SOLUTIONS MANUAL FOR

THERMAL RADIATION HEAT TRANSFER

by

John R. Howell
Robert Siegel
M. Pinar Menguc

CRC Press
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Solutions to Homework Problems

Table of Contents

SOLUTIONS- CHAPTER 1

1.1 What are the wave number range in vacuum and the frequency range for the visible spectrum (0.4 to 0.7 μm)? What are the wave number and frequency values at the spectral boundaries between the near and the far infrared regions?

SOLUTION: $c_0 = 2.99792458 \times 10^8$ m/s; $\lambda_1 = 0.4 \times 10^{-6}$ m, $\lambda_2 = 0.7 \times 10^{-6}$ m,

$\eta_1 = 1 / \lambda_1 = \underline{2.5 \times 10^6 \text{ m}^{-1}}$, $\eta_2 = 1 / \lambda_2 = \underline{1.428571 \times 10^6 \text{ m}^{-1}}$,

$\nu_1 = c_0 / \lambda_1 = 2.99792458 \times 10^8 / 0.4 \times 10^{-6} = \underline{7.494811 \times 10^{14} \text{ s}^{-1}}$,

$\nu_2 = c_0 / \lambda_2 = 2.99792458 \times 10^8 / 0.7 \times 10^{-6} = \underline{4.282749 \times 10^{14} \text{ s}^{-1}}$,

$\lambda_{boundary} = 25 \times 10^{-6}$ m, $\eta_{boundary} = 1 / \lambda_{boundary} = \underline{4 \times 10^4 \text{ m}^{-1}}$,

$\nu_{boundary} = c_0/\lambda_{boundary} = 2.99792458 \times 10^8 / 25 \times 10^{-6} = \underline{1.19917 \times 10^{13} \text{ s}^{-1}}$

Answer: 2.5×10^6 to 1.4286×10^6 m^{-1}; 4.2827×10^{14} to 7.4948×10^{14} s^{-1}; 4×10^4 m^{-1}; 1.1992×10^{13} s^{-1}.

1.2 Radiant energy at a wavelength of 2.0 μm is traveling through a vacuum. It then enters a medium with a refractive index of 1.24.

(a) Find the following quantities for the radiation in the vacuum: speed, frequency, wave number.

(b) Find the following quantities for the radiation in the medium: speed, frequency, wave number, and wavelength.

SOLUTION:

(a) $c_0 = 2.99792458 \times 10^8$ m/s; $\lambda_o = 2.0 \times 10^{-6}$ m, n = 1.24,

so speed in the vacuum = $\underline{2.99792458 \times 10^8 \text{ m/s}}$;

$\nu_0 = c_0 / \lambda_0 = 2.99792458 \times 10^8 / 2.0 \times 10^{-6} = \underline{1.49896229 \times 10^{14} \text{ s}^{-1}}$.

$\eta_0 = 1 / \lambda_0 = \underline{5 \times 10^5 \text{ m}^{-1}}$;

(b) Speed in the medium $c_m = c_0 /n = 2.99792458 \times 10^8 /1.24$

$= \underline{2.417681113 \times 10^8 \text{ m/s}}$;

$\nu_m = \nu_0 = \underline{1.49896229 \times 10^{14} \text{ s}^{-1}}$;

$\lambda_m = c_m / \nu_m = \underline{2.417681113 \times 10^8} / \underline{1.49896229 \times 10^{14}} = \underline{1.6129032 \times 10^{-6} \text{ m}}$;

$\eta_m = 1 / \lambda_m = \underline{6.20 \times 10^5 \text{ m}^{-1}}$.

Answer: (a) 2.9979 x10^8 m/s; 1.4990 x10^{14}s^{-1}; 5x10^5 m^{-1}
(b) 2.4176 x 10^8 m/s; 1.4990 x 10^{14} s^{-1}; 6.20 x 10^5 m^{-1}; 1.6129 x 10^{-6} m

1.3 Radiation propagating within a medium is found to have a wavelength within the medium of 1.570 μm and a speed of 2.500×10^8 m/s.

(a) What is the refractive index of the medium?

(b) What is the wavelength of this radiation if it propagates into a vacuum?

SOLUTION:

$C_0 = 2.99792458 \times 10^8$ m/s; $\lambda_m = 1.570 \times 10^{-6}$ m; $c_m = 2.500 \times 10^8$ m/s.

(a) $n_m = c_0/c_m = \underline{1.199}$

(b) $\lambda_0 = n_m \lambda_m = 1.882 \times 10^{-6}$ m = $\underline{1.882}$ μm.

Answer: (a) 1.199; (b) 1.882 μm.

1.4 What range of radiation wavelengths are present within a glass sheet that has a wavelength-independent refractive index of 1.33 when the sheet is exposed in vacuum to incident radiation in the visible range λ_0 =0.4 to 0.7 μm?

SOLUTION: $n_m = 1.33$; $\lambda_{m1} = 0.4/n_m = \underline{0.301\ \mu m}$;

$\lambda_{m2} = 0.7/n_m = \underline{0.526\ \mu m.}$

Answer: 0.301 to 0.526 μm.

1.5 A material has an index of refraction *n(x)* that varies with position x within its thickness. Obtain an expression in terms of c_o and *n(x)* for the transit time for radiation to pass through a thickness *L*. If $n(x) = n_i(1 + kx)$, where n_i and k are constants, what is the relation for transit time? How does wave number (relative to that in a vacuum) vary with position within the medium?

SOLUTION: $n(x) = c_o / c(x)$, so $c(x) = c_o / n(x) = dx/d\tau$. Then

$$t = \frac{1}{c_0}\int_{x=0}^{L} n(x)dx = \frac{n_i}{c_0}\int_{x=0}^{L}(1+kx)dx = \frac{n_i}{c_0}\left(L + \frac{kL^2}{2}\right)$$

$c = \lambda_m \nu$; $\nu = c / \lambda_m = c_o / \lambda_o$

$c_o / c = n = \lambda_o / \lambda_m$; $\lambda_m / \lambda_o = 1/n$ so

$\eta_m / \eta_o = n = \underline{n_i(1 + kx)}$.

Answer : $\dfrac{n_i}{c_o}\left(L + \dfrac{kL^2}{2}\right)$; $n_i(1 + kx)$.

1.6 Derive Equation 1.26 by analytically finding the maximum of the $E_{\lambda b}/T^5$ vs. λT relation (Equation 1.20).

SOLUTION: Take the derivative of $E_{\lambda b}/T^5$ with respect to λT. To simplify notation, let $\mu = \lambda T$.

$$\frac{d(E_{\lambda b}/T^5)}{d(\mu)} = \frac{d}{d(\mu)}\left[\frac{2\pi C_1}{(\mu)^5\left[\exp(C_2/\mu)-1\right]}\right] = 2\pi C_1 \frac{d}{d(\mu)}\left\{(\mu)^{-5}/\left[\exp(C_2/\mu)-1\right]\right\}$$

$$= 2\pi C_1\left\{[-5(\mu)^{-6}]/\left[\exp(C_2/\mu)-1\right] - (\mu)^{-5}\frac{\left(-C_2/(\mu)^2\right)\exp(C_2/\mu)}{\left[\exp(C_2/\mu)-1\right]^2}\right\}$$

$$= \frac{2\pi C_1}{\mu^6\left[\exp(C_2/\mu)-1\right]}\left\{[-5 + \frac{C_2/\mu}{\left[1-\exp(-C_2/\mu)\right]}\right\}$$

Setting the result = 0 to find the maximum,

$$\frac{C_2/\mu_{\max}}{1-\exp(-C_2/\mu_{\max})} = \frac{C_2/(\lambda T)_{\max}}{1-\exp\left[-C_2/(\lambda T)_{\max}\right]} = 5$$

Clearly, the product $(\lambda T)_{max}$ at the maximum of the Planck curve is equal to a constant, which must be found by iteration. Using iteration or a root-finding program gives $(\lambda T)_{\max} = C_3 = \underline{2897.8 \mu m \cdot K}$.

1.7 A blackbody is at a temperature of 1250 K (1250 K), and is in air.

(a) What is the spectral intensity emitted in a direction normal to the black surface at λ = 3.75 μm?

(b) What is the spectral intensity emitted at θ = 45° with respect to the normal of the black surface at λ = 3.75 μm?

(c) What is the directional spectral emissive power from the black surface at θ = 45° and λ = 3.75 μm?

(d) At what λ is the maximum spectral intensity emitted from this blackbody, and what is the value of this intensity?

(e) What is the hemispherical total emissive power of the blackbody?

SOLUTION:

(a) At λT = 4687.5 μm K, $I_{\lambda b,n} = \dfrac{2C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}$ = <u>7823 W/m^2•μm•sr</u>.

(b) Because the intensity from a blackbody is independent of angle of emission, the result is the <u>same as part (a)</u>.

(c) $E_{\lambda b}$(3.75 μm, 45°) = $I_{\lambda b}$ cos 45° = <u>7823</u> x 0.7071 = <u>5532 W/m^2•μm•sr</u>

(d) From Equation 1.26, $\lambda_{max}T = C_3$, and from Table A.4, C_3 = 2897.8 μm K. Thus, λ_{max} = 2318.2 / 1250 = <u>2.3182 μm</u>. At λ_{max},

$$I_{\lambda b,\max} = \frac{2C_1}{\lambda_{\max}^5\left(e^{C_2/\lambda_{\max}T}-1\right)} = \underline{12499.08\ \text{W/m}^2\bullet\mu\text{m}\bullet\text{sr}}$$

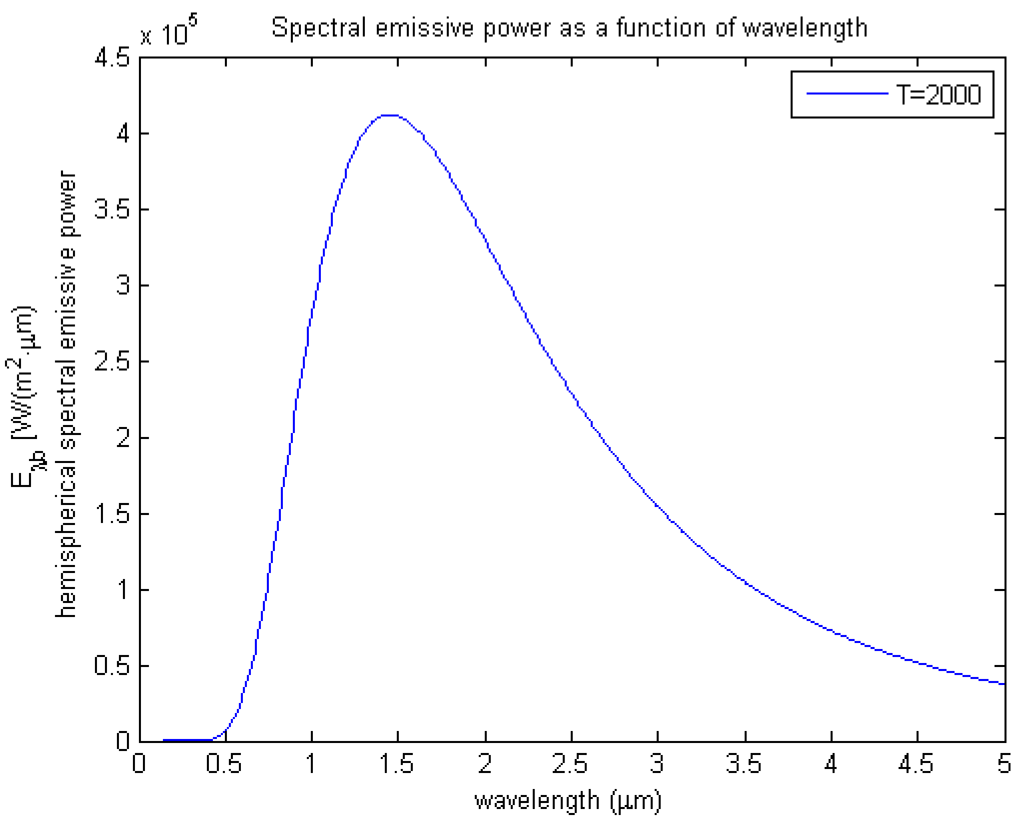
(e) From Equation 1.32 and Table A-4, $E_b = \sigma T^4$

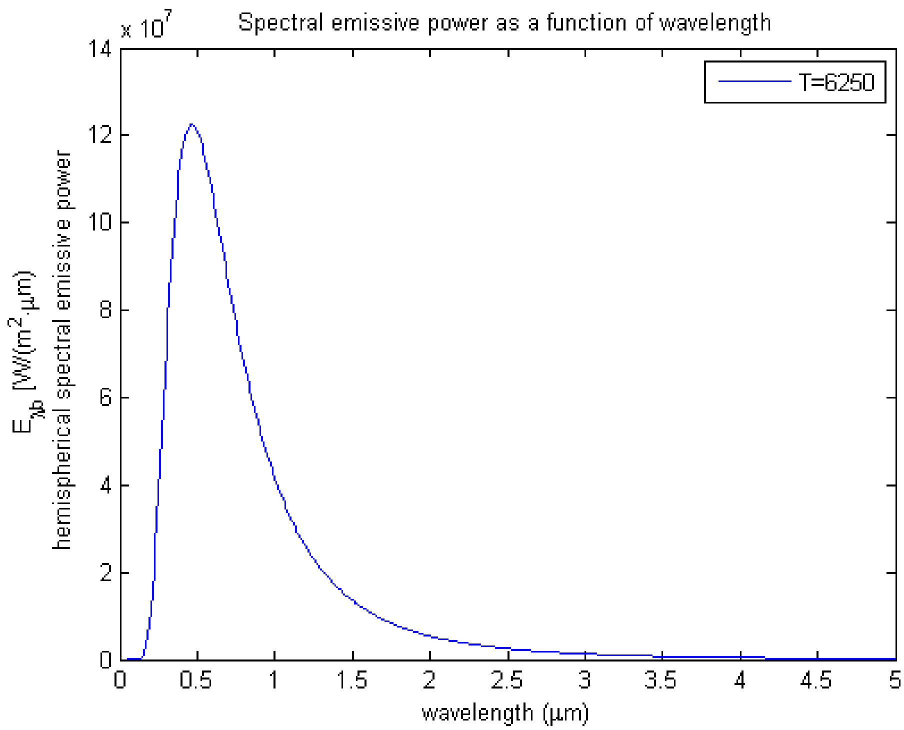
$= 5.67040\text{x}10^{-8}\ \text{W/m}^2\bullet\text{K}^4 \times (1250)^4\ \text{K}^4 = \underline{138{,}437.5\ \text{W/m}^2}$

Answers: (a) 7,823W/(m^2·μm·sr); (b) 7,823W/(m^2·μm·sr);
(c) 5532 W/(m^2·μm·sr); (d) 2.3182 μm, 12499 W/(m^2·μm·sr); (e) 138,437 W/m^2 .

1.8 Plot the hemispherical spectral emissive power $E_{\lambda b}$ for a blackbody in air [W/(m^2·μm)] as a function of wavelength (μm) for surface temperatures of 2000 and 6250K.

SOLUTION: $E_{\lambda b} = 2\pi C_1 / \{\lambda^5[e^{(C_2/\lambda T)} - 1]\}$ For a wavelength range of 0.01 to 5 μm, following figures are obtained for different surface temperatures .





1.9 For a blackbody at 2250 K that is in air, find :

(a) the maximum emitted spectral intensity (kW /m^2•μm•sr).

(b) the hemispherical total emissive power (kW /m^2).

(c) the emissive power in the spectral range between λ_o = 2 and 8μm.

(d) the ratio of spectral intensity at λ_o = 2 μm to that at λ_o = 8 μm.

SOLUTION:

(a) $I_{\lambda b,max} = C_4 T^5 = 4.09570\text{x}10^{-12} \text{ x } 2250^5$ = <u>236.17 kW / μm•m^2•sr</u>

(b) $E_b = \sigma T^4 = 5.67040\text{x}10^{-8}$(W/m^2•K^4) x (2250)^4 (K^4) = <u>1453.26 kW/m^2</u>

(c) Using Equation 1.37, for $\lambda_2 T$ = 18,000 μm•K, $F_{0\rightarrow \lambda 2T}$ = 0.97766 and for $\lambda_1 T$ = 4500 μm•K, $F_{0\rightarrow \lambda 1T}$ = 0.56429

$F_{\lambda 2T \rightarrow \lambda 1T}$ = 0.41336

$E_{b(\lambda 1 \rightarrow \lambda 2)} = 0.3482\, \sigma T^4$ = <u>600.730 kW/m</u>^{2}

(d) Using $I_{\lambda b,n} = \dfrac{2C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}$,

[$I_{\lambda b}(\lambda=2)$] / [$I_{\lambda b}(\lambda=8)$] = 1.5861x10^5/ 2.9695x10^3 = <u>53.4139</u>

Answers: (a) <u>236.17</u> kW/m^2•μm•sr; (b) <u>1453.26</u> kW/m^2;
(c) <u>600.730</u> kW/m^2; (d) 53.4139

1.10 Determine the fractions of blackbody energy that lie below and above the peak of the blackbody curve.

SOLUTION: At the peak, $\lambda_{max}T$ = 2893 μmK. Using Equation 1.37 with $\xi \equiv \frac{C_2}{\lambda T} = \frac{14388(\mu m \cdot K)}{2897.8(\mu m \cdot K)} = 4.965$ gives $F_{0-2897.8} = 0.25005$. Therefore about 25 percent of the blackbody energy is at wavelengths below the peak, and 75 percent is at longer wavelengths.

1.11 A blackbody at 1250K is radiating in the vacuum of outer space.

(a) What is the ratio of the spectral intensity of the blackbody at λ = 2.0μm to the spectral intensity at λ = 5μm?

(b) What fraction of the blackbody emissive power lies between the wavelengths of λ = 2.0μm and λ = 5μm?

(c) At what wavelength does the peak energy in the radiated spectrum occur for this blackbody?

(d) How much energy is emitted by the blackbody in the range $2.0 \le \lambda \le 5$ μm?

SOLUTION:

(a) $\lambda_1 T$ = 2.0 x 1250 = 2500 μm K; $\lambda_2 T$ = 5 x 1250 = 6250 μm K

(Note that these values bracket the peak in the blackbody curve.)

Using $I_{\lambda b} = \frac{2C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$,

$I_{\lambda 1b} / I_{\lambda 2b}$ = 11823.5 / 4237.4 = <u>2.7903</u> (b) From Equation 1.37, $F_{0\to\lambda_1 T} = F_{0\to 2500}$ = 0.16136,

$F_{0\to\lambda_2 T} = F_{0\to 7250}$ = 0.75792. The fraction between $\lambda_1 T$ and $\lambda_2 T$ is then= ((0.75792- 0.16136) = <u>0.59656</u>.

(c) From Table A.4, C_3 = 2897.77 μm K, so λ_{max} = 2897.77 / 1250 = <u>2.31824 μm</u>.

(d) From Table 1.2, the band emission is

$(F_{0\to\lambda_2 T} - F_{0\to\lambda_1 T})\,\sigma T^4$ = <u>0.59656</u> x 5.67040x10^{-8}(W/m^2•K^4) x (1250)^4 (K^4)

= <u>82587.2 W/m^2</u>

Answer: (a) <u>2.7903</u>; (b) <u>0.59656</u>.; (c) <u>2.31824</u>μm; (d) <u>82587.2</u> W/m^2.

1.12 Solar radiation is emitted by a fairly thin layer of hot plasma near the sun's surface. This layer is cool compared with the interior of the sun, where nuclear reactions are occurring. Various methods can be used to estimate the resulting effective radiating temperature of the sun, such as determining the best fit of a blackbody spectrum to the observed solar spectrum. Use two other methods (below), and compare the results to the oft-quoted value of T_{solar} = 5780 K.

a) Using Wien's Law and taking the peak of the solar spectrum as 0.50 μm, estimate the solar radiating temperature.

b) Given the measured solar constant in Earth orbit of 1368 W/m^2, and using the "inverse square law" for the drop in heat flux with distance, estimate the solar temperature. The mean radius of the Earth's orbit around the sun as 149x10^6 km and the diameter of the sun as 1.392x10^6 km.

SOLUTION:

a) $(\lambda T)_{max} = C_3 = 2898 \mu m \cdot K; \quad T_{solar} = \dfrac{2898 \mu m \cdot K}{0.50 \mu m} = \underline{5796 K}$

$$q_{solar} = 1353 (W/m^2) = q_{Sun} \frac{R^2_{Sun}}{R^2_{Earth\ orbit}} = \sigma T^4_{Sun} \frac{R^2_{Sun}}{R^2_{Earth\ orbit}} : \quad T_{Sun} = \left(\frac{q_{solar}}{\sigma \dfrac{R^2_{Sun}}{R^2_{Earth\ orbit}}} \right)^{1/4}$$

b)

$$= \left(\frac{1368 (W/m^2)}{5.6704 \times 10^{-8} (W/m^2 K^4) \dfrac{(1.392 \times 10^5 / 2)^2 (km^2)}{(149 \times 10^6)^2 (km^2)}} \right)^{1/4} = \underline{5766 K}$$

Answers: a) 5796K; b) 5766 K.

1.13 The surface of the sun has an effective blackbody radiating temperature of 5780 K.

(a) What percentage of the solar radiant emission lies in the visible range λ = 0.4 to 0.7 μm?

(b) What percentage is in the ultraviolet?

(c) At what wavelength and frequency is the maximum energy per unit wavelength emitted?

(d) What is the maximum value of the solar hemispherical spectral emissive power?

SOLUTION:

(a) $\lambda_1 T$ = 5780 K x 0.4 μm = 2312 μm K ;

$\lambda_2 T$ = 5780 K x 0.7 μm = 4046 μm K

$\Delta F = F_{0 \to \lambda 2T} - F_{0 \to \lambda 1T}$ = 0.48916 - 0.12240 = 0.36676, or <u>36.7 %</u>

(b) From Fig. 1.2, the UV range is taken as 0.01 to 0.4 μm.

$\lambda_1 T$ = 5780 K x 0.01 μm = 57.8 μm K; $\lambda_2 T$ = 5780 K x 0.4 μm = 2312 μm K.

$\Delta F = F_{0 \to \lambda 2T} - F_{0 \to \lambda 1T}$ = 0.12240 - 0 = 0.12240, or <u>12.2 %</u>

(c) The maximum energy is at λ_{max}, where, from Table A.4, C_3 = 2897.8 μm K, so λ_{max} = 2897.8 / 5780 = 0.50134 μm.
The corresponding frequency is
$\nu_{max} = c_o / \lambda$ = 2.9979x10^8 (m/s) / 0.50134 x 10^{-6} (m) = 5.9798x10^{14} Hz

(d) At $\lambda_{max} T = C_3$ = 2897.8 μm K, using

$$E_{\lambda\,max\,b} = \frac{2\pi C_1}{\lambda_{max}^5\left(e^{C_2/\lambda_{max}T} - 1\right)}$$ gives $E_{\lambda,max,b}$) = 8.301x10^7 W/(m^2·μm).

Answer: (a) 36.7%; (b) 12.2%; (c) 0.5013 μm, 5.98x10^{14} Hz;
(d) 8.301x10^7 W/(m^2·μm).

1.14 A blackbody radiates such that the wavelength at its maximum emissive power is 2.00 μm. What fraction of the total emissive power from this blackbody is in the range λ = 0.7 to λ = 5 μm?

SOLUTION: Wien's law gives $\lambda_{max} T = C_3$ = 2897.8 μm K, so
$T = C_3/\lambda_{max}$= 2897.8 / 2= 1448.9 K. Thus, $\lambda_1 T$ = 1448.9 K x 0.7 μm = 1014.2 μm K; $\lambda_2 T$ = 1448.9 K x 5 μm = 7244.5 μm K. Using Equation 1.37, $\Delta F = F_{0\to\lambda 2T} - F_{0\to\lambda 1T}$ =
0.82137 - 0.00038= 0.82099, or 82.09 %

Answer: 0.821.

1.15 A blackbody has a hemispherical spectral emissive power of 0.03500W/(m^2·μm) [0.0400W/(m^2·μm)] at a wavelength of 90 μm. What is the wavelength for the maximum emissive power of this blackbody?

SOLUTION: With $E_{\lambda b}$ and λ given, solve for T from Planck's equation,
$E_{\lambda b} = 2\pi C_1 / \{ \lambda^5 [e^{(C_2/\lambda T)} - 1]\}$ which becomes $T = C_2/\{\lambda \ln [1+(2\pi C_1/\lambda^5 E_{\lambda b})]\}$
Substituting numerical values gives T = 154.7 K, and Wien's displacement law, $\lambda_{max} T = C_3$, is used to find λ_{max} = 18.73 μm.

Answer: 18.73 μm.

1.16 A radiometer is sensitive to radiation only in the interval $3.6 \le \lambda \le 8.5$μm. The radiometer is used to calibrate a blackbody source at 1000K. The radiometer records that the emitted energy is 4600 W/m^2. What percentage of the blackbody radiated energy in the prescribed wavelength range is the source actually emitting?

SOLUTION:

λ, μm	λT, μm K	$F_{0-\lambda T}$
3.6	3600	0.4036
8.5	8500	0.87417

$F_{\lambda 1T \to \lambda 2T}$ = 0.87417 - 0.4036 = 0.47058

$E_{b,\Delta\lambda} = F_{\lambda 1T \to \lambda 2T}\, \sigma T^4 = 0.47058 \times 5.67040\text{x}10^{-8} \times 1000^4 = 26{,}684\ \text{W/m}^2$

Percentage sensed = (4600/26684) x 100 = 17.239%

Answer: 17.24 %.

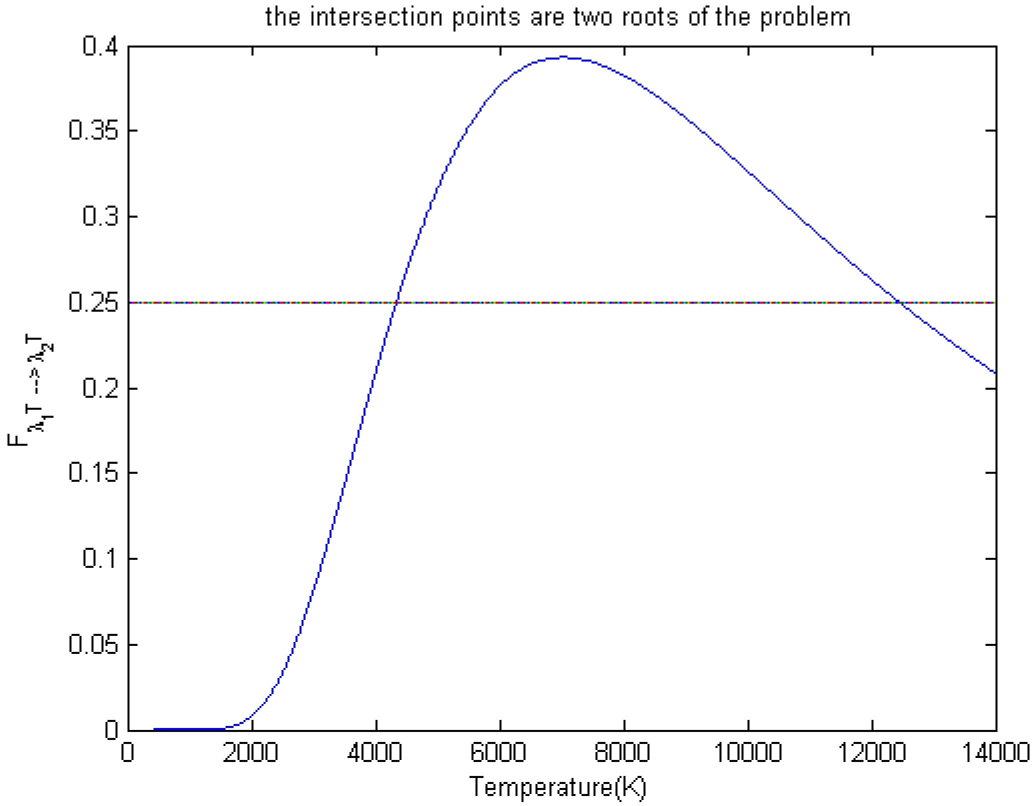
1.17 What temperature must a blackbody have for 25 percent of its emitted energy to be in the visible wavelength region?

SOLUTION: There are two solutions to this problem- one for the temperature at which the bulk of the energy is in the IR, and one for a higher temperature where the major portion is in the UV. We can assume a temperature and, by trial and error or by using a root solver, find the fraction in the visible ($0.4 \le \lambda \le 0.7$), and continue until the fraction is 0.25.

For example, for the lower temperature, try T = 4600 K. Then $\lambda_1 T$ = 4600 K x 0.4 μm = 1840 μm K; $\lambda_2 T$ = 4600 K x 0.7 μm = 3220 μm K. Using Equation 1.37, $\Delta F = F_{0\to\lambda 2T} - F_{0\to\lambda 1T}$ = 0.32253 - 0.04422 = 0.27831, or 27.8 %. Try a lower temperature and proceed to convergence.

Using a root solver in a computational software package, find T such that

$$\frac{\int_{\lambda=0}^{0.7} E_{\lambda b}(T)\,d\lambda}{\sigma T^4} - 0.25 = 0$$



This gives the two results T = 4,343 and 12,460 K.

Answer: 4,343 K, 12,460 K. (Note, two solutions are possible!)

1.18 Show that the blackbody spectral intensity $I_{\lambda b}$ increases with T at any fixed value of λ .

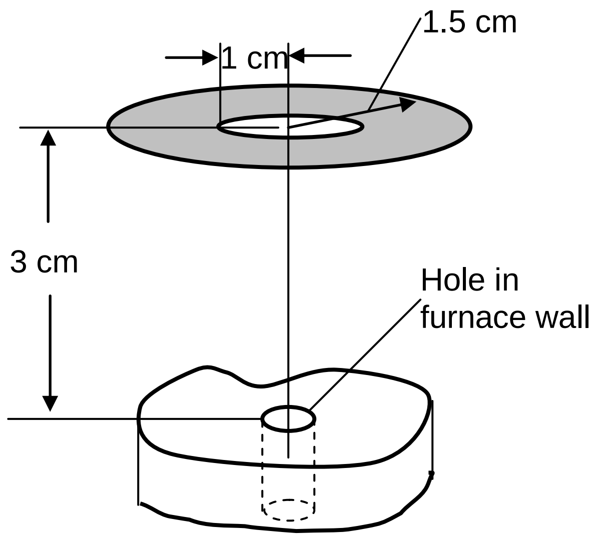
SOLUTION: From Equation 1.15, $I_{\lambda b} = 2C_1/\{\lambda^5 [e^{C_2/\lambda T} - 1] \}$. The change in $I_{\lambda b}$ with T for a fixed λ is:

$$\left(\frac{\partial I_{\lambda b}}{\partial T}\right)_\lambda = \frac{-2C_1 e^{\frac{C_2}{\lambda T}}\left(-\frac{C_2}{\lambda T^2}\right)}{\lambda^5\left[e^{\frac{C_2}{\lambda T}}-1\right]^2} = \frac{2C_1C_2}{\lambda^6}\frac{e^{\frac{C_2}{\lambda T}}}{T^2\left[e^{\frac{C_2}{\lambda T}}-1\right]^2}$$

The values of C_1, C_2, and λ are all positive, so that $(\partial I_{\lambda b} / \partial T)_\lambda$ is always positive. Hence, $I_{\lambda b}$ must increase with T for every λ.

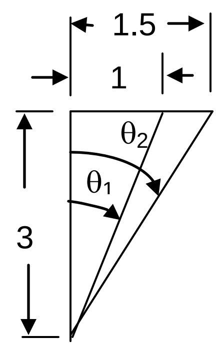
There are other ways to carry out this proof, such as plotting $I_{\lambda b}$ or showing that $I_{\lambda b}$ itself increases as T increases by noting that the denominator of Planck's spectral distribution of intensity decreases with increasing T at any fixed wavelength.

1.19 Blackbody radiation is leaving a small hole in a furnace at 1200 K (see the figure.) What fraction of the radiation is intercepted by the annular disk? What fraction passes through the hole in the disk?



SOLUTION: From Table 1.2, the fraction of radiation to an annular disk is:

$$\frac{2\pi}{\pi}\frac{\sin^2\theta_2 - \sin^2\theta_1}{2} = \sin^2\theta_2 - \sin^2\theta_1$$



where $\sin\theta_1 = 1/(3^2 + 1^2)^{1/2}$; $\sin\theta_2 = 1.5/(3^2 + 1.5^2)^{1/2}$.

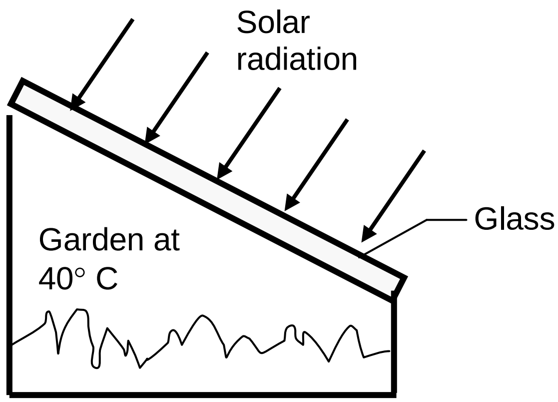
Then $F_{\text{annular disk}} = 1.5^2/(3^2 + 1.5^2) - 1/(3^2 + 1^2) = 0.2000 - 0.1000 = \underline{0.1000.}$

$F_{\text{hole}} = \sin^2\theta_1 = 1/(3^2 + 1^2) = \underline{0.1000}$

Answer: 0.1000; 0.1000.

1.20 A sheet of silica glass transmits 85% of the radiation that is incident in the wavelength range between 0.38 and 2.7μm, and is essentially opaque to radiation having longer and shorter wavelengths. Estimate the percent of solar radiation that the glass will transmit. (Consider the sun as a blackbody at 5780 K.)

If the garden in a greenhouse radiates as a black surface and is at 40 °C, what percent of this radiation will be transmitted through the glass?



SOLUTION: Part 1; $\lambda_1 T = 0.38\ \mu m \times 5780\ K = 2196.4\ \mu m\ K$; $\lambda_2 T = 2.7\ \mu m \times 5780\ K = 15606 \mu m\ K$. Using Equation 1.37, $\Delta F = F_{0\rightarrow\lambda 2T} - F_{0\rightarrow\lambda 1T} = 0.96951 - 0.10022 = 0.86929$. The percent transmitted is then 0.86929 x 0.85 x 100 = 73.9 %.

Part 2: $\lambda_1 T = 0.38\ \mu m \times (40 + 273)\ K = 118.94\ \mu m\ K$; $\lambda_2 T = 2.7\ \mu m \times 313\ K = 845.1 \mu m\ K$. Using Equation 1.37, $\Delta F = F_{0\rightarrow\lambda_2 T} - F_{0\rightarrow\lambda_1 T} = 0.000037 - 0.00000 = 0.000037$. The percent transmitted is then 0.000037 x 0.88 x 100 = 0.003 %.

Answer: 73.9%; 0.003%

1.21 Derive Wien's displacement law in terms of wave number by differentiation of Planck's spectral distribution in terms of wave number, and show that $T/\eta_{max} = 5099.4$ μm K.

SOLUTION: From Equation 1.17, $E_{\eta b} = 2\pi C_1 \eta^3/[e^{C_2\eta/T} - 1]$. To obtain the maximum, differentiate with respect to η,

$$\left(\frac{\partial E_{\eta b}}{\partial \eta}\right)_T = \frac{6\pi C_1 \eta^2 \left(e^{\frac{C_2\eta}{T}} - 1\right) - 2\pi C_1 \eta^3 \left(C_2/T\right) e^{\frac{C_2\eta}{T}}}{\left(e^{\frac{C_2\eta}{T}} - 1\right)^2}$$

Setting this equal to zero gives $3\left(e^{\frac{C_2\eta_{max}}{T}} - 1\right) = \frac{C_2\eta_{max}}{T} e^{\frac{C_2\eta_{max}}{T}}$

for which the solution is $(T/\eta_{max}) = \text{constant}$. Substitution of $(T/\eta_{max}) = 5099.4\ \mu m\ K$ shows this solution to be valid.

1.22 A student notes that the peak emission of the sun according to Wien's displacement law is at a wavelength of about $\lambda_{max} = C_3/5780$ K $= 2897.8/5780 = 0.501$ μm. Using $\eta_{max} = 1/0.501$ μm, the student solves again for the solar temperature using the result derived in Homework Problem 1.21. Does this computed temperature agree with the solar temperature? Why? (This is not trivial-- put some thought into *why.*)

SOLUTION: Solving for T using $\eta_{max} = 1/0.501$ μm in $(T / \eta_{max}) = 5099.4$ μm K gives T = 5099.4 x 0.501 = 2554.8 K, which is much too small for the solar radiating temperature. The reason is that E_λ is the energy per unit λ interval, while E_η is the energy per unit η interval. The peaks in the curves of E_λ vs λ and E_η vs η do not occur at corresponding values of η and λ as related by $\lambda = 1/\eta$. (See Homework Problem 1.24 for the correct relation.)

1.23 Derive the relation between the wave number and the wavelength at the peak of the blackbody emission spectrum. (You may use the result of Homework Problem 1.22.)

SOLUTION: At a given temperature, Wien's law in terms of wavelength (in SI units) is $T = 2897.8 / \lambda_{max}$ and, from the result of Homework Problem 1.18, in terms of wave number, $T = 5099.4\ \eta_{max}$. At a given T, these may be equated to give $\eta_{max} = 2897.8 / (5099.4\ \lambda_{max}) = 0.56826 / \lambda_{max}$ where η_{max} will be in μm^{-1}. For wave number in cm^{-1} and wavelength in μm, the relation is

$$\eta_{max}\ (cm^{-1}) = 5682.6 / \lambda_{max}\ (\mu m)$$

Answer: $\eta_{max}\ (cm^{-1}) = 5682.6 / \lambda_{max}\ (\mu m)$.

1.24 A solid copper sphere 3 cm in diameter has on it a thin black coating. Initially, the sphere is at 850 K, and it is then placed in a vacuum with very cold surroundings. How long will it take for the sphere to cool to 200K? Because of the high thermal conductivity of copper, it is assumed that the temperature within the sphere is uniform at any instant during the cooling process. (Properties of copper: density, $\rho = 8950$ kg/m^3; specific heat, c = 383 J/kg•K.)

SOLUTION: An energy balance on the sphere gives $-\rho c V \frac{dT}{d\tau} = \sigma T^4 A_s$, or

$d\tau = -\frac{\rho c V}{\sigma A_s}\frac{dT}{T^4}$. Integrating and using $V=\pi D^3/6$ and $A_s = \pi D^2$,

$$\tau = \frac{\rho c D}{18\sigma}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right) = 12430 \text{ s} = 3.453 \text{ hr}\cdot$$

Answer: 3.453 hr.

1.25 A black rectangular sheet of metal, 6 cm by 10 cm in size, is heated uniformly with 2350 Watts by passing an electric current through it. One face of the rectangle is well insulated. The other face is exposed to vacuum and very cold surroundings. At thermal equilibrium, what fraction of the emitted energy is in the wave number range from 0.33 to 2 μm^{-1}?

SOLUTION: The energy emitted must be equal to the electrical energy supplied: $Q_{elec} = \sigma T_A{}^4 A$, or $T_A = (Q_{elec}/A\sigma)^{1/4} = [2350/(6x10x10^{-4}x5.6704x10^{-8})]^{1/4}$ = 1621.2 K. The wavelengths corresponding to the wave numbers are $\lambda_1 = 1/2 = 0.5\ \mu m$; $\lambda_2 = 1/0.33 = 3.03\ \mu m$. Then $\lambda_1 T = 0.5x1621.2 = 810.58\ \mu m \bullet K$ and $\lambda_2 T = 2x1621.2 = 4912.6 \mu m \bullet K$. Using the series expression Equation 1.37 for $F_{0\to\lambda T}$ gives the fraction of emitted energy as <u>0.6225</u>.

Answer: <u>0.6225.</u>

1.26 Spectral radiation at $\lambda = 2.445\ \mu m$ and with intensity 7.35 kW/($m^2 \bullet \mu m \bullet sr$) enters a gas and travels through the gas along a path length of 21.5 cm. The gas is at uniform temperature 1000 K and has an absorption coefficient $\kappa_{2.445\ \mu m} = 0.557\ m^{-1}$. What is the intensity of the radiation at the end of the path? Neglect scattering, but include emission by the gas.

SOLUTION: Taking into account both absorption and emission gives

$I_\lambda(S) = I_\lambda(0)e^{-\kappa_\lambda S} + I_{\lambda b}\left(1 - e^{-\kappa_\lambda S}\right)$ and $e^{-\kappa_\lambda S} = e^{-0.557\times 0.215} = 0.8871$

For $I_{\lambda b}$ at 2.445 μm and 1000 K, Equation 1.15 gives $I_{\lambda b}$=3.803 kW/($m^2\ \mu m$ sr)

Substituting into the original equation gives

$$I_\lambda = 7.35 \times 0.8871 + 3.803(1 - 0.8871) = 6.950 kW / (m^2 \cdot \mu m \cdot sr).$$

Answer: 6.950 kW/($m^2 \bullet \mu m \bullet sr$).

1.27 Radiation from a blackbody source at 2000 K is passing through a layer of air at 12,000 K and 1 atm. Considering only the transmitted radiation (that is, not accounting for emission by the air), what path length is required to attenuate by 35% the energy at the wavelength corresponding to the maximum emission by the blackbody source? (At this λ, take $\kappa_\lambda = 1.200 \times 10^{-1}\ cm^{-1}$ for air at 12,000 K and 1 atm.

SOLUTION: From Wien's displacement law,

$$\lambda_{max} = \frac{C_3}{T} = \frac{2897 \mu m \cdot K}{2000K} = 1.4489 \mu m$$

Then $\dfrac{I_\lambda(S)}{I_\lambda(0)} = 0.65 = e^{-\kappa_\lambda S} = e^{-0.12S}$

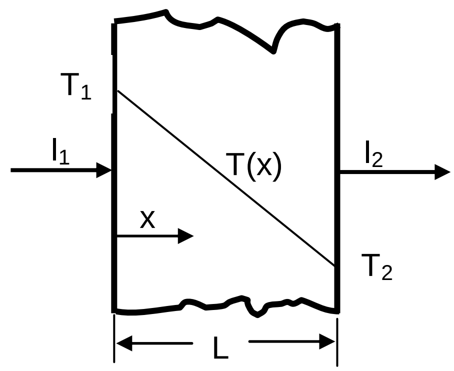
Solving for S yields S = <u>3.589 cm</u>.

Answer: 3.59 cm.

1.28 A gas layer at constant pressure P has a linearly decreasing temperature across the layer, and a constant mass absorption coefficient κ_m (no scattering). For radiation passing in a normal direction through the layer, what is the ratio I_2/I_1 as a function of T_1, T_2, and L? The temperature range T_2 to T_1 is low enough that emission from the gas can be neglected. The gas constant is R.

SOLUTION: For a linear temperature distribution,

$dT = -\frac{T_1 - T_2}{L} dx$, and for



absorption, $dI = -\kappa(x) I(x) dx = -\kappa_m \rho(x) I(x) dx = -\kappa_m \frac{P}{RT(x)} I(x) dx$ Integrating from x = 0 to x = L, $\int_{I_1}^{I_2} \frac{dI}{I(x)} = -\frac{\kappa_m P}{R} \int_0^L \frac{dx}{T(x)} = \frac{\kappa_m PL}{R(T_1 - T_2)} \int_{T_1}^{T_2} \frac{dT}{T(x)}$

Then, $\ln\left(\frac{I_2}{I_1}\right) = \frac{\kappa_m PL}{R(T_1 - T_2)} \ln\left(\frac{T_2}{T_1}\right)$ or $\frac{I_2}{I_1} = e^{-\frac{\kappa_m PL}{R(T_1 - T_2)} \ln\left(\frac{T_1}{T_2}\right)}$

Answer: $\frac{I_2}{I_1} = e^{-\frac{\kappa_m PL}{R(T_1 - T_2)} \ln\left(\frac{T_1}{T_2}\right)}$.

1.30 Hawking (1974) predicts that black holes should emit a perfect blackbody radiation spectrum in proportion to the surface gravity of the black hole. Further predictions are that the surface gravity of a black hole is inversely proportional to its mass. Thus, very small black holes should emit very large blackbody radiation. If a black hole with 1 Earth mass has a predicted blackbody temperature of 10^{-7} K, what fraction of an Earth mass black hole will emit at T = 2.7K (equal to the background temperature of space) and thus possibly be detectable?

SOLUTION: The relation between black hole mass and its emitted blackbody temperature is, from the problem statement,

$\frac{T_1}{T_2} = \frac{m_2}{m_1}$, so $m_2 = m_{Earth} \times \left(\frac{10^{-7}}{2.7}\right) = 3.7 \times 10^{-8} m_{Earth}$

(The Earth mass is about 5.97×10^{24} kg, so the smaller black hole will have an effective mass of 2.2×10^{17} kg.)

1.30 Observers at NASA noticed an unexpected small deceleration of the early deep space probes Pioneer 10 and 11 as they moved away from Earth. This was known as "the Pioneer Anomaly." After many speculations and false starts, researchers determined that thermal radiation pressure exerted by radiation from thermoelectric generators heated by small nuclear power sources plus radiated waste heat from the electronics was the culprit. Assuming a projected radiating area of 0.2 m^2 facing along the spacecraft trajectory, a spacecraft mass of 258 kg, and a mean radiating temperature on a the outward facing surface (effective emissivity = 0.3) of 375 K, what deceleration was exerted on the spacecraft? [Much more detailed analysis can be found in Turyshev et al. (2012). The detailed thermal model predicts a deceleration of 7 to 10 $\times 10^{-10}$ m/s^2.]

SOLUTION: Taking $a = F/m = (PA)/m$, the radiation pressure due to emission from the surface is needed. The radiation pressure by emission is (Equation 1.65): $P = 2\epsilon\sigma T^4/3c = [3 \times 0.3 \times 5.67 \times 10^{-8} \times 375^4 (W/m^2)/3 \times 2.998 \times 10^8 (m/s)]$
$= 1.12 \times 10^{-6}$ Pa.
Thus, the deceleration is
$a = [1.12 \times 10^{-6} (Pa) \times 0.2 (m^2)/ 258 (kg)] = \underline{8.68 \times 10^{-10} m/s.}$

SOLUTIONS-CHAPTER 2

2.1 A material has a hemispherical spectral emissivity that varies considerably with wavelength but is fairly independent of surface temperature (see, for example, the behavior of tungsten in Figures 3.31 and 3.32. Radiation from a gray source at T_i is incident on the surface uniformly from all directions. Show that the total absorptivity for the incident radiation is equal to the total emissivity of the material evaluated at the source temperature T_i.

SOLUTION: For a gray source, the incident radiation is proportional to blackbody radiation at the source temperature; that is, $CE_{\lambda b}(T_i)\, d\lambda$ in the spectral range $d\lambda$.

From Equation 2.31 the total hemispherical absorptivity is

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (T_A) dQ_{\lambda,i} d\lambda}{\int_{\lambda=0}^{\infty} dQ_{\lambda,i} d\lambda}$$

Substituting for the incident energy, $\alpha = \dfrac{\int_{\lambda=0}^{\infty} \alpha_\lambda (T_A) CE_{\lambda b}\left(T_i\right) d\lambda}{\int_{\lambda=0}^{\infty} CE_{\lambda b}\left(T_i\right) d\lambda}$

From Table 2.2, $\alpha_\lambda (T_A) = \epsilon_\lambda (T_A) = \epsilon_\lambda$, so that $\alpha = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b}\left(T_i\right) d\lambda}{\sigma T_i^4}$

For properties independent of surface temperature, $\epsilon = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b}\left(T_i\right) d\lambda}{\sigma T_i^4} = \alpha$

2.2 Using Figure 3.31, estimate the hemispherical total emissivity of tungsten at 2600 K.

SOLUTION: Use a numerical or graphical integration to find $\epsilon = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda}{\sigma T^4}$

A careful numerical integration with ϵ_λ ($\lambda > 2.65\ \mu m$) = 0.1 gives $\epsilon = 0.284$.

Answer: 0.284.

2.3 Suppose that ϵ_λ is independent of λ (gray-body radiation). Show that $F_{0\to\lambda T}$ represents the fraction of the total radiant emission of the gray body in the range from 0 to λT.

SOLUTION: The emission in a wavelength interval from λ = 0 to λ is
$\int_{\lambda^*=0}^{\lambda} \epsilon_\lambda E_{\lambda b} d\lambda^*$ and, for all wavelengths the emission is $\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda$.
The fraction of energy emitted for the range 0 → λ is, if ϵ ε_λ is independent of λ,

$$\text{Fraction } (0 \to \lambda) = \frac{\epsilon_\lambda \int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\epsilon_\lambda \int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda} = \frac{\int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda}$$

From Equation 1.33, this is $F_{0\to\lambda T}$.

2.4 For a surface with hemispherical spectral emissivity ϵ_λ, does the maximum of the E_λ distribution occur at the same λ as the maximum of the $E_{\lambda b}$ distribution at the same temperature? (*Hint*: examine the behavior of $dE_\lambda/d\lambda$.) Plot the distributions of E_λ as a function of λ for the data of Figure 2.9 at 600 K and for the property data at 700 K. At what λ is the maximum of E_λ? How does this compare with the maximum of $E_{\lambda b}$?

SOLUTION: $E_\lambda = \epsilon_\lambda E_{\lambda b}$

$$\frac{dE_\lambda}{d\lambda} = \frac{d\left\{\dfrac{2\pi C_1 \epsilon_\lambda}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}\right\}}{d\lambda} = 2\pi C_1 \epsilon_t \frac{d}{d\lambda}\left\{\frac{\lambda^{-5}}{\left[\exp(C_2/\lambda T)-1\right]}\right\}$$

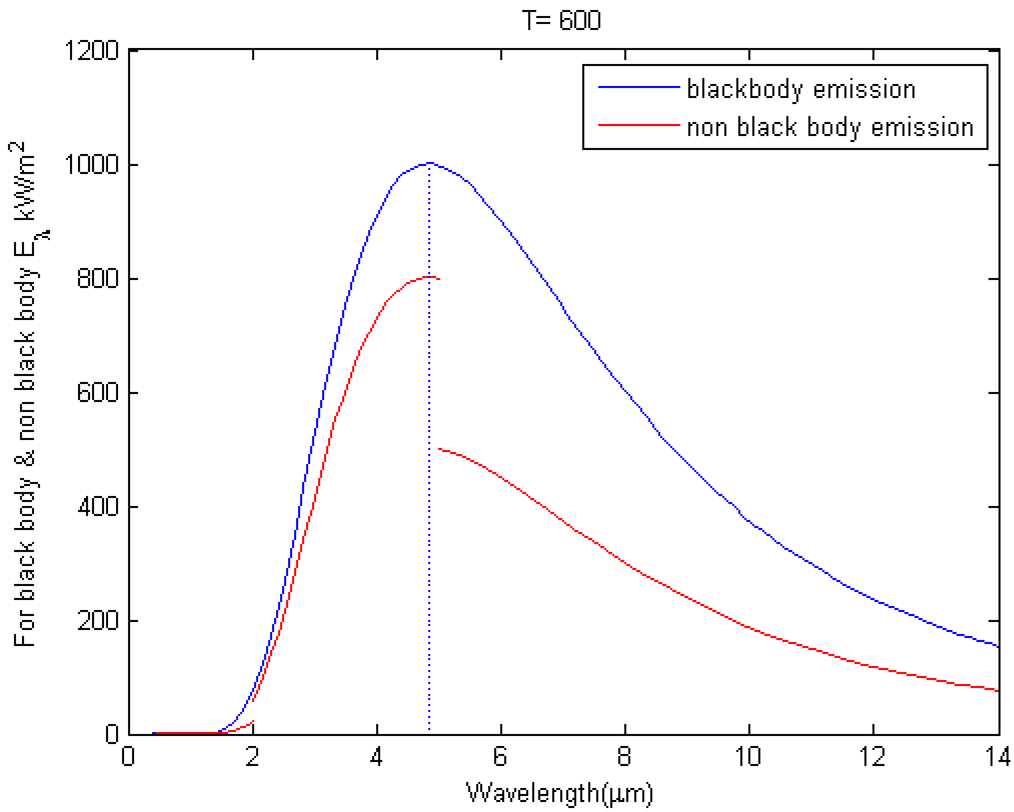
$$=2\pi C_1 \epsilon_t \left\{\frac{-5\lambda^{-6}}{\left[\exp(C_2/\lambda T)-1\right]} - \lambda^{-5}\frac{\left(-C_2/\lambda^2\right)\exp(C_2/\lambda)}{\left[\exp(C_2/\lambda T)-1\right]^2}\right\}$$

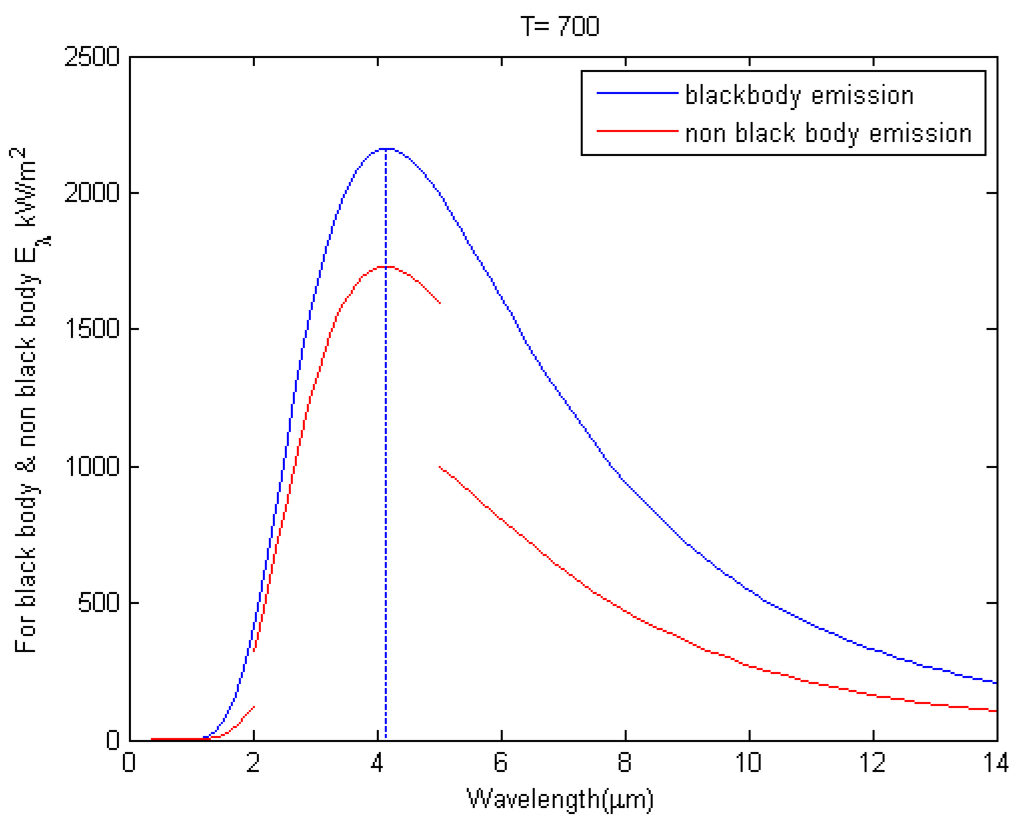
$$=\frac{2\pi C_1 \epsilon_t}{\lambda^6\left[\exp(C_2/\lambda T)-1\right]}\left\{-5 + \frac{(C_2/\lambda)}{\left[1-\exp(C_2/\lambda T)\right]}\right\}$$

where ϵ_t is the total hemispherical emittance.

Setting the result = 0 to find the maximum; $\dfrac{(C_2/\lambda)}{1-\exp(-C_2/\lambda T)} = 5$

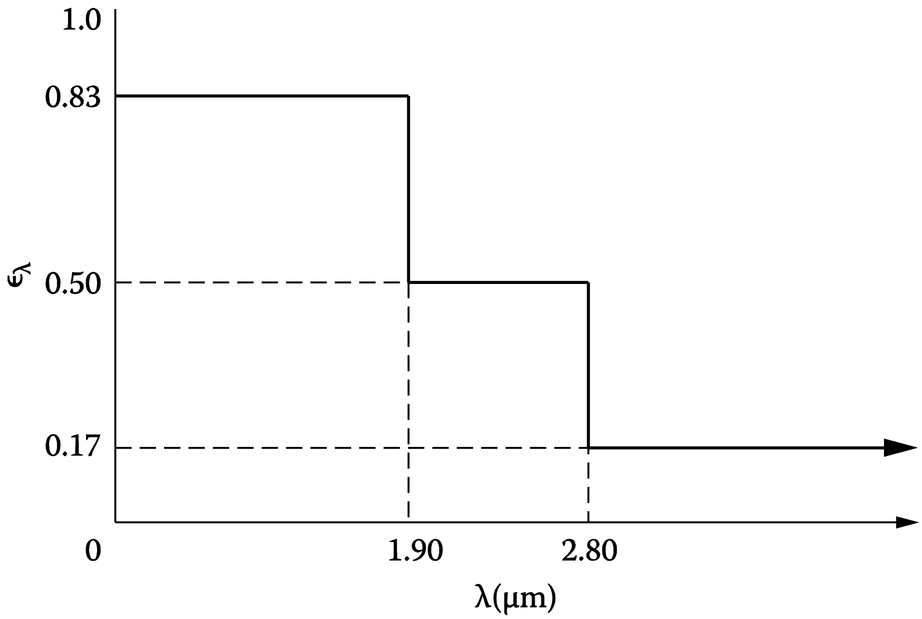
$(\lambda T)_{max}$=2897.8 μm.K. So for the same temperature, the maximum occurs at the same point as $E_{\lambda b}$. Only the intensity of this power is reduced by the numerical value of emittance. For data in Figure 2.9, the following figure is obtained


T= 600
blackbody emission
non black body emission
For black body & non black body E_λ kW/m^2
Wavelength(μm)


T= 700
blackbody emission
non black body emission
For black body & non black body E_λ kW/m^2
Wavelength(μm)

E_λ is maximum at 2897.8/600=4.83 μm which overlaps with the plot.
E_λ is maximum at 2897.8/700=4.14 μm which overlaps with the plot.

2-5 Find the emissivity at 400 K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure.



SOLUTION:
The emissivity using Equation 2.10 is

$$\epsilon(T)=\frac{\int_{\lambda=0}^{\infty}\epsilon_\lambda(T)E_{\lambda b}(T)d\lambda}{\sigma T^4}$$

$$=\frac{0.83\int_{\lambda=0}^{1.90}E_{\lambda b}(T)d\lambda}{\sigma T^4}+\frac{0.50\int_{\lambda=1.93}^{2.80}E_{\lambda b}(T)d\lambda}{\sigma T^4}+\frac{0.17\int_{\lambda=2.80}^{\infty}E_{\lambda b}(T)d\lambda}{\sigma T^4}$$

$$=0.83F_{0-1.90T}+0.50F_{1.90T-2.80T}+0.17F_{2.80T-\infty}$$

$$=0.83F_{0-1.90T}+0.50\left(F_{0-2.80T}-F_{0-1.90T}\right)+0.17\left(1-F_{0-2.80T}\right)$$

so only two blackbody fractions need be found. Using Equation 1.37 for the F values,

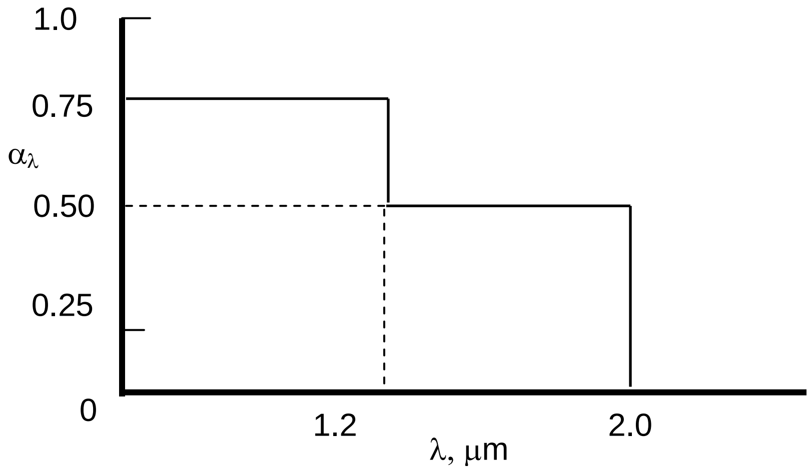
$$\epsilon(T=400)=0.83F_{0-1.90\times400}+0.5(F_{0-2.80\times400}-F_{0-1.90\times400})\quad+0.17(1-F_{0-2.80\times400})=$$
$$\approx 0+0.5\ \times 0.50\ \times(0.0011-\approx 0)+0.17\ \times(1-0.0011)=0.1704$$

$$\epsilon(T=5780K)=0.83F_{0-1.90\times5780}+0.50\left(F_{0-2.80\times5780}-F_{0-1.90\times5780}\right)+0.17\left(1-F_{0-2.80\times5780}\right)$$

$$=0.83\times0.9316+0.50\times\left(0.9745-0.9316\right)+0.17\times(1-0.9745)=\underline{0.7990}.$$

Answer: ϵ (T=400 K) = 0.17; α_S(T=5780 K) = 0.7990

2.6 The surface temperature-independent hemispherical spectral absorptivity of a surface is measured when it is exposed to isotropic incident spectral intensity, and the results are approximated as shown below. What is the total hemispherical emissivity of this surface when it is at a temperature of 1000 K?

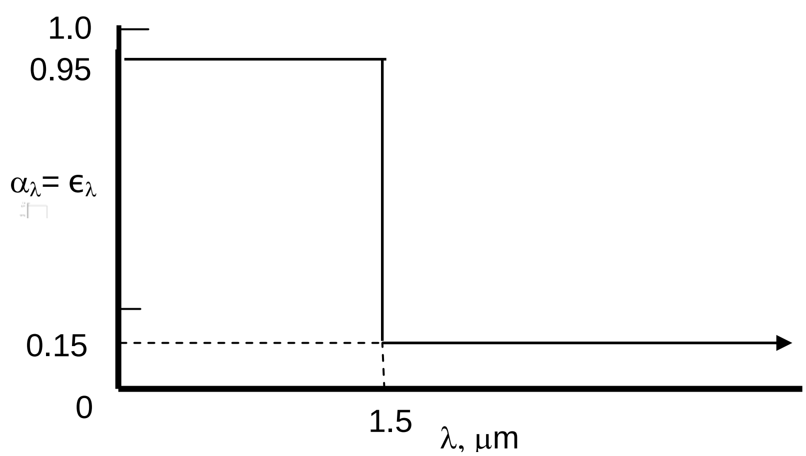


SOLUTION: $\epsilon_\lambda = \alpha_\lambda$ from Table 2.2 gives $\epsilon = \dfrac{\int_0^\infty \alpha_\lambda E_{\lambda b}(T_A)\,d\lambda}{\int_0^\infty E_{\lambda b}(T_A)\,d\lambda}$ so that

$$\epsilon = 0.75\, F_{0\to 1200} + 0.5\, F_{1200\to 2000}$$
$$= 0.75 \times 0.002134 + 0.5 \times (0.06673 - 0.002134)$$
$$= \underline{0.03390.}$$

Answer: <u>0.0339.</u>

2.7 (a) Obtain the total absorptivity of a diffuse surface with properties given in the figure for incident radiation from a blackbody with a temperature of 6200 K.
(b) What is the total emissivity of the diffuse surface with properties given in the figure if the surface temperature is 500 K?



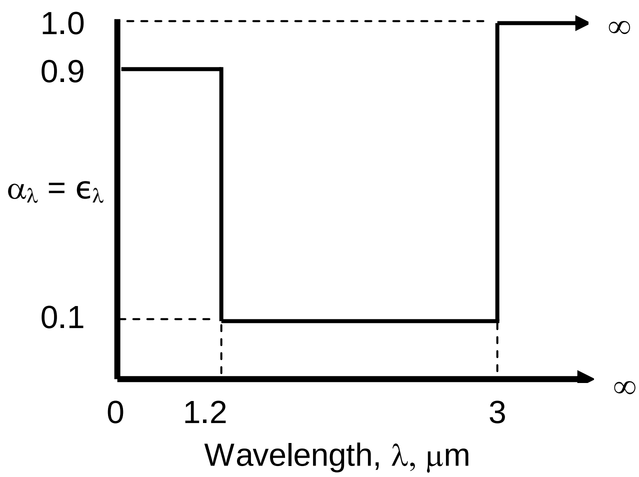
SOLUTION:

(a) Using $\alpha = \dfrac{\int_0^\infty \alpha_\lambda q_{\lambda,i} d\lambda}{\int_0^\infty q_{\lambda,i} d\lambda}$ with $q_{\lambda,i} = E_{\lambda b}(6200K)$ and Equation 1.37 for the blackbody fractions gives α=0.95 $F_{0\to 1.5x6200}$+0.15 (1-$F_{0\to 1.5x6200}$)
= 0.95 x 0.8975+0.15(1-0. 0.8975)= <u>0.8680</u>

(b) Similarly, for emission, ϵ=0.95 $F_{0\to 1.5x500}$+0.15 (1-$F_{0\to 1.5x500}$)
= 0.95 x ($\approx$0)+0.15[1-($\approx$0)] = <u>0.15.</u>

Answer: (a) 0.8680; (b) 0.15.

2.8 For the spectral properties given in the figure for a diffuse surface:
(a) what is the solar absorptivity of the surface (assume the solar temperature is 5800 K)?
(b) what is the total hemispherical emissivity of the surface if the surface temperature is 700 K?

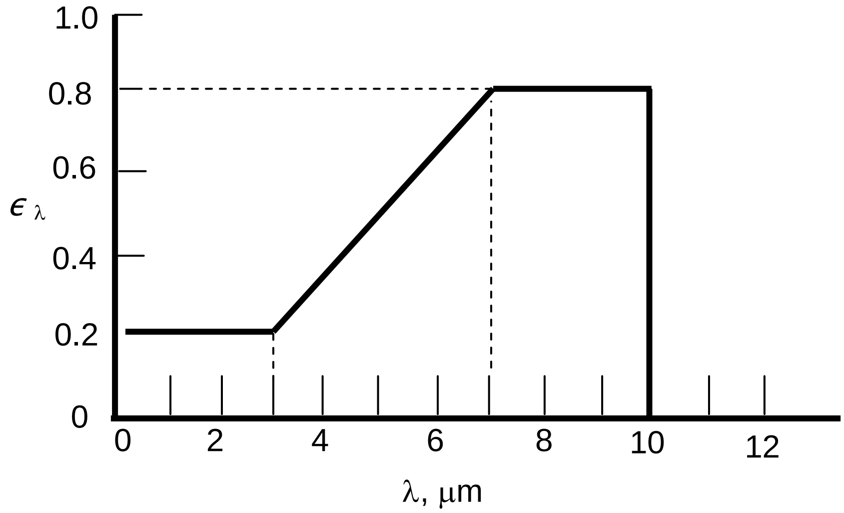


SOLUTION: The definitions of ϵ and α in terms of ϵ_λ and α_λ are used as in Problems 2.5 and 2.6. Using Equation1.37,

(a) α_λ = 0.9$F_{0\text{-}1.2x5800}$ + 0.1($F_{0\text{-}3x5800}$ – $F_{0\text{-}1.2x5800}$) + 1(1 – $F_{0\text{-}3x5800}$)
= 0.72505 + 0.01704 + 0.02399 = 0.76608
For a diffuse surface, Table 2.2 gives $\epsilon_\lambda = \alpha_\lambda$. Then
(b) ϵ = 0.9$F_{0\text{-}1.2x700}$ + 0.1($F_{0\text{-}3x700}$ – $F_{0\text{-}1.2x700}$) + 1(1-$F_{0\text{-}3x700}$)
= 0 + 0.00830 + 0.91695 = 0.92528.

Answer: (a) 0.766; (b) 0.925.

2.9 A white ceramic surface has a hemispherical spectral emissivity distribution at 1600 K as shown. What is the hemispherical total emissivity of the surface at this surface temperature?



SOLUTION: Numerical or graphical integratıon ıs necessary, as no analytical integration appears possible even for this simple variation in spectral emissivity. From Equation 2,10 with $E_{\lambda b}=\pi I_{\lambda b}$

$$\epsilon(1600K)=\frac{\int_{\lambda=0}^{\infty}\epsilon_\lambda(1600K)E_{\lambda b}(1600K)d\lambda}{\sigma(1600)^4}$$

Now,

$$\underset{\varepsilon\varepsilon}{\epsilon}(1600K)=\frac{1}{\sigma T_A^4}\int_{\lambda=0}^{\lambda_1}\epsilon_1 E_{\lambda b}d\lambda+\frac{1}{\sigma T_A^4}\int_{\lambda=\lambda}^{\lambda_2}\epsilon\left[\epsilon_1+(\epsilon_2-\epsilon_1)\left(\frac{\lambda-\lambda_1}{\lambda_2-\lambda_1}\right)\right]E_{\lambda b}d\lambda$$

$$(\quad+\frac{1}{\sigma T_A^4}\int_{\lambda=\lambda_2}^{\lambda_3}\epsilon_2 E_{\lambda b}d\lambda$$

where ϵ_1=0.2, ϵ_2=0.8, λ_1=3 μm, λ_2=7 μm, λ_3= 10 μm

Numerical Romberg integration of the ϵ (1600 K) equation gives 0.28128.

Answer: 0.281

2.10 A surface has the following values of hemispherical spectral emissivity at a temperature of 800K.

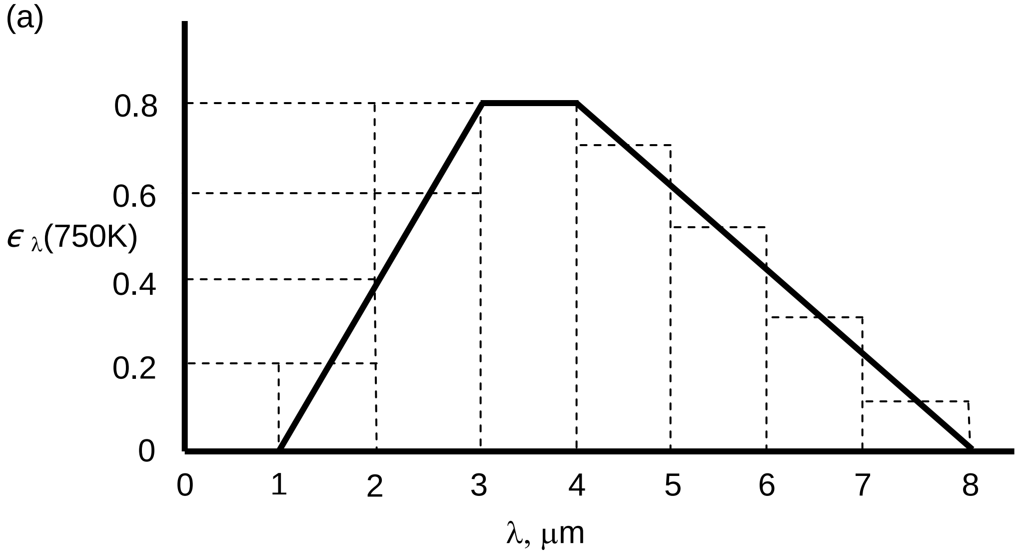
λ, μm	ϵ_λ(800 K)
<1	0
1	0
1.5	0.2
2	0.4
2.5	0.6
3	0.8
3.5	0.8
4	0.8
4.5	0.7
5	0.6
6	0.4
7	0.2
8	0
>8	0

(a) What is the hemispherical total emissivity of the surface at 800 K?

(b) What is the hemispherical total absorptivity of the surface at 800 K if the incident radiation is from a gray source at 1800 K that has an emissivity of 0.815? The incident radiation is uniform over all incident angles.

SOLUTION:

(a)



From Equation 2.10,

$$\epsilon(800K) = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda (800K) E_{\lambda b}(800K) d\lambda}{\sigma T_A^4}$$

$$= \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_1}^{\lambda_2} \left[\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right] E_{\lambda b}(800K) d\lambda$$

$$= \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_2}^{\lambda_3} \left[\epsilon_2 + (\epsilon_3 - \epsilon_2) \frac{\lambda - \lambda_2}{\lambda_3 - \lambda_2} \right] E_{\lambda b}(800K) d\lambda$$

$$= \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_3}^{\lambda_4} \left[\epsilon_3 + (\epsilon_4 - \epsilon_3) \frac{\lambda - \lambda_3}{\lambda_4 - \lambda_3} \right] E_{\lambda b}(800K) d\lambda$$

where T_A = 800K, ϵ_1=0, ϵ_2=0.8, ϵ_3=0.8, ϵ_4=0, λ_1=1 μm, λ_2 = 3 μm, λ_3 = 4 μm, λ_4 = 8 μm. Accurate numerical integration of the ϵ (800 K) equation gives <u>0.43826</u>.

(b) From Equation 2.25

$$\alpha(800K) = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (800K) 0.815 E_{\lambda b}(1800K) d\lambda}{\int_0^{\infty} 0.815 E_{\lambda b}(1800K) d\lambda} = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda (800K) E_{\lambda b}(1800K) d\lambda}{\sigma 1800^4}$$

$$= \frac{1}{\sigma 1800^4} \int_{\lambda=\lambda_1}^{\lambda_2} \left[\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right] E_{\lambda b}(1800K) d\lambda$$

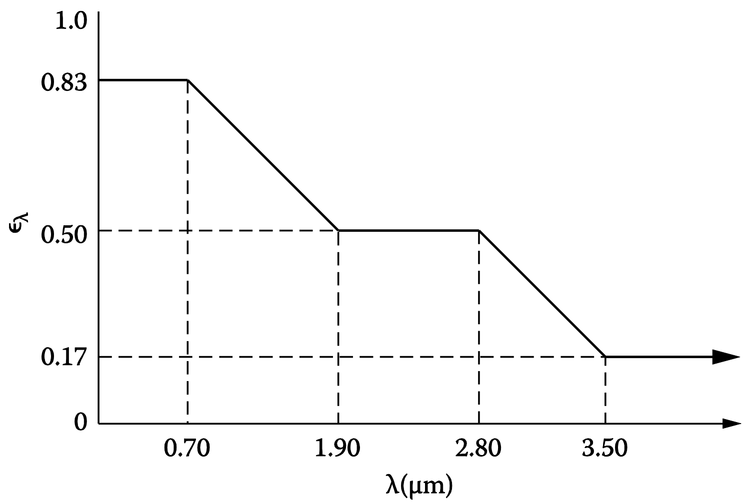
$$= \frac{1}{\sigma 1800^4} \int_{\lambda=\lambda_2}^{\lambda_3} \left[\epsilon_2 + (\epsilon_3 - \epsilon_2) \frac{\lambda - \lambda_2}{\lambda_3 - \lambda_2} \right] E_{\lambda b}(1800K) d\lambda$$

$$= \frac{1}{\sigma 1800^4} \int_{\lambda=\lambda_3}^{\lambda_4} \left[\epsilon_3 + (\epsilon_4 - \epsilon_3) \frac{\lambda - \lambda_3}{\lambda_4 - \lambda_3} \right] E_{\lambda b}(1800K) d\lambda$$

Numerical integration of the $\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda$ gives <u>0.42709</u>.

Answer: (a) 0.438; (b) 0.427.

2.11 Find the emissivity at 950 K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure. This will require numerical integration.



SOLUTION:
The emissivity in the various ranges can be expressed as

$$0 \le \lambda < 0.70 : \epsilon_\lambda = 0.83$$

$$0.70 \le \lambda < 1.90 : \epsilon_\lambda = 0.83 - 0.33\left(\frac{\lambda - 0.70}{1.20}\right)$$

$$1.90 \le \lambda < 2.80 : \epsilon_\lambda = 0.50$$

$$2.80 \le \lambda < 3.50 : \epsilon_\lambda = 0.50 - 0.33\left(\frac{\lambda - 2.80}{0.70}\right)$$

$$\lambda \ge 3.50; \epsilon_\lambda = 0.17$$

The total emissivity is then

$$\epsilon(T) = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda(T) E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$= \frac{0.83\int_{\lambda=0}^{0.70} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{\int_{\lambda=0.70}^{1.90}\left[0.83 - 0.33\left(\frac{\lambda - 0.70}{1.20}\right)\right] E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$+ \frac{0.50\int_{\lambda=1.90}^{2.80} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{\int_{\lambda=2.80}^{3.50}\left[0.50 - 0.33\left(\frac{\lambda - 2.80}{0.70}\right)\right] E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$+ \frac{0.17\int_{\lambda=3.50}^{\infty} E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

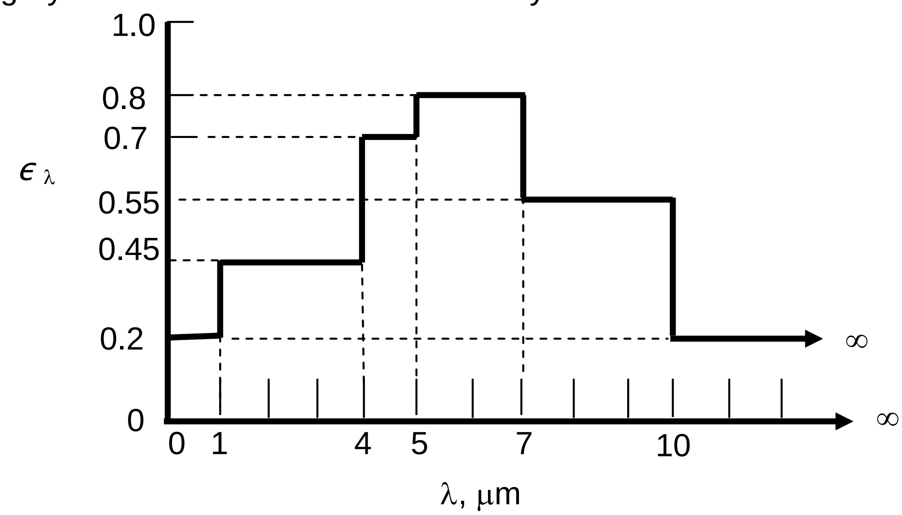
The blackbody fractions can be used to evaluate the first, third and fifth integrals, but the second and third probably require numerical integration. The results using numerical integration are ϵ (950 K) = 0.0000+0.0684+0.1185+0.0556+0.0823 = <u>0.3249</u>; ϵ (5780 K) = 0.4060+0.3225+0.0215+0.0041+0.0024 = <u>0.7564.</u>

Answer: ϵ *(*950 K) = 0.3248; ϵ (5780 K) = α_s(5780 K) = <u>0.7564.</u>

2.12 A diffuse surface at 1000 K has a hemispherical spectral emissivity that can be approximated by the solid line shown.

(a) What is the hemispherical-total emissive power of the surface? What is the total intensity emitted in a direction 60° from the normal to the surface?

(b) What percentage of the total emitted energy is in the wavelength range 5 < λ < 10 μm? How does this compare with the percentage emitted in this wavelength range by a gray body at 1000 K with an emissivity ϵ = 0.611?



SOLUTION:

(a): $\epsilon(T_A) = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda(T_A) E_{\lambda b}(T_A) d\lambda}{\sigma T_A^4}$; the blackbody fractions for each spectral band are obtained from Equation 1.37 as given in the table below:

λ range	ϵ_λ	λT range	$F_{0\to\lambda T}$ - $F_{0\to\lambda T}$	$\Delta F_{0\to\lambda T}$	$\epsilon_\lambda \Delta F_{0-\lambda T}$
0 - 1	0.2	0 - 1000	0.000032	0.00032	≈ 0
1 - 4	0.45	1000-4000	0.48086-0.00032	0.48054	0.21624
4 - 5	0.7	4000-5000	0.63371-0.48086	0.15285	0.10699
5 - 7	0.8	5000-7000	0.80793-0.63371	0.17422	0.13937
7 - 10	0.55	7000-10000	0.9134-0.80793	0.10547	0.05801
10 - ∞	0.2	10000 - ∞	1 - 0.9134	0.08660	0.017321

$\epsilon = \Sigma\, \epsilon_\lambda \Delta F_{0\to\lambda T}$ = <u>0.53800</u>

$E_{\lambda b} = \epsilon \sigma T^4 = 0.53800 \text{x}\ 5.6704\text{x}10^{-8} \text{ x } 900^4$ = <u>30506.9 W/m²</u>

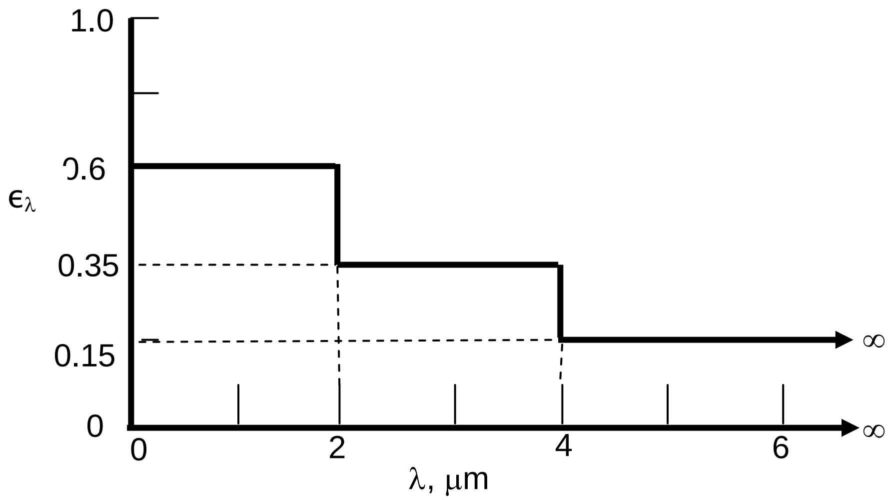
$I_{\lambda b} = E_{\lambda b}/\pi$ = <u>9710.7 W/m²•sr</u>

(b) For the range 5<λ<10, the percentage is [(0.13937+0.05801)/ 0.53800]x100=36.69%. For a gray surface, the percentage is the same as for a black surface, or

100 ($F_{0\to 9000}$ - $F_{0\to 4500}$) = 100 (0.91339 - 0.63371) = <u>27.97 %</u>.

Answer: (a) <u>30507</u> W/m²; <u>9711</u> W/m²•sr (b) 36.69 %; <u>27.97</u> %

2.13 The ϵ_λ for a metal at 1000 K is approximated as shown, and it does not vary significantly with the metal temperature. The surface is diffuse.



(a) What is α for incident radiation from a gray source at 1200 K with ϵ_{source} = 0.822?

(b) What is α for incident radiation from a source at 1200 K made from the same metal as the receiving plate?

SOLUTION:

(a) $$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (= \epsilon_\lambda) 0.822 E_{\lambda b}(1200K)\, d\lambda}{0.822 \sigma 1200^4} = \sum \epsilon_\lambda \Delta F$$

λ range	ϵ_λ	λT_i range	$F_{0\to\lambda T_i} - F_{0\to\lambda T_i}$	ΔF	ϵ_λ ΔF
0 - 2	0.6	0-2400	0.14026- 0	0.14026	0.08415
2 - 4	0.35	2400-4800	0.60753-0.14026	0.46727	0.16355
4 - ∞	0.15	4800-∞	1 - 0.60753	0.39247	0.05887

$\alpha = \Sigma\, \epsilon_\lambda \Delta F$ = 0.30657

(b) $\alpha = \Sigma\, \alpha_\lambda (\alpha_\lambda \Delta F) / (\Sigma\, \alpha_\lambda\, \Delta F)$

$= [\, 0.6^2 \times 0.14026 + 0.35^2 \times 0.46727 + 0.15^2 \times 0.39247\,] / 0.30657 = 0.38022$

Answer: (a) 0.30657; (b) 0.38022.

2.14 The directional total absorptivity of a gray surface is given by the expression $\alpha(\theta) = 0.450\cos^2\theta$ where θ is the angle away from the normal to the surface.

(a) What is the hemispherical total emissivity of the surface?

(b) What is the hemispherical- hemispherical total reflectivity of this surface for diffuse incident radiation (uniform incident intensity)?

(c) What is the hemispherical-directional total reflectivity for diffuse incident radiation reflected into a direction 75° from the normal?

SOLUTION:

$$\epsilon = \alpha = \frac{1}{\pi}\int_{\omega=0}^{4\pi} \alpha(\theta)\cos\theta d\omega$$

(a)

$$= \frac{2\pi}{\pi}\int_{\omega=0}^{4\pi} 0.450\cos^3\theta d\omega = \left.\frac{-0.9\cos^4\theta}{4}\right|_0^{\pi/2} = 0.2250$$

(b) $\rho = 1 - \alpha = 1 - \epsilon = 1 - 0.2250 = \underline{0.7750}$.

(c) Using Equation 2.43, $\rho(\theta_r,\phi_r) = \rho(\theta,\phi) = 1 - \alpha(\theta) = 1 - 0.450\cos^2(75°) = \underline{0.9699}$.

Answer: (a) 0.225; (b) 0.775; (c) 0.97.

2.15 Using Figure 3.24, estimate the total absorptivity of typewriter paper for normally incident radiation from a blackbody source at 1200 K.

SOLUTION: Use Equations 2.81 and 2.39 for a gray surface and assume that there is no dependence on circumferential angle ϕ. From the plot we have symmetry for the values of $\rho(\theta = 0, \theta_r, T_A)$.

$$\alpha(\theta = 0, T_A) = 1 - \rho(\theta = 0, T_A) = 1 - 2\pi\int_{\theta_r=0}^{\pi/2} \rho(\theta = 0, \theta_r, T_A)\cos\theta_r \sin\theta_r d\theta_r = 1 - 0.1079 =$$

$$= 0.8921$$

Evaluation requires numerical integration.

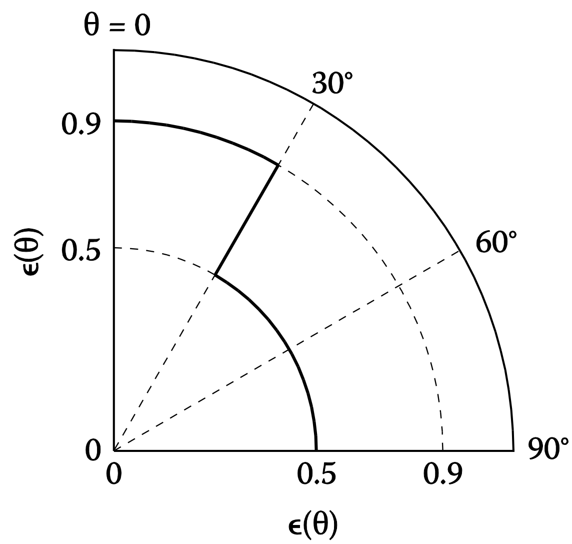
Answer: 0.8921.

2.16 A gray surface has a directional emissivity as shown in the figure. The properties are isotropic with respect to circumferential angle ϕ.

(a) What is the hemispherical emissivity of this surface?

(b) If the energy from a blackbody source at 650 K is incident uniformly from all directions, what fraction of the incident energy is absorbed by this surface?

(c) If the surface is placed in a very cold environment, at what rate must energy be added per unit area to maintain the surface temperature at 1000 K?



SOLUTION:
(a) From Equation 2.9,

$$\epsilon = 2\int_{\theta=0}^{\pi/2}\epsilon(\theta)\cos\theta\sin\theta\, d\theta = 2\left[\int_{\sin\theta=0}^{1/2}0.9\sin\theta\, d(\sin\theta)+\int_{\sin\theta=1/2}^{1}0.5\sin\theta\, d(\sin\theta)\right]$$

$$= 2\left[0.9\frac{\sin^2\theta}{2}\Bigg|_{\sin\theta=0}^{1/2}+0.5\frac{\sin^2\theta}{2}\Bigg|_{\sin\theta=1/2}^{1}\right] = 2\left[0.9\frac{\left(\frac{1}{4}-0\right)}{2}+0.5\left(\frac{1-\frac{1}{4}}{2}\right)\right] = \underline{0.600}$$

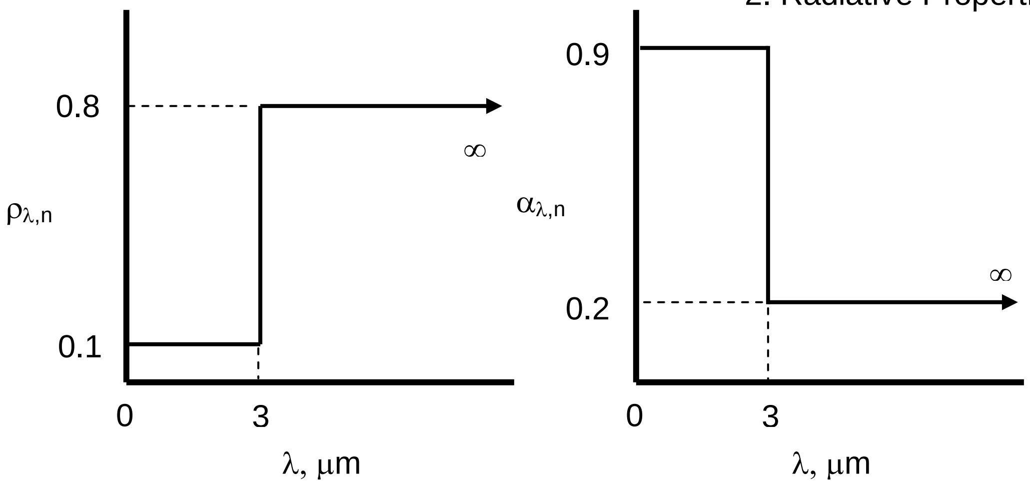
(b) Because the surface is gray, from Table 2.2, $\alpha = \epsilon$. Hence, α = 0.600 = fraction absorbed.

(c) Emissive power = $\epsilon\sigma T^4$ = 0.600 x 5.67040x10^{-8} x 1000^4 = 34,022W/m^2.

Answer: (a) 0.600; (b) 0.600; (c) 34,022 W/m^2 .

2.17 Using Figure 3.44, estimate the ratio of normal total solar absorptivity to hemispherical total emissivity for aluminum at a surface temperature of 650 K with a coating of 0.1-μm dendritic lead sulfide crystals. Assume the surface is diffuse. (The solar temperature can be taken as 5780 K.)

SOLUTION: From the figure, approximate the normal hemispherical spectral reflectivity as shown below. Using $\alpha_{\lambda,n} = 1 - \rho_{\lambda,n}$, the spectral normal absorptivity is approximated as shown below:



For $\alpha_{n,solar}$, use T_{solar} = 5780 K: then $(\lambda T)_{cutoff}$ = 3x5780 = 17340 μm•K, and $F_{0\to(\lambda T)cutoff}$ = 0.97880 [Equation 1.37].

Then $\alpha_{n,solar} = 0.9\, F_{0\to(\lambda T)cutoff} + 0.2\,(1 - F_{0\to(\lambda T)cutoff})$

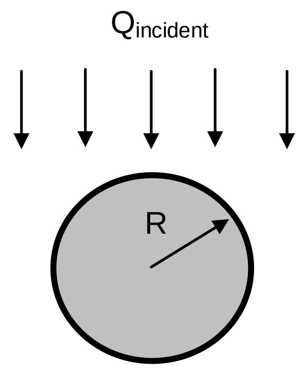
= 0.9 x 0.97880 + 0.2 (1 - 0.97880) = <u>0.8852</u>

ϵ (T = 650 K) $\approx \epsilon_n$(T = 650 K) = 0.9 $F_{0\to 1950}$ + 0.2 (1 - $F_{0\to 1950}$)

= 0.9 x 0.05920 + 0.2 (1 - 0.05920)= <u>0.2414</u> (NOTE: This assumes that the hemispherical $\epsilon \approx \epsilon_n$.) Thus, $\alpha_{n,solar}/\varepsilon$(T = 650 K) = 0.8852/0.2414 = <u>3.6662.</u>

Answer: 3.666.

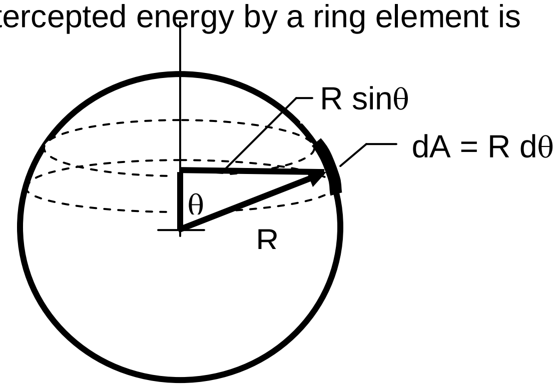
2.18 A gray surface has a directional total emissivity that depends on angle of incidence as $\epsilon(\theta) = 0.788 \cos\theta$. Uniform radiant energy from a single direction normal to the cylinder axis is incident on a long cylinder of radius R. What fraction of energy striking the cylinder is reflected? What is the result if the body is a sphere rather than a cylinder?



SOLUTION: $\alpha(\theta) = \epsilon(\theta) = 0.788 \cos\theta$. $\rho(\theta) = 1 - \alpha(\theta) = 1 - 0.788 \cos\theta$.
For the cylinder, the intercepted energy on dA per unit length of cylinder is $q\, dA \cos\theta = q\,(R d\theta)\cos\theta$, and the reflected energy per unit length is $q\,(R d\theta)\cos\theta\,(1 - 0.788\cos\theta)$. The ratio is

$$\frac{\text{reflected}}{\text{incident}} = \frac{q\int_{\theta=0}^{\pi/2} R\cos\theta\,(1-0.788\cos\theta)\,d\theta}{q\int_{\theta=0}^{\pi/2} R\cos\theta\, d\theta} = 1 - \frac{0.788\pi}{4} = 0.3811$$

For the sphere, the intercepted energy by a ring element is



q 2πR sinθ Rdθ cosθ, and the ratio of reflected to incident energy is

$$\frac{\text{reflected}}{\text{incident}} = \frac{q\int_{\theta=0}^{\pi/2} 2\pi R^2 \sin\theta\cos\theta\left(1-0.788\cos\theta\right)d\theta}{q\int_{\theta=0}^{\pi/2} 2\pi R^2 \sin\theta\cos\theta d\theta}$$

$$= \frac{\left(\frac{\sin^2\theta}{2} + \frac{0.788\cos^3\theta}{3}\right)\Big|_0^{\pi/2}}{\frac{\sin^2\theta}{2}\Big|_0^{\pi/2}} = \frac{\frac{1}{2} - \frac{0.788\pi}{3}}{\frac{1}{2}} = 0.4747$$

Answer: 0.381; 0.475.

2.19 A flat metal plate 0.1 m wide by 1.0 m long has a temperature that varies only along the long direction. The temperature is 900 K at one end, and decreases linearly over the one meter length to 350 K. The hemispherical spectral emissivity of the plate does not change significantly with temperature but is a function of wavelength. The wavelength dependence is approximated by a linear function decreasing from ϵ_λ = 0.85 at λ = 0 to ϵ_λ = 0.02 at λ = 10 μm. What is the rate of radiative energy loss from one side of the plate? The surroundings are at a very low temperature.

SOLUTION: The rate of energy loss is given by the total emissive power integrated over the plate length, or

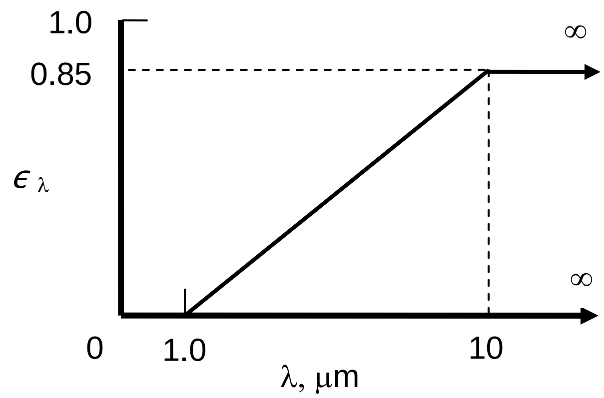
$$Q = 0.1(\text{m}^2)\int_{x=0}^{1}\int_{\lambda=0}^{10}\epsilon_\lambda E_{\lambda b}\left(T\right)d\lambda dx$$

$$= 0.1\int_{x=0}^{1}\int_{\lambda=0}^{10}\left(0.85 - \frac{0.83\lambda}{10}\right)\frac{2\pi C_1}{\lambda^3\left\{\exp\left[\frac{C_2}{\lambda\left(350+550x\right)}\right]-1\right\}}d\lambda dx = 416.6\text{W} \cdot$$

where the final result is obtained by numerical integration of the double integral.

Answer: 416.6 W.

2.20 A thin ceramic plate, insulated on one side, is radiating energy from its exposed side into a vacuum at very low temperature. The plate is initially at 1200 K, and is to cool to 300 K. At any instant, the plate is assumed to be at uniform temperature across its thickness and over its exposed area. The plate is 0.25 cm thick, and the surface hemispherical-spectral emissivity is shown in the figure and is independent of temperature. What is the cooling time? The density of the ceramic is 3200 kg/m^3, and its specific heat is 710 J/(kg•K).



SOLUTION: For an area A, the energy equation is

$$-V\rho c\frac{dT}{dt} = qA = A\int_{\lambda=0}^{\infty}\epsilon_\lambda \frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda$$

$$= A\int_{\lambda=0}^{10}\frac{0.85(\lambda-1)}{9}\frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda + A\int_{\lambda=10}^{\infty}0.85\frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda$$

Integrating gives

$$t = \frac{\rho c V}{A}\int_{T=300}^{1200}\frac{dT}{\int_{\lambda=1}^{10}\frac{0.85(\lambda-1)}{9}\frac{2\pi C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}d\lambda + \int_{\lambda=10}^{\infty}0.85\frac{2\pi C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}d\lambda}$$

Numerical integration is required, and results in <u>t = 1796 s = </u>29.9 <u>min</u>. (Note: V/A = 0.25x10^{-2} m.)

Answer: t = 29.9 min.

SOLUTIONS-CHAPTER 3

3.1 An electrical insulator has a refractive index of n = 1.332 and has a smooth surface radiating into air. What is the directional emissivity for the direction normal to the surface? What is it for the direction 60^o away from the normal?

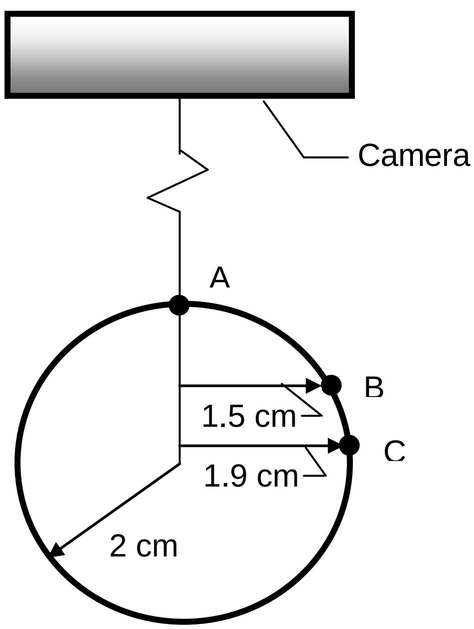
SOLUTION: From Equation 3.12 with $n_1 = 1$, $n_2 = 1.332$,
$\epsilon_n = 1 - [(n_2 - 1)/(n_2+1)]^2 = 1 - [(1.332- 1)/(1.332+ 1)]^2$ = <u>0.9797.</u>
Find χ from Equation 3.3 using $n_1 = 1$, $n_2 = 1.332$, $\theta = 60°$.

$\sin\chi = (n_1/n_2) \sin\theta = (1/1.332) \sin 60° = 0.6502$

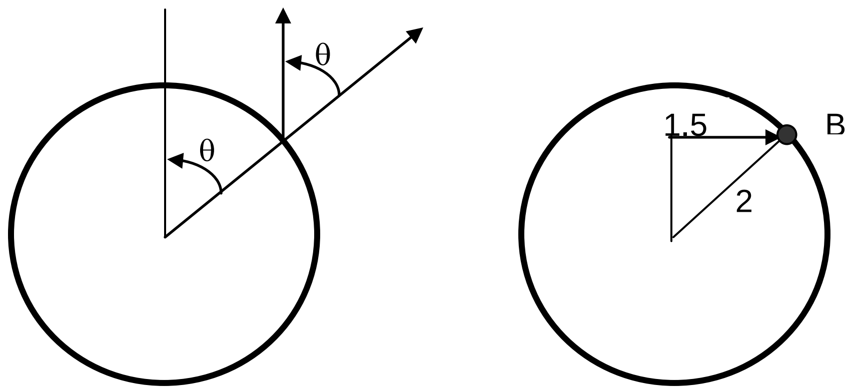
Then $\chi = \sin^{-1}(0.6502) = 40.55°$.
$\epsilon(60°) = 1 - \rho(60°)$; using Eq. 3.7a
$\epsilon(70°) = 1 - (1/2)[\sin^2(\theta - \chi)/\sin^2(\theta + \chi)]\ \{1 + [\cos^2(\theta + \chi)/\cos^2(\theta - \chi)]\}$
$= 1 - (1/2)[\sin^2(19.45°)/ \sin^2(100.55°)]\ \{1+[\cos^2(100.55°)/\cos^2(19.45°)]\}$
$= 1 - (1/2)(0.1108/0.9665)^2\ [1 + (-0.1831/0.8891)^2] =$ <u>0.9405.</u>

Answer: 0.9797; 0.9405.

3.2 A smooth hot ceramic dielectric sphere with an index of refraction n = 1.43 is photographed with an infrared camera. Calculate how bright the image is at locations B and C relative to that at A. (Camera is distant from sphere.)



SOLUTION: A blackbody has the same intensity for all directions. Hence, to compare intensities (brightness), compare the directional emissivities.



$\theta_B = \sin^{-1}(1.5/2) = 48.590°$; $\chi_B = \sin^{-1}(1.5/2n) = 31.633°$

$\theta_C = \sin^{-1}(1.9/2) = 71.805°$; $\chi_C = \sin^{-1}(1.9/2n) = 41.631°$

$\epsilon(\theta=0) = \varepsilon_A = 1 - [(n-1) / (n+1)]^2 = 1 - (0.43/2.43)^2 = 0.968.$

$$\epsilon = 1 - \frac{1}{2}\frac{\sin^2(\theta-\chi)}{\sin^2(\theta+\chi)}\left[1+\frac{\cos^2(\theta+\chi)}{\cos^2(\theta-\chi)}\right]$$

$$\epsilon_B = 1 - \frac{1}{2}\frac{\sin^2(16.957)}{\sin^2(80.223)}\left[1+\frac{\cos^2(80.223)}{\cos^2(16.957)}\right] = 0.9548$$

$$\epsilon_C = 1 - \frac{1}{2}\frac{\sin^2(30.174)}{\sin^2(113.436)}\left[1+\frac{\cos^2(113.436)}{\cos^2(30.174)}\right] = 0.8182$$

Thus, ϵ_B/ϵ_A = 0.9548/0.968 = <u>0.986</u>; ϵ_C/ϵ_A = 0.8182/0.968 = <u>0.845</u>.

Answer: 0.986; 0.845.

3.3 A particular dielectric material has a refractive index of n = 1.350. For a smooth radiating surface, estimate:

(a) the hemispherical emissivity of the material for emission into air.

(b) the directional emissivity at θ = 60° into air.

(c) the directional hemispherical reflectivity in air for both components of polarized reflectivity. Plot both components for n=1.350 on a graph similar to Figure 3.5. Let θ be the angle of incidence.

SOLUTION:

(a) From Equation 3.11,

ϵ = (1/2) - [(3 x 1.350 + 1) x (1.350 -1)]/[6 x (1.350 +1 $)^2$] - {1.350^2x(1.350^2 - $1)^2$/(1.350^2 + $1)^3$} ln[(1.350 - 1)/(1.350 + 1)]

+ 2 x 1.350 x (1.350^2 + 2x1.350 -1)/[(1.350^2 + 1) x (1.350^4 - 1)]

$- \{8 \times 1.350^4 \times (1.350^4 + 1)/[(1.350^2 + 1) \times (1.350^4 - 1)^2]\} \ln(1.350)$
$= \underline{0.9309}$
The result may also be found from Figure 3.3.

(b) From Figure 3.2, $\epsilon(\theta = 60°) = \underline{0.85}$. Could also use Equation 3.6 to obtain χ=39.9°, substitute χ into Equation 3.9 for $\rho(\theta)$=0.0629, and use $\epsilon = 1 - \rho$. This gives $\epsilon(\theta = 60°) = \underline{0.9371}$.
(For this particular set of parameters, the hemispherical emissivity and the directional emissivity at θ = 60° are nearly the same.)

Answer: (a) 0.9309, (b) 0.9371.

3.4 A smooth dielectric material has a normal spectral emissivity of $\epsilon_{\lambda,n}$ = 0.725 at a wavelength in air of 6 μm. Find or estimate values for:

(a) the hemispherical spectral emissivity ϵ_λ at the same wavelength.

(b) the perpendicular component of the directional hemispherical spectral reflectivity $\rho_{\lambda,\perp}(\theta)$ at the same wavelength and for incidence at θ = 40°.

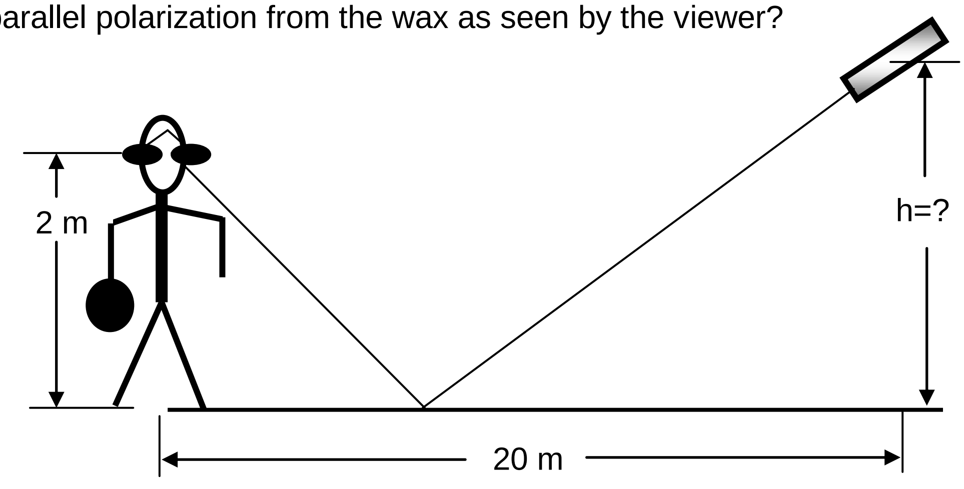
SOLUTION: From Equation 3.12
$\epsilon_n = 4n/(n+1)^2$; $n^2 + 2n + 1 = 4n/0.725$; so n = 3.2053 (or 0.3120, but n cannot be < 1).

a) From Figure 3.3: ϵ/ϵ_n = 0.965; so
$\varepsilon = 0.965 \times 0.725 = \underline{0.70}$. Equation 3.8 gives $\underline{0.7034}$

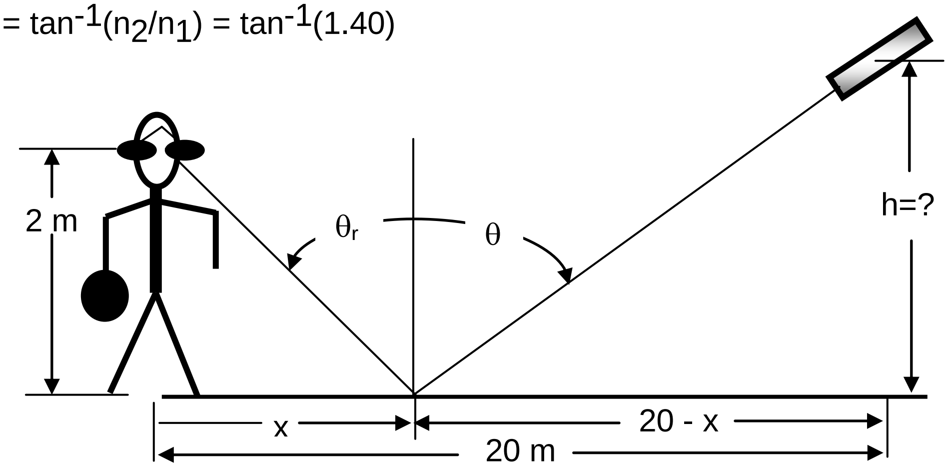
b) From Equation 3.8, $\rho_{\lambda,\perp}(\theta)$
$= \{[(3.2053^2 - 0.4132)^{1/2} - 0.7660]/[(3.2053^2 - 0.4132]^{1/2} + 0.7660]\}^2 = \underline{0.3694.}$

Answer: (a) 0.703; (b) 0.369.

3.5 An inventor wants to use a light source and some Polaroid glasses to determine when the wax finish is worn from her favorite bowling alley. She reasons that the wax will reflect as a dielectric with n = 1.40, and that the parallel component of light from the source will be preferentially absorbed and the perpendicular component strongly reflected by the wax. When the wax is worn away, the wood will reflect diffusely. At what height should the light source be placed to maximize the ratio of perpendicular to parallel polarization from the wax as seen by the viewer?

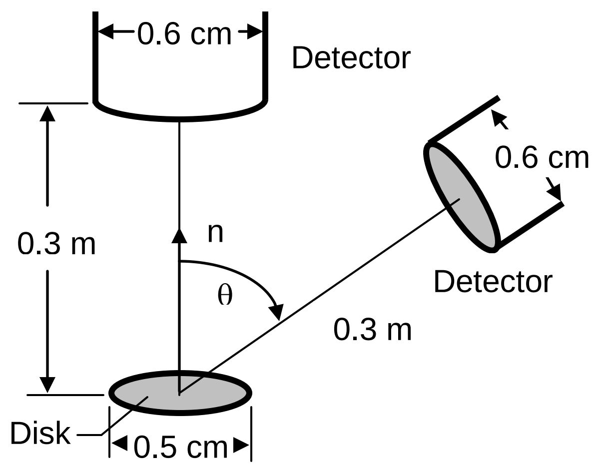


SOLUTION: To make $\rho_{\perp} / \rho_{||}$ a maximum, use Brewster's angle (Equation 3.7 et seq.):
$\theta = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.40)$



Since reflection is specular, $\theta_r = \theta$. Thus, $x/2 = \tan\theta = 1.40 = (20-x)/h$,
so x = 2.80 m, <u>h = 12.286 m</u> (a high ceiling!!)
Answer: 12.3 m.

3.6 A smooth ceramic dielectric has an index of refraction n = 1.58, that is independent of wavelength. If a flat ceramic disk is at 1100 K, how much emitted energy per unit time is received by the detector when it is placed at $\theta = 0°$ or at $\theta = 60°$? Use relations from electromagnetic theory.



SOLUTION: Neglect reflected energy from the surroundings. Then the energy reaching the detector is $I \cos\theta \, d\Omega \, dA$, where $dA = (\pi \times 0.005^2/4)$; $d\Omega = (\pi \times 0.006^2/4)(1/0.3^2)$.
In the normal direction, $\epsilon_n = 4n/(n+1)^2 = 4 \times 1.58/(2.58)^2 = 0.9495$.
Then the energy for ($\theta = 0$)
$= [\sigma\, 1100^4/\pi]\,(\pi \times 0.006^2/4)\,(1/0.3^2)\,(\pi \times 0.005^2/4) \times 0.9495$
= <u>1.5477×10^{-4} W</u>.
At $\theta = 60°$, $\chi = \sin^{-1}(\sin 60°/1.58) = 33.24°$, so

$$\rho(\theta = 75°) = \frac{1}{2}\frac{\sin^2(\theta-\chi)}{\sin^2(\theta+\chi)}\left[1+\frac{\cos^2(\theta+\chi)}{\cos^2(\theta-\chi)}\right] = 0.1021$$

$\epsilon(\theta) = 1 - \rho(\theta) = 0.8979$. The energy rate at θ =60° is then
=[$\sigma 1100^4/\pi$] cos(60°) (π x $0.006^2/4$) ($1/0.3^2$) (π x $0.005^2/4$) x 0.8979= $\underline{7.3183\text{x}10^{-5}}$ $\underline{\text{W}}$.

Answer: 15.48x10^{-5} W at θ = 0°; 7.318x10^{-5} W at θ = 60°.

3.7 At a temperature of 300 K these metals have the resistivities (Table 3.2):

Copper	1.72 x 10^{-6} Ohm-cm
Gold	2.27 x 10^{-6} Ohm -cm
Aluminum	2.73 x 10^{-6} Ohm -cm

What are the theoretical normal total emissivities and hemispherical total emissivities of these metals, and how do they compare with tabulated values for clean, unoxidized, polished surfaces?

SOLUTION: Using Equation 3.40 for the normal total emissivity :
$\epsilon_n = 0.578\ (r_eT)^{1/2} - 0.178\ r_eT + 0.0584\ (r_eT)^{3/2}$
and Eq. 3.45 for the total hemispherical emissivity:
$\epsilon = 0.776\ (r_eT)^{1/2} - [0.309 - 0.0889\ \ln(r_eT)]r_eT - 0.0175\ (r_eT)^{3/2}$

Material	r_eT	$(r_eT)^{1/2}$	$(r_eT)^{3/2}$	ϵ_n	ϵ	ϵ_n (Table B.1)
Copper	516x10^{-6}	22.7x10^{-3}	11.72x10^{-6}	0.0130	0.0171	0.02
Gold	681x10^{-6}	26.1x10^{-3}	17.7x10^{-6}	0.0150	0.0196	0.018
Aluminum	819x10^{-6}	28.6x10^{-3}	23.4 x10^{-6}	0.0164	0.0214	0.04

Alternatively, using Equation 3.39 gives for the normal value ϵ_{copper} = 0.0131;
ϵ_{gold} = 0.0150; $\epsilon_{aluminum}$ = 0.0164.

Answer: ϵ_n: 0.0131; 0.0157; 0.0168; ϵ: 0.0170. 0.0202, 0.0217.

3.8 A highly-polished metal disk is found to have a measured normal spectral emissivity of 0.095 at a wavelength of 12 μm. What is:

(a) the electrical resistivity of the metal (Ohm -cm) ?
(b) the normal spectral emissivity of the metal at λ = 10 μm?
(c) the refractive index n of the metal at λ = 10 μm?

(Note any assumptions that you make in obtaining your answers.)
SOLUTION:

(a) From Equation 3.35 (assumes n = κ for large wavelength.),
$\epsilon_{\lambda n} = 0.095 = 36.5\ (r_e/\lambda_o)^{1/2} - 464\ (r_e/\lambda_0)$; λ_0 = 12 μm
Solve for r_e = $\underline{8.7161 \text{ x } 10^{-5}}$ $\underline{\text{Ohm–cm.}}$

(b) Substitute the r_e from part (a) into Equation 3.35 with λ_0 = 10 μm to obtain $\epsilon_{\lambda n}$ (λ = 10 μm) = 0.1037.

(c) Can solve using Equation 3.33 or Figure 3.6:
Using the figure with n = κ, n = about 18.2. Using Equation 3.33 with $\epsilon_{\lambda n}$(λ = 10 μm) = 0.1037 and solving for n gives n = 18.26.

Answer: (a) 8.7161 x 10^{-5} Ohm -cm; (b) 0.104; (c) 18.3.

3.9 Show using Equation 3.7 that the parallel component of reflectivity becomes zero when $\theta = \tan^{-1}\left(\frac{n_2}{n_1}\right)$.

SOLUTION:

$$\rho_{\parallel}(\theta_i) = \left\{ \frac{\left(\frac{n_2}{n_1}\right)^2 \cos(\theta_i) - \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i\right]^{1/2}}{\left(\frac{n_2}{n_1}\right)^2 \cos(\theta_i) + \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i\right]^{1/2}} \right\}^2$$

To make $\rho_{\parallel}(\theta) = 0,$ the numerator must be = 0, so must prove that

$\left(\frac{n_2}{n_1}\right)^2 \cos(\theta) - \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta\right]^{1/2} = 0$. Substituting $\left(\frac{n_2}{n_1}\right) = \tan\theta$ gives

$$\left(\frac{n_2}{n_1}\right)^2 \cos(\theta) - \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta\right]^{1/2} = \tan^2\theta\cos(\theta) - \left[\tan^2\theta - \sin^2\theta\right]^{1/2}$$

$$= \frac{\sin^2\theta}{\cos(\theta)} - \left[\frac{\sin^2\theta}{\cos^2(\theta)} - \sin^2\theta\right]^{1/2} = \frac{\sin^2\theta}{\cos(\theta)} - \sin\theta\left[\frac{1}{\cos^2(\theta)} - 1\right]^{1/2}$$

$$= \sin\theta\left\{\tan\theta - \left[\frac{1}{\cos^2(\theta)} - 1\right]^{1/2}\right\} = \sin\theta\left\{\tan\theta - \left[\frac{1-\cos^2(\theta)}{\cos^2(\theta)}\right]^{1/2}\right\}$$

$$= \sin\theta\left\{\tan\theta - \left[\frac{\sin^2(\theta)}{\cos^2(\theta)}\right]^{1/2}\right\} = \sin\theta(\tan\theta \pm \tan\theta) = 0$$

(Must choose negative sign, or reflectivity can be < 0.)

3.10 A clean metal surface has a normal spectral emissivity of $\epsilon_{\lambda,n}$ = 0.06 at a wavelength of 12 μm. Find the value of the electrical resistivity of the metal.

SOLUTION: From Equation 3.23b