SOLUTIONS MANUAL FOR PROTECTIVE RELAYING Principles and Applications FOURTH EDITION

J. Lewis Blackburn Thomas J. Domin



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Protective Relaying Principles and Applications 4th Edition

J. Lewis Blackburn (Deceased) Thomas Domain

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Anthony D. Locatelli, P.E., MSEE Sr. Engineer-Relay & Protection Services Commonwealth Edison Company The classic work of J. Lewis Blackburn is well known to all of those practicing in the field of protective relaying and it is a humbling honor to have been invited to contribute to the latest revision of this work. The various collaborators to this solutions manual have worked many hours on the solutions to the exercise problems for this classic text. We hope that we have illustrated the solution techniques in a manner that is clear and educational for the users of this manual.

It is important to note that in working out these solutions, the collaborators have drawn upon their own years of experience working in this field. Some engineering assumptions and simplifying approximations are sometimes made and it is experience that guides one in the validity of the simplifying approximations and assumptions used. For the sake of those learning the craft, we have attempted to highlight approximations and assumptions when used, but there may be some occasions where we have forgotten to point out an assumption or approximation. We hope that this does not cause undue confusion. It is also possible that other justifiable assumptions may lead to somewhat different, equally valid solutions. Relay system design and settings selections are inherently a balance between competing considerations. Safety and protection being primary, other considerations of security, dependability, cost, and system performance may lead to different solutions depending upon which of these other considerations are emphasized in any particular application.

It is our hope that the users of this solutions manual will find it to be a useful educational tool and will benefit from it. If any errors are found, we apologize and would appreciate these being brought to our attention.

Sean Carr, Chicago, IL February 20, 2014

CHAPTER 2

- **2.1** A wye-connected generator has a nameplate rating of 200 MVA, 20 kV, and its subtransient reactance (X''_d) is 1.2 pu. Determine its reactance in ohms.
- 2.2 The generator of Problem 2.1 is connected in a power system where the base is specified as 100 MVA, 13.8 kV. What is the generator reactance $(X_d^{"})$ in per unit on this system base?
- **2.3** Convert the per-unit answer calculated in Problem 2.2 to ohms. Does this match the value determined in Problem 2.1?
- 2.4 Three 5 MVA single-phase transformers, each rated 8:1.39 kV, have a leakage impedance of 6%. These can be connected in a number of different ways to supply three identical 5 Ω resistive loads. Various transformer and load connections are outlined in Table P2.4. Complete the table columns. Use a three-phase base of 15 MVA.

TABL	E P2.4						
Case _	Transformer	connection	Load connection to	Line-to-line base kV	Load R in	Total Z as from the h	viewed ligh side
No. Pri	mary Se	condary	secondary HV	LV	per unit	Per unit	Ω
1	Wye	Wye	Wye				
2	Wye	Wye	Delta				
3	Wye	Delta	Wye				
4	Wye	Delta	Wye				
5	Delta	Wye	Wye				
6	Delta	Wye	Delta				
7	Delta	Delta	Wye				
8	Delta	Delta	Delta				

2.5 A three-phase generator feeds three large synchronous motors over a 16 km, 115 kV transmission line, through a transformer bank, as shown in Figure P2.5. Draw an equivalent single-line reactance diagram with all reactances indicated in per unit of a 100 MVA, 13.8 or 115 kV base.



- 2.6 In the system of Problem 2.5, it is desired to maintain the voltage at the motor bus of 1.∠0° per unit. The three motors are operating at full rating and 90% pf.
 - **a.** Determine the voltage required at the generator terminals assuming that there is no voltage regulating taps or similar equipment in this system.
 - b. What is the voltage required behind the subtransient reactance?
- 2.7 The percent impedance of a transformer is typically determined by a short circuit test. In such a test, the secondary of the transformer is shorted and the voltage on the primary is increased until rated current flows in the transformer windings. The applied voltage that produces rated current divided by the rated voltage of the transformer is equal to the per-unit impedance of the transformer.

A short circuit test on a 150 KVA, 7200–240 V transformer provides the following results: Primary voltage at 20.8 primary amperes = 208.8 V

- a. Determine the %Z of the transformer.
- b. Calculate the ohmic impedance of the transformer in primary and secondary terms.
- **c.** How much current would flow in the transformer if its secondary would become shorted during normal operating conditions? (Consider source impedance to be zero.)

. 2.1

Find impedance in ohms from per-unit.

 $MVA_B = 200, kV_B = 20, X''_d = Z_{PU} = 1.2$

Per Eq. (2.17)
$$Z_{\Omega} = \frac{k V_B^2 \times Z_{PU}}{M V A_B}$$
$$\frac{20^2 \times 1.2}{200}$$

Impedance $X''_d = 2.4$ ohms

2.2 - Method 1

Convert per-unit impedance from one base to another.

 $MVA_1 = 200$, $MVA_2 = 100$, $kV_1 = 20$, $kV_2 = 13.8$, $X''_d = Z_{1PU} = 1.2$

Per Eq. (2.33)
$$Z_{2 \text{ PU}} = Z_{1 \text{ PU}} \times \frac{\text{MVA}_2}{\text{MVA}_1} \times \frac{\text{kV}_1^2}{\text{kV}_2^2}$$
$$1.2 \times \frac{100}{200} \times \frac{20^2}{13.8^2}$$

1.26024

Per-unit impedance in new base $Z_{2PU} = 1.26024$

2.2 - Method 2

Convert impedance from ohms (Result 2.1) to per-unit.

 $MVA_B = 100, kV_B = 13.8, X''_d = Z_{\Omega} = 2.4$

Per Eq. (2.15)
$$Z_{PU} = \frac{Z_{\Omega}}{Z_B} = \frac{MVA_B \times Z_{\Omega}}{kV_B^2}$$

$$\frac{100 \times 2.4}{13.8^2}$$

1.26024

Per-unit impedance Z_{PU} = 1.26024

. 2.3

Convert impedance from per-unit (Result 2.2) back to ohms.

 $MVA_B = 100, kV_B = 13.8, X''_d = Z_{PU} = 1.26024$

Per Eq. (2.17)
$$Z_{\Omega} = \frac{kV_B^2 \times Z_{PU}}{MVA_B}$$

 $\frac{13.8^2 \times 1.26024}{100}$

2.4

Impedance $X''_d = 2.4$ ohms. This matches Result 2.1.

JKN 1/26/14 2.4 The transformer per unit impedance care be referred to either side, so when either windig P.1 122 It is connected in wye, the transformen impedance Will be 6% It quive let impelie in the line referred to the wage - Connected side. When both windings are connected in delta, the equivalent line impedence will be 1/3 of the winding impedence, or 276 IZ. Similarly, the load resistors, when connected in delta, here ar equivalent wye resistue of '3 the chine value of resistors used to make up the delta. Transforma impedance is 670 I 2 on transformer 1-plan Back of 5 MVA per phan, equivalent for 15 MVA 3-phase. The equivalent wye-connected load verifice can be expressed on per unit basit by dividing by the low-site base impedance, The Migh-side per unit impedance is the same as the locitside per und typedarer; the high side equivalent lupedane in shows can be found by miltiplying the permit impedance by the tiph-sile base impedue.

1º ca ERIV PUL Leakge Reasono Y Rollsh CO.Y EQ Y Ve ٧ß LOAD AN HVI LV HV LV j0.06 2,408 and and 5.8.51 YY YA 1.667 50.06 2.408 13.856 j0.06 1,390 13.857 Y Y 3 \triangle j0,06 1.667 12.94 1,390 13,856 \triangle Y \bigwedge 2,408 10.06 5 31.154 8 Y Y X Concern 1.667 2.408 10.06 10.38 8 AY \triangle (a Constanting of the second 1,390 10.02 8 AA Y ang t 1.667 1,390 (0.02 8 \triangle A P D

When $8kV \neq 13 = 13.856 kV$, $Z_B = \frac{(13.858 kV)^2}{15 MVA} = 1208.02$ A commonled HV 8kV, $Z_B = \frac{(8kV)^2}{15 MVA} = 4.267.02$ Wyeld $1.39 kV \neq 13 = 2.408 kV$, $Z_B = \frac{(2.408 kV)^2}{15 MVA} = 0.3864$ A connected LV 1.39 kV, $Z_B = \frac{(1.39 kV)^2}{15 MVA} = 0.1288.02$

$$\frac{5}{0.3864} = \frac{12.94}{5/3} = \frac{5/3}{0.1605} = \frac{4.31}{12.58}$$

$$\frac{5}{0.3864} = \frac{5/3}{0.1605} = \frac{4.31}{12.58}$$

$$\frac{5}{0.1288} = 38.82 \qquad \frac{5/3}{0.1288} = 12.94$$

See meal shut

JKN 1/26/14 2.92

						Intermediate calculations						
Col A	Col B	Col C	Col D	Col E	Col F	Col G	Col H	Col I	Col J	Col K	Col L	Col M
									equivalent			
									Y-connected	PU equivalent wye-		
								PU leakage	resistive load	connected load		
	Winding c	onnection		LL Voltage	e Base (kV)	Base Impeda	ance (ohms)	reactance	(ohms on LV side)	resistance	Total Z viewed	from HV side
			Load					equivalent			per unit	ohms
Case	HV	LV	connection	HV	LV	HV	LV	in line		(Col J / Col H)	(Col K + j*Col i)	(Col L * Col G)
1	wye	wye	wye	13.856	2.407	12.799	0.386	0.060	5.000	12.940	12.940 + j0.060	165.623 + j0.768
2	wye	wye	delta	13.856	2.407	12.799	0.386	0.060	1.667	4.313	4.313 + j0.060	55.208 + j0.768
3	wye	delta	wye	13.856	1.390	12.799	0.129	0.060	5.000	38.818	38.818 + j0.060	496.840 + j0.768
4	wye	delta	delta	13.856	1.390	12.799	0.129	0.060	1.667	12.939	12.939 + j0.060	165.613 + j0.768
5	delta	wye	wye	8.000	2.407	4.267	0.386	0.060	5.000	12.940	12.940 + j0.060	55.211 + j0.256
6	delta	wye	delta	8.000	2.407	4.267	0.386	0.060	1.667	4.313	4.313 + j0.060	18.404 + j0.256
7	delta	delta	wye	8.000	1.390	4.267	0.129	0.020	5.000	38.818	38.818 + j0.020	165.623 + j0.085
8	delta	delta	delta	8.000	1.390	4.267	0.129	0.020	1.667	12.939	12.939 + j0.020	55.208 + j0.085

Base	HV				
power	winding	HV delta			
(3-phase	voltage	LL voltage	HV wye LL	Zbase for HV	Zbase for HV
MVA)	(kV)	(kV)	voltage (kV)	delta	wye
15.000	8.000	8.000	13.856	4.267	12.799

Problem 2.4

Base	LV				
power	winding	LV delta			
(3-phase	voltage	LL voltage	LV wye LL	Zbase for LV	Zbase for LV
MVA)	(kV)	(kV)	voltage (kV)	delta	wye
15.000	1.390	1.390	2.407	0.129	0.386

JKN 2-1-14 P. 73 7-2 XA AYA MI) 115KV Transmissin Line 16km -53-X = 0.5 - 12/mi ZOMVA SBMVA SOMVA 115/13.XHV 13.8:115kv 13.24V x=0% X=12% x=20% M, : 10 MIA, 13.8101, Ka =20% M2: 5MVA, 13.8/eV, Xe = 20% M3: 5 MVA, 13.2 KV, X= 193 Changing Base : Zprinked = Zprord (MVANEW) (KVOLD)² MNAOLD (KVNEW)² $\chi a'' = 20\% \left(\frac{100}{50}\right) \left(\frac{13.2}{13.8}\right)^2 = 36.6\%, 0.366 \mu c$ Cenerator $\chi = 12\% \left(\frac{100}{50}\right) = 24\%, 0.24 pu$ F1 K = 10% (100) = 50%, 0.50 pu 7-2 Xa"= 20% (100) = 200%, 20 pu M1 xa"= 20% (100) = 400%, 4.0 pu MZ $\chi_{\theta}^{"} = 17\% \left(\frac{100}{5}\right) \left(\frac{13.2}{13.8}\right)^2 = 311\%, 3.11 \text{ pu}$ M3 Line impedance : Zpa= Zhano

Ebane INSEV = (15ku)2 102 MVA = 132.25-22 $Z_{\text{Line pu}} = \frac{(0.5 + 2.5)(16 \text{ mi})}{132.35 + 1} = 0.0605 \text{ pu}$

JEN 2/1/M P.2g2 Reactance diagram All impedances in PCL on 100 MVA base and wither 13-8 kV base or 115 KV base as appropriate. j2.0 j2.0 jo.366 jo.24 j 0.50 10.0605 14.0 ×0* 13.11 XT2 XLine XT1

JKN 2-1-14

 $MVA \equiv 10^{6} \cdot \text{volt-amp}$ $PU \equiv 1$

Problem 2.6

Prob 2.6.a - Voltage at generator bus to keep 1 PU voltage at motor bus

 $V_{motor} \approx 1.0 \cdot PU$

Load current - motors at rated power and 0.9 PF. Problem does not say explicitly, but we assume that the PF is lagging as this is the most probable.

Total motor bus power in per unit

$$S_{motor} := \frac{10 \cdot MVA + 5 \cdot MVA + 5 \cdot MVA}{100 \cdot MVA} \cdot e^{j \cdot a\cos(0.9)} = (0.18 + 0.087j) PU$$

$$|S_{motor}| = 0.2 PU$$
 $arg(S_{motor}) = 25.842 deg$

Current is found from the relation that complex power is equal to voltage times complex conjugate of current.

$$I_{\text{motor}} := \left(\frac{S_{\text{motor}}}{V_{\text{motor}}}\right) = (0.18 - 0.087) \text{PU} \qquad \left|I_{\text{motor}}\right| = 0.2 \text{PU} \qquad \arg(I_{\text{motor}}) = -25.842 \text{deg}$$

Line reactance in PU (from prob. 2.5)	$Z_{\text{Line}} := j \cdot 0.0605 \text{PU}$
T1 reactance in PU (from prob. 2.5)	$\mathrm{Z}_{T1}\coloneqq j\!\cdot\!0.24\mathrm{PU}$
T2 reactance in PU (from prob. 2.5)	$Z_{T2} := j \cdot 0.50 PU$

Generator bus voltage is motor bus voltage plus voltage drop in T1, line, and T2.

$$V_{\text{Gen.bus}} := V_{\text{motor}} + I_{\text{motor}} \cdot (Z_{\text{T1}} + Z_{\text{Line}} + Z_{\text{T2}}) = (1.07 + 0.144) \text{ PU}$$

$$|V_{\text{Gen.bus}}| = 1.079 \text{PU}$$
 $\arg(V_{\text{Gen.bus}}) = 7.671 \text{deg}$

<u>Prob 2.6 b - Generator internal voltage</u> behind subtransient reactance: add internal voltage drop to the generator terminal voltage

Generator subtransient reactance in PU (from prob. 2.5) $Z_{Gen} := j \cdot 0.366$

$$V_{\text{Gen.internal}} := V_{\text{Gen.bus}} + I_{\text{motor}} \cdot Z_{\text{Gen}} = (1.102 + 0.21) \text{ PU}$$

$$|V_{\text{Gen.internal}}| = 1.122 \text{PU}$$
 $\arg(V_{\text{Gen.internal}}) = 10.791 \text{deg}$

JKN 2-1-14	$kVA \equiv 1000 \cdot volt \cdot amp$
Problem 2.7	$MVA \equiv 10^6 \cdot volt \cdot amp$
Transformer rated power and rated voltages	$PU \equiv 1$

V_{rated.primary} := 7200·volt $V_{rated.secondary} := 240 \cdot volt$ Srated := 150·kVA

Transformer rated current

1

$$I_{rated.primary} := \frac{150 \cdot kVA}{7.2 \cdot kV} = 20.833 A$$

$$I_{rated.secondary} := \frac{150000 \cdot volt \cdot amp}{240 \cdot volt} = 625 A$$

This confirms that the short circuit test was in fact performed at rated current. The impedance voltage is therefore equal to the voltage measured during the test, or 208.8 volts.

2.7.a The transformer per cent impedance voltage (%IZ or %Z) is equal to the measured impedance voltage expressed as per cent of the rated voltage or in this case:

$$V_{\text{test}} \coloneqq 208.8 \cdot \text{volt} \qquad X_{\text{T}} \coloneqq \frac{V_{\text{test}}}{V_{\text{rated.primary}}} = 0.029 \text{ PU} \qquad X_{\text{T}} \equiv 2.9\%$$

$$\frac{208.8 \cdot \text{volt}}{7200 \cdot \text{volt}} = 0.029 \text{ PU}$$

2.7.b The equivalent ohmic impedance referred to the primary or secondary is equal to the corresponding base impedance times the per unit impedance.

 $Z_{\text{base.primary}} \coloneqq \frac{V_{\text{rated.primary}}^2}{S_{\text{rated}}} = 345.6 \,\Omega \qquad Z_{\text{base.secondary}} \coloneqq \frac{V_{\text{rated.secondary}}^2}{S_{\text{rated}}} = 0.384 \,\Omega$ $\frac{(7.2 \cdot kV)^2}{0.15 \cdot MVA} = 345.6 \,\Omega$

$$\frac{(0.24 \cdot kV)^2}{0.150 \cdot MVA} = 0.384 \,\Omega$$

 $X_{T.ohms.primary} := X_T Z_{base.primary} = 10.022 \Omega$

 $X_{T.ohms.secondary} := X_T Z_{base.secondary} = 0.0111 \Omega$

<u>2.7.c</u> There are a few ways to arrive at the short circuit current. One way is to divide the rated voltage by the equivalent ohmic impedance on either primary or secondary.

Alternatively, 1.0 PU voltage can be divided by the PU impedance to determine the short circuit current in PU of rated current. The PU short circuit current can then be multiplied by the base current (i.e., rated current) on either primary or secondary as desired. This method can be further simplified to "divide rated current by per unit impedance."

Both methods are illustrated here:

$$\frac{V_{\text{rated.primary}}}{X_{\text{T.ohms.primary}}} = 718.391 \text{ A} \qquad \frac{7200 \cdot \text{volt}}{10.022 \cdot \text{ohm}} = 718.419 \text{ A}$$

$$\frac{V_{\text{rated.secondary}}}{X_{\text{T.ohms.secondary}}} = 21551.724 \text{ A} \qquad \frac{240 \cdot \text{volt}}{.0111 \cdot \text{ohm}} = 21621.622 \text{ A}$$

(NOTE: this result can also be used to illustrate the concept of significant digits. The result is 21.6 kA, athough the computer internally carries out calculations to greater precision than is significant.)

Another method to arrive at the same result:

.

$$I_{SC.PU} \coloneqq \frac{1}{X_{T}} = 34.483 \text{ PU}$$

$$I_{SC.PU} \coloneqq I_{SC.PU} \cdot I_{rated.primary} = 718.391 \text{ A}$$

$$34.5 \cdot 20.8 \cdot \text{amp} = 717.6 \text{ A}$$

$$I_{SC.secondary} \coloneqq I_{SC.PU} \cdot I_{rated.secondary} = 21.552 \text{ kA}$$

And finally, another method, often the simplest:

$$\frac{I_{rated.primary}}{X_{T}} = 718.391 \text{ A} \qquad \frac{20.8 \cdot \text{amp}}{.029} = 717.241 \text{ A}$$
$$\frac{I_{rated.secondary}}{X_{T}} = 21551.724 \text{ A} \qquad \frac{625 \cdot \text{amp}}{0.029} = 21551.724 \text{ A}$$

CHAPTER 3

3.1 Four boxes represent an AC generator, reactor, resistor, and capacitor and are connected to a source bus XY as shown in Figure P3.1. From the circuit and phasor diagrams, identify each box.



3.2 Two transformer banks are connected to a common bus as shown in Figure P3.2. What are the phase relations between the voltages V_{AN} and $V_{A'N'}$; V_{BN} and $V_{B'N'}$, V_{CN} and $V_{C'N'}$?



3.3 Reconnect transformer bank 2 of Problem 3.2 with the left windings in wye instead of delta, and the right windings in delta instead of wye so that V_{AN} and $V_{A'N'}$ are in phase, V_i and $V_{B'N'}$ are in phase, and V_{CN} are in phase.



3.4 The power transformer connections shown in Figure P3.3 are nonstandard and quite unusual with today's standardization. However, this connection provides an excellent exercise in understanding phasors, polarity, and directional sensing relay connections.

Connect the three-directional phase relays *A*, *B*, *C* to line-side CTs and bus-side VTs for proper operation for phase faults out on the line. Use the 90° - 60° connection. Each directional relay has maximum torque when the applied current leads the applied voltage by 30° . The auxiliary VTs should be connected to provide the relays with equivalent line-side voltages.

PROBLEM 3.1

JKN SWC 2/15/14



I = RESISTOR Jz= INDUCTOR I3 = GENERATOR I4 = CAPACITOR