SOLUTIONS MANUAL FOR

MEASUREMENT AND DETECTION OF RADIATION

by

Nicholas Tsoulfanidis and
Sheldon Landsberger

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Note by the authors

This document contains solutions of the problems in the book "Measurement and Detection of Radiation", 4th edition, published by Taylor-Francis (2015).

The authors do not guarantee that all solutions are correct or that the solutions offered are the only correct ones; for some problems, different solutions (approaches) may be equally valid.

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The authors would appreciate any comments (corrections, different solutions, etc.) that may be used to improve this manual sent to

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Chapter 2

2.1 What is the probability when throwing a die three times of getting a four in any of the throws?

Answer: The probability to bring a four in one throw is $f(4)=\frac{1}{6}$. Using the binomial distribution, the probabilityto have <u>one</u> 4 in 3 throws is P(only one 4) =

$$\frac{3!}{(3-1)!\,1\,!}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2=\frac{75}{216}=0.3472$$

The same result can be obtained by enumerating all possible events.

Throw	1	2	3
Outcome	4	No – 4	No – 4
	No – 4	4	No – 4
	No – 4	No - 4	4

To bring only one 4, P(only one 4) = $\frac{1}{6}\bullet\frac{5}{6}\bullet\frac{5}{6}+\frac{5}{6}\bullet\frac{1}{6}\bullet\frac{5}{6}+\frac{5}{6}\bullet\frac{5}{6}\bullet\frac{1}{6}=\underline{\underline{\frac{75}{216}}}$

<u>Note:</u>: The probability to have at least one four is longer *P*(at least one 4) = P(one 4) + P(two 4's) + P(three 4's) =

$$\frac{75}{216}+\frac{3!}{(3-2)!2!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1+\frac{3!}{(3-3)!3!}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2=\frac{91}{216}=\underline{\underline{0.4213}}$$

Using possible outcomes, one should extend the table given above, as follows:

4	4	$No-4$	$\rightarrow$	$\Pr ob=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}$	$\frac{15}{16}$
4	$No-4$	4	$\rightarrow$	$\Pr ob=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}$	$\frac{15}{16}$
$No-4$	4	4	$\rightarrow$	$\Pr ob=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}$	$\frac{15}{16}$
4	4	4	$\rightarrow$	$\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$	$\frac{1}{216}$

Total probability = $\frac{75}{216}+3\bullet\frac{5}{216}+\frac{1}{216}=\underline{\underline{\frac{91}{216}}}$

2.2 What is the probability when drawing one card from each of three decks of cards that all three cards will be diamonds?

Answer: Probability of drawing a diamond from one deck is $\frac{13}{52}$. Probability that this event happens three times.

(Equation 2.15): $P(3-diamonds)=\frac{13}{52}\bullet\frac{13}{52}\bullet\frac{13}{52}=(0.25)^3=\underline{\underline{0.0156}}$

2.3 A box contains 2000 computer cards. If five faulty cards are expected to be found in the box, what is the probability of finding two faulty cards in a sample of 250?

Answer: The probability of finding a faulty card is *P*(faulty card) = $\frac{5}{2000} = 0.0025$. The probability of finding two faulty cards in a sample of 250 is (using Binomial distribution, Equation 2.45)

$$\frac{250!}{(250-2)!2!}(0.0025)^2(1-0.0025)^{250-2} = \frac{249\times 250}{2}(0.0025)^2(0.9975)^{248} = 31125\times 6.25\times 10^{-6}(0.5375) = \underline{\underline{0.104}}$$

2.4 Calculate the average and the standard deviation of the probability density function $f(x) = 1/(b-a)$ when $a \le x \le b$. (This pdf is used for the calculation to round off errors.)

Answer: Use Equation 2.27, or in this case Equation 2.26, for $\bar{x}$. Use Equation 2.33 for r^2

$$\bar{x} = \int_a^b x\frac{1}{b-a}dx = \underline{\underline{\frac{b+a}{2}}}$$

$$\sigma^2 = \int_a^b \left(x-\bar{x}\right)^2 \frac{1}{b-a}dx = \frac{(b-a)^2}{12} \quad , \qquad \sigma = \underline{\underline{\frac{b-a}{\sqrt{12}}}}$$

2.5 The energy distribution of thermal (slow) neutrons in a light-wave reactor follows very closely the Maxwell-Boltzmann distribution:

$$N(E)dE = A\sqrt{E}e^{-E/kT}dE$$

where *N(E) dE* = number of neutrons with kinetic energy between *E* and *E + dE*
k = Boltzmann constant = 1.380662 × 10^{-23}J/° K
T = temperature, *K*
A = constant

Show that

(a) The mode of this distribution is $E = \frac{1}{2}kT$.

(b) The mean is $\bar{E} = \frac{3}{2}kT$.

Answer:
(a) Use Equation 2.24

$$\frac{dN(E)}{dE} = 0 = A\frac{1}{2}E^{-1/2}e^{-E/kT} - AE^{1/2}\frac{1}{kT}e^{-E/kT} = Ae^{-E/kT}\left(\frac{1}{2\sqrt{E}} - \frac{\sqrt{E}}{kT}\right) \qquad \text{or} \quad E_{\text{mode}} = \underline{\underline{\frac{1}{2}kT}}$$

(b) Use equation 2.27

$$\bar{E} = \frac{\int_0^\infty EA\sqrt{E}e^{E/kT}\,dE}{\int_0^\infty A\sqrt{E}e^{-E/kT}\,dE} = \underline{\underline{\frac{3}{2}kT}}$$

2.6 If the average for a large number of counting measurements is 15, what is the probability that a single measurement will produce the result 20?

Answer: Use Poisson statistics, Equation 2.50

$$P(20)=\frac{15^{20}}{20!}e^{-15}=\frac{3.325\times10^{23}\times3.159\times10^{-7}}{2.433\times10^{18}}=\underline{\underline{4.18\times10^{-2}}}\ \text{or } 4.18\%.$$

2.7 For the binomial distribution, prove

$$(\mathrm{a})\ \sum_{n=0}^{N}P_{(N)}^{(n)}=1\quad(\mathrm{b})\ \bar{n}=pN\quad(\mathrm{c})\ \sigma^2=m(1-p)$$

Answer:

(a) $\displaystyle\sum_{N=0}^{N}P_N^{(n)}=\sum_{N=0}^{N}\frac{N!}{(N-n)!n!}P^n(1-P)^{N-n}=1$

because the binomial distribution represents the n^{th} term of the binomial expansion $(x+y)^N$. The n^{th} term in $\displaystyle\frac{N\left(N-(n-2)\right)\left(N-(n-1)\right)}{n!}x^{N-n}\bullet y^n=$(multiply and divide by (N-n)!) = $\displaystyle\frac{N!}{(N-n)!n!}\bullet x^{N-n}\bullet y^n$. In the present case

$x=p,\ y=1-p$ $\qquad\displaystyle\sum_{n=0}^{N}P_N^{(n)}=\left(p+(1-p)\right)^N=1^N=1$

(b):

$$\bar{n}=\sum_{n=0}^{N}nP_N^{(n)}=\sum_n n\frac{N!}{(N-n)!n!}p^n(1-p)^{N-n}=Np\sum_{n=0}^{N}\frac{(N-1)^y}{(N-n)!(n-1)!}p^{n-1}(1-p)=Np(p+1-p)^{N-1}=Np$$

(c) $\displaystyle\sigma^2=\sum_{n=0}^{N}(m-n)^2P_N^{(n)}=m^2\sum_n P_N^{(n)}-2m\sum_n nP_N^{(n)}+\sum_n n^2P_N^{(n)}=$

$\displaystyle m^2-2m\bullet m+\sum_n n^2P_N^{(n)}=\overline{n^2}-m^2$

$\displaystyle\overline{n^2}=\sum_n n^2P_N^{(n)}=\sum\left(n(n-1)+n\right)P_N^{(n)}=\sum_n n(n-1P_N^{(n)}+\sum_n nP_N^{(n)}$

$$=\sum_n n(n-1)\frac{N!}{(N-n)!n!}p^n(1-p)^{N-n}+m=N(N-1)p^2\sum_n\frac{(n-2)!}{(N-n)!(n-2)!}p^{n-2}(1-p)^{N-2}+m=$$

$$=N(N-1)p^2(p+1-p)^{N-2}+m=N(N-1)p^2+m$$

Combining terms: $\sigma^2=N(N-1)p^2+m-m^2=N^2p^2-Np^2+Np-N^2p^2=Np(1-p)=m(1-p)$

$\cancel{\sigma^2=N(N-1)p^2+m-m^2=N^2p^2-Np^2+Np-N^2p^2=Np(1-p)}$

$\underline{\underline{\sigma=\sqrt{m(1-p)}}}$

2.8 For the Poisson distribution, prove

$$(a)\ \sum_{n=0}^{\infty} P_n = 1 \quad (b)\ \bar{x} = m \quad (c)\ \sigma^2 = m$$

Answer:

(a) $\sum_{n=0}^{\infty} Pn = \sum_n \frac{m^n}{n!} e^{-m} = e^{-m} \sum_n \frac{m^n}{n!} = e^{-m}\ xe^m = \underline{\underline{1}}$

(b) $m = \sum_n nPn = \sum_n n \frac{m^n}{n!} e^{-m} = e^{-m} \sum_n \frac{m^n}{(n-1)!} = me^{-m} e^m = m$

(c) $r^2 = \sum_n (m-n)^2 Pn = m^2 \sum P_n - 2m \sum nP_n + \sum n^2 P_n =$

$$= m^2 - 2m^2 + \overline{n^2} = \overline{n^2} - m^2$$

$$n^2 = \sum_n n^2 P_n = \sum_n \left(n(n-1) + n\right) P_n = \sum_n n(n-1) \frac{m^n}{n!} e^{-m} + \sum_n nP_n =$$

$$= m^2 e^{-m} \sum_n \frac{m^{n-2}}{(n-2)!} + m = m^2 + m$$

combining terms: $\sigma^2 = m^2 + m - m^2 = m$ and $\sigma = \sqrt{m}$

2.9 For the normal distribution, show

$$(a)\ \int_{-\infty}^{\infty} P(x)\,dx = 1 \quad (b)\ \bar{x} = m \quad (c)\ \text{the variance is } \sigma^2$$

Answer:

(a) $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx = 1$

(look up tables; integral is of the form $\int_0^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}$)

(b) $\bar{x} = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx$

change variable to $y = x - m$, $\bar{x} = \int_{-\infty}^{\infty} \frac{y+m}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy = m \int_{-\infty}^{\infty} \frac{e^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} dx + \int_{-\infty}^{\infty} \frac{ye^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} dy =$

$= m + zero(odd\ function\ ) =\ m$

(c) $r^2 = \int_{-\infty}^{\infty} (x-m)^2 P(x)\,dx = \int_{-\infty}^{\infty} x^2 P(x)\,dx - 2m \int_{-\infty}^{\infty} xP(x)\,dx + m^2 \int_{-\infty}^{\infty} P(x)$

$= \overline{x^2} - 2m^2 + m^2 = \overline{x^2} - m^2 =$

$$\bar{x^2} = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx \text{, set } \frac{x-m}{\sqrt{2}\sigma} = y, x = \sqrt{2}\sigma y + m$$

$$= \int_{-\infty}^{\infty} \left(\sqrt{2}\sigma y + m\right)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2} \sqrt{2}\sigma dy = \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} y^2 e^{-y^2} dy + \frac{2\sqrt{2}\sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} y e^{-y^2} dy + \frac{m^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy =$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} 2 \frac{\sqrt{\pi}}{4} + (0) + \frac{m^2}{\sqrt{\pi}} 2 \frac{1}{2} \sqrt{\pi} = \sigma^2 + m^2$$

combining terms: $\sigma^2 = \sigma^2 + m^2 - m^2 = \sigma^2$

2.10 Show that in a series of N measurements, the result R that minimizes the quantity

$$Q = \sum_{i=1}^{N} \left(R - n_i\right)^2$$

is $R = \bar{n}$, where $\bar{n}$ is given by Eq. 2.31.

Answer: To find the minimum of Q, solve $\frac{\partial \alpha}{\partial R} = 0$.

$$\frac{\partial Q}{\partial R} = 2\sum_{i=1}^{N}\left(R - n_i\right) = 0 \rightarrow \sum R = \sum n_i \text{ , } NR = \sum_{i=1}^{N} n_i$$

$$R = \frac{1}{-}\sum^{N} n_i$$

2.11 Prove Eq. 2.62 using tables of the error function. (To find the tables of error function type in "error function" in the search window of your internet provider).

Answer: Equation 2.62 is $A = \int_{m-r}^{m+r} G(x)\, dx = \int_{m-r}^{m+r} \frac{dx}{\sqrt{2\pi}\sigma} e^{(x-m)^2 / 2\sigma^2}$

call $\frac{x-m}{\sigma} = y$, $x = \sigma y + m$

$$A = \int_{-F}^{1} \frac{\sigma dy}{\sqrt{2\pi}\sigma} e^{-y^2/2} = 2 \Big/ \sqrt{2\pi} \int_{0}^{1} e^{-y^2/2}\, dy$$

In tables, one finds the integral $\int_{0}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \frac{1}{2} erf\left(\frac{t}{\sqrt{2}}\right)$

$$2\int_{0}^{1} \frac{dy}{\sqrt{2\pi}} e^{-y^2/2} = 2 \times 0.3413 = 0.6826 \approx 0.683$$

2.12 As part of a quality control experiment, the lengths of 10 nuclear fuel rods have been measured with the following results in meters:

2.60	2.62	2.65	2.58	2.61
2.62	2.59	2.59	2.60	2.63

What is the average length? What is the standard deviation of this series of measurements?

Answer: Average length $\bar{\ell} = \sum_{i=1}^{N} \frac{\ell i}{10} = 2.609m$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{\infty}\left(\ell_i - \bar{\ell}\right)^2 = \frac{1}{9}(0.00409) = 0.000454, \underline{\underline{\sigma = 0.021}}m$$

2.13 The average number of calls in a 911 switchboard is 4 calls/h. What is the probability to receive 6 calls in the next hour?

Answer: Use the Poisson statistics equation:

$$P_n = \left[\frac{m^n}{n!}\right]e^{-m} = \left[\frac{4^6}{6!}\right]e^{-4} = 0.10 = 10\%$$

2.14 At a uranium pellet fabrication plant the average pellet density is 17 × 10^3 kg/m^3 with a standard deviation equal to 10^3 kg/m^3. What is the probability that a given pellet has a density less than 14 × 10^3 kg/m^3?

Answer: We have to find the probability that the density $\rho < 14\times10^3\, {kg}/{m^3}$ with $\bar{\rho} = 17\times10^3\, {kg}/{m^3}$ and

$\sigma = 10^3\, {kg}/{m^3}$ (See section 2.14).

$$\rho = \bar{\rho} - (17-14)\times10^3 = 17\times10^3 - 3\times10^3 = \bar{\rho} - 3\sigma$$

$$\underline{P}\left(\rho < \bar{\rho} - 3\sigma\right) = G(t>3) = \int_3^{\infty}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\,dt \quad \text{(Equation 2.78)}$$

From table 2.2, $\underline{P}\left(\rho < \bar{\rho} - 3\sigma\right) = \underline{\underline{0.0013}} = \underline{\underline{0.13\%}}$

2.15 A radioactive sample was counted once and gave 500 counts in 1 min. The corresponding number for the background is 480 counts. Is the sample radioactive or not? What should one report based on this measurement alone?

Answer: The answer depends on the standard error of the net counting rate using Equation 2.93:

$$r = g - b = 500 - 480 = 20\, {c}/{\min}$$

Using Equation 2.96, $\sigma_r = \sqrt{\frac{500}{i^2} + \frac{480}{i^2}} = 31.3\, {c}/{\min}$

$$\frac{\sigma_r}{r} = \frac{31.3}{20} = 156\%\ \text{error.}$$

Based on this measurement alone, one cannot tell whether or not the sample is radioactive.

<u>Note:</u> The counting rate is so low that dead time need not be considered.

2.16 A radioactive sample gave 750 counts in 5 min. When the sample was removed, the sealer recorded 1000 counts in 10 min. What is the net counting rate and its standard percent error?

Answer: Use Equation 2.93 → $r = \frac{750}{5} - \frac{1000}{10} = 150 - 100 = 50\, {}^{c}\!/_{\min}$

Using Equation 2.96 → $\sigma_r = \sqrt{\frac{750}{25} + \frac{1000}{100}} = \sqrt{30+10} = 6.32\, {}^{c}\!/_{\min}$

$\frac{\sigma_r}{r} = \frac{6.3}{50} = 0.126 = \underline{\underline{13\%}}$ standard error (again, dead time need not be considered).

2.17 Calculate the average net counting rate and its standard error from the data given below:

G	t_G(min)	B	t_B(min)
355	5	120	10
385	5	130	10
365	5	132	10

Answer: Use Equation 2.97 $\bar{r} = \frac{1}{3}\left(\frac{355+385+365}{5} - \frac{120+130+132}{10}\right) = \frac{1}{3}(221 - 38.2) = 60.9 \approx 61\, {}^{c}\!/_{\min}$

σ_r (Equation 2.98) → $\sigma_r = \frac{1}{3}\sqrt{\frac{1105}{25} + \frac{382}{100}} = \frac{6.93}{3} = 2.3\, {}^{c}\!/_{\min}$

$\frac{\sigma_r}{r} = \frac{2.3}{61} = 0.038 \approx 4\%$ standard error

2.18 A counting experiment has to be performed in 5 min. The approximate gross and background counting rates are 200 counts/min and 50 counts/min, respectively. (a) Determine the optimum gross and background counting times; (b) based on the times obtained in (a) what is the standard percent error of the net counting rate?

Answer: (a) Using Eq. 2.100 , $\frac{t_B}{t_G} = \sqrt{\frac{50}{200}} = 0.5$. Since $t_G+t_B=5$, $0.5t_G+t_G=5$, $t_G=3.33$ min, $t_B=1.67$ min,

(b) r=200-50=150 counts/min, $\sigma_r = \sqrt{\frac{g}{t_G} + \frac{b}{t_B}} = \sqrt{\frac{200}{3.33} + \frac{50}{1.67}} = 9.49\,\frac{counts}{min}$, $\sigma_r/r=9.49/150=0.063=6.3\%$

2.19 The strength of a radioactive source was measured with a 2 percent standard error by taking a gross count for time t min and a background for time $2t$ min. Calculate the time t if it is given that the background is 300 counts/min and the gross count 45,000 counts/min.

Answer: Since no information is given about dead time, consider it negligible (e.g. detector is scintillator). Using Equation 2.93 and 2.96,

$$\frac{\sigma_r}{r} = 0.02 = \frac{\sqrt{\frac{G}{t^2} + \frac{B}{2t^2}}}{\frac{G}{t} - \frac{B}{2t}} = \frac{\sqrt{\frac{g}{t} + \frac{b}{2t}}}{45000-300} = \frac{\sqrt{45000 + {}^{300}\!/_{2}}}{45000-300} * \frac{1}{\sqrt{t}}$$

$894\sqrt{t} = 212.48,\; t = 0.056\,\text{min} = 3.39s$

<u>Check:</u> $\sigma_r = \sqrt{\frac{45000}{0.056} + \frac{300}{0.112}} = 898\, {}^{c}\!/_{\min}$, $\frac{\sigma_r}{r} = \frac{898}{44700} = 0.02 = 2\%$

2.20 The strength of radioactive source is to be measured with a detector that has a background of 120 ± 8 counts/min. The approximate gross counting rate is 360 counts/min. How long should one count if the net counting rate is to be measured with an error of 2 percent?

Answer: Use Equation 2.102, $t_G = \dfrac{360}{(360-120)^2\left(\dfrac{2}{100}\right)^2 - 8^2} = \dfrac{360}{23-64} < 0$ there is no way to get 2 % accuracy under these conditions. The error in background alone is 3.3% !

(the best one can do is when : $(360-120)^2 \dfrac{x}{100^2} = 8^2$ which gives $x = \sqrt{\dfrac{8^2 * 100^2}{(360-120)^2}} = 5.7 \sim 6\%$

2.21 The buckling B^2 of a cylindrical reactor is given by

$$B^2 = \frac{2.405^2}{R^2} + \frac{\pi^2}{H^2}$$

where R = reactor radius

H = reactor height

If the radius changes by 2 percent and the height by 8 percent, by what percent will B^2 change? Take R = 1 m, H = 2 m.

Answer:

$$\Delta B^2 = \sqrt{\left(\frac{\partial B^2}{\partial R}\right)^2 (\Delta R)^2 + \left(\frac{\partial B^2}{\partial H}\right)^2 (\Delta H)^2} = \sqrt{4\left(\frac{2.405^2}{R^3}\right)^2 (\Delta R)^2 + 4\left(\frac{\pi^2}{H^3}\right)^2 (\Delta H)^2} =$$

$$= \sqrt{\frac{4\times 2.405^4}{R^4}\left(\frac{\Delta R}{R}\right)^2 + \frac{4\pi^4}{H^4} - \left(\frac{\Delta H}{H}\right)^2} = \sqrt{\frac{4\times 2.405^4}{1^4}(0.02)^2 + \frac{4\pi^4}{2^4}(0.08)^2} =$$

$$= \sqrt{0.05+0.156} = 0.454 m^{-2}, \; B^2 = 8.25 m^{-2}, \; \frac{\Delta B^2}{B^2} = \frac{0.45}{8.25} = 0.054$$

2.22 Using Chauvenet's criterion, should any of the sealer readings listed below be rejected?

115	121	103	151
121	105	75	103
105	107	100	108
113	110	101	97
110	109	103	101

Answer: N=20, 1-1/40=0.975. A reading may be rejected if it deviates from the average by more than the 97.5% error. By interpolation from Table 2.4, the number of standard deviations for this error is 2.23. $\bar{n} = \frac{\Sigma n_i}{N} = \frac{2158}{20} =$ 107.9~108 $\sigma = \sqrt{\frac{\Sigma(n_i-\bar{n})^2}{19}} = \sqrt{\frac{3741}{19}} = 14$

Since 2.23*14=31.2, the data 75 and 151 may be rejected.

Note: To obtain an exact value of σ's, one may use tables of the error function to find the tables of error function. (Type in "error function" in the search window of your internet provider). Proceed as follows: 1/2N=1/40=0.025, (1/2)(1/2N)=0.0125. Area listed in table: 0.5-0.0125=0.4875, number of σ's=2.24

2.23 As a quality control test in a nuclear fuel fabrication plant, the diameter of 10 fuel pellets has been measured with the following results (in mm):9.50, 9.80, 9.75, 9.82, 9.93, 9.79, 9.81, 9.65, 9.99, and 9.57. Calculate (a) the average diameter, (b) the standard deviation of this set of measurements, (c)the standard error of the average diameter, (d) should any of the results be rejected based on the Chauvenet criterion?

Answer: (a) d_{av}=(9.50+9.80+.....9.99+9.57)/10=9.76 mm

(b) $\sigma=\sqrt{\frac{\sum_1^{10}(d_i-d_{av})^2}{N-1}}=\sqrt{\frac{0.205}{9}}=0.15\,mm$ (c) $\sigma_{d,av}=\sigma/\sqrt{10}$=0.048 mm, $\sigma_{d,av}/d_{av}$=0.048/9.76=0.049~0.5%

(d) N=10, 1-1/20=0.95; check whether any measurements lie away from the average by more than the 95% error (see section 2.16) which leads to 1.96σ=0.294 mm. No measurement is farther from the average by that amount; no measurement should be rejected.

2.24 Using the data of Prob. 2.12, what is the value of accepted length x_a if the confidence limit is 99.4 percent?

Answer: From problem 2.12, $\overline{\ell}=2.609m$ and σ=0.21m. If the confidence limit is 99.4%, the accepted length, ℓ_a, should not deviate from the average length, $\overline{\ell}$, by more than 2.5 σ (see Table 2.2). or:

$$\ell_a<\overline{\ell}+2.5\sigma-2.609+2.5*0.021=2.661m$$

2.25 An environmental sample has been collected for determination of ^{210}Po content. The sample is chemically separated and counted in an instrument with the following results 60 days after sampling.

Chemical Yield	80%
Counting Efficiency	20%
Sample Counts (gross)	20 counts
Sample Count Time	30 minutes
Background Counts	10 counts
Background Count Time	30 minutes
Half-life of Po-210	138 days

(a) What was the sample ^{210}Po net counting rate at the time of the sampling?

(b) What is the standard error of value determined in part (a)?

(c) The lower limit of detection (MDA) at the 95% confidence level has been defined as:

$\mathrm{MDA}=k^2+2\mathrm{CDL}=2.71+4.653\sigma_B$ is the standard deviation. Calculate the MDA for this determination.

(d) Does the activity level of this sample exceed the MDA for this determination?

Answer:

(a) Let A_N, A_G, A_B and σ_N, σ_G, σ_B, represent net, gross and background count rates and standard deviations at the time of analysis: A_c, and S_c represent net activity and standard deviation, corrected for chemical yield and counting efficiency at the time of sampling. The net count rate at time of analysis is

$$A_N = A_G - A_B = \frac{20\,counts}{30\,min}-\frac{10\,counts}{30\,min}=0.33\,min^{-1}$$

which must be converted for radiological decay, counting efficiency, and chemical yield to determine the disintegration rate at the time of sampling. Corrections for radiological decay are made by using the decay constant λ (determined from the half-life; $\lambda=\ln 2/t_{1/2}$) and the time t from sampling to analysis. A overall efficiency term Y, can be defined as the product of individual efficiencies (such as the chemical yield and counting efficiency) and applied to the laboratory result. The chemical yield is defined, as the amount if the end-product recovered in a chemical process, is sometimes expressed as a percentage of expected recovery to indicate the efficiency of the process. If a correction factor C is defined as

$$C = \frac{\exp(\lambda t)}{Y} = \frac{\exp[\frac{(\ln 2)}{(138\,d)}\,(60\,d)]}{(0.2)(0.8)} = 8.45$$

Then the net counting rate in disintegrations per minute at the time of collection is

$$A_C = A_N C = (0.33\,min^{-1})(8.45) = 2.8\,min^{-1}$$

(b) The standard deviation of the net count rate obtained in the laboratory is

$$\sigma = \sqrt{\frac{20}{30^2} + \frac{10}{30^2}} = 0.18 \text{ min}^{-1}$$

The relative uncertainty for both is 0.18/0.33 = 0.54 =54%. Their associated standard deviations, S_c and S_n. differ by the correction factor C which accounts for yield, chemical efficiency and radioactive decay. The standard deviation in the original activity in the sample is

$$\sigma_N = \sigma_N C = (0.18\,min^{-1})\,(8.45) = 1.5\,min^{-1}$$

(c) The lower limit of detection for the counting portion of the analysis is

$$\text{MDA}=2.706+4.653\sigma_B=2.706+4.653\text{*sqrt}[(10)/(30^2)]=0.49 \text{ min}^{-1}$$

Correcting for yield, efficiency, and radioactive decay as in part)b), gives an MDA for the determination of the original activity of 4.1 min^{-1}

(d)The activity level of the sample does not exceed the MDA since A_N = 0.33 min^{-1} which is less than 0.49 min^{-1}

2.26 Prove that for radioactivity measurements the value of MDA is given by the equation MDA = k^2 + 2CDL, if $k_\alpha = k_\beta = k$. Hint: when n = MDA, the variance $\sigma_D^2 = MDA + \sigma_0^2$

Answer: Starting with Equation 2.104,

$$MDA = CDL + k_\beta \sigma_D = CDL + k_\beta \sqrt{MDA + \sigma_0^2} = CDL + k_\beta \sqrt{MDA + \frac{CDL^2}{k_\alpha^2}}$$

$$(MDA - CDL)^2 = k_\beta^2 \left(MDA + \frac{CDL^2}{k_\alpha^2} \right)$$

Now use: $k_\alpha = k_\beta$=k

$$MDA^2 + CDL^2 - 2(MDA)(CDL) = k^2 MDA + CDL^2$$

$$MDA^2 - 2(MDA)(CDL) = k^2 MDA$$

$$MDA(MDA - 2(CDL) - k^2) = 0 \quad \rightarrow$$

$$MDA = k^2 + 2(CDL)$$

2.27 A sample was counted for 5 min and gave 2250 counts; the background, also recorded for 5 min, gave 2050 counts. Is this sample radioactive? Assume confidence limits of both 95% and 90%.

Answer:

(a) 95% confidence limit; k=1.645, G=2250, B=2050, $\sigma_B = \frac{\sqrt{2050}}{5} = 99$. Using Equation 2.106:

$$MDA = 1.645^2 + 4.053\sigma_\beta = 44.8 \approx 45$$

$$n = \frac{2250}{5} - \frac{2050}{5} = 40 \, {}^c\!/_{\min}$$

40<45, not radioactive with 95% confidence level

(b) 90% confidence limit k=1.285

$$MDA = 1.285^2 + 2\sqrt{2} \times 1.285 \times 9 = 1.65 + 3.63\sigma_B = 34.3$$

40>34.2, yes it is, with 90% confidence limit

2.28 A sample thought to be radioactive was counted and gave 555 counts/min with a background of 450 counts/min. From a separate measurement it has been determined that the standard error of the background is 20 counts/min (it is assumed that the background is constant). Is this sample radioactive with a 97.5% confidence limit?

Answer: n=555-450=105 counts/min. For a 97.5 % confidence limit, k=1.96. Using Eq. 2.106:
MDA=1.96^2+2*SQRT(2)*1.96*20= 114.7~115.
Since MDA>105, one should report that the sample is not radioactive at the 97.5% confidence limit.

2.29 Determine the dead time of a detector based on the following data obtained with the two-source method:

g_2 = 14,000 counts/min $\quad$ g_{12} = 26,000 counts/min

g_2 = 15,000 counts/min $\quad$ b = 50 counts/min

Answer: Use Equation 2.110

$$(14000 \times 15000 \times 26000 + 14000 \times 15000 \times 50 - 14000 \times 26000 \times 50 - 15000 \times 26000 \times 50)\tau^2 -$$
$$-2(14000 \times 15000 - 26000 \times 50)\tau + 14000 + 15000 - 26000 - 50 = 0$$

$$5.43 \times 10^{12}\tau^2 - 4.174 \times 10^8 \tau + 2.95 \times 10^3 = 0$$

$$\tau = \frac{4.174 \times 10^8 = \sqrt{\left(4.174 \times 10^8\right)^2 - 4 \times 5.43 \times 10^{12} \times 2.75 \times 10^3}}{2 \times 5.43 \times 10^{12}} = 6.9 \times 10^{-5} \text{ min or } 1.78 \times 10^{-6} \text{ min}$$

$\tau_1 = 4.14$ ms $\qquad$ $\tau_2 = 472\mu s$? (τ_2 is the correct one since $\tau_1 g >> 1$)

Note: One could use the simpler equation 2.111, since $b << g_i$

2.30 If the dead time of a detector is 100 μs, what is the observed counting rate if the loss of counts due to dead time is equal to 5 percent?

Answer: $g\tau = 0.05 \rightarrow g = \frac{0.05}{100 \times 10^{-6}} = 500 \, {}^c\!/_s$