SOLUTIONS MANUAL FOR

ENGINEERING VIBRATIONS

SECOND EDITION

by

WILLIAM J. BOTTEGA
JOSEPH M. LAKAWICZ
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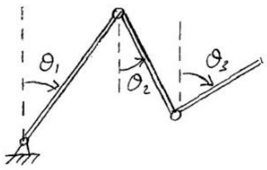
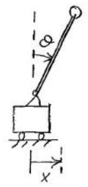
Chapter 1

1.1) 3 degrees of freedom

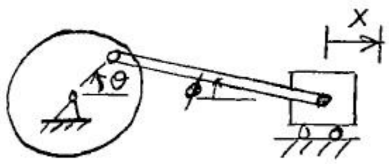


1.2) 2 degrees of freedom

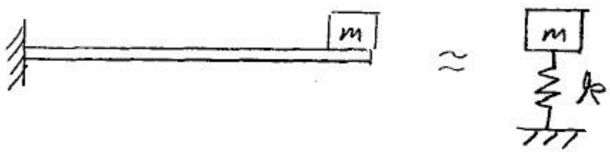


1.3) 1 degree of freedom

if θ is known, then ϕ and x are known



1.4) $\Delta = \frac{1}{2}''$, W = 200lb



$$k = \frac{W}{\Delta} = \frac{200\,\text{lb}}{\frac{1}{2}\,\text{in}} = 400\,\text{lb/in}$$

From Eq. (1.14), $k_{eff} = \frac{3EI}{L^3}, I = \frac{4 \cdot 2^3}{12} = \frac{8}{3}\text{in}^4$

$$E = \frac{kL^3}{3I} = \frac{400(10 \times 12)^3}{3(\frac{8}{3})} = 8.64 \times 10^7 \text{ psi}$$

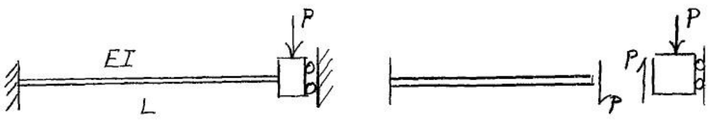
1.5) $k = \frac{W}{\Delta} = \frac{200\,\text{lb}}{\frac{1}{2}\,\text{in}} = 400\,\text{lb/in}, \quad I = \frac{4 \cdot 2^3}{12} = \frac{8}{3}\text{in}^4$

From Eq. (1.22), $k_{eff} = \frac{6EI}{L^3}$ where L= half length

$$E = \frac{kL^3}{6I} = \frac{400(5 \times 12)^3}{6(\frac{8}{3})} = 5.40 \times 10^6 \text{ psi}$$

1.6) $w(0) = 0$

$w'(0) = 0$



$EIw'''(L) = -P$

$w'(L) = 0$

$$w^{iv}(x) = 0$$
$$w'''(x) = c_0$$
$$w''(x) = c_0 x + c_1$$
$$w'(x) = c_0 \frac{x^2}{2} + c_1 x + c_2$$

$$w(x) = c_0 \frac{x^3}{6} + c_1 \frac{x^2}{2} + c_2 x + c_3 \quad (1)$$

$$w(0) = 0 \Rightarrow c_3 = 0$$
$$w'(0) = 0 \Rightarrow c_2 = 0 \quad (2a,b)$$

$$w'''(L) = -\frac{P}{EI} = c_0$$
$$w'(L) = 0 \Rightarrow c_1 = -c_0 L/2 = \frac{PL}{2EI} \quad (2c,d)$$

(2a,b,c,d) into (1) $\Rightarrow$ $w(x) = \frac{Px^2}{12EI}\left[3L - 2x\right]$

$\Delta_L = w(L) = \frac{PL^3}{12EI}$, thus $P = \frac{12EI}{L^3}\Delta_L$ or $P = k\Delta_L$ where

$$k = \frac{12EI}{L^3}$$

1.7) From Eq. (1.40): $k = \rho_f g A = \gamma_f A = 62.4 \frac{\text{lb}}{\text{ft}^3}(6' \times 6') = 2.25 \times 10^3$ lb/ft

$$\Delta = \frac{W}{k} = \frac{190}{2.25 \times 10^3} = 0.0844\,\text{ft} = 1.01\,\text{in}$$

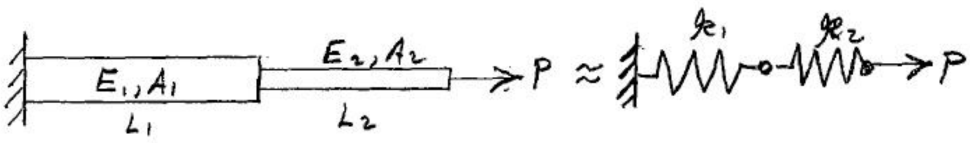
1.8) $\frac{1}{k_{eq}} = \frac{1}{k_1 + k_2} + \frac{1}{k_3} = \frac{k_1 + k_2 + k_3}{(k_1 + k_2)k_3}$

$$k_{eq} = \frac{(k_1 + k_2)k_3}{k_1 + k_2 + k_3}$$

1.9) $\frac{1}{k^*} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Rightarrow \quad k^* = \frac{k_1 k_2}{k_1 + k_2}$

$$\underline{\underline{k_{eq} = k^* + k_3 = \frac{k_1 k_2}{k_1 + k_2} + k_3}}$$

1.10)



$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Rightarrow \quad k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$ where, from Eq. (1.8), $k_1 = \frac{E_1 A_1}{L_1}$, $k_2 = \frac{E_2 A_2}{L_2}$

$E_{steel} = 29 \times 10^6 \text{ psi} = 20 \times 10^6 \text{ N/cm}^2$

$E_{alum} = 10 \times 10^6 \text{ psi} = 6.9 \times 10^6 \text{ N/cm}^2$

$$k_1 = \frac{20 \times 10^6 \cdot \pi (3)^2}{3 \times 100} = 18.8 \times 10^5 \text{ N/cm}$$

$$k_2 = \frac{6.9 \times 10^6 \cdot \pi (2)^2}{2 \times 100} = 4.34 \times 10^5 \text{ N/cm}$$

$$k_{eff} = \frac{\left(18.8 \times 10^5\right)\left(4.34 \times 10^5\right)}{\left(18.8 + 4.34\right) \times 10^5} = 3.53 \times 10^5 \text{ N/cm}$$

$$\underline{\underline{\Delta = \frac{P}{k_{eff}} = \frac{10\,\text{N}}{3.53 \times 10^5 \text{ N/cm}} = 2.83 \times 10^{-5} \text{ cm}}}$$

1.11)

$G_{steel} = 12 \times 10^6 \text{ psi} = 8.28 \times 10^6 \text{ N/cm}^2$

$G_{alum} = 4 \times 10^6 \text{ psi} = 2.76 \times 10^6 \text{ N/cm}^2$

$$J_1 = \frac{\pi R_1^{\,4}}{2} = \frac{\pi (3)^4}{2} = 127\,\text{cm}^4$$

$$J_2 = \frac{\pi R_2^{\,4}}{2} = \frac{\pi (2)^4}{2} = 25.1\,\text{cm}^4$$

From Eq. (1.32):

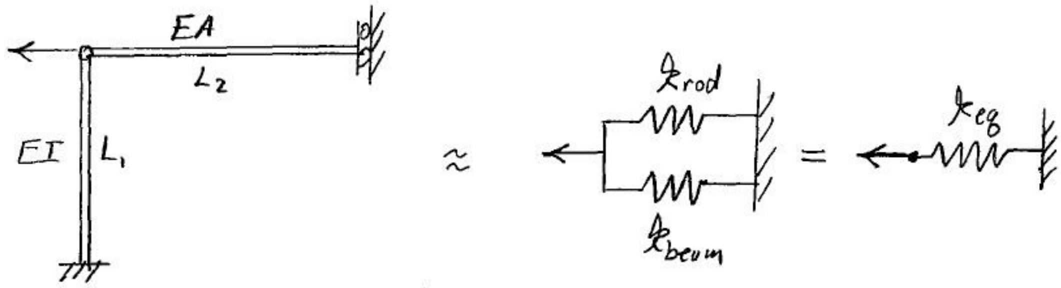
$$k_1 = \frac{G_1 J_1}{L_1} = \frac{8.28 \times 10^6 \cdot 127}{3 \times 100} = 3.51 \times 10^4 \text{ N} \cdot \text{m/rad}$$

$$k_2 = \frac{G_2 J_2}{L_2} = \frac{2.76 \times 10^6 \cdot 25.1}{2 \times 100} = 0.346 \times 10^4 \text{ N} \cdot \text{m/rad}$$

$$\frac{1}{k^*} = \frac{1}{3.51 \times 10^4} + \frac{1}{0.346 \times 10^4} = 3.18 \times 10^{-4} \quad \Rightarrow \quad k^* = 3140\,\text{N} \cdot \text{m/rad}$$

$$\underline{\underline{\vartheta = \frac{\text{T}}{k^*} = \frac{200\,\text{N} \cdot \text{m}}{3140\,\text{N} \cdot \text{m/rad}} = 0.0637\,\text{rad}}}$$

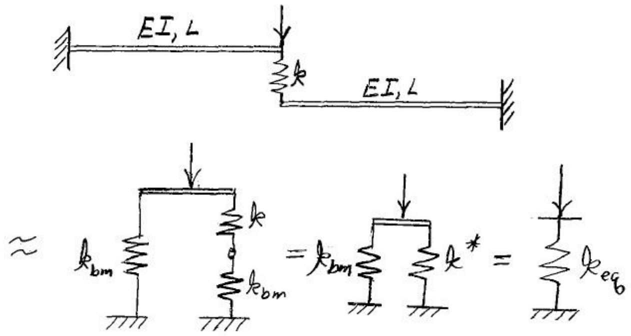
1.12)



Thus, from Eqs. (1.44), (1.8), and (1.14):

$$\underline{\underline{k_{eq} = k_{rod} + k_{beam} = \frac{EA}{L_2} + \frac{3EI}{L_1^3}}}$$

1.13)

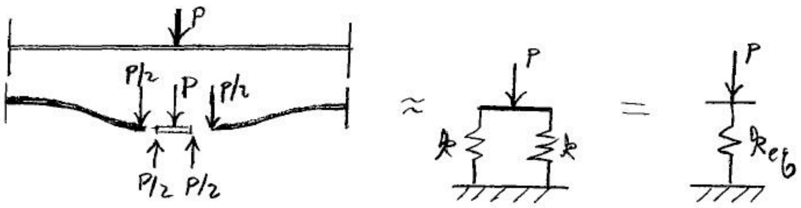


$$\frac{1}{k^*} = \frac{1}{k} + \frac{1}{k_{bm}} \quad \Rightarrow \quad k^* = \frac{kk_{bm}}{k + k_{bm}}$$

$$k_{eq} = k^* + k_{bm} = \frac{kk_{bm}}{k + k_{bm}} + k_{bm} = k_{bm}\left[\frac{k}{k + k_{bm}} + 1\right]$$

$$\underline{\underline{k_{eq} = \frac{3EI}{L^3}\left[\frac{k}{k + \frac{3EI}{L^3}} + 1\right]}}$$

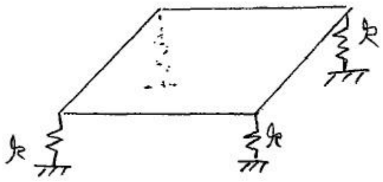
1.14)



From Eq. (1.17) or Prob. 1.6 $\qquad k = \frac{12EI}{\left(L/2\right)^3} = \frac{96EI}{L^3}$

$$\underline{\underline{k_{eq} = k + k = 2 \times \frac{96EI}{L^3} = 192\frac{EI}{L^3}}}$$

1.15) From Eq. (1.40) $k = \rho_f gA$

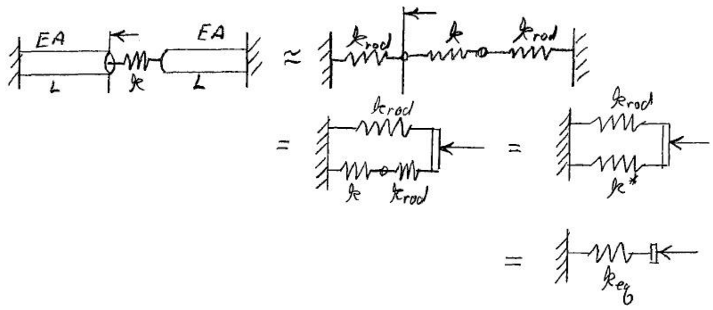


For seawater, $\gamma = \rho_f g = 64\,\text{lb/ft}^3$

Hence, from Eq. (1.44): $k_{eq} = 4k = 4\gamma A = 4(64)\pi(2)^2 = 3220\,\text{lb/ft}$

$$\Delta = \frac{W}{k_{eq}} = \frac{125}{3220} = 0.0373\,\text{ft} = 0.447\,\text{in}$$

1.16)



$$\frac{1}{k^*} = \frac{1}{k_{rod}} + \frac{1}{k} \Rightarrow k^* = \frac{kk_{rod}}{k + k_{rod}}$$

$$k_{eq} = k^* + k_{rod} = k_{rod}\left[\frac{k}{k + k_{rod}} + 1\right] = \frac{EA}{L}\left[\frac{kL}{kL + EA} + 1\right]$$

1.17) Same as Prob. 1.16 with linear springs replaced by torsional springs, and $k_{rod} = \dfrac{GJ}{L}$

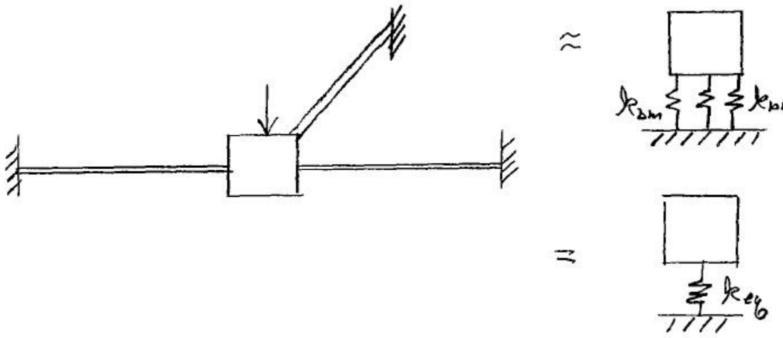
$$k_{eq} = k_{rod}\left[\frac{k_T}{k_T + k_{rod}} + 1\right] = \frac{GJ}{L}\left[\frac{k_T L}{k_T L + GJ} + 1\right]$$

1.18) Neglect rotation of block and hence, twisting of beams

From Eq. (1.17),

$k_{bm} = 12EI / L^3$

From Eq. (1.44),



$$k_{eq} = 3k_{bm} = 3 \cdot 12\frac{EI}{L^3} = 36\frac{EI}{L^3}$$

1.19) From Eq. (1.61) $e^{i\psi} = \cos\psi + i\sin\psi$
$e^{-i\psi} = \cos\psi - i\sin\psi$

$e^{i\psi} + e^{-i\psi} = 2\cos\psi \Rightarrow$ $\underline{\underline{\cos\psi = \dfrac{e^{i\psi} + e^{-i\psi}}{2}}}$

$e^{i\psi} - e^{-i\psi} = 2i\sin\psi \Rightarrow$ $\underline{\underline{\sin\psi = \dfrac{e^{i\psi} - e^{-i\psi}}{2i}}}$

1.20) From Eq. (1.63) $e^{\psi} = \cosh\psi + \sinh\psi$
$e^{-\psi} = \cosh\psi - \sinh\psi$

$e^{\psi} + e^{-\psi} = 2\cosh\psi \Rightarrow$ $\underline{\underline{\cosh\psi = \dfrac{e^{\psi} + e^{-\psi}}{2}}}$

$e^{\psi} - e^{-\psi} = 2\sinh\psi \Rightarrow$ $\underline{\underline{\sinh\psi = \dfrac{e^{\psi} - e^{-\psi}}{2}}}$

1.21) $f(\theta) = \tfrac{1}{2}(a+ib)e^{i\theta} + \tfrac{1}{2}(a-ib)e^{-i\theta}$

Expanding and regrouping terms

$$\begin{aligned} f(\theta) &= \tfrac{1}{2}ae^{i\theta} + \tfrac{1}{2}ibe^{i\theta} + \tfrac{1}{2}ae^{-i\theta} - \tfrac{1}{2}ibe^{-i\theta} \\ &= \tfrac{1}{2}a\left(e^{i\theta} + e^{-i\theta}\right) + \tfrac{1}{2}ib\left(e^{i\theta} - e^{-i\theta}\right)\cdot\tfrac{i}{i} \\ &= a\frac{e^{i\theta} + e^{-i\theta}}{2} - b\frac{e^{i\theta} - e^{-i\theta}}{2i} \end{aligned}$$

From Prob. 1.19 $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

Thus, $f(\theta) = a\cos\theta - b\sin\theta$ or $\underline{\underline{f(\theta) = c_1\cos\theta + c_2\sin\theta}}$ where

$\underline{\underline{c_1 = a}}$ and $\underline{\underline{c_2 = -b}}$

1.22) $z = \dfrac{a+ib}{c+id}\cdot\dfrac{c-id}{c-id} = \dfrac{(ac+bd) + i(bc-ad)}{c^2+d^2}$

Thus, $\underline{\underline{\mathrm{Re}(z) = \dfrac{ac+bd}{c^2+d^2}}}$ and $\underline{\underline{\mathrm{Im}(z) = \dfrac{bc-ad}{c^2+d^2}}}$

1.23) $W = \int \vec{F} \cdot \mathrm{d}\vec{r}$

(a) $W_{AC} = \int_0^L -ks\mathrm{d}s = -\frac{ks^2}{2}\bigg|_0^L = -\frac{kL^2}{2}$

$$\underline{\underline{W_{AC} = -\frac{k}{2}\left(a^2 + b^2\right)}}$$

(b) $W_{AB} = \int_0^a -kx\mathrm{d}x = -\frac{ka^2}{2}$

$$W_{BC} = \int -kL\vec{e}_L \cdot \vec{j}\mathrm{d}y = \int -kL\sin\theta\,\mathrm{d}y = -kL\int_0^b \frac{y}{L}\mathrm{d}y = -\frac{kb^2}{2}$$

$$W_{ABC} = W_{AB} + W_{BC} = -\frac{ka^2}{2} - -\frac{kb^2}{2}$$

$$\underline{\underline{W_{ABC} = -\frac{k}{2}\left(a^2 + b^2\right)}}$$

Comparison of (a) and (b) shows that $W_{AC} = W_{ABC}$ as it should be since the elastic force is conservative.

1.24) $f = -\mu N = -\mu W$

(a) $W_{A\to B} = \int_A^B -\mu W\,\mathrm{d}x = -\mu W\,\overline{AB} = -\mu W L_1$

$$W_{B\to A} = \int_B^A \mu W\left(-\mathrm{d}x\right) = -\mu W\,\overline{BA} = -\mu W L_1$$

$$\underline{\underline{W_{A\to B\to A} = -\mu W L_1 - \mu W L_1 = -2\mu W L_1 \neq 0}}$$

(b)

$$\begin{aligned} W_{A\to B\to C\to D\to A} &= -\mu W\,\overline{AB} - \mu W\,\overline{BC} - \mu W\,\overline{CD} - \mu W\,\overline{DA} \\ &= -\mu W\,L_1 - \mu W\,L_2 - \mu W\,L_1 - \mu W\,L_2 \end{aligned}$$

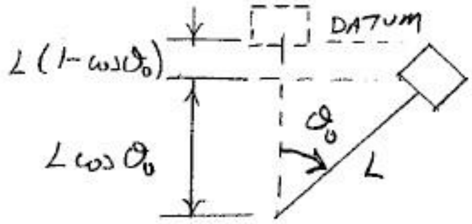
$$\underline{\underline{W_{A\to B\to C\to D\to A} = -2\mu W\left(L_1 + L_2\right) \neq 0}}$$

The work done is clearly dependent on the path traversed. Hence, the friction force is non-conservative.

1.25) -Since there are no dissipative forces acting on the rod, energy is conserved.

-Since the spring is not torqued when θ=0, let us choose this configuration as our datum to measure potential energy.

Then, $\left[KE+PE\right]_{\theta=0}=\left[KE+PE\right]_{\theta=\theta_0}$

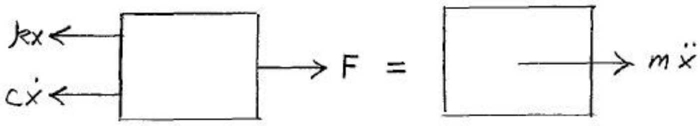


$\frac{1}{2}mv^2+0=0+\frac{1}{2}k_T\theta_0^{\ 2}-mgL\left(1-\cos\theta_0\right)$

Solving for *v:*

$$v=\sqrt{\frac{k_T}{m}\theta_0^{\ 2}-2gL\left(1-\cos\theta_0\right)}$$

1.26) (a) $\sum F=ma$:



$F-kx-c\dot{x}=m\ddot{x}$

$$m\ddot{x}+c\dot{x}+kx=F$$

(b) $W^{(NC)}=\Delta T+\Delta U$

$\int\left(F-c\dot{x}\right)\mathrm{d}x=\left[\frac{1}{2}m\dot{x}^2-\frac{1}{2}mv_0^{\ 2}\right]+\left[\frac{1}{2}kx^2-0\right]$ where $\mathrm{d}x=\frac{\mathrm{d}x}{\mathrm{d}t}=\dot{x}\mathrm{d}t$

$\int\left(F-c\dot{x}\right)\dot{x}\mathrm{d}t=\frac{1}{2}m\dot{x}^2-\frac{1}{2}mv_0^{\ 2}+\frac{1}{2}kx^2$ take derivative $\frac{\mathrm{d}}{\mathrm{d}t}$

$\left(F-c\dot{x}\right)\dot{x}=m\dot{x}\ddot{x}+kx\dot{x}$ or $\left(m\ddot{x}+c\dot{x}+kx-F\right)\dot{x}=0$ thus, for $\dot{x}\neq0$

$$m\ddot{x}+c\dot{x}+kx=F$$

1.27) Though a friction force is present to prevent slip, it acts only at the instantaneous stationary contact point. (Since the wheel is treated as rigid, contact with the road occurs at only one point of the wheel.) Since the point, and hence the force, does not move, the friction force does no work. Therefore, energy is conserved for this idealized rigid body model.

$\frac{1}{2}mv_{G_2}^{\ \ 2}+\frac{1}{2}I\omega_2^{\ 2}+0=0+0+mgh$

no slip: $v_G=\omega R$, noting that $I=mr_G^{\ 2}$

$\frac{1}{2}m\left(\omega_2R\right)^2+\frac{1}{2}mr_G^{\ 2}\omega_2^{\ 2}=mgh$ solving for ω_2 : $\omega_2=\sqrt{\frac{2gh}{R^2+r_G^{\ 2}}}$

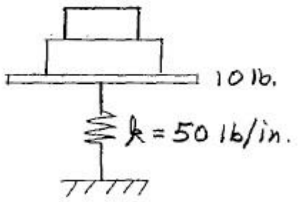
$$v_{G_2}=\omega_2R=R\sqrt{\frac{2gh}{R^2+r_G^{\ 2}}}$$

Chapter 2

2.1) $T = 2$ sec, $m = 20$ gm $\qquad \dfrac{k}{m} = \omega^2 = \left(\dfrac{2\pi}{T}\right)^2 = \left(\dfrac{2\pi}{2}\right)^2 = \pi^2$

$$\underline{\underline{k = \pi^2 m = \pi^2 (20) = 197.4 \text{ dynes/cm}}}$$

2.2) $m = \dfrac{k}{\omega^2}$



$$m_{p_1} + m_{pkg} = \frac{k}{\omega^2}$$

$$m_{pkg} = \frac{k}{\omega^2} - m_{p_1}$$

$$W_{pkg} = m_{pkg} g = \frac{kg}{\omega^2} - m_{p_1} g = \frac{kg}{(2\pi\nu)^2} - W_{p_1}$$

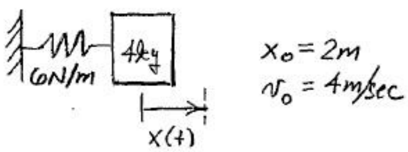
$$\underline{\underline{W_{pkg} = \frac{50(32.2 \times 12)}{\left[2\pi(3)\right]^2} - 10 = 44.4 \text{ lb}}}$$

2.3) Fig. 1.8 $\qquad \underline{\underline{\omega = \sqrt{\dfrac{k_{eff}}{m}} = \sqrt{\dfrac{12EI}{mL^2}} = 2\sqrt{\dfrac{3EI}{mL^2}}}}$

Fig. 1.9 $\qquad \underline{\underline{\omega = \sqrt{\dfrac{k_{eff}}{m}} = \sqrt{\dfrac{48EI}{mL^2}} = 4\sqrt{\dfrac{3EI}{mL^2}}}}$

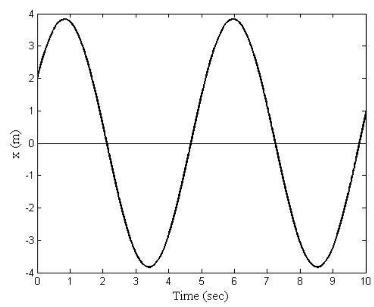
The effect of clamping the roof is to increase the stiffness and, in doing so, double the natural frequency of the structure.

2.4) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{6}{4}} = 1.225 \text{ rad/sec}$



From Eq. (2.16):

$$\underline{\underline{A = \sqrt{x_0^2 + (v_0/\omega)^2} = \sqrt{(2)^2 + (4/1.225)^2} = 3.830 \text{ m}}}$$



From Eq. (2.17):

$$\underline{\underline{\phi = \tan^{-1}\left(\frac{v_0}{\omega x_0}\right) = \tan^{-1}\left(\frac{4}{1.225(2)}\right) = 58.51° = 1.021 \text{ rads}}}$$

From Eq. (2.15): $\qquad \underline{\underline{x(t) = 3.830\cos(1.225t - 1.021)}}$ meters

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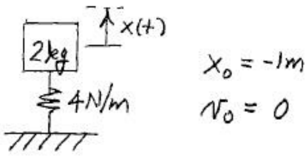
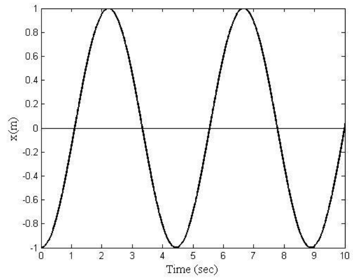
2.5) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4}{2}} = \sqrt{2}$ rad/sec





$A = 1\text{m},\ \phi = \pi$ rads

From Eq. (2.14): $\underline{\underline{x(t) = \cos\left(\sqrt{2}t - \pi\right) = -\cos\left(\sqrt{2}t\right)}}$ meters

---

2.6) $A = \pi R^2 = \pi(1.25)^2 = 4.91\,\text{cm}^2,\ E_{al} = 6.90\times10^6\ \text{N/cm}^2,\ m = 20\,\text{kg} = 2\times10^4\ \text{g}$

$$k_{eq} = \frac{E_{al}A}{L} = \frac{(6.90\times10^6)(4.91)}{30} = 1.13\times10^6\ \text{N/cm} = 1.13\times10^{11}\ \text{dynes/cm}$$

$$\underline{\underline{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.13\times10^{11}}{2\times10^4}} = 2380\,\text{rad/sec} = 378\,\text{cps}}}$$

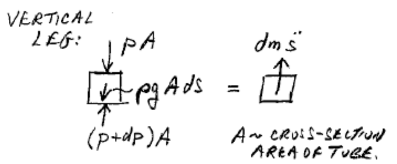
2.7) $I_\rho = m{r_G}^2 = 20(5)^2 = 500\,\text{kg}\cdot\text{cm}^2 = 5\times10^5\ \text{g}\cdot\text{cm}^2$

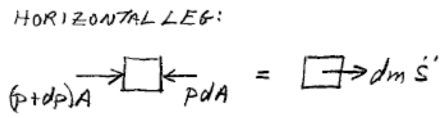
$$J = \frac{\pi R^4}{2} = \frac{\pi(1.25)^4}{2} = 3.83\,\text{cm}^4$$

From Eq. (1.29): $k_T = \dfrac{JG}{L} = \dfrac{(2.76\times10^6)(3.83)}{30} = 3.52\times10^5\ \text{N}\cdot\text{cm/ rad} = 3.52\times10^{10}\ \text{dyne}\cdot\text{cm/ rad}$

$$\underline{\underline{\omega = \sqrt{\frac{k_T}{I_\rho}} = \sqrt{\frac{3.52\times10^{10}}{5\times10^5}} = 265\,\text{rad/sec}}}$$

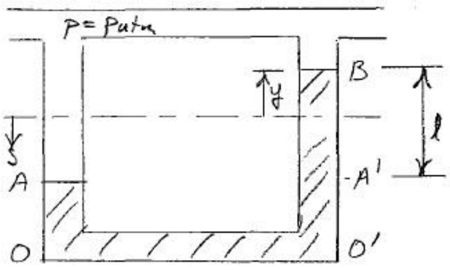
2.9) If we treat the fluid as incompressible, we may think of the confined fluid as an incompressible/inextensible rod with vanishing bending stress. We may then recognize that the components of displacement, velocity, and acceleration along the path of the fluid are uniform. That is, they are the same for each fluid particle. Let us isolate a typical fluid element, (1) moving vertically and (2) moving horizontally, and draw the kinetic diagrams for each. Hence,





$A\mathrm{d}p - \rho g A\,\mathrm{d}s = \ddot{s}\,\mathrm{d}m$ $\qquad A\,\mathrm{d}p = \ddot{s}\,\mathrm{d}m$

$$\int_A^0 A\mathrm{d}p - \int_A^0 \rho g A\,\mathrm{d}s = \int_A^0 \ddot{s}\,\mathrm{d}m \quad \text{(1a)}$$



$$\int_{0'}^{A'} A\mathrm{d}p - \int_{0'}^{A'} \rho g A\,\mathrm{d}s = \int_{0'}^{A'} \ddot{s}\,\mathrm{d}m \quad \text{(1b)}$$

$$\int_{A'}^{B} A\mathrm{d}p - \int_{A'}^{B} \rho g A\,\mathrm{d}s = \int_{A'}^{B} \ddot{s}\,\mathrm{d}m \quad \text{(1c)}$$

$$\int_{0}^{0'} A\,\mathrm{d}p = \int_{0}^{0'} \ddot{s}\,\mathrm{d}m \quad \text{(1d)}$$

Adding (1a)-(1d) and recognizing that $\int_A^0 \; = -\int_{0'}^{A}$ gives

$$A\int_0^{0'} \mathrm{d}p + A\int_{A'}^{B} \mathrm{d}p - \int_{A'}^{B} \rho g A\,\mathrm{d}s = \ddot{s}\int_A^B \mathrm{d}m$$

$$A\left(p_0 - p_0\right) + A\left(p_B - p_A\right) - \rho g A l = m\ddot{s} \quad \text{(2)}$$

where $m = \int_A^B \mathrm{d}m = \rho A L$ and L is the total given length of the manometer fluid. Let y measure the height of manometer fluid above its equilibrium position. Then, since acceleration of all particles along the path is the same, $\ddot{s} = \ddot{y}$ and further, $l = 2y$ then substituting into (2):

$$-2\rho g A y = \rho A L \ddot{y}$$

or

$$\ddot{y} + \frac{2g}{L} y = 0$$

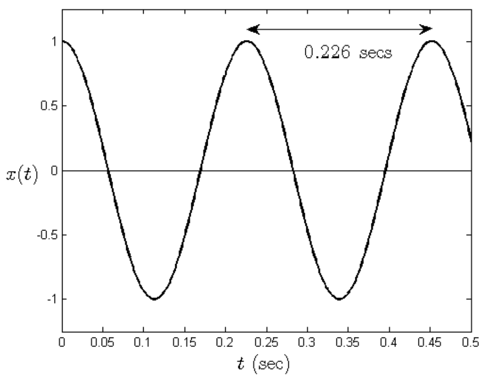
Thus,

$$\omega = \sqrt{\frac{2g}{L}}$$

2.10) $k = 400\,\text{lb/in},\; W = 200\,\text{lb}$

$$\omega = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{400}{200/32.2 \times 12}} = 27.8\,\text{rad/sec}$$

$$T = \frac{2\pi}{27.8} = 0.226\,\text{sec}$$



From Eq. (2.14): $x(t) = x_0 \cos\omega t + \dfrac{v_0}{\omega}\sin\omega t$

$$x(t) = (1)\cos 27.8t + \frac{0}{27.8}\sin 27.8t$$

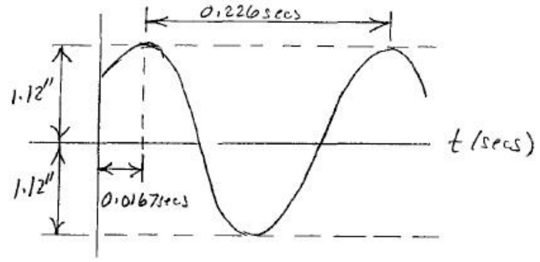
$$x(t) = \cos 27.8t \text{ inches}$$

2.11) $x_0 = 1\,\text{in},\;\; v_0 = 13.9\,\text{in/sec},\;\; \omega = 27.8\,\text{rad/sec},\;\; T = \dfrac{2\pi}{27.8} = 0.226\,\text{sec}$

$$A = \sqrt{(1)^2 + (13.9/27.5)^2} = 1.12\,\text{in}$$

$$\phi = \tan^{-1}\left(\frac{13.9}{27.8(1)}\right) = 0.464\,\text{rads}$$

$$t_\phi = \phi/\omega = 0.0167\,\text{sec}$$



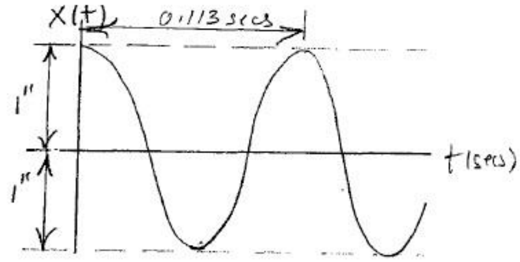
$$\underline{\underline{x(t) = 1.12\cos\left(27.8t - 0.464\right)}}\ \text{inches}$$

2.12) From Prob. 1.6:

$$k = \frac{12EI}{L^3} = 3\cdot\frac{4EI}{L^3} = 4\left(400\,\text{lb/in}\right) = 1600\,\text{lb/in}$$

$$\omega = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{1600}{200/32.2\times 12}} = 55.6\,\text{rad/sec}$$

$$T = \frac{2\pi}{55.6} = 0.113\,\text{sec}$$



$$\underline{\underline{x(t) = \cos 55.6t}}\ \text{inches}$$

2.13) The spring force is non-impulsive. Conservation of linear momentum:

$$m\cdot 0 + mv_1 = 2mv_2 \Rightarrow \quad v_2 = \tfrac{1}{2}v_1$$

Thus, the initial velocity for 2-car system is $v_0 = \tfrac{1}{2}v_1$

$$k_{eq} = \frac{4EA}{L} = \frac{4\pi R^2 E}{L}$$

$$\omega = \sqrt{\frac{k_{eq}}{2m}} = \sqrt{\frac{4\pi R^2 E}{2mL}} = \sqrt{\frac{2\pi R^2 E}{mL}}$$

$$x(t) = x_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t$$

$$x_0 = 0,\;\; \frac{v_0}{\omega} = \frac{\frac{1}{2}v_1}{\sqrt{2\pi R^2 E/mL}} = \frac{v_1}{2R}\sqrt{\frac{ml}{2\pi E}}$$

$$\underline{\underline{x(t) = \frac{v_1}{2R}\sqrt{\frac{ml}{2\pi E}}\sin\sqrt{\frac{2\pi R^2 E}{mL}}t}}$$

2.14) From Prob. 1.7: $k = 2.25\times10^3$ lb/ft

$x_0 = 1.01\,\text{in},\ v_0 = 0$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.25\times10^3}{200/32.2}} = 19.0\,\text{rad/sec}$$

$$\underline{\underline{x(t) = 1.01\cos 19t}}\ \text{inches}$$

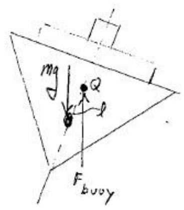
2.15) From Prob. 1.15: $k = 3220\,\text{lb/ft}$

$x_0 = 0.447\,\text{in},\ v_0 = 0$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3220}{200/32.2}} = 22.8\,\text{rad/sec}$$

$$\underline{\underline{x(t) = 0.447\cos 22.8t}}\ \text{inches}$$

2.16) $\sum M_G = I_G\ddot{\theta}$



$-lF_{buoy}\sin\theta = I_G\ddot{\theta}$

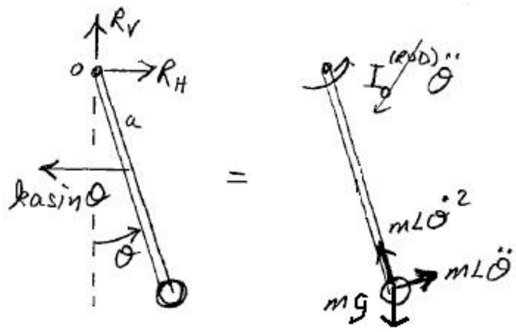
$I_G\ddot{\theta} + lF_{buoy}\theta \approx 0$

$$\underline{\underline{\omega^2 = \frac{lF_{buoy}}{I_G} = \frac{lmg}{I_G}}}$$

2.17) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{g/L}}$

$$\underline{\underline{L = \frac{gT^2}{4\pi^2} = \frac{32.2(3)^2}{4\pi^2} = 7.34\,\text{ft}}}$$

2.18) $\sum \vec{M}_0 = I_0^{(rod)}\vec{\alpha} + \vec{r}_{bob}\times m\vec{a}_{bob}$



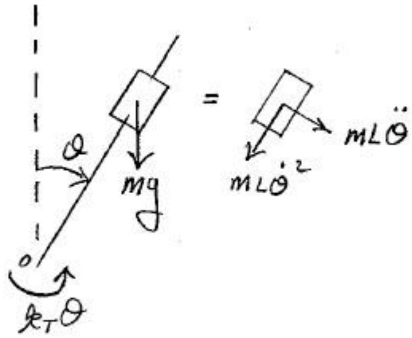
$-a\cos\theta\cdot ka\sin\theta - L\cos\theta\cdot mg\sin\theta = L\cdot mL\ddot{\theta}$

$mL^2\ddot{\theta} + \left[ka^2 + mgL\right]\sin\theta\cos\theta = 0$

$\sin\theta\cos\theta = \left[\theta - \dfrac{\theta^3}{3!} + \ldots\right]\cdot\left[1 - \dfrac{\theta^2}{2!} + \ldots\right] \approx \theta \ll 1$

$\ddot{\theta} + \left[\dfrac{ka^2 + mgL}{mL^2}\right]\theta = 0$ $\qquad \underline{\underline{\omega = \sqrt{\dfrac{ka^2}{mL^2} + \dfrac{g}{L}}}}$

2.19) $\sum \vec{M}_0 = I_0^{(rod)}\vec{\alpha} + \vec{r}_{cyl} \times m\vec{a}_{cyl}$



$-Lmg\sin\theta + k_T\theta = -L \cdot mL\ddot{\theta}$

$mL^2\ddot{\theta} + k_T\theta - Lmg\sin\theta = 0$

$\sin\theta \approx \theta \ll 1$

$\ddot{\theta} + \left[\dfrac{k_T - mgL}{mL^2}\right]\theta = 0$

$\omega = \sqrt{\dfrac{g}{L}\left[\dfrac{k_T}{mgL} - 1\right]}$ $\qquad$ $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\dfrac{g}{L}\left[\dfrac{k_T}{mgL} - 1\right]}}$

2.20) The solution follows that of Example 2.8, but with the moment of inertia now corresponding to a sphere rather than a disk. Thus, $I = \frac{2}{5}mR^2$

From Eqs. (h), (j), and (k) of Ex. 2.8:

$I_\rho\left(\dfrac{R-r}{r}\right)\ddot{\theta} + mgr\sin\theta = 0$

where $I_\rho = I_G + mr^2 = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$ and $\sin\theta \approx \theta \ll 1$

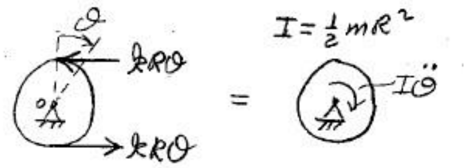
$\frac{7}{5}mr^2\left(\dfrac{R-r}{r}\right)\ddot{\theta} + mgr\theta = 0$ or $\ddot{\theta} + \dfrac{g}{L_{eff}}\theta = 0$

where $L_{eff} = \frac{7}{5}(R-r) = \frac{7}{5}(100-1.5) = 137.9\,\text{cm}$

$$\omega = \sqrt{\frac{g}{L_{eff}}} = \sqrt{\frac{981}{137.9}} = 2.67\,\text{rad/sec}$$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2.67} = 2.36\,\text{sec}$ $\qquad$ $t_{rise} = \dfrac{T}{2} = 1.18\,\text{sec}$

2.21) $\sum M_0 = I\alpha: \quad 2kR^2\theta = -I\ddot{\theta}$



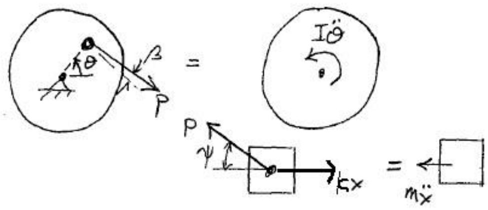
$\ddot{\theta} + \dfrac{2kR^2}{I}\theta = 0$

$$\omega = 2\sqrt{\frac{k}{m}}$$

2.22) $\theta(t) = \theta_0 \cos \omega t + \frac{\varpi_0}{\omega} \sin \omega t$

$$\underline{\underline{\theta(t) = \theta_0 \cos 2\sqrt{\frac{k}{m}}t}}$$

2.23) Wheel: $\sum M_0 = I\alpha$: $\quad -PR_0 \cos\beta = -I\ddot{\theta}$

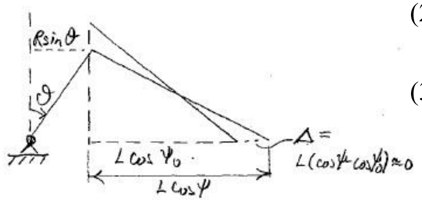


$I\ddot{\theta} + PR_0 \cos\beta = 0$

$\beta \ll 1$: $I\ddot{\theta} + PR_0 \approx 0$ (1)

Block: $\sum F_x = ma_x$: $\quad P\cos\psi - kx = m\ddot{x}$

$\psi \ll 1$: $m\ddot{x} + kx \approx P$ (2)



(2) into (1) $\quad I\ddot{\theta} + mR_0\ddot{x} + kR_0 x = 0$ (3)

Need $\theta - x$ relation →

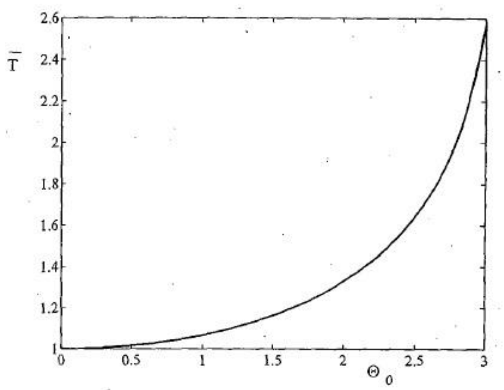
$x = R_0 \sin\theta + L\left(\cos\psi - \cos\psi_0\right) \approx R_0\theta$ (4)

(4) into (3) $\quad \left(\frac{I}{R_0} + mR_0\right)\ddot{x} + kR_0 x = 0$

$$\underline{\underline{\omega = \sqrt{\frac{kR_0^{\,2}}{I + mR_0^{\,2}}}}}$$

2.24) (a) Execute the following Matlab code:

```
theta=0.01:0.01:3;
q=sin(theta/2);
m=q.*q;
Tbar=(2/pi)*ellipke(m);
plot(theta,Tbar)
```



(b) $\bar{T} = \dfrac{1}{1-E} = \dfrac{2}{\pi}\mathbb{F}(m)$

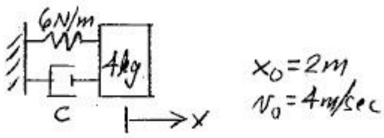
(i) $\bar{T} = \dfrac{1}{1-.05} = 1.05,\quad \mathbb{F}(m) = \dfrac{\pi}{2}\bar{T} = \dfrac{\pi}{2}(1.05) = 1.65$

$$\underline{\underline{\theta_0 = 0.873\,\text{rad}}}$$

(ii) $\bar{T} = \dfrac{1}{1-.01} = 1.01,\quad \mathbb{F}(m) = \dfrac{\pi}{2}\bar{T} = \dfrac{\pi}{2}(1.01) = 1.59$

$$\underline{\underline{\theta_0 = 0.400\,\text{rad}}}$$

2.25) (a) $c = 1\,\text{N}\cdot\text{sec/m}$



$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{4}} = 1.225\,\text{rad/sec}$$

$$\zeta = \frac{c}{2\omega m} = \frac{1}{2(1.225)(4)} = 0.1020$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 1.225\sqrt{1-(0.1020)^2} = 1.219\,\text{rad/sec}$$
$$\zeta\omega = (0.1020)(1.225) = 0.1250\,\text{rad/sec}$$

$$\underline{\underline{A = 2\sqrt{1+\frac{\left[\{4/(1.225)(2)\}+0.1020\right]^2}{1-(0.1020)^2}} = 4.020\,\text{m}}}$$

$$\underline{\underline{\phi = \tan^{-1}\left\{\frac{\dfrac{4}{(1.225)(2)}+0.1020}{\sqrt{1-(0.1020)^2}}\right\} = 1.050\,\text{rad}}}$$

$$\underline{\underline{x(t) = 4.020e^{-0.1250t}\cos(1.219t-1.050)}}\ \text{meters}$$

(b) $c = 5\,\text{N}\cdot\text{sec/m},\quad \omega = 1.225\,\text{rad/sec}$

$$\zeta = \frac{c}{2\omega m} = \frac{5}{2(1.225)(4)} = 0.5100$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 1.225\sqrt{1-(0.5100)^2} = 1.054\,\text{rad/sec}$$
$$\zeta\omega = (0.5100)(1.225) = 0.6248\,\text{rad/sec}$$

$$A = 2\sqrt{1+\frac{\left[\left\{4/(1.225)(2)\right\}+0.51\right]^2}{1-(0.51)^2}} = 5.368\,\text{m}$$

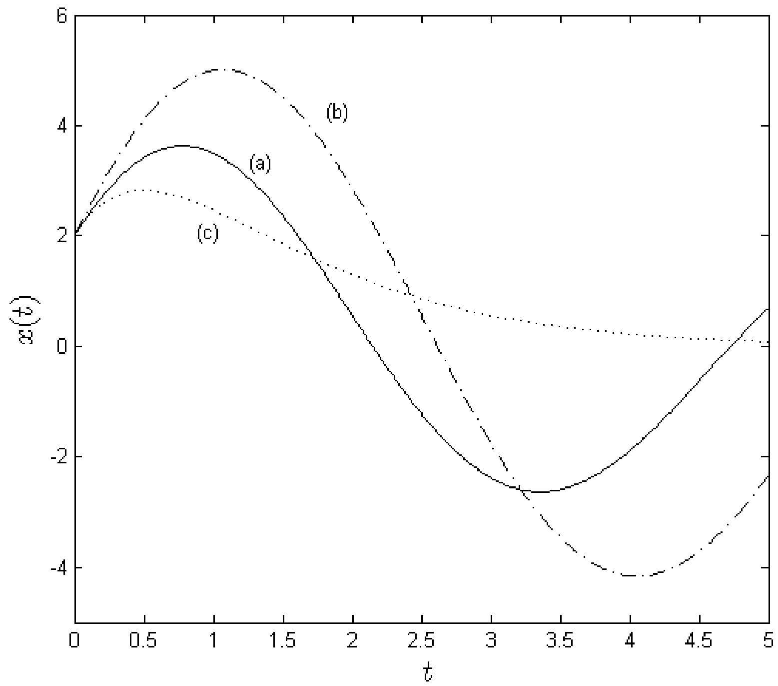
$$\phi = \tan^{-1}\left\{\frac{\dfrac{4}{(1.225)(2)}+0.51}{\sqrt{1-(0.51)^2}}\right\} = 1.189\,\text{rad}$$

$$x(t) = 5.368e^{-0.6248t}\cos(1.054t - 1.189) \text{ meters}$$

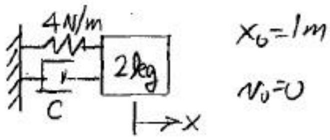
(c) $c = 10\,\text{N}\cdot\text{sec/m}, \quad \omega = 1.225\,\text{rad/sec}$

$$\zeta = \frac{c}{2\omega m} = \frac{10}{2(1.225)(4)} = 1 \quad \text{(critical damping)}$$

$$x(t) = \left[2+\left\{4+(1.225)(2)\right\}t\right]e^{-1.225t} = \left[2+6.45t\right]e^{-1.225t} \text{ meters}$$



2.26) (a) $c = 2\,\text{N}\cdot\text{sec/m}, \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{2}} = \sqrt{2} = 1.414\,\text{rad/sec}$



$$\zeta = \frac{c}{2\omega m} = \frac{2}{2\left(\sqrt{2}\right)(2)} = 0.3536$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 1.323\,\text{rad/sec}$$
$$\zeta\omega = 0.500\,\text{rad/sec}$$

$$A = x_0\sqrt{1+\frac{\zeta^2}{1-\zeta^2}} = \frac{x_0}{\sqrt{1-\zeta^2}} = \frac{1}{\sqrt{1-(0.3536)^2}} = 1.069\,\text{m}$$

$$\phi = \tan^{-1}\left\{\frac{\zeta}{\sqrt{1-\zeta^2}}\right\} = \tan^{-1}\left\{\frac{0.3536}{\sqrt{1-(0.3536)^2}}\right\} = 0.3614\,\text{rad}$$

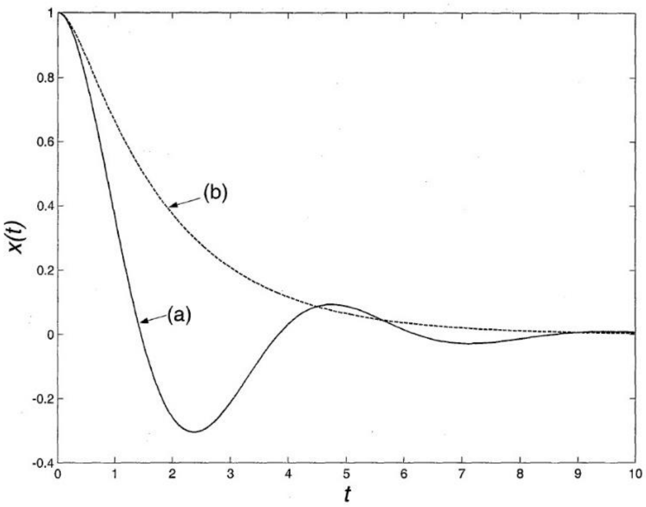
$$x(t) = 1.069e^{-0.5t}\cos\left(1.323t - 0.3614\right)\ \text{meters}$$

(b) $c = 8\,\text{N}\cdot\text{sec/m}, \quad \omega = \sqrt{2} = 1.414\,\text{rad/sec}$

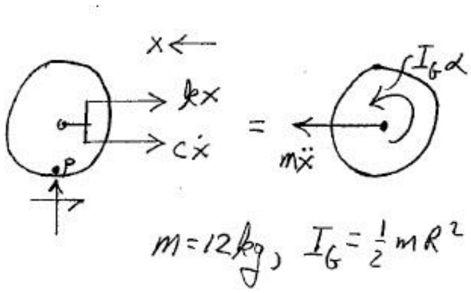
$$\zeta = \frac{c}{2\omega m} = \frac{8}{2\left(\sqrt{2}\right)(2)} = 1.414 > 1\,\text{(overdamped system)}$$

$$z = \sqrt{\zeta^2 - 1} = \sqrt{\left(\sqrt{2}\right)^2 - 1} = 1$$
$$\zeta\omega = 2\,\text{rad/sec}$$

$$x(t) = e^{-2t}\left[\cosh\sqrt{2}t + \sqrt{2}\sinh\sqrt{2}t\right]\ \text{meters}$$



2.27) Kinetics:



$$\sum \vec{M}_P = I_G\vec{\alpha} + \vec{r}_{PG} \times m\vec{a}_{PG}$$

$$-\left(kx + c\dot{x}\right)R = I_G\alpha + m\ddot{x}R \qquad (1)$$

Kinematics for no slip:

$$x = R\theta,\quad \dot{x} = R\dot{\theta},\quad \ddot{x} = R\ddot{\theta} \Rightarrow \quad \alpha = \ddot{\theta} = \frac{1}{R}\ddot{x} \qquad (2)$$

(2) into (1)

$$\begin{aligned} -\left(kx + c\dot{x}\right)R &= \frac{1}{R}\left(I_G + mR^2\right)\ddot{x} \\ &= \frac{1}{R}\left(\tfrac{1}{2}mR^2 + mR^2\right)\ddot{x} \\ &= \tfrac{3}{2}mR\ddot{x} \end{aligned}$$

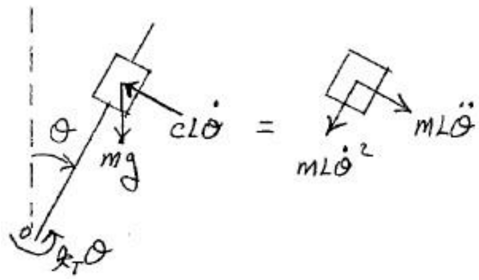
Thus, $\ddot{x} + \frac{2}{3}\frac{c}{m}\dot{x} + \frac{2}{3}\frac{k}{m}x = 0$ or $\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = 0$

where $\omega = \sqrt{\frac{2}{3}\frac{k}{m}} = \sqrt{\frac{2}{3}\left(\frac{8}{12}\right)} = 0.7454\,\text{rad/sec}$

$$2\omega\zeta = \frac{2}{3}\frac{c}{m} \Rightarrow \quad \zeta = \frac{c}{3\omega m} = \frac{8}{3\left(0.7454\right)\left(12\right)} = 0.2981$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 0.7454\sqrt{1-\left(0.2981\right)^2} = 0.7115\,\text{rad/sec}$$

2.28) $\sum \vec{M}_0 = I_0^{(rod)}\vec{\alpha} + \vec{r}_{cyl} \times m\vec{a}_{cyl}$



$$L \cdot cL\dot{\theta} - Lmg\sin\theta + k_T\theta = -L \cdot mL\ddot{\theta}$$

$$mL^2\ddot{\theta} + cL^2\dot{\theta} + k_T\theta - Lmg\sin\theta = 0 \text{ where } \sin\theta \approx \theta \ll 1$$

Thus, $\ddot{\theta} + \frac{c}{m}\dot{\theta} + \left[\frac{k_T}{mL^2} - \frac{g}{L}\right]\theta = 0$

$$\omega^2 = \frac{k_T}{mL^2} - \frac{g}{L}$$

$$2\omega\zeta = \frac{c}{m} \Rightarrow \quad \zeta = \frac{c}{2\omega m} = \frac{c}{2m\sqrt{\frac{k_T}{mL^2} - \frac{g}{L}}}$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = \sqrt{\frac{k_T}{mL^2} - \frac{g}{L} - \frac{c^2}{4m^2}}$$

2.29) (a) $T = 2.6\,\text{sec},\;\; T_d = 3\,\text{sec}$

$$\omega_d = \omega\sqrt{1-\zeta^2} \Rightarrow \quad \zeta^2 = 1-\left(\frac{\omega_d}{\omega}\right)^2 = 1-\left(\frac{2\pi/T_d}{2\pi/T}\right)^2 = 1-\left(\frac{T}{T_d}\right)^2$$

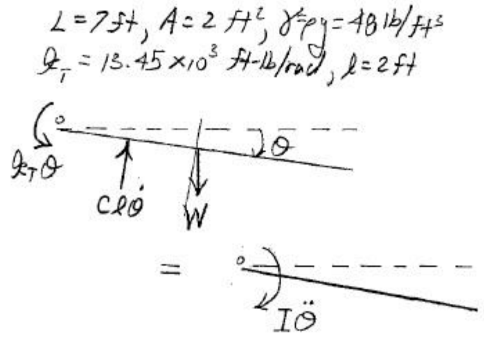
$$\zeta = \sqrt{1-\left(\frac{T}{T_d}\right)^2} = \sqrt{1-\left(\frac{2.6}{3}\right)^2} = 0.50$$

(b) $x_1 = 1\,\text{in},\;\; x_4 = 0.1\,\text{in},\;\; n = 3$

From Eq. (2.89), $\bar{\delta} = \frac{1}{n}\ln\frac{x_1}{x_{n+1}} = \frac{1}{3}\ln\frac{1}{0.1} = 0.7675$

$$\zeta = \frac{0.7675}{\sqrt{4\pi^2 + (0.7675)^2}} = 0.1213$$

2.30) $\sum M_0 = I\alpha: \quad k_T\theta + cl^2\dot{\theta} - \frac{L}{2}W\cos\theta = -I\ddot{\theta}$



$\cos\theta \approx 1$ for $\theta \ll 1$

$$\ddot{\theta} + \frac{cl^2}{I}\dot{\theta} + \frac{k_T}{I}\theta \approx \frac{WL}{2I}$$

or

$$\ddot{\beta} + \frac{cl^2}{I}\dot{\beta} + \frac{k_T}{I}\beta = 0 \text{ where } \beta(t) = \theta(t) - \theta_{static} = \theta(t) - \frac{WL}{2k_T}$$

$$I = \tfrac{1}{12}mh^2 + \tfrac{1}{3}mL^2 = \tfrac{1}{3}mL^2\left[\tfrac{1}{4}\left(\tfrac{h}{L}\right)^2 + 1\right] \approx \tfrac{1}{3}mL^2$$

$$I = \tfrac{1}{3}\tfrac{\gamma}{g}AL^3 = \tfrac{1}{3}\tfrac{(48)(2)(7)^3}{32.2} = 340.9\,\text{slug}\cdot\text{ft}^2$$

$$W = \gamma AL = (48)(2)(7) = 672\,\text{lb}$$

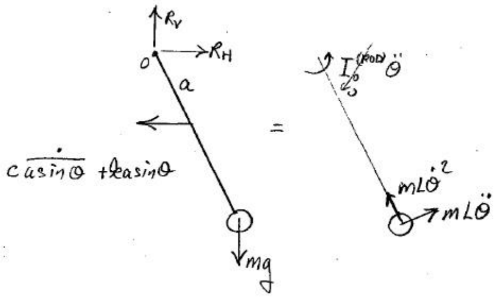
$$\omega = \sqrt{\frac{k_T}{I}} = = \sqrt{\frac{13.45\times10^3}{340.9}} = 39.45\,\text{rad/sec}$$

$$\bar{\delta} = \frac{1}{5}\ln\frac{1}{0.02} = 0.7824$$

$$\zeta = \frac{\bar{\delta}}{\sqrt{4\pi^2 + \bar{\delta}^2}} = \frac{0.7824}{\sqrt{4\pi^2 + (0.7824)^2}} = 0.1236$$

$2\omega\zeta = cl^2/I \Rightarrow \qquad c = \dfrac{2\omega\zeta I}{l^2} = \dfrac{2(39.45)(0.1236)(340.9)}{(2)^2} = 831.1\,\text{lb}\cdot\text{sec/ft}$

2.31) $\sum \vec{M}_0 = I_0^{(rod)}\vec{\alpha} + \vec{r}_{bob} \times m\vec{a}_{bob}$



$$-a\cos\theta\left[c\,\overline{a\sin\dot{\theta}} + ka\sin\theta\right] - L\cdot mg\sin\theta = L\cdot mL\ddot{\theta}$$

$$mL^2\ddot{\theta} + ca^2\cos^2\theta\dot{\theta} + ka^2\sin\theta\cos\theta + mgL\sin\theta = 0$$

$$\cos^2\theta = \left[1 - \frac{\theta^2}{2!} + \ldots\right]^2 \approx 1$$

$$\sin\theta\cos\theta = \left[\theta - \frac{\theta^3}{3!} + \ldots\right]\cdot\left[1 - \frac{\theta^2}{2!} + \ldots\right] \approx \theta \ll 1$$

$$mL^2\ddot{\theta} + ca^2\dot{\theta} + \left[ka^2 + mgL\right]\theta \approx 0$$

$$\ddot{\theta} + \frac{ca^2}{mL^2}\dot{\theta} + \left[\frac{ka^2}{mL^2} + \frac{g}{L}\right]\theta \approx 0$$

$$\omega = \sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}$$

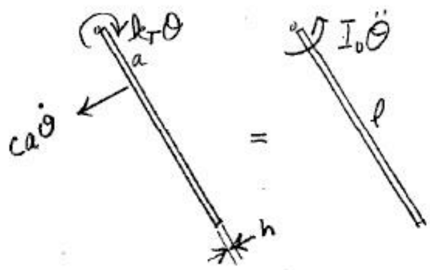
$$2\omega\zeta = \frac{ca^2}{mL^2} \Rightarrow \quad c = 2\zeta m\left(\frac{L}{a}\right)^2\sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}$$

$$c_{cr} = c\big|_{\zeta=1} = 2m\left(\frac{L}{a}\right)^2\sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}$$

2.32) $\zeta = 1$

$$c_{cr} = \frac{2\omega\zeta I}{l^2} = \frac{2(39.45)(340.9)}{(2)^2} = 6724\,\text{lb}\cdot\text{sec/ft}$$

2.33) $\sum M_0 = I\alpha: \quad -ca^2\dot{\theta} - k_T\theta = I\ddot{\theta}$



$$\ddot{\theta} + \frac{ca^2}{I}\dot{\theta} + \frac{k_T}{I}\theta = 0$$

$$I = \tfrac{1}{12}mh^2 + \tfrac{1}{3}ml^2 = \tfrac{1}{3}ml^2\left[\tfrac{1}{4}\left(\tfrac{h}{l}\right)^2 + 1\right] \approx \tfrac{1}{3}ml^2$$

$$\omega = \sqrt{k_T/I}$$

$$2\omega\zeta = ca^2/I \Rightarrow \quad c = 2\omega\zeta I/a^2$$

$$c_{cr} = c\big|_{\zeta=1} = \frac{2\omega I}{a^2} = \frac{2l}{a^2}\sqrt{\frac{k_T m}{3}}$$

2.34) $\frac{-4}{1.225(2)} < -1$ ✓

Eq. (2.100): $t_a = \frac{1}{\left\| -4/2 \right\| - 1.225} = 1.290\,\text{sec}$

Eq. (2.101): $t_{os} = t_a + \frac{1}{\omega} = 1.290 + \frac{1}{1.225} = 2.106\,\text{sec}$

Eq. (2.102):

$$\underline{\underline{x_{os} = -\frac{x_0}{\omega t_a} e^{-(\omega t_a + 1)} = -\frac{2}{(1.225)(1.290)} e^{-((1.225)(1.290)+1)} = -0.09588\,\text{m}}}$$

2.35) $\frac{-2}{1.414(1)} < -1$ ✓

Eq. (2.100): $t_a = \frac{1}{\left\| -2/1 \right\| - 1.414} = 1.706\,\text{sec}$

Eq. (2.101): $t_{os} = t_a + \frac{1}{\omega} = 1.706 + \frac{1}{1.414} = 2.413\,\text{sec}$

Eq. (2.102):

$$\underline{\underline{x_{os} = -\frac{x_0}{\omega t_a} e^{-(\omega t_a + 1)} = -\frac{1}{(1.414)(1.706)} e^{-((1.414)(1.706)+1)} = -0.01367\,\text{m}}}$$

2.36) $m = 4\,\text{kg},\ k = 6\,\text{N/m},\ \mu_s = \mu_k = 0.1,\ x_0 = 2\,\text{m},\ v_0 = 4\,\text{m/sec}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{4}} = 1.225\,\text{rad/sec}$, $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{1.225} = 5.130\,\text{sec}$

$f_\mu = \mu_k \frac{mg}{k} = (0.1)\frac{(4)(9.81)}{6} = 0.6540\,\text{m}$

$n_{cr} = \frac{1}{2}\left[\frac{2}{0.6540} - 1\right] = 1.029 \Rightarrow \quad n_{stick} = 2$

$$\underline{\underline{t_{stick} = t_2 = \frac{(2)(5.13)}{2} = 5.130\,\text{sec}}}$$

$$\underline{\underline{\left\| x_{stick} \right\| = x_2 = \left\| 2 - 2(2)(0.6540) \right\| = 0.616\,\text{m}}}$$

2.37) $m = 2\,\text{kg},\ k = 4\,\text{N/m},\ \mu_s = 0.12,\ \mu_k = 0.1$

$$\bar{\mu} = \frac{\mu_s}{\mu_k} = \frac{0.12}{0.10} = 1.2$$

Eq. (2.116): $$\Delta = 4f_\mu = 4\mu_k \frac{mg}{k} = 4(0.10)\frac{(2)(9.81)}{4} = 1.962\,\text{m}$$

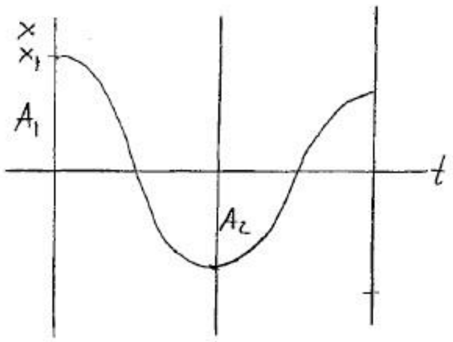
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{2}} = \sqrt{2} = 1.414\,\text{rad/sec}$$

2.38) Eq. (1.92): $W^{(NC)} = \Delta T + \Delta U$

$$\int_{x_1}^{x_2} F^{(NC)}\mathrm{d}x = \left[\tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}\right] + \left[\tfrac{1}{2}kx_2^{\,2} - \tfrac{1}{2}kx_1^{\,2}\right]$$

over ½ cycle: $-\int \mu_k mg\,\mathrm{d}x = [0-0] + \tfrac{1}{2}k\left[x_2^{\,2} - x_1^{\,2}\right]$

$$-\mu_k mg(x_2 - x_1) = \tfrac{1}{2}k(x_2 + x_1)(x_2 - x_1)$$



$$\begin{aligned} -\mu_k mg &= \tfrac{1}{2}k(x_2 + x_1) \\ &= \tfrac{1}{2}k(-A_c + A_1) \\ &= -\tfrac{1}{2}k(A_1 - A_c) \\ &= -\tfrac{1}{2}k\frac{\Delta}{2} \end{aligned}$$

$$\Delta = 4\mu_k \frac{mg}{k}$$
