Chapter 2

Mobility

2.1 (a) The mean free time between collisions using Equation (2.2.4b) is

$$\mu_n = \frac{q\tau_{mn}}{m_n} \quad \rightarrow \quad \tau_{mn} = \frac{\mu_n m_n}{q} = 2.85\times10^{-13}\ \text{sec}$$

where μ_n is given to be 500 cm^2/Vsec (= 0.05 m^2/Vsec), and m_n is assumed to be m_0.

(b) We need to find the drift velocity first:

$$v_d = \mu_n \mathcal{E} = 50000\, cm/\text{sec}.$$

The distance traveled by drift between collisions is

$$d = v_d \tau_{mn} = 0.14\, nm.$$

2.2 From the thermal velocity example, we know that the approximate thermal velocity of an electron in silicon is

$$v_{th} = \sqrt{\frac{3kT}{m}} = 2.29\times10^7\, cm/\text{sec}.$$

Consequently, the drift velocity (v_d) is (1/10)v_{th} = 2.29×10^6 cm/sec, and the time it takes for an electron to traverse a region of 1 μm in width is

$$t = \frac{10^{-4}\, cm}{2.29\times10^6\, cm/\text{sec}} = 4.37\times10^{-11}\ \text{sec}.$$

Next, we need to find the mean free time between collisions using Equation (2.2.4b):

$$\mu_n = \frac{q\tau_{mn}}{m_n} \quad \rightarrow \quad \tau_{mn} = \frac{\mu_n m_n}{q} = 2.10\times10^{-13}\ \text{sec}$$

where μ_n is 1400 cm^2/Vsec (=0.14 m^2/Vsec, for lightly doped silicon, given in Table 2-1), and m_n is 0.26m_0 (given in Table 1-3). So, the average number of collision is

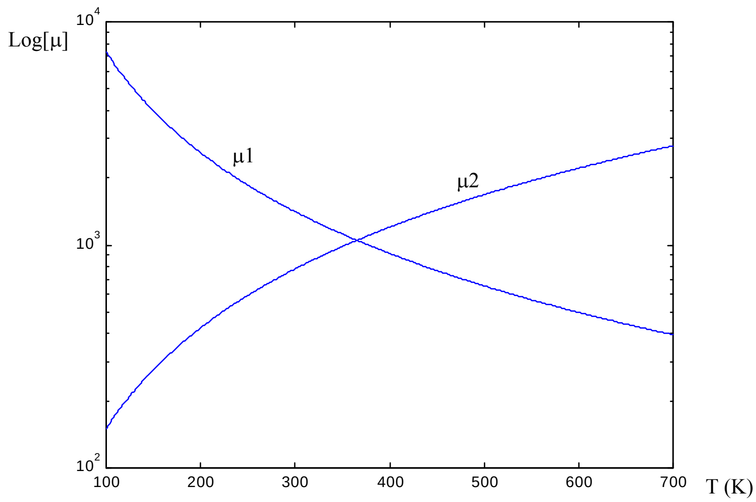
$$\frac{t}{\tau_{mn}} = 207.7\ collision \quad \Rightarrow \quad 207\ collisions\ .$$

In order to find the voltage applied across the region, we need to calculate the electric field using Equation (2.2.3b):

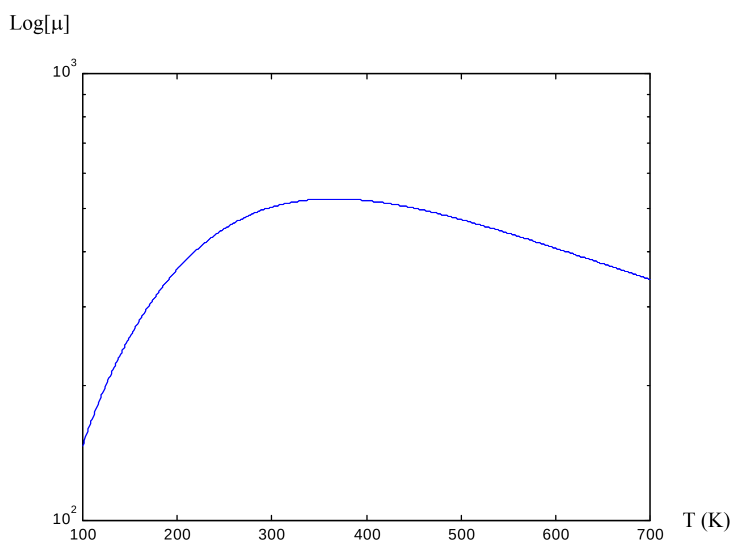
$$v_d = -\mu_n \mathcal{E} \quad \rightarrow \quad \mathcal{E} = \frac{v_d}{\mu_n} = \frac{2.29\times10^{6}\, cm/\sec}{1400\, cm^2/V \sec} = 1635.71\, Vcm^{-1}\ .$$

Then, the voltage across the region is
$V = \mathcal{E} \times width = 1635.71\, Vcm^{-1} \times 10^{-4}\, cm = 0.16V$.

2.3 (a)



(b) If we combine μ_1 and μ_2,



The total mobility at 300 K is

$$\mu_{TOTAL}(300\,K) = \left(\frac{1}{\mu_1(300\,K)} + \frac{1}{\mu_2(300\,K)}\right)^{-1} = 502.55\,cm^2/V\sec.$$

(c) The applied electric field is

$$\varepsilon = \frac{V}{l} = \frac{1V}{1mm} = 10V/cm.$$

The current density is

$$J_{ndrift} = q\mu_n n\varepsilon = q\mu_n N_d\varepsilon = 80.41A/cm^2.$$

Drift

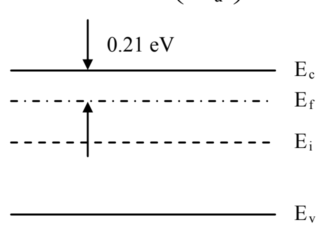
2.4 (a) From Figure 2-8 on page 45, we find the resistivity of the N-type sample doped with $1\times10^{16}cm^{-3}$ of phosphorous is 0.5 Ω-cm.

(b) The acceptor density (boron) exceeds the donor density (P). Hence, the resulting conductivity is P-type, and the net dopant concentration is $N_{net} = |N_d-N_a| = p = 9\times10^{16}cm^{-3}$ of holes. However, the mobilities of electrons and holes depend on the total dopant concentration, $N_T=1.1\times10^{17}cm^{-3}$. So, we have to use Equation (2.2.14) to calculate the resistivity. From Figure 2-5, $\mu_p(N_T=1.1\times10^{17}cm^{-3})$ is 250 cm^2/Vsec. The resistivity is

$$\rho = \frac{1}{\sigma} = \frac{1}{qN_{net}\mu_p} = \frac{1}{q\times9\times10^{16}cm^{-3}\times(250\,cm^2/V\sec)} = 0.28\,\Omega cm.$$

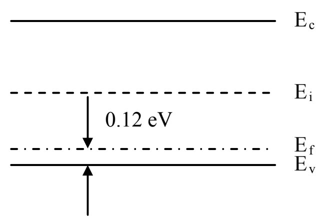
(c) For the sample in part (a),

$$E_c - E_f = kT\ln\left(\frac{N_c}{N_d}\right) = 0.026V\ln\left(\frac{2.8\times10^{19}cm^{-3}}{10^{16}cm^{-3}}\right) = 0.21eV.$$



For the sample in part (b),

$$E_f - E_v = kT\ln\left(\frac{N_v}{N_{net}}\right) = 0.026V\ln\left(\frac{1.04\times10^{19}cm^{-3}}{9\times10^{16}cm^{-3}}\right) = 0.12eV$$



2.5 (a) Sample 1: N-type □ Holes are minority carriers.
$p = n_i^2/N_d = (10^{10}cm^{-3})^2/10^{17}cm^{-3} = 10^2\ cm^{-3}$

Sample 2: P-type □ Electrons are minority carriers.
$n = n_i^2/N_a = (10^{10}cm^{-3})^2/10^{15}cm^{-3} = 10^5\ cm^{-3}$

Sample 3: N-type □ Holes are minority carriers.
$p = n_i^2/N_{net} = (10^{10}cm^{-3})^2/(9.9\times10^{17}cm^{-3}) \approx 10^2\ cm^{-3}$

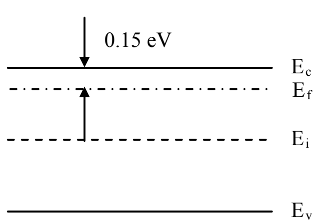
(b) Sample 1: $N_d = 10^{17}cm^{-3}$
$\mu_n(N_d = 10^{17}cm^{-3}) = 750\ cm^2/Vsec$ (from Figure 2-4)
$\sigma = qN_d\mu_n = 12\ \Omega^{-1}cm^{-1}$

Sample 2: $N_a = 10^{15}cm^{-3}$
$\mu_p(N_a = 10^{15}cm^{-3}) = 480\ cm^2/Vsec$ (from Figure 2-4)
$\sigma = qN_a\mu_p = 12\ \Omega^{-1}cm^{-1}$

Sample 3: $N_T = N_d+N_a = 1.01\times10^{17}cm^{-3}$
$\mu_n(N_T = 1.01\times10^{17}cm^{-3}) = 750\ cm^2/Vsec$ (from Figure 2-4)
$N_{net} = N_d-N_a = 0.99\times10^{17}cm^{-3}$
$\sigma = qN_{net}\mu_n = 11.88\ \Omega^{-1}cm^{-1}$

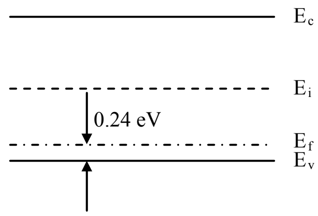
(c) For Sample 1,

$$E_c - E_f = kT\ln\left(\frac{N_c}{N_d}\right) = 0.026V\ln\left(\frac{2.8\times10^{19}cm^{-3}}{10^{17}cm^{-3}}\right) = 0.15eV.$$



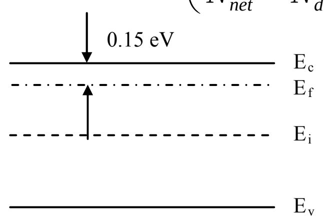
For Sample 2,

$$E_f - E_v = kT\ln\left(\frac{N_v}{N_a}\right) = 0.026V\ln\left(\frac{1.04\times10^{19}cm^{-3}}{10^{15}cm^{-3}}\right) = 0.24eV\ .$$



For Sample 3,

$$E_c - E_f = kT\ln\left(\frac{N_c}{N_{net} = N_d - N_a}\right) = 0.026V\ln\left(\frac{2.8\times10^{19}cm^{-3}}{9.9\times10^{16}cm^{-3}}\right) = 0.15eV\ .$$



2.6 (a) From Figure 2-5, $\mu_n(N_d = 10^{16}cm^{-3}$ of As) is 1250 cm^2/Vs. Using Equation (2.2.14), we find

$$\rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n} = 0.5\ \Omega cm\,.$$

(b) The mobility of electrons in the sample depends not on the net dopant concentration but on the total dopant concentration N_T:

$$N_T = N_d + N_a = 2\times10^{16}cm^{-3}\,.$$

From Figure 2-5,

$$\mu_n(N_T) = 1140\,cm^2/Vs \quad and \quad \mu_p(N_T) = 390\,cm^2/Vs\,.$$

$N_{net} = N_d - N_a = 0$. Hence, we can assume that there are only intrinsic carriers in the sample. Using Equation (2.2.14),

$$\rho = \frac{1}{\sigma} = \frac{1}{qn_i\mu_n + qp_i\mu_p} = \frac{1}{qn_i(\mu_n + \mu_p)}$$
$$= \frac{1}{q \times 1 \times 10^{10} cm^{-3} \times (1140 + 390)(cm^2 / V \sec)}.$$

The resistivity is 4.08×10^5 Ω-cm.

(c) Now, the total dopant concentration (N_T) is 0. Using the electron and hole mobilities for lightly doped semiconductors (from Table 2.1), we have

$$\mu_n = 1400 cm^2 / V \sec \quad and \quad \mu_p = 470 cm^2 / V \sec.$$

Using Equation (2.2.14),

$$\rho = \frac{1}{\sigma} = \frac{1}{qn_i\mu_n + qp_i\mu_p} = \frac{1}{qn_i(\mu_n + \mu_p)}$$
$$= \frac{1}{q \times 1 \times 10^{10} cm^{-3} \times (1400 + 470)(cm^2 / V \sec)}.$$

The resistivity is 3.34×10^5 Ω-cm. The resistivity of the doped sample in part (b) is higher due to ionized impurity scattering.

2.7 It is given that the sample is *n*-type, and the applied electric field ε is1000V/cm. The hole velocity υ_{dp} is 2×10^5cm/s.

(a) From the velocity and the applied electric field, we can calculate the mobility of holes:

$\upsilon_{dp} = \mu_p \varepsilon$, $\mu_p = \upsilon_{dp}/\varepsilon = 2 \times 10^5/1000 = 200 cm^2/V \cdot s$.

From Figure 2-5, we find N_d is equal to $4.5 \times 10^{17}/cm^3$. Hence,

$n = N_d = 4.5 \times 10^{17}/cm^3$, and $p = n_i^2/n = n_i^2/N_d = 10^{20} / 4.5 \times 10^{17} = 222/cm^3$.

Clearly, the minority carriers are the holes.

(b) The Fermi level with respect to E_c is

$E_f = E_c - kT\,ln(N_d/N_c) = E_c - 0.107$ eV.

(c) R = ρL/A. Using Equation (2.2.14), we first calculate the resistivity of the sample:

$\sigma = q(\mu_n n + \mu_p p) \approx q\mu_n n = 1.6 \times 10^{-19} \times 400 \times 4.5 \times 10^{17} = 28.8/\Omega$-cm, and
$\rho = \sigma^{-1} = 0.035$ Ω-cm.

Therefore, R = (0.035) × 20μm / (10μm× 1.5μm) = 467 Ω.

Diffusion

2.8 (a) Using Equation (2.3.2),

$J = qn\upsilon = qD(dn/dx)$.

Therefore,

$\upsilon = D(1/n)(dn/dx) = -D/\lambda$. **(**constant**)**

(b) $J = q\mu_n n\varepsilon = qn\upsilon$ and $\upsilon = \mu_n\varepsilon$.

Therefore, $\varepsilon = -D/\mu_n \lambda = -(kT/q)/\lambda$.

(c) ε = -1000V/cm = $-0.026/\lambda$. Solving for λ yields 0.25μm.

2.9 (a) $\varepsilon = -\dfrac{dV}{dx} = \dfrac{1}{q}\dfrac{dE_v}{dx} = \dfrac{1}{q}\dfrac{\Delta}{L} = \dfrac{\Delta}{qL}$.

(b) E_c is parallel to E_v. Hence, we can calculate the electron concentration in terms of E_c.

$$n(x) = n_0 e^{-(E_c(x)-E_c(0))/kT} \quad where \quad E_c(x) - E_c(0) = (\Delta / L)x.$$

Therefore, $n(x) = n_0 e^{-x\Delta / LkT}$.

(c) $J_n qn\mu_n\varepsilon + qD_n \dfrac{dn}{dx} = 0$

$$qn_i e^{-\Delta x / LkT}\mu_n \frac{\Delta}{qL} + qD_n n_i e^{-\Delta x / LkT}\left(-\frac{\Delta}{LkT}\right) = 0$$

Therefore,

$$\frac{\mu_n}{q} = \frac{D_n}{kT} \Rightarrow D_n = \frac{kT}{q}\mu_n.$$