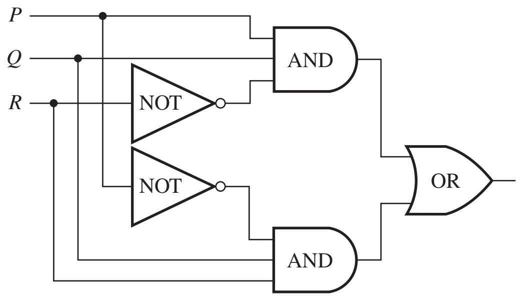


17.



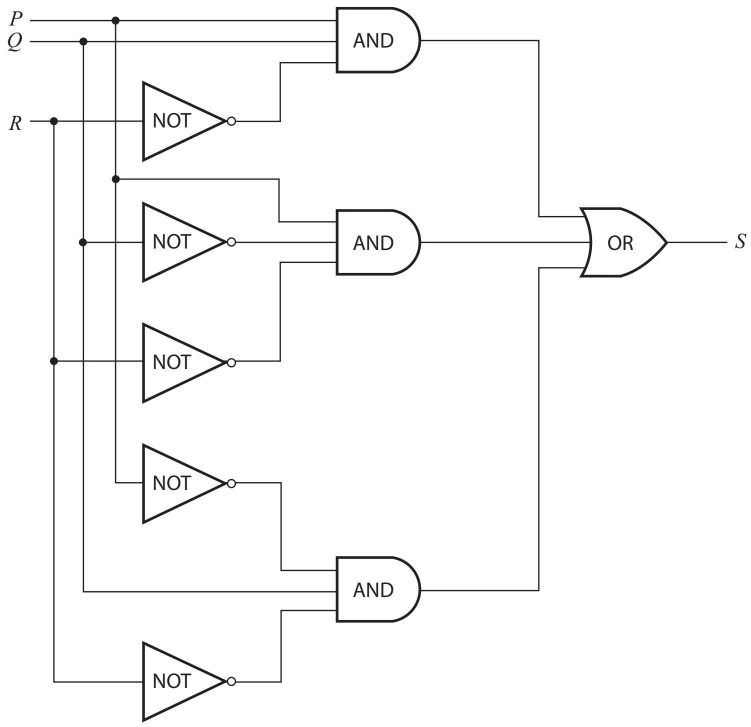
18. **a.** $(P \wedge Q \wedge {\sim}R) \vee ({\sim}P \wedge Q \wedge R)$

b.



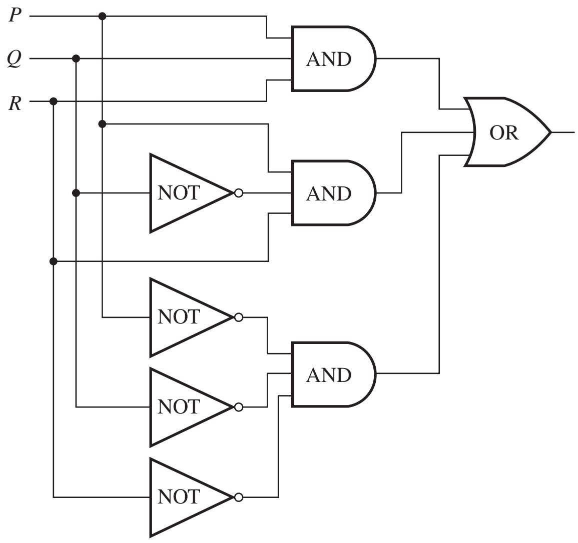
19. **a.** $(P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R)$

b. One circuit (among many) having the given input/output table is the following:



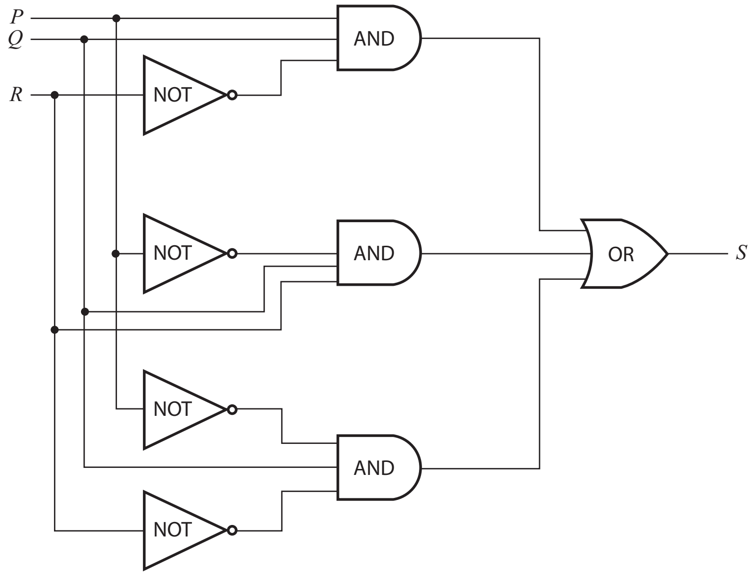
20. **a.** $(P \wedge Q \wedge R) \vee (P \wedge {\sim}Q \wedge R) \vee ({\sim}P \wedge {\sim}Q \wedge {\sim}R)$

b.



21. **a.** $(P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R)$

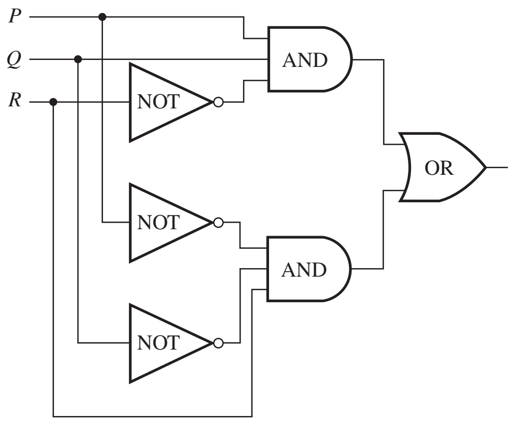
b. One circuit (among many) having the given input/output table is the following:



22. The input/output table is

Input			Output
P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

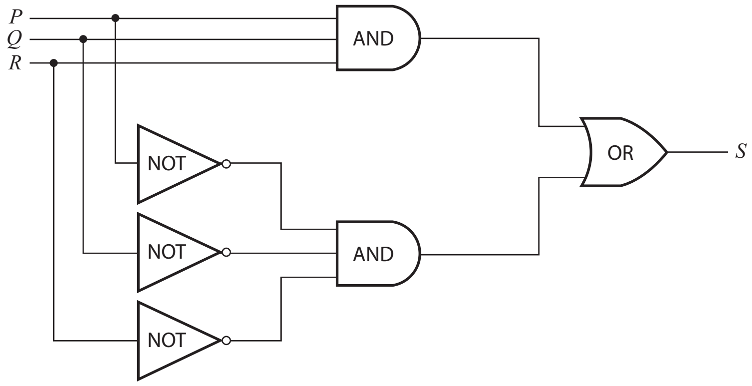
One circuit (among many) having this input/output table is shown below.



23. The input/output table is as follows:

Input			Output
P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

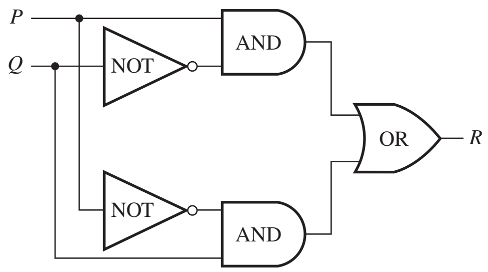
One circuit (among many) having this input/output table is the following:



24. Let P and Q represent the positions of the switches in the classroom, with 0 being "down" and 1 being "up." Let R represent the condition of the light, with 0 being "off" and 1 being "on." Initially, $P = Q = 0$ and $R = 0$. If either P or Q (but not both) is changed to 1, the light turns on. So when $P = 1$ and $Q = 0$, then $R = 1$, and when $P = 0$ and $Q = 1$, then $R = 1$. Thus when one switch is up and the other is down the light is on, and hence moving the switch that is down to the up position turns the light off. So when $P = 1$ and $Q = 1$, then $R = 0$. It follows that the input/output table has the following appearance:

Input		Output
P	Q	R
1	1	0
1	0	1
0	1	1
0	0	0

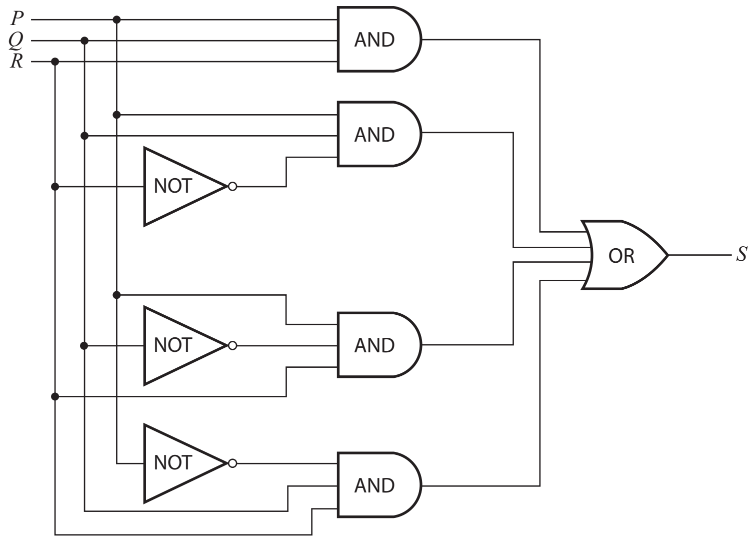
One circuit (among many) having this input/output table is the following:



25. Let P, Q, and R indicate the positions of the switches, with 1 indicating that the switch is in the on position. Let an output of 1 indicate that the security system is enabled. The complete input/output table is as follows:

Input			**Output**
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

One circuit (among many) having this input/output table is the following:



Note: One alternative answer interchanges the 1's and 0's.

26. The Boolean expression for (a) is $(P \wedge Q) \vee Q$, and for (b) it is $(P \vee Q) \wedge Q$. We must show that if these expressions are regarded as statement forms, then they are logically equivalent.

Now

$$\begin{array}{rcll}(P \wedge Q) \vee Q & \equiv & Q \vee (P \wedge Q) & \text{by the commutative law for } \vee \\ & \equiv & (Q \vee P) \wedge (Q \vee Q) & \text{by the distributive law} \\ & \equiv & (Q \vee P) \wedge Q & \text{by the idempotent law} \\ & \equiv & (P \vee Q) \wedge Q & \text{by the commutative law for } \wedge \end{array}$$

Alternatively, by the absorption laws, both statement forms are logically equivalent to Q.

27. The Boolean expression for circuit (a) is $\sim P \wedge (\sim (\sim P \wedge Q))$ and for circuit (b) it is $\sim (P \vee Q)$. We must show that if these expressions are regarded as statement forms, then they are logically equivalent. Now

$$\begin{array}{rcll}\sim P \wedge (\sim (\sim P \wedge Q)) & \equiv & \sim P \wedge (\sim (\sim P) \vee \sim Q) & \text{by De Morgan's law} \\ & \equiv & \sim P \wedge (P \vee \sim Q) & \text{by the double negative law} \\ & \equiv & (\sim P \wedge P) \vee (\sim P \wedge \sim Q) & \text{by the distributive law} \\ & \equiv & (P \wedge \sim P) \vee (\sim P \wedge \sim Q) & \text{by the commutative law for } \wedge \\ & \equiv & \mathbf{c} \vee (\sim P \wedge \sim Q) & \text{by the negation law for } \wedge \\ & \equiv & (\sim P \wedge \sim Q) \vee \mathbf{c} & \text{by the commutative law for } \vee \\ & \equiv & \sim P \wedge \sim Q & \text{by the identity law for } \vee \\ & \equiv & \sim (P \vee Q) & \text{by De Morgan's law.} \end{array}$$

28. The Boolean expression for circuit (a) is $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ and for circuit (b) it is $P \vee \sim Q$. We must show that if these expressions are regarded as statement forms, then they are logically equivalent. Now

$(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$$\begin{array}{cll}\equiv & ((P \wedge Q) \vee (P \wedge \sim Q)) \vee (\sim P \wedge \sim Q) & \text{by inserting parentheses (which is} \\ & & \text{legal by the associative law for } \vee) \\ \equiv & (P \wedge (Q \vee \sim Q)) \vee (\sim P \wedge \sim Q) & \text{by the distributive law} \\ \equiv & (P \wedge \mathbf{t}) \vee (\sim P \wedge \sim Q) & \text{by the negation law for } \vee \\ \equiv & P \vee (\sim P \wedge \sim Q) & \text{by the identity law for } \wedge \\ \equiv & (P \vee \sim P) \wedge (P \vee \sim Q) & \text{by the distributive law} \\ \equiv & \mathbf{t} \wedge (P \vee \sim Q) & \text{by the negation law for } \vee \\ \equiv & (P \vee \sim Q) \wedge \mathbf{t} & \text{by the commutative law for } \wedge \\ \equiv & P \vee \sim Q & \text{by the identity law for } \wedge. \end{array}$$

29. The Boolean expression for circuit (a) is $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$ and for circuit (b) it is $P \vee Q$. We must show that if these expressions are regarded as statement forms, then they are logically equivalent. Now

$(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

$$\begin{array}{cll}\equiv & ((P \wedge Q) \vee (\sim P \wedge Q)) \vee (P \wedge \sim Q) & \text{by inserting parentheses (which is} \\ & & \text{legal by the associative law for } \vee) \\ \equiv & ((Q \wedge P) \vee (Q \wedge \sim P)) \vee (P \wedge \sim Q) & \text{by the commutative law for } \wedge \\ \equiv & (Q \wedge (P \vee \sim P)) \vee (P \wedge \sim Q) & \text{by the distributive law} \\ \equiv & (Q \wedge \mathbf{t}) \vee (P \wedge \sim Q) & \text{by the negation law for } \vee \\ \equiv & Q \vee (P \wedge \sim Q) & \text{by the identity law for } \wedge \\ \equiv & (Q \vee P) \wedge (Q \vee \sim Q) & \text{by the distributive law} \\ \equiv & (Q \vee P) \wedge \mathbf{t} & \text{by the negation law for } \vee \\ \equiv & Q \vee P & \text{by the identity law for } \wedge \\ \equiv & P \vee Q & \text{by the commutative law for } \vee. \end{array}$$

30. $(P \wedge Q) \vee ({\sim}P \wedge Q) \vee ({\sim}P \wedge {\sim}Q) \equiv (P \wedge Q) \vee (({\sim}P \wedge Q) \vee ({\sim}P \wedge {\sim}Q)$ by inserting parentheses (which is legal by the associative law)

$\equiv (P \wedge Q) \vee ({\sim}P \wedge (Q \vee {\sim}Q))$	by the distributive law
$\equiv (P \wedge Q) \vee ({\sim}P \wedge \mathbf{t})$	by the negation law for $\vee$
$\equiv (P \wedge Q) \vee {\sim}P$	by the identity law for $\wedge$
$\equiv {\sim}P \vee (P \wedge Q)$	by the commutative law for $\vee$
$\equiv ({\sim}P \vee P) \wedge ({\sim}P \vee Q)$	by the distributive law
$\equiv (P \vee {\sim}P) \wedge ({\sim}P \vee Q)$	by the commutative law for $\vee$
$\equiv \mathbf{t} \wedge ({\sim}P \vee Q)$	by the negation law for $\vee$
$\equiv ({\sim}P \vee Q) \wedge \mathbf{t}$	by the commutative law for $\wedge$
$\equiv {\sim}P \vee Q$	by the identity law for $\wedge$

31. $(\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \equiv ((\sim P \wedge \sim Q) \vee (\sim P \wedge Q)) \vee (P \wedge \sim Q)$ by inserting parentheses (which is legal by the associative law)

$\equiv \quad (\sim P \wedge (\sim Q \vee Q)) \vee (P \wedge \sim Q)$	by the distributive law
$\equiv \quad (\sim P \wedge (Q \vee \sim Q)) \vee (P \wedge \sim Q)$	by the commutative law for $\vee$
$\equiv \quad (\sim P \wedge \mathbf{t}) \vee (P \wedge \sim Q)$	by the negation law for $\vee$
$\equiv \quad \sim P \vee (P \wedge \sim Q)$	by the identity law for $\wedge$
$\equiv \quad (\sim P \vee P) \wedge (\sim P \vee \sim Q)$	by the distributive law
$\equiv \quad (P \vee \sim P) \wedge (\sim P \vee \sim Q)$	by the commutative law for $\vee$
$\equiv \quad \mathbf{t} \wedge (\sim P \vee \sim Q)$	by the negation law for $\vee$
$\equiv \quad (\sim P \vee \sim Q) \wedge \mathbf{t}$	by the commutative law for $\wedge$
$\equiv \quad \sim P \vee \sim Q$	by the identity law for $\wedge$
$\equiv \quad \sim (P \wedge Q)$	by De Morgan's law.

32. $(P \wedge Q \wedge R) \vee ((P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$

$\equiv \quad ((P \wedge (Q \wedge R)) \vee (P \wedge (\sim Q \wedge R))) \vee (P \wedge (\sim Q \wedge \sim R)) \vee (P \wedge \sim Q)$ by inserting parentheses (which is legal by the associative law)

$\equiv \quad (P \wedge [(Q \wedge R) \vee (\sim Q \wedge R)] \vee (P \wedge [\sim Q \wedge \sim R])$	by the distributive law
$\equiv \quad P \wedge ([(Q \wedge R) \vee (\sim Q \wedge R)] \vee [\sim Q \wedge \sim R])$	by the distributive law
$\equiv \quad P \wedge ([(R \wedge Q) \vee (R \wedge \sim Q)] \vee [\sim Q \wedge \sim R])$	by the commutative law for $\wedge$
$\equiv \quad P \wedge ([(R \wedge (Q \vee \sim Q)] \vee [\sim Q \wedge \sim R])$	by the distributive law
$\equiv \quad P \wedge ([(R \wedge \mathbf{t}] \vee [\sim Q \wedge \sim R])$	by the negation law for $\vee$
$\equiv \quad P \wedge (R \vee [\sim Q \wedge \sim R])$	by the identity law for $\wedge$
$\equiv \quad P \wedge ((R \vee \sim Q) \wedge (R \vee \sim R))$	by the distributive law
$\equiv \quad P \wedge ((R \vee \sim Q) \wedge t])$	by the negation law for $\vee$
$\equiv \quad P \wedge (R \vee \sim Q)$	by the identity law for $\wedge$.

33. **a.**

$(P \mid Q) \mid (P \mid Q)$	$\equiv \quad \sim [(P \mid Q) \wedge (P \mid Q)]$	by definition of $\mid$
	$\equiv \quad \sim (P \mid Q)$	by the idempotent law for $\wedge$
	$\equiv \quad \sim [\sim (P \wedge Q)]$	by definition of $\mid$
	$\equiv \quad P \wedge Q$	by the double negative law.

b.

$P \wedge (\sim Q \vee R) \quad \equiv \quad (P \,|\, (\sim Q \vee R)) \,|\, (P \,|\, (\sim Q \vee R))$ by part (a)

$\equiv \quad (P \,|\, [(\sim Q \,|\!\sim Q) \,|\, (R \,|\, R)]) \,|\, (P \,|\, [(\sim Q \,|\!\sim Q) \,|\, (R \,|\, R)])$ by Example 2.4.7(b)

$\equiv \quad (P \,|\, [((Q \,|\, Q) \,|\, (Q \,|\, Q)) \,|\, (R \,|\, R)]) \,|\, (P \,|\, [((Q \,|\, Q) \,|\, (Q \,|\, Q)) \,|\, (R \,|\, R)])$ by Example 2.4.7(a)

34. *a.*

$$\begin{aligned}(P\downarrow Q)\downarrow(P\downarrow Q) &\equiv \sim(P\downarrow Q) && \text{by part (a)}\\ &\equiv \sim[\sim(P\vee Q)] && \text{by definition of } \downarrow\\ &\equiv P\vee Q && \text{by the double negative law}\end{aligned}$$

b.

$$\begin{aligned}P\vee Q &\equiv \sim(\sim(P\vee Q)) && \text{by the double negative law}\\ &\equiv \sim(P\downarrow Q) && \text{by definition of } \downarrow\\ &\equiv (P\downarrow Q)\downarrow(P\downarrow Q) && \text{by part (a).}\end{aligned}$$

c.

$$\begin{aligned}P\wedge Q &\equiv \sim(\sim P\vee \sim Q) && \text{by De Morgan's law and the double negative law}\\ &\equiv \sim P\downarrow\sim Q && \text{by definition of } \downarrow\\ &\equiv (P\downarrow P)\downarrow(Q\downarrow Q) && \text{by part (a).}\end{aligned}$$

d.

$$\begin{aligned}P\to Q &\equiv \sim P\vee Q && \text{by Exercise 13(a) of Section 2.2}\\ &\equiv (\sim P\downarrow Q)\downarrow(\sim P\downarrow Q) && \text{by part (b)}\\ &\equiv ((P\downarrow P)\downarrow Q)\downarrow((P\downarrow P)\downarrow Q) && \text{by part (a).}\end{aligned}$$

e.

$$\begin{aligned}P\leftrightarrow Q &\equiv (P\to Q)\wedge(Q\to P) && \text{by the truth table on page 46 of the text}\\ &\equiv ([(P\downarrow P)\downarrow Q]\downarrow[(P\downarrow P)\downarrow Q)])\wedge([(Q\downarrow Q)\downarrow P]\downarrow[(Q\downarrow Q)\downarrow P)])\\ & \qquad \text{by part (d)}\\ &\equiv (([(P\downarrow P)\downarrow Q]\downarrow[(P\downarrow P)\downarrow Q)])\downarrow([(P\downarrow P)\downarrow Q]\downarrow[(P\downarrow P)\downarrow Q)]))\\ &\quad \downarrow(([(Q\downarrow Q)\downarrow P]\downarrow[(Q\downarrow Q)\downarrow P)])\downarrow([(Q\downarrow Q)\downarrow P]\downarrow[(Q\downarrow Q)\downarrow P)]))\\ & \qquad \text{by part (c)}\end{aligned}$$

Section 2.5

1. $19_{10} = 16 + 2 + 1 = 10011_2$
2. $55 = 32 + 16 + 4 + 2 + 1 = 110111_2$
3. $287 = 256 + 16 + 8 + 4 + 2 + 1 = 100011111_2$
4. $458_{10} = 256 + 128 + 64 + 8 + 2 = 111001010_2$
5. $1609 = 1024 + 512 + +64 + 8 + 1 = 11001001001_2$
6. $1424 = 1024 + 256 + 128 + 16 = 10110010000_2$
7. $1110_2 = 8 + 4 + 2 = 14_{10}$
8. $10111_2 = 16 + 4 + 2 + 1 = 23_{10}$
9. $110110_2 = 32 + 16 + 4 + 2 = 54_{10}$
10. $1100101_2 = 64 + 32 + 4 + 1 = 101_{10}$
11. $1000111_2 = 64 + 4 + 2 + 1 = 71_{10}$
12. $1011011_2 = 64 + 16 + 8 + 2 + 1 = 91_{10}$
13.

$$\begin{array}{ccccc} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\ & 1 & 0 & 1 & 1_2\\ + & & 1 & 0 & 1_2\\ \hline 1 & 0 & 0 & 0 & 0_2\end{array}$$

14.
$$\begin{array}{rrrrr} & {\scriptstyle 0} & {\scriptstyle 1} & {\scriptstyle 1} & \\ & 1 & 0 & 0 & 1_2 \\ + & 1 & 0 & 1 & 1_2 \\ \hline 1 & 0 & 1 & 0 & 0_2 \end{array}$$

15.
$$\begin{array}{rrrrrrr} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & {\scriptstyle 1} & \\ & 1 & 0 & 1 & 1 & 0 & 1_2 \\ + & & 1 & 1 & 1 & 0 & 1_2 \\ \hline 1 & 0 & 0 & 1 & 0 & 1 & 0_2 \end{array}$$

16.
$$\begin{array}{rrrrrrrrrrrr} & & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & {\scriptstyle 1} & & \\ & & & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1_2 \\ + & & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0_2 \\ \hline & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1_2 \end{array}$$

17.
$$\begin{array}{rrrrr} & & & {\scriptstyle 1} & \\ & {\scriptstyle 1} & {\scriptstyle 10} & {\scriptstyle 10} & {\scriptstyle 1} \\ 1 & 0 & 1 & 0 & 0_2 \\ - & 1 & 1 & 0 & 1_2 \\ \hline & & 1 & 1 & 1_2 \end{array}$$

18.
$$\begin{array}{rrrrr} & {\scriptstyle 10} & & & \\ {\scriptstyle 0} & {\scriptstyle 0} & {\scriptstyle 10} & {\scriptstyle 0} & {\scriptstyle 10} \\ 1 & 1 & 0 & 1 & 0_2 \\ - & 1 & 1 & 0 & 1_2 \\ \hline & 1 & 1 & 0 & 1_2 \end{array}$$

19.
$$\begin{array}{rrrrrr} & & & {\scriptstyle 0} & {\scriptstyle 10} & \\ 1 & 0 & 1 & 1 & 0 & 1_2 \\ - & 1 & 0 & 0 & 1 & 1_2 \\ \hline & 1 & 1 & 0 & 1 & 0_2 \end{array}$$

20.
$$\begin{array}{rrrrrrr} & {\scriptstyle 1} & {\scriptstyle 10} & {\scriptstyle 1} & {\scriptstyle 10} & {\scriptstyle 1} & \\ {\scriptstyle 0} & {\scriptstyle 10} & {\scriptstyle 0} & {\scriptstyle 10} & {\scriptstyle 0} & {\scriptstyle 10} & {\scriptstyle 10} \\ 1 & 0 & 1 & 0 & 1 & 0 & 0_2 \\ - & & 1 & 0 & 1 & 1 & 1_2 \\ \hline & 1 & 1 & 1 & 1 & 0 & 1_2 \end{array}$$

21. **a.** $S = 0,\ T = 1$ *b.* $S = 0,\ T = 1$ *c.* $S = 0,\ T = 0$

22. Note that

$$\begin{array}{rr} & 11111111_2 \\ + & 1_2 \\ \hline & 100000000_2 \end{array}$$

and $100000000_2 = (2^8)_{10}$. Because $1_2 = 1_{10}$, we have that

$$\begin{array}{rrcrr} & 11111111_2 + 1_2 & = & & (2^8)_{10} \\ - & 1_2 & & - & 1_{10} \\ \hline & 11111111_2 & = & & (2^8 - 1)_{10} \end{array}$$

23. $|-23|_{10} = 23_{10} = (16 + 4 + 2 + 1)_{10} = 00010111_2 \xrightarrow{\text{flip the bits}} 11101000 \xrightarrow{\text{add 1}}$ 11101001. So the answer is 11101001.

24. $|-67|_{10} = 67_{10} = (64+2+1)_{10} = 01000011_2 \xrightarrow{\text{flip the bits}} 10111100 \xrightarrow{\text{add 1}} 10111101.$
So the 8-bit two's complement is 10111101.

25. $|-4|_{10} = 4_{10} = 00000100_2 \xrightarrow{\text{flip the bits}} 11111011 \xrightarrow{\text{add 1}} 11111100.$ So the answer is 11111100.

26. $|-115|_{10} = 115_{10} = (64+32+16+2+1)_{10} = 01110011_2 \xrightarrow{\text{flip the bits}} 10001100 \xrightarrow{\text{add 1}} 10001101$
So the 8-bit two's complement is 10001101.

27. Because the leading bit is 1, this is the 8-bit two's complement of a negative integer.
$11010011 \xrightarrow{\text{flip the bits}} 00101100 \xrightarrow{\text{add 1}} 00101101_2 = (32+8+4+1)_{10} = |-45|_{10}.$
So the answer is -45.

28. Because the leading bit is 1, this is the 8-bit two's complement of a negative integer. $10011001 \xrightarrow{\text{flip the bits}} 01100110 \xrightarrow{\text{add 1}} 01100111_2 = (64+32+4+2+1)_{10} = |-103|_{10}$. So the answer is -103.

29. Because the leading bit is 1, this is the 8-bit two's complement of a negative integer.
$11110010 \xrightarrow{\text{flip the bits}} 00001101 \xrightarrow{\text{add 1}} 00001110_2 = (8+4+2)_{10} = |-14|_{10}.$
So the answer is -14.

30. Because the leading bit is 1, this is the 8-bit two's complement of a negative integer. $10111010 \xrightarrow{\text{flip the bits}} 01000101 \xrightarrow{\text{add 1}} 01000110_2 = (64+4+2)_{10} = |-70|_{10}$. So the answer is -70.

31. $57_{10} = (32+16+8+1)_{10} = 111001_2 \to 00111001$
$|-118|_{10} = (64+32+16+4+2)_{10} = 01110110_2 \xrightarrow{\text{flip the bits}} 10001001 \xrightarrow{\text{add 1}} 10001010.$
So the 8-bit two's complements of 57 and -118 are 00111001 and 10001010. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 00111001 \\ + & 10001010 \\ \hline & 11000011 \end{array}$$

Since the leading bit of this number is a 1, the answer is negative. Converting back to decimal form gives $11000011 \xrightarrow{\text{flip the bits}} 00111100 \xrightarrow{\text{add 1}} 00111101_2 = (32+16+8+4+1)_{10} = |61|_{10}$.
So the answer is -61.

32. $62_{10} = (32+16+8+4+2)_{10} = 111110_2 \to 00111110$
$|-18|_{10} = (16+2)_{10} = 00010010 \xrightarrow{\text{flip the bits}} 11101101 \xrightarrow{\text{add 1}} 11101110$
Thus the 8-bit two's complements of 62 and -18 are 00111110 and 10110111. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 00111110 \\ + & 11101110 \\ \hline & 00101100 \end{array}$$

Truncating the 1 in the 2^8th position gives 00101100. Since the leading bit of this number is a 0, the answer is positive. Converting back to decimal form gives

$$00101100 \to 101100_2 = (32+8+4)_{10} = 44_{10}.$$

So the answer is 44.

33. $|-6|_{10} = (4+2)_{10} = 110_2 \xrightarrow{\text{flip the bits}} 00000110 \to 11111001 \xrightarrow{\text{add 1}} 11111010$

$|-73|_{10} = (64+8+1)_{10} = 01001001 \xrightarrow{\text{flip the bits}} 10110110 \xrightarrow{\text{add 1}} 10110111$

Thus the 8-bit two's complements of -6 and -73 are 11111010 and 10110111. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 11111010 \\ + & 10110111 \\ \hline & 10110001 \end{array}$$

Truncating the 1 in the 2^8th position gives 10110001. Since the leading bit of this number is a 1, the answer is negative. Converting back to decimal form gives

$$10110001 \xrightarrow{\text{flip the bits}} 01001110 \xrightarrow{\text{add 1}} 01001111_2 = (64+8+4+2+1) = 79_{10} = |-79|_{10}$$

So the answer is -79.

34. $89_{10} = (64+16+8+1)_{10} = 01011001_2$

$|-55|_{10} = (32+16+4+2+1)_{10} = 00110111_2 \xrightarrow{\text{flip the bits}} 11001000 \xrightarrow{\text{add 1}} 11001001$

So the 8-bit two's complements of 89 and -55 are 01001111 and 11010101. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 01011001 \\ + & 11001001 \\ \hline & 100100010 \end{array}$$

Truncating the 1 in the 2^8th position gives 00100010. Since the leading bit of this number is a 0, the answer is positive. Converting back to decimal form gives

$$00100010_2 = (32+2)_{10} = 34_{10}.$$

So the answer is 34.

35. $|-15|_{10} = (8+4+2+1)_{10} = 00001111_2 \xrightarrow{\text{flip the bits}} 11110000 \xrightarrow{\text{add 1}} 11110001$

$|-46|_{10} = (32+8+4+2)_{10} = 00101110_2 \xrightarrow{\text{flip the bits}} 11010001 \xrightarrow{\text{add 1}} 11010010$

So the 8-bit two's complements of -15 and -46 are 11110001 and 10100010. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 11110001 \\ + & 11010010 \\ \hline & 111000011 \end{array}$$

Truncating the 1 in the 2^8th position gives 11000011. Since the leading bit of this number is a 1, the answer is negative. Converting back to decimal form gives

$$11000011 \xrightarrow{\text{flip the bits}} 00111100 \xrightarrow{\text{add 1}} 00111101_2 = -(32+16+8+4+1)_{10} = |-61|_{10}.$$

So the answer is -61.

36. $123_{10} = (64+32+16+8+2+1)_{10} = 01111011_2$

$|-94|_{10} = (64+16+8+4+2)_{10} = 01011110_2 \xrightarrow{\text{flip the bits}} 10100001 \xrightarrow{\text{add 1}} 10100010$

So the 8-bit two's complements of 123 and -94 are 01111011 and 10100010. Adding the 8-bit two's complements in binary notation gives

$$\begin{array}{rr} & 01111011 \\ + & 10100010 \\ \hline & 100011101 \end{array}$$

Truncating the 1 in the 2^8th position gives 00011101. Since the leading bit of this number is a 0, the answer is positive. Converting back to decimal form gives

$$00011101_2 = (16+8+4+1)_{10} = 29_{10}.$$

So the answer is 29.

37. **a.** The 8-bit two's complement of -128 is computed as follows:

$|-128|_{10} = 128_{10} = (2^7)_{10}$

$= 10000000_2 \xrightarrow{\text{flip the bits}} 01111111 \xrightarrow{\text{add 1}} 10000000.$

So the 8-bit two's complement of -128 is 10000000. If the two's complement procedure is applied to this result, the following is obtained $10000000 \xrightarrow{\text{flip the bits}} 01111111 \xrightarrow{\text{add 1}} 10000000.$ So the 8-bit two's complement of the 8-bit two's complement of -128 is 10000000, which is the 8-bit two's complement of -128.

b. Suppose a, b, and $a+b$ are integers in the range from 1 through 128. Then

$$1 \le a \le 128 = 2^7 \qquad 1 \le b \le 128 = 2^7 \qquad 1 \le a+b \le 128 = 2^7.$$

Multiplying all parts of all three inequalities by -1 gives

$$-1 \ge -a \ge -2^7 \qquad -1 \ge -b \ge -2^7 \qquad -1 \ge -(a+b) \ge -2^7.$$

Thus

$$2^8-(a+b) \ \ge\ 2^8-2^7 \ \ge\ 2^7(2-1) = 2^7,$$

and

$$(2^8-a)+(2^8-b) = 2^8+(2^8-(a+b)) \ge 2^8+2^7.$$

Therefore, the 8-bit two's complement of $(2^8-a)+(2^8-b)$ has 1's in both the 2^8th and the 2^7th positions. The 1 in the 2^8th position is truncated, and the 1 in the 2^7th position shows that the sum is negative.

38. $A2BC_{16} = 10\cdot16^3 + 2\cdot16^2 + 11\cdot16 + 12 = 41{,}660_{10}$

39. $E0D_{16} = 14\cdot16^2 + 0 + 13 = 3597_{10}$

40. $39EB_{16} = 3\cdot16^3 + 9\cdot16^2 + 14\cdot16 + 11 = 14,827_{10}$

41. $0001\ 1100\ 0000\ 1010\ 1011\ 1110_2$

42. $B53DF8_{16} = 1011\,0101\,0011\,1101\,1111\,1000_2$

43. $4ADF83_{16} = 0100\,1010\,1101\,1111\,1000\,0011_2$

44. $2E_{16}$

45. $1011\,0111\,1100\,0101_2 = B7C5_{16}$

46. $1011\,0111\,1100\,0101_2 = B7C5_{16}$

47. **a.** $6{\cdot}8^4 + 1{\cdot}8^3 + 5{\cdot}8^2 + 0{\cdot}8 + 2{\cdot}1 = 25{,}410_{10}$

b. $20763_8 = 2{\cdot}8^4 + 0{\cdot}8^3 + 7{\cdot}8^2 + 6{\cdot}8 + 3 = 8,691_{10}$

c. To convert an integer from octal to binary notation:

(i) Write each octal digit of the integer in fixed 3-bit binary notation (and include leading zeros as needed). Note that

octal digit	0	1	2	3	4	5	6	7
3-bit binary equivalent	000	001	010	011	100	101	110	111

(ii) Juxtapose the results.

As an example, consider converting 61502_8 to binary notation:

$$6_8 = 110_2 \quad 1_8 = 001_2 \quad 5_8 = 101_2 \quad 0_8 = 000_2 \quad 2_8 = 010_2.$$

So in binary notation the integer should be $110\,001\,101\,000\,010_2$. This result can be checked by writing the integer in decimal notation and comparing it to the answer obtained in part (a):

$$110\,001\,101\,000\,010_2 = (1 \cdot 2^{14} + 1 \cdot 2^{13} + 1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^6 + 1 \cdot 2)_{10} = 25410_{10}.$$

This is the same as the answer obtained in part (a). So the two methods give the same result.

To convert an integer from binary to octal notation:

(i) Group the digits of the binary number into sets of three, starting from the right and adding leading zeros as needed:

(ii) Convert the binary numbers in each set of three into octal digits;

(iii) Juxtapose those octal digits.

As an example consider converting 1101011101_2 to octal notation. Grouping the binary digits in sets of three and adding two leading zeros gives

$$001\,101\,011\,101.$$

To convert each group into an octal digit, note that

$$001_2 = 1_8 \quad 101_2 = 5_8 \quad 011_2 = 3_8 \quad 101_2 = 5_8.$$

So the octal version of the integer should be 1535_8. To check this result, observe that

$$1\,101\,011\,101_2 = (1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1)_{10} = 861_{10}$$

and

$$1535_8 = (1 \cdot 8^3 + 5 \cdot 8^2 + 3 \cdot 8 + 5)_{10} = 861_{10}$$

also. So the two methods give the same result.