SOLUTIONS MANUAL FOR

Introduction to Compressible Fluid Flow

SECOND EDITION

by

Patrick H. Oosthuizen
William E. Carscallen

CRC Press
Taylor & Francis Group

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PREFACE

This solution manual contains complete solutions to all of the problems in the textbook “Introduction to Compressible Fluid Flow”, second edition. The problems have been solved using the same basic methodology as that adopted in the worked examples contained in the textbook. Each solution in the manual begins with the statement of the problem and each solution begins on a separate page. This arrangement will, we hope, prove to be convenient to those using this Solution Manual. The solutions are grouped by chapter and the solutions for each chapter are preceded by summaries of the major equations developed in the corresponding chapter in the textbook. Instructors may wish to provide copies of these equation summaries to their students.

The problems can all be solved using the equations and tables given in the textbook. However, the majority of the solutions in this manual have been obtained with the aid of the software **COMPROP**. The use of this software is described in an appendix in the textbook. The software is available free of charge through the publisher to adopters of the textbook.

The authors would like to express their sincere appreciation to Jane Paul for all of her help in preparing the Solution Manual.

Patrick H. Oosthuizen

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Chapter One

INTRODUCTION

SUMMARY OF MAJOR EQUATIONS

Perfect Gas Relations

$$\frac{p}{\rho} = RT = \left(\frac{R}{m}\right)T \tag{1.1}$$

$$\gamma = \frac{c_p}{c_v} \tag{1.2}$$

$$R = c_p - c_v \tag{1.3}$$

Conservation Equations for Steady Flow

Conservation of Mass:

(Rate mass enters control volume) = (Rate mass leaves control volume)

Conservation of Momentum:

(Net Force on Gas In Control Volume In Direction Considered) =
(Rate Momentum leaves Control Volume in Direction Considered)
− (Rate Momentum enters Control Volume in Direction Considered)

Conservation of Energy:

(Rate sum of Enthalpy and Kinetic Energy leave control volume)
- (Rate sum of Enthalpy and Kinetic Energy enter control volume) =
(Rate Heat is transferred into control volume) -
(RateWork is done by fluid in control volume)

PROBLEM 1.1

An air stream enters a variable area channel at a velocity of 30 m/s with a pressure of 120 kPa and a temperature of 10^o C. At a certain point in the channel, the velocity is found to be 250 m/s. Using Bernoulli's equation (i.e. $p + \rho\ V^2$ = constant), which assumes incompressible flow, find the pressure at this paint. In this calculation use the density evaluated at the inlet conditions. If the temperature of the air is assumed to remain constant, evaluate the air density at the point in the flow where the velocity is 250 m/s. Compare this density with the density at the inlet to the channel. On the basis of this comparison, do you think that the use of Bernoulli's equation is justified?

SOLUTION

Bernoulli's equation gives:

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

This can be rearranged to give:

$$p_2 = \rho \left[\frac{V_1^2}{2} - \frac{V_2^2}{2} \right] + p_1 \qquad \text{(a)}$$

The density, ρ , is evaluated using the initial conditions i.e. by using p, $p_1 / \rho = RT_1$, which, since air flow is being considered, gives:

$$\rho = \frac{p_1}{RT_1} = \frac{120 \times 10^3}{287 \times 283} = 1.478 \text{ kg/m}^3$$

Substituting this back into eq. (a) then gives:

$$p_2 = 1.478 \times \left[\frac{30^2}{2} - \frac{250^2}{2} \right] + 120 \times 10^3 = 7.448 \times 10^4 \text{ Pa} = 74.48 \text{ kPa}$$

Therefore the pressure at the point considered is 74.48 kPa.

Assuming that the changes in temperature can be neglected, the equation of state gives at the exit:

$$\rho_2 = \frac{p_2}{RT_2} = \frac{74.48 \times 10^3}{287 \times 283} = 0.917 \text{ kg/m}^3$$

Since this indicates that the density changes by about 38%, the incompressible flow assumption is not justified.

PROBLEM 1.2

The gravitational acceleration on a large planet is 90 ft/sec^2. What is the gravitational force acting on a spacecraft with a mass of 8000 lbm on this planet?

SOLUTION

Because:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

it follows that:

$$\text{Gravitational Force} = 8000 \times 90 \text{ lbm-ft/sec}^2$$

But 32.2 lbm-ft /sec^2 = 1 lbf, so this equation gives:

$$\text{Gravitational Force} = \frac{800 \times 90}{32.2} = 22{,}360 \text{ lbf}$$

Therefore the gravitational force acting on the craft is 22,360 lbf

PROBLEM 1.3

The pressure and temperature at a certain point in an air flow are 130 kPa and 30° C respectively. Find the air density at this point in kg/m^3 and lbm/ft.

SOLUTION

The density, ρ , is evaluated using the perfect gas law, i.e. by using $p/\rho = RT$. Using SI units this gives since air-flow is being considered:

$$\rho = \frac{130 \times 10^3}{287 \times 303} = 1.495 \text{ kg/m}^3$$

But 1 lbm = 0.4536 kg and 1 ft = 0.3048 m so:

$$1.495 \text{ kg/m}^3 = \frac{1.495/0.4536}{(1/0.3058)^3} = 0.0943 \text{ lbm/ft}^3$$

Therefore the density is 1.459 kg/m^3 or 0.0943 lbm/ft^3

PROBLEM 1.4

Two kilograms of air at an initial temperature and pressure of 30° C and 100 kPa undergoes an isentropic process, the final temperature attained being 850° C. Find the final pressure, the initial and final densities and the initial and final volumes.

SOLUTION

The isentropic relations give:

$$p_2 = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \times \ p_1 \ = \left(\frac{1123}{303}\right)^{1.4/(1.4-1)} \times 100 \ = \ 9801 \text{ kPa}_{\ 1}$$

The initial density is given by the perfect gas law as:

$$\rho_1 \ = \ \frac{100 \times 10^3}{287 \times 303} \ = \ 1.15 \text{ kg/m}^3$$

The final density is given in the same way by:

$$\rho_2 \ = \ \frac{9801 \times 10^3}{287 \times 1123} \ = \ 30.41 \text{ kg/m}^3$$

Also, using:

$$\text{Volume} \ = \ \frac{\text{Mass}}{\text{Density}}$$

It follows that the initial and final volumes are given by:

$$V_1 \ = \ \frac{2}{1.150} \ = \ 1.739 \text{ m}^3$$

and:

$$V_2 \ = \ \frac{2}{30.41} \ = \ 0.0658 \text{ m}^3$$

Therefore the initial and final volumes are 1.739 m^3 and 0.0658 m^3.

PROBLEM 1.5

Two jets of air, each having the same mass flow rate, are thoroughly mixed and then discharged into a large chamber. One jet has a temperature of 120° C and a velocity of 100 m/s while the other has a temperature of 50° C and a velocity of 300 m/s. Assuming that the process is steady and adiabatic, find the temperature of the air in the large chamber.

SOLUTION

Using:

$$\text{Rate Mass Enters Control Volume} = \text{Rate Mass Leaves Control Volume}$$

and because the flow is adiabatic:

$$\begin{bmatrix}\text{Rate Enthalpy plus}\\ \text{Kinetic Energy Leave}\\ \text{Control Volume}\end{bmatrix} - \begin{bmatrix}\text{Rate Enthalpy plus}\\ \text{Kinetic Energy enter}\\ \text{Control Volume}\end{bmatrix} = 0$$

If the subscripts 1 and 2 are used to refer to conditions in the two jets and if the subscript 3 is used to refer to conditions in the chamber then, if m refers to the mass flow rate in each jet, the first of the above equations gives:

$$m_3 = m_1 + m_2 = 2m_i$$

where m_i is the mass flow rate in each of the jets, i.e., $m_i = m_1 = m_2$.

The second of the above equations then gives if the velocity in the chamber is assumed to be small because the chamber is large:

$$m_3 h_3 = m_1\left(h_1 + \frac{V_1^2}{2}\right) + m_2\left(h_2 + \frac{V_2^2}{2}\right)$$

Because air flow is involved it can be assumed that $h = c_p T$. Therefore the above equations together give f it is assumed that c_p is constant:

$$2m_i c_p T_3 = m_i\left(c_p \times 393 + \frac{100^2}{2}\right) + m_i\left(c_p \times 223 + \frac{300^2}{2}\right)$$

Dividing through by $m_i\ c_p$ then gives:

$$2T_3 = \left(393 + \frac{100^2}{2c_p}\right) + \left(223 + \frac{300^2}{2c_p}\right)$$

i.e. since air flow is involved:

$$T_3 = \frac{1}{2}\left[\left(393 + \frac{100^2}{2 \text{ x } 1007}\right) + \left(223 + \frac{300^2}{2 \text{ x } 1007}\right)\right] = 332.8\text{K}$$

Therefore the air temperature in the chamber is 332.8 K (= 59.8° C).

PROBLEM 1.6

Two air streams are mixed in a chamber. One stream enters the chamber through a 5 cm diameter pipe at a velocity of 100 m/s with a pressure of 150 kPa and a temperature of 30^o C. The other stream enters the chamber through a 1.5 cm diameter pipe at a velocity of 150 m/s with a pressure of 75 kPa and a temperature of 30^o C. The air leaves the chamber through a 9 cm diameter pipe at a pressure of 90 kPa and a temperature of 30^o C. Assuming that the flow is steady find the velocity in the exit pipe.

SOLUTION

The flow situation being considered is shown in Fig. P1.6.

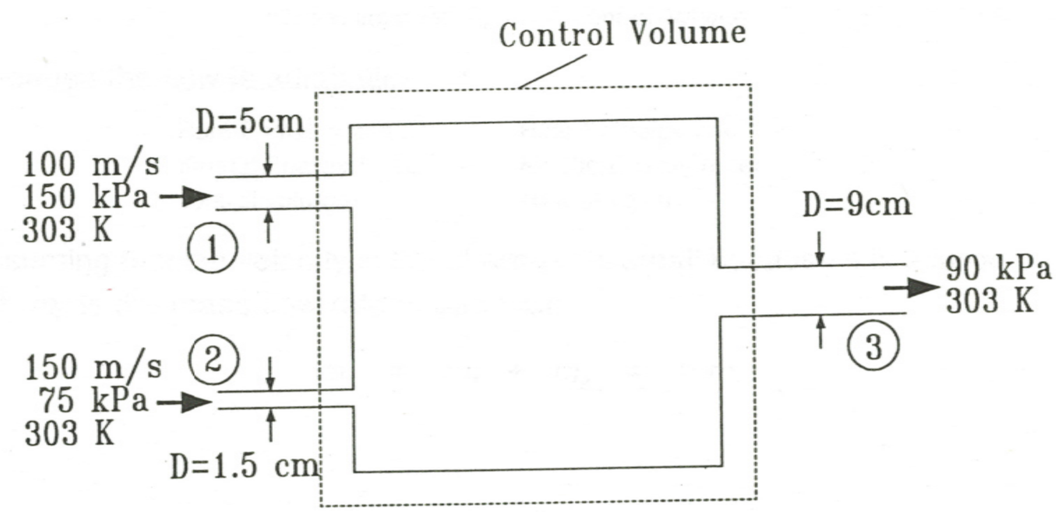

Figure P1.6

The densities in the three pipes are evaluated using the perfect gas law i.e. using $p/\rho = RT$. This gives:

$$\rho_1 = \frac{150 \times 10^3}{287 \times 303} = 1.725 \text{ kg/m}^3$$

$$\rho_2 = \frac{75 \times 10^3}{287 \times 303} = 0.863 \text{ kg/m}^3$$

$$\rho_3 = \frac{90 \times 10^3}{287 \times 303} = 1.035 \text{ kg/m}^3$$

Conservation of mass gives since the flow is steady:

$$m_1 + m_2 = m_3$$

i.e.:

$$\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = \rho_3 V_3 A_3$$

Hence:

$$1.725 \times 100 \times \frac{\pi}{4} \times 0.05^2 + 0.863 \times 150 \times \frac{\pi}{4} \times 0.015^2 = 1.035 \times V_3 \text{ x } \frac{\pi}{4} \times 0.09^2$$

which gives:

$$V_3 = 54.9 \text{m/s}$$

Therefore the velocity in the exit pipe is 54.9 m/s.

PROBLEM 1.7

The jet engine fitted to a small aircraft uses 35 kg/s of air when the aircraft is flying at a speed of 800 km/h. The jet efflux velocity is 590 m/s. If the pressure on the engine discharge plane is assumed to be equal to the ambient pressure and if effects of the mass of the fuel used are ignored, find the thrust developed by the engine.

SOLUTION

The flow relative to the aircraft is considered. The momentum equation applied to a control volume surrounding the aircraft then gives in the direction of flight:

$$\begin{array}{c}\text{Net Force on Gas in}\\ \text{Control Volume}\end{array} = \begin{array}{c}\text{Rate Momentum Leaves}\\ \text{Control Volume}\end{array} + \begin{array}{c}\text{Rate Momentum Enters}\\ \text{Control Volume}\end{array}$$

But the pressure is equal to ambient everywhere on the surface of the control volume and only the air that passes through the engine undergoes a velocity change, i.e., a momentum change. Hence if F is the force exerted on the fluid by the system (this will be equal in magnitude but opposite in direction to the force on the aircraft, i.e., to the thrust), it follows that:

$$T = \dot{m}\left(V_{\text{exit}} - V_{\text{inlet}}\right)$$

This gives because V_{inlet} = 800 km/hr = 222.2 m/s:

$$T = 35 \times (\ 590 \ - \ 222.2\ ) = 12873 \text{ N}$$

Therefore, the thrust developed by the engine is 12.87 kN.

PROBLEM 1.8

The engine of a small jet aircraft develops a thrust of 18 kN when the aircraft is flying at a speed of 900 km/h at an altitude where the ambient pressure is 50 kPa. The air flow rate through the engine is 75 kg/s and the engine uses fuel at a rate of 3 kg/s. The pressure on the engine discharge plane is 55 kPa and the area of the engine exit is 0.2 m^2 . Find the jet efflux velocity.

SOLUTION

The flow relative to the aircraft is considered and the momentum equation is applied a control volume surrounding the aircraft. This gives in the direction of flight:

$$\text{Net Force on Gas in Control Volume} = \text{Rate Momentum Leaves Control Volume} + \text{Rate Momentum Enters Control Volume}$$

But the pressure is equal to ambient everywhere on the surface of the control volume except on the nozzle exit plane. Hence if T is the force exerted on the flow by the system (this will be equal in magnitude but opposite in direction to the force on the aircraft, i.e., to the thrust). it follows that:

$$T - \left(p_{\text{exit}} - p_{\text{ambient}}\right)A_{\text{exit}} = \dot{m}V_{\text{exit}} - \dot{m}V_{\text{inlet}}$$

This gives because V_{inlet} = 900 km/hr = 250 m/s:

$$18000 - (55000 - 50000) \times 0.2 = (75 + 3)V_{\text{exit}} - 75 \text{ x } 250$$

Hence:

$$V_{\text{exit}} = 458.3 \text{ m/s}$$

Therefore the jet efflux velocity is 458.3 m/s.

PROBLEM 1.9

A small turbo-jet engine uses 50 kg/s of air and the air/fuel ratio is 90:1. The jet efflux velocity is 600 m/s. When the afterburner is used, the overall air/fuel ratio decreases to 50:1 and the jet efflux velocity increases to 730 m/s. Find the static thrust with and without the afterburner. The pressure on the engine discharge plane can be assumed to be equal to the ambient pressure in both cases.

SOLUTION

The momentum equation is applied to the control volume surrounding the engine and gives:

$$\begin{matrix}\text{Net Force on Gas in} \\ \text{Control Volume}\end{matrix} = \begin{matrix}\text{Rate Momentum Leaves} \\ \text{Control Volume}\end{matrix} + \begin{matrix}\text{Rate Momentum Enters} \\ \text{Control Volume}\end{matrix}$$

But the pressure is equal to ambient everywhere on the surface of the control volume and the static thrust, i.e., the thrust developed when the engine is at rest, is being considered. Hence if T is the force exerted on the flow by the system (this will be equal in magnitude but in the opposite direction to the force on the engine, i.e., to the thrust), it follows that:

$$T = \dot{m} V_{\text{exit}}$$

First consider the thrust without afterburning. In this case:

$$T = \left(\dot{m}_{\text{air}} + \dot{m}_{\text{fuel}}\right) V_{\text{exit}} = (50 + 50/90) \times 600 = 30333 \text{ N} = 30.333 \text{ kN}$$

Next consider the thrust with afterburning. In this case:

$$T = \left(\dot{m}_{\text{air}} + \dot{m}_{\text{fuel}}\right) V_{\text{exit}} = (50 + 50/50) \times 730 = 37230 \text{ N} = 37.23 \text{ kN}$$

Therefore the thrust without afterburning is 30.33 kN and the thrust with afterburning is 37.23 kN.

PROBLEM 1.10

A rocket used to study the atmosphere has a fuel consumption rate of 120 kg/s and a nozzle discharge velocity of 2300 m/s. The pressure on the nozzle discharge plane is 90 kPa. Find the thrust developed when the rocket is launched at sea-level. The nozzle exit plane diameter is 0.3 m.

SOLUTION

Applying the momentum equation to a control volume surrounding the engine gives:

$$T - \left(p_{\text{exit}} - p_{\text{ambient}}\right) A_{\text{exit}} = \dot{m} V_{\text{exit}}$$

Therefore:

$$T - \left(90000 - 101300\right) \times \frac{\pi}{4} \times 0.3^2 = 120 \times 2300$$

From this it follows that:

$$T = 275201 \text{ N} = 275.201 \text{ kN}$$

Therefore the thrust developed is 275.2 kN.

PROBLEM 1.11

A solid fuelled rocket is fitted with a convergent-divergent nozzle with an exit plane diameter of 30 cm. The pressure and velocity on this nozzle exit plane are 75 kPa and 750 m/s respectively and the mass flow rate through the nozzle is 350 kg/s. Find the thrust developed by this engine when the ambient pressure is (a) 100 kPa and (b) 20 kPa.

SOLUTION

Applying the momentum equation to a control volume surrounding the engine gives:

$$T - \left(p_{\text{exit}} - p_{\text{ambient}}\right) A_{\text{exit}} = \dot{m} V_{\text{exit}}$$

This gives:

$$T - \left(75000 - p_{\text{ambient}}\right) \times \frac{\pi}{4} \text{ x } 0.3^2 = 350 \times 750$$

From which it follows that:

$$T = \left(283706 - 0.0707\, p_{\text{ambient}}\right) \text{ N}$$

Hence, if p_{ambient} is 100 kPa:

$$T = \left(283706 - 7070\right) = 276636 \text{ N} = 276.6 \text{ kN}$$

Similarly, if p_{ambient} is 20 kPa:

$$T = \left(283706 - 1414\right) = 282292 \text{ N} = 282.29 \text{ kN}$$

Therefore the thrusts developed when the ambient pressure is 100 kPa and when it is 20 kPa are 276.6 kN and 282.29 kN respectively.

PROBLEM 1.12

In a hydrogen powered rocket, hydrogen enters a nozzle at a very low velocity with a temperature and pressure of 2000° C and 6.8 MPa respectively. The pressure on the exit plane of the nozzle is equal to the ambient pressure which is 10 kPa. If the required thrust is 10 MN, what hydrogen mass flow rate is required? The flow through the nozzle can be assumed to be isentropic and the specific heat ratio of the hydrogen can be assumed to be 1.4.

SOLUTION

If the subscripts 1 and 2 are used to denote conditions on the inlet and exit planes of the nozzle respectively, then the isentropic relations give:

$$T_2 = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \times T_2 = \left(\frac{10000}{6800000}\right)^{0.2857} \times 2273 = 352.7 \text{ K}$$

Because the flow is adiabatic there is no heat transfer to the hydrogen in the nozzle so the energy equation gives:

$$\begin{matrix}\text{Rate Enthalpy plus} \\ \text{Kinetic Energy Leave} \\ \text{Control Volume}\end{matrix} \quad - \quad \begin{matrix}\text{Rate Enthalpy plus} \\ \text{Kinetic Energy enter} \\ \text{Control Volume}\end{matrix} \; = \; 0$$

i.e.:

$$\left(c_p T_1 + \frac{V_1^2}{2}\right) - \left(c_p T_2 + \frac{V_2^2}{2}\right) = 0$$

i.e. since the kinetic energy at the inlet is negligible:

$$V_2^2 = 2c_p\left(T_1 - T_2\right)$$

But:

$$c_p - c_v = R$$

i.e.:

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times (8314/2)}{0.4} = 14550 \ \ \text{J/kg K}$$

Hence:

$$V_2^2 = 2 \times 14550 \times (2273 - 352.7)$$

i.e.:

$$V_2 = 7475 \ \text{m/s}$$

Because the pressure on the exit plane is ambient, the thrust is given by:

$$T = \dot{m} V_2$$

i.e.:

$$10000000 = \dot{m} \times 7475, \ \text{i.e.}, \ \ \dot{m} = 1338 \ kg/s$$

Therefore the required mass flow rate is 1338 kg/s.

PROBLEM 1.13

In a proposed jet propulsion system for an automobile, air is drawn in vertically through a large intake in the roof at a rate of 3 kg/s, the velocity through this intake being small. Ambient pressure and temperature are 100 kPa and 30° C respectively. This air is compressed and heated and then discharged horizontally out of a nozzle at the rear of the automobile at a velocity of 500 m/s and a pressure of 140 kPa. If the rate of heat addition to the air stream is 600 kW, find the nozzle discharge area and the thrust developed by the system.

SOLUTION

The energy equation applied to the system gives:

Rate Enthalpy plus Kinetic Energy Leave Control Volume	-	Rate Enthalpy plus Kinetic Energy enter Control Volume	=	Rate Heat is Transferred into Control Volume

But, because of the low inlet velocity, the kinetic energy at the inlet is negligible. The above equation therefore gives since air flow is being considered:

$$\left(c_p T_{exit} + \frac{V_{exit}^2}{2}\right) - \left(c_p T_{inlet} + 0\right) = \dot{q}$$

This equation gives assuming $c_p = 1007$ J/kg K:

$$\left(1007 \times T_{exit} + \frac{500^2}{2}\right) - \left(1007 \times 303\right) = 600000$$

which gives:

$$T_{exit} = 774.7 \text{ K}$$

The exit density is evaluated using the perfect gas law, i.e. by using $p/\rho = RT$. This gives since air flow is being considered:

$$\rho_{\text{exit}} = \frac{140 \times 10^3}{287 \times 774.7} = 0.63 \text{ kg/m}^3$$

Then using:

$$\dot{m} = \rho_{\text{exit}} V_{\text{exit}} A_{\text{exit}}$$

gives:

$$A_{\text{exit}} = \frac{3}{0.63 \times 500} = 0.00952 \text{ m}^2$$

Because no air enters the system in the direction of motion, the momentum equation gives:

$$T - \left(p_{\text{exit}} - p_{\text{ambient}}\right) A_{\text{exit}} = \dot{m} V_{\text{exit}}$$

which gives:

$$T - \left(140000 - p_{\text{ambient}}\right) \times 0.00952 = 3 \times 500$$

This equation then gives if the ambient pressure is assumed to be 101.3 kPa = 101300 Pa:

$$T = 1868.4 \text{ N}$$

Therefore the nozzle discharge area is 0.00952 m^2 and the thrust is 1868 N.

PROBLEM 1.14

Carbon dioxide flows through a constant area duct. At the inlet to the duct the velocity is 120 m/s and the temperature and pressure are 200° C and 700 kPa respectively. Heat is added to the flow in the duct and at the exit of the duct the velocity t. 240 m/s and the temperature is 450° C. Find the amount of heat being added to the carbon dioxide per unit mass of gas and the mass flow rate through the duct per unit cross-sectional area of the duct at inlet. Assume that for carbon dioxide γ= 1.3.

SOLUTION

The energy equation applied to the system gives:

Rate Enthalpy plus Kinetic Energy Leave Control Volume	-	Rate Enthalpy plus Kinetic Energy enter Control Volume	=	Rate Heat is Transferred into Control Volume

i.e., considering the changes per unit mass flow through the system:

$$\left(c_p T_{exit} + \frac{V_{exit}^2}{2}\right) - \left(c_p T_{inlet} + \frac{V_{inlet}^2}{2}\right) = \dot{q}$$

where $\dot{q}$ is here the rate of heat transfer to the gas per unit mass flow rate. Taking:

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.3 \times (8314/44)}{0.3} = 818.8 \quad \text{J/kg K}$$

then gives:

$$\dot{q} = 818.8 \text{ x } (450 - 200) + \left(\frac{240^2}{2} - \frac{120^2}{2}\right) = 226300 \text{ J/kg} = 226.3 \text{ kJ/kg}$$

Using:

$$\frac{\dot{m}}{A_{inlet}} = \rho_{inlet} V_{inlet} \quad \text{and} \quad \frac{p_{inlet}}{\rho_{inlet}} = R T_{inlet}$$

it follows that:

$$\frac{\dot{m}}{A_{\text{inlet}}} = \left(\frac{p_{\text{inlet}}}{RT_{\text{inlet}}}\right)V_{\text{inlet}} = \left[\frac{700000}{(8314/44)\times 473}\right]\times 120 = 939.9 \text{ kg/s m}^2$$

Therefore the amount of heat added per unit mass of gas is 226.3 kJ/kg and the mass flow rate per unit inlet cross-sectional area is 939.9 kg/s m^2.

PROBLEM 1.15

Air enters a heat exchanger with a velocity of 120 m/s and a temperature and pressure of 225^{o} C and 2.5 MPa respectively. Heat is removed from the air in the heat exchanger and the air leaves with a velocity 30 m/s at a temperature and pressure of 80^{o} C and 2.45 MPa. Find the heat removed per kg of air flowing through the heat exchanger and the density of the air at the inlet and the exit to the heat exchanger.

SOLUTION

The energy equation applied to the system gives:

Rate Enthalpy plus Kinetic Energy Leave Control Volume	-	Rate Enthalpy plus Kinetic Energy enter Control Volume	=	Rate Heat is Transferred into Control Volume

i.e. considering the changes per unit mass flow through the system:

$$\left(c_p T_{exit} + \frac{V_{exit}^2}{2}\right) - \left(c_p T_{inlet} + \frac{V_{inlet}^2}{2}\right) = \dot{q}$$

where $\dot{q}$ is here the rate of heat transfer to the gas per unit mass flow rate.

The specific heat, c_p , will be assumed constant and, because air flow is being considered, it value will be taken as c_p = 1007 J/kg K. Therefore:

$$\dot{q} = 1007 \times (80 - 220) + \left(\frac{30^2}{2} - \frac{120^2}{2}\right) = -134230\,\text{J/kg} = 134.2 \text{ kJ/kg}$$

using $p / \rho = RT$ it follows that:

$$\rho_{inlet} = \frac{2500 \times 10^3}{287 \times 498} = 17.49 \text{ kg/m}^3$$

$$\rho_{outlet} = \frac{2450 \times 10^3}{287 \times 353} = 24.18 \text{ kg/m}^3$$

Therefore the rate that heat is removed per kg of air is 134.2 kJ/kg and the density of air at the inlet and the exit is 17.49 kg/m^3 and 24.18 kg/m^3 respectively.

PROBLEM 1.16

The mass flow rate through the nozzle of a rocket engine is 200 kg/s. The areas of the nozzle inlet and exit planes are 0.7 m^2 and 2.4 m^2 , respectively. On the nozzle inlet plane, the pressure and velocity are 1600 kPa and 150 m/s respectively whereas on the nozzle exit plane the pressure and velocity are 80 kPa and 2300 m/s respectively. Find the thrust force acting on the nozzle.

SOLUTION

The momentum equation is applied to the control volume surrounding the nozzle and gives:

$$\begin{array}{c}\text{Net Force on Gas in}\\ \text{Control Volume}\end{array} = \begin{array}{c}\text{Rate Momentum Leaves}\\ \text{Control Volume}\end{array} + \begin{array}{c}\text{Rate Momentum Enters}\\ \text{Control Volume}\end{array}$$

It is assumed that the pressure is equal to ambient everywhere on the surface of the control volume except on the inlet and exit planes. Hence if *T* is the force exerted on the flow which will be equal in magnitude but in the opposite direction to the force on the nozzle, i.e., to the thrust, it follows that:

$$T - \left(p_{\text{exit}} A_{\text{exit}} - p_{\text{inlet}} A_{\text{inlet}} \right) = \dot{m} \left(V_{\text{exit}} - V_{\text{inlet}} \right)$$

Hence:

$$\begin{aligned} T &= \dot{m}\left(V_{\text{exit}} - V_{\text{inlet}}\right) + \left(p_{\text{exit}} A_{\text{exit}} - p_{\text{inlet}} A_{\text{inlet}}\right) \\ &= 200 \times (2300 - 150) + (80000 \times 2.4 - 1600000 \times 0.77) = -610000 \text{ N} = -610 \text{ kN} \end{aligned}$$

Therefore the thrust force on the nozzle is 610 kN this thrust force arising mainly as a result of the pressure difference across the nozzle.

Chapter Two

EQUATIONS FOR STEADY ONE-DIMENSIONAL COMPRESSIBLE FLUID FLOW

SUMMARY OF MAJOR EQUATIONS

Relation Between Fractional Changes in Flow Variables

Mass Conservation:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \tag{2.3}$$

Momentum:

$$-\frac{dp}{\rho} = V\,dV \tag{2.8}$$

Energy:

$$c_p\,dT + V\,dV = 0 \tag{2.17}$$

Equation of State:

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0 \tag{2.19}$$

Entropy:

$$\frac{ds}{c_p} = \frac{dT}{T} - \left(\frac{\gamma - 1}{\gamma}\right)\frac{dp}{p} \tag{2.24}$$

PROBLEM 2.1

Air enters a tank at a velocity of 100 m/s and leaves the tank at a velocity of 200 m/s. If the flow is adiabatic, find the difference between the temperature of the air at the exit and the temperature of the air at the inlet.

SOLUTION

Because the flow is adiabatic, the energy equation gives:

$$\left(c_p T_{exit} + \frac{V_{exit}^2}{2} \right) = \left(c_p T_{inlet} + \frac{V_{inlet}^2}{2} \right)$$

Hence:

$$T_{exit} - T_{inlet} = \left(\frac{1}{c_p} \right) \left(\frac{V_{inlet}^2}{2} - \frac{V_{exit}^2}{2} \right)$$

Since air flow is being considered the specific heat, c_p , will be assumed to be 1007 J/ kg ^{o}C. The above equation then gives:

$$T_{exit} - T_{inlet} = \left(\frac{1}{1007} \right) \left(\frac{100^2}{2} - \frac{200^2}{2} \right) = -14.9^{o}\text{C}$$

Therefore the temperature decreases by 14.9^{o}C.

PROBLEM 2.2

Air at a temperature of 25° C is flowing at a velocity of 500 m/s. A shock wave (see later chapters) occurs in the flow reducing the velocity to 300 m/s. Assuming the flow through the shock wave to be adiabatic, find the temperature of the air behind the shock wave.

SOLUTION

Because the flow is adiabatic, the energy equation gives if subscripts 1 and 2 refer to conditions before and after the shock wave respectively:

$$c_p T_2 + \frac{V_2^2}{2} = c_p T_1 + \frac{V_1^2}{2}$$

Hence:

$$T_2 = T_1 + \left(\frac{1}{c_p}\right)\left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) = 298 + \left(\frac{1}{1007}\right)\left(\frac{500^2}{2} - \frac{300^2}{2}\right) = 377\text{K} = 104.4^\circ\text{C}$$

Since air flow is being considered the specific heat c_p has been assumed to be 1007 J/ kg °C.

Therefore the temperature "behind" the shock wave is 104.4° C.

PROBLEM 2.3

Air being released from a tire through the valve is found to have a temperature of 15°C. Assuming that the air in the tire is at the ambient temperature of 30°C find the velocity of the air at the exit of the valve. The process can be assumed to be adiabatic.

SOLUTION

Because the flow is adiabatic, the energy equation gives if subscripts 1 and 2 refer to conditions in the tire and at the discharge from the valve respectively:

$$c_p T_2 + \frac{V_2^2}{2} = c_p T_1 + \frac{V_1^2}{2}$$

But the velocity in the tire can be assumed to be zero so this gives:

$$c_p T_2 + \frac{V_2^2}{2} = c_p T_1 \ , \quad \text{i.e.,} \ \frac{V_2^2}{2} = c_p (T_1 - T_2)$$

Hence, assuming that for air the specific heat c_p is 1007 J/ kg °C:

$$V_2 = \sqrt{2c_p (T_1 - T_2)} = \sqrt{2 \times 1007 \times (303 - 288)} = 173.8 \text{ m/s}$$

Therefore the air leaves the valve with a velocity of 173.8 m/s.

PROBLEM 2.4

A gas with a molecular weight of 4 and a specific heat ratio of 1.67 flows through a variable area duct. At some point in the flow the velocity is 180 m/s and the temperature is 10° C. At some other point in the flow, the temperature is -10° C. Find the velocity at this point in the flow assuming that the flow is adiabatic.

SOLUTION

Because the flow is adiabatic, the energy equation gives if subscripts 1 and 2 refer to conditions at the first and second points considered respectively:

$$c_p T_2 + \frac{V_2^2}{2} = c_p T_1 + \frac{V_1^2}{2}$$

Hence:

$$V_2^2 = V_1^2 + 2c_p\left(T_1 - T_2\right)$$

Assuming that the gas can be treated as a perfect gas:

$$R = c_p - c_v = c_p\left(1 - \frac{c_v}{c_p}\right) \quad \text{i.e.,} \quad c_p = \frac{\gamma R}{\gamma - 1}$$

Hence for the gas being considered:

$$c_p = \frac{1.67 \times (8314/4)}{1.67 - 1} = 5181 \text{ J/kg}\,^{\circ}\text{C}$$

The energy equation therefore gives:

$$V_2^2 = V_1^2 + 2c_p\left(T_1 - T_2\right) = 180^2 + 2 \times 6181 \times \left(283 - 263\right)$$

Which gives V_2 = 489.5 m/s. Therefore the velocity at the second point is 489.5 m/s.

PROBLEM 2.5

At a section of a circular duct through which air is flowing the pressure is 150 kPa , the temperature is 35 °C , the velocity is 250 m/s , and the diameter is 0.2 m. If, at this section, the duct diameter is increasing at a rate of 0.1 m / m find *dp/dx* , *dV/dx* , and *dρ/dx* .

SOLUTION

Because:

$$A = \frac{\pi D^2}{4}$$

it follows that:

$$\frac{1}{A}\frac{dA}{dx} = \frac{2}{D}\frac{dD}{dx}$$

Hence. at the section considered:

$$\frac{1}{A}\frac{dA}{dx} = \frac{2}{0.2} \times 0.1 = 1\text{m}^{-1}$$

But the continuity equation gives:

$$\frac{1}{A}\frac{dA}{dx} + \frac{1}{V}\frac{dV}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} = 0$$

But using the information supplied:

$$\rho = \frac{p}{RT} = \frac{150000}{287 \times 308} = 1.697 \text{ kg/m}^3$$

Therefore the continuity equation gives:

$$1 + \frac{1}{250}\frac{dV}{dx} + \frac{1}{1.697}\frac{d\rho}{dx} = 0 \qquad (1)$$

It is next noted that the conservation of momentum equation gives:

$$-\frac{1}{\rho}\frac{dp}{dx} = V\frac{dV}{dx} \quad , \text{ i.e.,} \quad \frac{dp}{dx} = -\rho V\frac{dV}{dx}$$

hence:

$$\frac{dp}{dx} = -1.697 \times 250 \times \frac{dV}{dx} = -424.3\frac{dV}{dx} \tag{2}$$

The conservation of energy equation gives:

$$c_p\frac{dT}{dx} + V\frac{dV}{dx} = 0$$

hence again assuming that c_p is equal to 1007 J / kg °C it follows that:

$$1007\frac{dT}{dx} + 250\frac{dV}{dx} = 0$$

i.e.:

$$\frac{dT}{dx} = -0.243\frac{dV}{dx} \tag{3}$$

Lastly, it is noted that from the perfect gas law, $p = \rho R T$, it follows that:

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{T}\frac{dT}{dx}$$

i.e.:

$$\frac{1}{150000}\frac{dp}{dx} = \frac{1}{1.697}\frac{d\rho}{dx} + \frac{1}{308}\frac{dT}{dx} \tag{4}$$

Using eq. (3), eq. (4) becomes:

$$\frac{1}{150000}\frac{dp}{dx} = \frac{1}{1.697}\frac{d\rho}{dx} - \frac{0.2483}{308}\frac{dV}{dx}$$

i.e.:

$$\frac{dp}{dx} = 88915\frac{d\rho}{dx} - 120.9\frac{dV}{dx} \tag{5}$$

Substituting from eqs. (1) and (2) into eq. (5) then gives:

$$-424.3\,\frac{dV}{dx} \;=\; -88915 \times 1.697 \times \left(1 + \frac{1}{250}\,\frac{dV}{dx}\right) - 120.9\,\frac{dV}{dx}$$

i.e.:

$$\frac{dV}{dx}\left(-424.3 + 120.9 + \frac{88915 \times 1.697}{250}\right) \;=\; -88915 \times 1.697$$

Hence:

$$\frac{dV}{dx} \;=\; 502.6\ \mathrm{s}^{-1} \tag{6}$$

Using this result in eq. (1) then gives:

$$1 \;+\; \frac{502.6}{250} \;+\; \frac{1}{1.697}\,\frac{d\rho}{dx} \;=\; 0$$

i.e.:

$$\frac{d\rho}{dx} \;= -1.687 \;+\; \frac{1.697 \times 502.6}{250} \;= 1.715\ \mathrm{kg/m^4}$$

Similarly, using eq. (6) in eq. (2) gives:

$$\frac{dp}{dx} \;=\; 424.3 \times 502.6 \;=\; 213250\ \mathrm{Pa/m}$$

Therefore the values of *dp/dx* , *dV/dx* and *d*ρ*/dx* are 213250 Pa/m, -502.6 m/s per m, and 1.715 kg/m^4 respectively.

PROBLEM 2.6

Consider an isothermal air flow through a duct. At a certain section of the duct, the velocity, temperature and pressure are 200 m/s, 25°, and 120 kPa respectively. If the velocity is decreasing at this section at a rate of 30 per cent per m find *dp/dx, ds/dx* and *dp/dx.*

SOLUTION

The flow considered in this problem is not adiabatic.

The conservation of momentum equation gives:

$$-\frac{1}{\rho}\frac{dp}{dx} = V\frac{dV}{dx} \quad , \text{ i.e.,} \quad \frac{dp}{dx} = -\rho V\frac{dV}{dx}$$

But:

$$\rho = \frac{p}{RT} = \frac{120000}{287 \times 298} = 1.403 \text{ kg/m}^3$$

and:

$$\frac{1}{V}\frac{dV}{dx} = -0.3$$

so:

$$\frac{dp}{dx} = +1.403 \times 200 \times 200 \times 0.3 = 16836 \text{ Pa/m}$$

Because the flow is isothermal, the perfect gas law, $p = \rho R T$, gives:

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\rho}\frac{d\rho}{dx}$$

Hence:

$$\frac{d\rho}{dx} = \frac{\rho}{p}\frac{dp}{dx} = \frac{1.403}{120000} \times 16836 = 0.1968 \text{ kg/m}^3\text{/m}$$

Lastly since:

$$\frac{1}{c_{\mathrm{p}}}\frac{ds}{dx} = \frac{1}{T}\frac{dT}{dx} - \left(\frac{\gamma - 1}{\gamma}\right)\frac{1}{p}\frac{dp}{dx}$$

it follows that for the isothermal situation being considered:

$$\frac{1}{c_{\mathrm{p}}}\frac{ds}{dx} = -\left(\frac{\gamma - 1}{\gamma}\right)\frac{1}{p}\frac{dp}{dx}$$

which gives:

$$\frac{1}{1007}\frac{ds}{dx} = -\left(\frac{0.4}{1.4}\right) \times \frac{1}{120000}\frac{dp}{dx}$$

i.e.:

$$\frac{ds}{dx} = -\left(\frac{0.4}{1.4}\right) \times \frac{1007}{120000} \times 16836 = -40.36 \text{ J/kg-K per m}$$

The entropy is changing because of the heat transfer at the wall.

Therefore, the values of *dp/dx, dp/dx*, and *ds/dx* are 16.84 kPa/m, 0.1968 kg/m^3 / m, and - 40.36 J / kg-K per m respectively.

PROBLEM 2.7

Consider adiabatic air flow through a variable area duct. At a certain section of the duct, the flow area is 0.1 m^2, the pressure is 120 kPa, and the temperature is 15°C and the duct area is changing at a rate of 0.1 m^2/m. Plot the variations of *dp/dx, dV/dx* and *dρ/dx* with the velocity at the section for velocities between 50 m/s and 300 m/s.

SOLUTION

The continuity equation gives:

$$\frac{1}{A}\frac{dA}{dx} + \frac{1}{V}\frac{dV}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} = 0$$

But using the information supplied:

$$\rho = \frac{p}{RT} = \frac{120000}{287 \times 288} = 1.452 \text{ kg/m}^3$$

Therefore the continuity equation gives:

$$\frac{1}{0.1} \times 0.1 + \frac{1}{V}\frac{dV}{dx} + \frac{1}{1.452}\frac{d\rho}{dx} = 0 \qquad (1)$$

It is next noted that the conservation of momentum equation gives:

$$-\frac{1}{\rho}\frac{dp}{dx} = V\frac{dV}{dx} \quad , \quad \text{i.e.,} \quad \frac{dp}{dx} = -\rho V\frac{dV}{dx}$$

hence:

$$\frac{dp}{dx} = -1.452V\frac{dV}{dx} \qquad (2)$$

The conservation of energy equation gives:

$$c_p\frac{dT}{dx} + V\frac{dV}{dx} = 0$$

hence again assuming that c_p is equal to 1007 J / kg °C it follows that:

$$1007 \frac{dT}{dx} + V \frac{dV}{dx} = 0 \tag{3}$$

Lastly, it is noted that from the perfect gas law, $p = \rho R T$, it follows that:

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{T}\frac{dT}{dx}$$

i.e.:

$$\frac{1}{120000}\frac{dp}{dx} = \frac{1}{1.452}\frac{d\rho}{dx} + \frac{1}{288}\frac{dT}{dx} \tag{4}$$

Using eq. (3), eq. (4) becomes:

$$\frac{1}{120000}\frac{dp}{dx} = \frac{1}{1.452}\frac{d\rho}{dx} - \frac{1}{1007 \times 288} V \frac{dV}{dx}$$

i.e.:

$$\frac{dp}{dx} = 82645 \frac{d\rho}{dx} - 0.4138 V \frac{dV}{dx} \tag{5}$$

Substituting from eqs. (1) and (2) into eq. (5) then gives:

$$-1.452 V \frac{dV}{dx} = -82645 \times 1.452 \times \left(1 + \frac{1}{V}\frac{dV}{dx}\right) - 0.4138 V \frac{dV}{dx}$$

i.e.:

$$\frac{dV}{dx}\left(0.4138V + 1.452V + \frac{82645 \times 1.452}{V}\right) = -82645 \times 1.452$$

i.e.:

$$\frac{dV}{dx} = -\frac{120000}{120000/V - 1.038V} \tag{6}$$

Using this result in eq. (1) then gives:

$$\frac{1}{1.452}\frac{d\rho}{dx} = -1 + \frac{120000}{120000 - 1.038V^2}$$

i.e.:

$$\frac{d\rho}{dx} = -1.452 + \frac{174240}{120000 - 1.038V^2} \qquad (7)$$

Similarly, using eq. (6) in eq. (2) gives:

$$\frac{dp}{dx} = \frac{124240V}{120000/V - 1.038V}$$

For any value of V (*m/s*), eqs. (6), (7) and (8) allow *dV/dx* (1 / m*), dρ/dx* (Kg / m^3 / m*)*, and *dp/dx* (Pa / m) to be found. Some results are given in the following table, these results also being shown in Figs. 2.7a, 2.7b and 2.7c that follow the table.

V **– m/s**	***dp/dx*** **– Pa/m**	***dρ/dx*** **- Kg / m^3 / m**	***dV/dx*** **- 1 / m**
50	3710	0.0321	-51.1
75	8585	0.0742	-78.8
100	15894	0.1374	-109.5
125	26230	0.2268	-144.5
150	40559	0.3506	-186.2
175	60480	0.5229	-238.0
200	88780	0.7675	-305.7
225	130716	1.1305	-400.1
250	197417	1.7077	-543.9
275	317159	2.7418	-794.3
300	588781	5.0900	-1315.7

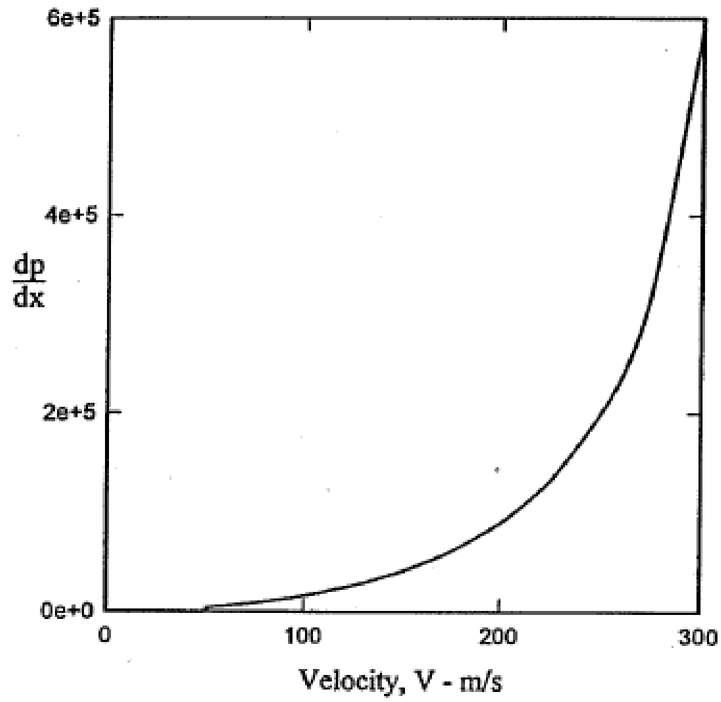

Figure P2.7a

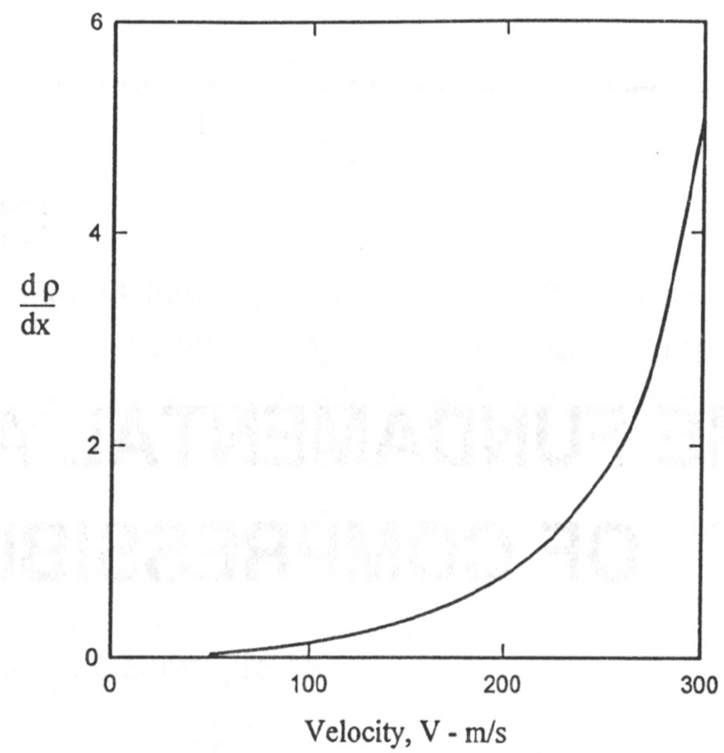

Figure P2.7b

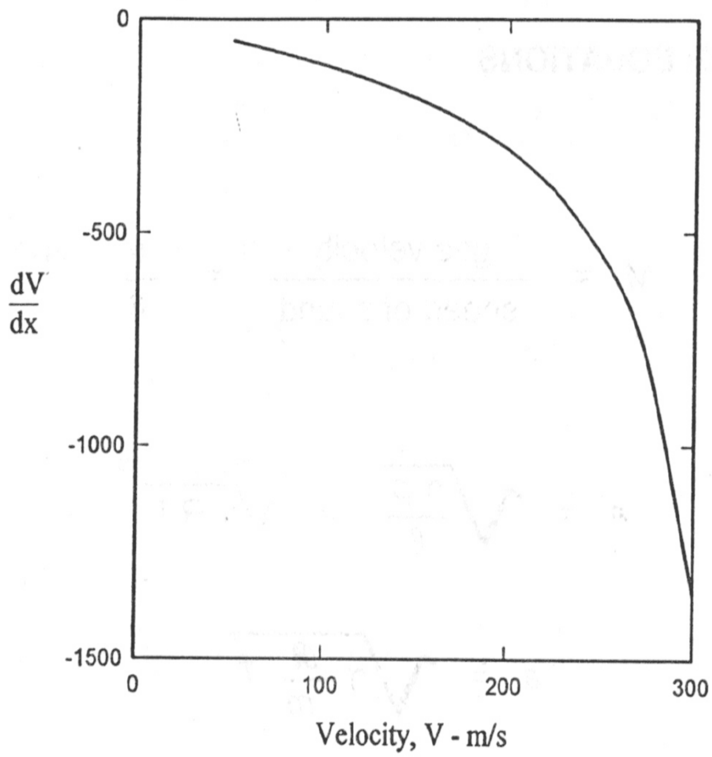

Figure P2.7c

PROBLEM 2.8

Methane flows through a circular pipe which has a diameter of 4cm. The temperature, pressure, and velocity at the inlet to the pipe are 200 K, 250 kPa, and 30 m/s respectively. Assuming that the flow is steady and isothermal calculate the pressure on the exit plane and the heat added to the methane in the pipe if the velocity on the pipe exit plane is 35m/s. Assume that the methane can be treated as a perfect gas with a specific heat ratio of 1.32 and a molar mass of 16.

SOLUTION

As shown in the textbook momentum conservation considerations give:

$$-\frac{dp}{\rho} = V\,dV$$

Using the perfect gas law this can be written as:

$$-RT\frac{dp}{p} = V\,dV$$

For an isothermal flow this can be integrated between any two points, 1 and 2, in the flow to give:

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = RT\left(\ln p_2 - \ln p_1\right) = RT\ln\left(\frac{p_2}{p_1}\right) \quad , \quad \text{i.e.,} \quad \ln\left(\frac{p_2}{p_1}\right) = \frac{1}{RT}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

Using the information provided:

$$R = \frac{8314}{16} = 519.6 \ \text{J/kg K}$$

Hence

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{1}{519.6 \times 200}\left(\frac{35^2}{2} - \frac{30^2}{2}\right) = 0.001564 \ , \quad \text{i.e.,}$$

$$p_2 = p_1\, e^{0.001564} = 250e^{0.001564} = 250.4 \ \text{kPa}$$

Now since using the perfect gas law gives:

$$\rho = \frac{p}{RT}\ ,\ i.e.\quad \rho_1 = \frac{p_1}{RT_1} = \frac{250000}{519.6 \times 200} = 2.406\ \text{kg/m}^3$$

Therefore the mass flow rate through the pipe is given by:

$$\dot{m} = \rho_1 V_1 A = 2.406 \times 30 \times \frac{\pi}{4} \times 0.04^2 = 0.091\ \text{kg/s}$$

The heat added per unit mass of air flow is given by the energy equation as:

$$q = \left(c_p T_2 + \frac{V_2^2}{2}\right) - \left(c_p T_1 + \frac{V_1^2}{2}\right)$$

i.e., since isothermal flow is being considered:

$$q = \frac{V_2^2}{2} - \frac{V_1^2}{2} = \frac{35^2}{2} - \frac{30^2}{2} = 162.5\ \text{J/kg}$$

Hence the rate of heat addition to the flow is given by:

$$Q = \dot{m} q = 0.091 \times 162.5 = 14.79\ \text{J/s}$$

Therefore the exit plane pressure is 250.4 kPa and the rate heat is added to the flow is 14.79 J/s.