Discrete Mathematics with Applications, 5th Edition
by Susanna S. Epp

Test Bank Questions

Chapter 1

1. Fill in the blanks to rewrite the following statement with variables: Is there an integer with a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7?

   (a) Is there an integer n such that n has ______?

   (b) Does there exist ______ such that if n is divided by 4 the remainder is 1 and if ______?

2. Fill in the blanks to rewrite the following statement with variables:

   Given any positive real number, there is a positive real number that is smaller.

   (a) Given any positive real number r, there is ______ s such that s is ______.

   (b) For any ______ , ______ such that $s < r$.

3. Rewrite the following statement less formally, without using variables:

   There is an integer n such that $1/n$ is also an integer.

4. Fill in the blanks to rewrite the following statement:

   For all objects T, if T is a triangle then T has three sides.

   (a) All triangles ______.

   (b) Every triangle ______ .

   (c) If an object is a triangle, then it ______.

   (d) If T ______ , then T ______.

   (e) For all triangles T, ______.

5. Fill in the blanks to rewrite the following statement:

   Every real number has an additive inverse.

   (a) All real numbers ______.

   (b) For any real number x, there is ______ for x.

   (c) For all real numbers x, there is real number y such that ______.

6. Fill in the blanks to rewrite the following statement:

   There is a positive integer that is less than or equal to every positive integer.

   (a) There is a positive integer m such that m is ______.

   (b) There is a ______ such that ______ every positive integer.

   (c) There is a positive integer m which satisfies the property that given any positive integer n, m is ______.

7. (a) Write in words how to read the following out loud $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 9\}$.

   (b) Use the set-roster notation to indicate the elements in the set.

8. (a) Is $\{5\} \in \{1,3,5\}$?
   (b) Is $\{5\} \subseteq \{1,3,5\}$?
   (c) Is $\{5\} \in \{\{1\},\{3\},\{5\}\}$?
   (d) Is $\{5\} \subseteq \{\{1\},\{3\},\{5\}\}$?

9. Let $A = \{a,b,c\}$ and $B = \{u,v\}$. Write a. $A \times B$ and b. $B \times A$.

10. Let $A = \{3,5,7\}$ and $B = \{15,16,17,18\}$, and define a relation R from A to B as follows: For all $(x,y) \in A \times B$,
$$(x,y) \in R \quad \Leftrightarrow \quad \frac{y}{x} \text{ is an integer.}$$
    (a) Is $3\ R\ 15$? Is $3\ R\ 16$? Is $(7,17) \in R$? Is $(3,18) \in R$?
    (b) Write R as a set of ordered pairs.
    (c) Write the domain and co-domain of R.
    (d) Draw an arrow diagram for R.
    (e) Is R a function from A to B? Explain.

11. Define a relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows: For all $(x,y) \in \mathbf{R} \times \mathbf{R}$, $(x,y) \in R$ if, and only if, $x = y^2 + 1$.
    (a) Is $(2,5) \in R$? Is $(5,2) \in R$? Is $(-3)\ R\ 10$? Is $10\ R\ (-3)$?
    (b) Draw the graph of R in the Cartesian plane.
    (c) Is R a function from $\mathbf{R}$ to $\mathbf{R}$? Explain.

12. Let $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$. Define a function $G\colon A \to B$ as follows:
$$G = \{(1,b),(2,c),(3,b),(4,c)\}.$$
    (a) Find $G(2)$.
    (b) Draw an arrow diagram for G.

13. Define functions F and G from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas:
$$F(x) = (x+1)(x-3) \quad \text{and} \quad G(x) = (x-2)^2 - 7.$$
    Does $F = G$? Explain.

Chapter 2

1. Which of the following is a negation for "Jim is inside and Jan is at the pool."
    (a) Jim is inside or Jan is not at the pool.
    (b) Jim is inside or Jan is at the pool.
    (c) Jim is not inside or Jan is at the pool.
    (d) Jim is not inside and Jan is not at the pool.
    (e) Jim is not inside or Jan is not at the pool.

2. Which of the following is a negation for "Jim has grown or Joan has shrunk."

   (a) Jim has grown or Joan has shrunk.
   (b) Jim has grown or Joan has not shrunk.
   (c) Jim has not grown or Joan has not shrunk.
   (d) Jim has grown and Joan has shrunk.
   (e) Jim has not grown and Joan has not shrunk.
   (f) Jim has grown and Joan has not shrunk.

3. Write a negation for each of the following statements:

   (a) The variable S is undeclared and the data are out of order.
   (b) The variable S is undeclared or the data are out of order.
   (c) If Al was with Bob on the first, then Al is innocent.
   (d) $-5 \leq x < 2$ (where x is a particular real number)

4. Are the following statement forms logically equivalent: $p \vee q \to p$ and $p \vee (\sim p \wedge q)$? Include a truth table and a few words explaining how the truth table supports your answer.

5. State precisely (but concisely) what it means for two statement forms to be logically equivalent.

6. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

   If Sam bought it at Crown Books, then Sam didn't pay full price.

   Sam bought it at Crown Books or Sam paid full price.

7. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

   If Sam is out of Schlitz, then Sam is out of beer.

   Sam is not out of beer or Sam is not out of Schlitz.

8. Write the converse, inverse, and contrapositive of "If Ann is Jan's mother, then Jose is Jan's cousin."

9. Write the converse, inverse, and contrapositive of "If Ed is Sue's father, then Liu is Sue's cousin."

10. Write the converse, inverse, and contrapositive of "If Al is Tom's cousin, then Jim is Tom's grandfather."

11. Rewrite the following statement in if-then form without using the word "necessary": Getting an answer of 10 for problem 16 is a necessary condition for solving problem 16 correctly.

12. State precisely (but concisely) what it means for a form of argument to be valid.

13. Consider the argument form:

$$\begin{array}{ll} & p \to \sim q \\ & q \to \sim p \\ \therefore & p \vee q \end{array}$$

Use the truth table below to determine whether this form of argument is valid or invalid. Annotate the table (as appropriate) and include a few words explaining how the truth table supports your answer.

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$	$p \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

14. Consider the argument form:

$$\begin{array}{ll} & p \wedge \sim q \to r \\ & p \vee q \\ & q \to p \\ \text{Therefore} & r. \end{array}$$

Use the truth table below to determine whether this argument form is valid or invalid. Annotate the table (as appropriate) and include a few words explaining how the truth table supports your answer.

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \to r$	$p \vee q$	$q \to p$	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

15. Determine whether the following argument is valid or invalid. Include a truth table and a few words explaining why the truth table shows validity or invalidity.

    If Hugo is a physics major or if Hugo is a math major, then he needs to take calculus.

    Hugo needs to take calculus or Hugo is a math major.

    Therefore, Hugo is a physics major or Hugo is a math major.

16. Determine whether the following argument is valid or invalid. Include a truth table and a few words explaining why the truth table shows validity or invalidity.

    If 12 divides 709,438 then 3 divides 709,438.
    If the sum of the digits of 709,438 is divisible by 9 then 3 divides 709,438.
    The sum of the digits of 709,438 is not divisible by 9.
    Therefore, 12 does not divide 709,438.

17. Write the form of the following argument. Is the argument valid or invalid? Justify your answer.

    If 54,587 is a prime number, then 17 is not a divisor of 54,587.
    17 is a divisor of 54,587.
    Therefore, 54,587 is not a prime number.

18. Write the form of the following argument. Is the argument valid or invalid? Justify your answer.

If Ann has the flu, then Ann has a fever.
Ann has a fever.
Therefore, Ann has the flu.

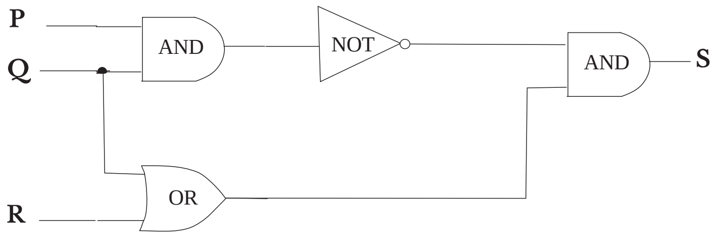
19. On the island of knights and knaves, you meet three natives, A, B, and C, who address you as follows:

A: At least one of us is a knave.

B: At most two of us are knaves.

What are A, B, and C?

20. Consider the following circuit.



(a) Find the output of the circuit corresponding to the input $P = 1,\ Q = 0$, and $R = 1$.

(b) Write the Boolean expression corresponding to the circuit.

21. Write 110101_2 in decimal form.

22. Write 75 in binary notation.

23. Draw the circuit that corresponds to the following Boolean expression: $(P \wedge Q) \vee (\sim P \wedge \sim Q)$.(*Note for students who have studied some circuit design*: Do not simplify the circuit; just draw the one that exactly corresponds to the expression.)

24. Find a circuit with the following input/output table.

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

25. Find $10111_2 + 1011_2$.

26. Write 100110_2 in decimal form.

27. Write the 8-bit two's complement for 49.

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Answers for Test Bank Questions: Chapters 1-4

Please use caution when using these answers. Small differences in wording, notation, or choice of examples or counterexamples may be acceptable.

Chapter 1

1. a. a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7

   b. an integer n; n is divided by 7 the remainder is 3

2. a. a positive real number; smaller than r

   b. positive real number r; there is a positive real number s

   Fill in the blanks to rewrite the following statement with variables:

3. There is an integer whose reciprocal is also an integer.

4. a. have three sides

   b. has three sides

   c. has three sides

   d. is a triangle; has three sides

   e. T has three sides

5. a. have additive inverses

   b. an additive inverse

   c. y is an additive inverse for x

6. a. less than or equal to every positive integer

   b. positive integer m; less than or equal to every positive integer

   c. less than or equal to n

7. (a) The set of all integers n such that n is a factor of 9.

   Or: The set of all elements n in **Z** such that n is a factor of 9.

   Or: The set of all elements n in the set of all integers such that n is a factor of 9.

   (b) $\{1, 3, 9\}$

8. (a) No

   (b) Yes

   (c) Yes

   (d) No

9. a. $\{(a, u), (a, v), (b, u), (b, v), (c, u), (c, v)\}$

   b. $\{(u, a), (v, a), (u, b), (v, b), (u, c), (v, c)\}$

10. a. Yes; No; No; Yes

    b. $\{(3, 15), (3, 18), (5, 15)\}$

    c. domain is $\{3, 5, 7\}$; co-domain is $\{15, 16, 17, 18\}$.

    d. Draw an arrow diagram for R.

    e. No: R fails both conditions for being a function from A to B. (1) Elements 5 and 7 in A are not related to any elements in B, and (2) there is an element in A, namely 3, that is related to two different elements in B, namely 15 and 18.

11. a. No; Yes; No; Yes

 b. Draw the graph of R in the Cartesian plane.

 c. No: R fails both conditions for being a function from $\mathbf{R}$ to $\mathbf{R}$. (1) There are many elements in $\mathbf{R}$ that are not related to any element in $\mathbf{R}$. For instance, none of 0, 1/2, and -1 is related to any element of $\mathbf{R}$. (2) there are elements in $\mathbf{R}$ that are related to two different elements in $\mathbf{R}$. For instance 2 is related to both 1 and -1.

12. a. $G(2) = c$

 b. Draw an arrow diagram for G.

13. $F \neq G$. Note that for every real number x,

$$G(x) = (x-2)^2 - 7 = x^2 - 4x + 4 - 7 = x^2 - 4x - 3,$$

whereas

$$F(x) = (x+1)(x-3) = x^2 - 2x - 3.$$

Thus, for instance,

$$F(1) = (1+1)(1-3) = -4 \quad \text{whereas} \quad G(1) = (1-2)^2 - 7 = -6.$$

Chapter 2

1. e

2. e

3. a. The variable S is not undeclared or the data are not out of order.

 b. The variable S is not undeclared and the data are not out of order.

 c. Al was with Bob on the first, and Al is not innocent.

 d. $-5 > x$ or $x \geq 2$

4. The statement forms are not logically equivalent.

 Truth table:

p	q	$\sim p$	$p \vee q$	$\sim p \wedge q$	$p \vee q \to p$	$p \vee (\sim p \wedge q)$
T	T	F	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	F	T
F	F	T	F	F	T	F

 Explanation: The truth table shows that $p \vee q \to p$ and $p \vee (\sim p \wedge q)$ have different truth values in rows 3 and 4, i.e, when p is false. Therefore $p \vee q \to p$ and $p \vee (\sim p \wedge q)$ are not logically equivalent.

5. *Sample answers:*

 Two statement forms are logically equivalent if, and only if, they always have the same truth values.

 Or: Two statement forms are logically equivalent if, and only if, no matter what statements are substituted in a consistent way for their statement variables the resulting statements have the same truth value.

6. *Solution 1:* The given statements are not logically equivalent. Let p be "Sam bought it at Crown Books," and q be "Sam didn't pay full price." Then the two statements have the following form:

$$p \to q \quad \text{and} \quad p \vee \sim q.$$

 The truth tables for these statement forms are

p	q	$\sim q$	$p \to q$	$p \vee \sim q$
T	T	F	T	T
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Rows 2 and 3 of the table show that $p \to q$ and $p \vee \sim q$ do not always have the same truth values, and so $p \to q \not\equiv p \vee \sim q$.

Solution 2: The given statements are not logically equivalent. Let p be "Sam bought it at Crown Books," and q be "Sam paid full price." Then the two statements have the following form:

$$p \to \sim q \quad \text{and} \quad p \vee q.$$

The truth tables for these statement forms are

p	q	$\sim q$	$p \to \sim q$	$p \vee q$
T	T	F	F	T
T	F	T	T	T
F	T	F	T	T
F	F	T	T	F

Rows 1 and 4 of the table show that $p \to \sim q$ and $p \vee q$ do not always have the same truth values, and so $p \to \sim q \not\equiv p \vee q$.

7. The given statements are not logically equivalent. Let p be "Sam is out of Schlitz," and q be "Sam is out of beer." Then the two statements have the following form:

$$p \to q \quad \text{and} \quad \sim q \vee \sim p.$$

The truth tables for these statement forms are

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \vee \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	T	T

The table shows that $p \to q$ and $\sim p \vee \sim q$ sometimes have opposite truth values (shown in rows 1 and 2), and so $p \to q \not\equiv \sim p \vee \sim q$.

8. *Converse*: If Jose is Jan's cousin, then Ann is Jan's mother

   Inverse: If Ann is not Jan's mother, then Jose is not Jan's cousin.

   Contrapositive: If Jose is not Jan's cousin, then Ann is not Jan's mother.

9. *Converse*: If Liu is Sue's cousin, then Ed is Sue's father.

   Inverse: If Ed is not Sue's father, then Liu is not Sue's cousin

   Contrapositive: If Liu is not Sue's cousin, then Ed is not Sue's father.

10. *Converse*: If Jim is Tom's grandfather, then then Al is Tom's cousin.

    Inverse: If Al is not Tom's cousin, then Jim is not Tom's grandfather

    Contrapositive: If Jim is not Tom's grandfather, then then Al is not Tom's cousin.

11. If someone does not get an answer of 10 for problem 16, then the person will not have solved problem 16 correctly.

    Or: If someone solves problem 16 correctly, then the person got an answer of 10.

12. *Sample answers:*

    For a form of argument to be valid means that no matter what statements are substituted for its statement variables, if the resulting premises are all true, then the conclusion is also true.

    Or: For a form of argument to be valid means that no matter what statements are substituted for its statement variables, it is impossible for all the premises to be true at the same time that the conclusion is false.

    Or: For a form of argument to be valid means that no matter what statements are substituted for its statement variables, it is impossible for conclusion to be false if all the premises are true.

13. The given form of argument is invalid.

				premises		*conclusion*
p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$	$p \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Row 4 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion.

14. The given form of argument is invalid.

					premises			*conclusion*
p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \to r$	$p \vee q$	$q \to p$	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

Row 2 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion.

15. Let p be "Hugo is a physics major," q be "Hugo is a math major," and r be "Hugo needs to take calculus." Then the given argument has the following form:

$$\begin{array}{ll} & p \vee q \to r \\ & r \vee q \\ \text{Therefore} & p \vee q. \end{array}$$

Truth table:

				premises		*conclusion*
p	q	r	$p \vee q$	$p \vee q \to r$	$r \vee q$	$p \vee q$
T	T	T	F	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	F	T	T	F
F	F	F	F	T	F	F

Row 7 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion. Therefore, the given argument is invalid.

16. Let p be "12 divides 709,438," q be "3 divides 709,438," and r be "The sum of the digits of 709,438 is divisible by 9." Then the given argument has the following form:

$$\begin{array}{ll} & p \to q \\ & r \to q \\ & \sim r \\ \text{Therefore} & \sim p. \end{array}$$

Truth table:

p	q	r	$\sim q$	$p \wedge \sim q$	$p \to q$ (premises)	$r \to q$ (premises)	$\sim r$ (premises)	$\sim p$ (conclusion)
T	T	T	F	F	T	T	F	F
T	T	F	F	F	T	T	T	F
T	F	T	T	T	F	F	F	F
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	F	F	T
F	F	F	T	F	T	T	T	T

Row 2 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion. Therefore, the given argument is invalid.

17. The argument has the form

$$\begin{array}{ll} & p \to q \\ & \sim q \\ \text{Therefore} & \sim p, \end{array}$$

which is valid by modus tollens (and the fact that the negation of "17 is not a divisor of 54,587" is "17 is a divisor of 54,587").

18. The argument has the form

$$\begin{array}{ll} & p \to q \\ & q \\ \text{Therefore} & p, \end{array}$$

which is invalid; it exhibits the converse error.

19. A and B are knights, and C is a knave.

Reasoning: A cannot be a knave because if A were a knave his statement would be true, which is impossible for a knave. Hence A is a knight, and at least one of the three is a knave. That implies that at most two of the three are knaves, which means that B's statement is true. Hence B is a knight. Since at least one of the three is a knave and both A and B are knights, it follows that C is a knave.

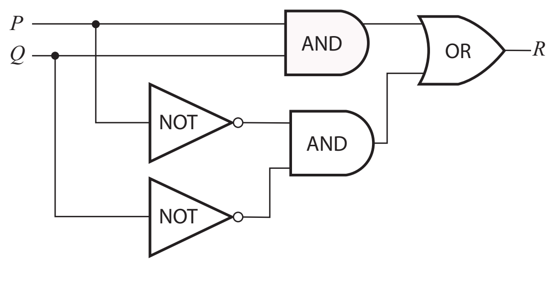
20. a. $S = 1$

b. $\sim (P \wedge Q) \wedge (Q \wedge R)$

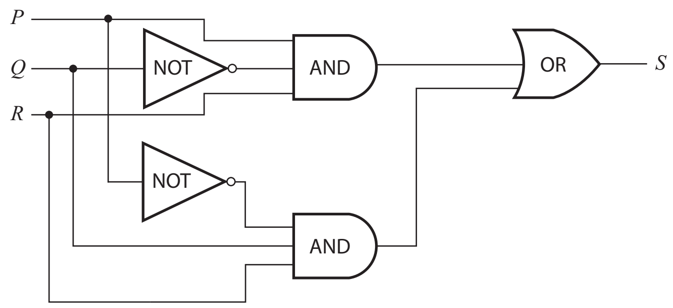
21. $110101_2 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^0 = 32 + 16 + 4 + 1 = 53_{10}$

22. $75_{10} = 64 + 8 + 2 + 1 = 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 1001011_2$.

23. The following circuit corresponds to the given Boolean expression:



24. One circuit (among many) having the given input/output table is the following:



25.

$$\begin{array}{rr} & 10111_2 \\ + & 1011_2 \\ \hline & 100010_2 \end{array}$$

26. $100110_2 = 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 4 + 2 = 38_{10}$

27. $49_{10} = (32 + 16 + 1)_{10} = 00110001_2 \longrightarrow 11001110 \longrightarrow 11001111.$

    So the two's complement is 11001111.

    Check: $2^8 - 49 = 256 - 49 = 207$ and

$$\begin{aligned} 11001111_2 &= 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \\ &= 128 + 64 + 8 + 4 + 2 + 1 \\ &= 207, \end{aligned}$$

    which agrees.

Chapter 3

1. ∀ valid argument x, if x has true premises, then x has a true conclusion.

2. a. ∀ odd integer n, n^2 is odd.

   b. ∀ integer n, if n is odd then n^2 is odd.

   c. ∃ an odd integer n such that n^2 is not odd.

   Or: ∃ an integer n such that n is odd and n^2 is not odd.

3. ∀ rational number r, ∃ integers u and v such that r is the ratio of u to v.

   Or: ∀ rational number r, ∃ integers u and v such that $r = u/v$.

4. ∀ even integer n that is greater than 2, ∃ prime numbers p and q such that $n = p + q$.

   Or: ∀ even integer n, if $n > 2$ then ∃ prime numbers p and q such that $n = p + q$.

5. e

6. a

7. d

8. g